Tracking Control for Directional Drilling Systems Using Robust Feedback Model Predictive Control

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Abstract: A rotary steerable system (RSS) is a drilling technology which has been extensively studied and used for over the last 20 years in hydrocarbon exploration and it is expected to drill complex curved borehole trajectories. RSSs are commonly treated as dynamic robotic actuator systems, driven by a reference signal and typically controlled by using a feedback loop control law. However, due to spatial delays, parametric uncertainties and the presence of disturbances in such an unpredictable working environment, designing such control laws is not a straightforward process. Furthermore, due to their inherent delayed feedback, described by delay differential equations (DDE), directional drilling systems have the potential to become unstable given the requisite conditions. This paper proposes a Robust Model Predictive Control (RMPC) scheme for industrial directional drilling, which incorporates a simplified model described by ordinary differential equations (ODE), taking into account disturbances and system uncertainties which arise from design approximations within the formulation of RMPC. The stability and computational efficiency of the scheme are improved by a state feedback strategy computed offline using Robust Positive Invariant (RPI) sets control approach and model reduction techniques. A crucial advantage of the proposed control scheme is that it computes an optimal control input considering physical and designer constraints. The control strategy is applied in an industrial directional drilling configuration represented by a DDE model and its performance is illustrated by simulations.

Keywords: Directional Drilling,Trajectory tracking, Robust MPC, LMI optimization

1. INTRODUCTION

The oil and gas industry has constantly searched for more economic and efficient technologies to exploit fossil energy resources. The process for obtaining and extraction of fossil energy resources such as oil and gas, which remain the major fuels for powering today's society, has two major difficulties. Firstly, access to energy resources requires boreholes with complex curves, which is not a simple task to achieve. Secondly, deep-seated and offshore hydrocarbon explorations commonly take place under an unpredictable environment and extreme working conditions while targeting resource locations in the crust of the Earth (Carpenter, 2013). These challenges are being addressed by Rotary Steerable Systems (Bayliss et al., 2012). This steering mechanism is a tool placed close to the drilling bit of a bottom hole assembly (BHA) as illustrated in Fig. 1. In this paper we study a push-the-bit RSS that controls the direction of borehole propagation via force actuated pads mounted close to the bit. Currently the control actuator commands are operated with major communication delays by professionals, where they are located at the surface close to the drilling rig, using complex data sets, such as location of the reservoir, rock layer geometry, etc. Human errors and communication delays could be minimized by automating the steering commands by developing a closed-loop controller using real-time data from sensors located in the drill string.

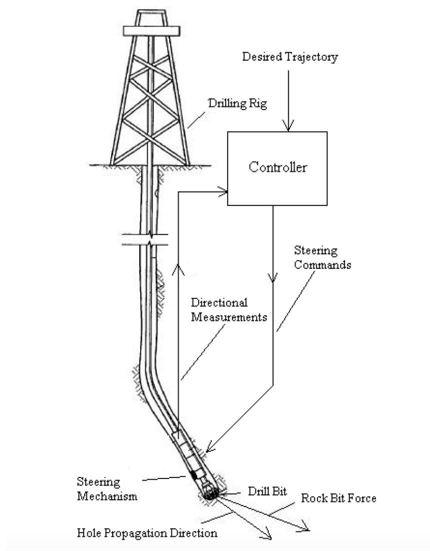

Fig. 1. Directional drilling system (Downton and Ignova, 2011).

The main difficulties of developing an automated RSS system are, firstly the unpredictable and harsh working environment, secondly, key parameters vary whilst drilling and lastly the

⋆ This work has been funded by an EPSRC Industrial CASE Studentship award in collaboration with Schlumberger (EP/R512540/1).

poor communication between surface and downhole. Previous research studies considered empirical or numerical kinematics models using the assumption that the curvature of the BHA is directly linked to the force applied by the RSS (Panchal et al., 2012), however these models could not fully reflect the dynamic behavior and variations of the system especially during transients. Downton and Ignova suggested various novel RSS dynamic models described by linear spatial delay equations based on reasonable simplifications and assumptions (Downton and Ignova, 2007, 2011). Based on the directional drilling model presented in Downton and Ignova (2007), an $\mathscr{L}_1$ adaptive controller alongside state prediction is presented in Sun et al. (2011). Recently, Kremers et al. investigated the behaviour of RSS system in directional drilling applications and have proposed a three-dimensional analytical model using non-linear delay differential equations (Kremers et al., 2016). However, in this approach it is assumed that all parameters remain constant while drilling, which is not generally a realistic assumption in drilling. Analytical models of RSS have been very promising since they can characterize the behavior of the system with minimum error.

By using the framework of RSS analytical modeling, the aim of this study is to develop an appropriate closed-loop feedback control law that can guarantee robustness and stability in the presence of the aforementioned uncertainties and disturbances. Since what is involved is a relatively slow dynamic system, and physical and design constraints which are very important for drilling operation safety, MPC type schemes are very suitable for designing controllers for this application (Agzamov, 2018; Bayliss et al., 2015).

The contribution of this work can thus be divided as follows. Firstly, a dynamic model of the directional drilling system is proposed in terms of ordinary differential equations by a closed-form state-space representation, unlike conventional representations in the literature that utilize either less accurate kinematics system models (Bayliss et al., 2015), or with comprehensive dynamic models that are presented in terms of delay differential equations (Downton and Ignova, 2011). Very importantly, the present model is validated successfully against a high-fidelity industry grade finite element model developed by Schlumberger. Secondly, the present work advances the control solutions available in the literature and to industry for directional drilling automation by proposing a robust control strategy that can handle disturbances and uncertainties. Even though the proposed control is synthesized from a combination of existing control strategies, this is the first time that these strategies are applied successfully to a complex industrial level problem. The particular methodologies employed are a Robust Model Predictive Control (RMPC) scheme, which is further combined with a Robust Positive Invariant (RPI) sets generated feedback control strategy (Tahir and Jaimoukha, 2012, 2013), to overcome the difficulties alluded to above regarding automating RSS systems, while minimizing the trajectory tracking error during drilling. Although the proposed combined strategy requires high-performance computation, it provides an optimal solution at each sample time with limited conservatism in the formulation, unlike the work in (Agzamov, 2018) which ignores disturbances.

The paper is organized as follows. In Section 2, the analytical model of the directional drilling system is introduced and a simplified discrete-spatial uncertain system is suggested. In Section 3, the overall control architecture is presented. A case study in directional drilling using the proposed control approach is illustrated in Section 4, in which robustness and tracking performance of the controller is demonstrated by simulations. Finally, a summary of the findings is given in Section 5, along with potential future work.

2. DIRECTIONAL DRILLING SYSTEM

The directional drilling system can be presented as a mechanical structure, where the centerline of the borehole can be expressed with respect to actuator stimuli by a quasi-polynomial transfer function. In this paper, the complex push-the-bit RSS drilling model presented by Downton and Ignova (2011) is used, where the average direction of drilling is normally assumed to be tangential to the m-*axis*, shown in Fig. 2. This assumption allow us to use a small angle approximation for displacements and angles in the system. The propagation of the BHA centerline can be computed by the lateral displacement rate ($\frac{dH(m)}{dm}$) with respect to distance drilled (m) as determined by:

$$\frac{dH(m)}{dm} = \tan(\alpha + \tan^{-1}(\frac{LWOR}{WOR} K_{anis})), \tag{1}$$

where α indicates the angle of the bit's rotation axis with respect to the m-*axis* (indicated by the slope of the blue line in Fig. 2), K_{anis} is the *anisotropy* of the bit which measures the rock removal capability ratio of the two axis (axial and lateral), and WOR and LWOR is the axial and lateral load on the rock, respectively. In this work the lateral displacement $H(m)$ is considered as the dependent variable and the distance drilled m as the independent variable.

By using the assumption that the deformation inside the borehole is small, the BHA can be statically treated as an Euler-Bernouli beam. Therefore, the general expression of a beam element under load for small angles is given by:

$$\frac{\partial^2}{\partial l^2}(EI(\frac{\partial y^2}{\partial l^2})) + \frac{\partial}{\partial l}(P(\frac{\partial y}{\partial l})) = w, \tag{2}$$

where y is the beam's lateral displacement, l is the length along the beam which is considered as the independent variable, EI is the bending stiffness, P is the axial-load along the beam and w is the beam's load per unit length.

The method of dividing the drilling string into smaller segments (Downton, 2014), according to the position of the stabilizers on the BHA, is followed. In this case, a BHA with four stabilizers is considered, with the parameters L_1, L_2, L_3 and L_4 defined as the distance of the first stabilizer from the bit and stabilizer i from stabilizer $i-1$, respectively (as shown in Fig. 2). As a result, four independent beam equations arise that are influenced by self-weights and applied load and moments for each element, with continuity constraints at the joints between any two beams.

After extensive algebra using these four equations, the parameter LWOR is expressed in terms of the forces and moments applied on the BHA. By substituting this in (1) and assuming constant terms for K_{anis}, end-moments (M_1 and M_2) and weight on the bit (WOB), as well as, that all stabilizers are located on the centerline of the borehole ($v_i = 0$ for $i = 2, \dots, 5$), the final expression for lateral borehole propagation is given by the delay differential equation (DDE):

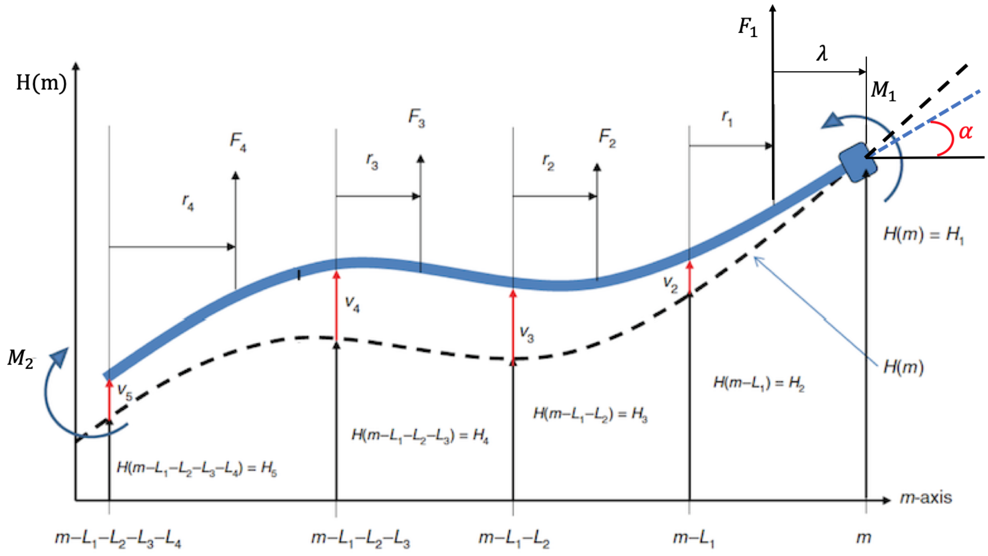

Fig. 2. Generic BHA drilling system formulation based on lateral displacement H(m) with respect to the drilled distance m expressed in a locally tangent coordinate system. The black dashed line represents the centerline of the borehole, while the blue line is the actual shaft shape and the dash blue line is the slope of the shaft. F_1 is the force applied by the steering mechanism (RSS) and is considered as the input of the system. F_2 up to F_4 and v_2 up to v_5 model the forces applied by the stabilizers to the sidewall of the borehole and the lateral displacement of each stabilizer with respect to the centerline, respectively. F_2 to F_4 and v_2 to v_5 are assumed zero for the present case study (Downton and Ignova, 2011).

$$\frac{dH(m)}{dm} = -\Bigg(\sum_{i=1}^{n_{stb}}(A_i \cdot H(m-\tau_i)) + \sum_{i=1}^{n_{beam}}(B_{w_i}\cdot w_i) + \sum_{i=1}^{2}(B_{M_i}\cdot M_i) + \sum_{i=1}^{n_{force}}(B_{F_i}\cdot F_{pad(m)_i}\Bigg), \tag{3}$$

where A_i, B_{w_i}, B_{M_i} and B_{F_i} are the coefficient vectors computed by the BHA configuration, τ_i denotes the distance of stabilizer i with respect to the bit (e.g. $\tau_3 = L_1 + L_2 + L_3$), n_{stb} is the number of stabilizers under consideration, $n_{beam} = n_{stb} - 1$, and n_{force} is the number of (control and reaction) forces applied to the system. F_{pad} is the force applied to the sidewall of the borehole by the steering mechanism (shown as F_1 in Fig. 2). In this paper we consider only one RSS system located at distance λ away from the bit and the effective stabilizers are the first four on the BHA (Fig. 2 shows up to the fourth stabilizer). Ideally, the supervisory trajectory control system should be embedded in the BHA to minimize communication delays. However, this method requires a high-performance processor with insignificant size due to the limited space on the BHA, that can work efficiently at extreme environments with minimum power consumption. Also, it is important to note that the drilled distance measurement (m) is available only at the surface of drilling, which implies that the control unit at present must be considered to be located at the surface, since drilled distance (m) is the dependent variable in the control scheme.

For the purposes of this work, the sensors are assumed to be located at some distance from the bit, close to the rear stabilizers of the BHA. The sensors measure the tilt of the beam, which is related to the inclination, $dH(m)/dm$, instead of lateral displacement, $H(m)$, at the drill bit. Therefore, the borehole propagation DDE in (3) can be modified as follows:

$$\frac{dH(m)}{dm} = -\Bigg(\sum_{i=1}^{n_{stb}}(A_i \cdot \frac{dH(m-\tau_i)}{dm}\cdot \tau_{n_{stb}-i}) + \sum_{i=1}^{n_{stb}-1}(B_{w_i}\cdot w_i) + \sum_{i=1}^{2}(B_{M_i}\cdot M_i) + B_F \cdot F_{pad(m)}\Bigg), \tag{4}$$

in which also the sum in the last term has been dropped, since in the present work only one steering mechanism (applying force F_{pad}) is considered.

2.1 *Simplified Model*

The general expression for lateral borehole propagation is transformed into an ODE and then reduced to a low order system in order to be computationally efficient for closed loop control formulation. The first step is to transform the DDE presented in (4) into state space form by considering the lateral displacement and inclination at the drill bit as the states of the system:

$$\mathbf{x}(m) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} H(m) \\ \frac{dH}{dm} \end{bmatrix}, \quad \dot{\mathbf{x}}(m) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{dH}{dm} \\ \frac{d^2H}{dm^2} \end{bmatrix},$$

$$\begin{aligned}\dot{\mathbf{x}}(m) = &\begin{bmatrix} 0 & 1 \\ 0 & -A_1G_1 \end{bmatrix}\begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -A_2G_2 \end{bmatrix}\begin{bmatrix} x_1(m-\tau_1) \\ x_2(m-\tau_1) \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 \\ 0 & -A_3G_3 \end{bmatrix}\begin{bmatrix} x_1(m-\tau_2) \\ x_2(m-\tau_2) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -A_4G_4 \end{bmatrix}\begin{bmatrix} x_1(m-\tau_3) \\ x_2(m-\tau_3) \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 \\ 0 & E \end{bmatrix}\begin{bmatrix} x_1(m-\tau_4) \\ x_2(m-\tau_4) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ BW & B_F \end{bmatrix}\begin{bmatrix} W \\ u_F \end{bmatrix},\end{aligned} \tag{5}$$

$$\mathbf{y}(m) = [0 \;\; 1]\begin{bmatrix} x_1(m) \\ x_2(m) \end{bmatrix}, \tag{6}$$

where $\dot{x}_2 = \frac{d^2H}{dm^2}$ represents the rate of inclination (curvature of the borehole trajectory calculated by differentiating (4)), G_1, G_2, G_3, and G_4 are constant coefficients depending on the structure of the BHA, and the parameter E is given by $E = A_1G_1 + A_2G_2 + A_3G_3 + A_4G_4$. BW and W represent the column vectors of B_{w_i} and of the spatial derivatives of w_i for $i = 1 \dots n_{stb-1}$, respectively, and u_F is the spatial derivative of F_{pad}. M_1 and M_2 are assumed constant and therefore do not appear in (5).

In the second step, the state space form of the directional drilling system with delays (5) is transformed into an ODE by a rational approximation method. In this paper the *Páde* approximation method is followed (AI-Amer and AL-Sunni, 2000). The accuracy of this method can be improved by increasing the order of the approximation, however, doing so also increases the number of states of the ODE, which is not desirable in terms of computational efficiency of the RMPC method that will be employed. In order to minimize the approximation error while keeping the system's state number low, several values of the approximation order were evaluated. In the particular system under study a 9th order *Páde* approximation method is chosen, since it keeps the approximation error low while the number of states is not excessive (38 states). However, using a RMPC scheme with a system of this order, the online computational time is extremely high. Therefore, model reduction by balanced truncation is employed to reduce the number of states (Antoulas, 2000), from 38 states to an ODE with 3 states, for the specific BHA configuration studied in this paper. The error of transforming the DDE system to a reduced order ODE is presented in Fig. 5 at Section 4.

In order to compensate such approximations and unmodeled dynamics which may be left out either at the design process or after delays approximation and model reduction, the directional drilling system is reformulated as a linear discrete-time uncertain system with disturbances (Balarkrishnan and Morari, 1996), as shown in (7). It is assumed that all the states of the system are fully measurable and the sample distance, λ, which is chosen to discretize the system, is equal to the distance between the bit and the RSS actuator, as shown in Fig. 2. The assumption of fully measurable states is not practically realistic since the states, after approximation and model reduction of the system, do not represent any physical quantities which can be measured. However, we can proceed with this assumption at this stage, by considering that an estimation strategy can be employed followed by linear transformations to provide the state's value of the simplify model with a minimum error using the available measurements.

$$\begin{bmatrix} x_{k+1} \\ q_k \\ f_k \\ z_k \end{bmatrix} = \begin{bmatrix} A & B_u & B_w & B_p \\ C_q & D_{qu} & D_{qw} & 0 \\ C_f & D_{fu} & D_{fw} & D_{fp} \\ C_z & D_{zu} & D_{zw} & D_{zp} \end{bmatrix} \begin{bmatrix} x_k \\ u_k \\ w_k \\ p_k \end{bmatrix}, \quad p_k = \Delta q_k,$$
$$\begin{bmatrix} q_N \\ f_N \\ z_N \end{bmatrix} = \begin{bmatrix} \hat{C}_q & 0 \\ \hat{C}_f & \hat{D}_{fp} \\ \hat{C}_z & \hat{D}_{zp} \end{bmatrix} \begin{bmatrix} x_N \\ p_N \end{bmatrix}, \quad p_N = \Delta q_N, \tag{7}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^{n_u}$, $w_k \in \mathbb{R}^{n_w}$, $f_k \in \mathbb{R}^{n_f}$, $z_k \in \mathbb{R}^{n_z}$, $p_k \in \mathbb{R}^{n_p}$ and $q_k \in \mathbb{R}^{n_q}$ are the state, input, disturbance, constraint, cost, and input and output uncertainty vectors, respectively, and where N is the prediction horizon. Furthermore, $\Delta \in \mathscr{B}\mathbf{\Delta}$ where the operator $\mathscr{B}$ denotes the unit ball of the structured uncertainty set $\mathbf{\Delta}$. All the coefficient matrices can be computed from the configuration of the BHA and its reduced order model approximation already explained. The constraints are imposed by physical factors, such as input actuator limits, or design preferences.

3. TRACKING CONTROL APPROACH

In this section, the RMPC methodology employed for the directional drilling tracking control problem is summarized. See Tahir and Jaimoukha (2013) for full details. Thus, the algebraic formulation of an online and offline controller, which are used to steer a system to an admissible reference signal, is explained. The online controller is based on the RMPC problem for an uncertain system subject to disturbances and the offline controller is calculated by a state-feedback law, where the gain of the controller is computed offline by the optimal RPI set problem (Tahir and Jaimoukha, 2012).The main advantages of using a combination of these controllers are insuring stability while robust properties are preserves with minimum of computation burden. At the end of this section an algorithm is presented to summarize the control strategy that is followed.

3.1 Robust MPC problem

First let us consider the dynamic system in (7) and define the prediction step k belonging to the time set $T_N = \{0, 1, \dots, N-1\}$, where N is the horizon length, and also consider a disturbance formulated as:

$$w_k \in \mathscr{W}_k := \{w_k \in \mathbb{R}^{n_w} : -d_k \leq w_k \leq d_k\}, \tag{8}$$

where $d_k > 0$ is given, and where the inequalities are interpreted component-wise. The requirement for RMPC is to find a control input u_k such that the cost function,

$$J = \max_{w \in \mathscr{W}_k, \Delta \in \mathscr{B}\mathbf{\Delta}} \sum_{k=0}^{N} (z_k - \bar{z}_k)^T (z_k - \bar{z}_k), \tag{9}$$

is minimized, while the future predicted outputs satisfy the constraints $f_k \leq \bar{f}_k$ and $f_N \leq \bar{f}_N$ for all $w_k \in \mathscr{W}_k$ and all $\Delta \in \mathscr{B}\mathbf{\Delta}$. The parameter $\bar{z}_k$ defines the reference trajectory and $\bar{f}_k$ and $\bar{f}_N$ are chosen to include polytopic constraints on input, state and output signals, and terminal signals respectively.

To simplify the presentation, we reparameterize the disturbance as uncertainty by redefining $\mathscr{W}_k := \{\Delta_k^w d_k : \Delta_k^w \in \mathbf{\Delta}^w\}$ where $\mathbf{\Delta}^w$ is a structured space and,

$$B_p := [B_p \ \ B_w], \ C_q := \begin{bmatrix} C_q \\ 0 \end{bmatrix}, \ D_{qu} := \begin{bmatrix} D_{qu} \\ 0 \end{bmatrix}, \ d_k := \begin{bmatrix} 0 \\ d_k \end{bmatrix},$$
$$q_k := \begin{bmatrix} q_k \\ w_k \end{bmatrix} = C_q x_k + D_{qu} u_k + d_k.$$

By defining the stacked vectors,

$$u = \begin{bmatrix} u_0 \\ \vdots \\ u_{N-1} \end{bmatrix} \in \mathbb{R}^{N_u}, \ x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \in \mathbb{R}^{N_n}, \ \zeta = \begin{bmatrix} \zeta_0 \\ \vdots \\ \zeta_N \end{bmatrix} \in \mathbb{R}^{N_\zeta},$$

where ζ stands for $f, \bar{f}, p, q, z, \bar{z}$ or d, and $N_n = Nn$, $N_u = Nn_u$ and $N_\zeta = (N+1)n_\zeta$, we get:

$$\begin{bmatrix} x \\ q \\ f \\ z \end{bmatrix} = \begin{bmatrix} \mathscr{A} & 0 & \mathscr{B}_p & \mathscr{B}_u \\ \mathscr{C}_q & I & \mathscr{D}_{qp} & \mathscr{D}_{qu} \\ \mathscr{C}_f & 0 & \mathscr{D}_{fp} & \mathscr{D}_{fu} \\ \mathscr{C}_z & 0 & \mathscr{D}_{zp} & \mathscr{D}_{zu} \end{bmatrix} \begin{bmatrix} x_0 \\ d \\ p \\ u \end{bmatrix}, \quad p = \hat{\Delta} q, \tag{10}$$

with $\hat{\Delta} \in \mathscr{B}\hat{\mathbf{\Delta}}$ where,

$$\hat{\mathbf{\Delta}} = \{\mathrm{diag}(\Delta, \Delta_0^w, \dots, \Delta, \Delta_{N-1}^w, \Delta) : \Delta \in \mathbf{\Delta}, \Delta_k^w \in \mathbf{\Delta}^w\},$$

and where the matrices in (10) are readily obtained from iterating the dynamics in (7).

The input signal u_i, as proposed by Skaf and Boyd (2010), is considered as a causal state feedback which depends only on state x_j for $j = \{0 \dots i\}$. Thus, the input signal can be given by:

$$u = K_0 x_0 + Kx + \upsilon, \tag{11}$$

where $K_0 \in \mathbb{R}^{N_u \times n}$ and $K \in \mathbb{R}^{N_u \times N_n}$ are the current and future state feedback gains (where, due to causality, $[\,K_0 \;\, K\,]$ is lower block diagonal with $n_u \times n$ blocks) and $\upsilon \in \mathbb{R}^{N_u}$ is the (stacked) control perturbation vector.

Substituting the expression of x in (10) into (11) gives,

$$u = \hat{K}_0 x_0 + \hat{K}\mathscr{B}_p p + \hat{\upsilon}, \tag{12}$$

where,

$$\left[\hat{K}_0 \;\, \hat{K} \;\, \hat{\upsilon}\right] = (I - K\mathscr{B}_u)^{-1}\left[K_0 \;\, K \;\, \upsilon + K\mathscr{A}x_0\right], \tag{13}$$

and note that $\hat{K}_0, \hat{K}$ and $\hat{\upsilon}$ have the same structure as K_0, K and υ, respectively. It follows that,

$$\begin{bmatrix} q \\ f \\ z - \bar{z} \end{bmatrix} = \begin{bmatrix} \mathscr{D}_{qp}^{\hat{K}} & \mathscr{D}_{q0}^{\hat{K},\hat{\upsilon}} \\ \mathscr{D}_{fp}^{\hat{K}} & \mathscr{D}_{f0}^{\hat{K},\hat{\upsilon}} \\ \mathscr{D}_{zp}^{\hat{K}} & \mathscr{D}_{z0}^{\hat{K},\hat{\upsilon}} \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix}, \tag{14}$$

where the submatrices can be deduced by eliminating u from (10). Finally,

$$\begin{bmatrix} f \\ z - \bar{z} \end{bmatrix} = \begin{bmatrix} \mathscr{D}_{f0}^{\hat{K}_0,\hat{K},\hat{\upsilon},\hat{\Delta}} \\ \mathscr{D}_{z0}^{\hat{K}_0,\hat{K},\hat{\upsilon},\hat{\Delta}} \end{bmatrix}, \tag{15}$$

where the submatrices can be deduced by eliminating p from (14) using $p = \hat{\Delta}q$. For convenience we write $f(\hat{K}_0, \hat{K}, \hat{\upsilon}, \hat{\Delta}) = \mathscr{D}_{f0}^{\hat{K}_0,\hat{K},\hat{\upsilon},\hat{\Delta}}$ and $f_c(\hat{K}_0, \hat{K}, \hat{\upsilon}, \hat{\Delta}) = (\mathscr{D}_{z0}^{\hat{K}_0,\hat{K},\hat{\upsilon},\hat{\Delta}})^T(\mathscr{D}_{z0}^{\hat{K}_0,\hat{K},\hat{\upsilon},\hat{\Delta}})$ to denote the constraints f and the cost function $(z-\bar{z})^T(z-\bar{z})$.

By following the procedure presented by Tahir and Jaimoukha (2013), the RPMC problem can be transformed to a min-max problem (Scokaert and Mayne, 1998), where the objective is to find a feasible triple $(\hat{K}_0, \hat{K}, \hat{\upsilon})$ that solve,

$$\mathbf{J} = \min_{(\hat{K}_0,\hat{K},\hat{\upsilon})\in\mathscr{U}} \; \max_{\hat{\Delta}\in\mathscr{B}\hat{\mathbf{\Delta}}} f_c(\hat{K}_0, \hat{K}, \hat{\upsilon}, \hat{\Delta}). \tag{16}$$

The set $\mathscr{U}$ is defined as shown in Tahir and Jaimoukha (2013) to be the set of all feasible control variables $(\hat{K}_0, \hat{K}, \hat{\upsilon})$ such that all the problem constraints are satisfied:

$$\mathscr{U} := \{(\hat{K}_0, \hat{K}, \hat{\upsilon}) : f(\hat{K}_0, \hat{K}, \hat{\upsilon}, \hat{\Delta}) \leq \bar{f}, \;\; \forall \hat{\Delta} \in \mathscr{B}\hat{\mathbf{\Delta}}\}. \tag{17}$$

It can be seen that the problem is non-convex, therefore the semi-definite relaxation procedure presented in Tahir and Jaimoukha (2013, Lemma 1), is used by introducing an upper bound on the cost function (16), defined by $\bar{\gamma}$. After some matrix manipulations the inequality $\mathbf{J} - \bar{\gamma} \leq 0$ holds $\forall \hat{\Delta} \in \mathscr{B}\hat{\mathbf{\Delta}}$, and $(\hat{K}_0 \;\, \hat{K} \;\, \hat{\upsilon}) \in \mathscr{U}$ if there exists a solution to the following nonlinear matrix inequalities:

$$\begin{bmatrix} I & \mathscr{D}_{z0}^{\hat{K}_0,\hat{\upsilon}} & \mathscr{D}_{zp}^{\hat{K}}G^T & \mathscr{D}_{zp}^{\hat{K}}S \\ * & \bar{\gamma} & (\mathscr{D}_{q0}^{\hat{K}_0,\hat{\upsilon}})^T & 0 \\ * & * & T + \mathscr{D}_{qp}^{\hat{K}}G^T + G(\mathscr{D}_{qp}^{\hat{K}})^T & \mathscr{D}_{qp}^{\hat{K}}S \\ * & * & * & S \end{bmatrix} \succ 0, \tag{18}$$

$$\begin{bmatrix} e_i^T(\bar{f} - \mathscr{D}_{f0}^{\hat{K}_0,\hat{\upsilon}}) & -(\mathscr{D}_{q0}^{\hat{K}_0,\hat{\upsilon}})^T + \frac{1}{2}e_i^T\mathscr{D}_{fp}^{\hat{K}}G_i^T & \frac{1}{2}e_i^T\mathscr{D}_{fp}^{\hat{K}}S_i \\ * & T + \mathscr{D}_{qp}^{\hat{K}}G_i^T + G_i(\mathscr{D}_{qp}^{\hat{K}})^T & \mathscr{D}_{qp}^{\hat{K}}S_i \\ * & * & S_i \end{bmatrix} \succ 0, \tag{19}$$

where $*$ denotes a term deduced from symmetry and e_i denotes the ith column of the identity matrix with appropriate dimension and where (S,T,G) and $(S_i,T_i,G_i), i = 1,\dots,N_f$ are slack variable matrices on the set $\hat{\Psi}$ which is defined as:

$$\begin{aligned} \hat{\Psi} = \{&(S,T,G) \,:\, S = S^T \succ 0, \, T = T^T \succ 0 \\ &S\hat{\Delta} = \hat{\Delta}S, \, \hat{\Delta}G + G^T\hat{\Delta}^T = 0, \, \forall \hat{\Delta} \in \mathscr{B}\hat{\mathbf{\Delta}}\}. \end{aligned} \tag{20}$$

The non-linearities appear with respect to $\hat{K}$ and the slack variables (S,T,G,S_i,T_i,G_i,R_0). By introducing two new slack variables Y, Y_i and using the extended S-procedure approach proposed by Tahir and Jaimoukha (2013), the problem can be linearized into a Linear Matrix Inequality (LMI) optimization described by:

$$\begin{bmatrix} S & * \\ -G & Y \end{bmatrix} \succ 0, \qquad \begin{bmatrix} S_i & * \\ -G_i & Y_i \end{bmatrix} \succ 0, \tag{21}$$

$$\begin{bmatrix} e_i^T & -(\mathscr{D}_{q0}^{\hat{K}_0,\hat{\upsilon}})^T & \frac{1}{2}e_i^T(\mathscr{D}_{fp}S_0 + \mathscr{D}_{fu}\bar{K}) & -\frac{1}{2}e_i^T\mathscr{D}_{fp}G_0^T \\ * & T_i + Y_i & \mathscr{D}_{qp}S_0 + \mathscr{D}_{qu}\bar{K} & Y_0 - \mathscr{D}_{qp}G_0 \\ * & * & S_0^T + S_0 - S_i & (G_i - G_0 - R_0)^T \\ * & * & * & Y_0 + Y_0^T + Y_i \end{bmatrix} \succ 0, \tag{22}$$

$$\begin{bmatrix} I & \mathscr{D}_{z0}^{\hat{K}_0,\hat{\upsilon}} & 0 & \mathscr{D}_{zp}S_0 + \mathscr{D}_{zu}\bar{K} & -\mathscr{D}_{zp}G_0^T \\ * & \bar{\gamma} & (\mathscr{D}_{q0}^{\hat{K}_0,\hat{\upsilon}})^T & 0 & 0 \\ * & * & T + Y & \mathscr{D}_{qp}S_0 + \mathscr{D}_{qu}\bar{K} & Y_0 - \mathscr{D}_{qp}G_0 \\ * & * & * & S_0^T + S_0 - S & (G - G_0 - R_0)^T \\ * & * & * & * & Y_0 + Y_0^T + Y \end{bmatrix} \succ 0, \tag{23}$$

where $\bar{K} = \hat{K}\mathscr{B}_pS_0$ and where the structure of the variables S_0, Y_0, G_0 and R_0 is chosen to ensure linearity; see Tahir and Jaimoukha (2013) for more details.

The above formulation shows that the initial non-convex and non-linear RMPC problem can be written as an LMI optimization problem (Boyd et al., 1994). Therefore, the control gains K_0 and K can be computed on-line and applied in a MPC manner, where the first input of the control sequence u is applied to the plant. Note that the variables K, K_0, υ can be recovered by the following expression:

$$[K_0 \;\, K \;\, \upsilon] = (I - \hat{K}\mathscr{B}_u)^{-1}\left[\hat{K}_0 \;\, \hat{K} \;\, \hat{\upsilon} - \hat{K}\mathscr{A}x_0\right]. \tag{24}$$

3.2 Offline controller using optimal RPI set

RPI sets found great success in robust analysis and synthesis of uncertain systems. In the case of RMPC, state feedback law based on RPI sets guarantees stability in uncertain systems and reduction of the computation time, since the feedback gain and the volume of the invariant set are computed off-line. A set is defined as an RPI set if the following statement is satisfied (Blanchini, 1999):

Definition 1. The set $Z \subset \mathbb{R}^n$ is a Robust Positively Invariant set of a system (7) if, by applying state-feedback control law $u = Kx$, then $A_\Delta Z \oplus B_\Delta KZ \oplus B_w\mathscr{W} \subseteq Z$ is satisfied for all Δ, where $\oplus$ denotes the Minkowski sum.

Consequently, if the current state is inside the set Z, by applying the state feedback control law $u = Kx$ all the future states lie in the set Z in the presence of model uncertainties defined by $A_\Delta = A + B_p\Delta C_q$, $B_\Delta = B_u + B_p\Delta D_{qu}$, and disturbances $w_k \in \mathscr{W}$. In this paper, a multi-objective problem is considered where the target is to maximize the volume of the polytopic invariant set Z of the form,

$$Z = \{x \in \mathbb{R}^n : -v \leq Ex \leq v\}, \tag{25}$$

where E is a matrix with appropriate dimension ($E \in \mathbb{R}^{n \times n}$) and v is column vectors with all entries one ($v \in \mathbb{R}^n$). The problem can be expressed as an optimization problem as follows:

$$\begin{aligned} &\max \quad Volume(Z) \\ &subject\ to \begin{cases} Z \subseteq \mathscr{X}_I \\ KZ \subseteq \mathscr{U}_I \\ A_\Delta Z \oplus B_\Delta KZ \oplus B_w W \subseteq Z \end{cases} \end{aligned} \tag{26}$$

where $\mathscr{U}_I := \{u \in \mathbb{R}^{n_u} : \underline{u}_I \leq u \leq \bar{u}_I\}$ and $\mathscr{X}_I := \{x \in \mathbb{R}^{n_x} : \underline{x}_I \leq x \leq \bar{x}_I\}$ define the input and state constraints sets, respectively.

By considering the dynamic system (7), the invariant set constraint condition for the polytopic set (25) can be rewritten as:

$$-e_i^T v \leq e_i^T E\left((A_\Delta + B_\Delta K)x + B_w w\right) \leq e_i^T v. \tag{27}$$

Since the polytopic invariant constraint is assumed to be symmetric, the relevant invariant conditions are computed using only the upper bound:

$$e_i^T E\left((A_\Delta + B_\Delta K)x + B_w w\right) - e_i^T v \leq 0.$$

Using the extended S-procedure (Pólik and Terlaky, 2007), the problem can be expressed as a convex LMI optimization problem, presented in Tahir and Jaimoukha (2012).

By following the description that is given for both offline and online controllers, the RMPC strategy that is proposed in this paper is summarized as follows:

Algorithm 1. RMPC controller strategy

Offline calculation:

1. Compute the polytopic RPI set Z and the corresponding gain matrix K, by solving the LMI problem described in Section 3-B.

Online calculation:

1. Read the current state x_k and set it as initial state x_0
2. **If** the initial state lies inside the RPI set Z, switch to offline control and apply the state feedback controller $u = Kx_k$.
3. **else** compute the triple (K_0, K, υ) through the LMI procedure outlined in Section 3-A and apply the first input of the control sequence (11).
4. Return to step 1.

4. CASE STUDY

In this section a directional drilling application with an industrial BHA configuration is considered. Simulation results are presented to demonstrate the effectiveness of the proposed control strategy for directional drilling. At first we validate that the chosen BHA configuration is successfully described by the DDE model in (5). Validation is presented in Fig. 3 using curvature steady state values from a finite element industrial model provided by Schlumberger, which accurately describes the borehole's propagation with respect to distance drilled. In both models the same normalized input force (F_{pad}) is applied. As it can be seen, the steady-state curvature predicted by the DDE model converges to the curvature values calculated by the industrial model.

In order to test the model approximation strategy presented in Section 2.1, the same input signal (F_{pad} shown in Fig. 3) is also applied in an open-loop manner to the DDE and simplified ODE models. As shown in Figs. 4 and 5, the open-loop inclination responses for the two systems are very similar and the error between them remains below 0.3 degrees. Therefore, it is sufficient to steer the directional drilling system with minimum error, by developing closed-loop control using the simplified model and considering uncertainties on the model.

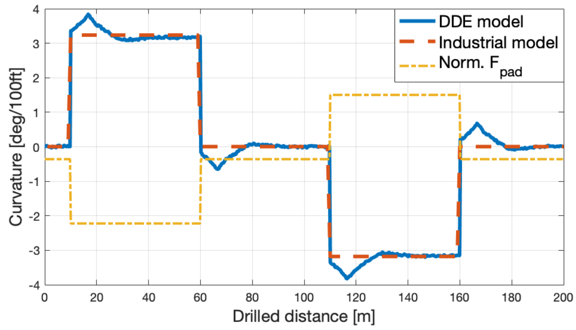

Fig. 3. Open-loop response of curvature versus measured drilled distance predicted by the DDE model (5) and industrial model. The normalized F_{pad} input force applied to both models is also shown.

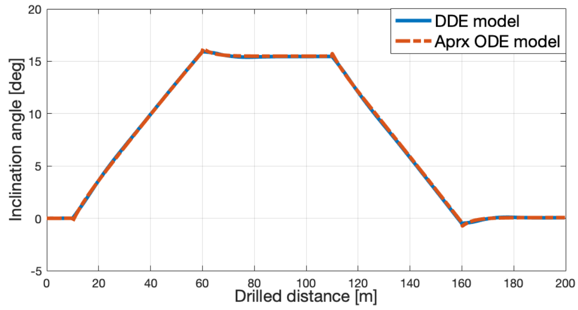

Fig. 4. Open-loop inclination response versus measured drilled distance predicted by the DDE model (5) and the ODE simplified model. The normalized input force applied to both models is as shown in Fig. 3.

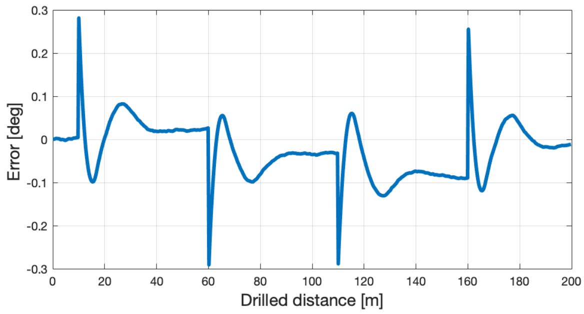

Fig. 5. Inclination error between the responses of the DDE model (5) and the ODE simplified model, for the input force shown in Fig. 3.

For the closed-loop control problem, the drilling system (DDE model in (5)) is required to track an inclination reference, while satisfying BHA bending limitations, which can be approximately translated to input constraints. Furthermore, it is assumed that the system is affected by disturbances at its input and output denoted by w_1 and w_2, respectively. Input disturbance w_1 aims to characterize the discrepancy between the desired and actual input value provided by the actuator due to physical losses and inability to measure the input directly by a sensor, and also to capture relevant signal noise. The disturbance signal w_1 is assumed to be bounded by 10% of the maximum input constraint value (u_{max}) that is chosen by design at the RPMC formulation. Therefore, the distribution of w_1 is assigned as white noise with zero mean and an appropriate standard deviation ($std = \frac{0.1 \cdot u_{max}}{3}$), such that 97% of the disturbance stays within the 10% of u_{max}. The output (inclination) disturbance w_2 is due to inertial sensors accuracy and it is also white noise with zero mean and 0.33 standard deviation, such that 97% of the disturbance stays within 1 degree. The main design uncertainties arise from the knowledge that the WOB and K_{anis} values fluctuate during drilling. Therefore, to demonstrate that our proposed control scheme can successfully steer the system to the reference trajectory under the presence of uncertain variables that describe the system, we assign the parameters WOB and K_{anis} as uniformly varying through out the simulation while staying inside the following sets: $5000 \leq WOB \leq 15000$ (lbf) and $0.018 \leq K_{anis} \leq 0.043$. The bounded sets that describe the uncertainty of WOB and K_{anis} are selected based on past experimental data obtained by Schlumberger for a given rock formation set and known bit design. The block diagram in Fig. 6 shows the closed-loop scheme of the controller (utilizing the simplified ODE model) and plant (complex DDE model in (5)), used for closed-loop simulations.

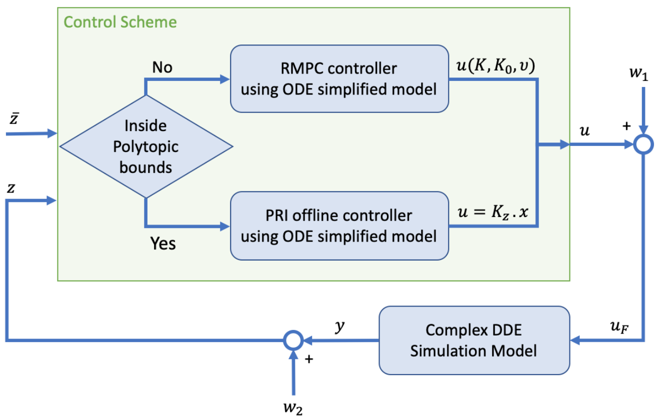

Fig. 6. Block diagram of directional drilling closed-loop control and simulation scheme.

Figure 7 shows the inclination response of the closed-loop system, while tracking a given reference value, for various constraint levels of the control input, to assess the performance of the proposed control scheme. It can be seen that the inclination response is stable and the reference is tracked well despite the controller only uses a simplified model of the plant, and despite the presence of disturbances and constraints. It can also be seen that by tightening the input constraints, there is slower convergence to the steady-state value, as would be expected by the more limited flexibility of the BHA. Moreover, it can be seen that the tracking performance when the response is inside the RPI set Z (close to steady-state values) is decreased due to the offline computed controller, however robust performance is guaranteed due to the RPI set properties.

Figure 8 shows the normalized control input for the case when its bounds are between [-3,3], demonstrating that constraints are satisfied.

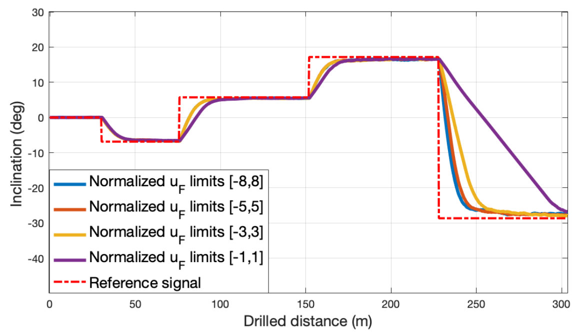

Fig. 7. Closed-loop system inclination response versus drilled distance for a predefined inclination reference trajectory and various levels of normalized control input constraints, using the proposed closed-loop RMPC controller.

In terms of comparison of the proposed method with other MPC based control methods, a conventional MPC scheme has also been tested for the same closed-loop task shown in Fig. 6. However, the inclination response is found to diverge from the reference trajectory due to the systems mismatch (ODE and DDE) caused by the presence of disturbances and uncertainties. Consequently, the problem's constraints are violated and the solver is not able to provide a feasible solution to the problem. By comparing the online RMPC proposed in the present work with tube-based MPC described in Mayne et al. (2009), the tube-based MPC can effectively reduce the computing time of the optimization problem, however the main drawback of this approach is the additional conservatism on the optimization solution due to the state-observer estimation error that is calculated offline. Since in the present work the states of the system are assumed to be fully measurable, further comparison with tube-MPC is left for future work.

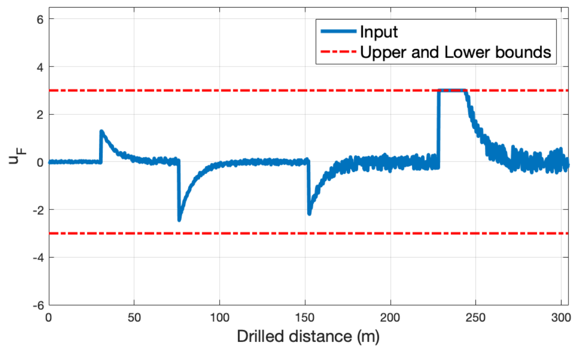

Fig. 8. Normalized control input evolution versus drilled distance using the proposed closed-loop RMPC controller, when the normalized input constraint limits are [-3,3].

5. CONCLUSIONS

In this paper, an effective way to simplify a directional drilling model which characterizes inclination and lateral displacement borehole assembly behavior is presented. On this basis, a robust model predictive control scheme is proposed that can effectively control the complex rotary steerable system using an uncertain system description, while system stability is preserved by the proposed robust positive invariant set. The work provides a promising method for effectively automating the inclination tracking control process in directional drilling applications, to replace the currently employed manual human-in-the-loop control processes.

Future work will focus on extending this work in 3-dimensional space by azimuth control, considering time delays on the steering input force, spatial delay on the output signals and estimation of inclination by the available measurements, at the formulation of the problem.

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