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Seismic migration with inverse Q filtering

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[1] Taking account the earth Q effect (i.e., frequency-dependent amplitude decay and velocity dispersion) during seismic migration simultaneously, the resultant seismic image is expected to be high resolution with true amplitude and correct timing. We present here a stabilized algorithm for such a "migration incorporating inverse Q filtering" scheme, so that the imaging technique is applicable to seismographs with long recording time. The earth model is assumed to be 1-D with continuous variations vertically in velocity and attenuation, and the imaging algorithm is implemented easily and efficiently in the frequency-wavenumber domain. Although such a 1-D model suits mostly to the cases in lithospheric-scale regional geophysics, it is sometimes also suitable to the exploration seismic studies. The method is demonstrated using a real data example from exploration seismics with an attempt to recover a target reflection underneath a group of strong coal-seam reflections. *INDEX TERMS:* 0902 Exploration Geophysics: Computational methods, seismic; 0910 Exploration Geophysics: Data processing; 0935 Exploration Geophysics: Seismic methods (3025); 3260 Mathematical Geophysics: Inverse theory; 5144 Physical Properties of Rocks: Wave attenuation. **Citation:** Wang, Y., and J. Guo (2004), Seismic migration with inverse Q filtering, *Geophys. Res. Lett.*, *31*, L21608, doi:10.1029/2004GL020525.

1. Introduction

[2] Seismic migration is an effective imaging technique used extensively in geophysics investigation. It is an inverse process of wave propagation in which the surface-recorded seismic waves are propagating back to the subsurface medium. Therefore, it should also consider the earth Q effect (i.e., the amplitude attenuation and velocity dispersion) during migration processing. By performing amplitude compensation and phase correction during migration simultaneously, the resultant seismic image is expected to have high resolution with true amplitude and correct timing, which are important for both regional geophysics investigation and oil/gas reservoir exploration. As a pilot research, we present here such a 'migration + inverse Q filtering' method that is applicable to the subsurface media with vertical variable velocity and Q functions. This 1-D earth model suits to most cases in regional geophysics, although not so common in the exploration seismics.

[3] When seismic waves propagate through viscoelastic media, due to the energy absorption and velocity dispersion, both amplitudes and arrival times are changed accordingly [*Kolsky*, 1953, 1956; *Mason*, 1958; *Futterman*, 1962; *Strick*, 1967; *Kjartansson*, 1979; *Ben-Menahem and Singh*, 1981]. Therefore, seismic migration without compensating for the earth Q effect would produce a migrated section with diffused image and incorrect position of reflectors. *Mittet et al.* [1995] demonstrated that even when migrating with a Q model deviating by 10% from the correct one, seismic images would still be better focused and of a higher quality than when no compensation was performed.

[4] If including inverse Q filtering in seismic migration, numerical instability is a problem of concern; errors from observed noise, numerical approximation and roundoff tend to be amplified with increasing depth because the higher frequency components grow faster in the extrapolation procedure of seismic migration. *Wang* [2003] presented an expression showing that the improvement of seismic resolution depends not only on the frequency bandwidth but also the signal-to-noise ratio. Seismic resolution will be improved only when both factors enforce each other. We present here a stabilized scheme for migration incorporating inverse Q filtering, so that the imaging technique becomes applicable to seismogram with long recording time in either regional or exploration seismics. The method is demonstrated using a real data example from exploration seismic with an attempt to recover a potential gas-reservoir underneath a group of strong coal-seam reflections.

2. The Algorithm

[5] As we assume the earth model to be 1-D varying with depth z or equivalently the travel time τ, the imaging algorithm may be implemented in the frequency-wavenumber domain: at the vertical axis, velocity and Q values within an extrapolation step are assumed to be constant stepwisely, thus wavefield downward extrapolation can be implemented in the frequency domain; at the horizontal axis, there is no spatial variations in velocity and Q, it may be implemented in the wavenumber domain. The algorithm is an extension of Stolt's migration algorithm for constant-velocity media [*Stolt*, 1978]. The extension includes two aspects: the use of a velocity function with continuous vertical variation and, even more importantly, considering the subsurface model as a viscoelastic medium.

[6] The derivation may be started with a 2-D scalar wave equation in the Fourier transform domain:

$$\frac{\partial^2}{\partial z^2} U(\omega, k_x, z) + k_z^2 U(\omega, k_x, z) = 0, \tag{1}$$

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where ω is the angular frequency, k_x is the horizontal wavenumber, k_z is the vertical wavenumber, and $U(\omega, k_x, z)$ is the 2-D Fourier spectrum of a wavefield $u(t, x, z)$. One of the solutions to equation (1) is

$$U(\omega, k_x, z + \Delta z) = U(\omega, k_x, z) \exp[\, \mathrm{i}\, k_z \Delta z], \tag{2}$$

where Δz is the step length for wavefield extrapolation. The vertical wavenumber k_z is defined as

$$k_z = \frac{\omega}{v} \sqrt{1 - \frac{k_x^2}{k^2}}, \tag{3}$$

where $k \equiv \omega/v$, and v is half of the constant wave propagation speed [*Loewenthal et al.*, 1976].

[7] In order to take account the earth Q effect, velocity v is taken as a complex value, $c(\omega)$, and solution (2) is rewritten as

$$U(\omega, k_x, z + \Delta z) = U(\omega, k_x, z) \exp\left[\mathrm{i}\, \frac{\omega \Delta z}{c(\omega)} \sqrt{1 - \frac{k_x^2}{k^2}}\right]. \tag{4}$$

The complex velocity $c(\omega)$ is defined as [*Wang*, 2002]

$$\frac{1}{c(\omega)} = \left(1 - \frac{\mathrm{i}}{2Q}\right) \left|\frac{\omega}{\omega_\mathrm{h}}\right|^{-\gamma} \frac{1}{v}, \tag{5}$$

where Q is the frequency-independent value within the seismic band, $\gamma = (\pi Q)^{-1}$ [*Kjartansson*, 1979], and ω_h is the high-limit of the possible frequency range [*Wang and Guo*, 2004]. Both v and Q are variable with depth z and so is the γ parameter.

[8] A time-domain migrated sample is obtained by replacing depth z in equation (4) with migration time τ. Consider the wavefield downward continuation starting from the recording surface, $\tau = 0$, the migrated wavefield at the current time level τ may be expressed as

$$U(\omega, k_x, \tau) = U(\omega, k_x, 0)\, M(\omega, k_x, \tau), \tag{6}$$

where $M(\omega, k_x, \tau)$ is the time-migration operator:

$$M(\omega, k_x, \tau) = \exp\left[\mathrm{i}\, \omega \int_0^\tau \left(1 - \frac{\mathrm{i}}{2Q(\tau')}\right) \left|\frac{\omega}{\omega_\mathrm{h}}\right|^{-\gamma(\tau')} \times \sqrt{1 - \frac{k_x^2}{k^2(\omega, \tau')}}\, d\tau'\right], \tag{7}$$

which takes account the vertical variations in velocity and Q functions. Thus, equations (6) and (7) are the basic time-migration formulae.

[9] By using the complex velocity $c(\omega)$, the migration process also compensates the amplitude attenuation and corrects the phase distortion due to the earth Q effect. However, as discussed in *Wang* [2002], a key problem with inverse Q filtering is the numerical instability, because of the exponential growth of the amplitude compensation operator with respect to the frequency and the travel time. In order to stabilize the operation, let us rewrite the basic migration equation (6) as

$$W(\omega, k_x, \tau) U(\omega, k_x, \tau) = U(\omega, k_x, 0), \tag{8}$$

where

$$W(\omega, k_x, \tau) = \exp\left[-\mathrm{i}\, \omega \int_0^\tau \left(1 - \frac{\mathrm{i}}{2Q(\tau')}\right) \left|\frac{\omega}{\omega_\mathrm{h}}\right|^{-\gamma(\tau')} \times \sqrt{1 - \frac{k_x^2}{k^2(\omega, \tau')}}\, d\tau'\right]. \tag{9}$$

A stabilized migration operator is then given by

$$M(\omega, k_x, \tau) = \frac{W^*(\omega, k_x, \tau) + \sigma^2}{W^*(\omega, k_x, \tau)\, W(\omega, k_x, \tau) + \sigma^2}, \tag{10}$$

where σ^2 is a small positive definite stabilization factor. That is, the stabilized migration scheme, instead of applying the migration operator (7) directly, uses the stabilized migration operator (10) evaluated in terms of the inverse migration operator W.

[10] Finally, the frequency-wavenumber domain migration formula is expressed as an inverse Fourier transform at $t = 0$, that is

$$u(k_x, \tau) = \frac{1}{\pi} \int_0^\infty U(\omega, k_x, 0) M(\omega, k_x, \tau) d\omega. \tag{11}$$

The migrated wavefield $u(k_x, \tau) \equiv u(t = 0, k_x, \tau)$ is presented here in the k_x domain, as the earth model (i.e., the velocity and Q functions) is assumed to be laterally invariant. An inverse Fourier transform with respect to k_x may be performed to produce a desired seismic section in the space-time domain.

3. The Implementation

[11] The migration formula, equation (11), can be understood as a nonstationary filter, implemented on each individual k_x trace separately.

[12] For a given wavenumber k_x, the input to the migration is a frequency domain seismic trace, $U(\omega)$, recorded at the surface $\tau = 0$, and the output is a migrated trace presented in the travel time domain, $u(\tau)$. Suppressing the dependency on wavenumber k_x, equation (11) can be discretized as

$$u_m = \frac{1}{N} \sum_{n=0}^{N} M_{m,n} U_n, \tag{12}$$

where m is the time sample index of the migrated trace, and n is the frequency sample index of the input trace.

[13] Equation (12) can be represented as

$$\begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_M \end{bmatrix} = \frac{1}{N} \sum_{n=0}^{N} \left\{ \begin{bmatrix} M_{0,n} \\ M_{1,n} \\ \vdots \\ M_{M,n} \end{bmatrix} U_n \right\}. \tag{13}$$

where the vector on the left-hand-side represents migrated samples, depending on the migration time τ, and the vector

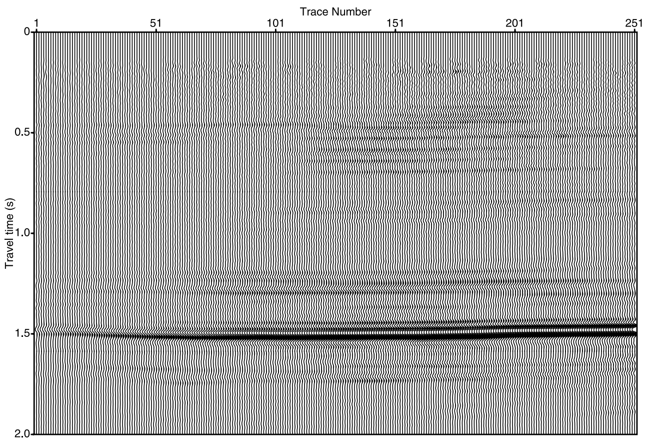

Figure 1. Seismic section used for testing the migration with inverse Q filtering algorithm. The objective is to recover the target reflection of a potential gas-reservoir immediately underneath the group of strong coal-seam reflections (at about 1.5 second).

on the right-hand-side, which is also time variant, is a migration response. Such a migration response is the function of the earth model and is data independent. But it is frequency dependent and thus can be called as the plane-wave migration response. Migration image is constructed by scaled superposition of the plane-wave migration responses.

[14] Each of these plane-wave migration responses is a 1-D function of time τ and is defined by equation (10) with fixed ω and k_x. That is, for a specific k_x trace, stabilized equation (10) gives a travel time range implicitly, within which a given frequency component may be fully recovered by inverse Q filtering and then migrated to right position. Beyond such a range, the amount of compensation decreases gradually. When a given high-frequency component was fully attenuated through wave propagation, the method automatically limits the attempt to recover it and to migrate it. Otherwise, it would boost the ambient noise.

4. Real Data Example

[15] We now show a real data example of applying migration with inverse Q filtering algorithm to improve the vertical resolution in migrated seismic section, in order to recover a weak reflection of the target gas-layer underneath the strong coal-seam reflections.

[16] Figure 1 is the sample seismic section. In this study area, reflection seismic surveys are often complicated by high-velocity layers that lie above deeper interfaces of interest. High-velocity hard layers sandwiching softer coal seams form a group of strong reflections that prevents the energy from penetrating to the deeper layers. Thus, deeper reflections are commonly very weak in this area. In addition, seismic data are generally observed to lose high frequencies with increasing travel time. The loss of high frequencies lengthens the dominant signal wavelength and thereby degrades the seismic resolution in general cases. In this particular area, the weak reflection of interest, a potential gas-reservoir, is so close to the group of strong reflections and is completely buried by the lengthened wavelet of strong reflections on the top. To recover the target reflection underneath the group of coal seams, we try now the algorithm of migration with inverse Q filtering described above.

[17] Figures 2 and 3 are the migrated seismic sections without consideration of the anelastic property and with inverse Q filtering, respectively. The earth Q model is estimated directly from the stack section shown in Figure 1, using a Q analysis method described by *Wang* [2004] which consists of four steps: (a) measuring time-frequency-dependent attenuation from seismic data, (b) generating a compensation curve based on the attenuation, (c) fitting compensation curve with a function in the least-squares sense to invert for the average Q function, and then (d) working out the interval Q values. The interval Q values obtained from such a Q analysis procedure are listed as follows:

time (s)	0.1–1.0	1.0–1.3	1.3–1.6	1.6–2.0
Q	39.5	48.9	98.9	163.8

[18] From Figure 3, in comparison with Figure 2, we can clearly see that the vertical resolution has been improved because of the higher frequency bandwidth, typically in the deep portion. More coherent events have appeared under-

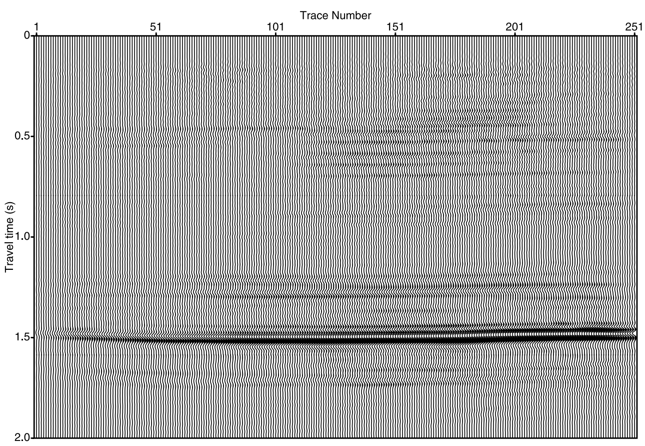

Figure 2. Seismic section after migration without considering the earth Q effect (i.e., attenuation and dispersion).

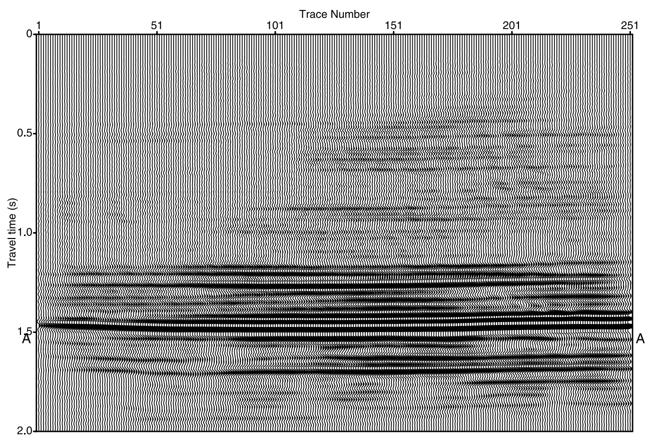

Figure 3. Seismic section after migration with inverse Q filtering, which has enhanced the vertical resolution and recovered the target reflection "A-A" (at time 1.52–1.56 second) underneath the coal-seam reflections.

neath the group of strong reflections at 1.5 second. The target reflection "A-A" (at time 1.52–1.56 second), immediately underneath the group of strong reflections, has been recovered. It was destroyed because of the phase distortion of the strong reflections on the top, and now stands clearly after a proper phase correction.

5. Conclusions

[19] The seismic imaging algorithm presented in this paper can produce a high resolution migrated section with true amplitudes and correct timings:

[20] (1) It successfully combines an inverse *Q* filter, which compensates for amplitudes and corrects for phases, into seismic migration processing. The success is due to the stabilization scheme applied to the 'combined' imaging operator.

[21] (2) The imaging algorithm is applicable to the subsurface model with vertical variations in velocity and attenuation. It is easy to implement and efficient in computation, and bound to have a great potential in the application of lithospheric-scale seismology and oil/gas reservoir seismics.

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