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A Method for Temporal Fault Tree Analysis Using Intuitionistic Fuzzy Set and Expert Elicitation

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ABSTRACT Temporal fault trees (TFTs), an extension of classical Boolean fault trees, can model time-dependent failure behaviour of dynamic systems. The methodologies used for quantitative analysis of TFTs include algebraic solutions, Petri nets (PN), and Bayesian networks (BN). In these approaches, precise failure data of components are usually used to calculate the probability of the top event of a TFT. However, it can be problematic to obtain these precise data due to the imprecise and incomplete information about the components of a system. In this paper, we propose a framework that combines intuitionistic fuzzy set theory and expert elicitation to enable quantitative analysis of TFTs of dynamic systems with uncertain data. Experts' opinions are taken into account to compute the failure probability of the basic events of the TFT as intuitionistic fuzzy numbers. Subsequently, for the algebraic approach, the intuitionistic fuzzy operators for the logic gates of TFT are defined to quantify the TFT. On the other hand, for the quantification of TFTs via PN and BN-based approaches, the intuitionistic fuzzy numbers are defuzzified to be used in these approaches. As a result, the framework can be used with all the currently available TFT analysis approaches. The effectiveness of the proposed framework is illustrated via application to a practical system and through a comparison of the results of each approach.

INDEX TERMS Fault tree analysis, reliability analysis, fuzzy set, intuitionistic fuzzy set theory, expert judgement, temporal fault trees.

I. INTRODUCTION

Over the years, we have seen a widespread use of safety critical systems in a wide variety of industries, including automotive, aerospace, maritime, medical, nuclear, and energy sectors. Such systems have one thing in common: if they fail, they can cause great harm to people and the environment. Accordingly, the reliability of these systems is held to a higher standard. Reliability is ''the probability that a piece of equipment or component will perform its intended function satisfactorily for a prescribed time and under stipulated environmental conditions'' [1].

Fault tree analysis (FTA) [2], [3] is widely used for the reliability analysis of systems. Although fault tree models are well-structured and easily understood, they are unable to model some aspects of system behaviour, e.g., priorities or functional and stochastic dependencies between events [4]. The modelling capability of classical fault trees has been enhanced through several extensions, such as dynamic fault trees (DFTs) [5] and Pandora TFTs [6]. For instance, in DFTs, dynamic gates like Functional Dependency (FDEP), Priority-AND (PAND), and SPARE gates are introduced to model the dynamic failure behaviour of systems. DFTs are primarily analysed quantitatively and for the analysis of fault trees, especially the DFTs, different approaches like algebraic [7], [8], Markov chain-based [9], [10], stochastic [11], [12], Bayesian network-based [13], [14], Sequential Binary Decision Diagram (SBDD)-based [15], [16] approaches have been developed.

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Pandora TFT is another dynamic fault tree extension. In addition to Boolean AND and OR gates, Pandora TFTs use temporal gates to capture the time-dependent behaviour of systems. One advantage of Pandora over other dynamic extensions of fault trees is that it can be integrated into model-based design and analysis processes and tools. That means Pandora TFTs can be synthesised from system models using popular modelling languages such as Matlab Simulink [17], EAST-ADL [18], or AADL [19]. Given the advantage of model-based dependability analysis of systems as described in [20] and the potential benefits of Pandora in this context, in this paper, we focus only on this particular extension. Therefore, although the authors are aware of other recent developments with other dynamic extensions of fault trees, for brevity, other developments related to DFTs are omitted in this paper.

Qualitative analysis of Pandora TFTs allows determining the minimal cut sequences (MCSQs). MCSQs are smallest sequences of basic events (BEs) that can cause system failure. Methodologies have been proposed in the past for quantitative analysis of TFTs. Such methodologies include algebraic solution [21], BN-based approach [22], and PN-based approach [23], [24]. All these approaches use precise failure probabilities/rates of system components for the purposes of quantification. It is generally problematic to collect exact failure rates or probabilities for all the components of complex and large systems, which can make it difficult to perform quantitative analysis.

Fuzzy set theory [25] has been developed to handle such uncertain scenarios by attributing a degree to which a certain object belongs to a set. It has been widely used for uncertainty quantification in reliability engineering applications. The first application of fuzzy set theory with FTA can be found in [26]. In [27], a comprehensive review of the concept of fuzzy set theory based FTA and their applications was presented and a review of the applications of fuzzy set theory in system safety and reliability analysis was presented in [28]. In [29], fuzzy set theory has been used with a stochastic computational model for the analysis of fuzzy systems. To acquire uncertain failure data, expert judgement has been used in association with fuzzy set theory. For instance, Lin and Wang [30] combined expert elicitation with fuzzy set theory for fault tree analysis with uncertain data.

In fuzzy set theory, the concept of a membership function is used to define the degree of membership of a particular object to a set, i.e., how strongly an object belongs to a set. In many cases it may not be possible to define this membership degree with certainty. Classical fuzzy set theory is not able to incorporate uncertainty or hesitation in the membership functions. As a potential solution to this problem, Atanassov [31] proposed the concept of intuitionistic fuzzy sets (IFS). As an extension to classical fuzzy set theory, IFS is useful in defining an imprecise quantity using fuzzy sets where classical fuzzy sets cannot define the quantity due to the inadequacy of available information. Unlike fuzzy set, the IFS uses the concept of a non-membership function in addition to membership function in such a way that their summation is less than 1 [32]. According to Biswas [33], in many situations where the determination of degree of membership of an object to a set with certainty is difficult, the use of IFS is preferable to handle uncertainty; moreover, in [34], it was pointed out that the vague set concept coincides with the IFS concept. As a result, the expectation is that IFS could be utilised to model uncertainties associated with any processes and/or activities involving human expertise and knowledge.

In the past, the IFS concept has been used in reliability engineering applications. For instance, Shu *et al.* [35] introduced a FTA method using IFS. Occurrence possibilities of BEs of fault trees were represented as IFSs and were computed through expert elicitation. A vague FTA method to determine the reliability of a weapon system has been proposed in [36]. IFS has been used by Cheng *et al.* [37] for reliability analysis of a liquefied natural gas terminal emergency shutdown system through FTA. The failure possibility of BEs were represented using triangular fuzzy numbers. These failure data were collected through expert judgement. Similarly, Kumar *et al.* [38] used triangular intuitionistic fuzzy numbers (IFN) and proposed an approach for reliability evaluation using IFS. They have also developed FTA using intuitionistic fuzzy set theory in [39]–[41]. Other researchers [42], [43] have also developed FTA approaches using IFS theory.

Although the potential applications of intuitionistic fuzzy set theory in classical static FTA has been investigated in the past, to the best of the authors' knowledge, it has not yet been investigated how IFS could be used with dynamic extensions of fault trees. In [4], [44], [45], classical fuzzy set theory has been used for addressing the issue of uncertain failure data during quantitative analysis of TFTs. These approaches can only be applied when sufficient information is available to define the failure probabilities of basic events using classical fuzzy sets. As a result, when using these approaches it is not possible to model scenarios where exact knowledge about the fuzziness of quantitative data is not expressible with a certain level of confidence. Given the additional uncertainty modelling capability offered by IFS theory over classical fuzzy set theory, integrating this technique in the TFT quantification process will open many possibilities.

In this paper, we propose a framework for integrating IFS theory with expert elicitation to enable the dynamic reliability analysis of systems through TFTs where exact failure data of system components are unavailable. In this context the contributions of this paper include:

- A framework to show how the concept of IFNs and expert elicitation can be integrated into the TFT quantification process to evaluate the reliability of dynamic systems.
- A method to use expert knowledge to compute failure possibilities of basic events (BEs) in TFTs as IFNs.
- Procedure for calculating weightings of the employed experts using a variant of the fuzzy analytical hierarchy process, in which together with other attributes (e.g., job

field, experience, education) the *confidence level* of the experts are taken into account for the first time.

- Definition of fuzzy operators for the TFT's logic gates to quantify TFT based on the failure probability of BEs represented as triangular IFNs.
- A process for calculating criticality of events based on IFNs.

The rest of the paper is organized as follows: Section II describes the fundamentals of intuitionistic fuzzy set theory. An overview of temporal fault tree analysis is also provided in this section. The proposed framework is described in Section III. The description includes the intuitionistic fuzzy data collection process through expert elicitation, formulas to evaluate the logic gates of TFTs with intuitionistic fuzzy data, and the defuzzification of IFNs to facilitate the TFT analysis via PN and BN-based approaches. Section IV provides a numerical example to illustrate the use of the proposed framework. Finally, Section V presents concluding remarks and future research directions.

II. BACKGROUND

A. INTUITIONISTIC FUZZY SET THEORY

Atanassov [32], [46] generalized the concept of fuzzy sets into IFS by introducing a non-membership value $v_{\tilde{A}}(x)$ representing the evidence against $x \in X$ along with the membership value $\mu_{\tilde{A}}(x)$ representing evidence for $x \in X$ and this admits an aspect of indeterminacy. This idea appears to be effective in modelling many practical scenarios.

1) INTUITIONISTIC FUZZY SET

If X is a universe of discourse, then an IFS $\tilde{A}$ in X is given by

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle : x \in X\} \quad (1)$$

where $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ is the membership function and $v_{\tilde{A}} : X \rightarrow [0, 1]$ is the non-membership function. These functions satisfy the following condition [47].

$$0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1, \quad \forall x \in X \quad (2)$$

For every value $x \in X$, the values $\mu_{\tilde{A}}(x)$ and $v_{\tilde{A}}(x)$ represent the membership and non-membership degrees of the element $x \in X$ to $\tilde{A} \subseteq X$, respectively. Additionally, the intuitionistic fuzzy (IF)-index (degree of uncertainty or hesitation level) of x in $\tilde{A}$ is defined as [47], [48]:

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x) \quad (3)$$

If $\pi_{\tilde{A}}(x) = 0, \forall x \in X$, then the IFS is reduced to a normal fuzzy set.

2) CONVEX AND NORMAL INTUITIONISTIC FUZZY SET

An IFS $\tilde{A}$ in X is IF-convex [49], [50] iff

1) Membership function $\mu_{\tilde{A}}(x)$ of $\tilde{A}$ is fuzzy-convex, i.e.,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \quad \forall x_1, x_2 \in X, 0 \leq \lambda \leq 1 \quad (4)$$

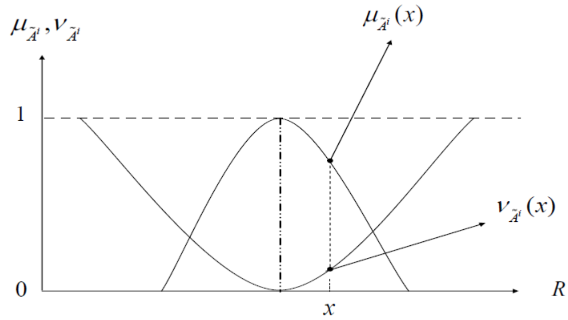

FIGURE 1. Intuitionistic fuzzy number.

2) Non-membership function $v_{\tilde{A}}(x)$ of $\tilde{A}$ is fuzzy-concave, i.e.,

$$v_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max(v_{\tilde{A}}(x_1), v_{\tilde{A}}(x_2)) \quad \forall x_1, x_2 \in X, 0 \leq \lambda \leq 1 \quad (5)$$

An intuitionistic fuzzy number $\tilde{A}$ in X is IF-normal [49], [50] if there exits at least two points $x_1, x_2 \in X$ such that $\mu_{\tilde{A}}(x_1) = 1$ and $v_{\tilde{A}}(x_2) = 1$.

3) INTUITIONISTIC FUZZY NUMBER

An intuitionistic fuzzy set $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle : x \in \mathbb{R}\}$ is called an intuitionistic fuzzy number iff [47]

1) $\tilde{A}$ is IF-convex and IF-normal.
2) $\mu_{\tilde{A}}(x)$ is upper semi continuous and $v_{\tilde{A}}(x)$ is lower semi continuous.
3) $Supp\ \tilde{A} = \{x \in X : v_{\tilde{A}}(x) < 1\}$ is bounded.

A triangular IFN (TIFN) is an IFN with membership function $\mu_{\tilde{A}}(x)$ and non-membership function $\mu_{\tilde{A}}(x)$ given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & otherwise. \end{cases} \quad (6)$$

and

$$v_{\tilde{A}}(x) = \begin{cases} \dfrac{b-x}{b-a'}, & a' \leq x \leq b \\ \dfrac{x-b}{c'-b}, & b \leq x \leq c' \\ 1, & otherwise. \end{cases} \quad (7)$$

where $a' \leq a \leq b \leq c \leq c'$. This TIFN is denoted by $\tilde{A} = (a, b, c; a', b, c')$.

B. PANDORA TEMPORAL FTA

In addition to the Boolean gates of the classical FTs, Pandora TFT uses temporal gates such as Priority-AND (PAND) and Priority-OR (POR) to model the temporal behaviour of systems. The graphical representation of the logic gates used in Pandora is shown in Fig. 2. A detailed description of the behaviour of these logic gates can be found in [24]. In a logical expression, the PAND and the POR gate is represented by the symbol '$\lhd$' and '$|$', respectively. The AND

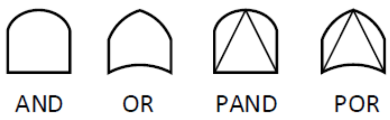

FIGURE 2. Graphical representation of Pandora's logic gates.

and OR gates are represented as '.' and '+', respectively. Pandora TFTs can be created through model-based analysis of systems. For example, using HiP-HOPS [51], a model-based safety analysis technique, Pandora TFTs can be semi-automatically generated from the system models [52]. Once a TFT is constructed, both qualitative and quantitative analysis can be performed on it. Through qualitative analysis the TFT is minimised to obtain MCSQs. For quantitative analysis of TFTs, both algebraic [21] and state-space based [22], [23] methodologies have been developed.

1) ALGEBRAIC SOLUTION TO TFTs

In an algebraic solution, mathematical formulas are proposed to quantify the temporal gates.

If the failure rates of the N input events $\{X_1, X_2, \ldots, X_N\}$ to a PAND gate are defined as $\{\lambda_1, \lambda_2, \ldots, \lambda_{N-1}, \lambda_N\}$, then the occurrence probability of the PAND gate at time t is quantified as [8]:

$$Pr\{X_1 \lhd X_2 \lhd \ \ldots \ \lhd X_{N-1} \lhd X_N\}(t) = \prod_{i=1}^{N} \lambda_i \sum_{k=0}^{N} \left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j \neq k}}^{N} (u_k - u_j)} \right] \quad (8)$$

where $u_0 = 0$ and $u_m = -\sum_{j=1}^{m} \lambda_j \, for \ m > 0$.

The following formula can be used to quantify an POR gate with N inputs [21]:

$$Pr\{X_1 \wr X_2 \wr \ldots \wr X_{N-1} \wr X_N\}(t) = \frac{\lambda_1 \left(1 - \left(e^{-(\sum_{i=1}^{N} \lambda_i)t}\right)\right)}{\sum_{i=1}^{N} \lambda_i} \quad (9)$$

The Boolean AND and OR gates can be quantified using the following equations.

$$Pr\{X_1 \,.\, X_2 \,.\, \ldots \,.\, X_{N-1} \,.\, X_N\}(t) = \prod_{i=1}^{N} P_r\{X_i\}(t) \quad (10)$$

$$Pr\{X_1 + X_2 + \ \ldots \ + X_{N-1} + X_N\}(t) = 1 - \prod_{i=1}^{N} \left(1 - P_r\{X_i\}(t)\right) \quad (11)$$

where $Pr\{X_i\}(t)$ is the probability of the event X_i at time t.

Note that the above formulas can only be used for the quantification of TFTs if the precise failure rates of BEs are available. To allow the use of imprecise/uncertain failure rates in the quantification process, a methodology has been proposed in [45] for fuzzy temporal fault tree analysis. In this method, the failure rates of BEs are considered as fuzzy numbers and represented as triangular fuzzy numbers. Formulas were defined to quantify the logic gates with fuzzy data.

2) STATE-SPACE BASED SOLUTIONS TO TFTs

In addition to the algebraic solution to TFTs, PN and BN-based approaches have also been developed for the quantification of TFTs. In the PN-based TFT quantification approach [23], [24], graph transformation rules are provided to translate the elements of the TFT to PNs. In the TFT to PN transformation process, each basic event and logic gate of a TFT is translated into a sub-net and then all the sub-nets are combined together to form the PN model of the whole TFT. For the sake of brevity, a detailed description of the TFT to PN transformation process is omitted in this paper. However, for a detailed description, interested readers are referred to [23], [24]. After the formation of a PN model, the precise failure rates of system components are used to characterize the timed transitions in the PN model. For unreliability evaluation (e.g. top event probability of a TFT), the PN model of a TFT can be simulated for a specific mission time using a continuous model of time. While this approach relies on precise failure data of system components, recently, in [4], a framework has been presented showing how classical fuzzy set theory can be used to address the issue of uncertain failure data in PN-based dynamic fault tree analysis.

In the BN-based approach [24], a discrete model of time is considered and a TFT is translated into a discrete-time BN. This translation is one-to-one, where the basic events of the TFT are translated as root nodes (the nodes without any parent) of BN and the logic gates are translated into internal nodes. As a result, the top event of the TFT is mapped as the only leaf node, i.e., a node without any children, in the BN model. For the purpose of quantifying the TFT while taking the order of occurrence of failure events into account, the mission time of a system is divided into n intervals. In order to be able to use the BN model of TFT for quantitative analysis, the root nodes of the BN are assigned with prior probabilities, which are calculated based on the precise failure rate of the basic events of the corresponding TFT. At the same time, the conditional probabilities of the internal nodes are deterministically defined based on the behaviour of the logic gates they represent.

III. THE PROPOSED FRAMEWORK

The framework proposed for integrating IFS theory and expert elicitation into the reliability analysis of systems through TFT analysis is shown in Fig. 3. As seen in the figure, a reliability analysis using this framework requires four steps: *A. TFT Modelling, B. Failure Data Collection, C. TFT Solutions,* and *D. Reliability Quantification.* The steps are explained in the following subsections.

A. TFT MODELLING

This step concentrates on modelling the dynamic failure behaviour of the system under study using temporal fault

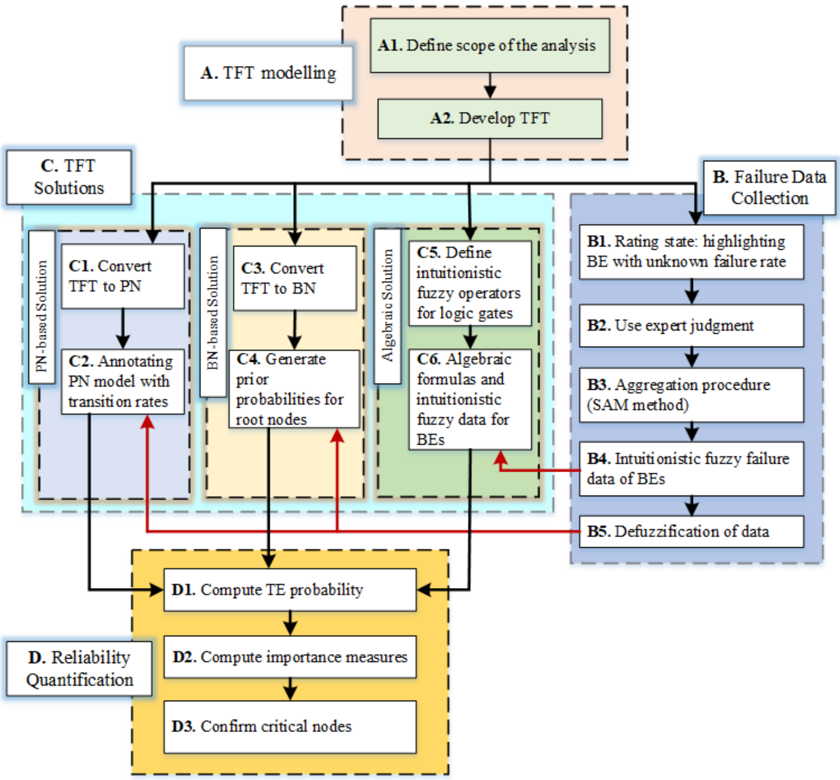

FIGURE 3. The proposed framework.

trees. In order to do so, the first step is to define the scope of the analysis. For instance, the analyst has to define which part of the system is going to be analysed and which part is omitted. Then it is necessary to determine the contributing factors and failure events which will be included in the analysis and which are not. Finally, the level of detail that is going to be covered in the root causes analysis of system failure is defined. Once all these are defined, the failure behaviour of the system is modelled as a TFT. The TFT development process follows a top-down approach. At first, a top event (system failure condition) is identified and the TFT development process starts with this top event. The top event is decomposed into a number of immediate events that can cause the top event. Each of these events are modelled using Boolean and dynamic gates to reflect the combinatorial and temporal relationships between events. The intermediate events are decomposed further until the basic events are reached.

B. FAILURE DATA COLLECTION

For the quantification of the TFT developed in the previous step, we need to obtain the failure data for the BEs. Note that, in this paper, it is assumed that the precise failure data for the BEs are unavailable; therefore, IFNs are used to represent the failure data of the BEs. As a result, it is necessary to obtain the unknown failure data of BEs as IFNs. In this paper, multi-expert knowledge is utilised to acquire failure possibility of BEs with uncertain data. As expert knowledge is affected by individual visions and purposes [53], it is difficult to ensure complete impartiality in expert knowledge. The experts can be from diverse backgrounds and they can have different levels of expertise and working experience. For this reason, the weighting of experts is different; therefore, for practical application, employing a heterogeneous group of experts is more realistic [54], [55]. Several criteria were taken into account to determine the weighting of an expert, e.g., the work experience, educational qualification and confidence of the opinions.

As seen in Fig. 4, the proper utilisation of the expert judgement system involves three main steps: eliciting opinions, expert weighting, and aggregation. The eliciting opinions procedure indicates that selecting a proper method to collect experts' opinion should satisfy rational consensus principles like fairness and accountability. In the second step, a suitable method should be utilised to obtain the relative importance of the employed experts in order to quantify the weights of different experts. Additionally, it should help to minimise subjective bias and improve the accuracy of failure possibility of each BE as much as possible. Finally, an aggregation procedure should be applied to combine all expert opinions by considering their different weights to obtain a single opinion for further computations. Each of these three step is explained in more detail below.

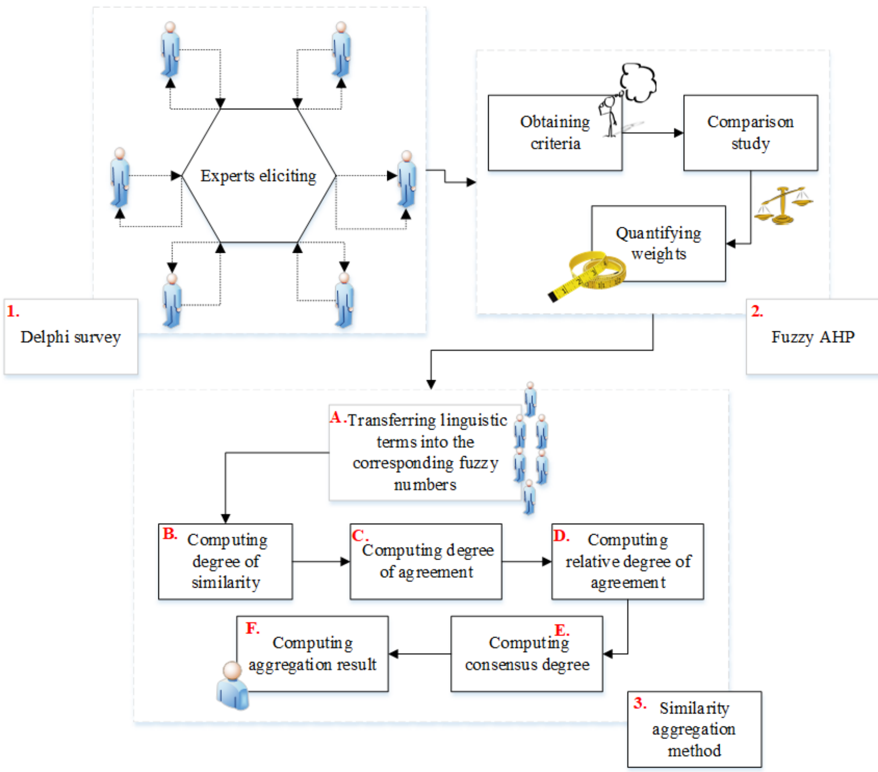

FIGURE 4. The steps of using expert judgment system.

1) ELICITING OPINIONS PROCEDURE

When the failure data collection is difficult or too costly, expert knowledge elicitation can be considered as means of data collection [56]. This means that, in certain circumstances, expert opinion can be used as an alternative and useful source of data; however, it should be noted that it is not a considerable source of rational consensus. Therefore, an improper methodology will not be able to handle and contribute to the rational consensus. The five principles noticed by Cooke [57], including reproducibility, accountability, empirical control, neutrality, and fairness, represent an attempt to formulate a uniformity guideline for using experts' opinions on expert system science.

In this regard, the Delphi method can be used to acquire the most reliable opinion of a group of experts [58]. To conduct a Delphi survey, a group of independent, experienced experts with relevant background are selected. Questionnaires were spread among experts and each expert fills the questionnaires by following strict ethical guidelines. All the information related to the survey can be communicated through mail, email, and fax. Such communications may help to avoid counter-productive negotiations and deviations that may occur in face-to-face group discussions. After collecting the opinions from the experts, the data are processed to reach a consensus. When evaluating the performance of employed experts, not all detailed behaviour information needed may be available; but is necessary to satisfy rational consensus principles like as empirical control, neutrality, and fairness. Other available methods described in the literature, like the classical method, can deal with expert opinions, but will not be able to satisfy all rational consensus principles [59]–[61].

2) EXPERT WEIGHTING

The calculation of expert weighting is a complicated task due to the large number of judgements required to fully quantify the relationships and compute the probability of BEs in large FTs. Thus, obtaining realistic weightings for the employed experts is important. Experts' judgments are subject to bias, especially in expressing their opinions about large and complex system [62]. Among many available methods, the analytical hierarchy process (AHP) [63] is a popular method in multi criteria decision making (MCDM). In AHP, a complex decision-making problem is broken down into several smaller problems and these problems are formulated in a hierarchical order to manage complexity. Afterwards, it is possible to concentrate on the smaller decision problems at a time to reach to the final decision. The classical AHP has limited capability to model human thinking and cognitive process, especially for situations where it is hard for experts to estimate precise values. To handle these cases, a new method named the fuzzy analytical hierarchy process (FAHP) [64], [65] has been developed. Among the different available variants of the FAHP, the methods proposed by Buckley [66] and Chang [67] are the two most important

TABLE 1. The corresponding fuzzy number for relative importance comparison to criterion.

Linguistic scales	Relative importance number	Triangular fuzzy scale	Relative importance number	Triangular fuzzy reciprocal scale
Just equal	1	(1, 1, 1)	1	(1, 1, 1)
Equally probable	$\tilde{1}$	(1/2, 1, 3/2)	$\tilde{1}^{-1}$	(2/3, 1, 2)
Weakly probable	$\tilde{3}$	(1, 3/2, 2)	$\tilde{3}^{-1}$	(1/2, 2/3, 1)
Strongly more probable	$\tilde{5}$	(3/2, 2, 5/2)	$\tilde{5}^{-1}$	(2/5, 1/2, 2/3)
Very strongly more probable	$\tilde{7}$	(2, 5/2, 3)	$\tilde{7}^{-1}$	(1/3, 2/5, 1/2)
Absolutely more probable	$\tilde{9}$	(5/2, 3, 7/2)	$\tilde{9}^{-1}$	(2/7, 1/3, 2/5)

ones. This section gives an illustration of how to use FAHP to obtain the weightings of the employed experts using the following stages.

Stage 1: For expert k, after comparing criterion i with criterion j, the relative fuzzy importance obtained is: $\tilde{a}_{ij}^k = (\tilde{a}_{ij1}^k, \tilde{a}_{ij2}^k, \tilde{a}_{ij3}^k)$. Using these values, the aggregated relative fuzzy importance $\tilde{a}_{ij}$ is calculated as follows.

$$\tilde{a}_{ij} = \left(\sum_{k=1}^{K} \delta_k.\tilde{a}_{ij1}^k, \sum_{k=1}^{K} \delta_k.\tilde{a}_{ij2}^k, \sum_{k=1}^{K} \delta_k.\tilde{a}_{ij3}^k\right) \tag{12}$$

where $\delta_k > 0$ and $\sum_{k=1}^{K} \delta_k = 1$.

Stage 2: Pairwise comparisons are made between all criteria and a matrix is formed as shown below. Qualitative terms are used to define the relative importance of criteria.

$$\tilde{A} = [\tilde{a}_{ij}] = [l_{ij}, m_{ij}, u_{ij}] = \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ 1/\tilde{a}_{21} & 1 & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{a}_{n1} & 1/\tilde{a}_{n2} & \dots & 1 \end{bmatrix} \tag{13}$$

When $\tilde{a}_{ij} = \tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$ represents a scenario where criterion i is of relative importance to criterion j and $\tilde{a}_{ij} = \tilde{1}^{-1}, \tilde{3}^{-1}, \tilde{5}^{-1}, \tilde{7}^{-1}, \tilde{9}^{-1}$ represents the opposite scenario. If $i = j$, then $\tilde{a}_{ij} = 1$.

Table 1 shows the linguistic terms used to represent relative importance criterion and their associated fuzzy values.

Stage 3: In this stage, the consistency of the fuzzy pairwise comparison matrix is examined by assuming that $A = [a_{ij}]$ is a positive mutual matrix and $\tilde{A} = [\tilde{a}_{ij}]$ is a fuzzy positive mutual matrix. According to [66], $\tilde{A} = [\tilde{a}_{ij}]$ will be consistent if $A = [a_{ij}]$ is consistent. If there exists any inconsistency, the weighting evaluation process should be reiterated to increase the consistency [68].

Stage 4: The fuzzy weights of fuzzy comparison values between criteria is computed by using the geometric mean method as follows:

$$\tilde{r}_i = \left(\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \cdots \otimes \tilde{a}_{in}\right)^{1/n} \tag{14}$$

Stage 5: For each criterion, the fuzzy weights $\tilde{w}_i = \left(w_i^L, w_i^M, w_i^U\right)$ are defined as follows [69]:

$$\tilde{w}_i = \tilde{r}_i \otimes \left(\tilde{r}_1 \oplus \tilde{r}_2 \oplus \cdots \oplus \tilde{r}_n\right)^{-1} \tag{15}$$

The above fuzzy weight vector can be defined as $\tilde{w}_i = (w_1, w_2, w_3, \cdots, w_i, \cdots, w_n)^T$ when the comparison matrix $\tilde{A}$ shown in equation (13) is perfectly consistent. Otherwise, the following constrained nonlinear optimization model can be solved to determine the weight vectors of $\tilde{A}$ [70], [71].

$$min\, j = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\left(ln\, w_i^L - ln\, w_j^U - ln\, l_{ij}\right)^2 + \left(ln\, w_i^M - ln\, w_j^M - ln\, m_{ij}\right)^2 + \left(ln\, w_i^U - lnw_j^L - lnu_{ij}\right)^2\right) \tag{16}$$

$$\text{Subject to} \begin{cases} w_i^L + \sum_{j=1, j\neq 1}^{n} w_j^U \geq 1, \\ w_i^U + \sum_{j=1, j\neq 1}^{n} w_j^L \geq 1, \\ \sum_{i=1}^{n} w_i^M = 1, \\ \sum_{i=1}^{n} \left(w_i^L + w_i^U\right) = 2, \\ 0 \leq w_i^L \leq w_i^M \leq w_i^U. \end{cases}$$

The model can be solved using the General Algebraic Modelling System (GAMS) [72] and the optimal solution to this model forms normalized fuzzy weights as mentioned earlier. In this study both types of consistency evaluation are used.

Stage 6: Defuzzification process. Defuzzification is an important step in the fuzzy MCDM process, which finds the best non-fuzzy performance (BNP) value. Different methods such as mean of maximum (MoM), center of area (CoA), and alpha cut are available for defuzzification. Out of these approaches, the application of CoA technique for finding the BNP is simpler and more practical. In addition, it is not necessary to take into account the preferences of any experts [73]; thus, we use CoA for the defuzzification. The following equation is utilised to obtain the BNP value of the fuzzy number w_i [71].

$$BNP_{w_i} = \frac{\left(w_i^U - w_i^L\right) - \left(w_i^M - w_i^L\right)}{3} + w_i^L, \quad \forall i = 1, 2, \cdots, n. \tag{17}$$

The normalized weight BNP_{w_i} is the corresponding weight of each expert.

3) AGGREGATION PROCEDURE

In this step, the employed experts expressed their opinions regarding the failure possibility of each BE. The expert judge-

TABLE 2. The linguistic terms and the corresponding IFNs.

Linguistic terms	Corresponding IFNs
Very Low (VL)	(0.00, 0.04, 0.08; 0.00, 0.04, 0.08)
Low (L)	(0.07, 0.13, 0.19; 0.06, 0.13, 0.20)
Relatively Low (RL)	(0.17, 0.27, 0.37; 0.15, 0.27, 0.39)
Medium (M)	(0.35, 0.50, 0.65; 0.32, 0.50, 0.68)
Relatively High (RH)	(0.62, 0.73, 0.82; 0.61, 0.73, 0.85)
High (H)	(0.81, 0.87, 0.93; 0.79, 0.87, 0.95)
Very High (VH)	(0.92, 0.96, 1.00; 0.92, 0.96, 1.00)

ment on failure possibility of BEs can be acquired by using linguistic terms provided in terms of IFNs. Table 2 presents the linguistic terms and their associated IFNs.

As the opinion of experts may vary widely due to their level of experience and expertise, aggregation of multi-expert opinion is needed to reach a consensus. Different methods for aggregation such as arithmetic averaging and the similarity aggregation method (SAM) can be used in this regard. Yazdi and Zarei [73] discussed the advantages and superiority of such common methods on conventional fuzzy FTA. In this paper, an extension of SAM is used for the aggregation of IFNs. The procedure of this technique in provided in detail below.

Step A. Transferring linguistic terms to corresponding IFNs:

Once each expert, $E_k(k = 1, \cdots, m)$ gives linguistic judgement about the failure possibility of each BE, it is converted into the corresponding IFNs.

Step B. Computing degree of similarity: The similarity $S_{uv}(\tilde{A}_u, \tilde{A}_v)$ between the opinions $\tilde{A}_u$ and $\tilde{A}_v$ of experts E_u and E_v is evaluated as:

$$S_{uv}(\tilde{A}_u, \tilde{A}_v) = \begin{cases} \dfrac{EV_u}{EV_v}, & \text{if } EV_u \leq EV_v \\ \dfrac{EV_v}{EV_u}, & \text{if } EV_v \leq EV_u \end{cases} \tag{18}$$

where $S_{uv}(\tilde{A}_u, \tilde{A}_v) \in [0, 1]$ is the similarity measure function, $\tilde{A}_u$ and $\tilde{A}_v$ are two standard intuitionistic fuzzy numbers, EV_u and EV_v represent the expectancy evaluation for $\tilde{A}_u$ and $\tilde{A}_v$. The EV of a triangular IFN $\tilde{A} = (a, b, c; a', b, c')$ is defined as:

$$EV(\tilde{A}) = \frac{(a + a') + 4 \times b + (c + c')}{8} \tag{19}$$

If a total of m experts is employed then a *similarity matrix* is generated as shown below.

$$SM = \begin{bmatrix} 1 & s_{12} & s_{13} & \dots & s_{1m} \\ s_{21} & 1 & s_{23} & \dots & s_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_{m1} & s_{m2} & s_{m3} & \dots & 1 \end{bmatrix} \tag{20}$$

where $s_{uv} = S_{uv}(\tilde{A}_u, \tilde{A}_v)$, if $u = v$ then $s_{uv} = 1$.

Step C. Computing degree of agreement:

The average agreement degree $AA(E_u)$ for each of the experts is computed as:

$$AA(E_u) = \frac{1}{m-1} \sum_{\substack{v=1 \\ v \neq u}}^{m} S_{uv} \tag{21}$$

where $u = 1, 2, \cdots, m$.

Step D. Computing the Relative Agreement:

The relative agreement degree $RAD(E_m)$ for all experts is obtained as:

$$RAD(E_u) = \frac{AA(E_u)}{\sum_{v=1}^{m} AA(E_v)} \tag{22}$$

where $u = 1, 2, \cdots, m$.

Step E. Computing Consensus degree:

The aggregation weight (w_i) of each expert E_i is the combination of the $RAD(E_i)$ and the weight of each expert BNP_{w_i} obtained by fuzzy AHP.

$$\alpha \odot BNP_{w_i}(E_i) + (1 - \alpha) \odot RAD(E_i) \tag{23}$$

where BNP_{w_i} is the weight of each expert, α $(0 \leq \alpha \leq 1)$ is a relaxation factor, and the operator '$\odot$' represents the multiplication of scalar number with IFNs. In this equation, α illustrates the importance of BNP_{w_i} over $RAD(E_i)$. When $\alpha = 0$, no weight has been given to BNP_{w_i}, therefore, it is better to employ a homogeneous group of experts. On the other hand, $\alpha = 1$ means no weight is given to $RAD(E_i)$. Yazdi [74] suggested that the consensus coefficient of each expert is better to be known when the comparative competency of each expert's opinion is estimated. Accordingly, it is important to allocate a proper value of α, otherwise sensitivity analysis should be applied to analyse the failure behaviour of system when α has given different value from zero to 1. To give equal weight to the variables in the equation (23), in this study the value of α is considered as 0.5.

Step F. Computing aggregation result:

The aggregation result for each BE can be calculated as follows:

$$\tilde{P}_j = \sum_{i=1}^{m} w_i \otimes \tilde{P}_{ij} \tag{24}$$

where $\tilde{P}_j$ is the aggregation possibility of BE_j in term of IFNs.

As seen in Section II-B, the quantification of TFT is performed based on failure rate/probability of the BEs. Therefore, failure possibilities of BEs obtained from expert elicitation need to be converted to corresponding failure probability. This can be accomplished by using the following formula proposed by Onisawa [75].

$$FP = \begin{cases} \dfrac{1}{10^K}, & CP \neq 0, \\ 0, & CP = 0. \end{cases} \tag{25}$$

where FP is the failure probability, CP is the crisp failure possibility and $K = \left(\dfrac{1 - CP}{CP}\right)^{\frac{1}{3}} \times 2.301$ [75].

Moreover, for the crisp value based TFT quantification approaches such as PNs and BNs we need crisp failure rates and/or probabilities of the BEs. Therefore, we need to defuzzify the IF failure probability of a BE to a crisp value. If the IF failure probability of a BE is represented by a triangular IFN as $\tilde{A} = \{a, b, c; a', b, c'\}$, then it can be defuzzified using the following formula to obtain a crisp value.

$$X = \frac{1}{3}\left[\frac{(c'-a')(b-2c'-2a')+(c-a)(a+b+c)+3(c'^2-a'^2)}{c'-a'+c-a}\right] \tag{26}$$

C. TFT SOLUTIONS

In this third step of the proposed framework (see Fig. 3), for the quantification of a TFT, the analysts can choose one or more of the available solution techniques for TFTs as mentioned in Section II-B. If the PN-based approach is selected, the TFT of the system has to be translated to a PN model by following the instructions available at [24]. Note that, as seen in Fig. 3, after the formation of the PN model, the transition rates of the timed-transitions of the PN model are annotated by the crisp failure rates of the BEs, which are obtained in the earlier step.

On the other hand, if the BN-based approach is selected, the TFT of the system has to be translated to a discrete-time BN by following the instructions available at [22]. In this case, the analysts have to decide the number of discrete time intervals to divide the mission time into and then populate the prior probability values for the root nodes in the BN model accordingly based on the crisp failure rates of the BEs. The conditional probabilities of the internal nodes in the BN are populated based on the behaviour of the logic gates they represent.

If the algebraic solution is chosen as a solution technique for TFT, then the equations (8)-(11) can be used to quantify the logic gates in the TFT based on the crisp failure rates of BEs obtained in the step described in the previous section. However, instead of crisp values, it is possible to directly use the IFNs for BE failure rates. To use the IFNs to quantify the TFTs, intuitionistic fuzzy (IF) operators for the logic gates need to be defined. In order to use intuitionistic fuzzy failure rates or probabilities in the quantification process, we have formulated operators for all the logic gates as described below.

1) INTUITIONISTIC FUZZY PROBABILITY OF AND GATE

If the failure probability of an event $X_i, i = 1, 2, 3, \ldots, N$ at time t is denoted by a TIFN $\tilde{P}_i = \{a_i, b_i, c_i; a'_i, b_i, c'_i\}$, then the intuitionistic fuzzy failure probability of an AND gate with N inputs $\{X_1, X_2, \ldots, X_N\}$ can be defined as:

$$P_{IF-AND}(t) = AND_{IF}\{P_1, P_2, \ldots, P_N\} = \left\{\prod_{i=1}^{N} a_i(t), \prod_{i=1}^{N} b_i(t), \prod_{i=1}^{N} c_i(t); \prod_{i=1}^{N} a'_i(t), \prod_{i=1}^{N} b_i(t), \prod_{i=1}^{N} c'_i(t)\right\} \tag{27}$$

2) INTUITIONISTIC FUZZY PROBABILITY OF OR GATE

The intuitionistic fuzzy failure probability of an OR gate (TE of fault tree) with N input events $\{X_1, X_2, \ldots, X_N\}$ can be defined as:

$$\begin{aligned} P_{IF-OR}(t) &= OR_{IF}\{P_1, P_2, \ldots, P_N\} = \Big\{1 - \prod_{i=1}^{N}(1 - a_i(t)), \\ &1 - \prod_{i=1}^{N}(1-b_i(t)), 1 - \prod_{i=1}^{N}(1 - c_i(t)); 1-\prod_{i=1}^{N}(1-a'_i(t)), \\ &1 - \prod_{i=1}^{N}(1 - b_i(t)), 1 - \prod_{i=1}^{N}(1 - c'_i(t))\Big\} \end{aligned} \tag{28}$$

3) INTUITIONISTIC FUZZY PROBABILITY OF PAND GATE

If there are N input events $\{X_1, X_2, \ldots, X_N\}$ with intuitionistic fuzzy failure rates $\lambda_1, \lambda_2, \ldots, \lambda_N$ respectively and $\lambda_i, i = 1, 2, 3, \ldots, N$ is represented by a triangular IFN $\{l_i, m_i, n_i; l'_i, m_i, n'_i\}$, then the intuitionistic fuzzy failure probability of PAND gate with these events can be defined as:

$$\begin{aligned} P_{IF-PAND}(t) &= PAND_{IF}\{\lambda_1, \lambda_2, \ldots, \lambda_N\} \\ &= \{l, m, n; l', m, n'\} \end{aligned} \tag{29}$$

where

$$l = \prod_{i=1}^{N} l_i \sum_{k=0}^{N}\left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j\neq k}}^{N}(u_k - u_j)}\right],$$

$$u_0 = 0 \text{ and } u_k = -\sum_{i=1}^{k} l_i \text{ for } k > 0 \tag{30}$$

$$m = \prod_{i=1}^{N} m_i \sum_{k=0}^{N}\left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j\neq k}}^{N}(u_k - u_j)}\right],$$

$$u_0 = 0 \text{ and } u_k = -\sum_{i=1}^{k} m_i \text{ for } k > 0 \tag{31}$$

$$n = \prod_{i=1}^{N} n_i \sum_{k=0}^{N}\left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j\neq k}}^{N}(u_k - u_j)}\right],$$

$$u_0 = 0 \text{ and } u_k = -\sum_{i=1}^{k} n_i \text{ for } k > 0 \tag{32}$$

$$l' = \prod_{i=1}^{N} l'_i \sum_{k=0}^{N}\left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j\neq k}}^{N}(u_k - u_j)}\right],$$

$$u_0 = 0 \text{ and } u_k = -\sum_{i=1}^{k} l'_i \text{ for } k > 0 \tag{33}$$

$$n' = \prod_{i=1}^{N} n'_i \sum_{k=0}^{N}\left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j\neq k}}^{N}(u_k - u_j)}\right],$$

$$u_0 = 0 \text{ and } u_k = -\sum_{i=1}^{k} n'_i \text{ for } k > 0 \tag{34}$$

4) INTUITIONISTIC FUZZY PROBABILITY OF POR GATE

If there are N input events $\{E_1, E_2, \ldots, E_N\}$ with intuitionistic fuzzy failure rates $\lambda_1, \lambda_2, \ldots, \lambda_n$ respectively and $\lambda_i, i = 1, 2, 3, \ldots, N$ is represented by a triangular IFN $\{l_i, m_i, n_i; l'_i, m_i, n'_i\}$, then the intuitionistic fuzzy failure probability of POR gate with these events can be defined as:

$$P_{IF-POR}(t) = POR_{IF}\{\lambda_1, \lambda_2, \ldots, \lambda_N\} = \{l, m, n; l', m, n'\} \tag{35}$$

where

$$l = \frac{l_1\left(1-\left(e^{-(\sum_{i=1}^{N} l_i)t}\right)\right)}{\sum_{i=1}^{N} l_i} \tag{36}$$

$$m = \frac{m_1\left(1-\left(e^{-(\sum_{i=1}^{N} m_i)t}\right)\right)}{\sum_{i=1}^{N} m_i} \tag{37}$$

$$n = \frac{n_1\left(1-\left(e^{-(\sum_{i=1}^{N} n_i)t}\right)\right)}{\sum_{i=1}^{N} n_i} \tag{38}$$

$$l' = \frac{l'_1\left(1-\left(e^{-(\sum_{i=1}^{N} l'_i)t}\right)\right)}{\sum_{i=1}^{N} l'_i} \tag{39}$$

$$n' = \frac{n'_1\left(1-\left(e^{-(\sum_{i=1}^{N} n'_i)t}\right)\right)}{\sum_{i=1}^{N} n'_i} \tag{40}$$

D. RELIABILITY QUANTIFICATION

In this final step of the framework, the probability of the occurrence of the TE of the TFT is computed and the criticality of the BEs is determined. Based on the TFT quantification approaches selected in the previous step, these computation processes will vary. In the PN-based method, the TE probability is determined by simulating the PN model. There are many tools available to simulate a PN model. In this paper, we used ORIS Tool [76] to simulate the PN model of a TFT. In the BN-based approach, a query is run on the BN model of the TFT to obtain the TE probability. For modelling and analysing a BN model of a TFT, we modified and used an open-source tool called JavaBayes [77]. For TE probability computation using the algebraic approach, the mathematical formulas provided in equations (27)-(40) are used together with IFNs. By using these formulas, the system unreliability is obtained as another triangular IFN.

Importance measures can be used to identify the critical component. It determines the various contributions of BEs to the occurrence probability of the TE, i.e., it can determine the change in the TE probability due to the change in BEs probability. The results of a criticality analysis can help the decision makers to improve the dependability of systems by taking necessary actions such as planning maintenance or upgrade. Different criticality analysis techniques such as the Birnbaum importance measure (BIM) and risk reduction worth (RRW) are widely used [2].

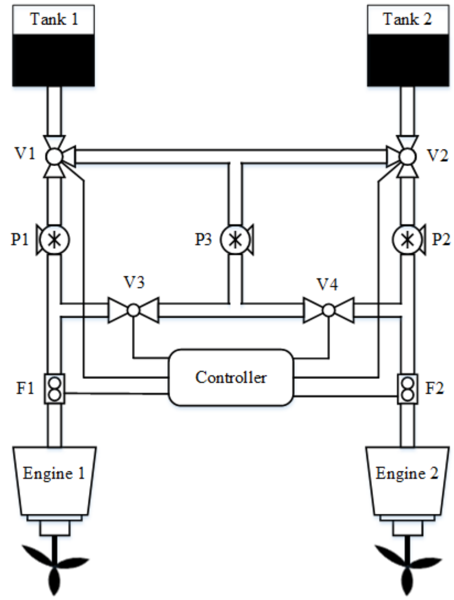

FIGURE 5. Fuel Distribution System of a ship [78].

Here, to rank the BEs in intuitionistic fuzzy TFT, the definition of fuzzy importance measure proposed in [45] is generalized and is defined as:

If $\tilde{P}_{T_{i=1}} = \{a_1, b_1, c_1; a'_1, b_1, c'_1\}$ and $\tilde{P}_{T_{i=0}} = \{a_2, b_2, c_2; a'_2, b_2, c'_2\}$ are two TIFNs representing the intuitionistic fuzzy failure possibility of the TE with the BE i fully unavailable and available, respectively, then the intuitionistic fuzzy importance measure (IFIM) for i^{th} BE is estimated as:

$$IFIM(BE_i) = \sqrt{\frac{(a'_1-a'_2)^2+(a_1-a_2)^2+2(b_1-b_2)^2+(c_1-c_2)^2+(c'_1-c'_2)^2}{2}} \tag{41}$$

IV. NUMERICAL EXAMPLE

To illustrate the application of the proposed framework, we use the TFT of a fuel distribution system shown in Fig.5. A detailed description of the system and its functional behaviour is available in [78]. This system consists of two tanks (Tank 1 and 2), four valves (V1, V2, V2, and V4), three pumps (P1, P2, and P3), two flowmeters (F1 and F2), a controller, and two engines (Engine 1 and 2). In the normal operating condition, Tank 1 and Tank 2 are responsible for supplying fuel to Engine 1 and 2, respectively, and the primary fuel flow paths are "*Tank* 1 $\rightarrow$ *V*1 $\rightarrow$ *P*1 $\rightarrow$ *F*1 $\rightarrow$ *Engine* 1" and "*Tank* 2 $\rightarrow$ *V*2 $\rightarrow$ *P*2 $\rightarrow$ *F*2 $\rightarrow$ *Engine* 2". In the case of failure, when the fuel flow through one or both of these primary paths is not possible, the controller can take action to restore the fuel flow by establishing secondary paths. For instance, if P1 fails then a secondary path can be formed as "*Tank* 1 $\rightarrow$ *V*1 $\rightarrow$ *P*3 $\rightarrow$ *V*3 $\rightarrow$ *F*1 $\rightarrow$ *Engine* 1" by activating P3 and V3 to provide fuel to Engine 1. In the same way, a secondary path "*Tank* 2 $\rightarrow$ *V*2 $\rightarrow$ *P*3 $\rightarrow$ *V*4 $\rightarrow$ *F*2 $\rightarrow$ *Engine* 2" can be

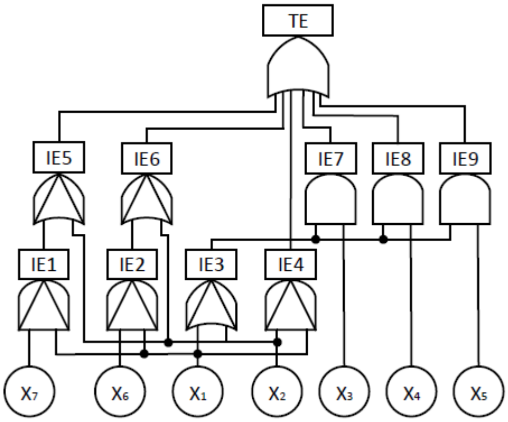

FIGURE 6. TFT of the fuel distribution system.

TABLE 3. BEs of the TFT in Fig. 6.

Basic Events	Description
X_1	Omission of fuel flow through P1 due to mechanical failure
X_2	Omission of fuel flow through P2 due to mechanical failure
X_3	Omission of fuel flow through P3 due to mechanical failure
X_4	Omission of fuel flow through V1 due to failure
X_5	Omission of fuel flow through V3 due to failure
X_6	Omission of output from F1 due to failure
X_7	Omission of control output from Controller due to failure

formed to continue supply to engine 2 if P2 fails. As pump P3 is present in both the secondary paths, it can replace either P1 or P3 at a time, not both. Therefore, if both P1 and P2 fail, then at least one of the engines will not get any fuel. By considering the fuel tanks as completely reliable, the TFT of Fig. 6 represents the causes of no fuel supply to Engine 1. As seen in the figure, the TFT contains seven basic events (represented as circles): $X_1, X_2, X_3, X_4, X_5, X_6$, and X_7. These basic events represent failure of different components of the fuel system (see Table 3). The TFT in Fig. 6 is analysed to obtain following six minimal cut sequences (MCSQs).

MCSQ #1: $X_1 \wr X_2 \,.\, X_3$
MCSQ #2: $X_1 \wr X_2 \,.\, X_4$
MCSQ #3: $X_1 \wr X_2 \,.\, X_5$
MCSQ #4: $X_6 \lhd X_1 \wr X_2$
MCSQ #5: $X_7 \lhd X_1 \wr X_2$
MCSQ #6: $X_2 \lhd X_1$

Using the Delphi survey, six independent experts were engaged to provide their opinions about the failure possibility of 7 BEs (step 1). To determine the occurrence possibility of a BE, the qualitative terms from table 2 were used and each expert was requested to provide his/her opinion as a linguistic term. For instance, in response to the question "how much do you believe that the BE will be in a failed state after time t?", an expert may provide his opinion as VH for failure possibility.

TABLE 4. Expert' opinions and corresponding aggregated IF probabilities of BEs.

Basic event	Experts' opinions						Aggregated IFNs	Defuzzified Crisp Probability
	E_1	E_2	E_3	E_4	E_5	E_6		
X_1	H	H	M	H	H	M	Possibility= (0.702, 0.783, 0.864; 0.679, 0.783, 0.886) Probability=(1.865E-2, 3.161E-2, 5.723E-2; 1.612E-2, 3.161E-2, 6.892E-2)	3.759E-2
X_2	H	H	M	H	H	M	Possibility= (0.702, 0.783, 0.864; 0.679, 0.783, 0.886) Probability=(1.865E-2, 3.161E-2, 5.723E-2; 1.612E-2, 3.161E-2, 6.892E-2)	3.759E-2
X_3	VH	H	L	RL	M	RH	Possibility=(0.553, 0.638, 0.719; 0.539, 0.638, 0.736) Probability=(7.187E-3, 1.245E-2, 2.078E-2; 6.544E-3, 1.245E-2, 2.318E-2)	1.379E-2
X_4	M	RL	RH	M	H	L	Possibility=(0.407, 0.522, 0.635; 0.384, 0.522, 0.660) Probability=(2.463E-3, 5.829E-3, 1.221E-2; 2.025E-3, 5.829E-3, 1.430E-2)	7.140E-3
X_5	M	RL	RL	H	H	RL	Possibility= (0.423, 0.522, 0.621; 0.400, 0.522, 0.643) Probability=(2.806E-3, 5.829E-3, 1.117E-2; 2.322E-3, 5.829E-3, 1.285E-2)	6.825E-3
X_6	RH	M	RL	M	RH	M	Possibility=(0.439, 0.568, 0.689; 0.418 , 0.568, 0.718) Probability=(3.184E-3, 7.937E-3, 1.718E-2; 2.695E-3, 7.937E-3, 2.065E-2)	9.993E-3
X_7	RL	M	L	RL	L	RL	Possibility=(0.165, 0.261, 0.356; 0.147, 0.261, 0.375) Probability=(1.121E-4, 5.556E-4, 1.572E-3; 7.331E-5, 5.556E-4, 1.870E-3)	7.945E-4

TABLE 5. Profile and weighting of selected experts.

No	Job field	Experience (years)	Education level	Confidence level	Weighing scores
Expert 1	Professor	17	PhD	Very high	0.324
Expert 2	Technician	15	BSc	Medium	0.125
Expert 3	Reliability and risk instructor	10	MSc	Medium	0.089
Expert 4	Safety analyst	18	BSc	Low	0.220
Expert 5	Assistant professor	14	PhD	High	0.143
Expert 6	System designer	7	MSc	Medium	0.098

TABLE 6. Aggregation calculations for the BE X_1.

Ex 1: (H)	(0.81, 0.87, 0.93; 0.79, 0.87, 0.95)				
Ex 2: (H)	(0.81, 0.87, 0.93; 0.79, 0.87, 0.95)				
Ex 3: (M)	(0.35, 0.50, 0.65; 0.32, 0.50, 0.68)				
Ex 4: (H)	(0.81, 0.87, 0.93; 0.79, 0.87, 0.95)				
Ex 5: (H)	(0.81, 0.87, 0.93; 0.79, 0.87, 0.95)				
Ex 6: (M)	(0.35, 0.50, 0.65; 0.32, 0.50, 0.68)				
S(Ex 1 & Ex 2)	1.000	$EV_1 = (0.81 + 0.79 + 0.93 + 0.95 + 4 \times 0.87)/8 = 0.87$			
S(Ex 1 & Ex 3)	0.575	$EV_2 = (0.81 + 0.79 + 0.93 + 0.95 + 4 \times 0.87)/8 = 0.87; EV_1 \geq EV_2 \rightarrow \frac{EV_1}{EV_2} = 1.000$			
S(Ex 1 & Ex 4)	1.000	S(Ex 2 & Ex 5)	1.000	S(Ex 4 & Ex 5)	1.000
S(Ex 1 & Ex 5)	1.000	S(Ex 2 & Ex 6)	0.575	S(Ex 4 & Ex 6)	0.575
S(Ex 1 & Ex 6)	0.575	S(Ex 3 & Ex 4)	0.575	S(Ex 5 & Ex 6)	0.575
S(Ex 2 & Ex 3)	0.575	S(Ex 3 & Ex 5)	0.575		
S(Ex 2 & Ex 4)	1.000	S(Ex 3 & Ex 6)	1.000		
AA(Ex 1)	0.830	$AA(E_u) = \frac{1}{m-1} \sum_{\substack{v=1 \\ v \neq u}}^{m} S_{uv} = \frac{(1.000 + 1.000 + 0.575 + 0.575 + 1.000)}{6 - 1} = 0.830$			
AA(Ex 2)	0.830				
AA(Ex 3)	0.660	AA(Ex 5)	0.830		
AA(Ex 4)	0.830	AA(Ex 6)	0.660		
RAD(Ex 1)	0.179	$RAD(E_u) = \frac{AA(E_u)}{\sum_{v=1}^{m} AA(E_v)} = \frac{0.830}{(0.830 + 0.830 + 0.830 + 0.830 + 0.660 + 0.660)} = 0.179$			
RAD(Ex 2)	0.179				
RAD(Ex 3)	0.142	RAD(Ex 5)	0.179		
RAD(Ex 4)	0.179	RAD(Ex 6)	0.142		
w_{Ex1}	0.252	$\alpha.BNP_{w_i}(E_i) + (1 - \alpha).RAD(E_i) = 0.5 \times 0.324 + 0.5 \times 0.179 = 0.252$			
w_{Ex2}	0.152				
w_{Ex3}	0.116	w_{Ex5}	0.161		
w_{Ex4}	0.200	w_{Ex6}	0.120		

Aggregation for $X_1 : \tilde{P}_j = \sum_{i=1}^{m} w_i \otimes \tilde{P}_{ij}$

$$= 0.252 \otimes (0.81, 0.87, 0.93; 0.79, 0.87, 0.95) \oplus 0.152 \otimes (0.81, 0.87, 0.93; 0.79, 0.87, 0.95) \oplus 0.116 \otimes (0.35, 0.50, 0.65; 0.32, 0.50, 0.68) \oplus 0.200 \otimes (0.81, 0.87, 0.93; 0.79, 0.87, 0.95) \oplus 0.161 \otimes (0.81, 0.87, 0.93; 0.79, 0.87, 0.95) \oplus 0.120 \otimes (0.35, 0.50, 0.65; 0.32, 0.50, 0.68) = (0.702, 0.783, 0.864; 0.679, 0.783, 0.886)$$

After collecting all opinions via the Delphi method, FAHP (step 2) is applied to obtain specific weightings for the employed experts. Several criteria based on literature and current case study conditions are considered, including job field, experience, education level, and confidence level. The confidence level is one of our new additions as part of the our IFS framework. Therefore, it is necessary to recognize the confidence level of the employed experts for further analysis. The profiles of the selected experts and their weightings are reported in Table 5. The qualitative terms based on experts' opinions for each BE, their corresponding aggregated intuitionistic fuzzy possibilities, equivalent intuitionistic fuzzy probabilities and the defuzzified crisp probabilities are shown in Table 4.

Take the event X_1 (Omission of fuel flow through P1 due to mechanical failure) as an example. According to the definition of IFNs shown in Table 2, the linguistic terms provided by six experts fall into 'H', 'H', 'M', 'H', 'H', and 'M' categories. The integrated IFNs attained and details of calculation are reported in table 6 (step3).

The data shown in table 4 are used in the TFT quantification approaches to evaluate the TE probability. Without loss of generality, and for the purposes of comparison, the probability of the TE is calculated for 10000 hours using algebraic,

TABLE 7. Intuitionistic fuzzy failure probabilities of MCSQs for t=10000 hours.

MCSQs	Intuitionistic fuzzy failure probabilities					
	a_{i1}	a_{i2}	a_{i3}	a'_{i1}	a_{i2}	a'_{i3}
$X_1 \wr X_2 . X_3$	1.328E-4	3.873E-4	1.155E-3	1.046E-4	3.873E-4	1.543E-3
$X_1 \wr X_2 . X_4$	4.545E-5	1.814E-4	6.788E-4	3.230E-5	1.814E-4	9.516E-4
$X_1 \wr X_2 . X_5$	5.192E-5	1.814E-4	6.210E-4	3.710E-5	1.814E-4	8.551E-4
$X_6 \lhd X_1 \wr X_2$	2.930E-5	1.230E-4	4.741E-4	2.154E-5	1.230E-4	6.810E-4
$X_7 \lhd X_1 \wr X_2$	1.013E-6	8.664E-6	4.321E-5	5.581E-7	8.664E-6	6.147E-5
$X_2 \lhd X_1$	1.739E-4	4.996E-4	1.638E-3	1.299E-4	4.996E-4	2.375E-3

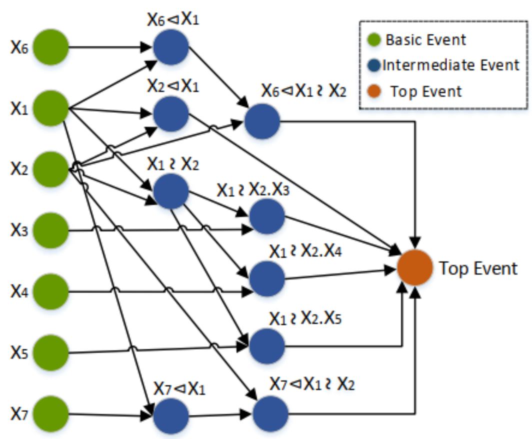

FIGURE 7. BN model of the TFT of Fig. 6.

TABLE 8. Comparison of system unreliability estimated by other approaches with the unreliability estimated by the proposed approach.

Approaches	Unreliability	
Algebraic	Based on IFNs	2.484E-3
	Based on classical FNs	2.139E-3
	Based on crisp values	1.927E-3
Petri Net Based	1.222E-3	
Bayesian Network Based	with 3 intervals	1.604E-3
	with 4 intervals	1.681E-3
	with 5 intervals	1.730E-3
	with 6 intervals	1.757E-3
	with 7 intervals	1.791E-3
	with 8 intervals	1.800E-3
	with 9 intervals	1.814E-3
	with 10 intervals	1.826E-3

PN, and BN-based approaches. Using the intuitionistic fuzzy probabilities of the BEs and the intuitionistic fuzzy operators defined for TFT gates in Section III-C, the intuitionistic failure probabilities of the MCSQs are obtained, and shown in table 7. Using the equation (28) for intuitionistic fuzzy probability of the OR gate and the intuitionistic fuzzy data from table 7, the intuitionistic fuzzy probability of the TE of the TFT of Fig. 6 is calculated. The values obtained are: (4.344E-4, 1.381E-3, 4.602E-3; 3.260E-4, 1.381E-3, 6.451E-3). Using equation (26), the IF possibility of the top event is defuzzified and the value obtained was 2.484E-3. In addition to that, we used the crisp values from table 7 and formulas presented in Section II-B.1 to calculate the TE probability: 1.927E-3.

To compare the results estimated by the IFNs-based algebraic approach, the TFT was quantified using both PN and BN-based approaches. Figs. 8 and 7 show the PN and BN models of the TFT of Fig. 6, respectively. In the PN model, the timed transitions (white rectangular bars named in the form $X_i.FR$) are annotated according to the crisp failure probability of the BEs shown in the last column of the Table 4. Similarly, the crisp failure data for the BEs is used to define prior probabilities of the root nodes (represented as green circles) of the BN model. The TE probabilities obtained by these approaches are reported in the table 8. Note that, in the algebraic approach, using the triangular intuitionistic fuzzy numbers and classical fuzzy numbers, the TE probability is estimated as another triangular intuitionistic fuzzy number and a classical fuzzy number, respectively. On the other hand, the crisp values are used in algebraic, PN and BN-based approaches to estimate the TE probability as a crisp value. Therefore, to compare the results estimated by the fuzzy approaches with the results estimated by the crisp value-based approaches, we defuzzified the intuitionistic fuzzy failure probabilities and classical fuzzy failure probabilities of the system. A comparison of the results of different approaches is shown in table 8. From table 8, it can be seen that the TE probabilities estimated by different approaches are close to each other. Although there exist small differences between the TE probabilities estimated by the different approaches, the important thing to note is that the use of intuitionistic fuzzy set theory with expert elicitation enables the analysis in cases where the available information about system components is insufficient to define their failure rate using classical fuzzy sets.

Based on the intuitionistic fuzzy possibility of the TE of the TFT, the IFIMs of the BEs are determined using equation (41). In addition, the fuzzy importance measure of the BEs are also calculated according to the process described in [45]. The BEs are ranked according to their criticality and the results are reported in Table 9. As can be seen from the table, both the intuitionistic fuzzy set based approach and the classical fuzzy set based approach ranked the basic events in the same order.

In summary, each of the approaches for quantitative analysis of TFTs have their own strength and weaknesses.

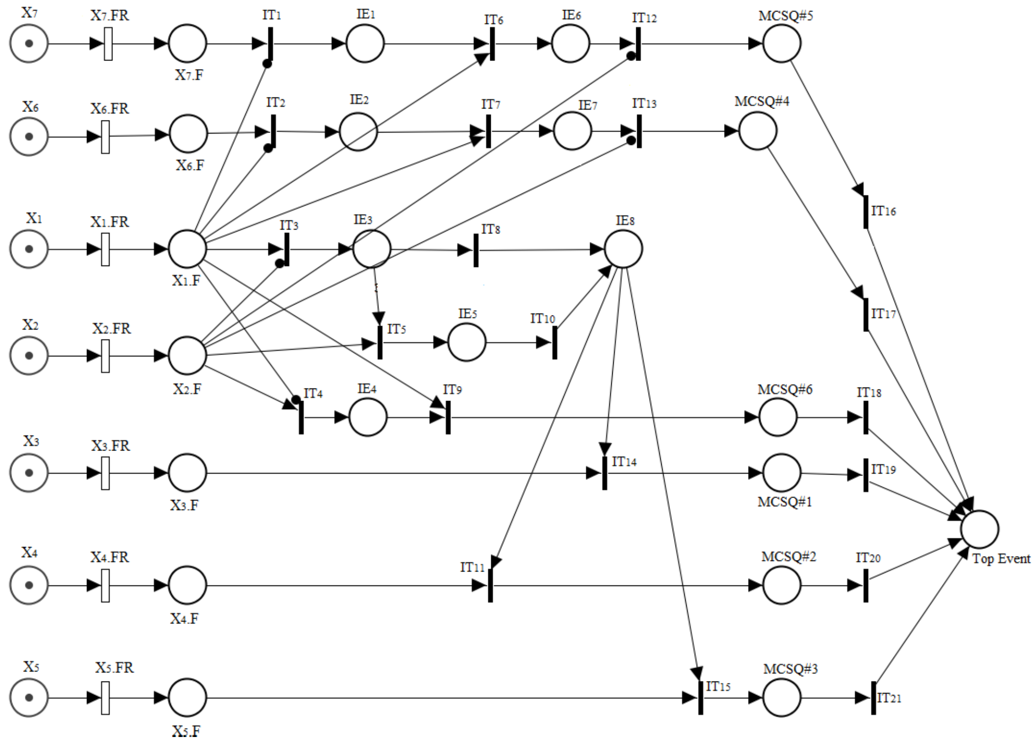

FIGURE 8. PN model of the TFT of Fig. 6.

TABLE 9. Ranking of BEs based on criticality.

Basic event	IF-TFT Approach		Fuzzy TFT Approach [45]	
	$\mathrm{IFIM(BE_i)}$	Rank	$\mathrm{FIM(BE_i)}$	Rank
X_1	1.594	1	1.597	1
X_2	0.077	7	0.079	7
X_3	0.301	2	0.296	2
X_4	0.257	4	0.253	4
X_5	0.260	3	0.255	3
X_6	0.202	5	0.195	5
X_7	0.187	6	0.181	6

However, most quantitative TFT analysis approaches depend on the availability of the precise failure data, and if this is not available, those approaches cannot be used. By contrast, the fuzzy set theory-based approach to TFT analysis enables us to evaluate system reliability in the absence of concrete failure data. Furthermore, the IFS theory-based method allows us to describe scenarios where knowledge about the fuzziness of quantitative data is subject to varying levels of confidence. For this reason, the combination of IFS theory with expert elicitation as proposed in this paper should provide more flexibility to the analysts in terms of expressing failure data as fuzzy numbers. The proposed framework is therefore more suitable for the quantification of TFTs when precise failure data are unavailable or insufficient.

V. CONCLUSION

In this paper, we presented a framework for temporal FTA to evaluate system reliability using intuitionistic fuzzy set theory where failure data for system components are unavailable or insufficient. The framework combined IFS theory and expert judgement to facilitate the collection of uncertain failure data. Intuitionistic fuzzy operators are defined to quantify the logic gates in a temporal fault tree where failure data of system components are represented by TIFNs. The primary difference of using IFSs over classical fuzzy sets is that IFSs separate the positive and negative evidence for membership of an element in the set. The efficacy of the proposed framework has been illustrated via a numerical example. The experiments show that the intuitionistic fuzzy TFT analysis approach provides a useful means of dynamic reliability evaluation when the fuzziness in the failure data cannot be expressed with high confidence.

In the future, we plan to explore the effects of different choices of membership functions, non-membership functions, and expert opinions on the system reliability approximated by the IFS-based TFT analysis approach. In addition, in the current study, the value of relaxation factor is assumed to be 0.5. As the amount of relaxation may have significant influence on the final results, it is imperative to apply a sensitivity analysis by varying the value of relaxation factor from 0 to 1 to understand the behaviour of the results. Indeed, it can lead to further investigation on how decision-makers can consider a viable and appropriate relaxation factor for their system under study.

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