School of Electrical Engineering Computing and Mathematical Sciences

Modeling, Control and Optimisation of Hybrid Systems in a Manufacturing Setting

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Abstract

This thesis comprises a body of work that investigates the performance of industrial hybrid manufacturing systems. We considered a shared resources manufacturing system, where products/customers compete for best service in each server in unprecedented fourth industrial revolution (Industries 4.0). To counterbalance the measure, we developed a mathematical logarithmic, exponential smoothing algorithm that balances the trade-off cost between product quality and completion time.

The result obtained is extended to model N-stage manufacturing and (re)manufacturing systems. We further investigated the optimal production rate and inventory level by deploying Hamiltonian equations and Markov Decision Process. Due to the stochastic nature of a manufacturing process, we adopted linear programming and converted it to a discrete-time Markov process to regulate the inventory level that minimises the system cost. For a decision-making process, we used a randomised Markov policy to select the best possible inventory level with respect to each manufacturing state.

Similarly,to remain in control we introduced a hybrid sliding mode controller to stabilise the manufacturing system. Furthermore, for an effective, flexible manufacturing system, we proposed sufficient conditions which are satisfied by our controller. The designed controller strategy helps to produce various products promptly to keep up with the demands and shorten the delay in the hybrid manufacturing system.

One of the aspects of our study in alignment with today Industrie 4.0, is the development of manufacturing firms equipped, with controllers that aid manufactures and engineers on how to run a thriving manufacturing industry. Therefore work developed in this framework is one step closer to the development of self-regulated, self optimised manufacturing systems.

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List of Publication

This thesis contains work that has been published and/or prepared for publication. Approval from the respective authorities has been granted and attached in Appendix.

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Chapter 1

Introduction

1.1 Overview of Manufacturing Systems

The manufacturing industry has developed rapidly over the past two centuries, due to the development of new technologies on the Fourth Industrial Revolution (Industries 4.0). The revolution is transforming the manufacturing industry at an unprecedented pace and many new methods and technologies emerge such as artificial intelligent in hybrid systems, Internet of Things (IoT), robotics and automation.

The industries 4.0 came with an imminent need to upgrade our resources and knowledge on how to operate manufacturing systems. As a result, the mismatch of human labour and the skills required for an open job manufacturing has led to the automation of the production industry. However, with the unprecedented technology growth [26], there is a need to close the gap in analytic outcome and production management of manufacturing systems.

It is evident that modern technology promotes job creation, from a recent report of world economic forum [14], [15] stated that algorithm and machine applications would break out from 29% in 2018 to 42% in 2022. The new technological flow and automation application and robots have played an essential role in bridging the gap labour in manufacturing. While the effort for continuous improvement led to the search for new tools and skills, according to [14], approximately 2.4 million manufacturing jobs might be un-filled between 2018 and 2028, while 60% of these positions were vacant in 2018.

According to [15] the pace of new job position is expected to accelerate at an estimate of 65% with a new workforce working completely in new jobs that has never existed. To keep up with the pace in such evolving employment environment, we need to work tirelessly to develop new tools and methods that are efficient and robust in today fourth industrial revolution. We have observed that, advanced manufacturing,robotics, artificial intelligence and 3D printing application came with a new wave of productivity and flexibility in work environment. And some of this applications capabilities have not been fully harnessed.

Manufacturing systems modeling plays a vital role in understanding the impact of decision making on the value function of the manufacturing model. While the complexity of manufacturing activities has grown over the years, and currently the manufacturing system is facing new emerging technologies, designed to address manufacturing problems. Therefore, there is a need to know and understand the new implementation and reduce costs to achieve quality and efficiency for continuous improvement of manufacturing systems.

Product quality is the key to most of the manufacturing firms to remain competitive in today open market. The trade-off between the manufacturing cost and the quality impacts manufacturing strategic decision making and influences the manufacturing system design at an early stage and its future development. Manufacturing system design modeling is the most critical and difficult task to attain due to the complexity of manufacturing systems dynamics.

Inventory plays an essential role in logistics and supply chain management. The right quantity at the right time reduces the manufacturing cost dramatically. The inventory provides support during product shortages and hence reduces lead time. Nevertheless, a manufacturing catastrophe is imminent with poor inventory management, and if the demand rate is greater than the production rate, then the manufacturing system will experience shortages, bottlenecks and missing due dates [32].

Uncontrolled shared manufacturing resources cause system imbalances and bottlenecks, system delays, long lead times of raw materials/products, missing due dates and consequently lead to a manufacturing trade-off. However, with the emerge of industry 4.0 in the area of hybrid systems, we can design controllers that mitigate this problem. With a new tool developed in this area, we can design stable manufacturing systems that can produce high-quality products and meet customer demands on time.

As a result, unprecedented control over the entire manufacturing system has become imminent. So the aim of optimal control and a new tool developed is primarily to gain insight on making production system faster, and aid manufacturers to make an informed decision, on operations and implementation of lean tools, and other optimisation methods. However, the industries 4.0 is more customer-oriented and is in line with socialization, with controls in handy and easily accessible remotely. The system is integrated with hybrid controls, cloud computing, mass data, 3D printing, robotics and automation, see Figure 1.1 for better insight on current manufacturing configuration and layout. It is found that for efficient, fast and safe production, a system is incorporated with automatic sensing, automatic identification and automatic control setting, with inventory and quality control, that enable self adjusting [12]. The need to achieve this level of control has stimulated the study of modeling, control and optimisation of the hybrid system in the manufacturing environment, which is the title of this thesis and has received little attention in the literature.

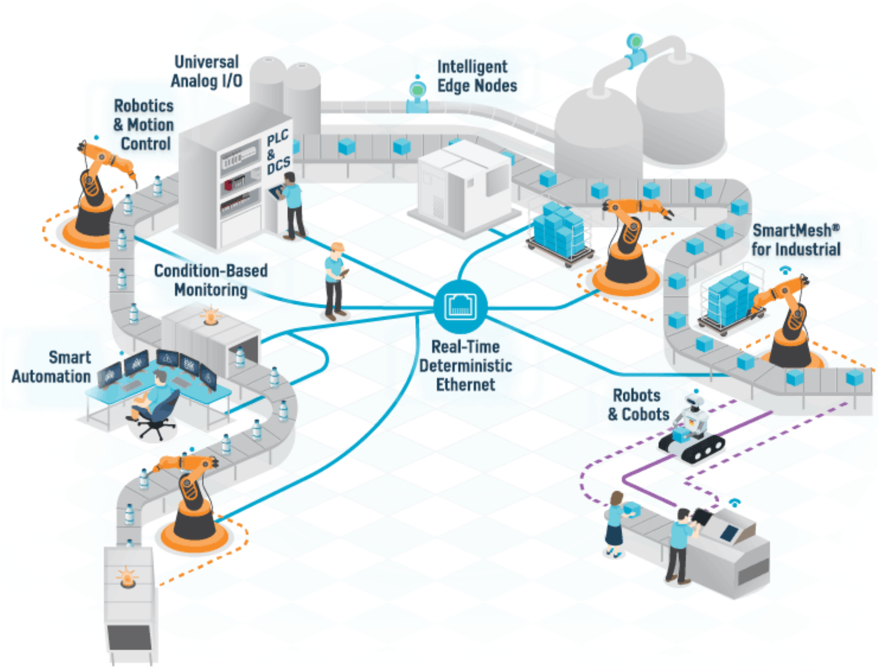

Figure 1.1: Modern manufacturing system configuration [13]

The digitization era has resulted in most of hybrid manufacturing systems to have a high level of automation. The term automation, refers to a single task, or process that is either discrete, continuous or both to be processed with minimal human interference. For instance, see the problem of filling in a two-tank system in Figure 1.2. The tank system is a typical hybrid system, with continuous dynamics to measure tank level, and discrete dynamics to regulate flow. [19], used sliding mode controller to control the flow and while in [18], used PI or PID controllers to control water level. For the system to be automatic, an artificial intelligent embedded with sensors and hybrid controllers is required to regulate the flow rate, fluid level in each tank system. Therefore automation has tremendous benefits on process improvement and product quality.

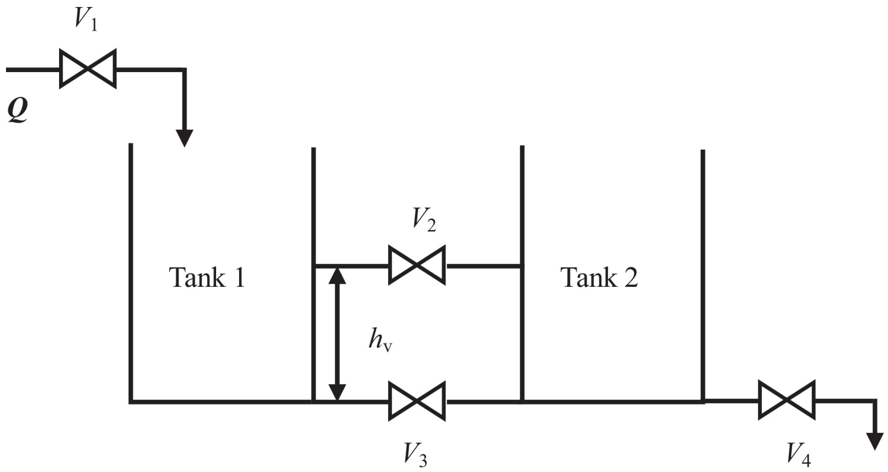

Figure 1.2: Two water tank system [19]

The water tank hybrid system consist of two tanks. The liquid in Tank 1 comes from outside source through valve V_1. Water can flow from Tank 1 to Tank 2 using either valve V_2 or V_3. However, the two tank can be filled at the same level if valve V_3 is opened, otherwise Tank 2 water can be regulated through valve V_2 once water reached height h_v. The outflow of water is controlled by valve V_4.Therefore, both valve can take the states on/off and as results we could have four possible discrete modes of hybrid system.

Optimisation of hybrid manufacturing systems is a link to smart manufacturing systems controlled by artificial intelligence. And this has resulted in a flexible system with self optimized process, that can adopt any manufacturing mode and condition at a given time. Due to optimisation capabilities, smart manufacturing systems are more reliable with predictable production capacity to meet customer demands on time. And they are able to increase production up-time and efficiency, with minimal human interaction, and thus reduce human error and minimize the cost of quality and production. Therefore, optimal hybrid manufacturing systems are proactive and agile, with the help of embedded sensors and controllers that can predict anomalies and product quality earlier. Flexibility and adaptability to scheduling and other manufacturing operations make hybrid manufacturing systems to be unprecedented in today industrial revolution 4.0 [11].

The flexibility and adaptability of smart hybrid manufacturing systems theory are still at an infant stage [27], as the self-organized systems and self optimized process may lead to unexpected results as we do not have matured mathematical models and algorithms for this process. And to keep up with the pace, we need to deduce self optimized dynamical mathematical equations and appropriate control methods [12]. Other methods available according to our knowledge for this framework include model checking and hybrid verification [20], [21]. Therefore our work is one of few works that is a tapping stone for developing controllers that control hybrid systems with minimal human interference in a manufacturing setting.

As an example, the hybrid manufacturing system, especially in the process industry, is made up of hierarchical structure, that consists of several functional layers that are tied to production decision making. Two of these layers are vital to our work, which is cash flow and material flow. In the layer of cash flow, the current manufacturing modeling does not support a supply chain distribution that matches market demand and for material flow layer the industry is not well equipped with decision-making mechanism that responds quickly to market changes or tuned to self balancing [25].

The hybrid manufacturing system is a key to the economic boom of 21^{1st} century industrial revolution 4.0. The flexibility control that is embedded in a hybrid dynamical system allows the interaction of continuous and discrete dynamics and this has resulted in a hybrid manufacturing systems that are competitive in nature. This result is more evident in the reduction of lead time, completion time and in the enhancement of product quality. The use of machine learning algorithms, has enabled the application of intelligent control, sliding mode control and impulsive switch control to stabilize manufacturing systems.

The earlier work on hybrid manufacturing system dynamic modeling and control is found in [5], [6], [94]. While in [94] a hybrid framework that involves a combination of time-driven and event-driven manufacturing systems dynamics were investigated. With the application of Bezier approximation technique, to smooth non-differentiable hybrid manufacturing systems, to find a balance between job tardiness and product quality.

Similarly, an integrated manufacturing system with traceable quality and cost-effective methods represents an opportunity to provide clear control over the implementation of the production line. Moreover, factory efficiency can be easily achieved through integrated planning and proper inventory management, which takes a long process to be implemented. Consequently, the cost of manufacturing can be reduced by considering the following manufacturing areas in Figure 1.3.

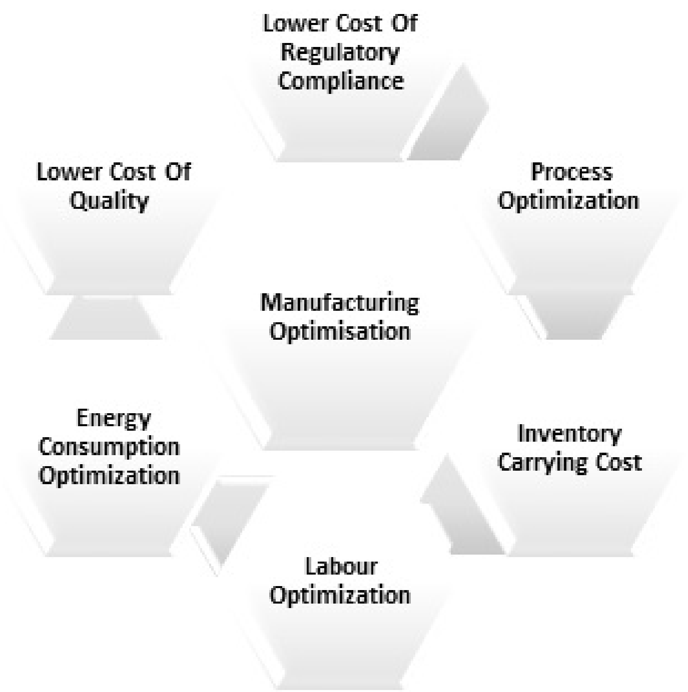

Figure 1.3: Manufacturing optimization areas

This thesis focuses more on issues related to process improvement and manufacturing system modeling, lower cost of quality and lower inventory cost. These measures aid in identification of stable manufacturing systems, and are able to establish a mechanism to prioritize manufacturing activities geared to customer satisfaction. Similarly with known factory design, production engineers can improve the efficiency of the plant with different manufacturing environments and product renewal periods. Therefore satisfying customers demands on time plays a crucial rule in making production decision.

1.2 Modeling of Hybrid Systems

Modeling is the concept of representing or imitating the function of a system. Therefore, the modeling of hybrid systems is the method of capturing and showing how continuous and discrete behavior are composed. Here mathematical models are used to describe the manufacturing hybrid system. The problem under manufacturing optimisation are resource contention, stability, and hybrid dynamical models are used to show the dynamics of continuous evolution and discrete events [24].

For a good representation of a manufacturing framework, a modeling language that is descriptive, composable and abstractable is vital for use, to demonstrate a hybrid behavior effectively. In addition, the model should capture both continuous and discrete state evolution over time [23]. Stochastic time state automaton constitutes of the above description and poses the properties of a hybrid automaton with rich mathematical language, and simple mathematical formalism and notation.

Petri nets and queuing models are popular manufacturing models and have been discussed heavily in the literature, and for further details for Petri nets and hybrid automata, see [22]. Petri nets use graphical features to show how the system interacts while queuing models use system performance measures. These models have the capability of capturing discrete and continuous behaviours that describe state transition mechanism in the state trajectory path. And it is observed that, during transition parts has to wait in the buffer while the machine is busy see Figure 1.4.

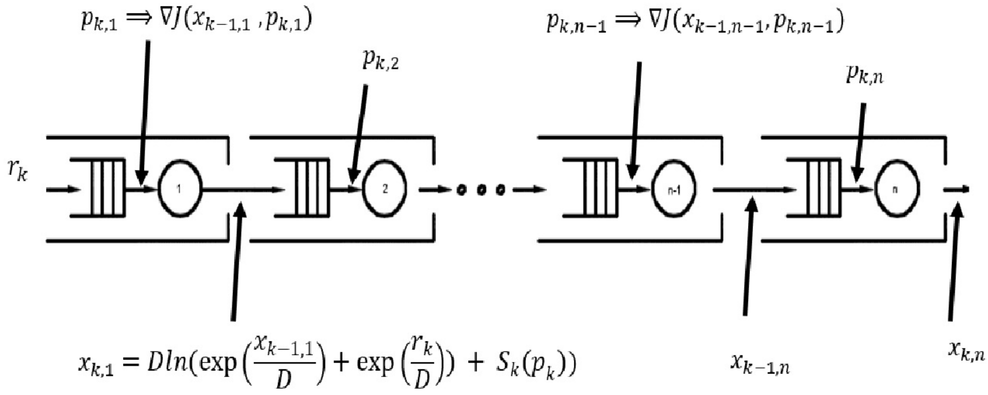

Figure 1.4: N-stage manufacturing process

$P_{k,1}$ is the processing of job k, in machine 1. For series configuration all jobs are processed in each machine in a sequentially manner.The N-stage model consist of several subsystems of single servers. The job arrival time r_k is scheduled and once the job arrive in the server, it is process according to first in first out service. But when the server is busy the job as to wait in the buffer until the server or machines becomes available. Each job is processed according to the optimal policy depicted by the algorithm developed in Chapter 3. Therefore each machine in N-stage process each job to minimize the total completion time x with respect to maximum processing time of each job to attain the best possible quality.

1.2.1 Control of Manufacturing Systems

Although flexibility is an essential problem in manufacturing uncertain environments. The striking balance between product quality and production cost is imperative to stay on the brink of competition in today's industry. The goal can be achieved through a manufacturing strategy designed to reduce production costs and ultimately reduces customer prices. However, this kind of work should be adopted more to ensure the effectiveness of production system. Moreover, it is not supposed to be limited to trade-off cost but examination of each method optimal to determine the most cost-effective approach that reaches the ultimate goal of manufacturing firms [17].

The work on integrated control of the product inventory is discussed in the literature review and will be further discussed in Chapter 2 and Chapter 4. While in the recent literature, different improvement methods were discussed with different control mechanisms. For instance, a refined, improved Hamilton Jacobi Bellman function was used in [4] to achieve the best ideal value function of controlled production inventory system. Similarly, re-manufacturing systems have been studied in [2] with an optimal control policy proposal for a multiple point hedging policy. Hedging policy support idea of keeping zero inventory.

The hybrid state trajectory path is coupled with N-stages of several jobs, which compete for the best service in common share resources. Therefore, the problem under consideration in this study is a multistage problem where each stage is associated with one-step cost and coupling all the jobs together to give the following cost in equation (1.1)-(1.4). We followed [81], [82], [83], [84] for optimal control problems in manufacturing settings. And we modeled single stage and N-stage manufacturing systems as follows.

1.2.2 Single stage manufacturing

$$\min_{p_1,\ldots,p_i} J_a(x,p) = \sum_{i=1}^{n} L_i(x_i,p_i), \forall i = 1,\ldots,n \tag{1.1}$$

$$x_i \in R^+, \quad p_i \in R^+,$$

subject to

$$x_i = f(x_{i-1},p_i,t) = \max(x_{i-1},r_i) + S_i(p_i) \tag{1.2}$$

$$Z_i = g_i(z_i,p_i,t) \tag{1.3}$$

1.2.3 N-stage manufacturing

$$\min_{p_1,\cdots,p_n} J_a(x,p) = \sum_{i=1}^{n}\sum_{j}^{N} L_{i,j}(x_{(i,j)},p_{(i,j)}), \forall i=1,\ldots,n; j=1,\ldots,N \tag{1.4}$$

$$x_i \in R^+, \quad p_i \in R^+,$$

subject to

$$x_{i,j} = f(x_{i-1,j},p_{i,j},t) = max(x_{i-1,j},r_i) + S_{i,j}(p_{i,j}) \tag{1.5}$$

$$Z_{i,j} = g_{i,j}\left(z_{i,j},p_{i,j},t\right)$$

where x and r is the completion time and arrival time respectively, with $S(p)$ the service time and Z is the physical state of the product.

1.2.4 Stability and stabilization of hybrid systems

The hybrid manufacturing systems composed of both manufacturing and (re)-manufacturing with n servers are classified as a switched system. A switched system is a dynamical system with several subsystems that can be described by differential or difference equations. Similarly, if the dynamics of the subsystems are state-dependent or time-independent, we then term the system a hybrid dynamical system with continuous and discrete dynamics.

A common Lyapunov function is employed to ensure the stability of the switched system under an arbitrary switching rule [8]. The existence of a common Lyapunov function is an important problem of control theory for stability analysis. Stable hybrid manufacturing systems have Lyapunov function properties. Both linear and non-linear systems can be exerted to this technique for stability verification. While for the non-linear system a common quadratic Lyapunov function is more favorable. And the results are sufficient to conclude that the manufacturing system can be operated if asymptotic stability of the system exists. However, this condition is not necessary but sufficient for stability criterion for hybrid dynamical systems. For example, consider a non-linear system given by

$$\begin{aligned} \dot{x} =& f(x,u) \\ \text{subject to}& \\ x =& \max\{x_{i-1},r_i\} + u \end{aligned} \tag{1.6}$$

where $r > 0$ is a job arrival time, $x \in \Phi$: state of the hybrid system, and $u \in R^n$ is the control input. $f : \Phi \subset R^n \to R^n$ is a continuous differentiable function, whereby the equilibrium point is defined by $x = 0$ such that $f(0, u) = 0$, and with $x = 0$ be an interior point in Φ. Hence Φ denote the domain around the equilibrium as shown in Figure 1.5. Therefore, we inferred that, the system stability is defined by the behavior of the system around the equilibrium point. While for systems with external signal (input), it can be an autonomous/non-autonomous system.

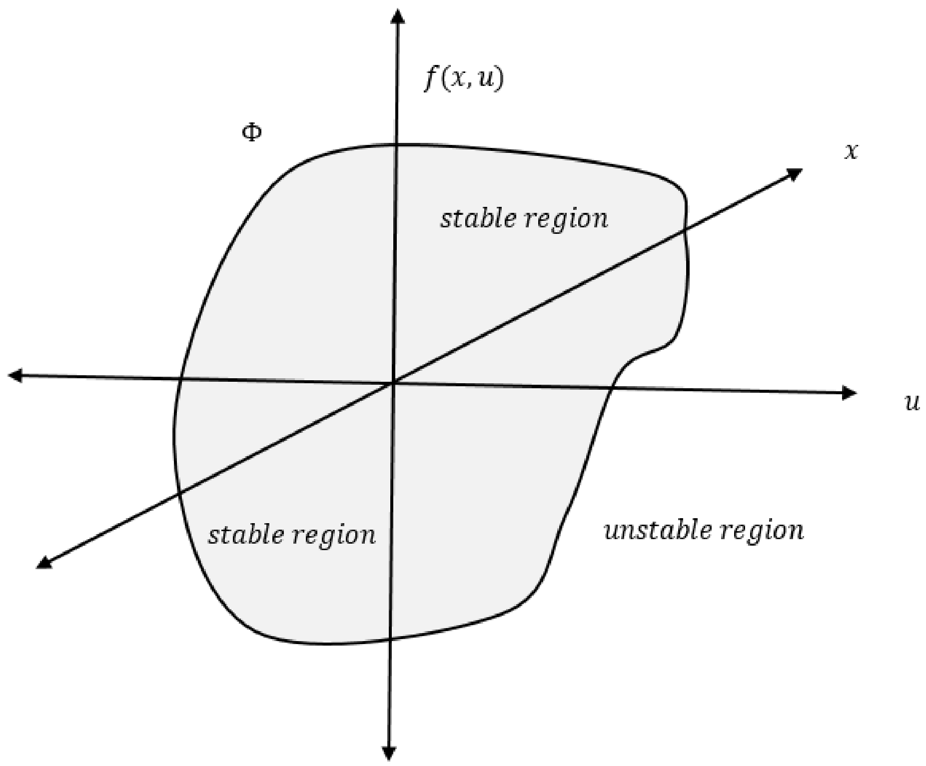

Figure 1.5: Stability overview

The structural properties of our manufacturing framework influence the system stability. Since our framework poses hybrid phenomena and the hybrid system has switching behaviors. If the system is not under control, undesirable results might be achieved such as 'Zeno" behavior, 'Live-lock' and 'Deadlock" [1], [23]. For stability of hybrid systems, we will exploit class of Lyapunov functions investigated in [24], such as

$$
\begin{aligned}
V(t) =& x^T(t) P_i x(t) \\
&\forall x \in R^n
\end{aligned}
\tag{1.7}
$$

An N-stage manufacturing process is shown in Figure 1.4 as Job k arrives in the system at time r_i and from the model, it can be inferred that the objective is to find controllers that minimises the cost and at the same time meets the desired target due dates.

1.3 Manufacturing Optimisation Methods

Controlled hybrid dynamical manufacturing systems can be easily optimised or improved. While uncontrolled dynamics cause the system to be unstable and result in long lead time, shortages and poor product quality. The primary hybrid optimisation goal is to develop efficient cost operations and controls, that can easily change the optimisation goal automatically with less human interference. For systems that are continuous and differentiable everywhere, gradient decent methods can be implemented for optimisation. And while for non-differentiable systems, approximation methods and smoothing aggregation techniques are considered.

Similarly, manufacturing systems with stochastic behavior, Markov and Maximum Principle with Bellman equations are more favorable. For a system with N-servers, impulsive and sliding mode control are more applicable to drive systems dynamics to the desired targets. Many of these techniques discussed do not admit an analytical solution and are known as numerical optimisation methods. For instance, a real-world complex manufacturing system does not admit analytical solution and are composed of many constraints, and makes numerical solutions to be handy and applicable in this framework.

1.4 Numerical Methods

1.4.1 Optimal Control

The aim of control optimization of the hybrid dynamical system is to maximize or minimize the value function. A hybrid manufacturing system consists of several subsystems or machines, and for instance, a manufactured part has to pass through one of the subsystems for processing. Most of the production machines are shared resources, and a part has to wait if the machine is busy, and thus it prolongs the completion time. Another challenge is meeting customer demand on time by producing as fast as possible without deteriorating product quality. To achieve the balance, or the optimal value function we model this as an optimisation problem.

In control theory, optimal control is defined or perceived as a control strategy, with a set of differential equations adjoined to find a control law. In most cases, the optimal solution is obtained if a necessary condition is met, using the principle of Pontryagin's maximum or by finding a sufficient condition, by solving the problem with the Hamilton Jacobi-Bellman equation.

An optimisation problem in manufacturing systems has similar attributions with optimisation problem in economics systems. And following constrained and unconstrained optimisation categories, the decision in each framework is subject to

Consumer Constraints:

- Affordability of the products.
- Quality of the products.
- Functionality of the products.

Firms Constraints:

- Decision on maximum inventory level kept due to unlimited production capacity.
- Decision on make or buy, as a results of the cost of production.

For a successful competitive manufacturing firm, a balance between consumer constraints and firm constraints is imperative. In the manner that the trade-off cost between the product quality and the product cost should be balanced for economic gains. We demonstrate this with an example in Equation (1.1) where a manufacturer has to minimize the production cost, by producing the good quality product at the shortest time possible.

$$\min_{u_1,\ldots,u_n} J\left(u,x\right) = \sum_{i=1}^{n} \left\{ \theta(u_i) + \varphi(x_i) \right\} \tag{1.8}$$

s. t.

$$x_i = \max(r_i, x_{i-1}) + s_i(u_i) \tag{1.9}$$

$$0 \leq x_i \leq r_{i+1}, s(u_i) \geq 0, i = 1, 2, \ldots, n$$

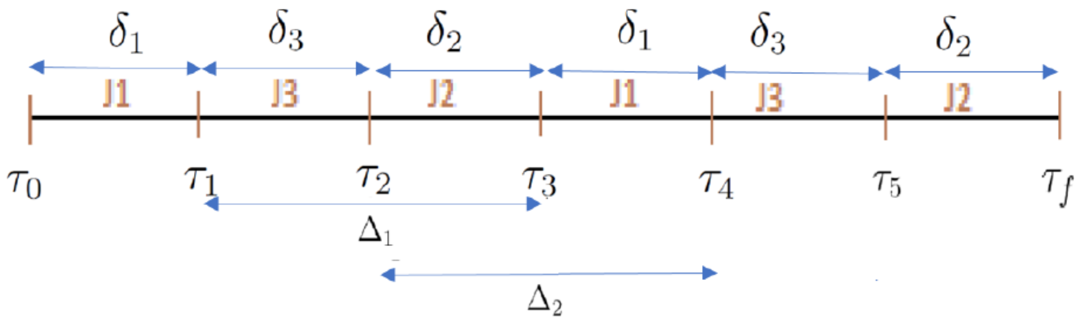

Figure 1.6: A hybrid manufacturing system with three modes [16].

Another typical hybrid system example can be adopted from [16], with three modes, and whereby the cost function is defined and minimised over a hybrid dynamical trajectory with a switching function among several subsystems shown in Figure 1.6. The time spent on each (mode) or subsystem is known as a minimum dwell time δ_i which can be stochastic, fixed or equidistant depending on problem under investigation for optimal control of hybrid system. And we have extended this example to hybrid manufacturing system in Chapter 4.

1.4.2 Smoothing Methods

Non-smoothness for a non convex optimization problems refer to state, where the objective function is non differentiable.The application of smoothing methods is most common in non-convex function on a convex set such as image restoration, optimal control and spherical approximation. And some typical problems that were studied for decades are not limited to complementary problems, variation inequalities,eigenvalues optimization and penalty methods with mathematical programming with non-smooth objective function.

The presence of 'max', 'min' in unconstrained optimisation problem is non-smooth. This problem is considered to be NP-hard, due to kinks in the objective function and the calculus of variation methods might fail to find the optimum solution. As a result smoothing approximation methods are used for optimisation. A smoothing method replaces a non-smooth, non-differentiable value function with a smooth differentiable function. We will further discuss the smoothing method in Chapter 3. For example, a non-smooth function in equation (1.10) can be approximated as in equation (1.11)

$$\max(m, a - x) \geq \begin{cases} m, & x > a \\ a - x^2, & t < x \leq a \\ x^2, & x \leq a \end{cases} \tag{1.10}$$

$$\max(m, a) = \log(\exp(m) + \exp(a)) \tag{1.11}$$

1.4.3 Markov Chain

Inventory control is the main aspect of manufacturing optimisation. And if the demand is known prior to the production schedule, then a hedging policy will be the most economical policy in the modern production system and with industrial revolution 4.0 . However, the market demand is stochastic in nature, and as a result, a Markov chain is the most useful technique to predict the future outcomes of inventory level to meet market demand.

A Markov chain consists of set of states, $\bar{S} = \bar{s}_1, \bar{s}_2, ...\bar{s}_n$. It is known that Markov process moves from one state to another with probability P_{ij}. These probabilities are known as transition probability and do not depend on past history to predict future outcomes. We depict that, Markov chain, rely on probabilistic automaton. Whereby, its probability distribution of state transition is represented by transition matrix. Such that, if the Markov chain has N possible states, then the transition matrix will be N $\times$ N. This topic will be further discussed in Chapter 4.

1.4.4 Sliding Mode Control

Due to the non-linearity of hybrid manufacturing dynamics, a sliding mode control is deployed to drive the system from a particular manifold into a sliding surface in the state space. A sliding mode control consists of two parts. The first part is concerned with driving the dynamics of the states to reach the manifold and the second part deals with keeping the trajectory on the sliding surface. These two parts have designed the specifications and attracted by attributing to the model condition, and this has resulted in a sliding mode control that has remarkable properties of robustness and accuracy.

The model mismatch in a manufacturing systems is handled by sliding mode control, whose primary function is to switch between different subsystems.And most of the system, that follows sliding mode control design turned to reject system disturbance . As a result, a sliding mode control is classified as a variable structure control. Chattering phenomena is more common characteristic in design phase, due to delay or infinite frequency of switching device. Therefore, when modeling this kind of system, engineers need to developed control structure that maneuver, or smooth motion that oscillates about the sliding manifold.
For a multi dimensional dynamics, a sliding surface is given by
$S_i = \{x(t) : s_i(x(t)) = 0\},\ x \in R^n, \quad i = 1, 2, ...m$

1.5 Motivation

The industrial revolution 4.0, with the need of self regulated, self optimised manufacturing systems, has motivated the undertake of this research, whereby a new control policy or tools is needed for decision making in industrial hybrid system. We were also motivated by problems that shared resources, and we were interested on how to balance the trade-off cost. For instance, manufacturing managers strive to produce products at the lowest possible cost. Where the cost criteria involve inventory build-up, product tardiness, control parameters and flow rates [83]. And from the literature it was observed that more work need to be done, to develop controllers that are inline with fourth industrial revolution. Therefore, this study aim to fill the gap as stated in [1], [85] by developing a general hybrid system model that can be applied in any manufacturing system to find an optimal policy to reduce the cost whereby a product quality and product tardiness are used as performance measures to balance manufacturing trade-off cost.

1.6 Significance of Research

Manufacturing systems and related industries are being transformed at a rapid pace. This change in the manufacturing setting is due to industries 4.0, where hybrid manufacturing systems control plays a vital role in optimisation. Moreover, it is significant to meet customer demand on time as a necessity to remain competitive in an open market. Control manufacturing systems have enormous benefits that include improved production stability, financial gains, and product quality.

General, the significance of our work liaise within a manufacturing firm as we have established that an adequate control manufacturing system is the backbone of the efficient production process, with quality and productivity being one of the main drives of the ingredients of a successful manufacturing system. Satisfactory product quality is essential as customers rely on a particular quality when purchasing a product. Therefore, proper system modeling and control strategy developed in our work will aid manufacturers to prevent product defects due to human error. Lastly, we have observed that our work give insight to managers, on how to run day-to-day production process, to make more efficient manufacturing systems, to reduce, manufacturing cost, lead times and manufacturing waste.

This study is more relevant, today as many manufacturing companies tend to keep less inventory to reduce manufacturing cost. Hence we have developed an algorithm or policies that can able to switch from one mode to another depending on the inventory level, and this is one of the step to develop self regulated, self optimised manufacturing systems.

1.7 Overview of the Thesis

Our work presents effective computational methods for modeling, and control of hybrid manufacturing systems, and it was observed that, hybrid manufacturing systems are linked to industrial revolution 4.0. For a self optimizing, self regulating manufacturing system, there is a need for handy controllers that stabilize manufacturing hybrid systems. And with system modeling, we have established and developed a mathematical algorithms for optimal control and optimisation of product cost and product quality trade-off. For this trade-off, we have proposed a single stage and a two stage manufacturing system where product quality, completion time are considered and stability have been further investigated for smooth operation of manufacturing system, in subsequent chapters.

Chapter 2 introduces previous work on manufacturing systems and some common optimisation methods. We have highlighted the trend of development of manufacturing systems from hand tool to smart factories. And we further emphasize that the success of hybrid manufacturing systems depends on production planning and control activities. And we have found that, for economic value gain, (re)manufacturing has economic leverage over pure production.

We further studied optimal control theory, optimisation methods and impulsive control optimisation of hybrid systems. Furthermore, we investigated deterministic approach, stochastic optimisation, decomposition of sub problems and relaxation of constraints to meet the objective function. After an insight on the literature review, a new algorithm was proposed to design and conceptualize the general hybrid system with N-servers in a manufacturing setting.

While in Chapter 3 we proposed a new control policies to solve the complex decision-making problems faced by industrial hybrid systems in a manufacturing environment where resources are shared. In such a setting, different dynamic systems communicate with each other and share common functions for seamless tasks. Entities that access shared resources compete for services. Interactions of industrial hybrid systems become more complex and require an appropriate controller for the best performance and the best possible service for each of the entities that access the system. To solve these challenges, we propose an optimal control policy to reduce the operating cost of the manufacturing system. Furthermore, we have developed a hybrid model with a new homogeneity algorithm to balance costs between the quality and the job delay, and to create an ideal service time for each function in the system.

The results obtained is further analyzed in N-stage manufacturing systems, and the service level and product quality is best achieved with proposed method. Therefore with, with the proposed method we have successfully overcame non-differentiable points, in the min-max plus algebra where critical jobs exist within the busy structure of the hybrid trajectory.

In Chapter 4 we establish a two-stage hybrid inventory re-manufacturing system model with Poisson demand rates. In the first stage, we formulate the uncertain demand rate as a Markov Decision Process (MDP). We further investigated the structural properties of the optimal policy and thus, the problem was converted to find optimal controllers for the discrete-time Markov process that regulates an optimal inventory level and minimize the system cost. In order to achieve cost-effective production-inventory system, we develop a smooth algorithm in the second stage and apply Hamilton-Jacobi-Bellman equations to determine the production rate of an optimal inventory trajectory of the hybrid system. Finally, we provide a numerical example to show how to obtain the optimal inventory trajectory and production rate.

In Chapter 5, we introduced a suitable controller that stabilizes the dynamical hybrid system in our framework. The goal of our framework is to find controllers for manufacturing and (re)manufacturing production mode. We developed theorems and sufficient conditions to guarantee the stability of a trivial solution. For a proper selection of a production mode, we employed a hybrid impulsive slide mode control. A sliding mode controller stabilizes the manufacturing hybrid system by confining systems dynamics within the selected manifold. While in the past, the stability of a manufacturing plant was achieved by introducing a moving assembly line. In a nutshell, the aim was to keep a minimum inventory at all times in an assembly line. Therefore the conceptual framework developed in this thesis mitigates the problem of backlog and inventory builds up within the production line. Moreover, the feasibility and viability of these techniques are shown with an example from inventory control and the technique proposed is in-line with hedging policy operation that supports the production of producing just-in-time.

Chapter 6, concludes work done in this thesis. And we have found that system feasibility is important parameter for stable manufacturing systems. And a master production plan can not be authorized if the systems is not stable, that means demand can not be meet with unstable manufacturing system. Therefore we have managed to develop a cost effective algorithms, and we have also presented some future work.

Finally, we have developed controllers that are able to mitigate unstable manufacturing system. And we have found that our work is still at infant stage, more mathematical algorithms in aligned with industries 4.0 needs to be developed for manufacturing firm to remain competitive. Moreover, with cloud computing there is a need to develop social manufacturing systems that are capable to communicate with each other with capabilities of self regulating and self optimizing.

1.8 Chapter Summary

In 21^{st} century, fourth industrial revolution is growing at unprecedented pace, with today technology embedded with sensors in hybrid manufacturing systems. We found that for a successful and competitive manufacturing firm, it needs to be incorporated with controllers that are handy to control inventory level and production rate to meet customer demands at unprecedented level. In an open market, the product quality and the cost are the main drives for manufacturing firms for turnovers and stability of manufacturing systems plays an important role on inventory optimisation and control.

The uncontrolled inventory level and the production rate lead to chaotic and unstable manufacturing systems. Consequently, a trade-off cost between product quality and production cost is inevitable for an unstable system. To mitigate it, we have proposed a smoothing algorithm that finds a balance between the product quality and the production cost. Markov chain and Bellman equations are applied to derive the decision on how to run a smooth optimal production system with optimal inventory level. Finally, we proposed, to run a manufacturing firm with a impulsive sliding mode control.

Chapter 2

Literature Review

2.1 Manufacturing Systems

Manufacturing revolution was brought by Britain in the earlier 18^{th} century from essential hand tools to powered machines for mass production. The need for mechanisation was invented to meet customer demands in a less costly manner. Manufacturing systems operations have changed drastically, and now with an open market. Internationalisation, with growing competition around the globe, has led to the emergence of the fourth industrial revolution.

The industries 4.0 concept allows automation and intelligent machines to communicate with each other. So this alone has brought massive changes in operations and manufacturing productivity. As a result, they are a tremendous increase in revenues across different firms that have already adopted the concept in their production system [10], [9]. While for full realisation of this ability, mathematical models are necessary to control manufacturing systems at an unprecedented level, embedded with sensors that respond to an uncertain manufacturing environment [46], [47], [48], and system interaction of discrete and continuous dynamics leads to the study of hybrid manufacturing systems.

The basic concept of hybrid manufacturing systems is to take advantage of the interaction of discrete and continuous dynamics to reduce production costs and lead times. Many factors lead to this optimal performance of hybrid systems are discussed in the literature [73]. Problems arising in manufacturing systems are not limited to the setup time reduction, variability reduction, in processing time and job arrival variability. Despite the effort to implement these factors, the success of hybrid manufacturing systems depends heavily on production planning and control activities. An appropriate planning and production control decision must be applied to concur benefits of hybrid systems.

Production systems involve the allocation of contention resources, human resources and inventory, to reduces product deadlines. While shared resource manufacturing systems may lead to waiting conditions, which leads to a dead-end if the task remains idle indefinitely. Without proper control, once a deadlock occurs, and in order for the system to continue, an external agency is needed to mitigate it and thus increases the completion time. Consequently, different control methods are needed in production planning, and new optimal controls policy are needed to address these challenges. While a decision has to be made on either to manufacture or re-manufacture, as shown in Figure 2.1.

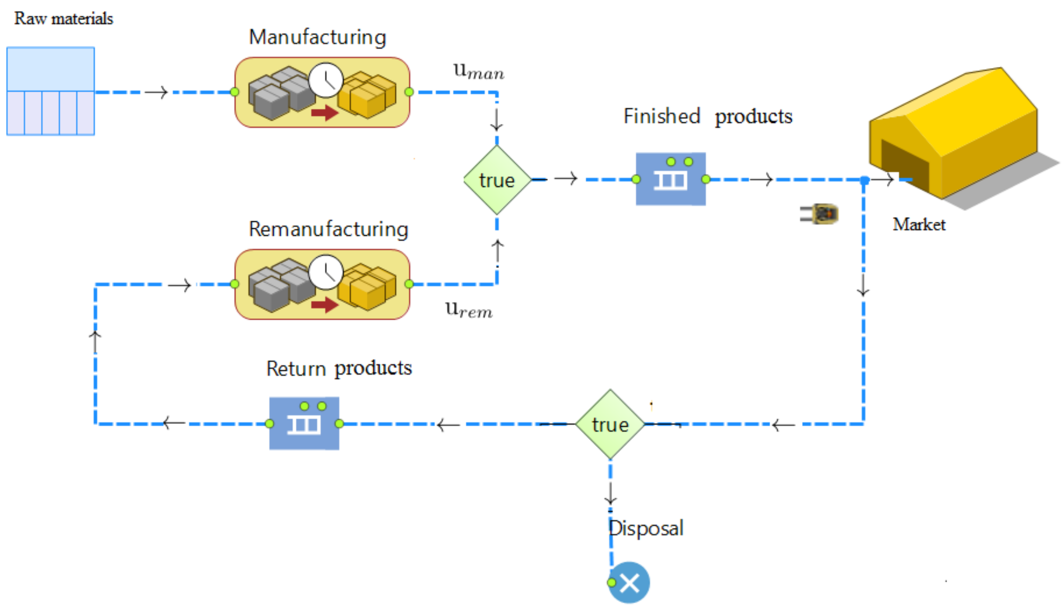

Figure 2.1: Manufacturing (re)manufacturing system model

2.2 Re-Manufacturing System

Other examples of hybrid systems found in the production system include a combination of recycled products, with manufactured parts. For feasibility and viability of this kind of re-manufacturing system, they should be a sufficient amount of recycled parts or material available [49]. And with today production trend, re-manufacturing is a common practice among different firms, because of its economic value.

Re-manufacturing is an industrial process whereby a product or component sold are returned for a reprocessing to a ”new” or ”better than new” condition, and whereby the satisfying quality is granted. Re-manufacturing experiences low cost compared to manufacturing as the customer has to pay less as a result of low automation in re-manufacturing, and small-batch sizes [51], [50].

There is a steady increase in the number of publications of re-manufacturing systems in the literature, with improved production models indicating the importance of this topic. While in the context of the hybrid system, reprocessing is complicated with the underlying assumption of uncertainty in the dynamics of production [52], [30].

The rapid instant change of inventory level, due to re-manufactured products is not continuous. Therefore, in this regard production rate is ramp up in both manufactured and re-manufactured modes to satisfy the demand. As a result, this articulates many threshold control behaviour; for instance, the production is ramped up to reduce the lead time, and to satisfy the demand on time. Similarly, in a chemical process, the rate of the reaction is increased to speed up the process, and in the pest control systems, pesticides are added to control pets. All this example have one thing in common, the control measure, and we define it as an impulse dependent feedback control.

The state dependents feedback control is achieved in both hybrid and semi-continuous dynamic system. Impulsive state-dependent feedback control is applicable to address this phenomenon as it has properties similar to the hybrid dynamical system, where the interaction of continuous and discrete dynamics is not inevitable. Impulsive control has severals application in other fields such as the health system for the pharmacological control system of tumours and diabetes [36]. For diabetic, an impulsive injection of insulin is administered to control type 1 and type 2 diabetes. Other combinations of the hybrid system are reported in 3D printing and CNC machine to form a single platform.

Cost optimisation of the hybrid system is reported in [31] by considering hybrid production systems that consist of re-manufactured products and develops an optimal operational strategy that focuses on manufacturing trades off between system uncertainty, capacity limitations, product quality and demand substitution. A Hessian matrix and multivariate optimisation methods were deployed to search for an optimal solution. And a balance between manufacturing and re-manufacturing systems was investigated. For example, [32] considered a hybrid manufacturing system with production switching mode for optimisation of the system and in [33], [34] multi-objective optimisation was used for decision making on return products.

An accurate analysis of production planning problems and optimal stock control for (re)manufacturing without failure is found in the literature. For instance, in [58], the analysis is performed at a fixed time, and optimum conditions are derived. In [59], the analysis is performed at a separate time and the importance of time limits for harmonisation of manufacturing and reprocessing policies is illustrated.

The open market and tight competition make decision-makers embark timelessly and endlessly on application and use of optimisation techniques. For outstanding performance and leverage over other firms, decision-makers integrate the application of economic models, simulations models with this optimisation technique with the objective to reduce manufacturing cost. For instance, proxy-based separate models are more useful in the analysis of inventory, and for better insight into operational planning [9] Also, Monte Carlo and random simulation are handy and are used in [55] to improve supply chain performance.

Manufacturing optimal policies have been presented as a critical policy in the inventory control literature and as a policy of hedging in the production control literature. An additional analysis of exemplary policies addressing arithmetic and previously neglected cases were presented in [60]. While in [75], authors have proposed a model of random dynamic control in a continuous-time to improve the global performance of a two-device system that is prone to failure in manufacturing and re-manufacturing, respectively. The optimisation requirements were developed in the form of the Hamilton-Jacob-Bellman equations (HJB) and for the design of coordinated manufacturing and reprocessing policies. Similarly, [4], [74] proposed a manufacturing / re-manufacturing policy that reduces the cost of inventory and implements random dynamic programming to find the best manufacturing /re-manufacturing values.

Proper design is important for effective control of the manufacturing process. For example, a proper scheduling and resource allocation can smooth the production flow. The waiting time and the cycle time reduces enormously if machines are not starved. While in contrary, the inventory rises, and machine bottlenecks and blockage are common for uncontrolled, unbalanced manufacturing system. System balancing constitutes to both on-line and off-line load balance. For recent work in this area see [35] and reference therein.

The problem of scheduling multi-component economic contracts for the production line that consists of manufacturing and (re)manufacturing were considered in [57], where a reliable algorithm is proposed to solve the problem. An example of a concrete industrial system using the same production line for both production and reprocessing was described in [56]. Further, a case study was conducted, with a company that manufactures auto parts, specifically, the water pump production line was analysed by expanding the economic scheduling and lottery technology to include the status of the products being returned. An ideal accounting solution was obtained for installation times and costs using the appropriate mixed linear programming technique. Using the same facilities for manufacturing and attractive processing seems to improve system flexibility.

Unprecedented modern manufacturing technologies have caused operational flexibility to be the heart of the design of hybrid systems. While this flexibility is achievable during the design strategy of horizon production system planning [113]. The design of production systems involves the allocation of contention resources, human resources and inventory, to reduce product deadlines. While shared resource manufacturing systems may lead to waiting conditions, which have a dead-end if the task remains idle indefinitely in the queue. Without proper control, once a deadlock occurs, and in order for the system to continue, an external agency is needed to mitigate it and thus increases completion time. Consequently, different control methods have been deployed in production planning and optimal controls are needed to address these challenges.

2.3 Hybrid Manufacturing System Optimization

With the new technological development in engineering and manufacturing, many research efforts have been devoted to the development of hybrid systems theory in recent years. The physical system, designed as a hybrid system, transcends the traditional system which consists of continuous and discrete dynamics. While the interaction between each of the dynamics is the reason for the study of hybrid systems optimisation.

optimisation of hybrid systems has remained a challenge over the years due to the combination of discrete dynamics with the interaction of infinite-dimensional continuous dynamics. Consequently, indirect methods (dynamic programming) and direct methods are employed to find the optimal or sub-optimal trajectory of the system. For globally optimal trajectory, indirect methods are normally deployed, but they suffer from the "curse of dimensionality, while direct methods (such as hybrid maximum principle, neighbourhood search) gives local optimisation as the solutions are easily trapped in minimum local search.

A hybrid switch is a controller that can depict an abrupt change in system dynamics and stabilise the trajectory with a switching controller [131], [124], [137]. While recent studies show the application of switching system in diverse areas, in high-level flexible manufacturing systems, power electronics, automotive engine management and for further application in this area, see [124]. It has been shown in the literature, switching controllers stabilising the unstable continuous system. Modelling manufacturing systems that exhibit impulsive effects, has remained challenging, despite a great desired performance that can be attained from this combination [131], [54].

Previous control methods have limitations to obtain satisfactory results required in production, scheduling and maintenance planning for manufacturing systems. While for shared resources, [2] policies were established to reduce the expected cost of demand and accumulation of inventory across the supply horizon. As a result, Hamilton Jacobi Bellman methods were used to find the best functional value.

Other work on hybrid control in manufacturing is found in the switched max-plus-linear system (SMPL) with primary control of determining, a feed rate of raw material or processing time, given due dates, or completion time. Model Predictive Control (MPC) is applicable to solve MPL problems with the approach of minimising the error of failure of meeting due dates [76], [77].

Over the years hybrid dynamical system became popular, for modelling physical process controlled by switching control to stabilise the dynamics of the system. The switching control is more attractive for the hybrid system due to a limitation of finding a perfect continuous system without disturbance, and uncertainties see [132] for application. Similarly, stabilization of nonlinear systems was investigated using a time-dependent switching rule by Mancilla-Aguilar and Garcia in [117], [138].

Despite enormous effort over the years for optimisation of continuous and discrete dynamics, few optimisation techniques are available for solving hybrid optimisation problem subjected to constrained or unconstrained conditions. For instance, continuous/discrete systems in [78] was solved using a Laplacian gradient algorithm in resource allocation problem, while for the hybrid system, brutally force method or penalty cost method are typically deployed to optimise the problem.

The theory of stability in Lyapunov, which is well known in the control literature was introduced over a century, and it is still a standard method among the researchers. Its success is due to its simplicity, generality and usefulness. Lyapunov theory of stability provides a means of testing the stability of nonlinear systems. The idea is that if we choose the appropriate Lyapunov function and design it in a way that its trajectory reduces, the resulting system will be regarded as a stable system. [135].

The hybrid manufacturing system stability plays an important role in running a successful production line. If the system converges to an equilibrium point and follows a Lyapunov function, then we can describe the set of equilibria of dynamical manufacturing system as pointwise asymptotically stable [37]. For the last decades, the interest of hybrid optimal control research has been increasing at a steady state. And asymptotic stability design of stabilizing feedback has remain important since then [37], [38], [39].

Industrial 4.0, came with a flexibility concept that aids in satisfying and meeting customer demand on time in an uncertain market environment. As a result, different scholars/ manufacturers have embarked on production systems that are stable and economical sounded [79], [80]. Some of factors that lead to stable manufacturing are in Figure 2.2, with stability production at the center.

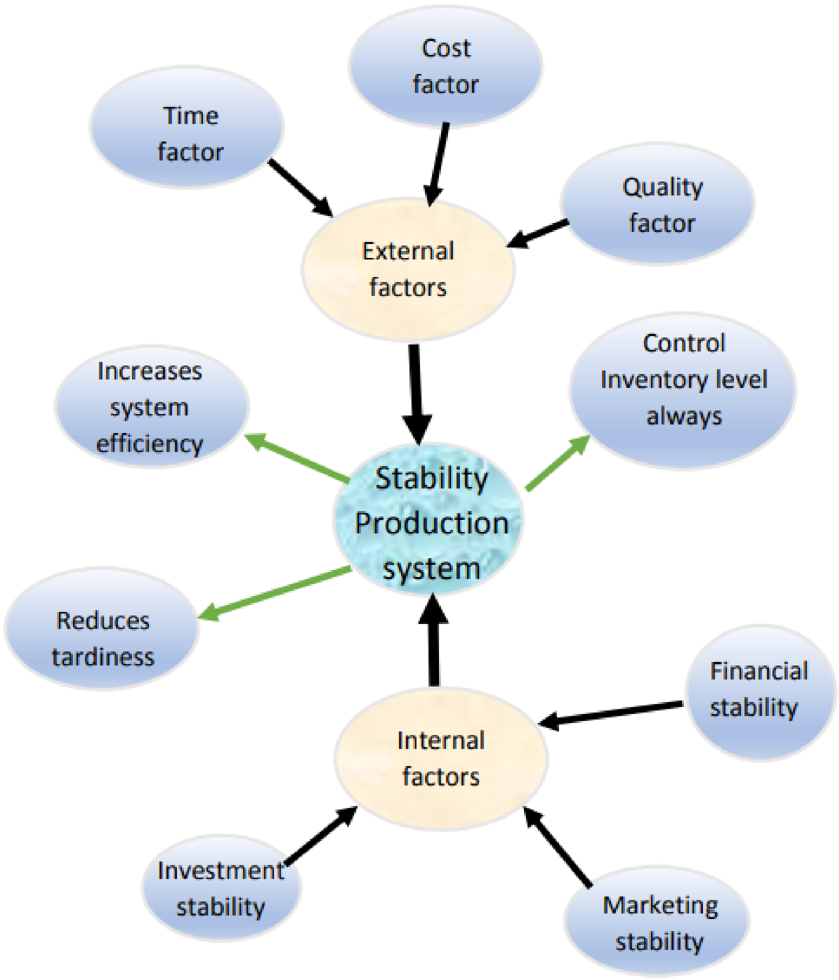

Figure 2.2: Factors that affect manufacturing stability

Other methods that track production systems performance and stability are Lean, Just-in-Time and Total Quality Management. Lean tools are responsible for eliminating waste and tool such as 5's, TPM, Adon are relevant, and for quality criteria, Kanban system, ABC-XYZ, FMEA are used and are inline with Just-in-Time, and statistical methods and engineering techniques are used for total quality management. Since our approach to stability is on mathematical models, we apply Lyapunov functions to verify stability.

The stability of the Lyapunov switching system ensures that the system remains stable within a specified range for a specified time horizon. Lyapunov Theorems for the stability of a dynamical system is centred on an idea that there exists a positive definite, norm-like function that reduces each time during system evolution along its trajectory. And as a result, most work on switch linear or nonlinear systems stability has adopted this notion and it is well established in the literature that an existence of a common Lyapunov function, shows the stability of the system under investigation, especially linear systems [137], [70], [5].

A hybrid dynamical switch system in the manufacturing framework consists of subsystems embedded with sensors and logic rules that controls switching between subsystems. Further, it is recalled that the evolution of continuous-time dynamic and discrete-time dynamic through state-space constraints constitute a hybrid system [63]. For further study and origins of hybrid systems, see [1], [68], [66], [67]. Moreover, the hybrid tutorials are found in [68].

The future trends of production system lie in modelling, control and optimisation of the hybrid manufacturing system. The production system is designed with the objective of the future intelligent manufacturing process, smart factories to reduce production delay and setup control system [41]. Discrete planning and continuous process are in the heart of a hybrid control system in future trends of smart factories, whereby the continuous states is controlled by discrete states that are linear time-invariant. While for more depth of understanding control of hybrid system see [43], [44] and their reference therein.

2.3.1 Manufacturing System Modeling and Design

the manufacturing system is an imperative task to understand the process flow. The work on the formalism of hybrid manufacturing system flow in automata and Petri nets are further analysed and discussed in [42]. The smart factories concept is a development towards control of hybrid manufacturing systems. As a result, smart factories constitute of production process intertwined with software modules that control schedule, variety of manufacturing process, work schedule and market demand [40]. Recently, [41] considered a holonic control architecture that goes under several switching during system change over and the authors evaluated its optimality and stability of the system, with the predictive-reactive algorithm for control and used mixed-integer linear programming model to find the optimal solution of the system. So these optimal solutions are used to design or improve a hybrid manufacturing system.

The design of smart hybrid manufacturing systems is rooted in engineering aspects of it, and that includes mechanical, electrical, mechatronics, instrumentation and control aspects with correct and meaningfully adjustments engineering technical requirement aid the manufacturing system to be more flexible and easily configurable. The flexibility allows the manufacturing workstation to increase production capacity or integrate new functionality with ease. The functionality is the most critical aspects of both the production of complex parts and the production of a variety of parts types on the same machine.

Many control architectures were discussed in the control community literature, that monitor hybrid manufacturing systems. The choice of selecting the control depends on either the system is a centralised, decentralised, single or multi-agent system or a hybrid system. For more information on this control architecture see [12], [72], [104] for and prevention of occurrence of deadlock. A proportional integral derivative (PID) controller is a classic control commonly used for engineering application. However, the PID controller is more sensitive to control design; hence, we can conclude that most control architecture has some tradeoffs. Apart from the PI and PID controller strategy, a sliding mode controller (SMC) is a control strategy that produces discontinuous on/off signal to slide along the desired behaviour.

It was observed that the sliding mode control is more robust than classic controls and it is gaining more popularity due to its practical solution and easy implementation, while two-stage on/off PID control outsmarts standard PI control, but it is not as precise as sliding mode control. SMC uses a discrete sliding decision rule that enables the system to flow through discrete and continuous mode which has resulted in a hybrid configuration.

2.3.2 Typical Examples of Control Systems

Example 1: Production Line

Consider hybrid manufacturing systems that consist of n-servers, n-buffers in both production and re-manufacturing mode. In a hybrid setting, parts transfer, inventory buildup, machine tool change over are not continuous. Therefore, a manufacturing system is a typical example of a discontinuous dynamical system known as a sliding mode in the control community. A slide mode considers a surface in n-dimensional space denoted by S_0 with $(n-1)$-dimensional differential manifold [1].

For a better insight on a sliding mode, consider a structural example given by a differential equation as shown below

$$\dot{x} = f(x(t)) \tag{2.1}$$

such that $f_+ \to S_+US_0$ and similarly $f_- \to S_-US_0$ so that $f_+ = f_-$ is a continuous function for $x \in S$. It can be depicted that the dynamics of this system changes abruptly when the state vector crosses the switching surface, and from a calculus of variation, it is known that this kind of system is not continuously differentiable at the boundary-crossing. We further discuss this kind of system in subsequent chapters. While in Chapter 5, we model the manufacturing system with impulsive sliding mode control and derive the stability of the system switching from one mode to another, until a certain threshold is attained.

Example 2: Self Regulating Thermostat

Other typically hybrid system examples are modeled as a hybrid automaton which is an extension to a finite state machine. In the early 1990s, hybrid automaton remained a popular choice for formal modelling discrete and continuous dynamic system (hybrid systems). For example, [20] considered a self-regulating thermostat with two modes on and off to model the temperature variation. The problem was modeled as hybrid automaton, with each mode following a differential equation given by

$$\begin{gathered} \dot{x} = K(70 - T) \\ k_1 \leq \dot{T} \leq k_2 \end{gathered} \tag{2.2}$$

where $k_1, k_2 \in K$ are constants. Bounds on the derivative control the dynamic evolution of this typical hybrid system. Similarly, a guard from each mode on or off determines system switching. This typical example is amplified in the inventory production system to control inventory level and to determine the production rate in consecutive chapters. For more insight in hybrid automata, see [45] and their reference therein.

2.4 Chapter Summary

We have observed that optimisation techniques of the hybrid system are not yet fully developed [95], more so with emergence of fourth industrial revolution and designing an optimal strategy for the system to respond to adverse circumstance is challenging, as solving the problem leads to minimal game theory which is hard to be solved. This is escalated by the fact that the optimisation of hybrid systems has not been studied to a satisfactory level. Recently, [86] suggested the need for further study on the topic of hybrid optimisation, as little work has been done and only heuristics methods have been developed, which fails to find an optimal solution [94].

As a result, we have merge existing literature and embarked on developing a coherent literature that shows the need to develop new mathematical algorithms in the area of optimisation of an industrial hybrid system. With the primary objective of developing effective mathematical algorithms in manufacturing setting that improves product quality and reduces processing time.

Through this study we have provides a valuable insight into the development of the optimisation models for classes of hybrid systems in the fourth industrial revolution. It is reported in [1] that hybrid control design is an open problem and developing a robust numerical algorithm is not expected for a general hybrid system. On the contrary, we have found that a simplified model with relaxed constraints could be developed to fit the range of hybrid systems, and that is what we intend to achieve through this study.

Chapter 3

Hybrid Manufacturing Systems [3]

This chapter introduces new optimal control policies for solving complex decision-making problems encountered in industrial hybrid systems in a manufacturing setting where critical jobs exist in a busy structure. In such setting, different dynamical systems interlink each other and share common functions for smooth task execution. Entities arriving at shared resources compete for service. The interactions of industrial hybrid systems become more and more complex and need a suitable controller to achieve the best performance and to obtain the best possible service for each of the entities arriving at the system. To solve these challenges, we propose an optimal control policy to minimize the operational cost for the manufacturing system. Furthermore, we develop a hybrid model and a new smoothing algorithm for the cost balancing between the quality and the job tardiness by finding optimal service time of each job in the system.

3.1 Introduction

In manufacturing industry, the facilities consist of different dynamical systems that are integrated to serve producing products or providing service. The manufacturing process aims to achieve the highest productivity at lowest possible cost, and a hybrid system can model its dynamics (called a hybrid manufacturing system) [81], [91] , [97]. Most of these systems share resources and may cause time conflicts to utilise the resources. All entities in the systems need a variety of resources to their optimal level, and this is unlikely to be achieved for unplanned

[3]Copyright permission see in Appendix C.
Kobamelo Mashaba, Jianxing Li, Honglei Xu, Xinhua Jiang. Optimal control of hybrid manufacturing systems by log-exponential smoothing aggregation. Discrete and Continuous Dynamical Systems - S, doi: 10.3934/dcdss.2020100

arrivals of the entities in the system. Since these entities involve both continuous dynamics and discrete states, optimal control design of this kind of hybrid manufacturing system has thereby remained an open problem [86].

Recently, a set of optimization methods have been applied to hybrid manufacturing systems to control the job completion time and the quality of products [6]. The selection of entities following a specific sequence is a scheduling problem. However, difficulties arise when the problem is nonlinear especially with critical jobs defined in the optimal trajectory. To overcome the non-smoothness of critical jobs, we develop a smooth hybrid algorithm that ensures continuous differentiable everywhere for cost balancing between the quality and the job tardiness by finding optimal service time of each job in the system.

Optimization and control problems of dynamical systems with hybrid nature have been discussed in [96], [99], [100], [101]. Challenges of optimizing the hybrid system in the manufacturing setting arise when critical jobs exist in the hybrid busy structure. Among them, challenges of the manufacturing system have been reported in [87], [88], [92], [98]. Scheduling plays an important role in production planning management of the hybrid manufacturing system. It is quite difficult to obtain the optimal schedule, due to the computational complexity to solve job shop scheduling problems which are NP-Hard [103]. Job arrivals to a manufacturing machine are discrete, as jobs arrive at stipulated times, while the processing of jobs and the change of jobs' physical characteristics is continuous. The arrivals of jobs in this manner cause the system states to jump, which converts the problem to be non-smooth, non-differentiable and cannot be solved by the gradient-based method [81], [102]. Therefore, the problem under investigation is not easy to be solved, and to date, only heuristics methods have been developed to find optimal solutions [91], [93], [94], [1].

Optimization theory of hybrid systems has been studied extensively over the past decades see [86] , [81], [93] and the references therein.The most profound work on this section covers an optimal control of single stage. And these results are extended to N-stage manufacturing systems. It should be noted that the former hybrid system framework problem formulation was non-smooth and hence non-smooth optimization methods were employed such as Lipschitz continuous functions were applied to find the sub-optimal solution.

Based on the work in [81], [81], [94] we formulate a hybrid manufacturing system and find an optimal solution for a single-stage manufacturing setting. Recent works in [81], [94] show that guaranteeing the quality of products and meeting customer demand due dates are imperative to the customers' satisfaction. However, producing high-quality jobs and meeting customer demand due dates are always in conflict especially when the product is in high demand. For decision makers, a rule of thumb should be provided as quickly as possible to avoid deteriorating the quality of products. The main contribution of this chapter is twofold. Firstly, we propose an optimal control policy to find the optimal solution for the hybrid manufacturing system. Our optimal control strategy is to minimize the trade-off cost between the jobs' completion times and product quality. Moreover, we develop a smooth hybrid optimization algorithm to design the optimal control and the smooth algorithm ensures that the system to be continuously differentiable everywhere even in the existence of critical jobs within the complex structure.

The remainder of this chapter is organized as follows. Section 3.2 proposes a hybrid manufacturing model and section 3.3 formulates a non-smooth optimization problem. Section 3.4 converts the non-smooth optimization problem of a single server to a smooth optimization problem by log-exponential smoothing aggregation. In section 3.5 and section 3.6 we develop necessary optimal conditions of the smooth hybrid optimization algorithm. Sections 3.7 provides its numerical solution, and section 3.8 summarizes Chapter 3.

3.2 Hybrid Manufacturing System Model

Consider n jobs arriving at a single machine one by one. The products' physical characteristics continuously change with time during the processing until the desired state reaches. The machine process follows the first come first out (FIFO) rule in a job non-preemptive environment. If the machine is busy, the jobs will queue in a buffer. Nevertheless, the longer the service level, the better the quality of the products, but the longer service may result in the product's tardiness, especially if all jobs' resource contention results in conflict between the quality of the product and the jobs' tardiness. Both the quality and the job tardiness are the measures of the customer satisfaction. Therefore, a balance between two performance measures is an imperative task in this manufacturing framework. For robustness and accuracy, penalty costs are associated with the poor quality and the job tardiness in the system. To minimize the cost, an optimal control policy is implemented to decide when to start to process a job and to finish it, for the desired quality. The machine is in working states when processing a job and it becomes idle after releasing a job unless jobs are waiting in the queue. In this model, we assume that the machines are always reliable and available.

A manufacturing system model consists of arrivals of jobs, machines, buffers and operators. During operation, the sum of machine failures times, the operator delay times and the job waiting time in the buffer and actual processing time contribute to work in process times to process a job to the desired quality. During the operation, the processing job cannot distinguish between work in the process times. Therefore there is no need to incorporate work in the process times, as this can be included in the total completion time of a job. We formulate the following hybrid manufacturing system to model the dynamic evolution of discrete events and continuous dynamics of producing the products.

$$\dot{z}_i = \ f_i(z_i, u_i, t),\ \ z_i \in R^p,\ \ u_i \in R^q,\ \ i = 1, 2, ..., n \tag{3.1}$$

where z_i is the physical state vector of the product and u_i is the input control vector associated with balancing quality and job tardiness. For simplicity, all variables in this paper are one dimensional variables. Other general cases can be extended accordingly. The initial and desired physical states of the system is given by

$$z_i\,(\tau_i) = \varsigma_i^o, \quad z_i(x_i) = \varsigma_i^d \tag{3.2}$$

where ς_i^o is the initial physical state of job i and ς_i^d is the desired physical state of job i. For simplicity, in this paper, we consider a one dimensional simplified hybrid manufacturing system in form of $\dot{z}_i = u_i$, $i = 1, 2, \ldots, n$, where u_i is a constant for job i.

For the ith job in the manufacturing system, the completion time of each job x_i is given by $x_i = \tau_i + s_i$, where τ_i is the processing start time for job i and s_k is the service time to complete job i to a physical states ς_i^d. Then, the control required to achieve the desired quality is

$$\int_{\tau_i}^{x_i} \dot{z}_i(t)dt = \int_{\tau_i}^{x_i} u_i\,(t)\,dt \to z_i(x_i){-}z_i\,(\tau_i) = \ u_i(t)s_i \to u_i\,(t) = \ \frac{\varsigma^d - \varsigma^o}{s_i} \tag{3.3}$$

Now, we define a cost on the physical state as

$$\theta\left(u_i\right) = \frac{1}{2}\int_{\tau_i}^{x_i} \gamma u_i^2 \; dt \tag{3.4}$$

where γ is the system parameter of the manufacturing process. Letting $\alpha_i = \gamma(\varsigma_i^d - \varsigma_i^o)^2/2$, then we have $\theta\left(u_i\right) = \alpha_i/s_i(u_i)$. The hybrid state trajectory of (3.1) is coupled with n jobs, which compete for the best service in shared resources. The problem under consideration is a multi-step cost problem and the jobs' arrival times are given as $r_1, r_2, \ldots, r_k$. The cost of completing a job at time x_i is defined by $\varphi(x_i)$.

$$\min_{u_1,\ldots,u_n} J\left(u, x\right) = \sum_{i=1}^{n}\left\{\theta(u_i) + \varphi(x_i)\right\}$$

subject to.

$$x_i = \tau_i + s_i(u_i) \; = \max(x_{i-1}, r_i) + s_i(u_i)$$

where $u = [u_1, \ldots, u_n]^T$ and $x = [x_1, \ldots, x_n]^T$.

3.3 Non-smooth Optimization Approach

This section introduces non-smooth optimization methods for hybrid manufacturing systems. We first start with Lipschitz continuous functions that are defined by the differential between the interval of the form $(\varepsilon^-, \varepsilon^+)$ [81], and associated with critical jobs in a hybrid busy structure. The subdifferentials exist because of decomposition of jobs into subproblems. In [94], the sub differential is defined as;

$$\varepsilon_{j,k} = \frac{d\theta_j}{du_j} + \frac{ds_j}{du_j}\sum_{i=j}^{k}\frac{d\varphi_i}{dx_i} \tag{3.5}$$

The corresponding algorithm finds an optimal solution by searching in a backwards recursive manner with the addition of earlier processed job at a time, and while every job is added, a sign test is performed to identify the structure of the block sequence and the busy period. When a critical job is identified, the algorithm will sweep back and forth to verify if other jobs have been affected by adding a critical job in the sequence. The sub-differential is then applied to decompose the optimal solution as follows [93]: $\varepsilon^- \times \varepsilon^+ \leq 0$ means job i is critical, $\varepsilon^- > 0,\ \varepsilon^+ > 0$ means increasing the control leads to increase the cost and $\varepsilon^- \leq 0,\ \varepsilon^+ \leq 0$ means increasing the control causes decreasing the cost.

Furthermore, in [86] reported the same observation found in [81] of mode switching within the hybrid system, which causes the problem to be non-smooth, and thus the inability to use gradient-based solvers. To handle this problem, [86] divided the problem into smooth sub-problems and a deterministic branch and cut framework were employed to find the optimal solution. Forward and backward heuristic algorithms were developed to find an optimal control policy that minimizes the cost and [93] investigated the structural properties of a single server system. Extension of the similar study was done by [81] by investigating optimal control of a two-stage hybrid system, in which the objective was to find optimal control policy that minimizes trade-offs cost between job completion deadline and achieving the desired target quality. The authors used Bezier approximation techniques to find the optimal control sequence for a two-stage problem, since the problem under investigation was not continuously differentiable everywhere [81], [94].

We see that the algorithm developed in [81], [82] are too expensive as they sweep through blocks forth and backwards in search of solution especially if the critical job exists. This implies that more memory storage is required to store data of a problem of large size. The computational complexity of the problem is in the dimension of 2^{n-1} busy different period. Therefore to redeem the computational complexity of the problem a new algorithm is developed in the next section, which is more efficient as compared to algorithm studied in [81], [82]and the references therein.

3.4 Smoothing Aggregation-Optimal Policy

The problems in [81], [82] are non-smooth and non-differentiable at $\max(x_i, r_{i+1})$ when $x_i = r_{i+1}$ (where the critical job exists in the busy period). A busy period is defined when at least two jobs are coupled in the system. It says that "when critical jobs exist in an optimal solution, a standard gradient-based algorithm for solving the two-point boundary value problem is not working in [81], [94]. In this paper, the non-smooth problem of $\max(x_i, r_{i+1})$ is overcome by introducing a log-exponential smoothing aggregation which is continuously differentiable at all points. The results of the smoothing aggregation function have been realized in [102], where the problem of non-smooth is reformulated as a second-order cone programming, which allows the use of interior-point method and is given by:

$$f(x, D) = D \ln \left(\sum_{i=1}^{m} \exp \left(\frac{g_i(x, D)}{D} \right) \right), \quad i = 1, 2, \ldots, m \tag{3.6}$$

Lemma 3.4.1 *[102]*
The function f (x; D) has the following properties:

- *For any* $x \in R^n$ *and* D_1, D_2 *satisfying* $0 < D_1 < D_2$*, then* $f(x, D_1) < f(x, D_2)$*.*
- *For any* $x \in R^n$ *and* $D > 0$*,* $f(x) < f(x; D) \leq f(x) + D(1 + lnm)$*.*
- *For any* $D > 0$*,*$f(x; D)$ *is continuously differentiable and strictly convex.*

This implies that $f(x, D_1)$ *is continuously differentiable and strictly convex. For the proof of this property, please refer to [102] and the references therein.*

3.4.1 No-waiting Time Scenario $F_1|$no-wait, sequence-dependent$|x_{\max}$

We will replace the non-smooth function by a smoothing log-exponential aggregation function. The non-smooth function is given by (3.5), where the function was non-differentiable at $(x_{i-1} = r_i)$. The optimization problem will be transformed to

$$\min_{u_1, \ldots, u_n} J(u, x) = \sum_{i=1}^{n} \left\{ \theta(u_i) + \varphi(x_i) \right\} \tag{3.7}$$

s. t.

$$x_i = D \ln \left(e^{\frac{x_{i-1}}{D}} + e^{\frac{r_i}{D}} \right) + s_i(u_i) \tag{3.8}$$

$$0 \leq x_i \leq r_{i+1}, s(u_i) \geq 0, D > 0, i = 1, 2, \ldots, n$$

We impose the no-wait restriction on the job arrivals, which means each job should be processed as soon as it is ready (r_i is the ready times of job i) and next job cannot start until it is released. In this scenario, once job arrives in the system it should be processed immediately. In other words, the job operation is non-preemptive with the first-come, first-serve sequence.

3.4.2 Waiting Time Scenario $F_1|$wait, sequence-dependent$|x_{\max}$

In the waiting time scenario, the smoothing optimization problem will be formulated as

$$\min_{u_1,\ldots,u_n} J(u,x) = \sum_{i=1}^{n} \left\{ \theta(u_i) + \varphi(x_i) \right\} \tag{3.9}$$

s. t.

$$x_i = D \ln \left(e^{\frac{x_{i-1}}{D}} + e^{\frac{r_i}{D}} \right) + s_i(u_i) \tag{3.10}$$

$$x_i \geq 0, x_i \geq r_{i+1}, s(u_i) \geq 0, D > 0, i = 1, 2, \ldots, n$$

Before each job departure in the server, its physical state changes with respect to its processing time. In this setting, the physical structure, size, temperature and surface should finish changes with the dynamic evolution of continuous states.

3.5 Necessary Optimality Condition

The manufactured poor quality job will incur a penalty cost associated with short service time and the objective function $\theta(u_i)$ is α/u^2 in section 2. While the penalty cost of missing a deadline $\varphi(x_i)$ is a quadratic function given by $(x_i - d_i)^2$, the processing time $s(u_i)$ is forced to be strictly convex and can be defined as $s_i(u_i) = u_i^2$. The objective of this work is to find a optimal controller that minimizes the cost function. Since the objective function and constraints are strictly convex, with unique optimal solution at the global minimum with a smooth factor $D > 0$.Then, the optimization problem will be

$$\min_{u_1,\ldots,u_n} J(u,x) = \sum_{i=1}^{n} \left\{ \frac{\alpha}{u_i^2} + (x_i - d_i)^2 \right\} \tag{3.11}$$

s. t.

$$x_i = D \ln \left(e^{\frac{x_{i-1}}{D}} + e^{\frac{r_i}{D}} \right) + s_i(u_i) \tag{3.12}$$

$$0 \leq x_i \leq r_{i+1}, s(u_i) \geq 0, D > 0, \text{(Non-waiting time senario)}$$

$$\text{or } x_i \geq 0, x_i \geq r_{i+1}, s(u_i) \geq 0, D > 0, \text{(Waiting time senario)}$$

For each scenario, the objective function should be

$$J = \sum_{i=1}^{n} \left(\frac{\alpha}{u_i^2} + \left(D \ln(e^{\frac{x_{i-1}}{D}} + e^{\frac{r_i}{D}}) + {u_i}^2 - d_i \right)^2 \right) \tag{3.13}$$

Then, the gradient should be vanishing,

$$\nabla J\left(u\right) = -2\frac{\alpha}{u_i^3} + 4u_i\left(D\ln\left(e^{\frac{x_{i-1}}{D}} + e^{\frac{r_i}{D}}\right) + u_i^2 - d_i\right) = 0$$

and the second order derivative should be positive definite,

$$\nabla^2 J(u) = 6\frac{\alpha}{u_i^4} + 12u_i^2 + 4D\ln\left(e^{\frac{x_{i-1}}{D}} + e^{\frac{r_i}{D}}\right) - 4d_i > 0, \tag{3.14}$$

where $d_i \geq 0$ is allowable job lateness for the waiting scenario while for no-waiting scenario, $d_i = 0$. $J\left(u\right)$ is strictly convex for $u > 0$.

Critical jobs

The critical jobs in the system effects, optimal solution by shifting the controller to process jobs faster or slower depending on the trajectory path of the optimal solution for each busy period and exist when $x(i) > \ r(i+1)$.

$x*$ is an optimal solution if and only if $u \ > \ 0$; $D \ > \ 0$; $x(i) \ \leq \ r(i+1)$ for no wait restriction, while for waiting restriction the $x*$ is achieved in the global solution, and this implies J(x*, u*) $\leq J(x,u)$ for $x_i \leq r_{i+1}$ or $x_i \geq r_{i+1}$, and for no- wait restriction, and if $x_i \geq r_{i+1}$, then a minimum cost will be moved, from its global point, to satisfy no wait restriction with J(x,u) $\geq J(x^{*}, u^{*})$.

3.6 Algorithms for No-waiting and Waiting

The computational algorithms for no-waiting and waiting scenarios can be described below.

Input:

n jobs arrives in the system with the arrival times $r_1, r_2, \ldots, r_n$ where $r_1 > r_2, \ldots, r_{n-1} > r_n$ and only one job can be served in the system at a time.

Initialization:

For the algorithm to be differentiable, choose a smoothing factor between $0.004 \leq D \leq 0.9$ (experiential values). Let $i = 1$, and $x_0 = 0$.

Step 1: Determine the hybrid trajectory for the given x_i , and find u_i by use of $\partial J = 0$.

Step 2: Check a single busy period, by identifying critical jobs in the system. Then, compute $x_i = D \ln \left(e^{\frac{x_{i-1}}{D}} + e^{\frac{r_i}{D}} \right) + u_i^2$, If $x_i > r_{i+1}$, then job i is critical, then set $x_i^* = r_{i+1}$ for the no-waiting scenario and find u_i^*. Otherwise, x_i^* and u_i^* will be obtained by $\partial J = 0$.

Step 3: increase $i = i + 1$, go to step 1.

Step 4: if $i = n + 1$, then stop.

3.6.1 Optimal Service Rate

To model N-stage manufacturing system, we should have first an insight on production cost of operating a single stage manufacturing system. This is done by finding minimum and maximum service rate, of both waiting and non waiting scenario using algorithm in developed in section 3.6. The minimum and maximum service rate act as minimum bound and maximum bound of operation, and the mean of the two bounds is selected and used as an optimal service rate. To test optimal performance of the system , jobs are coupled in series and parallel configuration, and the cost of each configuration is evaluated.

3.6.2 N - stage Manufacturing ($N > 1$)

The configuration designed for the manufacturing system plays an important role in system reliability, flexibility and product quality. By using the appropriate design, the system cost can be improved in such a way that the product quality is balanced with respect to customer demands. Using the results in optimal single-stage service rate , we further investigate the performance of manufacturing system systems coupled with n jobs and m machines.

First of all we recall that z_i is the physical state vector of the product and u_i is the input control vector associated with balancing quality and job tardiness. $x_{i,j}$ is completing time with $J(\cdot)$ as total running cost.

$$\begin{aligned}
&\min_{p_1,\ldots,p_i} J_a(x,p) = \sum_{i=1}^{N}\sum_{j}^{M} L_{i,j}(x_{(i,j)}, p_{(i,j)}), \quad \forall \;\; i = 1,\ldots,N \;\; and \;\; j = 1,\ldots,M \\
&x \in R^n, \quad p \in R^n \\
&subject\ to: \\
&x_{i,j} = f(x_{i-1,j}, p_{i,j}, t) = max(x_{1-i,j}, r_i) + S_{i,j}(p_{i,j}) \\
&Z_{i,j} = \dot{g}_{i,j}\left(z_{i,j}, p_{i,j}, t\right)
\end{aligned}$$

3.6.3 Series Configuration

$$\min_{u_1,\ldots,u_n} J\left(u,x\right) = \sum_{i=1}^{i=n}\left\{\theta(u_i) + \varphi(x_{i,3})\right\} \tag{3.15}$$

$$\begin{aligned}
&subject\ to: \\
&x_{i,1} = D\ln\left(e^{\frac{x_{i-1,1}}{D}} + e^{\frac{r_i}{D}}\right) + s_i(u_i) \\
&x_{i,2} = D\ln\left(e^{\frac{x_{i-1,2}}{D}} + e^{\frac{x_{i,1}}{D}}\right) + s_i(u_i) \\
&x_{i,3} = D\ln\left(e^{\frac{x_{i-1,3}}{D}} + e^{\frac{x_{i,2}}{D}}\right) + s_i(u_i) \\
&x_{i,j} \geq 0, \quad x_{i,j} \geq r_{i+1}, \quad s_i(u_i) \geq 0, D > 0, i = 1,2,\ldots,n, j = 1,2,3.
\end{aligned} \tag{3.16}$$

3.6.4 Parallel Configuration

$$\min_{u_1,\ldots,u_n} J\left(u,x\right) = \sum_{i=1}^{i=n}\left\{\theta(u_i) + \varphi(x_{i,3})\right\} \tag{3.17}$$

$$\begin{aligned}
&subject\ to: \\
&x_{i,1} = D\ln\left(e^{\frac{x_{i-1,1}}{D}} + e^{\frac{r_{i,1}}{D}}\right) + s_i(u_i) \\
&x_{i,2} = D\ln\left(e^{\frac{x_{i-1,2}}{D}} + e^{\frac{r_{i,2}}{D}}\right) + +s_i(u_i) \\
&x_{i,3} = D\ln\left(e^{\frac{x_{i,1}}{D}} + e^{\frac{x_{i,2}}{D}}\right) + s_i(u_i) \\
&x_{i,j} \geq 0, \quad x_{i,j} \geq r_{i+1}, \quad s_i(u_i) \geq 0, D > 0, i = 1,2,\ldots,n, j = 1,2,3.
\end{aligned} \tag{3.18}$$

3.7 Numerical Simulation

To have more insight on how the algorithm finds the optimal solution, we demonstrate the effectiveness of the algorithm with a numerical example. Two separated problems were considered with 6 jobs with release times $[0.4, 0.5, 0.7, 0.9, 1.3, 1.5]$ and $[1.1, 1.5, 1.7, 2.1, 2.3]$, respectively. Note that our problem definition is similar to [81], [82] and the algorithm developed is strictly convex and continuously differentiable everywhere for $\alpha = 0.22$, $u > 0$ and $D = 0.05$.

The results of our algorithm is tabulated in Table 3.1 and Table 3.1 for both no-waiting time and waiting time scenarios with different arrival time. For job arrival times between $0.4 \leq r_i \leq 1.5$, the algorithm has identified that jobs $1, 2, 3$ and 5 are critical. Jobs $1, 2, 3$ and 4 are coupled and job 5 and job 6 are coupled, with two busy block structures for both no-waiting and waiting scenarios. While for job arrival times between $1.1 \leq r_i \leq 2.3$, three busy periods are identified for both scenarios. The first busy period is given by job 1, while the second and the third are given by coupling jobs 2, 3, 4 and jobs 5, 6 respectively.

The algorithm developed computed the control time of each job by minimizing the cost of poor quality and minimizing the total completion time, with allowable job lateness. For instance for job arrival between $1.1 \leq r_i \leq 2.3$ the job lateness is given by the following interval $[0, 0, 0.0857, 0.0416, 0]$. The job lateness is calculated from global optimum completion time of each job in its single busy period. And for $i = 1$, the algorithm computed ∂J, following steps given in section 3.5 to find u^* for job $i = 1$ to $i = 6$. Since $x_0 = 0$, the algorithm find $u_1 = 0.4963$ and $x_1 = 1.3463$ from

$$\partial J\left(u\right) = -2\frac{\alpha}{u_i^3}+4\mathrm{u_i}\left(D\ln\left(e^{\frac{x_{i-1}}{D}}+e^{\frac{r_1}{D}}\right)+u_1^2-d_i\right).$$

Since $x_1 \leq r_2$, job 1 is not critical, then algorithm continues to find the service time for jobs 2, 3 and 4 and by increasing i and jobs are coupled together and finally the algorithm computes the service time for jobs 5 and 6 as shown in Table 2.

For the no-waiting and waiting scenarios, the optimal hybrid trajectories are calculated using the similar algorithms, with more relaxed constraints for the waiting scenario.

$F_1 \mid$ no-wait, seq-dependent $\mid x_{\max}$				$F_1 \mid$ wait, seq-dependent $\mid x_{\max}$		
Job Arrival time	Control input	Completion time	Processing cost	Control input	Completion time	Processing cost
0.4	0.3162	0.5000	2.4500	0.4404	0.5940	1.4872
0.5	0.4472	0.7000	1.5900	0.4404	0.7950	1.7664
0.7	0.4472	0.9000	1.9100	0.4404	0.9959	2.1262
0.9	0.5300	1.1809	2.1777	0.5132	1.2593	2.4211
1.3	0.4472	1.5000	3.3500	0.447214	1.5000	3.3500
1.5	0.4423	1.6956	3.9998	0.4423	1.6956	3.9998
Total cost: 15.4775				Total cost: 15.1507		

Table 3.1: Job arrival $0.4 \leq r_i \leq 1.5$

$F_1 \mid$ no-wait, seq-dependent $\mid x_{\max}$				$F_1 \mid$ wait, seq-dependent $\mid x_{\max}$		
Job Arrival time	Control input	Completion time	Processing cost	Control input	Completion time	Processing cost
1.1	0.4963	1.3463	2.7057	0.4963	1.3463	2.7057
1.4	0.4472	1.6000	3.6600	0.4472	1.6000	3.6600
1.6	0.3162	1.7000	5.0900	0.4310	1.7857	4.3733
1.7	0.4204	1.8767	4.7668	0.4121	1.9530	5.1098
2.2	0.3162	2.3000	7.4900	0.3652	2.3393	7.2480
2.3	0.3690	2.4361	7.5503	0.3633	2.4713	7.9814
Total cost: 31.2628				Total cost: 31.0782		

Table 3.2: Job arrival $1.1 \leq r_i \leq 2.3$

The log-exponential smoothing algorithm for a hybrid manufacturing system considered provides a solution that is optimal for balancing the system costs. From Table 3.1, for job time arrival between $0.4 \leq r_i \leq 1.3$ job 1, job 2, job 3 and job 5 are critical. For non-waiting and waiting scenarios, the algorithm provides a minimum cost that differs by 2.5 percent. The results further, shows that the overall quality of products manufactured under waiting restriction is better as this is indicated by longer service or control time and this has resulted in a minimum cost compared to no waiting restriction algorithm. Table 3.2 for job arrival time between $1.1 \leq r_i \leq 2.5$ indicates the same results and verify the effectiveness of the algorithm introduced, with longer service time, better the cost associated with the job quality. Job 2, Job 3 and Job 5 are critical and non-differentiability is overcome by the exponential smooth logarithmic algorithm.

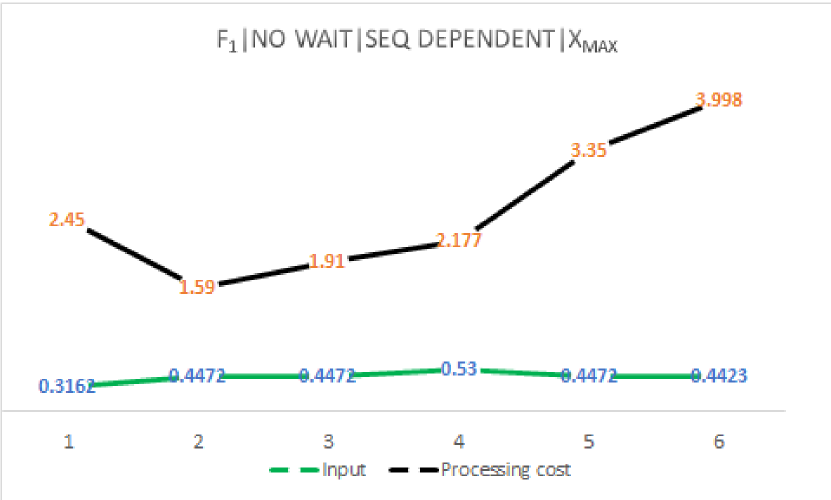

Figure 3.1: Manufacturing performance with no wait scenario

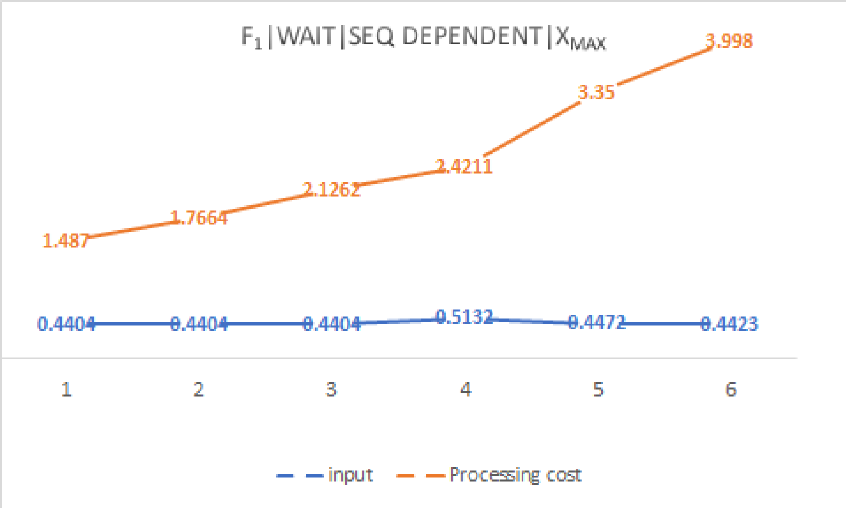

Figure 3.2: Manufacturing performance with wait scenario

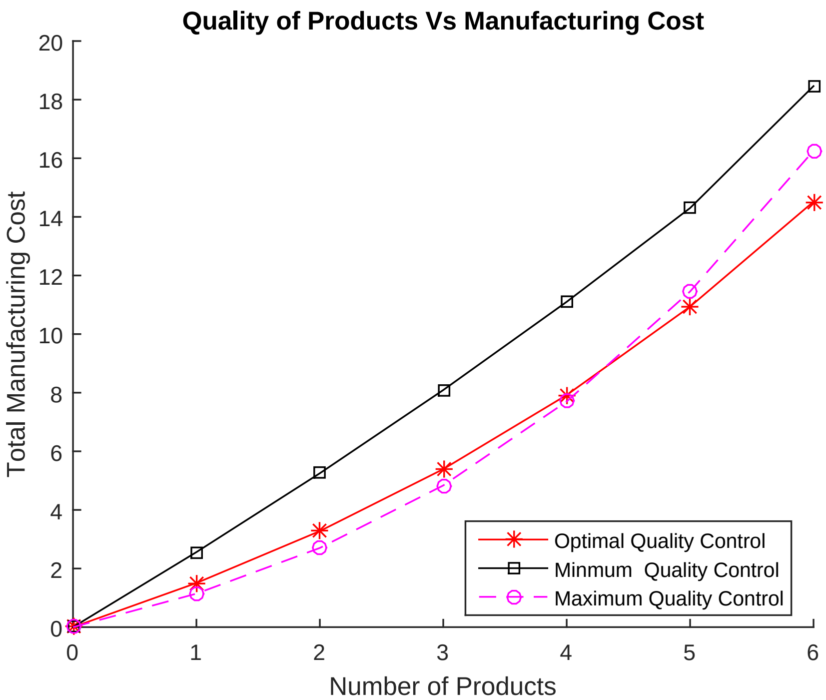

Figure 3.3: Manufacturing system with arrival time between 0.4 and 1.5

3.7.1 Manufacturing Product Standardization

Due to standardization of manufactured product, we have to select one optimal service rate, and apply it in both series and parallel configuration. We use [0.4,0.5,0.7,0.9,1.3,1.5] arrival times of single stage, for series configuration and assumed the following arrival times in machine 1 and machine 2 [0.4,0.5,0.7,0.9,1.3,1.5] and [0.4,0.6,0.7,1.0,1.3,1.4 1.5] respectively , connected for parallel configuration. Both machine 1 and 2 are in parallel configuration and feeds machine 3.

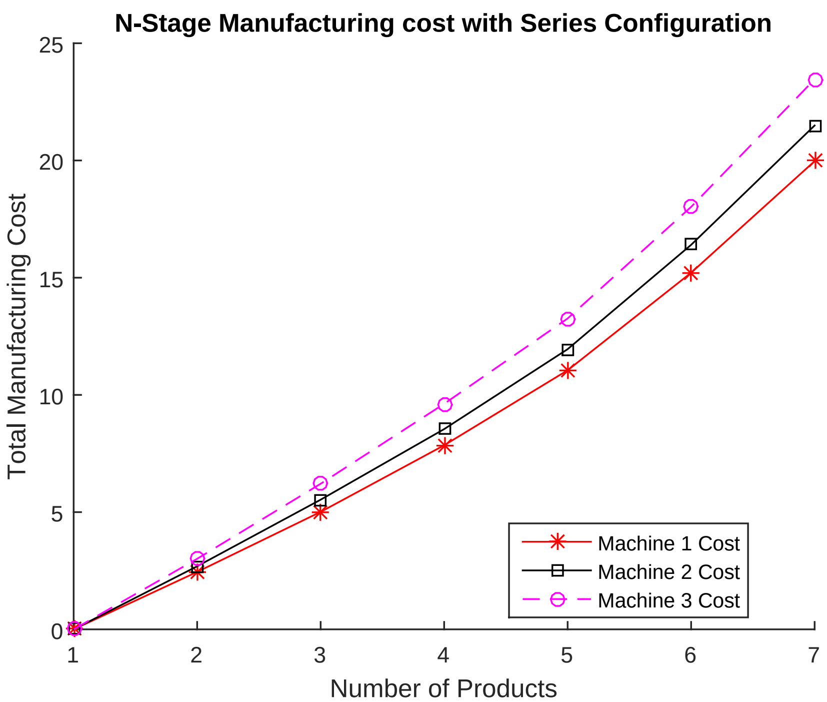

Figure 3.4: N-stage manufacturing system with series configuration for arrival times between 0.4 and 1.5

The total cost of series configuration is proportional to the number of productions machines in manufacturing facility. The total cost is exponential, since parts are produced in sequential order and machines has to wait for prior process to be completed. This setup is not ideal, for products with high demand and high penalty cost as it is depicted in Figure 3.4.

According to the results in Figure 3.3 it was observed that the minimum quality control attracts high manufacturing cost, followed by maximum quality controller which gives best performance for less than 4 thousands products. while optimal quality controller, attributes to minimum overall cost when processing more than 4 thousands products. We also observed that, optimal quality control outweigh other two, and adopted the optimal policy in N-stage for series and parallel configuration. We found that, the manufacturing cost is proportional to the number of machines, and the last machine determines the total completion time in both series and parallel configuration. Since completion time is a decision variable, we conclude that the manufacturing cost increases with the number of machines and with more than 4 batches (four thousands products), the manufacturing cost becomes non-linear.

We extended optimal quality controller results to parallel configuration and allow machine 1 and 2 to be parallel and in such setting both machines feed machine 3, with different products arrival time in machine 1 and machine 2, and We found that that this manufacturing set-up gives minimum cost as compared to series configuration with three machines.

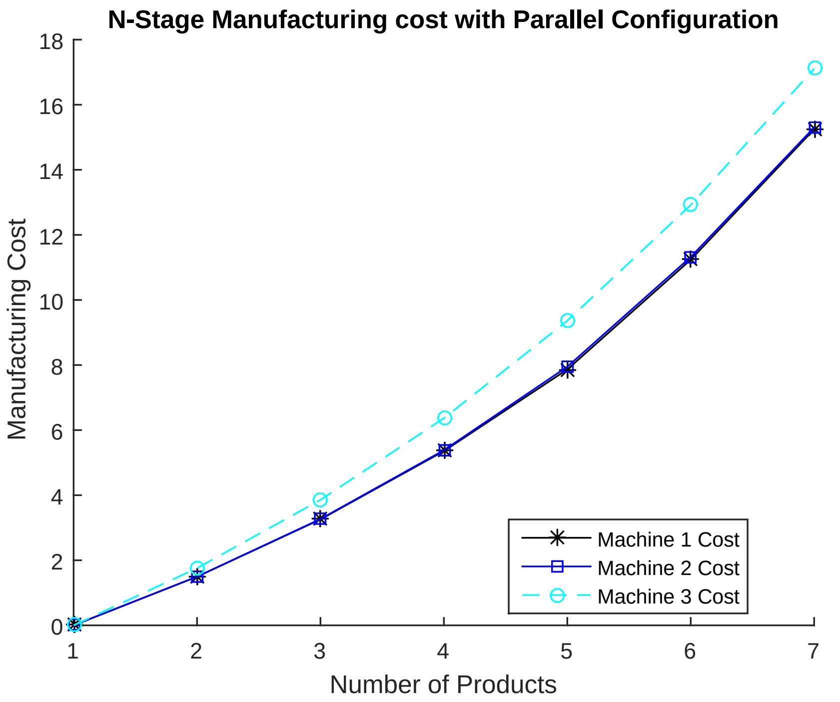

Figure 3.5: N-stage manufacturing system with parallel configuration for arrival time between 0.4 and 1.5

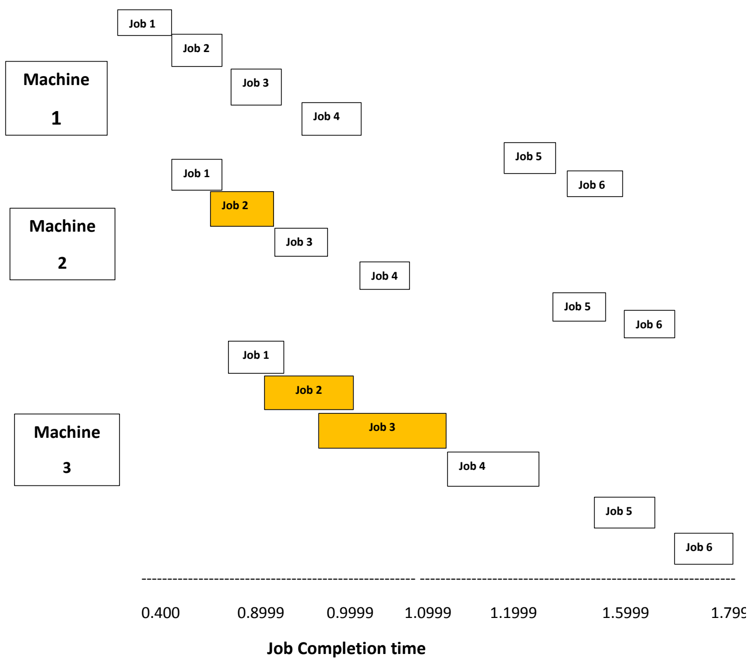

Figure 3.6: Optimal completion time of N-stage series configuration with arrival time between 0.4 and 1.5

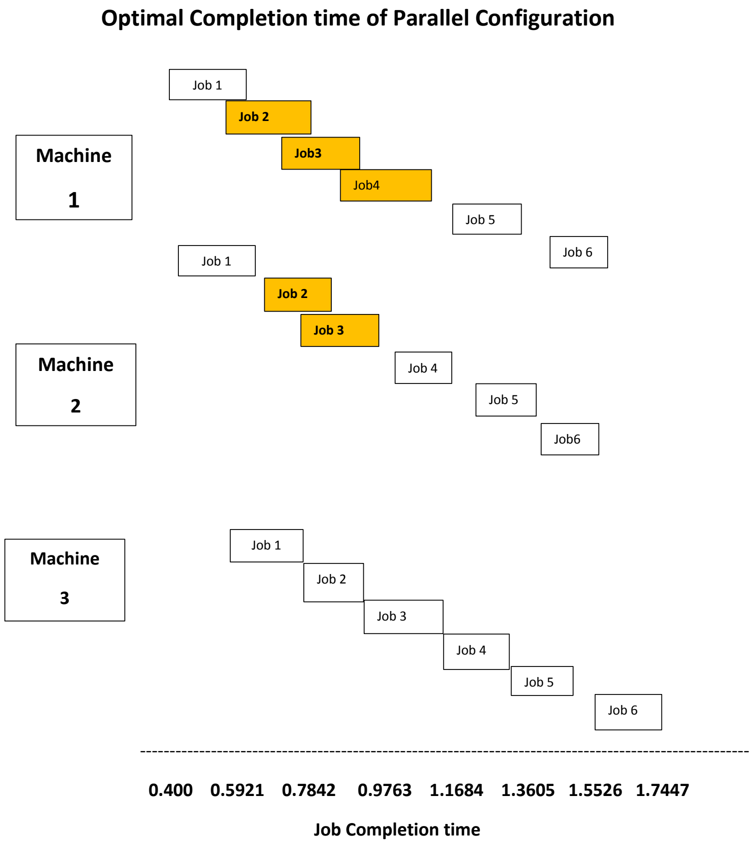

Figure 3.7: Optimal completion time of N-stage parallel configuration with arrival time between 0.4 and 1.5

In Figure 3.6 it is observed that Job 2 and 3 were critical in machine 1, while in machine 2 only Job 2 and 3 were found to be critical. The critical jobs did not affect the total completion time, with adoption of optimal quality control policy. While for N-stage parallel configuration Figure 3.7 we found that Job 2,3 and 4 were critical in machine 1, and only Job 2 and 3 were critical in machine 2 and no critical jobs were found in machine 3 which determines the total cost of manufacturing.

3.8 Chapter summary

This Chapter has provided an insight on the study of a hybrid system for a single server, in the context of using the logarithmic smoothing exponential algorithm to find the service times of jobs that balance the cost of desired quality with acceptable job tardiness. The study has provided a method that is used to overcome non-differentiable points, in the min-max plus algebra where critical jobs exist within the busy structure of the hybrid trajectory. The algorithm developed and the theory behind optimal control for balancing the quality and the job tardiness will be further applied to other systems such as manufacturing inventory control in subsequent chapters.

Chapter 4

Optimal Control of Hybrid Inventory System

In this chapter, we introduced a two-stage hybrid inventory (re)manufacturing system model with a Poisson arrival demand rate. We then formulate a (re)manufacturing system with uncertain demand rate as a Markov decision process (MDP). Furthermore, we investigated the structural properties of the system and found the optimal policy of running the hybrid inventory (re)manufacturing system. To obtain the satisfactory results, we converted the problem to discrete-time Markov process and regulate the inventory level that minimises the system cost. While in the second stage, we developed a smooth algorithm with the help of Hamilton Jacobi equations, for effective control of inventory level. Lastly, we determined the production rate of the system for optimal inventory level and provided a numerical example to show how our algorithm works.

4.1 Introduction

Managing the inventory and obtaining the optimal production rate for a manufacturing system that switches between several modes remains a challenge and have drawn research attention recently [110]. However, early researches on hybrid systems have demonstrated difficulties in obtaining solution analytically. Systems dynamics characteristics are captured using structural properties of the problem of interest and heuristic algorithm were developed [94]. Furthermore, event-driven dynamics in manufacturing that causes the system to switch in the time-driven dynamics are described by differential equations. Therefore considering hybrid manufacturing systems that evolve according to event time-driven dynamics and switch between several modes, (manufacturing, re-manufacturing, repairing) we are interested in finding controllers that regulate optimal inventory level with product demand rate and follows the Poisson distribution. While most of the controllers in dynamically systems are controlled internal or external and are classified as autonomous and control switch, respectively.

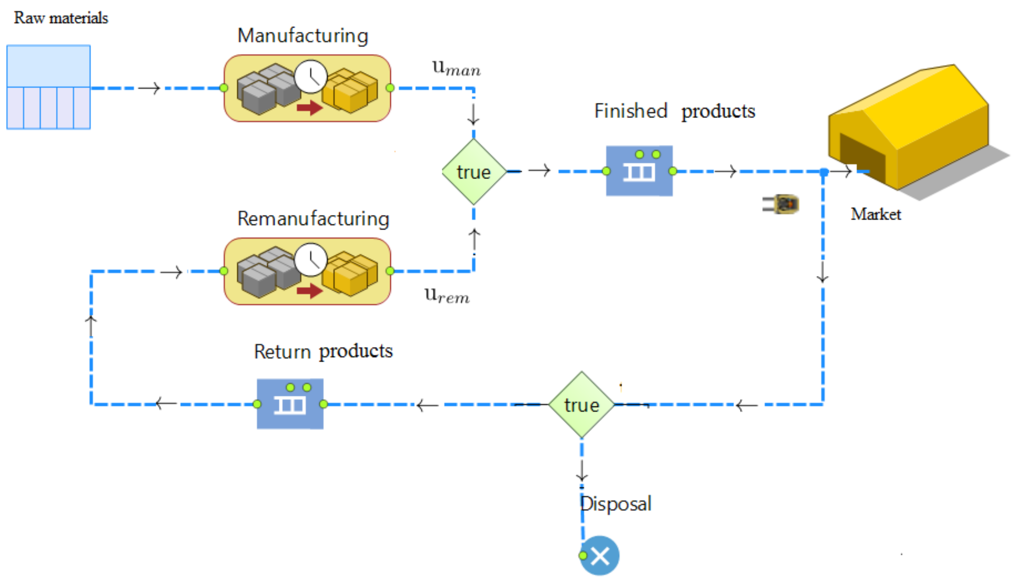

Figure 4.1: Manufacturing (re)manufacturing system model

(Re)manufacturing has been defined as the process of rebuilding, refurbishment or reconditioning [113] while in this framework, we refer to the (re)manufacturing of a worked parts into a single stage for a further processing for omitted or additional features in the manufactured components, caused by tool failure, or manufacturing uncertainties. These components are returned before being disposed to the customer. Therefore, (re)manufactured parts increases system cost and total completion time of processed components. In this framework (re)manufacturing process is undesirable events, that need to be avoided where ever possible. Due to uncertainties in hybrid-manufacturing systems, numerical methods are deployed to find the solution.

For control of production output, during uncertainties caused by a low and high volume of product demands, inventory fluctuations and production defects. Inventory regulators are employed to control the flow of material and implemented to prevent shortages and for storage during overproduction. It is well known that in manufacturing set-up low inventory level, reduces the storage cost, but lead to high risk of shortages during machines failures, and as a result, heavy penalties are imposed for failure of not meeting customer demand on time. Most of the work done in this area does not incorporate how production rate should be controlled during a certain inventory threshold but instead focus more on production rate being fixed at the beginning of production.

The paper [114] reported that the production rate and inventory level of most of the manufacturing systems are set before the start of an operation. Consequently, the optimal strategy of the inventory system is typically computed by employing dynamic programming. For instance, a single manufacturing inventory model was proposed by [106] with stochastic demand rate, where production rate was set as a decision variable, to minimize the cost of manufacturing supply chain.

Manufacturing systems inventory models with machine prone to failure were considered by [105], where the decision-maker determines the production rate and the maintenance action to be taken in each period to maximize the overall system effectiveness with an application of partially observed Markov decision process.

It suffices in the literature that, solving hybrid systems that are non-linear and not continuously differentiable imposed problems in finding optimal solutions. Different scholars have used the approximation method, for instance, [108] have employed stochastic methods that utilize the gradient estimators, with receding horizon to find optimal service time. Also, an improved numerical method based on infinitesimal perturbation were employed to approximate the solution. [109] also used numerical method for a predefined hybrid mode sequence, based on a differential transformation of a two-point boundary value problem. It sufficed that two-point-boundary-value-problem solution has discontinuous switching times, that causes instability of controller input. So the intent of differential application was to overcome discrete, discontinuous states, and to reduce computational time of numerical methods.

Mixed integer linear programming was used by [111] to examine the effects of trade-off among different quality of returns, with optimal production rate and inventory level in (re)manufacturing system. It was concluded that higher flexibility in the hybrid system is more viable due to lower re-adjustment cost in manufacturing, as the system hold minimum inventory. In general, the hybrid system outperforms pure manufacturing operations, and thus it has potential to generate more profit, especially if it designed with flexibility features of re-adjustment of production levels while effective manufacturing models eliminate defective parts as soon as there are produced.

Further reading on inventory model see [107], [112] and reference therein, whereby two states machine with constant demand rate and cost function that is associated with long term manufacturing storage and backlog cost in its optimal policy is defined by (a hedge policy) critical inventory model. The policy is maintained as a hedge against machine failures, as it allows the build-up of manufactured components. To capture the structural properties and for a complete analysis of the manufacturing system quality of components produced are incorporated in the production model.

We formulated an inventory model with product demand rate that follows Poisson distribution process as a hybrid-switch system. The inventory problem is modeled as a Markov decision process associativity with optimal strategy required to switch between inventory level to minimize the manufacturing cost. We take advantages of the inventory dynamics equations to develop optimal production rate with respect to the inventory level switch and product demand rate using Hamiltonian Jacobi Equations (HJB).

We first introduce the inventory model with return products due to wrong product specification and poor quality in section 4.2, while in section 4.3, we discussed MDP, with its steady states characteristics. A randomized markov method will be used to allow inventory problem to be continuous, and we then employee linear programming method in section 4.4 to find the best decision that corresponds to an optimal inventory level. In section 4.5, we use the HJB equation to find the optimal production rate, according to optimal policy obtained in section 4.4. A numerical example is given in section 4.6 followed by a chapter summary in section 4.7.

4.2 Inventory Manufacturing System Model

For single-stage manufacturing system, we consider an inventory model that experiences a product demand that follows a Poisson distribution, with a mean demand rate of ζ. During production or inventory accumulation, It is assumed that the machine can switch between four modes, manufacturing at maximum production rate, manufacturing at minimum production rate, preventive maintenance scheduling and switching to re-manufacturing. Every time a production enters into a new mode, an extra cost is incurred. Therefore it is undesirable for a system to jump from one state to another, and to avoid this a heavy penalty cost is associated with each jump of the system. Similarly, machine failure is undesirable for this systems, therefore to capture the dynamics of the system, we assume that the machine reliability is 100 per cent, while is availability is what is needed to be determined in optimal policy, that determined when the system need to switch to preventative maintenance after some operational mode. And to avoid shortages during preventative machine maintenance, an inventory is kept at an optimal level, and the optimal production rate is obtained at each level to meet customer demand rate during production. However, during operation, parts are return at the rate of $u\alpha$ where $0 \leq \alpha \leq 1$, is the return rate and u is the number of products produced to satisfy demand.

4.2.1 Problem Formulation

Consider a single-stage manufacturing hybrid control system defined on the time horizon [0 T], on which the dynamics depend on the mode of the states, within the state space.

$$\dot{x}(t) = \begin{cases} f^1(x(t), u(t)), & \text{if } x \in \Omega_1 \\ f^2(x(t), u(t)), & \text{if } x \in \Omega_2 \\ \dots \quad \dots & \dots \\ f^n(x(t), u(t)), & \text{if } x \in \Omega_n \end{cases} \tag{4.1}$$

where $x(t)$ is the state vector at a time t while $u(t)$ is a control vector at time t and Ω_1 ,$\Omega_2, \dots, \Omega_n$ are state mode of the system with disjoint interiors. While $f^1, f^2, \dots, f^n$ are continuously differentiated functions. The function switches from mode 1, 2,…, m. In each mode we have m internal mode described by the inventory model. And only repair mode does not have internal switching mode.

The problem is to find an optimal policy that minimizes the total cost of manufacturing systems with respect to optimal inventory service level with switching mode and to determine the production rate in each mode. We use a two-stage approach to solve this problem, in the first stage we find an optimal decision for mode switching, while for the second stage we find optimal production rate, that satisfies the product demand, in each mode. Inventory level build up depends on the profitability. Therefore following inventory system that experience Poisson demand and exponential distribution constitute of Markov decision process, and using fundamental properties of Markov states that the transition and discounted sum of rewards associated with inventory level does not depend on past states and action, This property helps to decompose overall optimisation problem in two separate stages, thus enables us to solve inventory problem in stage 1 and optimal production rate in stage 2. Using each optimal level as an inventory hedging policy and given continuous demand probability density function $f(\zeta)$, the expected inventory optimal cost is given by [116]

$$C(s) = E[c(d,s)] = c(s) + \int_{s}^{\infty} pQ(x,s)dx + \int_{0}^{s} hq(x,s)dx \tag{4.2}$$

The above equation for optimal service level is simplified to

$$s^{\star} = \frac{1}{\zeta} ln \frac{h+p}{h+c} \tag{4.3}$$

given that the cumulative density function

$$F(\zeta) = \int_{0}^{d} f(x)dx = 1 - e^{-s/d} \tag{4.4}$$

Inventory level dynamic

$$x_1 - x_2 = I_{max} = S^{\star} \tag{4.5}$$

$x_2 \geq 0$, returns products can not be negative, while $x_1 \leq 0$ is caused by shortages and backlog of products. $x_2 = \alpha x_1$ where α is return rate of product x_1.

Return products wait in the buffer for further processing, and the production follow optimal inventory policy discussed in section 4.2. We have observed that return products should be processed as soon as they emerge in the manufacturing system to mitigate holding cost. And the problem becomes similar to one discussed in Chapter 3, ”finding the balance between service times and quality to minimize the cost of manufacturing”.

4.3 Markov Decision Process

In the first stage we deals with the optimisation of inventory level, with respect to jump process due to stochastic demand that follows the Poisson process. The maximum storage of inventory is defined by equation 4.5. The dynamics and uncertainties of our problem are captured by stochastic models with Markov decision process employed to find an optimal policy that minimizes the system cost. Every jump or mode switching is associated with an optimal decision taken to influence the state of the system for optimal performance criterion. And each action incur a cost or reward, therefore the ultimate goal of this work is to maximize the sum of reward over a time horizon that is finite or infinite denoted by decision epoch T, we recall that $x(t)$ is a state at time t and Bellman equation is used to find discounted sum of rewards, to find the best choice of action at state $x(t)$. The action affects the transition probability and both immediate and subsequent rewards cost. It is known that the Bellman equation fails to satisfy optimal equations in the presence of the max operator. Therefore randomized Markov policy is introduced, to make MDP function to be continuously differentiable. Switching mode depend on the current state/ inventory level $x(t)$, production rate $u(t)$ and demand rate $d(\zeta)$ given by $\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_n$.

For inventory greater than allowable optimal inventory $s^\star$ without demand, then the production is set to zero $u(t) = 0$, an set $x(t) = s^\star$, and if $x(t) \leq s^\star$. Then the next inventory level is defined by

$$x(t+1) = \begin{cases} \max(s^\star - d_{t+1}, 0), & \textit{if x(t)=0} \\ \max(x(t) + u(t) - d_{t+1}, 0), & \textit{if } 1 \leq x(t) \leq s^\star \end{cases} \tag{4.6}$$

The product demand and arrivals follow a Poisson distribution with parameter ζ, then the transition probability, the matrix of demand rate is given by

$$P(d_{t+1}) = \zeta^n e^{-\zeta}/n! \tag{4.7}$$

And assuming the homogeneous state transition probability P_{ij} that satisfies the following conditions.

1. $0 \leq P_{ij} \leq 1$
2. $\sum_j Pij = 1 \quad i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, N$

Given $x(0) = i$ and using total probability theory and memory less property of Markov and for large N-state, then

$$\begin{aligned} \sum_{i=1}^{N} Px(0) = 1 \\ P[x(n) = j] = \sum_{i=1}^{\infty} P[x(0) = i]P_{ij} \end{aligned} \tag{4.8}$$

This implies that for large N-state process, it can be clearly seen that as $n \to \inf$,

$$\lim_{n\to\infty} P[x(n) = j] = \pi_j \tag{4.9}$$

Under the above conditions, the transition probability is given by P_{ij}^n in state i through j in n steps. As we increase the number of steps, the system achieves steady states and the long-expected total cost of the inventory model is described by the steady-state vector $\pi = [\pi_1, \pi_2, ...]$

$$\begin{aligned} \pi_j &= \sum \pi_k P_{kj} \\ 1 &= \sum_j \pi_j \\ \pi &= \pi P \end{aligned} \tag{4.10}$$

The inventory level evolves according to dynamics, and optimal policy, with the cost structure given by

$$Cost = C[x(t), u(t)] \tag{4.11}$$

The cost of being in states $x(t)$ with production rate $u(t)$ is shown in (4.11) and the expected average cost under policy π is calculated as follows;

$$V_\pi(x(0)) = \lim_{T\to\infty} E_\pi \left[\int_0^T C(x(t), u(t)) \right] \tag{4.12}$$

with $x(t)$ and $u(t)$ optimized in the first stage, and second stage respectfully. And to get more insight on switching behaviour and the structure of inventory problem under policy π, a uniformization method is employed to convert continuous transitions probabilities to discrete as follows; for $\gamma \geq max(\Lambda(i))$

$$P_{ij} = \begin{cases} \frac{\Lambda(i)}{\gamma} & if\, i \neq j \\ \frac{1-\Lambda(i)}{\gamma} & if\, i = j \end{cases} \tag{4.13}$$

where $\Lambda(i) = \zeta_1, \zeta_2, \zeta_3.....$
It is recalled that, a control action is taken when a new state is reached, at time T_k, with associated cost, however for a long run under policy π for steady states, the cost remains unchanged

$$E_\pi \left[\int_0^\infty e^{-\beta t} C(x(t), u(t)) dt \right] = \sum_{k=0}^{\infty} E_\pi \left[\int e^{-\beta t} C(x(t), u(t)) dt \right] \tag{4.14}$$

With discount factor $\alpha = \frac{\gamma}{\beta+\gamma}$.

$$V_\pi(i) = E_\pi \left[\sum_{k=0}^{\infty} \alpha^k C(x(t), u(t)) \right] \textit{given } x(o) = x0 \tag{4.15}$$

The problem is to determine a policy π to minimize V_π, and with a discount factor of 1, a optimisation in the first stage depends on $x(t)$

$$\lim_{x \to \infty} E \frac{1}{n} \sum_{t=1}^{n} Cx(t) = \sum_{j=0}^{m} \pi_j C_j \tag{4.16}$$

4.4 Randomized Markov Policy

Markov deals with the selection of different policy for the evaluation of our manufacturing system. Since several policies are available, and a decision is needed to select the best policy. To avoid dynamic enumeration, we randomized Markov inventory system with a specific policy (R), where $D_i(R)$ corresponds to the decision to be made in state i with decision possible values, $D_i(R) = 1, 2, 3, \ldots, k$ and state i. And each specific policy is characterized by $D_0(R)$, $D_1(R)$, $D_2(R), \ldots, D_m(R)$.

$$D_{ik} = \begin{cases} 1 & \textit{if decision } k \textit{ is made in state } i \\ 0 & \textit{otherwise} \end{cases} \tag{4.17}$$

for $i = 0, 1, 2, ...m$ and $k = 1, 2, ...K$ decision matrix can take the following form; $0 \leq D_{ik} \leq 1$ Since π is the steady states of the inventory system of Markov chain to be in mode i, we define

$$z_{ik} = \pi_i D_{ik} \pi = \sum_{k=1}^{K} D_{ik} = \frac{z_{ik}}{\sum z_{ik}} \tag{4.18}$$

Since D_{ik} is continuous in randomized policy, a linear programming method is used to find an optimal solution.

4.4.1 Linear Programming

For linear programming to be feasible in Markov modelling, transition probabilities need to recurrent, that means a steady states condition need be achieved. Therefore z_{ik} is a steady-state with an unconditional probability that the system is in state i with decision k and equations (4.19) to (4.22) find z_{ik}.

$$\min \sum_{i=0}^{M} \sum_{k=1}^{K} C_{ik} z_{ik} \tag{4.19}$$

subject to

$$\sum_{i=0}^{M} \sum_{k=1}^{K} z_{ik} = 1 \tag{4.20}$$

$$\sum_{i=0}^{M} z_{ik} - \sum_{i=0}^{M} \sum_{k=1}^{K} z_{ik} P_{ij}(k) = 0 \tag{4.21}$$

$$Z_{ik} \geq 0 \tag{4.22}$$

4.5 Hamiltonian Jacobi Equation

Production switch mode is active when the inventory level reaches a certain threshold and evolve according to policy depicted in stage 1. Manufacturing system dynamics evolve according to production mode, manufacturing mode, and preventive maintenance mode.

Production Mode Dynamics:

$$\begin{aligned} \dot{x}_1(t) =& u(t)(1-\alpha) - d(\zeta(t)) \\ \dot{x}_2(t) =& \alpha d(\zeta(t)) \end{aligned} \tag{4.23}$$

Remanufacturing Mode Dynamics:

$$\begin{aligned} \dot{x}_1(t) =& -d(\zeta(t)) \\ \dot{x}_2(t) =& u(t) - \alpha x_1(t) \end{aligned} \tag{4.24}$$

Preventive Maintenance Mode Dynamics:

$$\begin{aligned}\dot{x}_1(t) =& -d(\zeta(t)) \\ \dot{x}_2(t) =& 0\end{aligned} \tag{4.25}$$

Following the trajectory path of optimal inventory in each state, for the infinite horizon, we have the following cost function and determined the production to achieve minimum manufacturing cost. The production rate is determined from the following equations cost function in the second stage.

$$\begin{aligned}&\min \sum_{t=1}^{k} \left(\hat{L}(x(t), u(t)) + g(v(s(t)), u(t))\right) \\ &\textit{subject to}: \\ &\dot{x}(t) = f^T(x(t), u(t), d(t, \zeta))\end{aligned}$$

where $\hat{L}(x(t), u(t)$ is the production cost and $g(v(s(t))), u(t)) = max(v(s(t)) - d(t, \zeta), u(t) + I(t) - d(t, \zeta))$ is the shortage/holding cost. Using difference equation modeling for discrete inventory level. the sate equation below, where b is the ramp factor to achieve optimal inventory level $v(s^\star(t))$ from stage 1 and $R(u(t)) = 1/u(t)$ is the number of products produced during period t to satisfy production rate $u(t)$. Similarly $Q(\zeta(t)) = 1/\zeta(t)$ is the number of product required to satisfied demand rate ζ. The second objective of the second stage is to minimize, the manufacturing cost due to production cost, holding and shortage cost. It is noted that $g(\cdot)$ follows the optimal trajectory from first stage optimisation for switching of inventory level. It turns out that if we apply the gradient decent method for our objective function, we can find a feasible solution which is optimal with respect to inventory level from stage 1.

$$\min \sum_{t=1}^{k} x^2(t) + bu^2(t) + \max(v(s(t)) - d(\zeta(t)), u(t) + I - d(\zeta(t))) \tag{4.26}$$

Our second stage function is not differentiated because of the maximum operator and we use the smoothing algorithm to overcome this with a smoothing factor D. Substituting log exponential function from Chapter 3 into equation eq:smooth , and smoothing operator, the manufacturing systems total cost is described by

$$\min \sum_{t=1}^{k} x^2(t) + bu(t)^2 + \\ Dln(e^{\frac{c^{+}(v(s(t))-d(\zeta(t)))}{D}} + e^{\frac{c^{-}(x(t)+u-d(\zeta(t)))}{D}}) \tag{4.27}$$

using HJB defined by the following equations to find, optimal production rate.

$$\begin{aligned} \frac{\partial V}{\partial t} + H_{opt} = 0 \\ H_{opt} = L(x(t), u(t)) + g(v(s(t)), u(t)) + \zeta(f(x(t), u(t))) \\ \zeta = f^T(x(t), u(t), d(\zeta(t))) \end{aligned} \tag{4.28}$$

To obtain equation 4.29 to equation 4.30 and from equation 4.28 if ζ define the optimal inventory level following an optimal policy obtained in stage 1, then optimal production rate can be calculated from Hamiltonian as follows;

Production mode

$$\begin{aligned} H_{opt} =& \frac{1}{2}(x^2(t) + bu^2(t)) + Dln(e^{\frac{v(s)-d(\zeta)}{D}} + e^{\frac{u(t)+I(t)-d(\zeta)}{D}}) \\ &+ \lambda_1(u(t)(1-\alpha) - d(\zeta(t))) + \lambda_2 \alpha d(\zeta(t)) \end{aligned} \tag{4.29}$$

Remnufaturing mode

$$\begin{aligned} H_{opt} =& \frac{1}{2}(x^2(t) + bu^2(t)) + Dln(e^{\frac{v(s)-d(\zeta)}{D}} + e^{\frac{u(t)+I(t)-d(\zeta)}{D}}) \\ &- \lambda_1 d(\zeta) + \lambda_2(u(t) - \alpha x_1(t)) \end{aligned} \tag{4.30}$$

4.5.1 Optimal Conditions

Stage 1 and stage 2 are joined by HJB functions by assuming that our manufacturing system follows optimal jumps of inventory rate from stage 1 solved by Markov decision process, and if running for a long time from stage 1 and we let

$$\begin{cases} \frac{\partial V}{\partial t} = 0, \\ \frac{e^{\frac{u(t)+I(t)-d(\zeta)}{D}}}{e^{\frac{v(s)-d(\zeta)}{D}} + e^{\frac{u(t)+I(t)-d(\zeta)}{D}}} = v(s^{\star}) \end{cases} \tag{4.31}$$

And using results from stage 1, we obtain optimal production rate in stage 2 if conditions in equation 4.30 are satisfied . It should be noted that both problem in stage 1 and stage 2 are continuously differentiable after using randomized method and smoothing algorithm. A smoothing factor is defined between $D_1 < D < D_2$.

Since demand rate and production rate can not be negative, and inventory rate can take any form due to backlog/shortages in production system. From stage 1 jumps are avoided, but are not inevitable, and hence Production rate is set as a decision variable in stage 2, and is calculated from HJB equation as follows for b=1 with boundary condition $x^T(0) = [0 \;\; 0]$ and $x^T(1) = [d(\zeta) \;\; 0]$ for production mode and $x^T(0) = [0 \;\; 0]$ and $x^T(1) = [0 \;\; d(\zeta)]$ for (Re)manufacturing.

Production mode

$$\begin{aligned} \dot{H}_{opt}(u) &= u(t) + \frac{e^{\frac{u(t)+I-d(\zeta}{D}}}{e^{\frac{v(s)-d(\zeta)}{D}} + e^{\frac{u(t)+I-d(\zeta)}{D}}} + \lambda_1(1-\alpha) = 0 \\ u(t) &= -(v^{\star}(s) + \lambda_1(1-\alpha)) \\ -\frac{\partial H_{opt}}{\partial x_1(t)} &= \dot{\lambda}_1 = (x_2(t) - x_1(t)) \\ -\frac{\partial H_{opt}}{\partial x_2(t)} &= \dot{\lambda}_2 = (x_1(t) - x_2(t)) \end{aligned} \tag{4.32}$$

Substituting u(t) into our state equation we find that,

$$\begin{aligned} x_1(t) &= \frac{(1-\alpha)^2 v^{\star}(s) t^2}{2} \\ x_2(t) &= \alpha d(\zeta) t \end{aligned} \tag{4.33}$$

Optimal production rate is given as follows

$$u = \begin{cases} -v^{\star}(s)(1-\alpha)(\alpha-1)d(\zeta) \\ -v^{\star}(s) + [(x_1 - x2) - v^{\star}(s)(\alpha-1)d(\zeta)](1-\alpha) \end{cases} \tag{4.34}$$

Remnufaturing Mode

$$\begin{aligned} \dot{H}_{opt}(u) &= u + v(s) + \lambda_2 = 0 \\ u &= -(v(s) + \lambda_2) \\ -\frac{\partial H_{opt}}{\partial x_2} &= \dot{\lambda}_2 = (x_1 - x_2) \end{aligned} \tag{4.35}$$

Similarly solving HJB function, we find the optimal production rate from state equation as follows;

$$\begin{aligned} x_1(t) &= -dt \\ x_2(t) &= -3v^{\star}(s) + 0.5v^{\star}(s)t^2 - d(\zeta) - d(\alpha-1) \end{aligned} \tag{4.36}$$

Optimal production rate during (re)manufacturing

$$u = v^{\star}(s)(1+t) + d(\alpha+1) \tag{4.37}$$

Maintenance mode

$$\begin{aligned} \dot{H}_{opt}(u) &= u + v(s) = 0 \\ u &= -v^{\star}(s) = 0 \end{aligned} \tag{4.38}$$

4.6 Numerical Results

We consider inventory problem with switching between optimal inventory to satisfy uncertain demand that follows a Poisson distribution with demand rate ζ, and given that for every setup the product can be manufactured in batches, and each batch consists of 1000 products. And we MDP to control and determine the optimal inventory level associated with each state and adopt the following inventory management decision policy shown in Table 4.1.

Table 4.1: Decision matrix.

Decision k	Action	Results
1	Max production	Jump to state 3 or 4
2	Minimum production	Balance inventory level
3	Maintenance	Jump to state 0 or 1
4	Remaunfacturing	Jump to state 0 or 1

From Table 4.1 we inferred that inventory manufacturing system should operate as follows

- Keep zero inventory from the beginning of the operation and operate according to hedging policy according to the optimal level in each state.

- Operate below or at least at the optimal inventory level as depicted in stage 1.

- Ramp the production rate after achieving optimal production rate to jump according to inventory policy.

Table 4.2: Inventory Probabilities.

States i	0	1	2	3	4
0	9/960	9/271	129/971	203/573	453/959
1	506/959	453/959	0	0	0
2	95/548	203/573	453/573	453/959	0
3	16/395	129/971	203/573	453/959	0
4	7/960	9/271	129/971	203/573	453/959

For manufacturing and (re)manufacturing mode, The inventory jumps within four modes depending on the optimal trajectory path due to decision k in-state i. And the cost of making decision k in-state i is given in table 4.3. The total cost consists of production cost (45.5), holding cost (35.5) and shortage cost (235). With above cost, and using equation 2 to 5, we calculate the optimal size as follows

$$v^{\star}(s(t)) = \zeta(t) \ln \frac{h+p}{h+c} \tag{4.39}$$

We use equation(4.8) and (4.9) to find probabilities of our inventory system, with mean $\zeta(t) = 0.75$ tabulated in Table 4.2. Equation 13 shows manufacturing cost that depends on both inventory level and production rate, that evolves to satisfy the demand rate. Using a discount factor of 1 for the Markov decision process, the optimal policy is obtained using Equation(4.21)-(4.24), where a decision is made on how to switch between optimal inventory level to lower manufacturing cost.

The optimal batch size determined maximum allowable space, required to set the manufacturing systems, and thus our inventory system can switch between the following values $s = (0, 1, 2, 3, 4)$. that cross-ponds to optimal inventory level in batches of thousands.

From stage 1 optimisation, we found the optimal policy on how to switch between modes to minimises the inventory cost is obtained by using equation (4.19). A decision that corresponds to best value function from MDP is made, and we use linear programming and simplex method in second stage to formulate our decision problem and find optimal decisions that minimize the expected cost.

Table 4.3: Decision Matrix Cost

States i	1	2	3	4
0	470	0	0	0
1	551	0	551	470
2	162	162	0	0
3	0	81	81	0
4	0	42	42	42

$$\min(470z_{01} + 551z_{11} + 551z_{13} + 470z_{14} + 162z_{21} + 162z_{22} + 81z_{32} + 81z_{33} + 42z_{42} + 42z_{43} + 42z_{44})$$

$$\textit{subject to} \begin{cases} z_{01} + z_{11} + z_{13} + z_{14} + z_{21} + z_{22} + z_{32} + z_{33} + z_{42} + z_{43} + z_{44} = 1 \\ z_{01} - 9/27z_{14} - 129/971z_{23} - 203/573z_{33} - 453/959z_{43} = 0 \\ z_{11} + z_{z14} - 203/573z_{21} = 0 \\ z_{21} + z_{22} + z_{24} - 203/503z_{22} - z_{44} = 0 \\ z_{32} + z_{33} - 203z_{01} - 453/959z_{32} = 0 \\ z_{43} + z_{44} - 9/271z_{11} - z_{21} = 0 \end{cases} \tag{4.40}$$

After obtaining equation (4.40), we get z_{ik}, and using Equation(4.20), to find the decision that corresponds to the optimal policy. According to our framework, we need to produce at maximum production when inventory level is in state 0, and switch to state 3, where the production is maintained. While if the decision 2 is made then, a system should run at minimum production rate that satisfies the demand rate. Also, at state 2 decision 1 is also optimal if it is employed, which allow the system to be operated at maximum production rate to reach states 4, which is equivalent to maximum optimal inventory level within the system. And we have observed that the system switch to the (re)manufacturing mode at state 1 and state 4.

After determining switching mode, now the problem is to find controllers that minimize the total expected cost in each state. To do this, we use the system dynamics discussed in equation (4.25) to equation (4.39) obtained using HJB equations. We let u be the production defined by the following function.

$$u = \begin{cases} u_{min} & = -v^{\star}(s) + [(x_1 - x2) - v^{\star}(s)(\alpha - 1)d(\zeta)](1 - \alpha) \\ u_{max} & = -v^{\star}(s)(1 - \alpha)(\alpha - 1)d(\zeta) \end{cases} \tag{4.41}$$

At states 0, we operate according to u_{max} as in MDP until state 3 is reached, where inventory level is balanced. Due to demand uncertainties, our inventory level can switch to any states, and if it jumps to sates 2, u_{max} is employed until states 4 is reached. Therefore decision for production mode is taken at state 0, 2 and 3, where we can either run at maximum production rate when inventory level reaches 0 or 2 and run at minimum production rate when inventory level is at mode 3. Figure 4.2 show how production rate, demand rate change over time. From the graph, it is depicted that, if demand rate exceeds, optimal inventory level and production rate, then the production starts to decline. This is an indication that the system should switch to different mode.

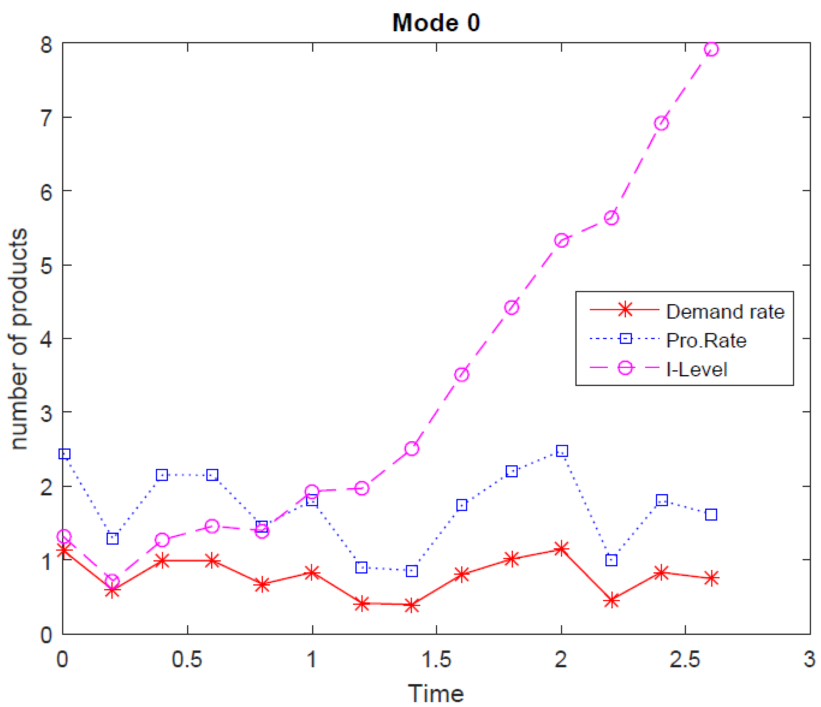

Figure 4.2: Manufacturing system dynamics with minimum inventory at mode 0

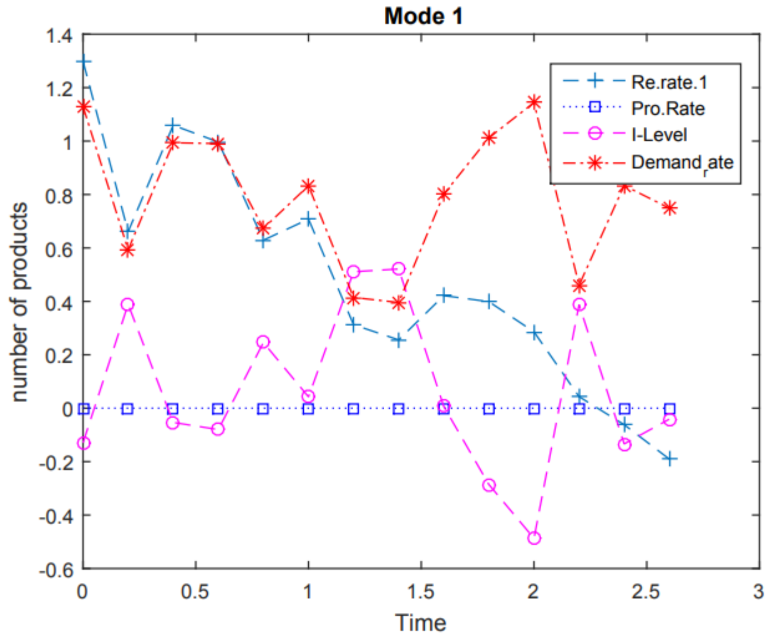

Figure 4.3: Remanufacturing system dynamics at maximum production rate at mode 1

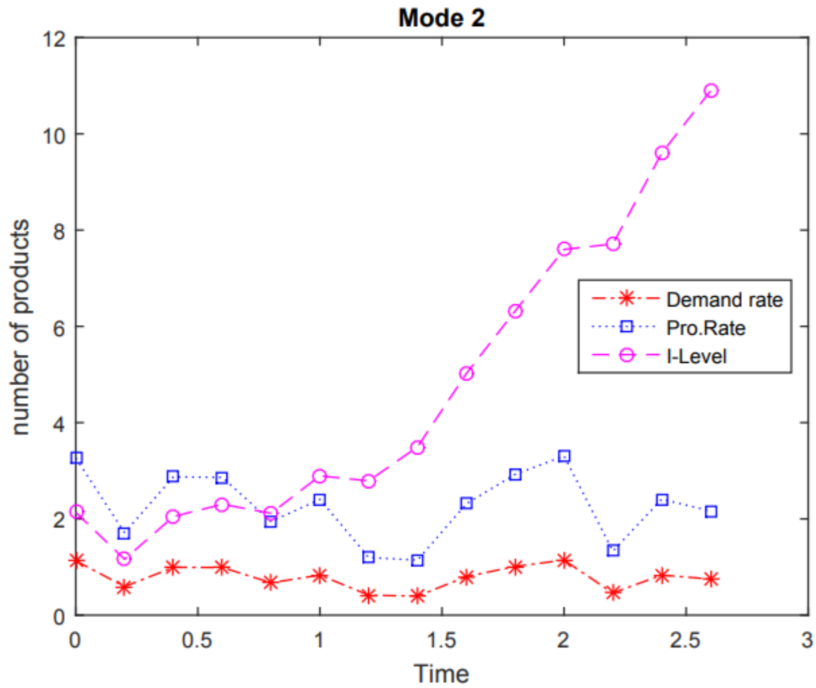

Figure 4.4: Manufacturing system dynamics with minimum inventory at mode 2

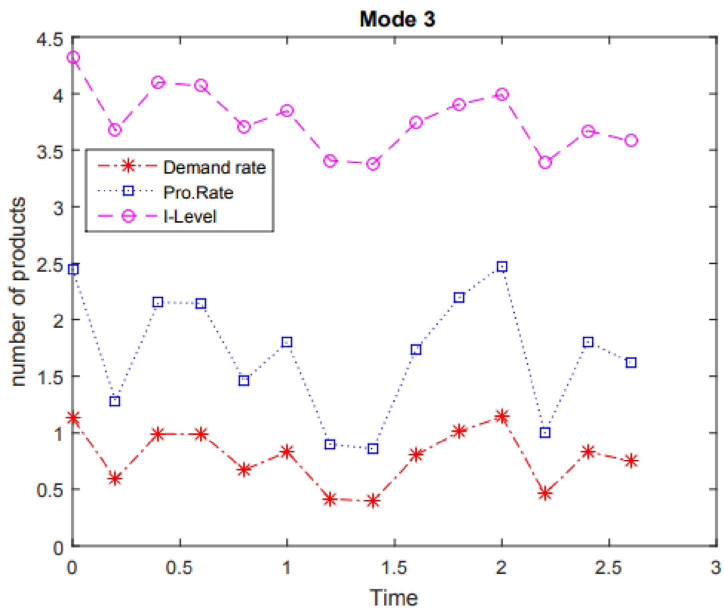

Figure 4.5: Manufacturing system dynamics with minimum production rate at mode 3

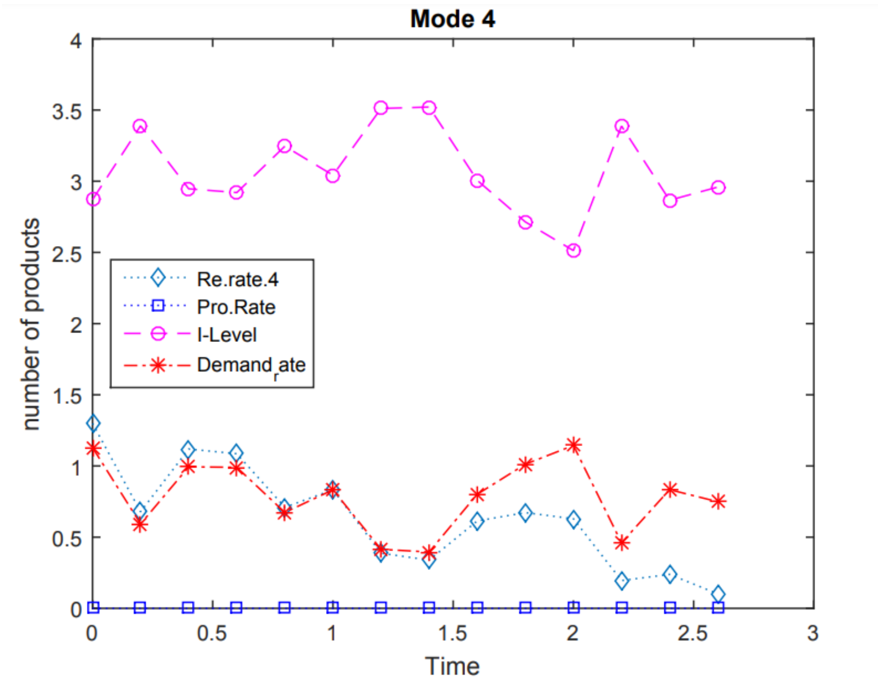

Figure 4.6: Re-Manufacturing System dynamics at maximum production rate at mode 4

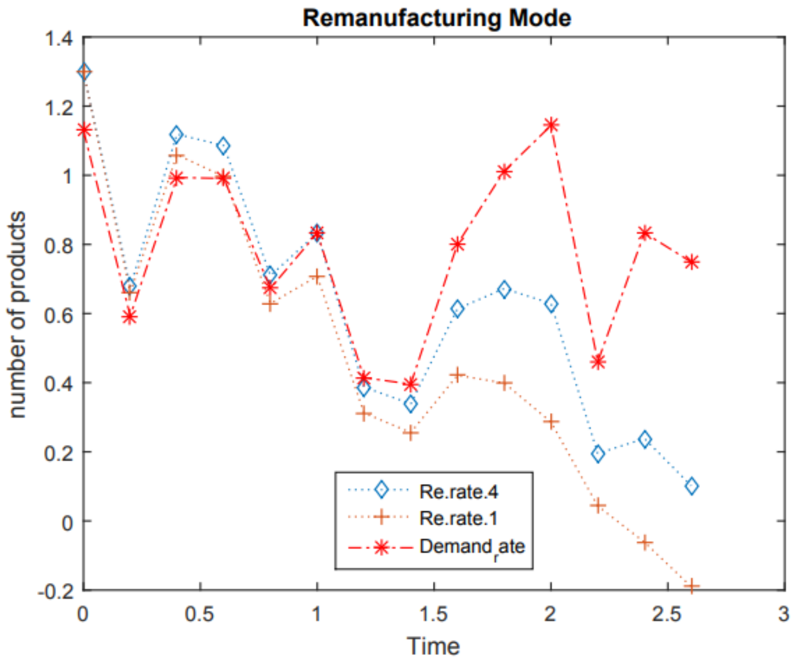

Figure 4.7: Remanufacturing system dynamics rate

following MDP policy develop in stage 1, the results in figure 2 depict that, if the system was not controlled, the system would experience shortages and surplus, in manufacturing mode and also from state 1 respectfully. However, with the results from HJB and MDP, as the system reaches states 3, the inventory level is maintained until the end of the production period. Furthermore if the system reaches states 2, it evolves according to production policy till it reaches states 4, where the production is halted, this is also experienced in state 1, where the manufacturing inventory system switch to (re)manufacturing mode and stays there until the inventory level falls to zero and the process restart again. Without manufacturing at state 4, the system can go for preventive maintenance; this is an indication that before beginning or at the end of a production period the system should be inspected and maintained to satisfactory high level.

4.7 Chapter Summary

We have combined the results of MDP and HJB to find controllers that minimize the cost of inventory hybrid (re)manufacturing system with states jumps. Our problem was coveted to discrete and continuous sates respectfully, and a smoothing method was employed to aid us to apply the differential equation to find the optimal solution of HJB. We have normalized our MDP to approximate the solutions of inventory system, and we have employed linear programming to find mode switching of steady states and run HJB function to find optimal production rate that satisfies product demand that follows poison distribution. The numeral results show how production changed, with uncertain demand. Also, it shows the effectiveness of our method in the application of manufacturing inventory system which is kept within internal states until the end of the manufacturing period.

Chapter 5

Hybrid Systems and Sliding Control [5]

This Chapter proposes a hybrid sliding mode controller to stabilize a complex manufacturing system with impulsive phenomena. Newly developed sufficient conditions ensure the proposed control to effectively work on the multi-mode manufacturing system. A manufacturing/re-manufacturing system is presented as an example to show the effectiveness of the proposed controller. Numerical solutions are developed to set up the controller to govern the two mode manufacturing system. The designed manufacturing control strategy will help produce various products in a timely manner to keep up with the demands and shorten the delay in current competitive and global market.

5.1 Introduction

Systems modeling of a manufacturing system plays an important role in understanding stability impact of decision making on the value function of the manufacturing model. Its aim is to understand the behaviors of the production process and to effectively control the flows of output, goods and information [123], [128]. In order to run smoothly, the manufacturing system's stability must be ensured. Otherwise, the manufacturing plant will not be operated normally if system stability is not guaranteed. When the system does not have these stability properties, then it will

[5] Copyright permission see in Appendix A.
Kobamelo Mashaba, Honglei Xu, Jianxiong Ye. Stabilization Of Complex Manufacturing Systems With State Impulsiveness By Hybrid Sliding Mode Control, Dynamics of Continuous, Discrete and Impulsive Systems, Series B: Applications and Algorithms 26 (2019) 291-302, Copyright c 2019 Watam Press

experience unexpected manufacturing dynamics [128]. Furthermore, if a chaotic or irregular behaviour is not controlled, it results in fluctuation of the processing [126].

Demand fluctuations raised by globalization and intense competition in the market require to design various control policies to make a manufacturing system stable. Thus, the stabilized manufacturing system can produce products in a timely manner, keep pace with demand requests, and shorten the lead time. Independent effective control can increase production flexibility in a timely manner and there are challenges in design complex control mechanism for modern manufacturing systems.

In many situations, the manufacturing system experiences instant and rapid changes of its states at some working times, it can be modeled by an impulsive differential equation or a differential equation with discontinuous right-hand side for its stability analysis. From the control perspective, an impulsive hybrid control can be applied to a nonlinear system with distributed delays, nonlinear perturbations and impulsive effects [132]. Furthermore, a bisection method was used in the application of high frequency switching control of a hybrid system and a minimum-rule algorithm was proposed to stabilize the hybrid dynamical system [132]. Other types of hybrid switching controls can stabilize the trajectory caused by abrupt changes of system dynamics [124], [132], [137]. Recent studies show hybrid controls can be applied in many areas, such as high level flexible manufacturing systems, power electronics, automotive engine management. For example, a time-dependent switching rule can stabilize nonlinear systems [117], [138]. Moreover, the hybrid controllers can stabilize many unstable nonlinear systems, for which traditional controllers have limited effects. For manufacturing systems that exhibit impulsive effects, the design of hybrid controls remain a challenge [144], [145], [146].

Stabilization of nonlinear dynamics, and systems with time delays have been studied extensively in the literature. Time-dependent rules, feedback controllers and impulsive controllers are applied to stabilize the controlled systems [121]. For instance, both both switching rules and impulsive controls are applied to unstable nonlinear systems with distributed delays and impulsive effects [132]. Nonlinear system stability is traceable with the application of a piecewise quadratic Lyapunov function [129]. A converse theorem can be established for a hybrid system with a smooth compact set to construct Lyapunov functions, without any insight or knowledge of system trajectories [122].

In this chapter, we aim to design a new hybrid sliding mode control that drives the state's trajectory to its stable state. Furthermore we want the states' trajectory to be confined in the manifold and leaves the selected surface only during impulsive time window. And we will design controllers that restrict the state to leave this manifold, for global stability.

The remainder of this chapter is organized as follows. Section 5.2 provides the control problem of a manufacturing system, and necessary definitions and lemmas. A hybrid sliding mode control is designed for the manufacturing system and the corresponding stability criteria. In Section 5.4, a numerical example of the manufacturing/re-manufacturing system is provided to show the effectiveness of proposed method. Finally, the chapter summary is given in Section 5.

5.2 Problem Formulation

We consider a complex manufacturing system modelled by

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i(u(t) + f(t,x)), & t \neq \tau_k \\ \Delta x(t) = F_i s_i(x(t)), & t = \tau_k \\ x(0) = x_0, s_i(x(t)) = B_i^T P_i x(t) \end{cases} \tag{5.1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is a control vector, and $f : R_+ \times R^n \to R^n$ is a continuous function to represent the system non-linearity or disturbance. $s_i \in S_i \subset R^m$ is the sliding mode function of the i-th subsystem on a sliding surface $S_i = \{x(t) : s_i(x(t)) = 0\}$. Moreover, A_i and B_i are known matrices with appropriate dimensions for each operating mode i, while P_i and F_i are to be determined to ensure the set of sliding surfaces $S_1, ..., S_m$ satisfies system performance requirements of system (5.1). When an operating mode i is detected, system (5.1) switches to the appropriate surface S_i and then the system state evolves according to the overall characteristics of selected manifold. The impulsive time sequence $\{\tau_k\}$ satisfies $\tau_0 = 0 < \tau_1 < \tau_2 < ... < \tau_i < \tau_{i+1} < ... < \tau_N$ for $i = 1, 2, ..., N$. At impulsive time points, we have $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-) = F_i B_i^T P_i x(\tau_k)$, where $x(\tau_k^-) = x(\tau_k)$. Hence, the solution of system (5.1) is a piecewise continuous function.

We assume that there does not exist the Zeno behaviour (i.e., $t_{t+1} - t_k \geq \delta_k$, where δ_k is a given positive real number), (A_i, B_i) is controllable and the nonlinear function in (5.1) is bounded [127], i.e.,

$$||f(t, x(t))|| \leq \rho_0, \tag{5.2}$$

where $\rho_0 > 0$ is a maximum bound for the nonlinear function.
Our objective of this paper is to develop an hybrid sliding mode control law to stabilize the complex manufacturing system (5.1). If the sliding surfaces exist, there should be symmetric positive definite matrices $P_i > 0$ and appropriate matrices $F_i > 0$ that ensure system (5.1) gets to a sliding surface $s_i \in S_i$ given by

$$S_i = \{x(t) : s_i(x(t)) = B_i^T P_i x(t) = 0\}. \tag{5.3}$$

5.2.1 Preliminaries

To proceed, we need the following definitions and assumptions:

Definition 5.2.1 *The state vector $x \in R^n$ is called a* solution *of the complex manufacturing system (5.1) if there exists a $x(\tau_0 + \epsilon) \in R^n$, $\epsilon > 0$ such that the following are satisfied:*

1) *$x(\tau_0^+) = x_0$;*

2) *$x(t)$ is continuously differentiable for the complex manufacturing system (5.1), where $t \in (\tau_0, \tau_0 + \epsilon)$ where $t \neq \tau_k$ and*

3) *when $t = \tau_k$, $x(\tau_k^+) = x(\tau_k) + F_i B_i^T P_i x(\tau_k)$. Furthermore, $[A_i x(t) + B_i(u_i(t) + f(t, x(t)))] \in PC[R^+, R^n]$, so there exists a unique solution of the complex manufacturing system (5.1) for $t > \tau_0$, where PC is a set of piecewise right continuous functions with $x(\tau_0^+) = x_0$ and $I + F_i B_i^T P_i \neq 0$ for every τ_k.*

Definition 5.2.2 *An* equilibrium *state of the complex manufacturing system (5.1) is said to be* stable *if for any $\epsilon > 0$, there exists a $\delta > 0$ such that $||x(t_0)|| < \delta$ implies $||x(t)|| < \epsilon$ for all $t \geq t_0$.*

Definition 5.2.3 *[140] An* impulsive time window *is said to be the time interval $\tau_{k+1} - \tau_k$ between successive occurrences of impulses.*

Lemma 5.2.4 *[143] Given real matrices Y and Z of appropriate dimensions for any $\epsilon > 0$, we have*

$$YZ + Z^T Y \leq \epsilon^{-1} Y^T Y + \epsilon Z^T Z \tag{5.4}$$

5.3 Main Results

5.3.1 Control Design

In this section, we develop an hybrid sliding mode controller to ensure that the overall system stability is achieved. Once a sliding surface is reached, the state will slide along the manifold till system (5.1) accomplishes stability.
Designing a sliding mode controller should decide the selection of an appropriate sliding surface and the design of a controller that satisfies the switching condition [139]. We set manifold S_i to be a sliding surface with a control in the form of [127].

$$u_i(t) = u_{eq(i)}(t) + u_{r(i)}(t) \tag{5.5}$$

where $u_{eq(i)} = -(B_i^T P_i B_i)^{-1} B_i^T P_i A_i x(t)$ and

$$u_{r(i)}(t) = \begin{cases} -(P_i + \rho_0 I) B_i^T P_i x(t), & ||s_i(t)|| \neq 0 \\ -P_i B_i^T P_i x(t), & ||s_i(t)|| = 0 \end{cases} \tag{5.6}$$

Remark 5.3.1 *When we do not consider the nonlinear term in system (5.1), the switching feedback control on the sliding surface will satisfy*

$$\dot{s}_i(t) = B_i^T P_i \dot{x}(t) = B_i^T P_i A_i x(t) + B_i^T P_i B_i u_i(t) = 0 \tag{5.7}$$

Then we design the switching feedback control

$$u_{eq(i)}(t) = -(B_i^T P_i B_i)^{-1} B_i^T P_i A_i x(t) \tag{5.8}$$

Remark 5.3.2 *We recall (*5.3*) to find the equivalent control that keeps the states on the manifold S_i. And for a closed loop feedback controller where by $(A_i + B_i u_{eq(i)})$ is stable, so for proper design of these feedback controller, we establish the following results in the next section. We assume that under an impulsive control, the system switches between surfaces and remains in the neighborhood of sliding surface. Then the system slips towards the target in the presence of disturbance.*

5.3.2 Stabilization Criteria

We first present sufficient conditions of asymptotically stability of the complex manufacturing system (5.1) and then study its corresponding stabilization criteria.

Theorem 5.3.3 *Suppose that there exists a continuous positive semi-definite function $V(t, x(t))$, and the following are satisfied:*

$$\frac{\partial V(t,x(t))}{\partial t} + \frac{\partial V(t,x(t))}{\partial x}\left[A_i x(t) + B_i(u_i(t) + f(t,x(t)))\right] \leq 0 \tag{5.9}$$

$$V[x(t) + \Delta x(t)] \leq V(t, x(t)) \tag{5.10}$$

Then the complex manufacturing system (5.1) is asymptotically stable.

Proof: *In the k-th impulsive time window, $V(t, x(t))$ can be written as*

$$\begin{aligned} V(t,x(t)) = {} & V(\tau_k, x(\tau_k)) + V(\tau_k, x(\tau_k) + \Delta x(\tau_k)) - V(\tau_k, x(\tau_k)) \\ & + \int_{\tau_k}^{t} \frac{\partial V(t,x(t))}{\partial t} + \frac{\partial V(t,x(t))}{\partial x}\left[A_i x(t) + B_i(u_i(t) + f(t,x(t)))\right] ds \end{aligned} \tag{5.11}$$

At the impulsive times τ_k, $k = 1, 2, \ldots,$ we obtain

$$\begin{aligned} V(\tau_k, x(\tau_k)) = {} & V(\tau_{k-1}, x(\tau_{k-1})) + [V(\tau_{k-1}, x(\tau_{k-1}) + \Delta x(\tau_{k-1})) - V(\tau_{k-1}, x(\tau_{k-1}))] \\ & + \int_{\tau_{k-1}}^{\tau_k} \frac{\partial V(t,x(t))}{\partial t} + \frac{\partial V(t,x(t))}{\partial x}\left[A_i x(t) + B_i(u_i(t) + f(t,x(t)))\right] ds \end{aligned} \tag{5.12}$$

Hence it follows from (5.9) and (5.10) that

$$V(\tau_k, x(\tau_k)) \leq V(\tau_{k-1}, x(\tau_{k-1})) \text{ and } V(t, x(t)) \leq V(\tau_k, x(\tau_k))$$

Thus we have $V(\tau_0, x(\tau_1)) \leq V(\tau_1, x(\tau_1)), \ldots, \leq V(t, x(t))$. Since $V(t, x(t))$ is monotonically decreasing and has the lower bound zero, it will converge to zero, i.e., the complex manufacturing system (5.1) is asymptotically stable. This completes the proof.

If a Lyapunov function $V(t, x(t))$ satisfying (5.9) and (5.10) for the complex manufacturing system (5.1), then system (5.1) is asymptotically stable, and this implies that $V(t, x(t))$ must continuously diminish along each orbit as t increases, i.e., $V(t, x(t))$ keeps an orbit that starts near the origin, without crossing its level sets.

Suppose that there exist symmetric positive definite matrices P_i and Q_i with appropriate dimensions and a scalar $\epsilon > 0$ satisfying $\eta_i = \lambda_{\max}(P_{i+1}^{-1}(I + F_i B_i^T P_i)^T P_i (I + F_i B_i^T P_i)) \leq 1$ and the following linear matrix inequalities:

$$\begin{bmatrix} P_i(A_i - B_iK_i) + (A_i - B_iK_i)^T P_i + \phi_i & I \\ I & -Q_i^{-1} \end{bmatrix} \leq 0 \tag{5.13}$$

where $K_i = (B_i^T P_i B_i)^{-1} B_i^T P_i A_i$ and $\phi_i = \epsilon^{-1} E^T E P_i + \epsilon H^T H$. Then the hybrid sliding mode control can stablize the complex manufacturing system (5.1).

Proof: We choose a Lyapunov function candidate

$$V_i(t, x(t)) = x(t)^T P_i x(t) + \int_0^t x^T(r) Q x(r) dr \tag{5.14}$$

and we design the sliding surface as $S = \{x(t) \in R^n : s_i(x) = B_i^T P_i x(t) = 0\}$. Next, we take the differential of Lyapunov candidate $V_i(t, x(t))$ along the trajectory of the complex manufacturing system (5.1) to get

$$\begin{aligned} \dot{V}_i(t, x(t)) &= \dot{x}^T(t) P_i x(t) + x^T(t) P \dot{x} + x(t)^T Q_i(t) x(t) \\ &= x^T(t)(A_i - B_iK_i)^T P_i + x^T(t) P_i (A_i - B_iK_i)^T x(t) \\ &\quad + x^T(t)(u_{r(i)}(t) + f(t, x))^T B_i^T P_i \\ &\quad + (P_i(u_{r(i)}(t) + f(t, x)) B_i^T x(t) + x(t)^T Q_i(t) x(t) \end{aligned} \tag{5.15}$$

Substituting (5.13) to (5.15) yields

$$\dot{V}_i(t, x(t)) \leq 2x^T(t) P_i B_i (u_{r(i)}(t) + f(t, x)) \tag{5.16}$$

From (5.6) and (5.15) we have

$$\begin{aligned} & 2x^T(t) P_i B_i u_{r(i)}(t) + 2x^T(t) P_i B_i f(t, x) \\ = \ & 2s_i^T u_{r(i)}(t) + 2s_i^T f(t, x) \\ = \ & -2s_i^T P_i s_i - 2\rho_0 ||s_i|| + 2s_i^T f(t, x) \\ \leq \ & -2s_i^T P_i s_i < 0 \end{aligned} \tag{5.17}$$

which ensures that (5.9) holds. Moreover, at impulsive time points τ_k, $V(\tau_k, x(\tau_k))$

satisfies

$$\begin{aligned} V(\tau_k^+, x(\tau_k^+)) &= (x(\tau_k) + \Delta x(\tau_k))^T P_i (x(\tau_k) + \Delta x(\tau_k)) \\ &= x(\tau_k)^T (I + F_i B_i^T P_i)^T P_i (I + F_i B_i^T P_i) x(\tau_k) \end{aligned} \tag{5.18}$$

Then we have

$$V(\tau_k^+, x(\tau_k^+)) \le \eta_i V(\tau_k, x(\tau_k)) \le V(\tau_k, x(\tau_k))$$

We see that (5.9) and (5.10) are guaranteed. Thus, by Theorem 5.3.3, we conclude that the complex manufacturing system (5.1) is asymptotically stable under the given hybrid sliding mode controller (5.5). The complex manufacturing system (5.1) changes the system dynamics instantaneously at impulsive time points. So, impulsive time windows are decision variables that are selected during operation to achieve optimal system dynamics. Here, we adopt the results in [137] and use Lyapunov function and selected cone as follows;

$$\begin{aligned} \Omega(i) = \{&x(t) \in R^n : x^T(t)(P_i(A_i - B_i K_i) + (A_i - B_i K_i)^T P_i) x(t) \\ &\le -\frac{1}{N} x^T(t)(Q_i + \epsilon^{-1} N E_i^T E_i P_i + \epsilon N H_i^T H_i) x(t)\} \end{aligned} \tag{5.19}$$

The right hand side $-\frac{1}{N} x^T(t) Q_i x(t)$ leads to an asymptotically stable system, where N denotes the number of system modes. Therefore in the presence of impulses and if we apply Theorem 5.3.3 and impulsive time window

$$\tau_{i+1} - \tau_i \ge \delta_k \tag{5.20}$$

So the minimal impulsive switch time can be obtained by

$$T_j := \arg\min \left\{ \delta_k, -\frac{1}{N} x^T(t)(Q_i + \epsilon^{-1} N E_i^T E_i P_i + \epsilon N H_i^T H_i) x(t) \right\} \tag{5.21}$$

5.4 Numerical Results

We validate the effectiveness of the proposed method with a numerical example in a manufacturing setting, which is composed of production mode and (re)manufacturing mode. A demand rate can be governed by a function $f(t, x_1, x_2)$, where x_1 is associated with products of good quality while x_2 represents goods that need (re)processing. Rework increases the production cost and the produce excess of x_1, and incur more cost on inventory holding cost. Therefore the objective of this problem is to minimize the manufacturing cost, by eliminating excess inventory and producing at an optimal production rate to meet customers' demands.

It is also known that the plant mode is controlled by an impulsive time window δ_k to switch between the production mode and the re-manufacturing mode. Furthermore we know that returns goods in the system cause shortages, and from economic approach, inventory models are used for balancing, and to cut the manufacturing cost, we model a manufacturing plant that produces according to a hedging policy and a Just-in-Time rule.

We consider a two mode manufacturing plant that consists of

Production mode:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t) + u_1(t) + f_1(x(t)) \\
\dot{x}_2(t) &= -rx_1(t) \\
\Delta x(t) &= F_1 B_1^T P_1 x(t),\ x(t) = [x_1(t)\ x_2(t)]^T
\end{aligned} \tag{5.22}$$

Re-manufacturing mode:

$$\begin{aligned}
\dot{x}_1(t) =& x_1(t) - f_1(x(t)) \\
\dot{x}_2(t) =& x_2(t) - rx_2(t) + u_1(t)
\end{aligned} \tag{5.23}$$

where x_1 is the quantity of finished goods, x_2 is the quantity of returned goods, r is the rate of return goods with demand rate $f_1(t,x) = \bar{a}\sin^2(x(t))$ and $u_1(t)$ is the production rate. The system parameters can be rewritten and set as

$$A_1 = \begin{bmatrix} 1 & 0 \\ -r & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 0 & (1-r) \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, Q_2 = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 2 & 5 \\ 5 & 5 \end{bmatrix}, E_2 = \begin{bmatrix} 2 & 25 \\ 25 & 25 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.2700 & 0 \\ -0.1542 & 0 \end{bmatrix}, F_2 = \begin{bmatrix} 0.3084 & 0 \\ -0.1542 & 0 \end{bmatrix}, 0 < \epsilon \leq 0.05,$$

$$0 < \bar{a} < \rho_0 = 4.5, \quad 5\% < r < 85\%.$$

Solving the linear matrix inequalities (5.13), we obtain

$$P_1 = \begin{bmatrix} 2.3323 & -04269 \\ 0.4269 & 0.9043 \end{bmatrix}, P_2 = \begin{bmatrix} 2.7151 & 0.2420 \\ 0.2420 & 3.3674 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 2.3323 & 0.4269 \\ 0.4269 & 0.9043 \end{bmatrix}, K_2 = \begin{bmatrix} 2.7151 & 0.2420 \\ 0.2420 & 3.3674 \end{bmatrix}$$

And from LMI (5.13) it can be found that $||\phi_1|| > 0$ and $||\phi_2|| > 0$ and thus meet criteria of our theorems. Furthermore the simulation results verify our main results. This also supported by a hedging policy to be implemented in inventory manufacturing system.

Example 2: We extended and validate the effectiveness of hybrid impulsive slide mode controller with a well known chaotic system presented in [53]. The system investigated experienced chaos that need to be suppressed by impulsive slide mode control method proposed.

We consider Lorenzo Chaotic system

$$\begin{aligned} \dot{x}_1(t) &= (2a+10)(x_2(t) - x_1(t)) \\ \dot{x}_2(t) &= (28-35a)x_1(t) - x_1(t)x_3(t) + (29a-1)x_2(t) \\ \dot{x}_3(t) &= (x_1(t)x_2(t) + \frac{(a+8)x_3(t)}{3} \end{aligned} \tag{5.24}$$

where a={-2,2} Since B_i is a gain matrix that need o be designed, we choose this gain matrix to be equivalent to $B_i u(t)$ and similarly if we consider $f(t, x(t))$ as disturbance, and choose $B_i = I$ we can introduce impulsive slide mode controller according to the main theorem, and transform equation 5.24 into;

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i(u_i(t) + f(t,x)) \quad \forall\, t \neq \tau_k \\ \Delta x(t) &= F_i B_i^T P_i x(t) = x^+(\tau_k) - x^-(\tau_k) \qquad\quad t = t_k \end{aligned} \tag{5.25}$$

where

$$A = \begin{bmatrix} -(2*a+10) & (25*a+10) & 0 \\ 28-35*a & 29*a-1 & 0 \\ 0 & 0 & -(a+8)/3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} 5 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix},$$

$$P = \begin{bmatrix} 25.9036 & -22.6945 & 0.0037 \\ -22.6945 & 60.4087 & 0.0435 \\ 0.0037 & 0.0435 & 0.2873 \end{bmatrix}, F = \begin{bmatrix} 0 & 0.0258 & 1.0000 \\ 0 & 0.0258 & 0 \\ 0 & 0.0258 & 1.0000 \end{bmatrix},$$

$$f(t, x(t)) = \begin{bmatrix} 0 & -x_1(t)x_3(t) & x_1(t)x_2(t) \end{bmatrix}^T,$$

If there exist $P = P^T$ that solve quadratic Lyapunov function and satisfied theorem (2) such $||B^T f(t, x(t))||$ is bounded above, with $\delta_k = \tau_{k+1} - \tau_k$. Then the anticipated results of theorem 5.3.3 hold. Then this implies that the trivial solution of (5.24) is asymptotically stable with proper design of matrix F.

The standard design of matrix F should be investigated further. As it has been seen in example 5.2 that wrong design of matrix F can led to unstable dynamic system stability. Furthermore and from our results, we can inferred that the proper design of the impulsive slide mode controller, can stabilize and suppress the chaos of Lorenzo system (5.24) such that the system is globally asymptotically stable see Fig. 5.13. while without control, the system shows chaotic behavior see Fig. 5.15.

Figure 5.1 shows dynamical trajectories of the manufacturing system (5.22) without controllers. It is clear to see that the manufacturing system (5.22) is unstable if not controlled. And Fig 5.2 depicts unstable controller without sliding mode control. And from Figure 5.3 a hybrid impulsive sliding mode control stabilize our manufacturing systems. The adopted controller was also effective when a manufacturing system experiences high demand, as shown in Figure 5.4. We further shows the selected manifold in Figure 5.7 to Figure 5.9.

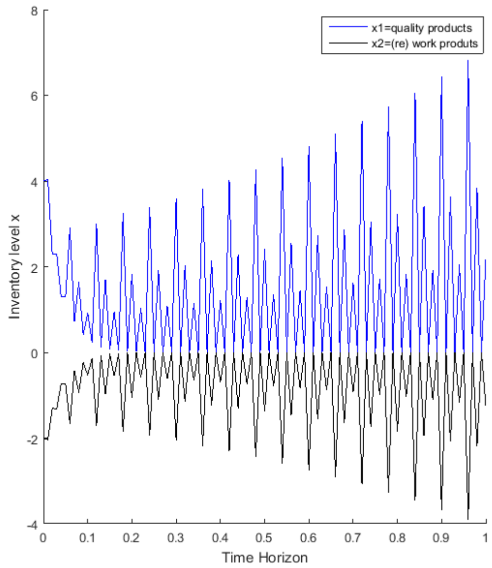

Figure 5.1: Unstable inventory levels without hybrid controllers

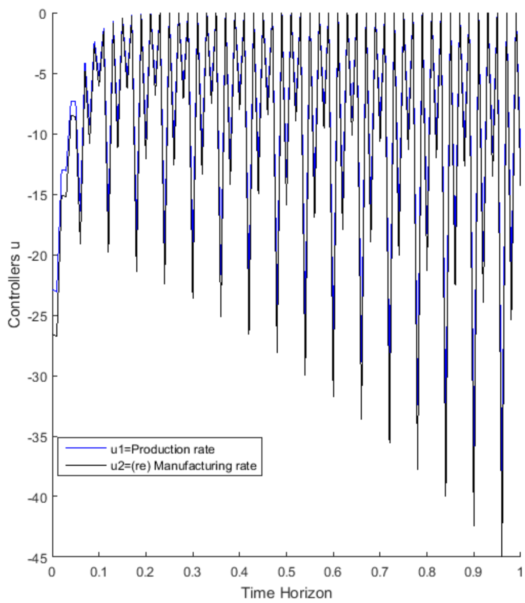

Figure 5.2: Unstable system with impulsive control only

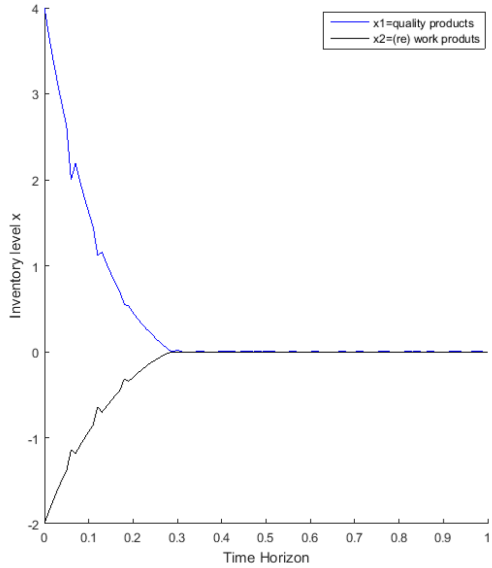

Figure 5.3: Stable manufacturing system trajectory with Hybrid impulsive sliding mode control

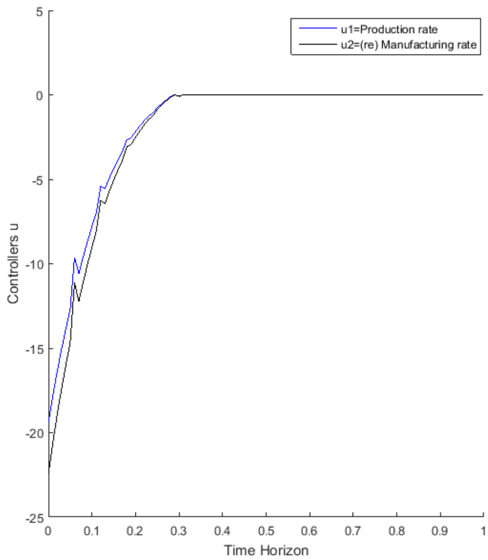

Figure 5.4: Manufacturing system controllers with hybrid impulsive sliding mode control

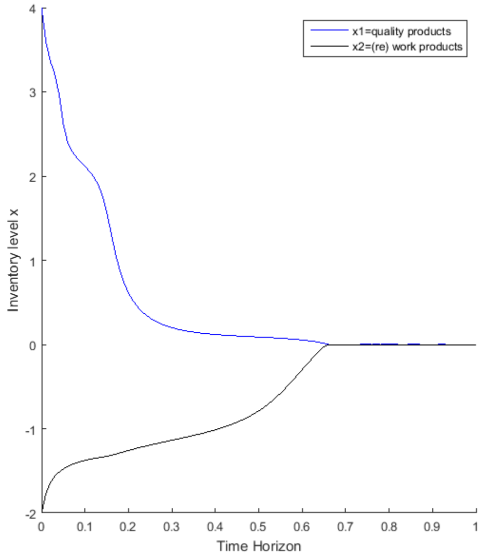

Figure 5.5: Stable manufacturing system inventory level with high demand

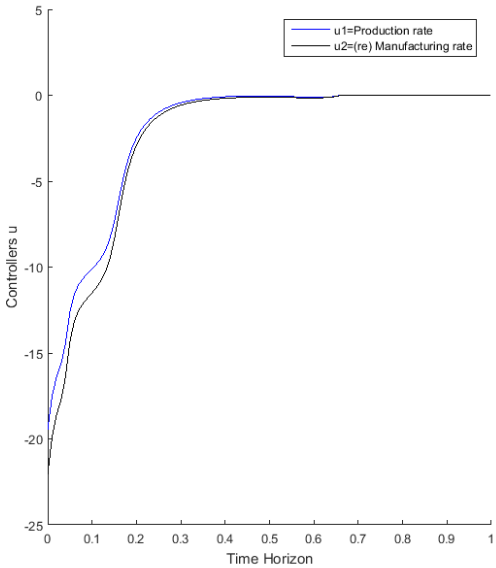

Figure 5.6: Stable manufacturing controllers with high demand

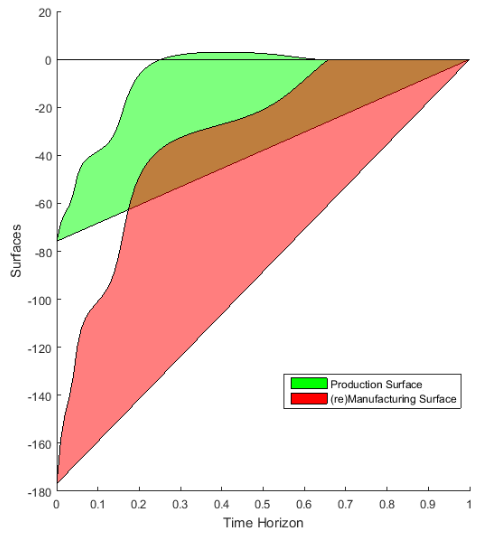

Figure 5.7: Selected manufacturing surfaces with sliding mode control

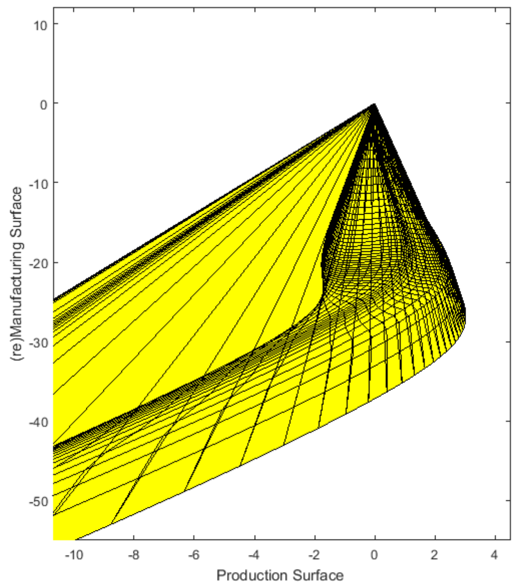

Figure 5.8: Selected manifold with sliding mode control

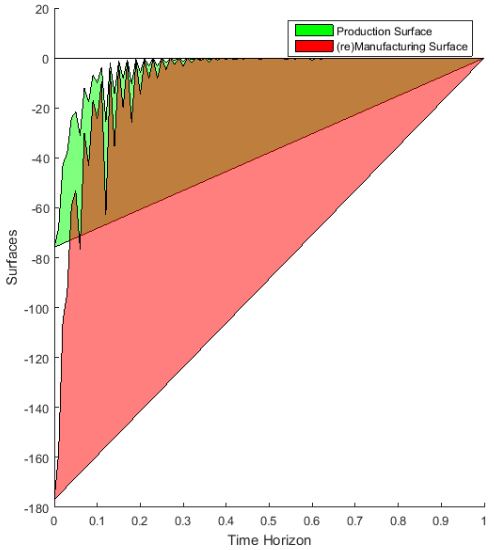

Figure 5.9: Selected manufacturing system surfaces

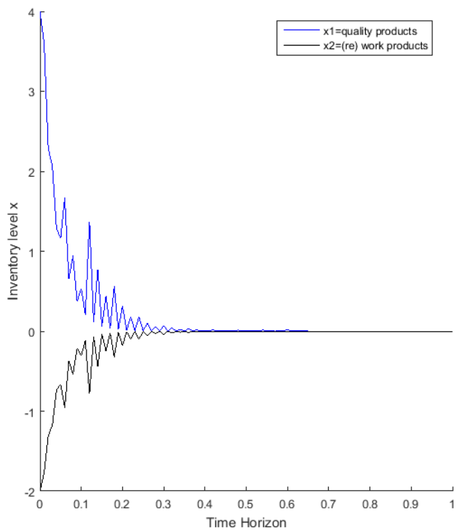

Figure 5.10: Impulsive slide mode inventory level with high demand rate

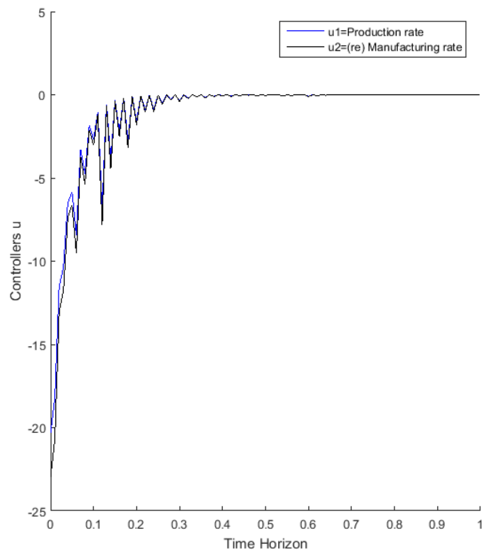

Figure 5.11: Impulsive sliding mode controllers with high demand rate

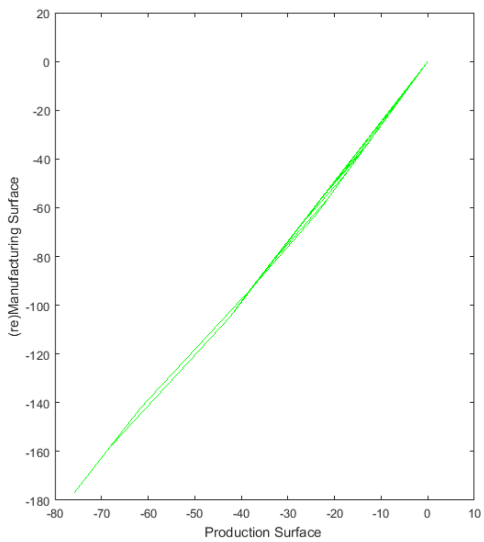

Figure 5.12: Selected manufacturing system surfaces controllers

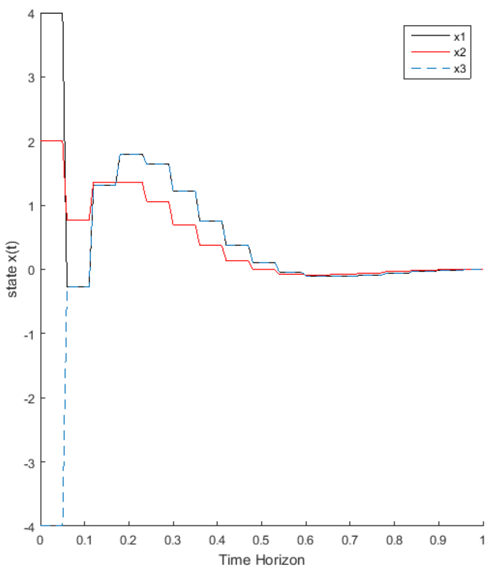

Figure 5.13: Lorenzo system with stable states-with impulsive sliding mode controller

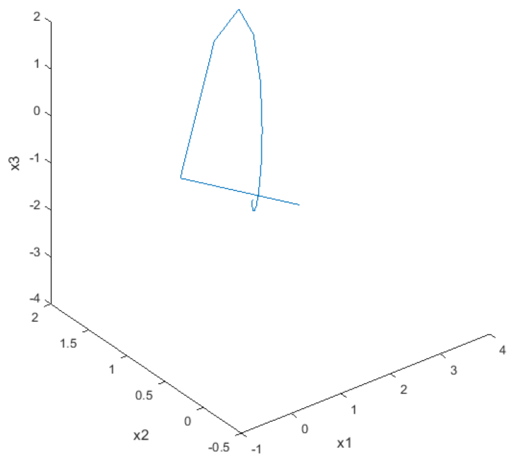

Figure 5.14: Lorenzo system with stable states with hybrid impulsive controller

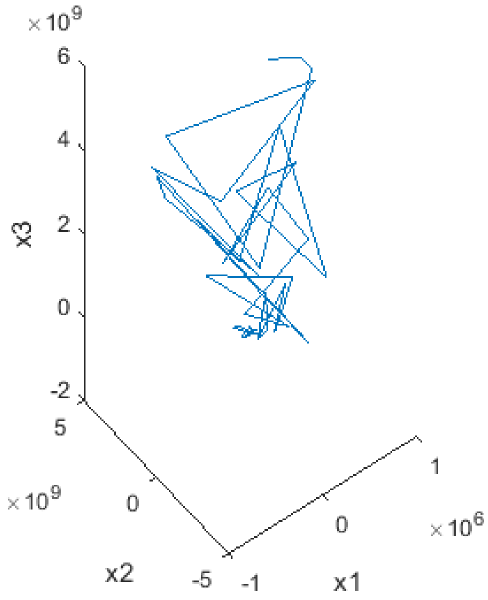

Figure 5.15: Lorenzo system unstable - no impulsive sliding mode controller

5.5 Chapter Summary

The problem of finding suitable controller that stabilizes dynamical hybrid systems has been considered in this Chapter. The goal of proposed method was to select controllers for manufacturing and (re)manufacturing mode that stabilize the production line. Main theorems results and sufficient conditions to grantee stability of trivial solution are given. And a hybrid impulsive slide mode control is adopted to stabilize the manufacturing hybrid system by confining systems dynamics within selected manifold. While in the past, especially with Toyota production line, the stability of manufacturing plant was achieved by introducing a moving assembly line. That is to say the plan was to keep a minimum products at all times in an assembly line. Therefore the conceptual framework developed in this chapter mitigate the problem of backlog or inventory build up within the production line. Therefore the feasibility and viability of these technique is shown with an example from inventory control and we found that the technique proposed is in-line with hedging policy operation that supports production of producing Just- In-Time. Furthermore, as an application to other system, we have developed a new hybrid impulsive slide mode control strategy, that suppresses chaos in a well known Lorenzo system. And one future direction is to investigate controllers that stabilize the manufacturing system with delay using impulsive sliding mode control.

Chapter 6

Conclusions and Future Research Directions

6.1 Main Contributions of the Thesis

In this thesis, we managed to show the imminent need of adopting hybrid manufacturing system framework, in today's industrial revolution 4.0 whereby technological development has an enormous impact on the creation of wealth and socio-political stability among many countries. The industries 4.0 combines technological development and human capabilities in an unprecedented way through the integration of controllers that enable the application of self- learning algorithm, self-regulating machines, big data and Internet of Things.

We found that the most vital parameter on production decision making in manufacturing setting is flexibility, which is seen as a promising factor for industrial performance capabilities for a system to reach an optimum objective value. We set a hybrid manufacturing model and develop algorithms that balance manufacturing firm constraints and customer constraints by developing a hybrid model and a new smoothing algorithm for the cost balancing between the quality and the job tardiness by finding optimal service time of each job in the system.

From chapter 3, we inferred that the presence of maximum operator in the objective function, makes the problem to be non-smooth, and thus makes the decent gradient method to fail especially if critical jobs exist in the busy structure. Also we concluded that the optimum service time of each job is directly proportional to job quality such that the longer service time incur more cost, and shorter service time led to poor job quality. We further found that the waiting scenario, gives minimum cost function with optimum service time and optimum job quality. And a balance between trade-off cost was achieved through development and selection of the best controller from proposed algorithm.

It is observed that the quantity of products manufactured plays important role on the manufacturing optimisation. Therefore, in order to achieve cost-effective production-inventory system, we have developed a smooth algorithm and apply Hamilton-Jacobi-Bellman equations to determine the production rate of an optimal inventory trajectory of the hybrid system. And then we find controllers that minimize the cost of inventory hybrid (re)manufacturing system with states jumps in chapter 4.

We further observed that, for a successful and competitive manufacturing firm, it needs to be incorporated with controllers that are handy, to control inventory level and production rate to meet customer demands at unprecedented level. And a two stage approach was required and we incorporated the application of Markov Decision Process, Bellman equation and with the steady state probabilities, we used mixed integer linear programming to select best decision of how to operate a manufacturing firm, and found that a minimal inventory should be kept at all time to reduce production cost.

We further introduced, a sliding mode control that switches between manufacturing and (re)manufacturing systems. And we develop a conceptual framework to mitigate the problem of backlog or inventory build up within the production line. Therefore the feasibility and viability of these techniques are effective for inventory control and we find that the technique proposed is in-line with hedging policy operation that supports production of producing Just-in-Time.

The stability of manufacturing systems using Lyapunov feedback function was further discussed in chapter 5, and we found that, the design of sliding mode controller proposed was more effective, as we were able to find a balance between manufacturing and (re)manufacturing mode. And we have observed that manufacturing systems that inherent, this behavior of stability are more robust and flexible especially in chaotic environment.

6.2 Future Research Directions

We have successfully managed to tap on controllers of hybrid manufacturing systems stability and developed mathematical algorithms which are still at an infant stage. Therefore, more work needs to be done for the development of advanced mathematical models and algorithms that will allow the integration of cloud computing, artificial intelligence and optimal allocation of machines and determination of service time of processing each job. Our future work will expand to modelling real-time manufacturing systems, with the aid of RFID, machine learning and other concepts from industries 4.0. We intend to develop self-regulating or self optimizing manufacturing systems with capabilities of being data-driven decision making.

We will further investigate

- The stability of manufacturing systems using Lyapunov feedback functions after the design of our controller.
- Controllers that stabilize the manufacturing system with delay using impulsive sliding mode control.
- Our future work will incorporate n stage and stochastic arrivals of jobs in a re-manufacturing hybrid system.

We also intend to extend our methods to other applications such as power system, autonomous and hybrid vehicles. And since we have observed that electric vehicles have been entering market recently and are considered to be the future mode of transport after successful of development of lithium-ion batteries that would able to power electric cars more efficiently and effectively as compared to gasoline vehicles. Currently other researches are working on lithium-ion battery development, and we foresee that a sliding mode control developed in our work can be used to regulate the charging systems of lithium batteries.

Appendices

Appendix A

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November 25, 2019

TO WHOM IT MAY CONCERN

Subject: Percentage of Paper Contributions

I would like to inform you that I worked as a co-author for the following two papers:

1. Kobamelo Mashaba, Jianxing Li, Honglei Xu, Xinhua Jiang. Optimal control of hybrid manufacturing systems by log-exponential smoothing aggregation. Discrete and Continuous Dynamical Systems - S, doi: 10.3934/dcdss.2020100
2. Kobamelo Mashaba, Honglei Xu, Jianxiong Ye. Stabilization Of Complex Manufacturing Systems With State Impulsiveness By Hybrid Sliding Mode Control, Dynamics of Continuous, Discrete and Impulsive Systems, Series B: Applications and Algorithms 26 (2019) 291-302, Copyright c 2019 Watam Press

This letter is written to address Mr Kobamelo Mashaba responsible contribution. Mr Mashaba developed all mathematical models, and algorithms and simulated the results. He also wrote the first draft of the manuscript with the contribution of 70% in paper 1 and 75% in paper 2

Sincerely,

[redacted]

Honglei Xu, Senior Lecturer

November 25, 2019

TO WHOM IT MAY CONCERN

I am writing to address Mr. Kobamelo Mashaba responsible contribution. He developed all mathematical models, and algorithms and simulated the results.He also wrote the manuscripts and contributed to the manuscript revising processes, and the overall quality of the manuscripts.

I am happy to inform of his contribution of 75% in "Kobamelo Mashaba, Honglei Xu, Jianxiong Ye. Stabilization Of Complex Manufacturing Systems With State Impulsiveness By Hybrid Sliding Mode Control, Dynamics of Continuous, Discrete and Impulsive Systems, Series B: Applications and Algorithms 26 (2019) 291-302, Copyright c 2019 Watam Press"

Sincerely ,

Dr Jianxiong Ye
College of Mathematics and
Computer Science

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Fujian Normal University
Fuzhou, China

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