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A conceptual model of core dynamics and the earth's magnetic field

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Abstract. A conceptual model of core dynamics and the earth's magnetic field is presented. It differs from previous investigations in the use of an estimated core viscosity of $2 \times 10^7\ \mathrm{cm^2\,s^{-1}}$. The simplified derivations predict the correct order of magnitude for the external magnetic field and for the westward drift of the nondipole field.

Key words: Earth core – Magnetic field of earth – External magnetic field – Dipole field – Fluid dynamo – Westward drift of nondipole field

Introduction

The theory of magnetic field generation by a fluid dynamo in the core of the earth has received a great deal of attention over the past several years by a number of investigators. The early investigations concentrated on demonstrating that solutions to the magnetic induction equation would lead to an external dipole field, e.g., Elsasser (1941, 1946a, b, 1947, 1950, 1956) and Bullard (1948, 1949a, b). Later investigations concentrated on the core dynamics and the interactions and controls between the hydrodynamic flow and the magnetic field, e.g., Bullard and Gellman (1954), Parker (1955), Hide (1956), Herzenberg (1958), Hide and Roberts (1961), Rikitake (1966), Roberts (1971), Busse (1975, 1976, 1983), Gubbins (1974, 1976), Levy (1976), Braginsky (1976), Watanabe (1977) and Soward (1982). Concurrently, there have been presentations of various possible mechanisms that could lead to the observed polarity reversals, e.g., Parker (1969), Levy (1972a, b) and Robbins (1976, 1977).

There are, indeed, various ways in which an external dipole field can be generated by thermal convection of an electrical conductive fluid in a rotating earth. An important consideration has to be the values of the core parameters which delineate the hydrodynamic flow and the magnetic field, particularly the viscosity.

As discussed in the next section, the kinematic viscosity of the core is one of the least known parameters, with estimated values ranging from $10^{-3}\ \mathrm{cm^2\,s^{-1}}$ to $10^{11}\ \mathrm{cm^2\,s^{-1}}$. Most of the previous investigations have used a core viscosity of around $10^{-2}\ \mathrm{cm^2\,s^{-1}}$, based on theoretical estimates, or have ignored core viscosity effects. This value is not in accord with values determined from various observable geophysical parameters. In particular, a direct interpretation of the damping of seismic waves propagating through the fluid core leads to a value of $2 \times 10^7\ \mathrm{cm^2\,s^{-1}}$.

The purpose here is to develop a conceptual model of core dynamics and the earth's magnetic field using this value of kinematic viscosity. The derivations are simplified and apply to steady state conditions and the central regime of the convective flow.

The results are encouraging in that they lead to correct order of magnitude estimates for the external field strength and the westward drift, and correctly predict that the nondipole components are related to boundary layer flow effects near the mantle-outer core boundary.

With a viscosity of $2 \times 10^7\ \mathrm{cm^2\,s^{-1}}$ there is a global circulation including a predominant boundary layer flow; with a viscosity of $10^{-2}\ \mathrm{cm^2\,s^{-1}}$ the flow breaks up into small scale cyclonic circulations. In essence, the former resembles a worldwide oceanic circulation; and the latter resembles small scale, atmospheric geostrophic circulations.

Outer core parameters

Table 1 lists the parameter values used in the subsequent derivations and calculations. They include d_1, inner core radius; d_2, outer core radius; Ω, angular rotational velocity of the earth; g, gravity for outer core; ρ_o, outer core density; α, thermal coefficient of expansion; c_p, specific heat; k, thermal conductivity; μ, magnetic permeability; σ, electrical conductivity; ν, kinematic viscosity for outer core; ϕ, latitude; H_o, magnetic dipole field at mantle-outer core boundary; and Q, heat conduction at mantle-outer core boundary. The table also includes the dependent parameter values d, outer core thickness; a, Coriolis parameter; D, frictional depth for Ekman-type flow; R, ratio of Lorentz- to Coriolis-forcing terms in the motion equations; λ, horizontal density gradient; and κ, thermal diffusivity.

The values for α, c_p, k, μ and σ are the same as those used by Bullard and Gellman (1954) and are the same or similar to the values used by subsequent investigators. The value for H_o is taken from the consideration that magnetic field strength was about one-half

Table 1. Core parameter values used in derivations and calculations

$d_1 = 1{,}200$ km	$d_2 = 3{,}500$ km
$\Omega = 7.3 \times 10^{-5}\ \mathrm{s}^{-1}$	$g = 800\ \mathrm{cm\,s}^{-2}$
$\alpha = 4.5 \times 10^{-6}\ {}^{\circ}\mathrm{C}^{-1}$	$\rho_o = 10.6\ \mathrm{g\,cm}^{-3}$
$\mu = 1$ emu	$\sigma = 3 \times 10^{-6}$ emu
$c_p = 0.16\ \mathrm{cal\,g}^{-1}\,{}^{\circ}\mathrm{C}^{-1}$	$k = 0.10\ \mathrm{cal\,cm}^{-1}\,{}^{\circ}\mathrm{C}^{-1}\,\mathrm{s}^{-1}$
$\nu = 2 \times 10^{7}\ \mathrm{cm}^2\,\mathrm{s}^{-1}$	$Q = 0.1 \times 10^{-6}\ \mathrm{cal\,cm}^{-2}\,\mathrm{s}^{-1}$
$H_o = 1.9$ gauss	$\phi = 30°$

$$d = d_2 - d_1 = 2{,}300\ \mathrm{km}$$

$$a = \left[\frac{\Omega \sin\phi}{\nu}\right]^{1/2} = 1.35 \times 10^{-6}\ \mathrm{cm}^{-1}$$

$$D = \frac{\pi}{a} = 23.3\ \mathrm{km}$$

$$R = \frac{\sigma \mu^2 H_o^2}{2\Omega \rho_o \sin\phi} = 0.0140$$

$$\lambda = \left[\frac{8\alpha a^3 \rho_o \nu Q}{c_p g d}\right]^{1/2} = 0.253 \times 10^{-15}\ \mathrm{g\,cm}^{-4}$$

$$\kappa = \frac{k}{\rho_o c_p} = 0.0590\ \mathrm{cm}^2\,\mathrm{s}^{-1}$$

its present value throughout the Phanerozoic, e.g., Merrill and McElhinny (1983). The value for Q is taken from the discussion by Verhoogen (1980); this heat flow value per unit area at the mantle-outer core boundary is approximately one-tenth that at the earth's outer surface.

The critical parameter in Table 1 is the viscosity, ν, of the outer core. It is one of the least well known parameters with estimated values differing by several orders of magnitude, ranging from 10^{-3} to $10^{11}\ \mathrm{cm}^2\,\mathrm{s}^{-1}$, e.g., summaries given by Hide (1956), Hide and Roberts (1961) and Jacobs (1975). The lower values have come from theoretical estimates and the higher values from seismological determinations.

The general practice of those who have made theoretical investigations of various aspects of the earth's magnetic field and core dynamics has been to assume a viscosity of around $10^{-2}\ \mathrm{cm}^2\,\mathrm{s}^{-1}$, e.g., Bullard Roberts (1961), Roberts (1967), Gubbins (1974) and Busse (1975, 1976, 1983). In the latter studies this viscosity value was based on the theoretical estimate of Gans (1972). Understandably, the choice of the low viscosity estimate leads to a very different form of core dynamics and delineation of the controlling factors between the hydrodynamic flow and magnetic field than if the assumed viscosity had been several orders of magnitude larger.

The procedure here uses a viscosity based on seismological observations. The assumption is made that the Navier-Stokes equation applies not only to hydrodynamic flow in the outer core but also to seismic wave propagation through the outer core. This method is the same as that used by Jeffreys (1926, 1952), which gives an upper limit estimate for core viscosity. The viscous damping term for seismic wave propagation is of the form, e.g., Lamb (1932),

$$e^{-\frac{2\nu\omega^2}{3c^3}s} \tag{1}$$

Table 2. Seismological determinations of the kinematic viscosity of the outer core

Source	Q'	P (s)	ν ($\mathrm{cm}^2\,\mathrm{s}^{-1}$)
Sacks (1971)	10,000	1	1×10^7
Buchbinder (1971)	4,000	1	2×10^7
Adams (1972)	$>2{,}200$	1	$<4 \times 10^7$
Qamar and Eisenberg (1974)	5,000–10,000	1	$1\text{–}2 \times 10^7$

where ω is the circular frequency, c the compressional wave velocity and s the ray path distance. The corresponding relation between ν and the seismic dissipation parameter, Q', is, e.g., Stacey (1977),

$$\nu = \frac{3c^2 P}{8\pi Q'} \tag{2}$$

where P is the period of the seismic wave. There have been several determinations of the attenuation of P waves propagating through the core. These results are summarized in Table 2, along with the estimates for ν made from Eq. (2) using a value of $c = 9 \times 10^5\ \mathrm{cm\,s}^{-1}$. A value of $\nu = 2 \times 10^7\ \mathrm{cm}^2\,\mathrm{s}^{-1}$ was chosen as representative of the outer core viscosity.

There are a number of other observable geophysical parameters from which estimates of core viscosity can be made. Several lead only to an upper limit for the viscosity and, in general, these estimates appear somewhat less definitive than the procedures used in the previous paragraph.

Free oscillations of the earth caused by large earthquakes also undergo damping, and permit estimates for the seismic dissipation factor, Q', to be made as a function of earth radial distance. The damping in the outer core, however, is quite small compared with that in the mantle and precludes more than an approximate estimate of the outer core Q' value. Anderson and Hart (1978a, 1978b) quote a value of $Q' = 10^6$ for the outer core in their earth model. The same relation (2), between ν and Q', applies to free oscillations as well as to seismic wave propagation. Using an appropriate value of $P = 1{,}000$ s, this Q' value converts to $\nu = 14 \times 10^7\ \mathrm{cm}^2\,\mathrm{s}^{-1}$.

Sato and Espinosa (1967) and Suzuki and Sato (1970) have estimated values of the viscosity in the outer core at the mantle-outer core boundary from reflected shear waves. Their results are $\nu = 8 \times 10^{10}\ \mathrm{cm}^2\,\mathrm{s}^{-1}$ and $5 \times 10^9\ \mathrm{cm}^2\,\mathrm{s}^{-1}$, respectively.

Upper limit estimates of core viscosity from a variety of observed geodetic parameters, particularly the damping of the Chandler wobble, have had a long and complex history. Molodenskiy (1981) has recently given upper limit estimates of $10^6\ \mathrm{cm}^2\,\mathrm{s}^{-1}$ from the amplitudes of the forced nutation of the earth, $2 \times 10^9\ \mathrm{cm}^2\,\mathrm{s}^{-1}$ from the damping of the Chandler wobble, and $10^7\ \mathrm{cm}^2\,\mathrm{s}^{-1}$ from tidal variations in the length of day.

Theoretical estimates include those by Bondi and Lyttleton (1948) of $10^7\ \mathrm{cm}^2\,\mathrm{s}^{-1}$, Gans (1972) of $10^{-2}\ \mathrm{cm}^2\,\mathrm{s}^{-1}$, Bukowinski and Knopoff (1976) of $10^0\ \mathrm{cm}^2\,\mathrm{s}^{-1}$, and Watanabe (1977) of $10^3\ \mathrm{cm}^2\,\mathrm{s}^{-1}$.

Derivations and results

The equations defining the magnetohydrodynamics of the outer core are the Navier-Stokes hydrodynamic equation, the heat equation and the magnetic induction equation,

$$\frac{\partial v}{\partial t}+(v\cdot\nabla)\,v=-\frac{1}{\rho_o}\nabla p+\frac{1}{\rho_o}\rho g+\nu\nabla\cdot\nabla v-2\Omega\times v+\frac{\mu}{4\pi\rho_o}(\nabla\times H)\times H \tag{3}$$

$$\frac{\partial T}{\partial t}+(v\cdot\nabla)\,T=\frac{q}{\rho_o c_p}+\kappa\nabla\cdot\nabla T \tag{4}$$

$$\frac{\partial H}{\partial t}+(v\cdot\nabla)\,H=(H\cdot\nabla)\,v+\frac{1}{4\pi\sigma\mu}\nabla\cdot\nabla H \tag{5}$$

where v is the flow velocity, p the pressure, ρ the density, T the temperature, q the internal heat generated per unit volume, H the magnetic field strength and the other parameters are as defined in the previous section. In addition there are the flow and magnetic flux continuity relations and the equation of state relating density and temperature,

$$\nabla\cdot v=0 \tag{6}$$

$$\nabla\cdot H=0 \tag{7}$$

$$\frac{\partial\rho}{\partial T}=-\alpha\rho_o. \tag{8}$$

They form a set of coupled, nonlinear partial differential equations. The first two terms on the right hand side of Eq. (3) are the forcing terms, the third the viscous resistance term, the fourth the Coriolis or geostrophic term, and the fifth the Lorentz term. The ultimate driving force for the system is related to the heat flux through the core. The resultant temperature, or density, gradient provides the driving force for the hydrodynamic flow of Eq. (3), and the flow, itself, provides the generating term for the magnetic field of Eq. (5).

The interest, here, is in obtaining solutions to these equations under conditions of an assumed outer core viscosity of $\nu=2\times10^7\ \mathrm{cm^2\,s^{-1}}$, in order to delineate the resultant core dynamics as well as to ascertain whether such an outer core viscosity is plausible considering the observed external magnetic field strength and secular changes. Further, it is of interest to retain all three of the terms that may affect the flow, viscous, Coriolis and Lorentz, in the solutions.

We assume steady state conditions. We assume the flows are sufficiently small for the nonlinear term, $(v\cdot\nabla)\,v$, to be neglected in consideration of the other terms in Eq. (3). We assume that the flow is hemispherically symmetrical across the equator and that the first mode of the convection flow is dominant. In the absence of Coriolis and Lorentz effects, the flow pattern would be as shown in Fig. 1. We shall solve the equations in Cartesian coordinates, thus ignoring sphericity effects. For the northern hemisphere we take a coordinate system as shown in Fig. 2.

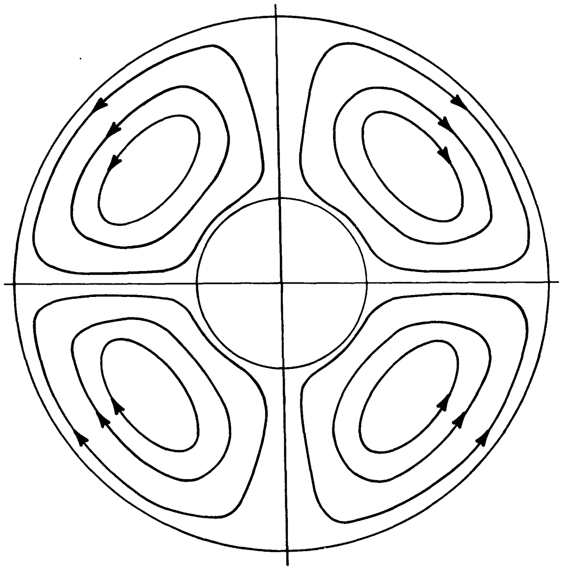

Fig. 1. Thermal circulation in the outer core in the absence of Coriolis and Lorentz forcing terms

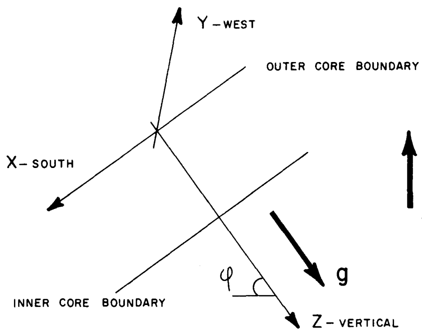

Fig. 2. Coordinate system

We look for solutions applicable to the central, or mid-latitude, region. Under these conditions it is reasonable to assume that the horizontal density gradient, $\partial\rho/\partial x$, is constant. This form of reduction to a description of the flow in its central regime is the same as that applied to gravitational circulation in the upper mantle by Officer and Drake (1983), and in estuaries, by Officer (1976). Since the system is closed in the longitudinal direction, $\partial\rho/\partial y$ is necessarily zero. Under these conditions v and H are functions of z only, and p, ρ and T are the pressure, density and temperature differences from adiabatic, or static, conditions. The flow boundary conditions are $v_x=v_y=v_z=0$ at $z=0$ and $z=d$ which give, from relation (6), $v_z=0$. The magnetic field conditions for a nonconducting mantle and inner core are $H_x=H_y=0$ at $z=0$ and $z=d$ which give, from relation (7), $H_z=H_o=$ constant. The choice of a conducting inner core would change the H_x and H_y fields near the outer core-inner core boundary.

Equations (3) and (5), thus, reduce to

$$\frac{\partial^2 v_x}{\partial z^2}=\frac{1}{\rho_o\nu}\frac{\partial p}{\partial x}+\frac{2\Omega v_y\sin\phi}{\nu}-\frac{\mu}{4\pi\rho_o\nu}H_z\frac{\partial H_x}{\partial z} \tag{9}$$

$$\frac{\partial^2 v_y}{\partial z^2}=-\frac{2\Omega v_x\sin\phi}{\nu}+\frac{\mu}{4\pi\rho_o\nu}H_z\frac{\partial H_y}{\partial z} \tag{10}$$

$$0=H_z\frac{\partial v_x}{\partial z}+\frac{1}{4\pi\sigma\mu}\frac{\partial^2 H_x}{\partial z^2} \tag{11}$$

$$0=H_z\frac{\partial v_y}{\partial z}+\frac{1}{4\pi\sigma\mu}\frac{\partial^2 H_y}{\partial z^2} \tag{12}$$

and the flow and magnetic flux continuity relations are

$$\int_0^d v_x\,dz=0 \tag{13}$$

$$\int_0^d H_x\,dz=0. \tag{14}$$

It is appropriate to take an isodynamic condition at the mantle-outer core boundary. The pressure will be given by

$$p=\rho g(z-\xi)+p_0 \tag{15}$$

where ξ is the dynamic depression of the boundary and p_0 a constant. We have, then, for the horizontal pressure gradient,

$$\begin{aligned}\frac{\partial p}{\partial x}&=g\int_0^z\frac{\partial\rho}{\partial x}dz-\rho_0 g\frac{\partial\xi}{\partial x}\\ &=g\lambda z-\rho_0 g i\end{aligned} \tag{16}$$

where we have ignored second order terms and where $i=\partial\xi/\partial x$ is the slope of the mantle-outer core surface and $\lambda=\partial\rho/\partial x$ the horizontal density gradient. Equations (9) through (12) are in the form of four coupled, ordinary differential equations which, with relation (16), may be easily solved. Equation (13) will provide a defining relation between i and λ, and Eq. (14) will provide a defining relation between H_0 and the other parameters of the system.

Equations (9) through (12) may be combined to give the following two equations, separable in v_x and v_y,

$$\frac{d^4 v_x}{dz^4}+\left(\frac{4\Omega^2\sin^2\phi}{v^2}-\frac{\sigma^2\mu^4 H_0^4}{\rho_0^2 v^2}\right)v_x=\frac{\sigma\mu^2 H_0^2}{\rho_0^2 v^2}\frac{\partial p}{\partial x} \tag{17}$$

$$\frac{d^4 v_y}{dz^4}+\left(\frac{4\Omega^2\sin^2\phi}{v^2}-\frac{\sigma^2\mu^4 H_0^4}{\rho_0^2 v^2}\right)v_y=-\frac{2\Omega\sin\phi}{\rho_0 v^2}\frac{\partial p}{\partial x}. \tag{18}$$

Using the parameter values listed in Table 1 we have for the ratio, R, of the square root of the terms in the parentheses

$$R=\frac{\dfrac{\sigma\mu^2 H_0^2}{\rho_0 v}}{\dfrac{2\Omega\sin\phi}{v}}=\frac{\sigma\mu^2 H_0^2}{2\Omega\rho_0\sin\phi}=0.0140. \tag{19}$$

We may neglect the second term in the parentheses with respect to the first term, reducing Eqs. (17) and (18) to

$$\frac{d^4 v_x}{dz^4}+\frac{4\Omega^2\sin^2\phi}{v^2}v_x=\frac{\sigma\mu^2 H_0^2}{\rho_0^2 v^2}\frac{\partial p}{\partial x} \tag{20}$$

$$\frac{d^4 v_y}{dz^4}+\frac{4\Omega^2\sin^2\phi}{v^2}v_y=-\frac{2\Omega\sin\phi}{\rho_0 v^2}\frac{\partial p}{\partial x}. \tag{21}$$

Using relation (16), the complementary and particular integral solutions to Eqs. (20) and (21) are

$$\begin{aligned}v_x=&-\frac{Rgi}{2a^2 v}+\frac{Rg\lambda}{2a^2\rho_0 v}z+C_1\cosh az\cos az\\ &+C_2\sinh az\cos az+C_3\cosh az\sin az\\ &+C_4\sinh az\sin az\end{aligned} \tag{22}$$

$$\begin{aligned}v_y=&\frac{gi}{2a^2 v}-\frac{g\lambda}{2a^2\rho_a v}z-(2C_1+RC_4)\sinh az\sin az\\ &-(2C_2+RC_3)\cosh az\sin az\\ &+(2C_3-RC_2)\sinh az\cos az\\ &+(2C_4-RC_1)\cosh az\cos az.\end{aligned} \tag{23}$$

The constants C_1 through C_4 are determined from the boundary conditions that $v_x=v_y=0$ at $z=0$ and $z=d$. Using the condition from Tables 1 and 2 that

$$D=\frac{\pi}{a}\ll d \tag{24}$$

and the continuity condition, Eq. (13), which gives

$$i=\frac{1}{2}\frac{\lambda d}{\rho_0} \tag{25}$$

we obtain

$$\begin{aligned}v_x=\frac{g\lambda d}{4a^2\rho_0 v}\Big[&(1-R)e^{-az}\sin az+2Re^{-az}\cos az\\ &-2R+\frac{4Rz}{d}-e^{-a(d-z)}\sin a(d-z)\\ &-2Re^{-a(d-z)}\cos a(d-z)\Big]\end{aligned} \tag{26}$$

$$\begin{aligned}v_y=\frac{g\lambda d}{4a^2\rho_0 v}\Big[&1-\frac{2z}{d}-e^{-az}\cos az+\frac{3R}{2}e^{-az}\sin az\\ &+e^{-a(d-z)}\cos a(d-z)+\frac{R}{2}e^{-a(d-z)}\sin a(d-z)\Big].\end{aligned} \tag{27}$$

Let us look for the moment at the type of core dynamics that these solutions delineate. The flow may be considered to consist of three parts, an Ekman-type boundary layer current defined by the frictional depth, $D=\pi/a$, near the mantle-outer core boundary and near the outer core-inner core boundary and a mid-depth drift current. For v_x the upper boundary layer flow is defined by the $\exp(-az)\sin az$ term and the lower boundary layer flow in the opposite direction by the $\exp[-a(d-z)]\sin a(d-z)$ term. In the mid-depth region there is a small drift current related to the Lorentz term in the original equations and given by the expression $[-2R+(4Rz)/d]$. For v_y there are corresponding boundary layer flows and a large mid-depth drift current related to the Coriolis term in the defining equations and given by the expression $[1-(2z)/d]$. In the limit of no Coriolis or Lorentz effects the solutions (26) and (27) reduce to

$$v_x=\frac{g\lambda}{12\rho_0 v}[d^2 z-3dz^2+z^3] \tag{28}$$

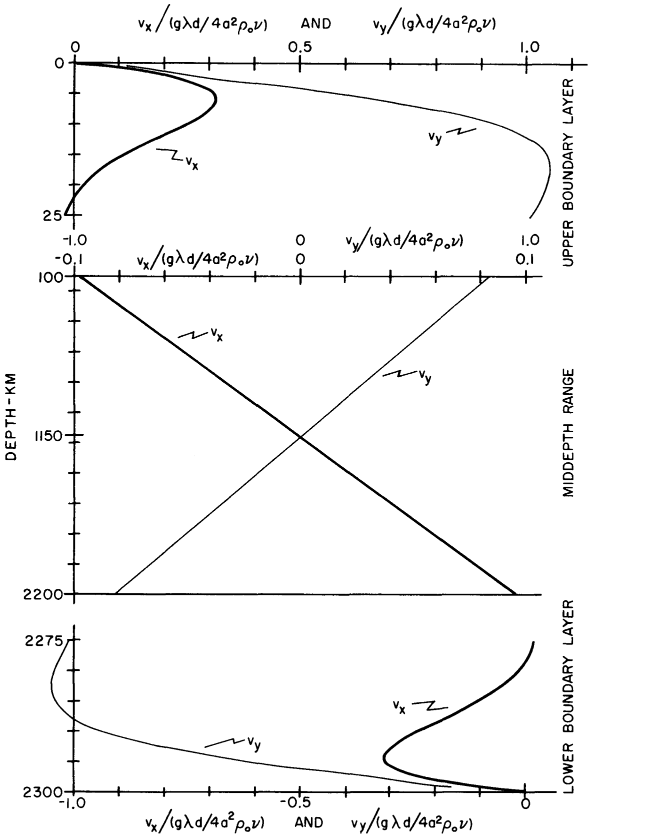

Fig. 3. Graphs of v_x and v_y for boundary layer flows and mid-depth flow. Note changes in ordinate and abscissa scales

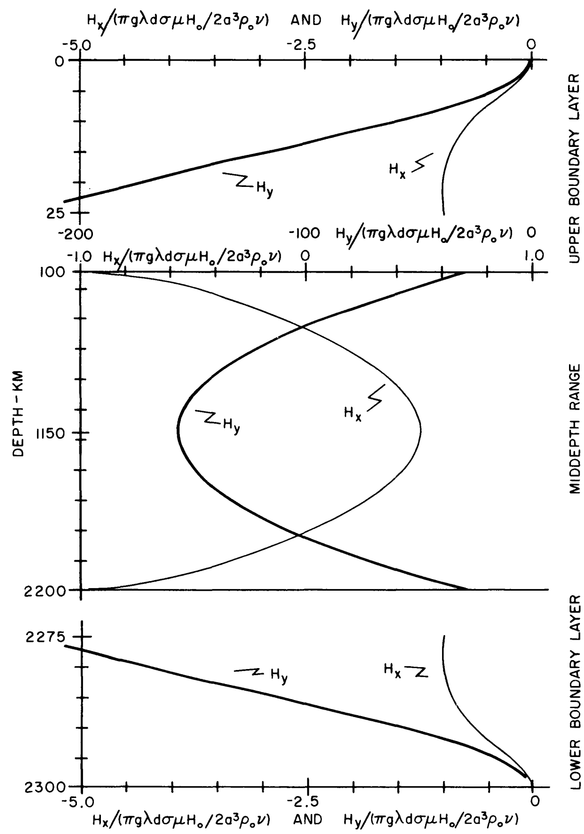

Fig 4. Graphs of H_x and H_y for boundary flows and mid-depth flow. Note changes in ordinate and abscissa scales

$$v_y=0 \tag{29}$$

which are the solutions for gravitational, or horizontal density gradient, circulation of Officer and Drake (1983).

From Eqs. (11) and (12) the corresponding solutions for H_x and H_y may be obtained with the boundary conditions $H_x=H_y=0$ at $z=0$ and $z=d$, giving

$$H_x=-\frac{\pi g\lambda d\sigma\mu H_o}{2a^3\rho_o\nu}\Big[1-e^{-az}(\cos az+\sin az)-4Raz+\frac{4Raz^2}{d}-e^{-a(d-z)}(\cos a(d-z)+\sin a(d-z))\Big] \tag{30}$$

$$H_y=\frac{\pi g\lambda d\sigma\mu H_o}{2a^3\rho_o\nu}\Big[1-e^{-az}(\cos az-\sin az)-2az+\frac{2az^2}{d}-e^{-a(d-z)}(\cos a(d-z)-\sin a(d-z))\Big]. \tag{31}$$

The magnetic flux relation of Eq. (14), then, gives from solution (30) the condition for H_o in terms of the other parameters of the system that

$$\tfrac{2}{3}Rad=1. \tag{32}$$

From the definition (19) for R, we have

$$H_o=\left[\frac{3\Omega\rho_o\sin\phi}{\sigma\mu^2ad}\right]^{1/2}=1.1\text{ gauss} \tag{33}$$

using the parameter values in Table 1. It is, indeed, interesting and encouraging that this simplified exposition does give a value for the external field strength which is the same order of magnitude as the value of 1.9 gauss for the average field strength during the Phanerozoic.

Both the H_x and H_y fields have a depth variation similar to that of the hydrodynamic flow. There is a rapidly varying portion near the mantle-outer core boundary and near the outer core-inner core boundary defined by the frictional depth, D. Within the mid-depth region both components vary slowly. The H_x field lies in a northerly direction in the northern hemisphere near the mantle-outer core boundary. In spherical coordinates, the H_x and H_z components form the poloidal field, and the H_y component determines the toroidal field. The mid-depth portion of the H_x field differs from that of the H_y field by the factor R. In other words, the toroidal field will be about two orders of magnitude greater than the poloidal field. In this formulation a strong toroidal field is predicted for the outer core.

Figures 3 and 4 are graphs of the depth-variable portions of the v_x and v_y flows and the H_x and H_y fields, respectively. The ordinate and abscissa scale changes should be noted in these figures. They illustrate quantitatively the depth variations for each component discussed in the previous paragraphs.

Figures 5 and 6 are schematic representations of the core dynamics assuming a hemispherically symmetrical

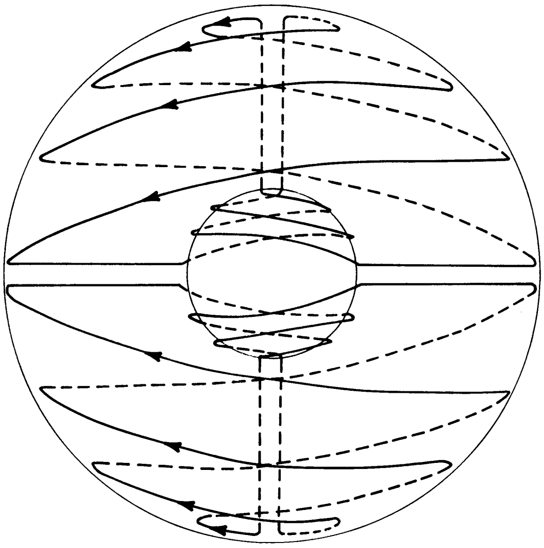

Fig. 5. Schematic representation of global boundary layer circulation in the outer core

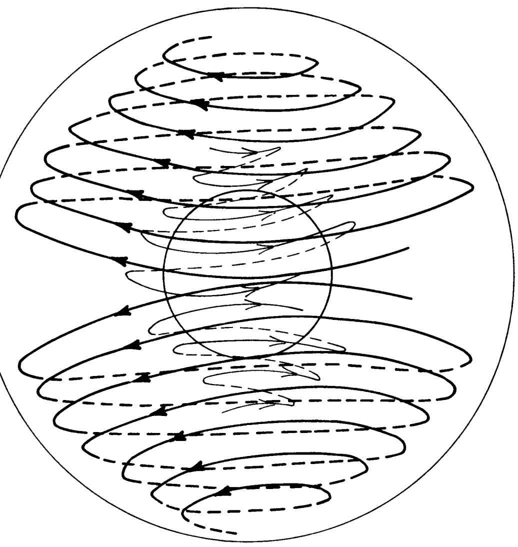

Fig. 6. Schematic representation of global mid-depth circulation in the outer core

circulation. The flows consist of spiral motions in a westerly direction from each pole toward the equator near the mantle-outer core boundary and spiral motions in the opposite direction near the outer core-inner core boundary. For the mid-depth portion the motion is dominantly westerly with a small poleward component in the upper portion, and dominantly easterly with a small equatorward component in the lower portion.

It is necessary, next, to obtain some reasonable estimate for the horizontal density gradient, $\lambda = \partial\rho/\partial x$. The horizontal heat fluxes related to the mid-depth portion of the v_x flow will balance out. For the upper boundary layer flow, the conductive heat flux out at the mantle-outer core boundary per unit distance must be equal to the decrease in the convective heat flux per unit distance and, correspondingly, for the heat flux in at the outer core-inner core boundary, assuming no internal heat generation. We have, then, from Eq. (26)

$$Q = -c_p \rho_o \frac{\partial T}{\partial x} \int_0^\infty \frac{g\lambda d}{4a^2 \rho_o \nu} e^{-az} \sin a z\, dz \tag{34}$$

or, using relation (8),

$$\lambda = \left[\frac{8\alpha a^3 \rho_o \nu Q}{c_p g d}\right]^{1/2} = 0.253 \times 10^{-15}\ \mathrm{g\,cm^{-4}} \tag{35}$$

using the parameter values of Table 1.

The multiplying factors for the v_x and v_y flows of Eqs. (26) and (27) and for the H_x and H_y fields of Eqs. (30) and (31) are, then,

$$A = \frac{g\lambda d}{4a^2 \rho_o \nu} = 0.0301\ \mathrm{cm\,s^{-1}} \tag{36}$$

and

$$B = \frac{\pi g \lambda d \sigma \mu}{2a^3 \rho_o \nu} = 0.420. \tag{37}$$

From relations (33) and (35) and the definition for a in Table 1 it is to be noted that the various dependences on the viscosity, ν, are λ proportional to $\nu^{-1/4}$; v_x and v_y proportional to $\nu^{-1/4}$; and H_x, H_y and H_z proportional to $\nu^{1/4}$. All the components show a relatively weak dependence on the viscosity.

Let us look next at the nondipole components of the earth's magnetic field. Elsasser (1941, 1946b) demonstrated that the nondipole components must originate as a skin effect in the outer core adjacent to the mantle-outer core boundary at a depth not in excess of 150 km from the boundary. This is in accord with the core dynamic formulation given here. The nondipole components could originate from spatial variations in the boundary layer flow near the mantle-outer core boundary. Further, the observed secular variations in the nondipole field would originate from temporal variations in the boundary layer flow. Although the simple formulation given here does not include consideration of such effects, spatial and temporal variations are, indeed, an important characteristic of Rayleigh-Benard boundary layer circulation.

It is possible to estimate the drift of these irregularities from the formulation given here. Over the frictional depth of the upper boundary layer flow there is a strong westward drift in both hemispheres, gives by

$$V_w = \frac{1}{D} \int_0^D v_y\, dz = 0.206 \frac{g\lambda d}{a^2 \rho_o \nu} = 0.025\ \mathrm{cm\,s^{-1}} \tag{38}$$

using the parameter values of Table 1. As with the deduced magnitude of the main field strength, H_o, from relation (33), it is encouraging that the predicted value for the westward drift from this simplified formulation is in accord with the observed westward drift of the main field, e.g., Bullard et al. (1950) as well as others. This formulation also predicts that there should be an additional but much smaller northerly drift in the northern hemisphere and southerly drift in the southern hemisphere given by

$$V_s = \frac{1}{D}\int_0^D v_x\,dz = 0.036\,\frac{g\,\lambda\,d}{a^2\,\rho_o\,\nu} = 0.004\ \mathrm{cm\,s^{-1}}. \qquad (39)$$

Speculations as to the origin of the secular variations and polarity reversals

The principal purpose of this investigation has been do delineate the gross features of core dynamics and the geomagnetic field under conditions of an outer core viscosity of $2 \times 10^7\ \mathrm{cm^2\,s^{-1}}$. The analytic solutions are for steady state and apply only to the central portion of the flow regime. It is of interest to pursue, in a qualitative manner, the implications of this formulation for an understanding of other features of the secular variations and the polarity reversals. The following discussion is admittedly speculative.

Special and temporal instabilities are an inherent characteristic of Rayleigh-Benard circulation, since the defining equations are coupled and nonlinear. Various aspects of these instabilities have been examined by a number of investigators, e.g., Howard (1966), Welander (1967), Krishnamurti (1970a, b), Busse and Whitehead (1971), Moore and Weiss (1973), Nield (1975), Busse and Riahi (1980) and Krishnamurti and Howard (1981). It is possible that both the observed temporal variations in the magnetic moment of the dipole field and the observed movement of the magnetic pole around the geographic pole might be related to such temporal and spatial variations of the global core dynamics about equilibrium. The nondipole field and the temporal variations in its magnitude could be related to smaller scale spatial irregularities and temporal variations in the boundary layer flow near the mantle-outer core boundary.

Of particular interest to a consideration of polarity reversals are the numerical experiments of Welander (1967). His numerical computations are related to a very simple, boundary-layer type flow. It consists of a vertical tube of fluid forming a closed loop that is heated from below and cooled from above. The fluid motion is defined by the usual coupled hydrodynamic and heat continuity equations and there are two equilibrium flow conditions, viz., clockwise or counterclockwise flow. His calculations show that depending on the relative magnitude of the resistive and driving forces, flow instabilities will result, leading in the extreme to flow reversals. The interesting result is that for even this very simple system of heat-driven, boundary-layer type convection flow, reversals are an *inherent* characteristic of the system. Robbins (1977) has extended the calculations for the Welander loop and those for a reversing disc dynamo, defined by similar equations, towards an understanding of the polarity reversals.

Following the results from Robbins (1977), Officer and Lynch (unpublished data, 1985) have continued the numerical calculations for the Welander loop for increasing flow resistance and the ratio of resistive to driving force (Fig. 7). The progression is from (A) steady state, to (B) steady state with oscillatory damping, to (C) nearly neutral oscillations about one of the equilibrium positions, to (D) gradual buildup of the oscillations to a reversal, and then repetition of the sequence about the other equilibrium position, to (E) periodic oscillation sequences, to (F) aperiodic oscil-

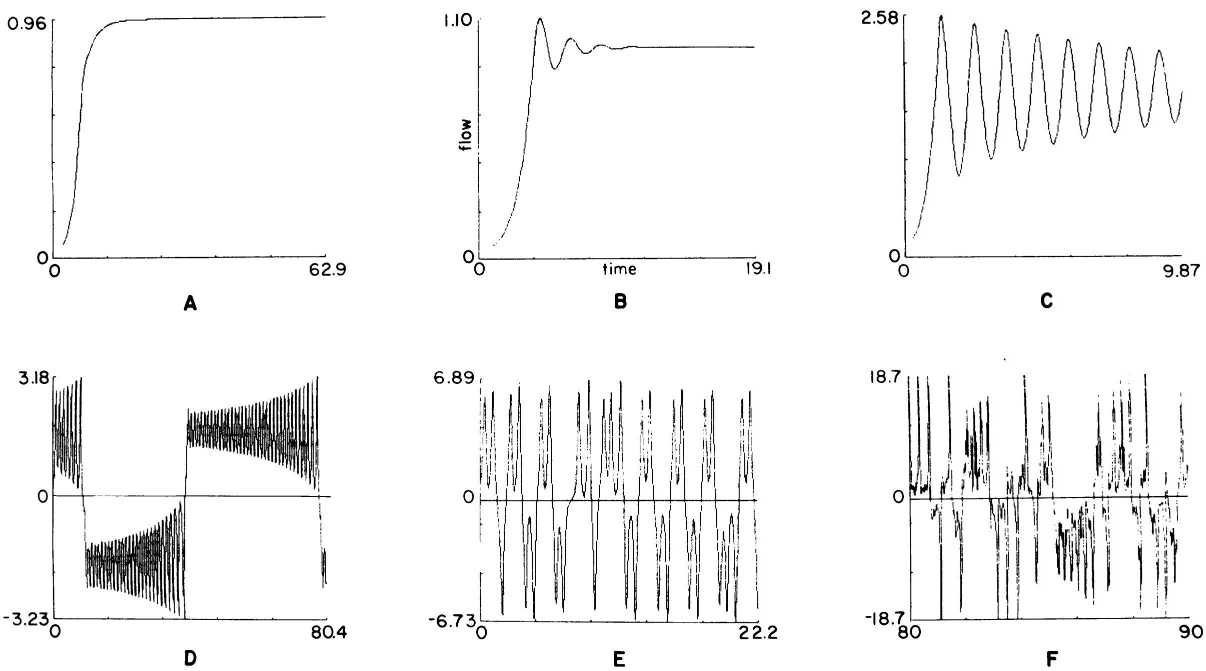

Fig. 7A–F. Plots of fluid flow (nondimensional) versus time (nondimensional) for the Welander loop numerical experiment for various values of the parameters, a and ε. **A** 0.4, 0.2; **B** 2, 1; **C** 20, 3; **D** 40, 6; **E** 102.4, 3.2; **F** 10,240, 96 lations. For the last case the spectral distribution of the reversal time intervals has a peak near the time for one circuit of the loop, dropping off steeply for shorter time intervals and more gradually for longer time intervals. This type of sequential behaviour appears to be characteristic of a number of similar systems, specifically the Lorenz equations, Lorenz (1963) and Sparrow (1982), and the nonlinear oscillator studied by Moore and Spiegel (1966). For the core dynamics formulated herein, the comparison with the Welander loop is that both are heat-driven, boundary-layer type circulations with heat-convective effects dominant over heat-conductive effects. For the core dynamics the mid-depth flow is geostrophically controlled. The boundary layer flow is characterized by a large resistance term and a small driving force, with a time for one complete cycle of about 10000 years.

Assuming that the magnetic polarity reversals are related to reversals in the global circulation in the outer core and that the present core conditions may correspond to Fig. 7F, a number of consequences follow. Throughout the Phanerozoic there would be essentially equal periods of westward and eastward drift of the nondipole field. The circulation, itself, forms a coupled northern and southern hemisphere system. The specifics of the flow reversals may not be exactly the same in each hemisphere, leading to a quadrupole field during the transition. Further, there is an inherent indeterminacy in the determination of the direction of a new dipole field. With flow reversals the new magnetic field may be in the same or the opposite direction to the old field. Thus, we should expect equal episodes of magnetic field excursions toward a zero field strength but return to the original field direction and of episodes of field reversals. Finally, if the core viscosity has increased substantially with geologic time, the flow reversal sequence would progress from Fig. 7D to E to F, leading to a possible explanation for the extended periods of either normal or reversed polarity during the earlier Phanerozoic.

All the above comments are, of course, speculative. The important point is that a more detailed understanding of the Welander loop type of flow instabilities as applied to core dynamics may lead toward an understanding of magnetic polarity reversals, as also concluded by Robbins (1977).

Conclusions

An alternative model of core dynamics and explanation for the origin of the earth's magnetic field has been given. The formulation differs from previous models in the application of an estimated kinematic viscosity for the outer core of $2 \times 10^7\, \mathrm{cm^2\, s^{-1}}$. The simplified derivations assume a hemispherically symmetrical, global circulation in the core. The resulting boundary layer flow is thermally driven, and is controlled in a latitudinal direction by the Coriolis force and in a longitudinal direction by the Lorenz and Coriolis forces. The theory predicts the correct order of magnitude for the dipole field strength, 1.1 gauss, and the correct order of magnitude for the westward drift of the nondipole field, $0.025\, \mathrm{cm\, s^{-1}}$. It is suggested that both the secular variations in the dipole and nondipole fields and the polarity reversals may be related to the instabilities in the Rayleigh-Benard, boundary-layer hydrodynamic flow in the outer core.

References

Adams, R.D.: Multiple inner core reflections from a Novaya Zemlya explosion. Bull. Seismol. Soc. Am. **62**, 1063–1072, 1972

Anderson, D.L., Hart, R.S.: Attenuation models of the earth. Phys. Earth Planet. Inter. **16**, 289–306, 1978a

Anderson, D.L., Hart, R.S.: *Q* of the earth. J. Geophys. Res. **83**, 5869–5882, 1978b

Bondi, H., Lyttleton, R.A.: On the dynamical theory of the rotation of the earth: I, the secular retardation of the core. Proc. Cambridge Philos. Soc. **44**, 345–359, 1948

Braginsky, S.I.: On the nearly axially symmetrical model of the hydromagnetic dynamo of the earth. Phys. Earth Planet. Inter. **11**, 191–199, 1976

Buchbinder, G.G.R.: A velocity structure of the earth's core. Bull. Seismol. Soc. Am. **61**, 429–456, 1971

Bukowinski, M.S.T., Knopoff, L.: Electronic structure of iron and models of the earth's core. Geophys. Res. Lett. **3**, 45–48, 1976

Bullard, E.C.: The secular change in the earth's magnetic field. Mon. Not. R. Astron. Soc., Geophys. Suppl. **5**, 248–257, 1948

Bullard, E.C.: The magnetic field within the earth. Proc. R. Soc. London, Ser. A, **197**, 433–453, 1949a

Bullard, E.C.: Electromagnetic induction in a rotating sphere. Proc. R. Soc. London, Ser. A, **199**, 413–443, 1949b

Bullard, E., Gellman, H.: Homogeneous dynamos and terrestrial magnetism. Philos. Trans. R. Soc. London, Ser. A, **247**, 213–278, 1954

Bullard, E.C., Freedman, C., Gellman, H., Nixon, J.: The westward drift of the earth's magnetic field. Philos. Trans. R. Soc. London, Ser. A, **243**, 67–92, 1950

Busse, F.H.: A model of the geodynamo. Geophys. J. R. Astron. Soc. **42**, 437–459, 1975

Busse, F.H.: Generation of planetary magnetism by convection. Phys. Earth Planet. Inter. **12**, 350–358, 1976

Busse, F.H.: Recent developments in the dynamo theory of planetary magnetism. Annu. Rev. Earth Planet. Sci. **11**, 241–268, 1983

Busse, F.H., Riahi, N.: Nonlinear convection in a layer with nearly insulating boundaries. J. Fluid Mech. **96**, 243–256, 1980

Busse, F.H., Whiteheed, J.A.: Instabilities of convection rolls in a high Prandtl number fluid. J. Fluid Mech. **47**, 305–320, 1971

Elsasser, W.M.: A statistical analysis of the earth's internal magnetic field. Phys. Rev. **60**, 876–883, 1941

Elsasser, W.M.: Induction effects in terrestrial magnetism: Part I, theory. Phys. Rev. **69**, 106–116, 1946a

Elsasser, W.M.: Induction effects in terrestrial magnetism: Part II, the secular variation. Phys. Rev. **70**, 202–212, 1946b

Elsasser, W.M.: Induction effects in terrestrial magnetism: Part III, electric modes. Phys. Rev. **72**, 821–833, 1947

Elsasser, W.M.: The earth's interior and geomagnetism. Rev. Mod. Phys. **22**, 1–35, 1950

Elsasser, W.M.: Hydromagnetic dynamo theory. Rev. Mod. Phys. **28**, 135–163, 1956

Gans, R.F.: Viscosity of the earth's core. J. Geophys. Res. **77**, 360–366, 1972

Gubbins, D.: Theories of the geomagnetic and solar dynamos. Rev. Geophys. Space Phys. **12**, 137–154, 1974

Gubbins, D.: Observational constraints on the generation process of the earth's magnetic field. Geophys. J. R. Astron. Soc. **47**, 19–39, 1976

Herzenberg, A.: Geomagnetic dynamos. Phil. Trans. R. Soc. London, Ser. A, **250**, 543–583, 1958
Hide, R.: The hydrodynamics of the earth's core. Phys. Chem. Earth **1**, 94–137, 1956
Hide, R., Roberts, P.H.: The origin of the main geomagnetic field. Phys. Chem. Earth **4**, 27–98, 1961
Howard, L.N.: Convection at high Rayleigh number. Proc. Eleventh Int. Cong. Appl. Math., 1109–1115, 1966
Jacobs, J.A.: The earth's core. London: Academic Press 1975
Jeffreys, H.: The viscosity of the earth. Mon. Notices Roy. Astron. Soc., Geophys. Suppl. **1**, 412–424, 1926
Jeffreys, H.: The earth, third edition. London: Cambridge University Press 1952
Krishnamurti, R.: On the transition to turbulent convection: Part 1, the transition from two to three dimensional flow. J. Fluid Mech. **42**, 295–307, 1970a
Krishnamurti, R.: On the transition to turbulent convection: Part 2, the transition to time dependent flow. J. Fluid Mech. **42**, 309–320, 1970b
Krishnamurti, R., Howard, L.N.: Large scale flow generation in turbulent convection. Proc. Natl. Acad. Sci. USA **78**, 1981–1985, 1981
Lamb, H.: Hydrodynamics. London: Cambridge University Press 1932
Levy, E.H.: Kinematic reversal schemes for the geomagnetic dipole. Astrophys. J. **171**, 635–642, 1972a
Levy, E.H.: On the state of the geomagnetic field and its reversals. Astrophys. J. **175**, 573–581, 1972b
Levy, E.H.: Generation of planetary magnetic fields. Annu. Rev. Earth Planet. Sci. **4**, 159–185, 1976
Lorenz, E.N.: Deterministic nonperiodic flow. J. Atmos. Sci. **20**, 130–141, 1963
Merrill, R.T., McElhinny, M.W.: The earth's magnetic field. London: Academic Press 1983
Moore, D.W., Spiegel, E.A.: A thermally excited nonlinear oscillator. Astrophys. J. **143**, 871–887, 1966
Moore, D.R., Weiss, N.O.: Two dimensional Rayleigh-Benard convection. J. Fluid Mech. **58**, 389–312, 1973
Molodenskiy, S.M.: Upper viscosity boundary of the earth's core. Izvestiya Earth Phys. **17**, 903–909, 1981
Neild, D.A.: The onset of convective instability. J. Fluid Mech. **71**, 441–454, 1975
Officer, C.B.: Physical oceanography of estuaries. New York: John Wiley 1976
Officer, C.B., Drake, C.L.: Plate dynamics and isostasy in a dynamic system. J. Geophys. **54**, 1–19, 1983
Parker, E.N.: Hydromagnetic dynamo models. Astrophys. J. **122**, 293–314, 1955
Parker, E.N.: The occasional reversal of the geomagnetic field. Astrophys. J. **158**, 815–827, 1969
Qamar, A., Eisenberg, A.: The damping of core waves. J. Geophys. Res. **79**, 758–765, 1974
Rikitake, T.: Electromagnetism and the earth's interior. Amsterdam: Elsevier 1966
Robbins, K.A.: A moment equation description of magnetic reversals in the earth. Proc. Natl. Acad. Sci. USA **73**, 4297–4301, 1976
Robbins, K.A.: A new approach to subcritical instability and turbulent transitions in a simple dynamo. Math. Proc. Cambridge Philos. Soc. **82**, 309–325, 1977
Roberts, P.H.: An introduction to magnetohydrodynamics. London: Longmans 1967
Roberts, P.H.: Dynamo theory. In: Mathematical problems in the geophysical sciences, II, inverse problems, dynamo theory and tides, Reid, W.H. Ed., pp. 129–206, Providence: American Mathematical Society 1971
Sacks, I.S.: Anelasticity of the outer core. In: Annu. Rep. Dep. Terr. Mag. 1969–1970, Carnegie Inst., Washington, pp. 414–426, 1971
Sato, R., Espinosa, A.F.: Dissipation in the earth's mantle and rigidity and viscosity in the earth's core determined from waves multiply reflected from the mantle-core boundary. Bull. Seismol. Soc. Am. **57**, 829–856, 1967
Soward, A.M., Ed.: Stellar and planetary magnetism. New York: Gordon and Breach 1982
Sparrow, C.: The Lorenz equations: Bifurcations, chaos and strange attractors. Berlin, Heidelberg, New York: Springer 1982
Stacey, F.D.: Physics of the earth. New York: John Wiley 1977
Suzuki, Y., Sato, R.: Viscosity determination in the earth's outer core from ScS and SKS phases. J. Phys. Earth **18**, 157–170, 1970
Verhoogen, J.: Energetics of the earth. Washington: National Academy of Sciences 1980
Watanabe, H.: Bounds on the fluid velocity and magnetic field in the earth's core imposed by hydromagnetic consideration of an $\alpha\omega$-dynamo. J. Geomagn. Geoelectr. **29**, 191–209, 1977
Welander, P.: On the oscillatory instability of a differentially heated fluid loop. J. Fluid Mech. **29**, 17–30, 1967

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