# An Adaptive Tabu Search Heuristic for the Location Routing Pickup and Delivery Problem with Time Windows with a Theater Distribution Application 

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AN ADAPTIVE TABU SEARCH HEURISTIC FOR THE
LOCATION ROUTING PICKUP AND DELIVERY PROBLEM WITH TIME WINDOWS WITH A THEATER DISTRIBTUTION APPLICATION

DISSERTATION

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The views expressed in this dissertation are those of the author and do not reflect the official policy or position of the United States Air Force, United States Army, Department of Defense, or the U.S. Government.

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## DISSERTATION

Presented to the Faculty<br>Graduate School of Engineering and Management<br>Air Force Institute of Technology<br>Air University<br>Air Education Training Command<br>in Partial Fulfillment of the Requirements for the<br>Degree of Doctor of Philosophy

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#### Abstract

The time constrained pickup and delivery problem (PDPTW) is a problem of finding a set of routes for a fleet of vehicles in order to satisfy a set of transportation requests. Each request represents a user-specified pickup and delivery location. The PDPTW may be used to model many problems in logistics and public transportation. The location routing problem (LRP) is an extension of the vehicle routing problem where the solution identifies the optimal location of the depots and provides the vehicle schedules and distribution routes.

This dissertation seeks to blend the PDPTW and LRP areas of research and formulate a location scheduling pickup and delivery problem with time windows (LPDPTW) in order to model the theater distribution problem and find excellent solutions. This research utilizes advanced tabu search techniques, including reactive tabu search and group theory applications, to develop a heuristic procedure for solving the LPDPTW. Tabu search is a metaheuristic that performs an intelligent search of the solution space. Group theory provides the structural foundation that supports the efficient search of the neighborhoods and movement through the solution space.

This research evaluates the robustness of the developed adaptive tabu search algorithm. A linear program is developed to calculate lower bounds for the generated problem objectives.


## Dedication

To Angie, Sophie, Anna, Joseph and Leo

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Robert E. Burks Jr.

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Table 7-8 ATS Objective Function Values.

## Glossary of Terms

| AF | Air Force |
| :---: | :---: |
| AO | area of operation |
| AOR | area of responsibility |
| APOD | aerial port of debarkation |
| APOE | aerial port of embarkation |
| ASP | ammunition supply point |
| ATP | ammunition transfer point |
| BSA | brigade support activity |
| C-17 | strategic and theater cargo aircraft |
| C-130 | theater cargo aircraft |
| CINC | commander in chief |
| CENTCOM | Central Command |
| CSA | corps support activity |
| CSS | combat service support |
| DARPTW | dial a ride problem with time windows |
| DCSLOG | Deputy Chief of Staff for Logistics |
| DoD | department of defense |
| DSA | division support activity |
| ETDD | early time definite delivery |
| FIFO | first- in first-out |
| GTTS | group theoretic tabu search |
| GVRP | generalized vehicle routing and scheduling problem |
| FDL | fixed depot list |
| H | Hub, a transshipment node |
| ISB | intermediate staging base |
| LOC | lines of communication |
| LPDPTW | location pickup and delivery problem with time windows |

LRP location routing problem
MOG maximum on the ground capacity (either working or parking)
MOGA maximum on the ground capacity for aircraft
MOGG maximum on the ground capacity for ground vehicles
MTMS multiple trip multiple services TDVRSP instance
MTMS $\quad$ w/hub multiple trip multiple services with hub TDVRSP instance
MTW multiple no delivery time windows
POD port of debarkation
PDPTW pickup and delivery problem with time windows
SA support activity
SDL set depot list
Sn symmetric group on $n$ letters
SPOD seaport of debarkation
SSA supply support activity
TAA tactical assembly area
TDD time definite delivery
TDMS theater distribution management system
TDP theater distribution problem
TDVRSP theater distribution vehicle routing and scheduling problem
Tier
TMCA transportation movement control agency
TPFDD time phased force deployment document
TS tabu search
TSA theater support activity
TSPTW traveling salesman problem with time windows
TTP
trailer transportation point
UCP unified command plan
USEUCOM united states european command
VRSP vehicle routing and scheduling problem

# AN ADAPTIVE TABU SEARCH HEURISTIC FOR THE <br> LOCATION ROUTING PICKUP AND DELIVERY PROBLEM WITH TIME WINDOWS WITH A THEATER DISTRIBTUTION APPLICATION 

## I Introduction

Vehicle routing problems are common in many logistics management situations. Researchers have developed different versions of vehicle routing problems over the years to address various practical situations. Despite the variety of practical situations, most of the research focuses on solving a common problem -- the efficient use of a fleet of vehicles that must service a collection of transportation requests.

The pickup and delivery problem with time windows (PDPTW) covers the general situation where a fleet of vehicles must service a set of transportation requests. These requests specify pickup and delivery locations. Potential solutions route each vehicle to service all requests, satisfying time window and vehicle capacity constraints while optimizing a desired objective function. The PDPTW is a generalization of the well known vehicle routing problem with time windows (VRPTW) but it has received much less research focus in the literature than the VRPTW.

Logistics managers utilize the location routing problem (LRP) to facilitate decisions concerning the location for factories/warehouses, allocation of customers to service areas and the development of transportation plans connecting customers and goods. The LRP is a VRP where the optimal number and locations of the warehouses is
determined simultaneously with the vehicle schedules and routes to minimize the total system operation cost.

This dissertation seeks to blend these two areas of research and formulate a location pickup and delivery problem with time windows (LPDPTW) to solve the theater distribution problem (TDP). This research utilizes advanced tabu search techniques, including adaptive tabu search and group theory, to develop a heuristic procedure for solving the LPDPTW.

### 1.1 Background

The Department of Defense's (DoD) Unified Command Plan (UCP) is the document that lays out the unified command structure and establishes combatant command missions, responsibilities, force structure, and delineates geographic areas of responsibility (AOR). The latest version of this document establishes five combatant commanders (Figure 1.1) with geographic responsibility. These combatant commanders are responsible for all operations within their designated areas. Regardless of the task or the nature of threat, these combatant commanders are responsible for the employment of air, sea, space, and special operations forces, and the coordination with multinational partners to achieve their assigned strategic and operational objectives (FM 3-0, 2000). To achieve these objectives, the combatant commander is required to synchronize all elements of the logistics system to ensure the correct delivery of the "right things" to the "right place" and at the "right time" (JP 4-01.4, 2001). For example, the commander of United States European Command (USEUCOM) is responsible for all operations in an area that covers more than 21 million square miles and includes 91 countries. The $1^{\text {st }}$

Transportation Movement Control Agency (TMCA) is the USEUCOM commander's agency charged with the responsibility of synchronizing the pickup and delivery of logistics to meet customer demands for over 6,000 addresses in USECOM's area of responsibility. This flow synchronization of personnel, equipment, and material within the theater to meet the combatant commander's mission is known as the theater distribution problem (TDP).


Figure 1-1 Combatant Commander's Geographic Area of Responsibility

### 1.2 The Theater Distribution Problem

### 1.2.1 The Operational Framework

The logistics distribution pipeline represents the critical link between the national industrial base and the tactical war fighter in the supported theater (Figure 1-2). This pipeline is comprised of strategic and operational components. The strategic portion of the pipeline represents the flow of material and support from points of origin external to the theater and culminates at the theater's points of entry (JP 4-01.4, 2001). The theater portion of the logistics system comprises all of the networks within the theater and generally extends from the port of debarkation to the final destination at the tactical level.


Figure 1-2 The Distribution System

Theater distribution begins at the point where material and personnel are transferred from strategic lift assets to the control of theater assets. For this research, these points are represented as the aerial ports of debarkation (APOD) and sea ports of debarkation (SPOD). At the operational level of war, the senior combatant commander is responsible
for development of the distribution system and ultimately for all transportation operations. Joint Publication 4-01.4 (2001) describes the theater distribution system as comprised of four networks: physical, financial, information, and communication systems. The interest of this research is in the physical network of the theater distribution system.

This physical network consists of the quantity, capacity, and capability of fixed structures and established facilities in support of distribution operations. This network comprises the actual infrastructure of the area of operation (AO) or theater of war and includes all roads, railroads, warehouses, supply depots, ports (air and sea), waterways, and pipelines. The network encompasses available resources such as personnel, equipment, material and the inherent ability to move these resources. The network also includes the organic distribution capability of assigned military units, commercial enterprises, host nation and multinational partners (JP 4-01.4, 2001). Figure 1-3 provides an illustrated example of a theater's physical network. This network represents a theater or area of responsibility (AOR) that has been sub-divided by the combatant commander with a theater of war. A combatant commander's AOR may contain multiple theaters of war if required by the situation. The ongoing operations in Afghanistan and Iraq provide an example of multiple theaters of war in the CENTCOM commander's AOR.

The physical components of the network are critical to the flow of material and personnel in the theater. These components, in modeling terms and in this research, are categorized as nodes, edges and modes. Figure 1-3 illustrates the six node types utilized


Figure 1-3 The Theater's Physical Network
in this research. These nodes represent the entry points of logistics into the theater of war (APOD, SPOD), the originating point of transportation assets (Depots), the point where supplies are stored and processed for forward movement (Transshipment) and the termination point of requirements (Demands). The edges represent the "lines of communication" connecting the various nodes in the theater. As represented in Figure 13, the network is not necessarily fully connected and the connection between any two nodes is not necessarily a straight line. The theater's transportation assets are categorized as one of three mode types: air, ground, and water. Each of these modes may contain various vehicle types at the commander's disposal for distribution planning. Logistics
planners utilize this collection of nodes, edges and modes to develop a distribution plan that satisfies customer demands.

The doctrinal operational logistic system possesses a hierarchical structure where a logistics node at a given echelon feeds logistics resources to subordinate units at lower echelons. Logistics flow is delivered from strategic sources (plants, depots) to receiving points in either the theater of operations or theater of war, such as a port of debarkation. Theater forces deliver these resources to forward bases (Corps, Divisions) and then additional forces deliver these resources to subordinate combat service support units at the tactical level.

This physical network possesses the basic form of a tree that is rooted at the strategic level and has its leafs at the tactical level. The number of associated levels in this tree depends on the number of logistic command levels. In general, the tree could possess as many as five layers corresponding to the rear theater facilities, forward theater facilities, and division, brigade, and battalion combat service support (CSS) units.

### 1.2.2 Motivation

Logistics lessons of the Gulf War forced military planners to reexamine the current "Cold War" logistics process. Foss (1994) pointed out that General Schwarzkopf's requirement for 60 days of supplies in the country was a huge mistake and poor decision. This stockpile requirement increased the logistics force structure and presented a large vulnerable footprint in country. These lessons initiated an Army effort to streamline how it conducts logistics support.

The Army is currently in the midst of a transformation effort that shifts sustainment focus from the warehouse-based logistics of the "Cold War" era to distribution-based logistics. The Deputy Chief of Staff, G-4 (Logistics), through an Army logistics white paper (2003) provided a transformation charter establishing three combat service support transformation goals. The first goal is the reduction of CSS footprint in the combat zone or theater of war. The second goal is focused on strategic mobility and the reduction of deployment timelines. The last goal is reducing the cost (required assets) of logistics without reducing war fighting capability. In conjunction with this transformation, Future Force operations require the blending of strategic and operational sustainment flows in the theater to provide continuous sustainment without requiring an extensive logistical buildup or risking an operational pause. TRADOC Pam 525-3-0 (2001) outlines the overarching goal of sustainment as the "continuous, precise, assured provisioning of deployed Army and supported forces in any environment, guaranteeing their ability to generate and maintain combat power throughout the campaign".

However, based on lessons from Iraq and Afghanistan, the DCSLOG in his Army logistics white paper (2003) commented that today's Army is not able to respond rapidly and precisely when support requirements are identified. He also noted that "we cannot provide time-definite delivery schedules and we cannot effectively control physical movements across the new battlefield environment." The fact of the matter is the Army currently does not have a working mechanism in place to establish a distribution system and develop a delivery schedule that ensures delivery on time, every time.

### 1.2.3 The Problem Statement

The main thrust of this research is to develop an efficient tabu search approach to the LPDPTW as applied to the TDP. The objectives of the theater distribution model change over time as the operational logistics plan transitions through the deployment, employment, sustainment and redeployment phases of the operation. However, all objectives possess a common element of determining how to best configure the distribution system to effectively and efficiently support the combat units over time. The commander's operational logistics plan establishes the physical network structure of the distribution system, provides planned locations and demands of combat forces, designates the set of all potential supply and depot unit locations, and designates the set of available transportation assets. Demands represent a multi-commodity request for delivery at a given time and transportation assets represent a fleet of heterogeneous vehicles.

The problem's objective is to determine the sequence of vehicle depot locations, allocation of transportation assets to these depots, the selection of appropriate supply points (APOD, SPOD, warehouses) and the development of a vehicle routing and scheduling plan that achieves the time definite delivery (TDD) of demands while minimizing total system costs.

The logistics footprint of the distribution network represents the system's cost in the TDP. These costs consist of the vehicle depot and supply point establishing cost, transportation cost and necessary depot and supply point operating cost. An entity's logistics footprint (cost) is based on three factors: the weight of assigned equipment, the
entity's square footage, and the number of personnel required for operations in the combat zone.

In addition to cost, the model must address many other considerations in the TDP. The establishment of a vehicle depot or supply point provides a fixed operating capacity which cannot be violated. For example, opening an airport (aircraft depot) only allows so many aircraft to operate at any given time or opening a supply point only provides a fixed (capacitated) amount of a certain commodity. Additional constraints include limiting the operation day of vehicle drivers. A driver's operating day begins with the preparation of the vehicle for operations and concludes at the end of after operations maintenance. Army policy limits the vehicle operator's operational day to 10 hours. Vehicle operations beyond 10 hours require the addition of a second vehicle operator. Many other constraints apply to the TDP and are defined in Section 4.2. These constraints and regulations combined with the underlying combinatorial nature of the LPDP make the TDP a difficult problem to solve.

### 1.3 Research Contributions

The primary objective of this research is to develop an efficient and effective tabu search heuristic to solve an instance of the LPDPTW. The theater distribution problem provides the practical operational framework for testing and evaluating the heuristic. Tabu search provides the ability to overcome local optimality traps providing more effective distribution plans and group theory provides the basic mechanism to direct the search process. This research accomplishes several supporting goals in achieving its primary objective.

The first goal is the development and formulation of the LPDP through the successful blending of the PDPTW and LRP class problems for modeling the TDP.

The second goal is the development of the theater distribution problem as a practical LPDPTW with all of its associated characteristics. These characteristics include multi-commodities, multi-depots, heterogeneous vehicles and the requirement for multiple visits. The resulting LPDPTW is formatted using the symmetric group on n letters as the framework for the solution representation and is solved with an adaptive tabu search heuristic.

The third supporting goal is combining adaptive tabu search (ATS) with group theory and symmetric groups as a means to structure the heuristic search process. Achieving this goal required research on utilizing group theory to effectively partition the solution space.

The last goal is testing the robustness of the tabu search process through designed experiments and benchmark test problems. Finally, the LPDPTW algorithm is coded in the Java ${ }^{\text {TM }}$ software language, in support of the $2^{\text {nd }}$ and $3^{\text {rd }}$ goals and provides a necessary portability element for the heuristic and makes it more accessible to military planners.

### 1.4 Organization of Dissertation

The remainder of this dissertation is organized as follows. Chapter II provides a tabu search and group theory primer. Chapter III provides a review of the literature relevant to the LPDPTW. Chapter IV provides the mathematical formulation of the LPDPTW. Chapter V provides the ATS methodology developed to solve the LPDPTW. Chapter VI lays out the design of experiment and statistical analysis of the ATS
methodology. Chapter VII applies the developed ATS approach to several large LPDPTW and theater distribution cases. Chapter VIII wraps up this research with a discussion of the contributions and potential future research.

## II Tabu Search and Group Theory Primer

This chapter provides an introduction to the basic concepts of tabu search and group theory. This lays the foundation necessary for understanding the adaptive tabu search approach developed in Chapter V.

### 2.1 Tabu Search

The present form of tabu search was first described by Glover in 1986 (Glover, 1986) but the process can trace its heritage back to the 1970s. Tabu search belongs to a general class of optimization procedures that utilize iterative techniques to find the optimal solution. The general procedure for these iterative techniques is to construct a new solution $s_{j}$ from a current solution $s_{i}$ and to check whether or not the procedure should stop or perform another step. Neighborhood search methods are iterative techniques that first define a neighborhood $N\left(s_{i}\right)$ for each current feasible solution $i$, and the next solution $s_{j}$ is selected from among the solutions in $N\left(s_{i}\right)$. The neighborhood $N\left(s_{i}\right) \subset S$ represents the set of all solutions $s_{i}^{\prime} \in N\left(s_{i}\right)$ that can be directly reached from the current solution $s_{i}$ by a single move operation. The descent method is the most famous of the basic neighborhood search methods. Tabu search uses a more dynamic version of the neighborhood search.

### 2.1.1 Basic Tabu Search

Tabu search improves the efficiency of the exploration process by not only keeping track of the current objective function value but information related to the exploration. This use of memory in the search process is a cornerstone element of the
tabu search process. Most exploration procedures track the objective function value, $f\left(s_{i}\right.$ ${ }^{*}$ ), of the best solution $s_{i}{ }^{*}$. Tabu search takes it a step further and tracks the itinerary or corresponding moves of the last set of solutions. The search process uses this information to guide the move from $s_{i}$ to the next solution $s_{j}$ in $N\left(s_{i}\right)$. Tracking this search history allows for the creation of a strategic dynamic neighborhood search method that selects distinctly different neighborhood definitions from an available set based on the current search itinerary.

Consider a tabu search problem where you are given a set $S$ of solutions and some function $\boldsymbol{f}: \boldsymbol{S} \rightarrow \square$. The objective is to find an excellent solution $s_{i} *$ quickly in S where $\boldsymbol{f}\left(\mathbf{i}^{*}\right) \leq \boldsymbol{f}(\mathbf{i}) \forall \boldsymbol{i}$ in $\boldsymbol{S}$. The tabu search procedure would represent a minimization algorithm if the process could guarantee that $i^{*}$ is reached after a finite number of iterations. However, tabu search makes no guarantee of optimality but does provide its own set of operating rules to guide and orient the local search process.

Once the search process generates an initial solution $i$ in $S$, tabu search must generate a means to travel from this incumbent solution to another solution within the solution space. Tabu search accomplishes this by creating a rules-driven neighborhood of the initial solution $i$. This neighborhood $N(i, \sigma)$ of a solution $i$ consists of all solutions that can be reached from $i$ by a single operation $\sigma$. The operation $\sigma$ is generally referred to as a move.

To help clarify this neighborhood concept, consider an initial arrangement of three colored blocks where $i=$ \{red, green, blue $\}[(\mathrm{R}, \mathrm{G}, \mathrm{B})]$. A swap move provides one example of a rule-driven neighborhood for exploring the solution space in this example.

Therefore, define $\sigma$ as exchanging block $x$ with block $y$. The neighborhood $N(i, \sigma)$ consists of all possible exchanges of block $x$ with block $y$ and for this example would consist of the following three ordered solutions: $(G, R, B),(B, G, R),(R, B, G)$.

A straightforward descent method generally evaluates the solutions in $N(i, \sigma)$ and selects a solution $\boldsymbol{j} \in \boldsymbol{N}(\mathbf{i}, \sigma)$ if $\boldsymbol{f}(\boldsymbol{j}) \leq \boldsymbol{f}(\mathbf{i})$. However, tabu search utilizes memory to exploit knowledge beyond the function $\boldsymbol{f}: \boldsymbol{S} \rightarrow \square$ and the neighborhood $N(\mathbf{i}, \sigma)$. Tabu search uses the problem's objective function and constraints in the function $f: S \rightarrow \square$ when evaluating the neighborhood $N(i, \sigma)$. These elements allow tabu search to implement various rules in selecting the next solution from $N(\mathbf{i}, \sigma)$. For example, the process might implement a rule that selects the first improving move or another rule that selects the best move from $N(i, \sigma)$. These rules provide an ability for the search process to accept non-improving moves (i.e., $\boldsymbol{f}(\boldsymbol{j})>\boldsymbol{f}(\boldsymbol{i})$ ) during the exploration process by selecting the best $\boldsymbol{j}$ in $\boldsymbol{N}(\mathbf{i}, \sigma)$. The selection of non-improving moves allows tabu search to escape local optima but also introduces the risk of cycling or re-visiting a solution. Tabu search uses short term memory structure to prevent cycling.

An early form of memory structure was to use a tabu list T (a finite list) of the last $|\mathrm{T}|=$ cardinality of T solutions which allowed tabu search to maintain for $k$ iterations (tabu tenure) a recently visited solution as forbidden for selection. This list T prevents cycles of at most size $|\mathrm{T}|$ during the search process. Additional early memory structures include tracking the last $|\mathrm{T}|$ moves selected or the inverses of the last $|\mathrm{T}|$ moves selected.

The type of memory structure is problem specific and an area of research in any tabu search application.

One drawback to using a memory structure is the prevention of the search process from selecting an unvisited solution because that solution possesses an attribute that renders the solution tabu. However, aspiration criteria allow tabu search to overcome the tabu status of a move that leads to an attractive solution. For example, consider a move $m$ that generates a tabu solution better than the best found solution so far. The search process allows $m$ in spite of the solution's tabu status since its aspiration level exceeds a pre-defined threshold value. This aspiration criterion can allow tabu search to achieve superior performance.

### 2.1.2 Tabu Search and the Java ${ }^{\text {TM }}$ Architecture

This research uses the Java ${ }^{\mathrm{TM}}$ programming language to develop the tabu search procedure. Prior research by Barnes, Wiley, Moore and Ryer (2004) on the aerial fleet refueling problem (AFRP), Crino, Moore, Barnes and Nanry (2004) on the theater distribution vehicle routing and scheduling problem (TDVRSP), and Combs and Moore (2004) on the crew scheduling problem (CSP) demonstrate the effectiveness of both Java ${ }^{\mathrm{TM}}$ and tabu search in providing a robust and effective solution procedure for large combinatorial optimization problems.

Harder's Java ${ }^{\text {TM }}$ based tabu search engine, OpenTS, provides the framework for constructing the tabu search procedure in this research (Harder, 2001). Figure 2-1 illustrates an iteration of the OpenTS Architecture.

OpenTS starts with a user-developed initial incumbent solution. The search engine then generates the neighborhood of moves, $\boldsymbol{N}(\mathbf{i}, \sigma)$ for consideration. The search manager then sends this list of moves to the objective function object for evaluation. The process selects the best non-tabu move from the list. The search manager may select high quality solutions that are tabu if they meet some aspiration criteria. The selected move from $N(i, \sigma)$ is used to operate on the current solution. This solution becomes the new incumbent solution completing an iteration of the tabu search process.


Figure II-1 An Iteration of OpenTS

### 2.1.3 Summary

The information presented here only represents an introduction to the concepts of tabu search. Interested readers are referred to Glover and Laguna (1997) and Glover et al. (2005) for a more complete discussion of available advanced tabu search concepts. The next section provides the foundation for understanding the group theoretic portion of this research.

### 2.2 Group Theory

Colletti's (1999) research explores group theory as a unifying mathematical framework for the study of metaheuristic methods. He demonstrates that the symmetric group on $n$ letters provides a natural setting for studying combinatorial optimization problems. His research utilizes group theory and tabu search to solve an instance of the Traveling Salesman Problem (TSP). Combs and Moore (2004) and Crino et al. (2004) expand upon Colletti's research and demonstrate the use of group theory and tabu search for solving instances of a Vehicle Routing Problem (VRP), specifically the crew scheduling problem (CSP) and the theater distribution vehicle routing and scheduling problem (TDVRSP). This collection of literature serves as this research's foundation for group theoretic metaheuristics.

The next section presents the concepts of group theory and its applications to an instance of the LPDP. The concepts are presented in a combined manner to maintain the underlying mathematical structure and language of the combinatorial optimization community.

### 2.2.1 Fundamental Concepts

A natural first question is to ask, what is group theory? First, recall that one of the primary concerns of algebra is the study of sets and operations on sets. The basic sets most students considered in algebra were the reals $(\square)$, the integers $(\square)$, and the rationals ( $\square$ ) with the operations of addition and multiplication. Abstract algebra focuses study on the structure of sets with operations on them. Consider the equation

$$
\begin{equation*}
5 x=2 \tag{2-1}
\end{equation*}
$$

from algebra. What is the solution to the equation? The answer depends on what the problem allows $x$ to be. There is no solution to the problem if $x$ is a member of the set of integers; however, if $x$ is from the set of rational numbers, then the solution is $x=2 / 5$. Abstract algebra allows a researcher to gain additional insights from this problem with the following equation:

$$
\begin{equation*}
a \bullet x=b \tag{2-2}
\end{equation*}
$$

Group theory is concerned with systems in which Equation (2-2) has a unique solution. Group theory is interested in the structure of the problem and does not care what $a, b$ and the operation symbolized by $\bullet$ actually represent in the problem. One purpose of abstract algebra is to compare and contrast different sets with different operations on them and determine ways they are the same and the ways they are different. This abstract approach allows group theory to deal with many mathematical systems at once. Group theory requires only that the mathematical system obey a few axioms. Let $G$ be a non-empty set:
(a) (closure) for any $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{G}$ then $\boldsymbol{a} * \boldsymbol{b} \in \boldsymbol{G}$
(b) (associative law) for all $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \boldsymbol{G}$,

$$
(a * b) * c=a *(b * c)
$$

(c) (existence of an identity element) there exists an element $\boldsymbol{e} \in \boldsymbol{G}$ such that

$$
(\boldsymbol{a} * \boldsymbol{e})=\boldsymbol{a}=(\boldsymbol{e} * \boldsymbol{a})
$$

$$
\text { for all } \boldsymbol{a} \in \boldsymbol{G}
$$

(d) (existence of inverses) for each $\boldsymbol{a} \in \boldsymbol{G}$, there exists an $\boldsymbol{a}^{\prime} \in \boldsymbol{G}$ such that

$$
\left(a * a^{\prime}\right)=e=\left(a^{\prime} * a\right)
$$

Any mathematical system that obeys these four rules is called a group. The study of systems that obey these four rules is the basis of group theory.

Definition: A group is a nonempty set $G$ together with a binary operation * defined on $G$ satisfying the above four axioms.

Definition: A semi-group is a nonempty set $G$ together with a binary operation * defined on $G$ that only satisfies the axioms (a) and (b) and (c) above.

### 2.2.2 Group Examples

One familiar group is the group of integers under addition. First, define $\mathbf{Z}$ as the set of integers $\{\ldots,-2,-1,0,1,2, \ldots\}$, and let the symbol " + " indicate the operation of addition. Then $(\square,+)$ is a group. If $a, b$ and $c$ are integers (i.e. $a, b, c \in \square)$, then:
(a) (Closure):
$a+b=$ integer $\in \square$
(b) (associative law): $(a+b)+c=a+(b+c)$
(c) (existence of identity): $0+a=a+0=a$
(d) (existence of inverses): $\exists$ an integer $b:=-a$ such $a+b=b+a=0$

### 2.2.3 The Symmetric Group

Cayley's Theorem [Fraleigh, 1976: 64] states that every finite group consists of a set of permutations. This set of permutations allows researchers to utilize group theory, the "algebra of permutations", in developing an understanding of using metaheuristics to solve combinatorial optimization problems.

Consider for a moment a more abstract example of a group. Consider three colored blocks $(\mathrm{R}=$ red, $\mathrm{G}=$ green, and $\mathrm{B}=$ blue $)$, initially placed in the order RGB . If a
researcher wanted to look at all the ways he could rearrange these three blocks, there would be a total of six potential permutations.
(1) $\mathrm{RGB} \rightarrow \mathrm{RGB}$
(2) $\mathrm{RGB} \rightarrow$ GRB
(3) RGB $\rightarrow$ RBG
(4) RGB $\rightarrow$ BRG
(5) RGB $\rightarrow$ GBR
(6) RGB $\rightarrow$ BGR

Group theory provides the mechanism to examine this set of six permutations. Define $a$ as the action of "swapping the first and second block", and define $b$ as the action of "swapping the second and third block". Traditional algebraic notation utilizes, $x y$ to signify the composition of actions $(x \circ y)$ or "first do $y$, then do $x$ ". In the above example, $a b$ becomes the action of "swap the second and third block" then "swap the first and second block" or simply rewritten as "take the last block and move it to the front" $(\mathrm{RGB} \xrightarrow{\mathrm{a}} \mathrm{RBG} \xrightarrow{\mathrm{b}} \mathrm{BRG})$. This set of permutations also has an identity action, $e$, for "leave the blocks as they are arranged". A researcher may use these three actions, $a, b$, and $e$, to develop all six permutation arrangements:

$$
\begin{array}{ll}
e: & \mathrm{RGB} \rightarrow \mathrm{RGB} \\
a: & \mathrm{RGB} \rightarrow \mathrm{GRB} \\
b: & \mathrm{RGB} \rightarrow \mathrm{RBG} \\
a b: & \mathrm{RGB} \rightarrow \mathrm{BRG} \\
b a: & \mathrm{RGB} \rightarrow \mathrm{GBR} \\
a b a: & \mathrm{RGB} \rightarrow \mathrm{BGR}
\end{array}
$$

Each of the above actions also possesses a valid inverse. For example, the combined action of $a a$ has the effect of leaving the blocks as they are currently arranged (RGB $\xrightarrow{\mathrm{a}} \mathrm{GRB} \xrightarrow{\mathrm{a}} \mathrm{RGB}$ ) so obviously $a a=e$ the identity action. Similarly,

- bb=e,
- $(\boldsymbol{a b})(b a)=\boldsymbol{e}$,
- $(\boldsymbol{b a})(\boldsymbol{a b})=\boldsymbol{e}$, and
- $(a b a)(a b a)=e ;$

By inspection, the set of permutations $\{\mathrm{a}, \mathrm{b}, \mathrm{e}\}$ also possesses the associative and closure axioms:

- $(\boldsymbol{a b}) \boldsymbol{a}=\boldsymbol{a}(\boldsymbol{b a})=\boldsymbol{a b a}$, and
- $(b a) b=b(a b)=a b a$.

This set of permutations $\{a, b, e\}$ obeys the four axioms listed in Section 2.2 above so this is a group, i.e. specifically, $(\{a, b, e\}, o)$ is called a symmetric group on 3 letters, or $\boldsymbol{S}_{3}$. This group has order 6, and is non-abelian since $\boldsymbol{a} \boldsymbol{b} \neq \boldsymbol{b} \boldsymbol{a}$. The work above showed that $\boldsymbol{S}_{3}$ is built up from the basic actions $a$ and $b$, so the set $\{\boldsymbol{a}, \boldsymbol{b}\}$ generates the group. Therefore, assuming that a set $A$ consists of $n$ objects labeled as $\{1,2,3, \ldots, n\}$, then $\boldsymbol{S}_{\boldsymbol{n}}$ is the group of all permutations of $n$ objects and has the order $\boldsymbol{n}$.. (Fassler and Stiete, 1992:8)

Definition: A permutation of a finite set $A$ is a function from $A$ into $A$, which is bijective, that is one to one and onto (Fraleigh, 1976).

Definition: A symmetric group on $n$ letters, $\boldsymbol{S}_{\boldsymbol{n}}$, is the group of all permutations of a finite set $A$ if $A$ is the finite set $\{1,2,3, \ldots, n\}$ (Fraleigh, 1976).

The symmetric group on $n$ letters is the specific group used in this research.

### 2.2.4 Standard Form Permutations and Cycles

There are two notation schemes for an element of $\boldsymbol{S}_{\boldsymbol{n}}$, the standard form and the cyclic form. The standard form notation is a 2 by $n$ matrix that represents a one-to-one and onto function whose domain (top row) and range (bottom row) are the integers $\{1,2$, $3, \ldots, n\}$. Assume that $m$ represents a one to one mapping of the letters $\{1,2,3, \ldots, n\}$ onto itself. The standard form notation is the array

$$
\begin{array}{ccccc}
1 & 2 & 3 & \ldots & n \\
m(1) & m(2) & m(3) & \ldots & m(n) .
\end{array}
$$

The cyclic form notation of $m$ is a streamlined notation of $\boldsymbol{S}_{\boldsymbol{n}}$ and is a single rowed array (Sagan, 1991: 1). Given $\boldsymbol{i} \in\{1,2, \ldots, \boldsymbol{n}\}$, and $\boldsymbol{m}^{\boldsymbol{p}}(\boldsymbol{i})=\boldsymbol{i}$, the cycle is represented as:

$$
\left(i, m(i), m^{2}(i), \ldots, m^{p-1}(i)\right) .
$$

For example, the cycle ( $i, j, k, m$ ) means $m$ sends $i$ to $j, j$ to $k, k$ to $m$, and $m$ back to $i$. "A cycle of length $k$ is a cycle containing $k$ elements" (Sagan, 1991:2). Cycles with only one element are called unit cycles. Unit cycles can be implied and dropped from the cyclic notation.

Examples of the standard form and cyclic form permutations are presented below (Sagan, 1991).

If $\boldsymbol{m} \in \boldsymbol{S}_{5}$ is

$$
\boldsymbol{m}(1)=2, \quad \boldsymbol{m}(2)=3, \quad \boldsymbol{m}(3)=1, \quad \boldsymbol{m}(4)=4, \quad \boldsymbol{m}(5)=5,
$$

then the standard form is $\quad \boldsymbol{m}=\binom{\boldsymbol{x}}{\boldsymbol{m}(\boldsymbol{x})}=\left(\begin{array}{lllll}1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5\end{array}\right)$
and the cyclic form is

$$
\boldsymbol{m}=(1,2,3)(4)(5)=(1,2,3) .
$$

A $k$-cycle or cycle of length $k$ is a cycle containing $k$ elements (Sagan, 1991: 2). The length of the first cycle is 3 and the two unit cycles are (4) and (5) in $\boldsymbol{m}$. The cycle type or structure of $\boldsymbol{m}$ is an expression of the form $\left(\boldsymbol{n}^{\boldsymbol{m}_{n}}, \ldots, 2^{\boldsymbol{m}_{2}}, 1^{\boldsymbol{m}_{1}}\right)$ where $\boldsymbol{m}_{\boldsymbol{k}}$ is the number of cycles of length $k$ in $m$. The example above has a cycle structure $\left(5^{0}, 4^{0}, 3^{1}, 2^{0}, 1^{2}\right)$.

The binary operation associated with the symmetric group on $n$-letters is function composition (Colletti, 1999:11). The product of permutations $\boldsymbol{m}$ and $\boldsymbol{p}, \operatorname{denoted} \boldsymbol{m} \oplus \boldsymbol{p}$, is function composition. That is, $(\boldsymbol{m} \oplus \boldsymbol{p})(\boldsymbol{x})=(\boldsymbol{p}(\boldsymbol{m}(\boldsymbol{x}))$.

For example, let $\boldsymbol{m}=\binom{\boldsymbol{x}}{\boldsymbol{m}(\boldsymbol{x})}=\left(\begin{array}{lllll}1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4\end{array}\right)$ and $\boldsymbol{p}=\binom{\boldsymbol{x}}{\boldsymbol{p}(\boldsymbol{x})}=\left(\begin{array}{lllll}1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 5 & 2 & 1\end{array}\right)$ then

$$
\left.\boldsymbol{m} \oplus \boldsymbol{p}=\left(\begin{array}{lllll}
1 & 2 & 3 & 4 & 5 \\
2 & 3 & 1 & 5 & 4
\end{array}\right)\left(\begin{array}{lllll}
1 & 2 & 3 & 4 & 5 \\
4 & 3 & 5 & 2 & 1
\end{array}\right)=\left(\begin{array}{lllll}
1 & 2 & 3 & 4 & 5 \\
3 & 5 & 4 & 1 & 2
\end{array}\right)=\left(\begin{array}{llll}
1 & 3 & 4
\end{array}\right)(2) 5\right) .
$$

A tabu search approach to solving the location PDP combinatorial problem will move from one solution to the next solution using methods that alter the customer sequence. Conjugation is one group theory operation that enables tabu search to build neighborhoods.

Definition: Let $(G,+)$ be a group. Conjugation by $k \in G$ is the operation defined by $\boldsymbol{x}^{\boldsymbol{k}}=\boldsymbol{k}^{-1} \oplus \boldsymbol{x} \oplus \boldsymbol{k}, \quad \forall \boldsymbol{x} \in \boldsymbol{G}$. (Crino, 2004).

A two-letter swap move on an incumbent solution is one example of a conjugation operation. In $\boldsymbol{S}_{5}$, consider an incumbent solution $p=(1,3,2)(4)(5)$ and a two-
letter swap $\boldsymbol{q}=(2,4)$. Conjugation results in the cycle solution $\boldsymbol{p}^{\boldsymbol{q}}=\boldsymbol{q}^{-1} \oplus \boldsymbol{p} \oplus \boldsymbol{q}=(1,3,4)(2)(5)$, where the cycle structure is maintained and only the letters in the permutation are changed. Notice that conjugation preserves the cycle structure.

### 2.2.5 The LPDP in Terms of $\boldsymbol{S}_{\boldsymbol{n}}$

The cyclic form of the symmetric group on $n$ letters, $S_{n}$, provides a compact solution representation for the LPDPTW. Figure 2-2 provides a graphical representation of a LPDP to aid further discussion on the topic.

The graph in Figure 2-2 represents a problem consisting of two potential depot locations $\{1,2\}$, that can both support the same type vehicle, three demands or delivery points $\{7,8,9\}$ and three potential supply pickup points $\{A, B, C\}$. Notice that pickup point A has two identification letters $\{3,4\}$. This multiple letter identification allows multiple vehicles to visit the same supply pickup point. Vehicle identification letters


Figure 2-2 Graphical Example of LPDPTW
begin in sequence after the last demand identification letter or 9 in this example. Assuming for the moment that there are two potential vehicles available for assignment, their identification letters become 10 and 11 in the solution structure.

A solution to the problem is denoted by a vector indicating the order in which customers are served by the unique depot vehicles to which they are assigned. One potential solution vector is as follows:

$$
\boldsymbol{S}=\left[\pi_{1}, \pi_{k}, \ldots, \pi_{n+m-1}, \pi_{n+m}\right]
$$

where $\pi_{i}$ is the index of the depot, customer, vehicle node or point in the $i$ th position of the route. The example has two depots so $k=2$, and $i=1,2$ represents the depots in the structure. The pick-up locations, $\mathrm{P}^{+}$, occupy the positions from $\mathrm{k}+1$ to $\left|\mathrm{k}+\mathrm{P}^{+}\right|$or 3-6 in the example. The delivery locations, $\mathrm{P}^{-}$, occupy positions $\left(\left|\mathrm{P}^{+}\right|+\mathrm{k}+1\right)$ to $\left(\left|\mathrm{P}^{+}\right|+\left|\mathrm{P}^{-}\right|+\mathrm{k}\right)$ and finally the vehicle letters, V , occupy positions $\left(\left|\mathrm{P}^{+}\right|+\left|\mathrm{P}^{-}\right|+\mathrm{k}+1\right)$ to $\left(\left|\mathrm{P}^{+}\right|+\left|\mathrm{P}^{-}\right|+\mathrm{k}+\right.$ $|\mathrm{V}|)$ in the vector. Group theory allows the solution structure for the depicted problem and information provided above to be represented in standard or cyclic form.

The following is a representation of the solution in standard permutation form:
Case 1: $\left(\begin{array}{lllllllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 10 & 2 & 7 & 8 & 9 & 6 & 4 & 5 & 1 & 3 & 11\end{array}\right)$
The second method utilizes disjoint cycles, where each cycle represents a subtour.
Case 1: $(1,10,3,7,4,8,5,9)(2)(6)(11)$
In the above example, each subtour represents a route of a vehicle type. It is obvious when comparing the two representations that the disjoint cycle form is much easier to interpret. In this representation, only one depot (1) was selected and vehicle 10
traveled to customers 3-7-4-8-5-9 in order, where the bold numbers represent pickup locations, and then back to the depot. The second vehicle depot, supply pickup point C , and vehicle 11 were not required to solve this problem and were not selected for opening. One advantage of utilizing the disjoint cycle notation is that you can clearly see and track the set of depots and supply points that are currently opened and closed. A second advantage is the ability to assign identification letters to each vehicle. This ability facilitates tracking the status of every vehicle in the problem.

This research utilizes Wiley's (2000) Java ${ }^{\mathrm{TM}}$ class for the symmetric group on $n$ letters. Since this Java ${ }^{\mathrm{TM}}$ class structures cycles by lexicographic ordering, vehicle letters follow customer letters. For example, given a $\boldsymbol{S}_{8}$ problem containing 3 vehicles and five customers, the group represents the vehicles by the letters 6,7 , and 8 and represents the customers with the letters $1,2,3,4$ and 5 .

### 2.2.6 Partitioning and Ordering the Solution Space

Group theory provides a structural and mathematical foundation for the LPDP. Tabu search can take advantage of this structure in developing the neighborhoods and moves while searching the solution space.

For example, consider a symmetric group with 8 letters, that is, $\boldsymbol{S}_{8}$. The number of permutations contained in the group is $8!=40,320$ potential solutions. Group theory provides tabu search an effective means of partitioning this solution space and focusing only on the most profitable regions.

Conjugacy class or cycle structure as mentioned earlier provides an effective means of partitioning the 40,320 solutions into sets of similar cycle structure.

Definition: Let $(G, \oplus)$ be a group. A conjugacy class of $g \in G$ is the set $\left\{\boldsymbol{h}^{-1} \oplus \boldsymbol{g} \oplus \boldsymbol{h}: \boldsymbol{h} \in \boldsymbol{G}\right\} \subset \boldsymbol{G}$. The conjugacy class of $\boldsymbol{g} \in \boldsymbol{G}$ is CClass $(\boldsymbol{G}, \boldsymbol{g})=\left\{\boldsymbol{g}^{\boldsymbol{h}}: \boldsymbol{h} \in \boldsymbol{G}\right\}$. The conjugacy classes of any finite group are mutually exclusive and exhaustive (Colletti, 2002).

A conjugacy class contains a number of conjugates of a permutation in the symmetric group. Once the search process selects a cycle type, conjugation preserves the cycle type or conjugacy class. The number of permutations contained in a conjugacy class is then based on the cycle type. Consider the following $S_{8}$ example:

$$
\begin{equation*}
\left(\pi_{1}, \pi_{2}\right)\left(\pi_{3}, \pi_{4}, \pi_{5}\right)\left(\pi_{6}, \pi_{7}\right)\left(\pi_{8}\right) \tag{2-3}
\end{equation*}
$$

There are 8 ! or 40,320 ways to arrange the numbers $\{1,2, \ldots, 8\}$ to get a permutation of this cycle type. However, the conjugacy class represented by the cycle structure $3^{1} 2^{2} 1^{1}$ does not contain 40,320 permutations. For example, the cycle $\left(\pi_{3}, \pi_{4}, \pi_{5}\right)$ is the same as the cycles $\left(\pi_{4}, \pi_{5}, \pi_{3}\right)$ and $\left(\pi_{5}, \pi_{3}, \pi_{4}\right)$. This cycle redundancy implies that our count of 40,320 represents an over-count by at least a factor of 3 . In fact, each $k$-cycle provides an over count by a factor of $k$ for the number of permutations in the conjugacy class. The $3^{1} 2^{2} 1^{1}$ conjugacy class provides an over-count by a factor of $3 \cdot 2 \cdot 2 \cdot 1$. This cycle redundancy does not represent the only needed source of correction. The counting process needs to account for the different ways disjoint cycles can commute. The 2cycles in the above example provide the same permutation for either $\left(\pi_{1}, \pi_{2}\right)\left(\pi_{3}, \pi_{4}, \pi_{5}\right)\left(\pi_{6}, \pi_{7}\right)\left(\pi_{8}\right)$ or $\left(\pi_{6}, \pi_{7}\right)\left(\pi_{3}, \pi_{4}, \pi_{5}\right)\left(\pi_{1}, \pi_{2}\right)\left(\pi_{8}\right)$. These two arrangements require a correction by a factor of 2 .

The size of the conjugacy class (Crino, 2002:67), based on the above considerations having $\boldsymbol{c}_{\boldsymbol{i}}$ k-cycles, where $g$ is a specific cycle structure is

$$
|\operatorname{CClass}(\boldsymbol{G}, \boldsymbol{g})|=\frac{\boldsymbol{c}!}{\prod c_{\boldsymbol{i}}!\boldsymbol{k}^{c_{i}}}
$$

Table 2.1 provides an example of our $\boldsymbol{S}_{8}$ example and its 22 conjugacy classes. The table provides a distribution of the 40,320 potential solutions.

Table 2-1 Conjugacy Classes for $\mathrm{S}_{\mathbf{8}}$

| CClass | Cycle | Cycle Structure | Size |
| :---: | :--- | :--- | :---: |
| CC 1 | $1^{8}$ | $(1)(1)(1)(1)(1)(1)(1)(1)$ | 1 |
| CC 2 | $2^{1} 1^{6}$ | $(2)(1)(1)(1)(1)(1)(1)$ | 28 |
| CC 3 | $2^{2} 1^{4}$ | $(2)(2)(1)(1)(1)(1)$ | 210 |
| CC 4 | $2^{3} 1^{2}$ | $(2)(2)(2)(1)(1)$ | 420 |
| CC 5 | $2^{4}$ | $(2)(2)(2)(2)(2)(2)(2)(2)$ | 105 |
| CC 6 | $3^{1} 1^{5}$ | $(3)(1)(1)(1)(1)(1)$ | 112 |
| CC 7 | $3^{1} 2^{1} 1^{3}$ | $(3)(2)(1)(1)(1)$ | 1120 |
| CC 8 | $3^{1} 2^{2} 1^{1}$ | $(3)(2)(2)(1)$ | 1680 |
| CC 9 | $3^{2} 1^{2}$ | $(3)(3)(1)(1)$ | 1120 |
| CC 10 | $3^{2} 2^{1}$ | $(3)(3)(2)$ | 1120 |
| CC 11 | $4^{1} 1^{4}$ | $(4)(1)(1)(1)(1)$ | 420 |
| CC 12 | $4^{1} 2^{1} 1^{2}$ | $(4)(2)(1)(1)$ | 2520 |
| CC 13 | $4^{1} 2^{2}$ | $(4)(2)(2)$ | 1260 |
| CC 14 | $4^{1} 3^{1} 1^{1}$ | $(4)(3)(1)$ | 3360 |
| CC 15 | $4^{2}$ | $(4)(4)$ | 1260 |
| CC 16 | $5^{1} 1^{3}$ | $(5)(1)(1)(1)$ | 1344 |
| CC 17 | $5^{1} 2^{1} 1^{1}$ | $(5)(2)(1)$ | 4032 |
| CC 18 | $5^{1} 3^{1}$ | $(5)(3)$ | 2688 |
| CC 19 | $6^{1} 1^{2}$ | $(6)(1)(1)$ | 3360 |
| CC 20 | $6^{1} 2^{1}$ | $(6)(2)$ | 3360 |
| CC 21 | $7^{1} 1^{1}$ | $(7)(1)$ | 5760 |
| CC 22 | $8^{1}$ | $(8)$ | 5040 |

The characteristics of the LPDP provide an additional reduction of the feasible solution space. For example, suppose there were two potential depots and two available vehicles in the $\boldsymbol{S}_{8}$ example. This implies that the cyclic structure must contain at most three disjoint cycles. This reduces the number of feasible conjugacy classes in $\boldsymbol{S}_{8}$ to only 10 classes. A feasible LPDP solution requires a letter for each depot, vehicle and a linked letter for each pickup and delivery job. This implies that any feasible solution to the problem contains disjoint cycles with at least four letters but no more than seven letters. A feasible solution would not have an eight letter cycle, since the cycle would contain both depot letters. This consideration reduces the number of feasible conjugacy classes in the solution space to only four classes (CC15, CC16, CC19, and CC21).

The reduced number of feasible conjugacy classes reduces the number of solutions the tabu search has to traverse. Limiting the search to only the feasible conjugacy classes reduces the number of potential solutions from the original 40,320 to only 11,724 .

### 2.2.7 Templates

Colletti (1999) coined the word template in his research of group theory and its application to the m-TSP. A template is a mechanism that either fragments a permutation or joins several smaller permutations into a single larger permutation. This research utilizes templates to traverse between conjugacy classes throughout its search process.

Definition: A welding template, w , is an $m$-cycle that joins smaller disjoint $k$ cycles according to the welding template's letter sequence (Colletti and Barnes, 2004).

For example, the following $S_{8}$ consisting of three disjoint cycles $(1,2,3)(4,5,6)(7,8)$ and the welding template $w=(1,4,7)$ create the following united cycle:

$$
(1,2,3)(4,5,6)(7,8) \oplus(1,4,7)=(1,2,3,4,5,6,7,8)
$$

Definition: A splitting template, s , is an $m$-cycle that splits larger cycles into smaller disjoint cycles according to the splitting template's letter sequence (Colletti and Barnes, 2004).

For example, the following $S_{8}$ solution $(1,2,3,4,5,6,7,8)$ and the splitting template $s=(2,5,7)$ create the following disjoint cycle solution:

$$
(1,2,3,4,5,6,7,8) \oplus(2,5,7)=(2,3,4)(5,6)(7,8,1)
$$

This research capitalizes on these two templates to help traverse the various conjugacy classes that partition the solution space.

### 2.2.8 Summary

Group theory provides a potential advantage over traditional tabu search procedures. Group theory provides the ability to completely partition the solution space. This partitioning allows the tabu search process to traverse the solution space and avoid unprofitable regions. Group theory also enhances the understanding of the search structure of traditional neighborhood moves.

### 2.3 Chapter Wrap-up

This chapter provided an overview of tabu search and group theory and laid the foundation for understanding the tabu search approach developed in this research. The
next chapter provides an overview of the literature pertinent to solving the PDP and LRP classes of problems.

## III Literature Review

Research in the General Pickup and Delivery Problem (GPDP) dates back more than three decades to the 1970s (Mitrovic-Minic, 1998:38). Despite 30 years of research, the GPDP is still not nearly as researched as the General Vehicle Routing Problem (GVRP). Savelsberg (1995) provides the most resent survey of the GPDP. The survey divides the GPDP into four categories: the static and dynamic single-vehicle PDP (1PDP) and the static and dynamic multi-vehicle PDP (m-PDP). Savelsberg introduces the notation and formulation for the GPDP to isolate for discussion the various complicating characteristics found in practical pickup and delivery problems, such as type of transportation requests, time constraints, and objective functions. There exist no known references in the literature of a tabu search solution or other heuristic approach for a multi-depot heterogeneous m-LPDPTW.

The literature on Location-Routing Problems (LRP) is far more abundant than PDP literature, but the LRP is still not as researched as the VRP. Research on the LRP dates back to the 1970s and has been primarily focused on simultaneously determining the optimal number and location of warehouses and the vehicle routes to satisfy a set of demands. There exist no known references in the literature for solving a location routing problem that seeks to locate vehicle depots and supply points while simultaneously solving a PDP routing and scheduling problem.

This review focuses on the exact and heuristic methods for solving the time constrained multi-vehicle pickup and delivery problems (m-PDPTW) and the LRP. The reader is referred to the reviews of Carlton (1995) and Crino et al. (2004) for alternate
time window constrained problems, like the traveling salesman problem with time windows (TSPTW) and the vehicle routing problem with time windows (VRPTW). Nanry (1998) provides another review of the VRPTW as part of his research into the single depot homogeneous m-PDPTW.

The intent of this chapter is not to provide a complete discussion of all references to the PDPTW or LRP but to provide a sufficient review of information that is relevant to this research. Section 3.1 covers the literature and basic definitions of the PDP. Section 3.2 is partitioned into three subsections. The first section reviews the literature relevant to the Dial-a-Ride problem (DARP). The second section reviews the literature relevant to the Handicapped Person Transportation Problem (HTP). The last section reviews the literature relevant to the PDP. The objective of this section is to discuss methods that researchers have used to solve the PDPTW. Section 3.3 looks at relevant literature for the location routing problem. The last section, 3.4, provides a review of relevant tabu search literature for routing and scheduling problems.

### 3.1 GPDP Literature

The GPDP is a combinatorial optimization problem that has attracted more interest from researchers in the last two decades. The PDP is a generalization of the vehicle routing problem, which is a generalization of the traveling salesman problem, a well known NP-hard combinatorial optimization problem. Despite its application to many real world situations, the PDP has received much less attention in the vehicle scheduling literature than the VRP. There is no definitive source that enumerates the major characteristics of the PDP such as Bodin (1990) did for the VRP. However, this
research extends many of those characteristics identified by Bodin (1990) to the PDP problem. Savelsbergh (1995:2) does characterize three special cases of the GPDP; the PDP, DARP, and VRP. Mitrovic-Minic (1998) categorizes literature for practical pickup and delivery problems into one of three classes: the Dial-a-Ride problem (DARP), the Handicapped person Transportation Problem (HTP), and the Pick-up and Delivery Problem (PDP). The first two instances deal with the transportation of individuals, while the third class of problems considers the transportation of objects.

### 3.1.1 Defining the PDP

The GPDP is a problem of finding a set of optimal routes for a given fleet of vehicles, in order to satisfy a set of transportation requests. Each vehicle has a set capacity, and a start and end location. Each request specifies the load to be transported, an origin and a destination location. The set of side constraints for the GPDP is more complex than those of the General Vehicle Routing Problem (GVRP). The GPDP includes a set of precedence and pairing constraints in addition to the capacity and visiting constraints in the GVRP. Precedence constraints add the restriction that a vehicle must pickup a request before dropping it off. The pairing, or coupling, constraints further restrict the solution by requiring that the same vehicle visit both the pickup and delivery locations.

A graphical representation of a simple PDP is given in Figure 3.1. The large circle represents a depot with assigned transportation assets. The squares represent pickup locations and the small circles represent delivery locations. The directed arcs
represent vehicle routes. This example contains a single vehicle that departs a depot and


Figure 3-1 Graphical PDP Example
services three job requests between five unique customers.
This research looks at extending the categories Bodin, et al. (1983) presented for the GVRP to the GPDP. Categorizing the PDP as a routing problem, scheduling problem or routing and scheduling problem provides a clearer link to the VRP literature. A PDP routing problem is purely spatially oriented with no time considerations. A PDP scheduling problem includes both spatial and temporal factors. This additional temporal consideration converts the PDP into a pickup and delivery problem with time windows (PDPTW). The PDP routing and scheduling problem considers both spatial and temporal factors characterized by task precedence and time window constraints (Bodin, et al. 1983). If there is only one vehicle in the problem, the corresponding problem is the single-vehicle pickup and delivery problem with time windows (1-PDPTW). If there is a
set of vehicles, the problem becomes the multi-vehicle pickup and delivery problem with time windows (m-PDPTW). This research is primarily concerned with the nonhomogeneous m-PDPTW.

### 3.2 GPDP Practical Problem Class Instances

This section provides specific instances of the three practical problem classes presented earlier. Section 3.2.1 covers the DARP. Section 3.2.2 provides examples of the HTP and Section 3.2.3 covers the PDP of most interest to this research.

Exact or optimization methods have been used to solve the PDPTW. These methods search for a set of routes that generate the best objective function values among the set of all possible routes. Exact methods to solve the PDPTW generally fall into either a dynamic programming or Dantzig-Wolfe type decomposition approach.

### 3.2.1 Dial-a-Ride Problems

The dial-a-ride problem is an instance of the GPDP that deals with transporting people instead of cargo and is the most studied class of PDP in the literature. Customers provide a request for transportation from a specific origin to a specific destination. Vehicles provide a shared service since many customers may be in a vehicle at the same time. Dial-a-Ride problems generally have two conflicting objectives: minimizing operating costs and minimizing user inconvenience. User inconvenience is often measured as a deviation from a desired pick-up or delivery time or customer max riding time. This research is most interested in the DARPTW, which is a multi-vehicle pickup and delivery problem with the added temporal concerns of time windows (m-PDPTW).

Psaraftis (1983) published the first exact dynamic programming approach for solving the static 1-PDPTW. This approach also represents the first optimization technique applied to the PDPTW and was based on Psaraftis' (1980) backward recursion dynamic programming algorithm for solving the 1-PDP. The algorithm utilizes a forward recursion algorithm that minimizes a weighted combination of the time needed to serve all customers and the total degree of customer dissatisfaction. The algorithm was able to solve problems to optimality with up to 10 customers.

Sexton and Bodin (1985) developed an algorithm that utilizes a Benders' decomposition approach applied to a mixed binary non-linear formulation to solve a 1PDPTW. The algorithm minimizes customer inconvenience by solving the routing master problem and scheduling sub-problems separately. The authors formulated the master problem as an integer program and the sub-problem as a linear program. Customer inconvenience is then measured as a weighted sum of the two functions. The first function captures excess ride time and the second function is the delta between desired and actual drop off time. Sexton and Bodin use a heuristic version of Benders' decomposition to solve the master and sub-problem individually. The scheduling subproblem is actually a dual of the maximum network flow problem which can be solved quickly. The authors report good results for several real-life problems in Baltimore, MD, with 7 to 20 customers.

Desrosiers, Dumas, and Soumis (1986) provided an intriguing forward dynamic programming approach that minimizes total distance traveled. The algorithm uses states ( $\mathbf{S}, \mathbf{i}$ ) with $\boldsymbol{S} \subseteq \boldsymbol{V}$ and $\boldsymbol{i} \in \boldsymbol{V}$, where $\boldsymbol{V}$ is a set of all pickup and delivery locations.

The algorithm only defines a state $(\mathbf{S}, \boldsymbol{i})$ if there is a feasible route that covers all pickup and delivery locations in $\boldsymbol{S}$ and ends in $\boldsymbol{i}$. The process includes a state elimination criteria based on both $\boldsymbol{S}$ and ( $\boldsymbol{S}, \boldsymbol{i})$. The power of the algorithm is based on nine elimination criteria that remove any states that are not compatible with the capacity, precedence, and time window constraints, effectively reducing the number of examined states. The algorithm provided good results on problems with up to 40 customers with tight time windows and small vehicle capacities.

Jaw, et al. (1986) solved a version of the m-PDPTW where windows are imposed on the pickup time of requests. The heuristic utilizes a greedy approach to solve the problem by sequentially inserting customers into routes that yield the smallest possible increase in the objective function. The authors tested the heuristic, with good results, on a generated data set with up to 250 customers.

Dumas, Derosiers, and Soumis (1991) provide an improvement to their two-phase cluster first, route second heuristic to solve the HTPTW. The authors adjusted the heuristic to incorporate part of the clustering phase into the routing phase.

Cordeau (2003) proposed a Branch-and-cut algorithm for solving the m-PDPTW. The procedure consists of two main phases. The first phase is a preprocessing phase followed by the main branch-and-cut phase. The authors designed the preprocessing phase to reduce the overall problem size. Preprocessing includes window tightening, arc elimination and variable fixing steps. Each of these steps is designed to reduce the required search space for the algorithm. The algorithm solves an LP relaxation of the problem after completing the preprocessing. The solution to the problem is optimal if it
is integer. If not, the algorithm generates an implicit enumeration tree. Cordeau utilizes a tabu search heuristic (Cordeau and Laporte 2003) to develop an upper bound for the problem. The branch-and-cut procedure uses this upper bound to prune the tree. Cordeau conducted computational experiments on generated instances with up to 32 customers.

Cordeau and Laporte (2003) developed a tabu search heuristic for the homogeneous m-PDPTW. Their heuristic utilizes a random construction phase that assigns every request to a randomly selected vehicle at the end of the vehicle's route. This approach does not guarantee a feasible initial solution. The algorithm then selects the best non-tabu solution based on a linear combination of the cost and penalty function. The heuristic utilizes an intra-route exchange to improve the solution. The authors achieved good results on six real-life instances from Denmark with up to 295 customers.

### 3.2.2 Handicapped Person Transportation Problems

The handicapped person transportation problem is a recent line of research that is similar to the DARP except that the problem consists of different types of passengers where each type (e.g., able to walk, needs a wheelchair, etc.) requires suitable space on a vehicle from an origin to a destination (Mitrovic-Minic, 1998:4). The service at each location occurs at a given time window and the customer's trip duration should not exceed a maximum travel time. This problem class is also an m-PDPTW but with a multi-commodity requirement based on customer type. This makes the HTP a generalization of the PDPTW and NP-hard.

Mitrovic-Minic (1998) provides an overview of heuristic methods to deal with the HTPTW. There are no known exact methods that address the HTPTW. The survey provides a comparison of both classical and modern heuristics. Classical heuristics contain both construction and improvement type heuristics. Mitrovic-Minic (1998) contains a set of construction (decomposition) heuristics that deal with the HTPTW. Mitrovic-Minic (1998) shows that Desrosiers, Dumas, and Soumis (1988) present a miniclustering/routing construction method that constructs mini-clusters based on the vehicles, then an exact dynamic programming algorithm (Desrosiers, Dumas, Soumis 1986) is used to solve each 1-PDPTW. This method found good solutions for the HTPTW with up to 880 requests and 53 homogeneous vehicles. Desrosiers, Dumas, Soumis, and Tallefer (1991) improved their mini-clustering method (Desrosiers, Dumas, Soumis, 1988) by applying a parallel insertion heuristic to direct the mini-clustering phase.

Ioachim, Desrosiers, Dumas and Solomon (1995) proposed an approximation algorithm for the homogeneous m-DARPTW, where maximum travel time constraints are not present. The algorithm is based on the mini-clustering approach of Dumas, Desrosiers and Soumis (1988). The algorithm builds a large set of small trip clusters through column generation. The algorithm develops the final routes by solving an mTSPTW utilizing a delayed column generation for each route. The authors compared their approach against the insertion algorithm and real life problems containing 50-250 customers.

The only modern heuristics referenced in Mitrovic-Minic (1998) is a tabu threshold improvement procedure by Toth and Vigo (1997). The procedure utilizes a parallel insertion heuristic, called TV by the authors, to improve the initial set of routes and then a tabu threshold procedure to help refine the routes by eliminating short routes. The heuristic is based on relaxing the desired service time constraints. The authors achieve this by introducing a linear user inconvenience penalty in the objective function. The construction algorithm consists of a procedure that iteratively assigns un-routed trips to existing routes. The heuristic utilizes a cost matrix to solve each iteration's assignment problem at each iteration. The cost matrix is generated by using a modified cheapest insertion criterion based on local choices. The heuristic starts a new route when it cannot feasibly insert a trip in a current route. The tabu threshold phase is based on alternating between an improve phase and a mixed phase. The heuristic uses the improve phase to find the local optima and the mixed phase to escape local optima. The heuristic uses three neighborhood procedures; trip insertion, trip exchange and trip double insertion. The neighborhood procedures essentially move trips from one route to another route or an exchange of trips between routes. The authors successfully applied their approach to several real-life problems from Bologna, involving approximately 300 trips. The method was able to achieve substantial improvement over the hand-made real world schedules.

### 3.2.3 Pickup and Delivery Problems

The pickup and delivery problem deals with the transportation of cargo. This cargo may include a diverse set of items, such as messages, packages, liquid, etc. These
problems are also classified as m-PDPTW. Unlike the DARPTW, very little heuristic research has been extended to solve the m-PDPTW.

Sexton and Choi (1986) introduced a Benders' decomposition method that utilized a two-phase routing and scheduling procedure. Their approach seeks to minimize a combination of total vehicle operating time and customer penalty for missing their time windows. The algorithm uses Benders' decomposition to generate an optimal solution for the scheduling sub-problem. The procedure then improves the route using Lagrangian relaxation based on the coefficients of the Benders' cut. Sexton and Choi (1986) suggested that their algorithm is efficient for problems with up to 17 customers.

Dumas, Desrosiers, and Soumis (1991) developed a Dantzig-Wolfe exact decomposition for an m-PDPTW for transporting goods. Their approach utilizes a column generation procedure coupled with a constrained shortest path sub-problem. The algorithm solves the shortest path problem with a forward dynamic programming algorithm. The algorithm's objective is to minimize total travel cost. The authors touted that the algorithm was very robust in its ability to handle different objective functions, the number of depots and heterogeneous vehicles. The algorithm appears to work well with up to 55 customers and large demand sizes, i.e., tight vehicle constraints.

Mitrovic-Minic's (1998) survey provides several heuristic methods for solving the m-PDPTW. Jaw, Odoni, Psaqraftis, and, Wilson (1986) developed a sequential insertion heuristic and Van Der Bruggen, Lenstra, and, Schurr (1993) produced a two-phase variable depth local search procedure for the 1-PDPTW. The construction phase starts with an infeasible tour and utilizes a penalized objective function that seeks to reduce
infeasibility. The algorithm only allows solutions to violate time window constraints. The algorithm implements the improvement phase once the construction phase develops a feasible solution. Both phases use a variable depth arc-exchange based on a neighborhood search strategy. The authors were able to show that their algorithm provided near optimal solutions to real life problems. Van Der Bruggen, et al. (1993) updated their variable depth procedure with the development of a penalized simulated annealing procedure to the PDPTW. This procedure provided the power to escape local optima by accepting inferior solutions. The heuristic provided good results, but they came with a high computational cost.

Bodin, Golden, Assad and Ball (1983) developed a route first cluster second approximation approach for the m-PDPTW. The procedure constructs a giant route that services all requests and then divides this route into feasible vehicle routes. The procedure develops the giant route by solving a postman problem. The application appears to provide good solutions for problems with up to 800 requests.

Nanry and Barnes (2000) present a reactive tabu search heuristic to solve a homogenous m-PDPTW. The heuristic utilizes a greedy insertion method to generate the initial solution. This solution is then improved by a reactive tabu search phase utilizing one of three proposed neighborhoods. These neighborhoods consist of a single paired insertion (SPI), swapping pairs between routes (SBR), and within route insertion (WRI). The heuristic utilizes a hierarchical search methodology based on average time window length to dynamically alternate between the three neighborhoods. This methodology
allows the heuristic to traverse different regions of the solution space and adjust its search strategy.

Xu , and Rajaopal (2003) proposed a set partitioning type formulation containing an exponential number of columns to solve a practical heterogeneous m-PDPTW. The authors solve a linear relaxation of the set partitioning problem with a standard column generation procedure. This procedure results in a linear program master problem that is solved with an LP solver. The sub-problems are too difficult to solve to optimality, so the authors propose two fast heuristics to solve the sub-problem. Once the master problem is solved, each trip in the basis has zero reduced cost. The heuristic seeks to generate new trips with a negative reduced cost by modifying existing trips with a zero reduced cost. The heuristic uses two phases to first insert a request into the route of a trip and then second to delete a request from the route of a trip. The rule for inserting and deleting a request is based on a greedy strategy tied to the cost for inserting and deleting the request. The authors successfully applied the heuristic to several generated test problems based on real world operations. The tests indicate that the heuristic can quickly handle problems with up to 210 requests and "up to 500 requests within an acceptable computational time" (Xu and Rajaopal, 2003: 363).

Bent and Hentenryck (2004) present a two-stage hybrid algorithm for the homogeneous m-PDPTW. The first stage of the algorithm utilizes a simulated annealing procedure to decrease the number of routes. The second stage uses large neighborhood search (LNS) to decrease the travel cost. The simulated annealing procedure only looks to relocate pairs of customers. Its objective function contains three criterions: minimize
the number of routes, maximize the sum of squares of the route sizes and minimize the travel cost of the entire routing plan. Once a routing plan is developed, LNS seeks to minimize travel cost by randomly selecting a set of customers for insertion. The procedure then utilizes a modified branch-and-bound process to determine the best insertion point. Bent and Hentenryck (2004) provide experimental results of their algorithm for 100, 200 and 600 customers.

### 3.3 Location Routing Problem

The location of supply facilities may be among the most critical management decision in developing an efficient supply chain. Decisions concerning the number, size and location of these supply facilities directly impact the distribution system's cost and customer support. Supply chain literature generally views these facility location decisions as long term strategic decisions, while routing and scheduling decisions are viewed as operational decisions (Perl and Daskin, 1985: 3). Location decisions that only consider the location of supply facilities are known as fixed charge facility location problems (FLP). The FLP seeks to determine the location of facilities and the shipping pattern between customers and facilities to minimize the total facility location and shipment costs. The location routing problem (LRP) represents an extension of the FLP.

### 3.3.1 Defining the LRP

Traditional research defines the LRP as a VRP where the solution procedure determines the optimal number and locations of the depots simultaneously with solving the vehicle schedules and distribution routes to minimize total system costs (Tuzun and Burke 1999). The LRP is a NP-hard problem because it merges two known NP-hard
problems: facility location and vehicle routing. Any solution attempt of the LRP must address the following three sub-problems: facility location, demand or customer allocation, and vehicle routing. Solving these sub-problems separately produces an overall non-optimal solution and solving a combined problem is computationally impractical. Figure 3-2 provides a graphical representation of a simple LRP example. The example includes two potential facility locations and three customers. The larger circles represent the facilities and the three smaller circles represent the customers. The LRP solution selects the appropriate set of facilities and develops the routes to service the customers. The example shows two vehicles servicing the three customers from a single facility.


Figure 3-2 Graphical LRP Example

### 3.3.2 LRP Literature

Laporte (1988) provides a survey of early location routing procedures and provides a summary of the formulation types, solution procedures and computational results of work published prior to 1988. Min, Jayaraman, and Srivastava (1998) is a more recent survey that provides a hierarchical and classification scheme for reviewing
existing LRP literature. Their work categorizes papers based on problem characteristics and solution methodology.

A review of Laporte (1988) and Min et al. (1998) provides several general aspects that characterize the location-routing problem (LRP):

1. Depot Type: Does a vehicle have to start and return to the depot?
2. Depot Number: How many depots are available and do they have capacities?
3. Transportation Assets: How many vehicles (types) are available and do they have capacities?
4. Customer Data: Are the requirements deterministic or stochastic?
5. Facility Layers: Does the problem include a distribution center?

The LPDPTW considered in this dissertation differs slightly in its definition of depots from the LRP literature. The literature assumes that a depot is a logistics supply point. This research considers a depot to be a home location of transportation assets without supplies and replaces the term depot with supply point in all discussions concerning the LRP. Based on the definition for a supply point, vehicles in this research do not start or return to a logistic supply point but are required to start and return to their assigned depots. This provides one distinction from the classic VRP where vehicles are assigned to a supply point. The m-PDPTW solved in this dissertation contains many logistics supply points that may or may not have a capacity. The definition of capacity refers to the amount of material that a supply point can process or throughput in a given period of time. The research assumes there is a set of potential unconstrained depot locations with
an associated fixed opening and operating cost. The m-PDPTW also contains multiple heterogeneous capacitated vehicles. The research also assumes that the problem has a given set of customers (suppliers / demanders) with deterministic demands.

One major difference between an LRP and PDP problem is the requirement to also consider the costs associated with opening and operating a depot. For example, the decision to open an airfield with the ability to stage, maintain and deploy a set of C-17s has an associated fixed overhead operating cost. This fixed operating cost might also include an opening cost. For example, establishing an airfield inside a theater of operations might require the movement of support assets to provide the ability to stage, maintain and deploy the aircraft. Therefore, the requirement to open this airfield requires a one time expenditure of resources. The problem also possesses the associated traveling cost between any two points. The heuristic's goal is to determine the number and locations of depots to open and a design of the routes from each depot to meet all customer demands satisfying the traditional PDPTW constraints while minimizing cost and lateness of deliveries.

This research is most interested in LRP solution methods that take an integrative approach to solving the LRP. This approach combines a determined location planning problem with a determined route planning problem and then employs a heuristic technique to solve the combined problem. These methods are essentially location allocation - routing (LAR) or allocation - routing - location (ARL) methods. Integrative examples from the literature include sequential, iterative and parallel methods.

Sequential methods seek to first develop a solution to either the location or routing problem and then utilize the information from this process to determine a good solution to the follow-on routing or location problem. Or and Pierskalla (1979), and Nambier, Gelder and Wassenhove (1989) provide examples of a locate first and route second process. Jacobsen and Madsen (1980) provide an example of the route first and locate second process.

The iterative method fluctuates between a pure location problem and one or more route problems. The location problem addresses potential customer allocation by including approximations of delivery costs. The solution of a sub-problem provides input for another sub-problem. Perl $(1983)$ and Perl and Daskin $(1985,1993)$ present several approaches that represent the iterative method. Perl solves a single commodity problem with a three phase heuristic. The first solves the routing problem. The second phase solves the location problem and assigns the routes from the first phase to the supply points. The last phase attempts to improve the solution by resolving the routing problem. The improvement phase continues until the improvement reaches a user defined threshold. Wu, Low, and Bai (2002) provide another iterative example with two phases. Their approach solved a single commodity multi-supply point problem with up to 150 customers. Salhi and Fraser (1996) provides the only known heterogeneous vehicle example. Their process iterates between the location phase and routing phase until a suitable stopping criterion is achieved.

The parallel solution method represents the last integrative approach. The parallel solution approach attempts to nest the location and route planning phases where routes
are a direct component of the location phase. Chien (1993) and Nagy and Salhi (1996) provide examples of the parallel solution method. Their approaches treat the routing problem as a sub-problem within the larger location problem.

Tuzun and Burke (1999) present an integrated two-phase tabu search approach for solving the location-routing problem. Their procedure aims to integrate the location and routing phase of the problem. The procedure decomposes the problem into two subproblems based on the location and routing variables. The location phase utilizes tabu search to determine a good configuration of the depots for the distribution network. The procedure passes this location configuration to the routing phase. The routing phase utilizes a second tabu search process to develop a good routing plan for the configuration. The overall heuristic coordinates these two search phases so that each time the location phase makes a move, the routing phase updates its solution to account for the new move. The advantage of the authors' method is that the routing phase only needs to account for the changes from the move and not conduct a global update. The location and routing phases utilize a set of exchange and insert moves to improve their specific configurations. The authors compared their algorithm with Srivastava's (1993) SAV1 algorithm on 360 generated problems with up to 200 customers with comparably good results.

Wu , Low and Bai (2002) present a simulated annealing sequential and iterative method for solving the heterogeneous multi-depot location-routing problem. The authors decompose the problem into a location-allocation and vehicle routing sub-problem. The heuristic then utilizes simulated annealing to sequentially and iteratively solve each subproblem. The heuristic determines the number and locations of the depots and allocation
of customers to each selected depot in the location-allocation sub-problem. The locationallocation phase consists of an initial solution and solution improvement module. The improvement module utilizes an exchange and insertion procedure with a tabu list to improve the initial solution. This process is repeated until the heuristic meets a stopping criterion. The heuristic then passes this information to the VRP sub-problem. The VRP module consists of a between route and within route improvement phase. These phases use a set of exchanges and insert procedures to improve the objective function. This process continues until the VRP heuristic meets some stopping criterion. The authors tested their heuristic method with good results on four generated cases with up to 150 customers.

Wasner and Zapfel (2004) present a modified parallel integrated multi-depot hub-location vehicle routing model (VRM) for solving an instance of the LRP. The authors utilize a local search procedure to successively solve a series of sub-problems. The solution to a sub-problem then serves as a constraint for the next sub-problem. The process is iterative in that the heuristic feeds back the solution for each sub-problem to re-solve the sub-problem during the next iteration. The first sub-problem determines the number and then the location of the depots. The heuristic uses this information to develop the line haul routes. This sub-problem also assigns customers (postal zones in this case) to the depots and determines the depot and line haul costs. The final subproblem assigns transportation assets to the routes to service the customers. The heuristic then feeds this information back into the previous sub-problem to see if any improvements (lower costs) in the route construction or customer assignment are
possible. The heuristic ends the iteration by calculating the overall costs and then all information is feed back into the define depot location sub-problem for the next iteration. VRM continues this process until it meets some stopping criterion. The authors tested their model on the Austrian postal system with 10 depots and over 2000 customers.

LRP research is much more limited than either facility location or VRP research. The inherent difficulties of combining strategic or long term supply point location decisions with operational routing decisions is likely one reason that accounts for the limited attention from researchers. Wu et al. (2002) and Salhi and Fraser (1996) represent the only known works to address heterogeneous vehicles. Most literature also assumes unlimited vehicles with the same capacity. There is little research that captures the practical aspects of the LRP and no known research that captures the pickup and delivery aspects associated with the TDP.

### 3.4 Tabu Search

Chapter II provided an introduction to the basic concepts of tabu search (TS) as outlined by Glover (1989). Since this introduction almost two decades ago, well over a hundred papers have appeared in the literature addressing various applications of TS to combinatorial problems. Many of these TS methods provide solutions very close to optimality and are shown to be effective at tackling complex combinatorial problems. These successes have made TS extremely popular for solving practical problems. Despite this wealth of TS approaches in the literature, GPDP class problems have not received near the attention of the GVRP class.

Current trends in TS research focus on more advanced concepts and techniques to make the search more effective. This focus includes methods that look to exploit the information that becomes available during the search process, the development of more powerful neighborhood operators and the creation of better starting points. Reactive tabu search (RTS), adaptive tabu search (ATS), and group theoretic tabu search (GTTS) provide examples of recent successful implementations of TS in routing and scheduling problems of interest in this research.

### 3.4.1 Reactive Tabu Search

Battiti and Tecchiolli (1994) designed their RTS procedure as an enhancement to the classical tabu search procedure. RTS utilizes a simple feedback scheme that updates the prohibition parameters (tabu tenure) in TS. The algorithm automatically adjusts the search parameters based on the current state and quality of the search. This feedback and update process provides the heuristic with a balance of exploration versus exploitation. This process requires maintaining a history of previously visited solutions. Typically the TS process uses some type of imbedded hashing function to identify the previously visited solutions.

Nanry and Barnes (2000) provide the only known example of using a RTS procedure for solving a case of the single-depot PDPTW. The authors utilize a greedy insertion procedure to develop an initial feasible solution. The RTS mechanism utilizes a hierarchical neighborhood scheme that alternates between three proposed neighborhoods based on the characteristics of the problem to negotiate different regions of the solution space. The proposed neighborhoods consist of the single paired insertion (SPI),
swapping pairs between routes (SBR) and within route insertion (WRI). The authors proved the quality and efficiency of the heuristic on three problem sets of 25,50 and 100 customers.

### 3.4.2 Adaptive Tabu Search

ATS represents another enhanced version of basic TS that seeks to exploit diversification and intensification strategies by changing tabu parameters during the search process. Typical ATS adjustments include updating the tabu tenure or objective function penalty weights based on the current solution. For example, if the current solution is the best solution found, ATS resets the tabu tenure or penalty weights to the original default value. However, if the current solution is not the best found but either better or worst than the previous solution, ATS makes pre-determined changes to the tabu parameters. Glover and Laguna (1997) indicate that this dynamic updating is what promotes diversification and intensification during the search process. Recent work by Combs and Moore (2004), Wiley (2001) and Chamber and Barnes (1996) provide examples of ATS in solving large scale problems. However, they do not address the PDP or LRP nature of the TDP. In fact, there appears to be no ATS approach applied to a TDP type problem.

### 3.4.3 Group Theoretic Tabu Search

Group theory is a fundamental building block of abstract algebra and has been applied over the years to a host of applications. Colletti's (1999) dissertation paved the way for utilizing group theory as a unifying framework for combinatorial optimization problems. His work demonstrates the usefulness of group theory as a tool in
understanding metaheuristic approaches to combinatorial optimization problems (Barnes and Colletti, 2000). Wiley (2001), Crino et al. (2004) and Combs and Moore (2004) provide excellent examples of utilizing GTTS in solving a large scale problem but none of them address the PDP or LRP nature of the TDP. The inclusion of the PDP and location requirements in this research represents an extension of Crino's et al. (2004) VRP work.

### 3.5 Current Theater Distribution Tools

The operational logistics (OPLOG) planner and the logistics estimation worksheet are the two tools used widely by military logisticians in planning theater distribution. These tools are designed to assist staff officers in developing information to facilitate the preparation of estimates of supportability throughout the planning process. These models are management information systems designed to manipulate data and provide statistics. They are not decision models and do not have the ability to construct the routes or schedules necessary to ensure delivery of demands.

### 3.6 Summary

The literature for the Pickup and Delivery problem with Time Windows (PDPTW) classifies practical pick-up and delivery problems into one of three classes: the Dial-a-Ride problem (DARP), the Handicapped Person Transportation Problem (HTP), and the Pick-up and Delivery Problem (PDP). The first two instances deal with the transportation of individuals, while the third class of problems considers the transportation of objects. Most of the literature deals with versions of the DARP class of
problems and attention to the cargo PDP is a focus of the last ten years. The concentration of this recent effort is on the single-depot multiple vehicles PDPTW with little attention to the multiple depot version. There are no known tabu search heuristics for solving the multi-depot multiple heterogeneous vehicles PDPTW.

Research interest in the Location Routing Problem has really picked up speed in the last 30 years. Over $75 \%$ of the literature is concerned with solving versions of the LRP with heuristics. The integrative approaches appear to provide the most promise in addressing the problem of solving the theater distribution problem. While tabu search approaches exist for solving the classic LRP, there are no known tabu search approaches for solving a LRP with pickup and delivery.

The next chapter constructs a mathematical programming formulation for the LPDPTW covered in this research and presents the characteristics of the TDP.

## IV Location Pickup and Delivery with Time Windows Formulation

The goal of this chapter is to present a problem formulation to solve instances of the LPDPTW and to characterize the TDP solved in this dissertation as an LPDPTW. The LPDPTW contains many characteristics in common with both the LRP and PDPTW class problems but neither problem class is robust enough to sufficiently describe the TDP on its own. This chapter is organized as follows. Section 4.1 defines the TDP utilized in this research and characterizes it as a LPDPTW. Section 4.2 defines the multidepot LPDPTW and provides a corresponding mathematical programming formulation.

### 4.1 Theater Distribution as a Location Pickup and Delivery Problem

The general LPDPTW has the following specifications: vehicles (transportation assets), depots (home location of vehicles), supply facilities, requests, routes, and time considerations (both delivery and pickup). Natural extensions to the LPDPTW include: the number of trips per vehicle, the number of visits per customer (traditionally held at one in the literature), hubs (locations for consolidation and transshipment), and commodities (traditionally held at single commodity).

### 4.1.1 LPDP Hierarchy Development

This research characterizes the LPDP and TDP in the classification scheme of Barnes and Carlton (1996). Carlton's (1996) work provided a multi-tiered framework that characterized the GVRP hierarchy and represents an extension of Bodin et al. (1983) vehicle routing and scheduling classification. Carlton's (1995) hierarchy represents the TSP, VRP and PDP class problems as succeeding levels. The TSP and VRP levels are connected in the presence of vehicle capacity constraints and the VRP and PDP levels are
connected if precedence and coupling constraints exist. Each level includes the common characteristics of the GVRP including:

1. Number of vehicles - single vehicle (SV)

- multiple vehicles (MV)

2. Type of vehicles - homogeneous vehicles (H)

- non-homogeneous vehicles $(\overline{\mathrm{H}})$

3. Number of depots - single depot (SD)

- multiple depots (MD)

4. Route Length (RL) constraints - distance or time a vehicle may travel
5. Time Windows (TW) - service must occur within a specified time window

The TDP, under Carlton's classification hierarchy, is partially classified as a MV $\bar{H}$, MD, PDP, with RL and TW constraints. Crino et al. (2004) extends Carlton's (1995) classification with the addition of the following four characteristics:

1. Trips per vehicle - single trip (ST)

- multiple trips (MT)

2. Services per customer - single service (SS)

- multiple services (MS)

3. Types of commodity - single commodity (SC)

- multiple commodities (MC)

4. Existence of transshipment points (T)

These four additions are typically assumed to be single or non-existent in the case of transshipment points in the TSP, VRP and PDP class problems but are necessary
to capture the true nature of the TDP. A TDP vehicle is expected to make multiple trips during the course of any given operation. This is especially true in any effort to reduce the logistics footprint so multiple trips is a needed characteristic of the TDP. Logistics requirements for a unit in the theater typically exceed the capacity of a single vehicle and require multiple services to complete the delivery of all requirements to the unit. Any given TDP scenario is characterized by a limited number of APOD/SPOD locations that all logistics flow through. This restriction requires multiple visits to a supply point to meet the demands of the problem. These situations make multiple services a characteristic of the TDP. TDP logistics requirements are multiple commodity requests that either have different vehicle fill efficiencies or require different vehicle types. For example, a Class III (Fuel) demand requires an entirely different vehicle type (tanker) than a Class I (Food) demand (flatbed trailer). Additionally, a Class V (Ammunition) demand generally exceeds a vehicle's weight limit before its volume limit is exceeded. Conversely, Class I demands generally exceed the vehicle's volume limit. This multicommodity aspect impacts the fleet mix assigned to each depot. In addition, the required travel distances between APOD/SPOD and customers may exceed a vehicle's allowed travel limit. This shortcoming may require the establishment of a transshipment point in the theater to serve as a temporary storage point prior to additional forward movement of supplies to a customer. This transshipment point becomes both a pickup and delivery point. The above four conditions represent Crino's et al. (2004) four additional characteristics and are included as part of the dissertation solved in this TDP. The TDP, under Carlton's and Crino's classification hierarchy, is then partially classified as a

MV̄̄ , MD, PDP, RL, TW, MT, MS, MC, T problem. However, this classification does not capture the TDP's requirement of selecting the required depots and supply facilities and assigning vehicles to the depot for the distribution network.

The classification schemes of Carlton (1995) and Crino et al. (2004) make the assumption that the set of depots are given and fixed. The TDP's objective is to minimize the distribution footprint of the theater and therefore selects, based on cost, the required depots and supply facilities from an available set of potential sites. This cost based selection adds a new location (L) characteristic to the problem description hierarchy. The TDP is classified as a traditional PDP if it has a given fixed depot list (FDL) and becomes the LPDP if it contains a set depot list (SDL). The set depot list implies that the distribution network consists of a selected subset of the list. Figure 4-1 provides a representation of the LPDP classification hierarchy.

The LPDP hierarchy (Figure 4-1) displays 32 characteristic description combinations for the physical network of the TDP. The first column (level 5) is the basic PDPTW, with standard assumptions that Carlton (1995) developed in his research effort. The hierarchy moves from left to right becoming more general as Crino's (2004) additional constraints are added with the requirement of locating the assigned depots, supply facilities and allocating vehicles to the depot. The TDP is now classified under the LPDP as a MV $\bar{H}$, MD, LPDP - SDL, RL, TW, MT, MS, MC, T problem.

Specific TDP scenarios contain several additional characteristics not described by the above LPDP hierarchy. Constraints at a depot or transshipment point may impact a


Figure IV-1 LPDPTW Hierarchy for the TDP
vehicle's route timing. These constraints include such items as on-hand commodity availability or throughput capacity. A vehicle may be forced to wait at a node for supplies to become available (delivered from another location) or for throughput capacity to become available for servicing. A typical throughput constraint is the maximum on the ground (MOG) ability of any given airfield or the number of container spaces in a port. This constraint limits the number of vehicles that can be processed at any given time and therefore limits the amount of supplies that can process through the point. These constraints place time restrictions on vehicles and impact on when they travel along routes. This research refers to these restrictions as timing constraints (TC) to separate them from time window and route constraints.

This section has presented a characterization of the TDP solved in this research. The next section provides a formal presentation of the mathematical formulation of the LPDPTW.

### 4.2 The Location Pickup and Delivery Problem with Time Windows

This section characterizes and provides a mathematical formulation for the LPDPTW. The problem assumes the following characteristics are known in advance; the number, location and type and quantity of demand of all customers, the location of all potential supply and vehicle depots, and the fleet type and size. The LPDPTW in this research is further characterized as a multi-depot, multi-commodity temporal problem. A logistics distribution plan (solution) is then created so that:
(1) the time definite delivery of each customer's demand is satisfied,
(2) each vehicle's route starts and ends at the same vehicle depot,
(3) each demand is serviced exactly once (multiple visits to customer required),
(4) the vehicle load at any given time never exceeds the vehicle's capacity,
(5) the same vehicle both picks up and delivers the demand,
(6) the vehicle visits the pickup location prior to visiting the delivery location,
(7) a vehicle's route does not exceed allowable travel time or distance,
(8) the arrival time and departure time of any given depot location is satisfied,
(9) the service time windows at the supply and demand locations are not violated.

The problem simultaneously determines the number and locations of both supply points and vehicle depots, the allocation of customers to supply points and vehicles to depots, and the assignment of vehicle distribution routes and schedules, so that total system costs are minimized. These system costs consist of supply point and depot establishing costs, supply point and depot operating costs, and vehicle fixed and operating costs. Vehicle operating costs is based on the vehicle's cost per mile of operation and a solution should seek to minimize the vehicle's travel distance.

The LPDPTW formulation developed in this section represents a modification and combination of both the three-layer location routing formulation of Perl and Daskin (1985) and the pickup and delivery problem with time windows formulation of Dumas et al. (1991). Sections 4.2.1-4.2.3 sequentially builds upon Perl and Daskin's (1985) LRP formulation to create the LPDPTW.

### 4.2.1 Generalization of the LRP

Perl and Daskin (1985) utilize a fixed charge location problem as the basis for their LRP formulation. Their work integrates this location problem with a three-layer
routing problem. Typical three-layer problems include flows from plants (commodity manufacturing sites) to a distribution center and then finally to customers. This section presents a more general mathematica programming formulation that differs slightly from Perl's (1985) LRP on five points:
(1) multi-commodity demands are included,
(2) logistics flow from the plant directly to the customer is possible,
(3) the flexibility of heterogeneous fleets is incorporated,
(4) a limitation on the size (number) of the vehicle fleet,
(5) the establishment of supply point storage capabilities.

Perl and Daskin's (1985) work assumes single commodity demands to reduce problem complexity. The inclusion of multi-commodities in the formulation is necessary to address real world practical problems and the TDP. The following formulation allows flow from the plant directly to the customer providing an opportunity to eliminate a transshipment center from the distribution network. Perl and Daskin's (1985) vehicle fleet represents a homogeneous fleet with an unlimited number of vehicles. These assumptions reduce computational complexity but do not represent practical situations and reduce the ability to explore effects associated with limited fleet sizes. The five additions in this section represent a generalization of the standard three-layer LRP. The following mathematical model incorporates the above five modifications creating a more robust LRP.

## Sets:

$J \quad$ set of all customer locations (demands); $\quad 1,2, \ldots, j, \ldots,|J|$.
$S \quad$ set of all source supply points; $\quad 1,2, \ldots, s, \ldots,|S|$.
$H \quad$ set of all transshipment or hub points; $\quad|S|+1,|S|+2, .,|S|+h, .,|S|+|H|$.
$I \quad$ set of all potential supply points $(\boldsymbol{S} \cup \boldsymbol{H}) ; \quad 1,2, \ldots, i, \ldots,|S|+|H|$.
$N \quad$ set of all nodes $(\boldsymbol{S} \cup \boldsymbol{H} \cup \boldsymbol{J}) ; \quad 1,2, \ldots, n, \ldots,|N|$.
$V \quad$ set of all vehicles;
$1,2, \ldots, v, \ldots,|V|$.
$P \quad$ set of all commodities demanded $\quad 1,2, \ldots, p, \ldots,|P|$.
$\boldsymbol{C}_{\boldsymbol{v}} \subseteq \boldsymbol{P}$ set of commodities $\boldsymbol{p} \in \boldsymbol{P}$ able to be carried on vehicle $\boldsymbol{v} \in \boldsymbol{V}$

## Parameters:

$\boldsymbol{d}_{i j} \quad$ distance between point $\boldsymbol{i}$ and point $\boldsymbol{j}(\boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{N})$.
$\boldsymbol{c}_{i j v} \quad$ cost of traveling in vehicle $\boldsymbol{v} \in \boldsymbol{V}$ between point $\boldsymbol{i}$ and $\boldsymbol{j}(\boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{N})$.
$\boldsymbol{q}_{p v} \quad$ capacity in configured loads for commodity $\boldsymbol{p} \in \boldsymbol{P}$ on vehicle $\boldsymbol{v} \in \boldsymbol{V}$
$\tau_{v} \quad$ maximum allowable length of route served by vehicle $\boldsymbol{v} \in \boldsymbol{V}$.
$\sigma_{p j} \quad$ quantity demanded for commodity $\boldsymbol{p} \in \boldsymbol{P}$ at customer $\boldsymbol{j} \in \boldsymbol{J}$.
$\boldsymbol{p}_{\boldsymbol{j}} \quad$ commodity $\boldsymbol{p} \in \boldsymbol{P}$ demanded by customer $\boldsymbol{j} \in \boldsymbol{J}$.
$\boldsymbol{f}_{\boldsymbol{i}} \quad$ fixed cost of selecting supply point $\boldsymbol{i} \in \boldsymbol{I}$ (i does not have to be selected).
$\boldsymbol{f}_{v} \quad$ fixed cost of selecting vehicle $\boldsymbol{v} \in \boldsymbol{V}$ ( $\boldsymbol{v}$ does not have to be selected).
$\boldsymbol{s}_{p i} \quad$ storage capacity for commodity $\boldsymbol{p} \in \boldsymbol{P}$ at candidate supply point $\boldsymbol{i} \in \boldsymbol{I}$.
$\boldsymbol{m}_{p i} \quad$ max daily throughput of commodity $\boldsymbol{p} \in \boldsymbol{P}$ through supply point $\boldsymbol{i} \in \boldsymbol{I}$.
$\boldsymbol{v}_{\boldsymbol{i}} \quad$ variable cost per unit processed at supply point $\boldsymbol{i} \in \boldsymbol{I}$.

M large number.

## Decision Variables:

$Z_{i j v}=\left\{\begin{array}{l}1 \text { if vehicle } \boldsymbol{v} \in \boldsymbol{V} \text { goes from } \boldsymbol{i} \text { to } \boldsymbol{j}(\boldsymbol{i}, \boldsymbol{j}, \in \boldsymbol{N}) \\ 0 \text { otherwise }\end{array}\right.$
$\boldsymbol{X}_{\boldsymbol{i}}=\left\{\begin{array}{l}1 \text { if supply point } \boldsymbol{i} \in \boldsymbol{I} \text { is selected } \\ 0 \text { otherwise }\end{array}\right.$
$\boldsymbol{Y}_{i j}=\left\{\begin{array}{l}1 \text { if demands at } \boldsymbol{j} \in \boldsymbol{J} \text { are satisfied by } \boldsymbol{i} \in \boldsymbol{I} \\ 0 \text { otherwise }\end{array}\right.$
$\boldsymbol{W}_{\text {psh }}$ quantity commodity $\boldsymbol{p} \in \boldsymbol{P}$ shipped from point $\boldsymbol{s} \in \boldsymbol{S}$ to point $\boldsymbol{h} \in \boldsymbol{H}$.

This notation provides the following modified location routing problem:
Minimize:

$$
\begin{equation*}
\sum_{i \in I} f_{i} X_{i}+\sum_{i \in I} v_{i} \sum_{j \in J} \sum_{p \in P} \sigma_{p j} Y_{i j}+\sum_{v \in V} f_{v} \sum_{i \in I} \sum_{j \in J} Z_{i j v} \tag{4-1}
\end{equation*}
$$

Subject to:

$$
\begin{array}{ll}
\sum_{v \in V} \sum_{i \in N} Z_{i j v}=1 & \forall \boldsymbol{j} \in J \\
\sum_{j \in J} \sigma_{p j} \sum_{i \in I \cup J} Z_{i j v} \leq \boldsymbol{q}_{p v} & \forall \boldsymbol{v} \in \boldsymbol{V} ; \forall \boldsymbol{p} \in \boldsymbol{P} \\
\sum_{j \in N} \sum_{i \in N} \boldsymbol{d}_{i j} \mathbf{Z}_{i j v} \leq \tau_{v} & \forall \boldsymbol{v} \in \boldsymbol{V} \\
\sum_{j \in N} Z_{i j v}-\sum_{j \in N} Z_{j i v}=0 & \forall \boldsymbol{i} \in N ; \forall \boldsymbol{v} \in \boldsymbol{V} \\
\sum_{i \in I} \sum_{j \in J} Z_{i j v} \leq 1 & \forall v \in V
\end{array}
$$

$$
\begin{array}{ll}
\sum_{j \in J} \sigma_{p j} \boldsymbol{Y}_{i j}-\boldsymbol{m}_{p i} \boldsymbol{X}_{\boldsymbol{i}} \leq 0 & \forall \boldsymbol{i} \in \boldsymbol{I} ; \forall \boldsymbol{p} \in \boldsymbol{P} \\
\sum_{\boldsymbol{u} \in \boldsymbol{I} \cup \boldsymbol{J}}\left(\boldsymbol{Z}_{i u v}+\mathbf{Z}_{u j \mathrm{v}}\right)-\boldsymbol{Y}_{i j} \leq 1 & \forall \boldsymbol{j} \in \boldsymbol{J} ; \forall \boldsymbol{i} \in \boldsymbol{I} ; \forall \boldsymbol{v} \in \boldsymbol{V} \\
\sum_{j \in J} \sigma_{p j} \boldsymbol{Y}_{s j}+\sum_{h \in \boldsymbol{H}} \boldsymbol{W}_{p s h}-\boldsymbol{m}_{p s} \boldsymbol{X}_{s} \leq 0 & \forall \boldsymbol{s} \in \boldsymbol{S} ; \forall \boldsymbol{p} \in \boldsymbol{P} \\
\sum_{s \in S} \boldsymbol{W}_{p s h}-\sum_{j \in J} \sigma_{p j} \boldsymbol{Y}_{h j}-\boldsymbol{s}_{p h} \boldsymbol{X}_{\boldsymbol{h}} \leq 0 & \forall \boldsymbol{s} \in \boldsymbol{S} ; \forall \boldsymbol{p} \in \boldsymbol{P} ; \forall \boldsymbol{h} \in \boldsymbol{H} \\
\boldsymbol{X}_{\boldsymbol{i}} \in\{0,1\} & \forall \boldsymbol{i} \in \boldsymbol{I} \\
\boldsymbol{Z}_{i j v} \in\{0,1\} & \forall \boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{N} ; \forall \boldsymbol{v} \in \boldsymbol{V} \\
\boldsymbol{Y}_{i j} \in\{0,1\} & \forall \boldsymbol{i} \in \boldsymbol{I} ; \forall \boldsymbol{j} \in \boldsymbol{J} \\
\boldsymbol{W}_{p s h} \geq 0 & \forall \boldsymbol{p} \in \boldsymbol{P} ; \forall \boldsymbol{s} \in \boldsymbol{S} ; \forall \boldsymbol{h} \in \boldsymbol{H} \tag{4-14}
\end{array}
$$

The objective function minimizes the sum of the supply point establishing cost, supply point variable operating cost and the fixed cost for any assigned vehicles. Constraint (4-2) ensures that each customer demand is assigned to exactly one route. Constraint (4-3) imposes a capacity restriction for each vehicle, while constraint (4-4) limits the traveling distance of each vehicle. Constraint (4-5) is a conservation of flow constraint which requires any route entering node $\boldsymbol{i} \in \boldsymbol{N}$ to also exit that node. Constraint (4-6) ensures that each route is served only once. Constraint (4-7) represents a capacity constraint or throughput limit for each selected supply point. Constraint (4-8) specifies that a customer's demand is assigned to a supply point only if there is a route from that supply point through the customer. This constraint serves as the link between the vehicle routing variables $\left(\boldsymbol{Z}_{i j v}\right)$ and the assignment variables. Constraint (4-9) represents flow
from a source supply point to a transshipment point. The constraint ensures that flow from a source point to either customers or a transshipment point does not exceed its capacity or throughput limit. Constraint (4-10) establishes a maximum storage capability for each transshipment supply point. This constraint provides the capability for exploring the effect of creating distribution and holding centers versus pure transshipment points. Constraints (4-11 to 4-14) are standard integrality and non-negativity constraints.

The formulation above represents a general version of Perl's (1985) LRP formulation. The formulation requires the inclusion of PDP and temporal characteristics to transform it into the LPDPTW formulation of interest in this research. The next section extends the formulation to include common PDP requirements.

### 4.2.2 Extending the LRP to the LPDP

This section extends the formulation presented in Section 4.2.1 by incorporating several common PDP characteristics found in Dumas et al. (1991) pickup and delivery formulation. This section incorporates the following two additional elements into the formulation:
(1) the selection of candidate vehicle depots for assigning vehicles,
(2) the same vehicle must both pick up and deliver the demand (coupling).

This section defines the following additional notation:

## Sets:

$K \quad$ set of all potential depots; $\quad 1,2, \ldots, k, \ldots,|K|$.
$N \quad$ set of all nodes $(\boldsymbol{K} \cup \boldsymbol{I} \cup \boldsymbol{J})$
$1,2, \ldots, n, \ldots,|N|$.

## Parameters:

$\boldsymbol{f}_{\boldsymbol{k}} \quad$ fixed cost of selecting supply point $\boldsymbol{k} \in \boldsymbol{K}$.
$\boldsymbol{g}_{v k} \quad$ variable cost of assigning vehicle $\boldsymbol{v} \in \boldsymbol{V}$ to depot $\boldsymbol{k} \in \boldsymbol{K}$

## Decision Variables:

$\boldsymbol{D}_{\boldsymbol{k}}=\left\{\begin{array}{l}1 \text { if } \operatorname{depot} \boldsymbol{k} \in \boldsymbol{K} \text { is selected } \\ 0 \text { otherwise }\end{array}\right.$
With this notation and the notation defined in Section 4.2.1, the following adjustments and additions are made to the general LRP formulation:

Minimize:

$$
\begin{equation*}
\sum_{k \in K} f_{k} D_{k}+\sum_{i \in K} g_{i} \sum_{v \in V} \sum_{j \in I} Z_{i j v}+\sum_{i \in I} f_{i} X_{i}+\sum_{i \in I} v_{i} \sum_{p \in P} \sum_{j \in J} \sigma_{p j} Y_{i j}+\sum_{v \in V} f_{v} \sum_{i \in I} \sum_{j \in J} Z_{i j v} \tag{4-15}
\end{equation*}
$$

Subject to:

$$
\begin{array}{ll}
\sum_{j \in I \cup J} Z_{i j v}-\sum_{j \in I \cup J} Z_{j i v}=0 & \forall i \in I \cup J ; \forall v \in V \\
\sum_{k \in K} Z_{k i v}-\sum_{j \in J} Z_{i j v}=0 & \forall i \in I ; \forall v \in V \\
\sum_{i \in I} Z_{k i v}-\sum_{j \in J} Z_{j k v}=0 & \forall \boldsymbol{k} \in \boldsymbol{K} ; \forall \boldsymbol{v} \in \boldsymbol{V} \\
\sum_{i \in I} Z_{k i v}-\boldsymbol{M} X_{k} \leq 0 & \forall \boldsymbol{k} \in \boldsymbol{K} ; \forall v \in V \\
\sum_{j \in J} Z_{i j v}-\sum_{j \in J} Z_{j i v} \leq 0 & \forall i \in I ; \forall v \in V \tag{4-20}
\end{array}
$$

The updated objective function (4-15) now incorporates the requirement to minimize the establishing and operating cost of selected vehicle depots. Constraint (416 ) is the familiar conservation of flow constraint and updates Constraint (4-5). This
update eliminates arcs that would not be part of a feasible solution. Constraint (4-17) states that if there is flow from a supply point, then there must be flow in from some depot, while constraint (4-18) specifies that if there is flow from a depot, there must be flow back to the depot. Constraint (4-19) serves as the link between routing variables and the selection of depots. It specifies that if there is flow from a depot, then the depot must be selected. Constraint (4-20) is the coupling constraint. This constraint in conjunction with constraint (4-16) ensures that the same vehicle both picks up and delivers the demand. The inclusion of these new constraints transforms the LRP formulation from Section 4.2.1 into a LPDP. However, to better model practical problems and the TDP, this formulation requires the addition of temporal considerations to the formulation.

### 4.2.3 Incorporating Temporal Requirements

This section extends the LPDP formulation to account for the temporal requirements of many practical problems. It incorporates the following additional elements into the formulation:
(1) the vehicle must visit the pickup point prior to the delivery point (precedence),
(2) the vehicle's route does not exceed allowable travel time or distance,
(3) the vehicle arrival and departure time of any given node is satisfied.

This section defines the following additional notation:

## Parameters:

$\boldsymbol{t}_{i j v} \quad$ travel time for vehicle $\boldsymbol{v} \in \boldsymbol{V}$ between nodes $\boldsymbol{i} \in \boldsymbol{N}$ and $\boldsymbol{j} \in \boldsymbol{N}$.
$\boldsymbol{t a}_{\boldsymbol{v k}}$ time vehicle $\boldsymbol{v} \in \boldsymbol{V}$ is available for mission at depot $\boldsymbol{k} \in \boldsymbol{K}$.
$\boldsymbol{e d}_{\boldsymbol{k}} \quad$ earliest allowed departure time from depot $\boldsymbol{k} \in \boldsymbol{K}$.
$\boldsymbol{I d}_{\boldsymbol{k}} \quad$ latest allows departure time from depot $\boldsymbol{k} \in \boldsymbol{K}$.
$\boldsymbol{e r}_{\boldsymbol{k}} \quad$ earliest desired return time to depot $\boldsymbol{k} \in \boldsymbol{K}$.
$\boldsymbol{I r}_{\boldsymbol{k}} \quad$ latest desired return time to depot $\boldsymbol{k} \in \boldsymbol{K}$.
$\boldsymbol{e}_{\boldsymbol{i}} \quad$ earliest arrival time at node $\boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J}$.
$\boldsymbol{I}_{\boldsymbol{i}} \quad$ latest arrival time at node $\boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J}$.
$\boldsymbol{s}_{i v} \quad$ required service time of vehicle $\boldsymbol{v} \in \boldsymbol{V}$ at node $\boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J}$.

## Decision Variables:

$\boldsymbol{T}_{\boldsymbol{i}} \quad$ is the time at which service at node $\boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J}$ begins.
$\boldsymbol{T D}_{v k}$ is the departure time for vehicle $\boldsymbol{v} \in \boldsymbol{V}$ from depot $\boldsymbol{k} \in \boldsymbol{K}$.
$\boldsymbol{T R}_{\boldsymbol{v k}} \quad$ is the return time for vehicle $\boldsymbol{v} \in \boldsymbol{V}$ to $\operatorname{depot} \boldsymbol{k} \in \boldsymbol{K}$.
$\boldsymbol{L}_{v i} \quad$ is the vehicle load after departing node $\boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J}$.
With this notation and the notation defined in Sections 4.2.1 and 4.2.2, the following adjustments and additions are made to the formulation (note: no changes in objective function from 4-15):

Minimize:

$$
\begin{equation*}
\sum_{k \in K} f_{k} D_{k}+\sum_{i \in K} g_{i} \sum_{v \in V} \sum_{j \in I} Z_{i j v}+\sum_{i \in I} f_{i} X_{i}+\sum_{i \in I} v_{i} \sum_{p \in P} \sum_{j \in J} \sigma_{p j} Y_{i j}+\sum_{v \in V} f_{v} \sum_{i \in I} \sum_{j \in J} Z_{i j v} \tag{4-21}
\end{equation*}
$$

Subject to:

$$
\begin{array}{ll}
\boldsymbol{T}_{i}+s_{i}+t_{i, n+i} \leq \boldsymbol{T}_{n+i} & \forall i \in I ; \forall v \in V \\
Z_{i j v} \times\left(T_{i}+s_{i v}+t_{i j v}\right) \leq \boldsymbol{T}_{j} & \forall i \in I ; \forall j \in J ; \forall v \in V \\
Z_{k i v} \times\left(\mathbf{T D}_{v k}+t_{k i v}\right) \leq \boldsymbol{T}_{i} & \forall k \in K ; \forall i \in I ; \forall v \in V \tag{4-24}
\end{array}
$$

$$
\begin{align*}
& Z_{j k v} \times\left(\boldsymbol{T}_{j}+\boldsymbol{s}_{j v}+\boldsymbol{t}_{j k}\right) \leq \boldsymbol{T} \boldsymbol{R}_{v k} \quad \forall k \in K ; \forall \boldsymbol{j} \in \boldsymbol{J} ; \forall v \in V  \tag{4-25}\\
& e_{i} \leq T_{i} \leq I_{i}  \tag{4-26}\\
& e d_{k} \leq \boldsymbol{T D}_{v k} \leq \boldsymbol{l} d_{k} \quad \forall k \in K ; \forall v \in V  \tag{4-27}\\
& e r_{k} \leq \boldsymbol{T} R_{v k} \leq \boldsymbol{l} r_{k} \quad \forall k \in K ; \forall v \in V  \tag{4-28}\\
& T R_{v k}-T_{v k} \leq w \quad \forall v \in V ; \forall k \in K  \tag{4-29}\\
& \boldsymbol{Z}_{i j v} \times\left(\boldsymbol{L}_{v i}+\sigma_{p j}\right)=\boldsymbol{L}_{\boldsymbol{j}} \quad \forall \boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J} ; \boldsymbol{j} \in \boldsymbol{I} ; \boldsymbol{v} \in \boldsymbol{V} ; \boldsymbol{p} \in \boldsymbol{P}  \tag{4-30}\\
& \boldsymbol{Z}_{i j v} \times\left(\boldsymbol{L}_{v i}-\sigma_{p, j-n}\right)=\boldsymbol{L}_{v j} \quad \forall \boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J} ; \boldsymbol{j} \in \boldsymbol{J} ; \boldsymbol{v} \in \boldsymbol{V} ; \boldsymbol{p} \in \boldsymbol{P}  \tag{4-31}\\
& \boldsymbol{Z}_{k i v} \times\left(\boldsymbol{L}_{v k}+\sigma_{p i}\right)=\boldsymbol{L}_{v i} \quad \forall \boldsymbol{k} \in \boldsymbol{K} ; \boldsymbol{i} \in \boldsymbol{I} ; \boldsymbol{v} \in \boldsymbol{V} ; \boldsymbol{p} \in \boldsymbol{P}  \tag{4-32}\\
& \boldsymbol{L}_{\mathbf{v k}}=0 \quad \forall \boldsymbol{v} \in \boldsymbol{V} ; \forall \boldsymbol{k} \in \boldsymbol{K}  \tag{4-33}\\
& \sigma_{p i} \leq \boldsymbol{L}_{i v} \leq \boldsymbol{q}_{p v} \quad \forall \boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J} ; \forall \boldsymbol{v} \in \boldsymbol{V} ; \forall \boldsymbol{p} \in \boldsymbol{P}  \tag{4-34}\\
& \boldsymbol{X}_{\boldsymbol{i}} \in\{0,1\} \quad \forall \boldsymbol{i} \in \boldsymbol{I}  \tag{4-35}\\
& Z_{i j v} \in\{0,1\} \quad \forall i, j \in N ; \forall v \in V  \tag{4-36}\\
& \boldsymbol{Y}_{i j} \in\{0,1\} \quad \forall \boldsymbol{i} \in \boldsymbol{I} ; \forall \boldsymbol{j} \in \boldsymbol{J}  \tag{4-37}\\
& \boldsymbol{L}_{\mathbf{v i}} \geq 0 \\
& \forall \boldsymbol{v} \in \boldsymbol{V} ; \forall \boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J} \tag{4-38}
\end{align*}
$$

Constraint (4-22) is the precedence constraint which forces vehicles to pickup a demand prior to delivering it. Constraints (4-23) - (4-25) describe the compatibility requirements between routes and schedules. Constraints (4-26) - (4-28) represent the time window constraints. These constraints impose increasing times at the nodes of the route. This restriction eliminates the potential of routes with cycles. Constraint (4-29) ensures that a vehicle's route does not exceed the total authorized travel time for a
vehicle. Constraints (4-30) - (4-32) ensures that a vehicle travels from a supply point to a customer with a compatible load. Constraint (4-33) ensures that the vehicle departs the depot with an empty load. Constraint (4-34) is an update to the vehicle capacity constraint (4-3). Constraints (4-35) - (4-38) represent the standard integer and nonnegativity constraints.

The formulation includes a route duration restriction $\left(\boldsymbol{T R}_{v k}-\mathbf{T D}_{v k}\right)$. This restriction coupled with the time window constraints and the compatibility constraints (23) - (25) allow waiting time before visiting a node. The formulation does not penalize this waiting time, but it does represent an opportunity for evaluation of routes that might otherwise be equal.

This section's additional constraints and decision variables complete the LPD with time windows formulation. This mathematical formulation represents a more general version of either the LRP or PDP. However, even for small problems, the formulation contains a tremendous number of decision variables and constraints. For example, consider a simple two vehicle, two commodities, 10 node problem containing two depots, four supply points and four demands. This problem has 120 decision variables and over 1,000 constraints. As the size of the problem grows, it becomes extremely difficult to obtain an optimum solution. The following section eases the size of the problem slightly by removing some of the un-needed decision variables and constraints.

### 4.2.4 Tightening the LPDP Formulation

The LPDP formulation presented in Sections 4.2.1 - 4.2.3 represents a fully connected graph. To tighten the formulation, this section removes arcs and variables from the problem that cannot form part of a feasible solution. It is reasonable to expect that some portion of the supply points and vehicles do not support all commodity types and therefore cannot support a demand for that commodity. Redefining their sets removes some decision variables and constraints from the problem. The following sets are defined to eliminate incompatible vehicles and supply points from the formulation.

## Sets:

$$
\begin{aligned}
& \boldsymbol{P}_{i}^{\prime} \subseteq \boldsymbol{P} \quad \text { set of all commodities provided by supply point } \boldsymbol{i} \in \boldsymbol{I} . \\
& \boldsymbol{I S}_{\boldsymbol{j}} \subseteq \boldsymbol{I} \quad \text { set of all supply points that can satisfy the demand } \boldsymbol{j} \in \boldsymbol{J} . \\
& \boldsymbol{I}_{j}^{\prime}=\left\{\boldsymbol{i}:\left(\boldsymbol{i} \in \boldsymbol{I}_{\boldsymbol{j}}\right)\right\} \cap\left\{\boldsymbol{i}:\left(\boldsymbol{I}_{\boldsymbol{j}}-\boldsymbol{e}_{\boldsymbol{i}} \geq \boldsymbol{t}_{i j}\right)\right\} . \\
& \boldsymbol{V}_{\boldsymbol{j}}^{\prime}=\left\{\boldsymbol{v}:\left(\boldsymbol{p}_{\boldsymbol{j}} \in \boldsymbol{C}_{\boldsymbol{v}}\right)\right\} \cap\left\{\boldsymbol{v}:\left(\sigma_{p j} \leq \boldsymbol{q}_{p v}\right)\right\} .
\end{aligned}
$$

The use of $\boldsymbol{I}_{\boldsymbol{j}}^{\prime}$ instead of the original set of supply points, $\boldsymbol{I}$, reduces the size of the problem by eliminating all supply points that cannot satisfy the demand of customer $\boldsymbol{j}$, or are so far from the customer that no vehicle can make the trip in time to achieve the customer's service time. Replacing the vehicle set $\boldsymbol{V}$ to $\boldsymbol{V}_{\boldsymbol{j}}^{\prime}$ has the same effect of eliminating vehicles that cannot satisfy the customer's demand.

A characteristic of the TDP is that most customer demands represent multiple truckload multi-commodity demands. The impact is that any truck servicing a demand is usually full and carrying a single commodity. Therefore, defining a demand $\boldsymbol{j} \in \boldsymbol{J}$ as a
truckload request, $\sigma_{j}$ coupled with the above redefinition of the vehicle set, $\boldsymbol{V}_{\boldsymbol{j}}^{\prime}$ allows for the removal of the vehicle capacity constraint (4-34) from the formulation.

The current formulation contains variables that allow vehicle movement directly from a depot to a customer location and from a supply point back to a depot. The formulation's constraints prevent the selection of these variables since a vehicle must first visit a supply point when departing a depot and return to a depot from a customer location. These variables may be eliminated from consideration by rewriting the constraints that direct movement from and to the depots. The inclusions of the above reductions help tighten the final mathematical formulation.

### 4.2.5 Complete LPDPTW Mathematical Programming Formulation

The following completed formulation represents the integer linear program for the LPDPTW utilized in this research, and is a consolidation of what has been constructed in previous sections in this chapter.

Minimize:

$$
\begin{equation*}
\sum_{k \in K} f_{k} D_{k}+\sum_{i \in K} g_{i} \sum_{v \in V} \sum_{j \in I} Z_{i j v}+\sum_{i \in I} f_{i} X_{i}+\sum_{j \in J} \sum_{i \in I_{j}^{\prime}}\left(v_{i} \times \sigma_{j} Y_{i j}\right)+\sum_{j \in J} \sum_{v \in V_{j}} \sum_{i \in I_{j}^{\prime}}\left(f_{v} \times Z_{i j v}\right) \tag{4-35}
\end{equation*}
$$

Subject to:

$$
\begin{array}{ll}
\sum_{v \in V_{j}^{\prime}} \sum_{i \in I_{j}^{\prime}} \mathbf{Z}_{i j v}=1 & \forall \boldsymbol{j} \in J ; \boldsymbol{i} \neq \boldsymbol{j} \\
\sum_{k \in K} \sum_{i \in I} \boldsymbol{d}_{k i} \boldsymbol{Z}_{k i v}+\sum_{j \in I \cup J} \sum_{i \in I \cup J} \boldsymbol{d}_{i j} Z_{i j v}+\sum_{j \in J} \sum_{k \in K} \boldsymbol{d}_{j k} Z_{j k v} \leq \tau_{v} \quad \forall \boldsymbol{v} \in \boldsymbol{V} \\
\sum_{j \in J} \sum_{i \in I_{j}} \boldsymbol{Z}_{i j v} \leq 1 & \forall \boldsymbol{v} \in \boldsymbol{V} \tag{4-38}
\end{array}
$$

$$
\begin{align*}
& \sum_{j \in J} \sigma_{p j} \boldsymbol{Y}_{i j}-\boldsymbol{m}_{p i} \boldsymbol{X}_{\boldsymbol{i}} \leq 0 \quad \forall \boldsymbol{i} \in \boldsymbol{I} ; \forall \boldsymbol{p} \in \boldsymbol{P}  \tag{4-39}\\
& \sum_{u \in I \cup J}\left(\mathbf{Z}_{i u v}+\mathbf{Z}_{u j v}\right)-\boldsymbol{Y}_{i j} \leq 1 \quad \forall \boldsymbol{j} \in \boldsymbol{J} ; \forall \boldsymbol{i} \in \boldsymbol{I} ; \forall \boldsymbol{v} \in \boldsymbol{V}  \tag{4-40}\\
& \sum_{j \in J} \sigma_{p j} \boldsymbol{Y}_{s j}+\sum_{\boldsymbol{h} \in \boldsymbol{H}} \boldsymbol{W}_{p s h}-\boldsymbol{m}_{p s} \boldsymbol{X}_{s} \leq 0 \quad \forall \boldsymbol{s} \in \boldsymbol{S} ; \forall \boldsymbol{p} \in \boldsymbol{P}  \tag{4-41}\\
& \sum_{s \in S} W_{p s h}-\sum_{j \in J} \sigma_{p j} \boldsymbol{Y}_{h j}-\boldsymbol{s}_{p h} \boldsymbol{X}_{\boldsymbol{h}} \leq 0 \quad \forall \boldsymbol{s} \in \boldsymbol{S} ; \forall \boldsymbol{p} \in \boldsymbol{P}  \tag{4-42}\\
& \sum_{k \in K} Z_{k i v}-\sum_{j \in J} Z_{i j v}=0 \quad \forall i \in I ; \forall v \in V  \tag{4-43}\\
& \sum_{i \in I} Z_{k i v}-\sum_{j \in J} Z_{j k v}=0 \quad \forall \boldsymbol{k} \in \boldsymbol{K} ; \forall \boldsymbol{v} \in \boldsymbol{V}  \tag{4-44}\\
& \sum_{i \in I} Z_{k i v}-M X_{k} \leq 0 \quad \forall \boldsymbol{k} \in \boldsymbol{K} ; \forall \boldsymbol{v} \in \boldsymbol{V}  \tag{4-45}\\
& \sum_{i \in I_{j}^{\prime}} \mathbf{Z}_{i j v}-\sum_{i \in I_{j}^{\prime}} \mathbf{Z}_{j i v} \leq 0 \quad \forall \boldsymbol{j} \in \boldsymbol{J} ; \forall \boldsymbol{v} \in \boldsymbol{V}  \tag{4-46}\\
& Z_{i j v} \times\left(T_{i}+s_{i v}+t_{i j v}\right) \leq T_{j} \quad \forall i \in I ; \forall j \in J ; \forall v \in V  \tag{4-47}\\
& Z_{k i v} \times\left(\boldsymbol{T A}_{v k}+\boldsymbol{t}_{\text {kiv }}\right) \leq \boldsymbol{T}_{\boldsymbol{i}} \quad \forall k \in K ; \forall i \in I ; \forall v \in V  \tag{4-48}\\
& Z_{j k v} \times\left(T_{j}+s_{j v}+T_{j k}\right) \leq T Z_{v k} \quad \forall k \in K ; \forall j \in J ; \forall v \in V  \tag{4-49}\\
& e_{i} \leq T_{i} \leq l_{i}  \tag{4-50}\\
& e_{k} \leq T D_{v k} \leq I_{k}  \tag{4-51}\\
& e_{k} \leq T R_{v k} \leq I_{k}  \tag{4-52}\\
& T R_{v k}-T D_{v k} \leq w  \tag{4-53}\\
& \boldsymbol{Z}_{i j v} \times\left(\boldsymbol{L}_{v i}+\sigma_{j}\right)=\boldsymbol{L}_{\boldsymbol{j}} \quad \forall \boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J} ; \boldsymbol{j} \in \boldsymbol{I} ; \boldsymbol{v} \in \boldsymbol{V}  \tag{4-54}\\
& \boldsymbol{Z}_{i j v} \times\left(\boldsymbol{L}_{\mathbf{v i}}-\sigma_{j}\right)=\boldsymbol{L}_{v j} \quad \forall \boldsymbol{i} \in \boldsymbol{I} \cup \boldsymbol{J} ; \boldsymbol{j} \in \boldsymbol{J} ; \boldsymbol{v} \in \boldsymbol{V} \tag{4-55}
\end{align*}
$$

$$
\begin{array}{ll}
\boldsymbol{Z}_{\text {kiv }} \times\left(\boldsymbol{L}_{v k}+\sigma_{i}\right)=\boldsymbol{L}_{v i} & \forall \boldsymbol{k} \in \boldsymbol{K} ; \boldsymbol{i} \in \boldsymbol{I} ; \boldsymbol{v} \in \boldsymbol{V} \\
\boldsymbol{L}_{v k}=0 & \forall \boldsymbol{v} \in \boldsymbol{V} ; \forall \boldsymbol{k} \in \boldsymbol{K} \\
\boldsymbol{X}_{i} \in\{0,1\} & \forall \mathbf{i} \in \boldsymbol{I} \\
\boldsymbol{Z}_{i j v} \in\{0,1\} & \forall \boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{N} ; \forall \boldsymbol{v} \in \boldsymbol{V} \\
\boldsymbol{Y}_{i j} \in\{0,1\} & \forall \mathbf{i} \in \boldsymbol{I} ; \forall \boldsymbol{j} \in \boldsymbol{J} \\
\boldsymbol{W}_{p s h} \geq 0 & \forall \boldsymbol{p} \in \boldsymbol{P} ; \forall \boldsymbol{s} \in \boldsymbol{S} ; \forall \boldsymbol{h} \in \boldsymbol{H} \tag{4-61}
\end{array}
$$

The objective function (4-35) now considers the establishing and variable operating cost for both the depot and supply point and the fixed cost of assigning vehicles to a depot. Constraint (4-36) assigns each customer demand to a route. Constraint (4-37) limits the travel distance of each vehicle. Constraint (4-38) ensures that each route is served only once. Constraint (4-39) limits the flow of supplies through each selected supply point. Constraint (4-40) specifies that a customer's demand is assigned to a supply point only if there is a route from that supply point through the customer. This constraint serves as the link between the vehicle routing variables $\left(Z_{i j v}\right)$ and the assignment variables. Constraint (4-41) accounts for flow from a source supply point to a transshipment point. The constraint ensures that flow from a source point does not exceed its capacity or throughput limit. Constraint (4-42) establishes a maximum storage capability for each transshipment supply point. This constraint provides the capability for exploring the effect of creating distribution and warehouse holding centers versus pure crossdocking points. Constraint (4-43) ensures that if there is flow out of a supply point, then there must be flow in from some depot, while constraint (4-44) specifies that
if there is flow from a depot there must be flow back to the depot. Constraint (45) serves as the link between routing variables and the selection of depots. It specifies that if there is flow from a depot, then the depot must be selected. Constraint (4-46) is the coupling constraint. This constraint in conjunction with constraint (4-43) ensures that the same vehicle both picks up and delivers the demand Constraints (4-47) - (4-49) describe the compatibility requirements between routes and schedules. Constraints (4-50) - (4-52) represent the time window constraints. These constraints impose increasing times at the nodes of the route. Constraint (4-53) ensures that a vehicle's route does not exceed the total authorized travel time for a vehicle. Constraints (4-54) - (4-56) ensure that a vehicle travels from a supply point to a customer with a compatible load. Constraint (457) ensures that the vehicle departs the depot with an empty load. Constraints (4-58) -(4-61) represent the standard integrality and non-negativity constraints.

### 4.2.6 Summary

Section 4.2.1 establishes the TDP characteristics of interest to this research. The
 MS, MC, T problem. Sections 4.2.1 - 4.2.4 sequentially construct the LPDPTW mathematical formulation based first on the location routing formulation of Perl and Daskin (1985) and then the pickup and delivery problem with time windows formulation of Dumas et al. (1991). Section 4.2.5 provides the complete LPDPTW formulation utilized in this research. Thanks to the complexity of the LPDPTW, there is little chance that this formulation can be solved to optimality for any realistic sized problem.

Therefore, Chapter V presents a metaheuristic approach for solving the LPDPTW characterized in this chapter.

## V A Tabu Search Approach to the Theater Distribution Problem

This chapter outlines the ATS process developed in this research to solve the LPDPTW and TDP. The ATS approach developed in this research incorporates several features designed to enhance the search process. These features include adaptive tabu tenure to control ATS memory, the incorporation of a dynamic neighborhood scheme, the utilization of objective function penalty weights and ATS restart using solutions from an elite list. Section 5.1 outlines the tabu search architecture of the ATS process. This section represents an extension of the basic tabu search discussion started in Chapter II. Section 5.2 discusses the preprocessing phase necessary to initiate the ATS process. Section 5.3 outlines the objective function used to evaluate each move and solution permutation. Section 5.4 describes the move neighborhood schemes developed in this dissertation. Section 5.5 covers the ATS tabu structure. Section 5.6 covers the ATS algorithm. Section 5.7 describes the ATS move manager and Section 5.8 provides a brief summary.

### 5.1 Adaptive Tabu Search Architecture

. This section opens with a discussion of the tabu search architecture and describes the ATS pre-processing

### 5.1.1 JAVA ${ }^{\text {TM }}$ Programming Language and OpenTS

This dissertation utilized the Java ${ }^{\mathrm{TM}}$ software programming language to develop the ATS tabu search heuristic for solving the TDP. Prior research by Barnes et al. (2004), Crino et al. (2004) and Combs and Moore (2004) illustrate the effectiveness of

Java ${ }^{\text {TM }}$ in developing tabu search heuristics. Harder's (2001) OpenTS architecture provides the basic framework and bookkeeping services for the ATS process. Figure 5-1


Figure 5-1 Tabu Search Architecture
graphically illustrates a basic iteration of OpenTS. The ATS starts with an incumbent solution constructed from an initial solution heuristic of Section 5.1.3.2. The neighborhood move generator creates a neighborhood or list of restricted moves as described in Section 5.4. The search manager invokes these moves in turn on the incumbent solution. The solution evaluator described in Section 5.2 evaluates each neighborhood search for the best available move. The best non-tabu move is selected, based on current selection criteria, and performed on the incumbent solution. The determination of best move is an adaptive process and is described in Section 5.2. ATS uses either conjugation or function composition on the incumbent solution to move to a new solution. For example, if the selected move is a swap move, ATS uses conjugation
and if it is an insert move, ATS uses function composition. This move operation completes the iteration and the process is ready to repeat.

### 5.1.2 Solution Structure

As described in Chapter IV, a solution to an instance of the TDP is a set of vehicle routes meeting the problem's demand and timeline requirements. This dissertation uses the cyclic form of the symmetric group, $S_{n}$, as the solution structure for the TDP. This solution structure is written as a set of disjoint cyclic factors, where each disjoint cycle represents a vehicle depot with all assigned vehicles and their associated routes and schedules. The first letter in the cycle represents the depot identification letter. Subsequent letters identify the vehicle assigned to the depot and the associated pickup and delivery points along that vehicle's route. For example, consider the example given in Section 2.2.5 (Figure 5-2). This problem contains four main elements consisting of


Figure 5-2 Graphical Representation of a LPDPTW
three multi-commodity demands (A, B, and C), three supply points (S1, S2, and S3), two depots (D1 and D2) and two vehicles (V1 and V2). The two potential depot locations support both vehicles but the supply points do not have the capability to support all demands. Supply point S1 can support demands A and B, while supply points S2 and S3 can only support demand C. This solution representation maps each of the above four elements to a letter sequentially from depots, supply points, demands and vehicles. Under this scheme, the four example elements are mapped to the following letters: depots to letters $0-1$, supply points to letters $2-5$, demands to letters $6-8$ and vehicles to letters 9 10. This problem is represented using the symmetric group on 11 letters or $S_{11}$. Two potential solutions from $S_{11}$ that select the depots, supply points and vehicles necessary to cover each demand are: $(0,9,2,7,3,6,10,4,8)$, and $(0,10,2,6)(1,9,3,7,5,8)$.

Determining the necessary network structure and associated routes requires an examination of each disjoint cycle. For example, $(0,10,2,6)(1,9,3,7,5,8)$ represents (D1,V2, S1, A)(D2, V1, S1, B, S3, C) in the symmetric group solution and indicates that vehicle V2 is assigned to depot D1 and travels to supply point S1 to pickup the demand for delivery at location A. Also, vehicle V1 is assigned to depot D 2 and travels a route between points S1, B, S3, C and back to the depot. Supply point S2 is not necessary for this operation and not included in the network distribution structure.

As the number of TDP nodes and demands increases, the size of the solution structure and problem increases. Storing full solutions or searched conjugacy class information for later retrieval becomes computationally expensive as the problem size increases. Glover and Laguna (1997) indicate that a typical solution to avoid storing all
this information is the development of a hashing function that maps the desired element to an integer value. These values are stored in hash maps for utilization by the ATS search process. Sections 5.5.2-5.5.4 describe the hash functions developed in this dissertation.

### 5.1.3 ATS Preprocessing Phase

The preprocessing phase contains a number of events necessary to initialize the ATS search process including importing a group of text files that characterize the TDP. These text files initialize a set of data arrays and create a series of Java ${ }^{\mathrm{TM}}$ objects for the ATS process. The final preprocessing step develops the initial solution for the ATS process.

### 5.1.3.1 ATS Data Structures

The Java ${ }^{\text {TM }}$ objects created during the ATS preprocessing phase and their key attributes are summarized in Table 5-1. A depot object is created for each type depot location available in the distribution network. A vehicle object is created for each vehicle available for assignment in the problem. A masterSuppliers object is created for each physical supply point location in the problem. This object maintains the bulk of the bookkeeping and tracking for the suppliers object. A suppliers object is created for each demand that a supply point node is capable of supporting. This support capability is based on both commodity type and distance from the supply point to the demand location.

Table 5-1 ATS Objects and Attributes

| Object | Key Attributes |
| :---: | :--- |
| Depot | Depot ID, Problem ID, Classification, Vehicle Capacity, <br> Fixed Cost, Variable Cost, Open Status, <br> Location Coordinates |
| Master Suppliers | Supply Point ID, Supply ID, Point Type, Source Node, <br> Commodity Type, Commodity Capacity, Availability, <br> Fixed Cost, Variable Cost, Vehicle Type Throughput, <br> Vehicle Support Capability, Service Window, <br> Location Coordinates |
|  | Supply Point ID, Supply ID, Problem ID, Availability, <br> Service Window |
|  | Demand Point ID, Demand ID, Problem ID, <br> Commodity Type, Commodity Demand, <br> Support Capability, Service Window, <br> TDD Requirement, Location Coordinates |
| Vehicle | Vehicle ID, Problem ID, Classification, Veh Type ID, <br> Commodity Capacity, Load Time, Unload Time, Speed, <br> Crusing Length, Route Length, Service Time, Fixed <br> Cost, Variable Cost, Initial Assigned Depot, Available <br> Time, Current Assigned Depot |

These restrictions eliminate unnecessary letters from the solution structure reducing the overall problem size. A suppliers object is generated only if the following two conditions are met:

1. supplyPoint[i].commodityType[j] = demanders[k].commodityType[ $j$ ]
distance.supplyPoint[i].demanders[d] $\leq$
2. $\frac{1}{2}$ distance. $[\max (\operatorname{depot}[d]$. .vehicle[v].routeLength)
$-2 \times$ distance.depot[d].supplyPoint[i]]
where, $d \in$ set of depots, $i \in$ set of supply points, $j \in$ set of commodities, $k \in$ set of demands, and $v \in$ set of vehicles assignable to depot $d$

Condition one ensures that the supply point actually carries the commodity type requested by the customer. The second condition ensures that there is at least one vehicle in the fleet that possesses the ability to travel a route from the depot to the customer, through a supply point and return home without violating its route length constraints. Each of these suppliers objects are linked to a specific supply point (masterSupplier) in the distribution network. Consider the example in Figure 5-3 consisting of five demands.


Figure 5-3 Example Suppliers Object Assignment

The ATS process generates a total of five suppliers objects to cover this example. Supply point S 1 possesses two suppliers objects for demands A and B and supply point S2 receives three suppliers objects for demand $\mathrm{B}, \mathrm{C}$, and D . The ATS process assigns a
suppliers object to both supply points for demand B. This allows ATS to determine the best supply point and vehicle route to satisfy the demand while minimizing the overall objective function. It is interesting to note that demand E is not supportable from either supply point. This implies that there is another supply point in the problem that can cover this customer or there is not a feasible solution to the problem. If this situation is encountered, this research assigns a suppliers object to both supply points and then seeks the best near feasible solution to the problem. This near feasible solution implies that the decision maker is required to loosen a violated constraint. For example, the decision maker may have to allow a vehicle to travel further or operate longer than desired to satisfy the demand.

A demanders object is created for each demand location node based on the amount of a specific commodity requested and the capacity of the smallest available vehicle in the fleet. The number of demanders objects assigned to a demand location node is:

$$
\sum_{\substack{j \in \text { Conmodity } \\ \text { Type Requested }}} \operatorname{roundup}\left(\frac{\text { commodityDemanded }[j]}{\min (\text { vehicle }[i] . c o m m o d i t y C a p a c i t y[j])}\right) \quad \forall i \in \text { Vehicles }
$$

where roundup rounds the ratio to the next greatest integer. For example, assume demand A requires 18 pallets of commodity type one and 12 pallets of commodity type two and there are two vehicle types in the available fleet of vehicles that can respectfully carry a total of 6 and 12 pallets. The ATS process assigns a total of five demanders objects to cover this customer based on: demanders objects $=\frac{12}{6}+\frac{18}{6}=5$

This process of generating demands does not imply that a vehicle is only able to satisfy a single demand. For example, the vehicle above that had a total capacity of 12 pallets has the capability to load two demands at a single supply point before departing to satisfy those two demands.

The preprocessing phase generates several information arrays and matrices (Table 5-2) that are used by the ATS process. The first three matrices in Table 5-2 represent the ground, air, and water distance between connected nodes in the problem. The ATS solution evaluator uses these matrices to determine vehicle variable cost and route feasibility. The three ID array lists and the problem structure matrix provide unique identifiers for all entities in the problem. The neighborhood move manager uses these data elements to construct the various neighborhood schemes discussed in Section 5.4. The tabu structure manager utilizes randMoveHash to track tabu moves and randSolutionHash for solutions in constructing the tabu lists for the ATS process. These

Table 5-2 Preprocessing Constructed Data Elements

| Constructed Data Element | ATS User |
| :--- | :--- |
| problemDistanceMatrix | Solution Evaluator |
| problemAirDistanceMatrix | Solution Evaluator |
| problemSeaDistanceMatrix | Solution Evaluator |
| vehicleIDArrayList | Neighborhood Move Manager |
| demandIDArrayList | Neighborhood Move Manager |
| suppliersIDArrayList | Neighborhood Move Manager |
| problemStructureMatrix | Neighborhood Move Manager |
| randMoveHash | Tabu Structure Manager |
| randSolutionHash | Tabu Structure Manager |

two data elements represent an $n$-sized array list and $n \times n$ sized matrix of random numbers.

### 5.2 Initial Solution Construction

The final preprocessing element is the creation of an initial solution. Past researchers (Potvin and Rousseau (1995), Kontrovdis and Bard (1995) and Carlton (1995)) have shown that it is desirable to start a heuristic with an initial feasible solution. This is even more critical in the multi-depot LPDPTW as a poor initial clustering of requests to a depot may prove difficult to overcome during the route improvement phase. This dissertation utilizes both a greedy and insertion heuristic to construct the initial solution. The two procedures are used in the design of experiments discussed in Chapter VI to test the robustness of the ATS process in overcoming problems associated with a poor initial solution.

The first initial solution construction technique is a simple distance based greedy heuristic. This heuristic assigns demands, supply points, and depots to a vehicle route based on minimum distance criteria. The first step assigns each demand to the closest supportable supply point. Supply point supportability is based on commodity and vehicle type constraints. The second step assigns the supply point to its closest supportable depot, where supportability is based on vehicle constraints. The final step completes the route by ordering available vehicles based on capacity and assigning the first vehicle to the depot and pickup and delivery customers. This process is repeated until all demands have been assigned to a route. Since none of the associated LPDPTW constraints are considered in the assignment process, this simple approach does not guarantee a feasible
solution but it does provide a reasonable initial distribution of demands. The following insertion heuristic represents the second construction procedure used in this dissertation, and it does guarantee a better initial solution in comparison to the greedy method.

The insertion heuristic is a sequential route construction heuristic that builds routes one at a time adding demands until the route's resources (e.g., capacity, max driving time, and distance) are consumed without violating time window constraints. At this point, the heuristic initializes another tour and the process continues. Demands are initially sorted and assigned to a depot based on an average travel time for the pickup job from its nearest depot, where avgSpeed represents the average speed of all vehicles that might be assigned to the depot.

$$
\text { DemandDepotAssignment }=\min \left(\frac{\text { custDist }_{i}}{\text { avgSpeed }_{i}}\right) \quad i=1, \ldots, d
$$

Assigned depot vehicles are ordered based on their capacity and average trip time, which includes travel and all necessary service times.

$$
\text { vehCapPerAvgTripTime }=\frac{\text { vehCap }}{(2 * \text { avgDist } / \text { speed })+(\text { loadTime }+ \text { unloadTime }+ \text { servTime })}
$$

| where $d$ | $=$ number of depots |
| :--- | :--- |
| custDist $_{i}$ | $=$ pickup customer distance $(\mathrm{km})$ from depot $i$ |
| avgSpeed $_{i}$ | $=$ average speed $(\mathrm{km} / \mathrm{min})$ of assigned vehicles at depot $i$ |
| vehCap | $=$ vehicle capacity (stons) |
| avgDist | $=$ average distance $(\mathrm{km})$ of all customers to their depot |
| speed | $=$ vehicle cruising speed $(\mathrm{km} / \mathrm{min})$ |

$$
\begin{array}{ll}
\text { loadTime } & =\text { time }(\min ) \text { to load vehicle } \\
\text { unloadTime } & =\text { time }(\min ) \text { to unload vehicle } \\
\text { servTime } & =\text { time }(\min ) \text { to service vehicle }
\end{array}
$$

The heuristic assigns the first job pair (a pickup and delivery request) to the first depot vehicle. The algorithm based on Solomon's (1987) famous insertion procedure, selects the next job pair and inserts it in the vehicle's route based on $\boldsymbol{c}_{1}(\mathbf{i}, \boldsymbol{u}, \boldsymbol{j})$ which determines the best insertion point for each unassigned request (job pair) as

$$
\boldsymbol{c}_{1}(\boldsymbol{i}, \boldsymbol{u}, \boldsymbol{j})=\min \left(\boldsymbol{c}_{1}\left(\boldsymbol{i}_{p}, \boldsymbol{u}, \boldsymbol{j}_{\boldsymbol{p}+1}\right)\right) \quad \boldsymbol{p}=1, \ldots, \boldsymbol{m}
$$

with

$$
\begin{aligned}
\boldsymbol{c}_{1}\left(\boldsymbol{i}_{p}, \boldsymbol{u}, \boldsymbol{j}_{p+1}\right) & =\pi_{1}\left(\boldsymbol{c}_{11}\left(\boldsymbol{i}_{\boldsymbol{p}}, \boldsymbol{u}, \boldsymbol{j}_{p+1}\right)\right)+\pi_{2}\left(\boldsymbol{c}_{12}\left(\boldsymbol{i}_{\boldsymbol{p}}, \boldsymbol{u}, \boldsymbol{j}_{p+1}\right)\right) \\
\pi_{1}, \pi_{2} & \geq 0 \text { and } \pi_{1}+\pi_{2}=1 \\
\boldsymbol{c}_{11}\left(\boldsymbol{i}_{p}, \boldsymbol{u}, \boldsymbol{j}_{p+1}\right) & =\boldsymbol{d}_{i u}+\boldsymbol{d}_{u j}-\alpha \cdot \boldsymbol{d}_{i j} ; \quad \alpha \geq 0 \\
\boldsymbol{c}_{12}\left(\boldsymbol{i}_{p}, \boldsymbol{u}, \boldsymbol{j}_{p+1}\right) & =\boldsymbol{b}_{j u}-\boldsymbol{b}_{\boldsymbol{j}}
\end{aligned}
$$

where $\boldsymbol{i}=\operatorname{depot}$; and $\boldsymbol{u}, \boldsymbol{j}=$ request (job pairs)


Figure 5-4 Example of Request Insertion

As an example, consider the partially completed route at the top of Figure 5-4. A vehicle departs the depot, $i$, at $\boldsymbol{e}_{\boldsymbol{i}}=0$ (earliest available time) to complete request (job pair) $j$. The vehicle requires $\boldsymbol{t}_{i j}=20$ time units to reach the pickup location of request $j$. The service time, $\boldsymbol{s}_{\boldsymbol{j}}$, for request $\boldsymbol{j}$ is $\boldsymbol{s}_{\boldsymbol{j}}=\operatorname{loadTime}+\boldsymbol{t}_{\boldsymbol{j}}+$ unloadTime $=10$ where $t_{j}$ is the travel time from pickup to delivery location for request (job pair) $j$. The earliest available time to begin service, $b_{j}$, for request $j$ is at $e_{j}=90$. This provides the vehicle an allowable time, $\boldsymbol{a t} \boldsymbol{j}_{\boldsymbol{j}^{*}}=\boldsymbol{c}_{\boldsymbol{i}}-\boldsymbol{b}_{\boldsymbol{j}}-\boldsymbol{s}_{\boldsymbol{j}}=120-90-10=20$, to service another request or return home in this case ( $c_{i}$ is the required return time to the depot). The vehicle also has a wait time of 70 time units $\left(\boldsymbol{w}_{i j}=\boldsymbol{b}_{\boldsymbol{i}}+\boldsymbol{b}_{\boldsymbol{j}}-\boldsymbol{t}_{\boldsymbol{i j}}=0+90-20=70\right)$ available for satisfying another request if possible. This wait time accounts for the remaining time after a vehicle departs the last job (or depot) and before the available time window opens for the next job. The insertion process now attempts to insert another job request, $u$, in the current route (bottom of Figure 5-4). The service time, $s_{u}$, for request $u$ is 15 time units and the travel time, $t_{u j}$, from request $u$ to request $j$ is 35 time units. After accounting for the earliest arrival time, $b_{u}$, the vehicle now has an adjusted wait time of 20 time units and there is no change in the earliest arrival time, $b_{j}$, for request $j$. The insertion process attempts to find another request to fill the vehicle's current wait time, and if none exists, closes this route and moves to the next vehicle.

This insertion heuristic takes advantage of Solomon's (1987) Lemma that states that if a customer is inserted into a tour that is time feasible, the tour remains time
feasible if the insertion does not result in a delay in servicing the following customer. This Lemma reduces the heuristic's computational requirement since it only requires checking from the insertion point onward. The heuristic stops the evaluation process once it reaches a customer where service has not been delayed and need not evaluate the remaining tour. The heuristic ensures that all constraints are maintained during the construction.

The preprocessing phase ends with the evaluation of the completed initial solution. The initial solution represents a feasible solution and its evaluation provides an upper bound for the ATS search process. The next section discusses the ATS solution evaluation used for the initial solution and all incumbent solutions.

### 5.3 Objective Function Evaluation

Solution and move evaluation in the ATS process consists of determining a solution's feasibility and assigning it an objective function value. Solution feasibility is based on satisfying all constraints in the problem. The objective is to minimize the fixed cost, variable cost and penalty cost of the selected distribution network. Fixed cost is the total cost of establishing the set of depots and supply points plus the cost of assigning vehicles to the depots. Variable cost is the total operating cost for the selected depots and supply points plus the cost for vehicles to travel their routes. Penalty cost includes six linear violation penalties for time definite delivery (TDD) violations, demand shortfalls, route length or time violations, queue violations and storage violations. The objective function includes a solution infeasibility penalty that penalizes selecting an infeasible solution. The ATS objective function object calculates the objective function value for
each solution permutation based upon the following equation, where $\alpha_{i}$ provides the ability for adjusting the weight (default setting is one) of the associated factor:

$$
\begin{align*}
\text { Soln }_{\text {eval }} & =\alpha_{f c}(\text { fixed cost })+\alpha_{v c}(\text { variable cost }) \\
& +\alpha_{T D D}(\text { TDD penalties })+\alpha_{d s}(\text { demand shortfall }) \\
& +\alpha_{r l}(\text { route penalties })+\alpha_{Q}(\text { queue penalties })  \tag{5-1}\\
& +\alpha_{s}(\text { storage penalties })+\alpha_{t w}(\text { time window penalties }) \\
& +\alpha_{s i}(\text { solution infeasibility })
\end{align*}
$$

Depot, supply point and vehicle assignment fixed costs represent the cost (stons) of adding the selected entity to the distribution network. Depot variable cost (stons) is an operating cost based on the number of vehicles assigned to the depot. This cost represents the overhead structure (men and equipment) necessary to manage and maintain the fleet. Supply point variable cost (stons) is an operating cost based on the supply point's daily supply throughput requirements. This represents the addition of men and equipment necessary to process increasing amounts of supplies. Vehicle variable cost (stons) is based on a vehicle's average daily route distance (km). This cost represents the 2nd and 3rd order support infrastructure necessary to logistically maintain the fleet of vehicles.

TDD and demand shortfall violations penalize solutions that fail to meet all customer requests. A TDD violation represents a demand not reaching its respective location prior to the specified delivery time. The TDD violation is based on the total weight of the entire demand. There is no credit given for a partial delivery of a TDD demand requirement. Demand shortfall is the total weight (stons) of all demands not delivered by the routing plan. TDD and demand shortfall violations are weighted
according to the amount of time (mins) the demand is late. The time for a demand shortfall is the problem's planning horizon.

Route, queue, storage and time window violations measure the feasibility of a solution. ATS considers a solution containing a vehicle route that violates one or more of these violations a near feasible solution. A near feasible solution represents a condition or case that additional decision maker guidance may resolve. For example, the best solution found may contain a route that violates a supply point's closing time by 15 minutes. This solution is near feasible and requires decision maker intervention to have the supply point remain open for an additional 15 minutes. This decision renders the solution feasible. ATS tracks both the best feasible (no violations) and near feasible solutions found during the search. This provides the decision maker a potential opportunity to improve a solution by loosening a violated condition or constraint.

Vehicle route violations are based on both route length (km) and route travel time (mins). Any given vehicle has a maximum allowed travel distance (km). This travel distance is the minimum distance of either the vehicle's on board fuel capacity (vehicle.crusingLength) or a doctrinally constrained limit (vehicle.routeLength). A vehicle.crusingLength represents a hard constraint that a decision maker can overcome by allowing refueling to occur outside the vehicle's assigned depot. A travel distance violation is the difference between the actual route's distance and maximum allowed distance for the assigned vehicle and is weighted by the vehicle's capacity (stons). A given vehicle's crew possesses an operating day constraint (vehicle.operatingTime) that limits the total operating time (mins) for the vehicle in an operating day. Operating day
violation is based on the difference of the time necessary to complete the route and a crew's operating day and is weighted by the assigned vehicle's capacity (stons). A route length violation represents the sum of both the travel distance and operating day violations. Queue violation is the sum of all queue violations at all nodes in a planning period. Queue violation is based on a node's ability or capacity to concurrently process vehicles at the site. This queue capacity is usually represented as a maximum on the ground or MOG for an airfield but may also represent the maximum number of vehicles a warehouse can concurrently load or offload. The queue violation is weighted by the capacity (stons) of the vehicle arriving at a node with a full queue and the length of time (mins) the node's queue capacity is exceeded. The default ATS setting forces vehicles to wait (FIFO) until a space opens at a site. Storage violation is the sum of all storage violations (stons) at all supply nodes in a planning period and the sum of all supply point throughput violations (stons). A storage violation represents a routing plan that delivers more commodities to a transshipment point than the point can actually support. This violation usually occurs in a given time period to alleviate distribution burdens in a following time period. A storage violation is the difference between the current on-hand quantity and the supply point's storage capacity for a given planning period. Storage violation also includes a penalty for violating the throughput capability of a supply point. This violation represents a supply point processing more supplies than the sites facilities or assigned personnel could actually support.

Solution infeasibility indicates that a solution permutation violates one of the following critical constraints: vehicle capacity, supply point and demand mismatch, or
vehicle route mismatch. Solution infeasibility provides ATS the opportunity to temporally select an infeasible solution if it allows the search process to traverse to a better region of the solution space. These constraints represent hard infeasibilities, which are situations where the solution is meaningless based on current conditions. For example, a vehicle capacity violation indicates that a solution contains a vehicle carrying a load that exceeds its capacity. This violation represents a situation that cannot be resolved by decision maker input. A supply point and demand mismatch violation indicates that a vehicle is expected to pickup a given demand from a supply point that does not support the demanded commodity. The final infeasibility constraint represents trying to assign a vehicle to an incompatible route. For example, a C-17 aircraft cannot support a route that contains a non-airfield node.

The design of experiment presented in Chapter VI includes solving the TDP problem with and without penalty parameters in the objective function. Equation (5-1) (objective function) contains nine weighted penalty parameters that are continuously updated by the ATS process. ATS controls the search movement through the feasible, near feasible and infeasible portions of the solution space by adjusting these nine parameters. This research implements a version of the self-adjusting scheme proposed by Combs and Moore (2004) for the Crew Scheduling Problem. The ATS process adjusts the value of the penalty parameter between the values of $1 / 2$ and 2 based on solution characteristics. The ATS adjusts the nine penalty parameters every tenth iteration as follows:

$$
\begin{equation*}
\alpha_{i}:=2^{\text {(iteration } / 5)-1}, \text { where } i \in \text { penalty parameters } \tag{5-2}
\end{equation*}
$$

The iteration value represents a summation count based on the penalty parameter and the feasibility of the solution. The iteration value for the fixed cost $\left(\alpha_{f c}\right)$ and variable cost $\left(\alpha_{v c}\right)$ parameters are adjusted for every feasible and near feasible solution in the last ten iterations. For example, if the last ten iterations represented feasible solutions, then iteration $=10$ and the ATS process doubles the weight $\left(\alpha_{f c}=2^{(10 / 5)-1}=2^{2-1}=2\right)$ for the solution's logistics footprint penalty parameters $\left(\alpha_{f c}\right.$ and $\left.\alpha_{v c}\right)$. This implies the search process is currently exploring a promising region and seeks to find a smaller logistics footprint. However, if all ten iterations were infeasible, then iteration $=0$ and the ATS process halves the values $\left(\alpha_{f c}=2^{(0 / 5)-1}=2^{0-1}=2^{-1}\right)$ of the penalty parameter to drive the solution toward a larger logistics footprint and feasibility.

ATS adjusts the parameters for the six violation penalties ( $\left.\alpha_{T D D}, \alpha_{d s}, \alpha_{r l}, \alpha_{Q}, \alpha_{S}, \alpha_{T W}\right)$ based on constraint violations during the last ten iterations. ATS updates the iteration value for each individual violation penalty each time a solution violates the constraint during the last ten iterations. For example, if the storage constraint is violated each of the last ten iterations, then iteration $=10$ and ATS doubles the value for the storage violation parameter $\left(\alpha_{s}\right)$. This forces the solution process to seek solutions that satisfy the storage constraint. ATS halves the value of the violation parameter if there were no violations in the last ten iterations (iteration $=0$ ) allowing the search process to explore new regions of the solution space.

### 5.4 The Move Neighborhoods

The neighborhood move generator manages the various neighborhood structures used to traverse the solution space. The LPDPTW and TDP are natural partitioning and ordering problems. Partitioning represents the assignment of a vehicle and its set of customers to a depot, while ordering represents the sequence each customer is visited by the vehicle. A solution consists of a fixed number of partitions and each of these partitions contains a set of letters. Group theory plays a role in neighborhood development by describing the partitioning, orderings and ATS methods that manipulate or change these partitionings and orderings. Each move neighborhood is designed to target elements of the TDP to obtain feasibility or improve the routing and scheduling of the distribution network. This section describes the various move neighborhoods developed for the ATS.

### 5.4.1 Between Cycle Swap (BCS) Move Neighborhood

BCS moves exchange two letters from disjoint sets or cycles of letters. The ATS utilizes a restricted candidate list strategy specific to the TDP to reduce the size of the BCS move neighborhood. Without these restrictions, the individual swap neighborhood becomes computationally large as the problem increases in size. For example, the swap neighborhood for a problem with 500 letters consists of $500(499) / 2=124,750$ individual swap moves. ATS implements the following restrictions to create the BCS move neighborhood:

1. Only exchange elements between disjoint cycles or depots.
2. Only exchange like element letters.
3. Only exchange between different supply point and vehicle types.

The BCS move neighborhood is an intensification scheme designed to improve routing within the same conjugacy class. This neighborhood is designed as a coarse grain move exploration on the current cycle structure. The first swap restriction forces the ATS process to consider unutilized elements within a conjugacy class solution to improve the routing and assignment plan. A typical incumbent solution consists of four major elements: depots, vehicles, suppliers, and demanders. The final BCS move restriction is based on supply point ID and vehicle type. Swapping two letters representing the same supply point does not change the solution and is not allowed. Likewise, a swap between two vehicles of the same type is a null move and is not allowed.

As an example of BCS, consider the problem presented in Table 5-3. This example consists of two depots, two supply points, three demands and four vehicles. One solution to this problem is:

$$
(0,17,3,13,12,19,7,15,2,16)(1,18,8,14)(4)(5)(6)(9)(10)(11)(20) .
$$

This solution contains a total of 210 individual swap moves. However, under the BCS move neighborhood, there are only 18 available swap moves. BCS moves maintain the current conjugacy class and serve as a reordering search scheme. These moves target unutilized elements and adjust routes between depot assignments in an attempt to construct better routes and improve the incumbent solution.

Table 5-3 Example TDP Problem

| Entity ID | Entity | Problem ID |
| :---: | :---: | :---: |
| 0 | Depot - Airfied | 0 |
| 1 | Depot - Airfied | 1 |
| A | Supply Point | $2,3,4,5,6$ |
| B | Supply Point | $7,8,9,10,11$ |
| 1 | Demand | 12 |
| 2 | Demand | 13 |
| 3 | Demand | 14 |
| 4 | Demand | 15 |
| 5 | Demand | 16 |
| A | Vehicle - C130 | 17 |
| B | Vehicle - C130 | 18 |
| C | Vehicle - C17 | 19 |
| D | Vehicle - C17 | 20 |

### 5.4.2 Within Cycle Swap (WCS) Move Neighborhood

WCS moves swap two letters within a cycle. WCS moves target the sequencing of pickups and deliveries within a selected vehicle's route seeking to improve scheduling efficiency. The WCS move neighborhood is a reordering neighborhood scheme that intensifies the search by maintaining the conjugacy class of the incumbent solution. The ATS utilizes a candidate list strategy specific to the incumbent TDP solution to construct the WCS move neighborhood. The following restrictions define the WCS move neighborhood:

1. Only exchange letters within the same cycle or depot assignment.
2. Only exchange letters that sustain pickup and delivery ordering constraints.
3. Only exchange vehicles if they have different vehicle type letters.

The WCS move neighborhood targets route scheduling efficiency. Restriction 1 forces the search to focus on the collection of entities assigned to a depot. Restriction 2 allows exchanges between supply point and demand elements that maintain the ordering restriction targeting vehicles making multiple pickups and deliveries. Restriction 3 explores the possibility that a different vehicle might be more efficient on a selected route.

Continuing with the example of Table 5.3 , where the solution was:

$$
(0,17,3,13,12,19,7,15,2,16)(1,18,8,14)(4)(5)(6)(9)(10)(11)(20) .
$$

The WCS move neighborhood for this solution contains 10 allowable moves, all resident in the first cycle $(0,17,3,13,12,19,7,15,2,16)$, as presented in Table 5-4. The second cycle $(1,18,8,14)$ contains no allowable moves.

Table 5-4 Allowable WCS Moves

| Allowable WCS Swap Moves |  |  |
| :--- | :--- | :---: |
| Element | Letters | Swaps |
| Vehicles | 17,19 | 1 |
| Supply Point | $2,3,7$ | 3 |
| Demands | $12,13,15,16$ | 6 |

### 5.4.3 Complete Route Insert (CRI) Move Neighborhood

The CRI move neighborhood removes a complete assigned route, which consists of the vehicle and its assigned pick-up and delivery locations, from one depot and inserts it in a different depot cycle. A CRI move targets reducing the logistics footprint of assigned depots by seeking reductions in the variable operating cost of a depot or by
eliminating a depot. The CRI neighborhood diversifies the search by changing the solution conjugacy class structure. CRI moves are designed to reduce the number of depots in the distribution network. The number of CRI moves in the move neighborhood depends on the number of depots in the problem and the number of vehicles with an assigned route. A problem will contain no more than
(numberDepots - 1)(numberAssignedVehicles) CRI moves.
Figure 5-5 illustrates the three potential CRI moves for the incumbent solution from Section 5.4.1. The ATS search process examines all available CRI moves and selects the non-tabu move that provides the best improvement in the distribution


Figure 5-5 Allowable CRI Moves
network's overall cost.

### 5.4.4 Demand Reallocation Insert (DRI) Move Neighborhood

The DRI move removes a single assigned demand from one vehicle route and inserts it into the route of another vehicle. DRI moves can change the conjugacy class of the incumbent solution and provides additional diversification. The DRI neighborhood can eliminate routes or vehicles from the distribution network. A DRI move often results in an un-improving or infeasible solution. A given solution's unrestricted DRI move neighborhood could become quite large as the potential insertion points for a given demand depends on the number of demands, number of vehicles and number of utilized
supply points. This research limits the number of DRI moves by enforcing the following two conditions:

1. Only allow demand insertions to vehicles with an assigned route.
2. Only insert a demand in front of a demand on the new vehicle route.

Condition one ensures that the demand is only inserted in a current viable vehicle route. The second condition limits the placement location of the demand. This limitation reduces the potential insertion points to only (numberDemands)(numberDemands-1) and ensures that the assigned vehicle visits a pickup point prior to visiting the demand. This limitation is consistent with the DRI objectives of reducing the logistics footprint and relies on the WCS move neighborhood to improve route efficiency.


## Figure 5-6 Example DRI Moves

Figure 5-6 illustrates seven of the twenty potential DRI moves for the incumbent solution from Section 5.4.1. Five of the seven displayed moves change the current conjugacy class.

### 5.4.5 Between Route Insert (BRI) Move Neighborhood

BRI moves address infeasible routes and attempt to regain feasibility by inserting unutilized supply points and vehicles. BRI moves also attempt to improve route efficiency by inserting pickup and delivery pairs into a different route. The BRI neighborhood becomes prohibitively large in large problems. The following three conditions limit the size of the BRI neighborhood:

1. Only insert unutilized supply points into a current route.
2. Only insert different supply points into a current route.
3. Only insert the new point in front of the current supply point.

Figure 5-7 illustrates three of nine potential BRI supply point insert moves. All BRI moves diversify by changing the current conjugacy class structure. There are no insert points for supply point 4 on either vehicle route 17 or 19 because point letters of 3 and 2 are the identical supply point (masterSupplier) .


Figure 5-7 Example BRI Moves

### 5.4.6 Route Extraction Insert (REI) Move Neighborhood

The REI move neighborhood removes unutilized supply points and vehicles augmenting the DRI neighborhood discussed in Section 5.4.4. While changing the conjugacy class structure, the REI neighborhood generally yields a better logistics footprint. The solution of Section 5.4.1 contains no potential REI moves since all vehicles are currently necessary for a feasible solution. However, as indicated in Figure 5-8, implementing a DRI move that inserts demand 14 into vehicle 19 's route creates a potential REI move candidate in the terms of vehicle 18. The REI neighborhood scheme selects vehicle 18 during the next iteration as a candidate for removal from depot 1 . The

$$
(0,17,3,13,12,19,7,15,2,16)(1,18,8,14)(4)(5)(6)(9)(10)(11)(20)
$$

Figure 5-8 Example REI Move

REI neighborhood is only implemented after several parameters are triggered by the search process.

### 5.4.7 Combined Move Neighborhoods

The ATS creates four insert and two swap neighborhoods for specific search purposes. One current methodology uses these six neighborhoods sequentially reflecting current search conditions. However, Combs and Moore, (2004) state that considering all neighborhoods simultaneously provides better performance than the sequential method. Chapter VI compares the combined swap and insert neighborhood scheme encompassing all elements described in Sections 5.4.1 - 5.4.6 against the sequential method to determine if there is significant difference in this implementation.

### 5.5 Tabu Structure

The tabu search manager uses a tabu memory structure to prevent cycling and to escape local optima. The tabu structure imposes constraints on allowable moves based on the current search status. Moves restricted under the current tabu structure are tabu for the duration of the tabu tenure. Tenure is defined as the number of iterations a particular move remains tabu. The ATS implements both a solution and move based tabu structure.

The solution tabu structure is adaptive and adjusts to recent search conditions. The solution tenure structure stores all solutions visited during the search. This enables ATS to determine whether current search encounters solutions that were explored earlier. The solution tabu tenure adjusts according to two rules:

1. If current solution is revisited, the solution tabu tenure is doubled
2. If current solution is previously unvisited, the solution tabu tenure is decreased by 1 .

This tabu tenure scheme provides a mechanism for escaping cycling but is not overly restrictive. The effectiveness of this tenure scheme is evaluated in Chapter VI.

The move based tabu tenure structure is the second structure ATS utilized and prevents the reversal of recent moves. The move tabu tenure is adjusted based on the following four rules:

1. If current solution is an un-improving solution, the solution tabu tenure is increased by 1 .
2. If the current solution represents no change in the solution, the move tabu tenure is decreased by 1 .
3. If the current solution represents an improving solution, but not the best solution found, the move tabu tenure is decreased by 1 .
4. If current solution represents the best solution found, the move tabu tenure is reset. The current default reset is size 7 .

Utilizing this move based scheme in conjunction with the solution tabu tenure provides ATS with a mechanism that can explore a local region of the solution space yet quickly escape if cycling is encountered.

The ATS algorithm allows a tabu active move if it results in a solution that is better than the current global best solution.

### 5.5.1 Hash Function Development

Storing and comparing information concerning moves and solutions becomes computationally expensive as the size of the problem increases. Glover and Laguna (1997) suggest creating hashing functions that map objects of interest to an integer "hash" value to reduce the computational burden. These values are stored in hashing tables for easy retrieval by the tabu list manager. Sections 5.5.2-5.5.4 describe the hash function developed and utilized in this research.

### 5.5.2 Move Hash Function

The move hash function tracks each implemented move. The initial step is the generation of randMoveHash, which is an $n$-sized vector mapping of the problem's $n$ letters to a uniformly distributed random integer ranging from 1 to $3.00 \times 10^{7}$ with a maximum integer value of $4.295 \times 10^{9}$. For example, the randMoveHash for the $S_{11}$ example of Section 5.1.2 contains 11 elements randomly generated from the uniform $\left(1,3.00 \times 10^{7}\right)$ distribution. Therefore, for a selected move cycle $k$ containing $j$ letters, the hashing function value becomes:

$$
\begin{equation*}
\text { moveHashValue }=\sum_{j=0}^{j-1} k_{j} \times \operatorname{randMoveHash}\left[k_{j}\right] \tag{5-3}
\end{equation*}
$$

Given a randMoveHash =
[ $810,4000,920000,56000,820000,83,5100,320000,530000,121000,19000]$
for the $S_{11}$ example of Section 5.1.2 and a route insert move $k=(3,7,2)$, the resulting hash value stored in the hash map is

$$
\begin{aligned}
\text { randMoveHash } & =3 * 56,000+7 * 320,000+2 * 920,000 \\
& =4,248,000
\end{aligned}
$$

The randMoveHash value for each move is specific to the group theoretic nature for generating the solutions for this research.

### 5.5.3 Solution Hash Function

The solution tabu list is critical to the adaptive scheme of the ATS search process. The ATS solution hash function reduces the computational burden and is used for comparison of solutions within the solution tabu list. The ATS solution hash function is an extension of the move hash function. The preprocessing phase generates a randSolutionHash table, which is a $n \times n$-sized mapping of the problem's $n$ letters to a uniformly distributed random integer ranging from 1 to $1.00 \times 10^{8}$ with an unsigned maximum integer value of $4.295 \times 10^{9}$. The $n \times n$ table is necessary since letter order within the solution is important. For example, the randSolutionHash table for the $S_{11}$ example of Section 5.1 .2 contains 121 integers randomly generated from the uniform $\left(1,1.00 \times 10^{8}\right)$ distribution. Therefore, for a selected solution permutation $p$ containing $j$ letters and $d$ depots, the hashing function value becomes:

$$
\begin{equation*}
\text { solutionHashValue }=\sum_{d=1}^{d} \prod_{j=0}^{j-1} \text { randSolutionHash }\left[p_{j}\right]\left[p_{j+1}\right] \tag{5-4}
\end{equation*}
$$

Wiley's (2000) symmetric group class contains a group theoretic method to calculate the solution hash values. Crino, et al. (2004) and Combs and Moore (2004) demonstrate this group method and present the theoretical hash value collision rates in there work. Table 5-5 provides a comparison of the collision rate and calculation time of

Table 5-5 Comparison of Equation 5-4 and Group Theory Hash functions

|  |  | Hash Function Collision Rate $^{c}$ Total Calculation Time (milliseconds) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $n$ | ${\text { Size of } S_{n}}^{\text {Group Theory }^{1}}$ | Equation 5-4 | Group Theory $^{1}$ | Equation 5-4 |  |
| 3 | 6 | 0 | 0 | 0 | 0 |
| 4 | 24 | 0 | 0 | 0 | 0 |
| 5 | 120 | 1 | 0 | 0 | 0 |
| 6 | 720 | 11 | 0 | 0 | 0 |
| 7 | 5040 | 555 | 9 | 46 | 18 |
| 8 | 40320 | 9801 | 32 | 153 | 121 |

Note: 1. Group Theory (Combs and Moore: 2004)
the group theoretic method and the ATS approach. Table 5-5 demonstrates that at least for smaller problem sizes, the ATS approach outperforms the group theoretic method.

### 5.5.4 Conjugacy Class Hash Function

The conjugacy class tabu list represents the last list utilized in this research. Since conjugacy classes completely partition the symmetric group solution space, information concerning which conjugacy classes are visited may prove useful to ATS search parameters. Intensification schemes tend to focus on a single or small number of conjugacy classes while diversification schemes seek to explore a large number of conjugacy classes.

Conjugacy class information is used in Chapter VI to evaluate the scope of diversification during the ATS search process. The conjugacy class hash function serves a role similar to that of the solution hash function and is based on the solution's cyclic form structure. Therefore, for a selected solution permutation $p$ containing $c$ cycles, the hashing function value becomes:

$$
\begin{equation*}
\text { conjugacyHashValue }=\sum_{c=1}^{c} 10^{\operatorname{size}_{c}-1} \tag{5-5}
\end{equation*}
$$

For example, the solution $(0,10,2,6)(1,9,3,7,5,8)$ from Section 5.1.2 is a member of the $1^{1} 4^{1} 6^{1}$ conjugacy class. The assigned integer value for this cyclic form structure is:

$$
\begin{aligned}
\text { conjugacyHashValue } & =10^{4-1}+10^{6-1}+10^{1-1} \\
& =1000+100000+1 \\
& =101001
\end{aligned}
$$

### 5.6 ATS Algorithm

This section provides the pseudo code for the ATS TDP (LPDPTW) algorithm. The algorithm consists of three phases: pre-processing, ATS search, and the post processing phase.

## ATS Preprocessor

1. Import necessary problem characteristics from text files
2. Build required Java ${ }^{\mathrm{TM}}$ objects and initialize data arrays (Section 5.1.3.1)
3. Construct user defined initial solution (Section 5.1.3.2):
a. Greedy heuristic
b. Insertion heuristic
4. Evaluate Initial solution (Section 5.2).

## ATS Search (for each iteration)

1. Select Move Neighborhood Phase (Section 5.6):
a. Combined Neighborhood scheme (Section 5.3.7)
b. Sequential Neighborhood scheme (Section 5.3.1-5.3.6)
2. Evaluate Move (Section 5.2)
3. Select best non-tabu move based on objective function value and search conditions:
a. Phase I: Select first improving move in neighborhood
b. Phase II: Select best move after evaluating complete neighborhood
4. Perform appropriate symmetric group function and generate new solution
5. Update necessary feasible and near feasible solutions and objective function values
6. Update feasible and near feasible elite lists as appropriate and check termination criteria
7. Update objective penalty weights as appropriate (Section 5.2)
8. Repeat steps $1-7$ until achieving termination criteria

## ATS Post processing

1. Save solution parameters to text file (Section 6.1)
2. Output solution (Node Selection, Vehicle Assignments, and Schedules)

### 5.7 ATS Strategy Move Manager

The tabu search manager determines the search path for the ATS process. The strategy reviews the current parameters and determines the phase and search context for the next iteration. The manager controls movement between the request assignment (partitioning) group and the route improvement (ordering) group based on the number of iterations since the last iteration resulting in the best solution found to that point in the
search. The search context is based on the current status of tracking parameters and transitions between super-diversify, diversify, and intensify. Intensification searches are focused within conjugacy classes that contain good solutions. Diversification occurs when the current search path fails to find good solutions or the process has detected an attractor basin. Phase I ends by establishing the current solution as the new incumbent solution.

The tabu search strategy manager determines the appropriate phase search context and move neighborhood to apply to the incumbent solution. These decisions are based on collected search data, objective function values and pre-defined search parameters. Two move manager schemes are used. Both search manager schemes are examined in Chapter VI. The first search scheme does not utilize elite lists during the second phase or intensification portion of the search process. The second search scheme maintains two elite lists to store solution data for both the best feasible and near feasible solutions found during the search process. The elite list size is a user pre-defined parameter and has a default size of five for this dissertation. Each elite list is sequenced in descending order based on solution objective function values. ATS utilizes the elite list as a restart mechanism during the second phase of the search process. The search manager monitors the iteration since last good solution (ISGS) parameter that maintains the number of iterations since the search process last found a new best (feasible or near feasible) solution. The consecutive infeasible iterations (CI) parameter tracks the consecutive number of iterations resulting in infeasible solutions and is used by the search manager to diversify. The choose first improving move (CFIM) is a boolean parameter that controls
the move manager's selection process. The move manager selects the first improving move from the move neighborhood scheme when the CFIM parameter is true. This serves as a diversification scheme allowing visits to areas of the solution space the ATS process might not normally visit. When the CFIM parameter is set to false, the move manager examines all moves in the neighborhood scheme before selecting the best move.

Figure 5-9 illustrates the move manager process for establishing the search phase and search context where ATS is utilizing elite lists. The search process contains two main phases consisting of an exploration and intensification phase. Once the move manager selects the appropriate search phase, it determines the search context (intensify, diversify, or super-diversify) based on solution characteristics.

The ATS search process begins in an exploration phase and the CFIM parameter is set to true. This allows the search process to traverse a greater number of regions in the solution space. The search manager tracks both the best feasible and best near feasible solutions discovered during the search process. Each iteration, if the solution is a new best solution (feasible or near feasible), the manager updates the appropriate elite list and resets the ISGS and CI counters. The discovery of a new best solution might be an indication of a promising region so the search process enters a mini-intensification search phase to explore the local area. If the incumbent solution is not a new best solution, the search manager increments the ISGS counter and checks to see if the solution is infeasible. The move manager increments the CI parameter for an infeasible incumbent solution and resets the CI parameter if the solution is feasible or near feasible. The
search manager next determines if the ISGS counter has surpassed a pre-defined tolerance or number of poor solutions.


Figure 5-9 ATS Strategy Move Manager

The search manager checks the CI tolerance level if the ISGS counter has not surpassed its tolerance. Surpassing the CI tolerance indicates that the search process is exploring an undesirable region of the solution space. This triggers a solution repair phase that targets infeasibility. Once the ISGS parameter surpasses this tolerance, the search process enters a more focused search phase. The CFIM parameter is set to false allowing the move manager to examine all moves and the elite list solutions are activated. The ATS process selects the first solution in the elite list as the new incumbent solution and repeats the phase one search process until the ISGS parameter surpasses the predefined tolerance. At this point, the next solution is selected from the elite list and the cycle begins again until exhausting all elite list solutions.

### 5.8 Summary

This chapter described the algorithm for the ATS process. The ATS consists of two main phases: the preprocessing phase and the ATS search phase. The preprocessing phase initializes the ATS and provides the initial solution to the ATS phase. The ATS search phase is comprised of five components. The five components are the neighborhood move generator, solution evaluator, tabu structure manager, perform move operator and search manager. The neighborhood move generator generates one of several available move neighborhoods based on guidance from the search manager. The search manager is the heart of the process and controls the search process through a cycle of the first four components. The search manager monitors a set of counters and parameters to dynamically select the appropriate search phase and search context for each iteration.

The next chapter describes the design of experiment conducted in this research to determine the main characteristics that impact the ATS process.

## VI Analysis of the ATS and Experimental Results

Researchers often conduct experiments to examine how their particular heuristic performs compared to other heuristics for a certain problem class and how well it solves a set of problem instances for the problem class. Myers and Montgomery (1995) and many others have traditionally proposed a factorial design process for this type of experimentation. This research utilizes a process outlined by Combs and Moore (2004) based on an empirical science approach proposed by Hooker $(1994,1995)$.

This empirical science approach evaluates the heuristic's performance based on the characteristics of the problem instead of testing its ability to solve a set of benchmark problems. Hooker's (1995) approach suggests running a controlled experiment over a variety of parameter settings to evaluate the heuristic's performance. This research utilizes a fractional factorial experiment to determine how the characteristics of adaptive tabu search affect problem performance measures such as the number of depots, the number of supply points and the number of vehicles or collectively the distribution network's logistics footprint and how LPDPTW problem changes affect tabu search performance. The set of benchmark problems in this research serve as a common platform to evaluate how changes to the above ATS characteristics affect solution performance.

### 6.1 Research Objectives and Problem Statement

Research objectives define the variables of interest in the research and serve to guide the data collection process on performance measures. Factorial analysis defines these variables of interest as factors and the desired performance measures as responses.

The analysis in this research addressed the following objectives:

1. Determine how the characteristics of the TDP (LPDPTW) and the ATS process affect the following set of responses:
a. The network's logistics footprint including the number of depots, supply points and vehicles of the best and near feasible solutions.
b. The number of ATS iterations necessary to solve the problem.
c. The number of conjugacy classes (repartitioning) visited during the search process.
d. The average neighborhood size utilized in the search process.
e. The average tabu tenure (move and solution) used for memory.
2. Determine if the initial solution impacts the above responses.
3. Determine if the advanced tabu search techniques of combined neighborhoods, adaptive objective penalty weights and elite list utilization significantly improve the solution process.
4. Determine which tabu tenure strategy provides the best performance.
5. Measure the quality of the ATS process by comparing the best solution found to the known optimal solution for several small TDP.
6. Measure the quality of the ATS process by comparing the best solution found to lower bounds for the TDP.
7. Demonstrate the performance of the ATS process on a benchmark data set.

### 6.2 Design of Experiments

The following sections cover the experimental design utilized in this research. The next section (6.2.1) describes the 16 design factors used for the evaluation. Section 6.2.2 contains a discussion of the ten responses $(a-e)$ introduced in objective 1 . The final section (6.2.3) discusses the fractional factorial design used for the experiment.

### 6.2.1 Design Factors

The experimental design uses 16 design factors (Table 6-1) divided into three main categories: scheduling, routing and tabu search. The scheduling factors are intended to capture the significant influencers in establishing the timings of a vehicle's route. The routing factors relate directly to the physical size of the required distribution network and the final grouping represents standard tabu search components used to initiate and direct the tabu search process.

TDP scenarios represent military operations across the full spectrum of operations and across various terrains. For example, the Third Infantry Division (Mechanized) during Operation IRAQI FREEDOM (OIF) fought an asymmetric enemy in the midintensity and low-intensity arena and transitioned to a stability and support operation (SASO) at the conclusion of the attack. These operations were conducted over open desert, urban and restrictive terrain. The values for the scheduling and routing factors were set to describe contingency operations at both ends of the full spectrum of
operations. The first operation (low) represents a stability type operation with a much slower pace of operation. The second operation (high) represents a high tempo operation characterized by rapid continuous movement. This research utilizes elements of the fourteen standardized scenarios outlined in the TRADOC Scenario GIST Book (TRAC-TR-05-016, 2005) to construct the representative low and high operations.

Table 6-1 Experimental Design Factors

| Source | Factor | Factor ID | Low | High |
| :---: | :--- | :---: | :---: | :---: |
| Scheduling | Time Window Size | TWS | 360 min | 60 min |
| Scheduling | Crew Operational Tempo | COT | 0.45 | 0.90 |
| Scheduling | Allow Early Waiting | AEW | Yes | No |
| Scheduling | Demand Distribution | DD | Loose | Tight |
| Routing | Theater of War Size | BoxSize | 200 km | 1000 km |
| Routing | Number Demands | NumDds | 5 | 200 |
| Routing | Max Distance Between Nodes | MDBN | 100 km | 500 km |
| Routing | Number Depots | NumDepots | 4 | 8 |
| Routing | Number Supply Points | NumSplyPts | 3 | 8 |
| Routing | Number of Theater Points | NumThPts | 0.25 | 0.5 |
| Routing | Allow Refuel | Refuel | Yes | No |
| Tabu Search | Tabu Tenure | Tenure | Single | Dual |
| Tabu Search | Initial Solution | IntSoln | Greedy | Insertion |
| Tabu Search | Combined Neighborhood | CmbNeigh | No | Yes |
| Tabu Search | Elite List Utilization | ELU | No | Yes |
| Tabu Search | Adaptive Obj Penalty Weights | OPWgts | No | Yes |

In the following explanation, the experimental design factors appear in bold type.
The following scheduling factors describe the expected speed and tempo of operations.
Time window size, TWS, represents the time span in minutes of a selected pickup or delivery point's service window. The two levels allow study of a fast pace higher tempo mid-intensity type operation versus a more established predictable lower tempo SASO type operation. Crew operational tempo, COT, refers to the expected amount of time a
vehicle is actually in operation. Crew operational tempo is affected by many factors from number of available crewmen to scenario conditions. For example, given a crew operational day of 20 hours, a given vehicle and crew is expected to be in operation for approximately nine hours at the low level. The low level for COT represents a slower paced operation over a smaller operational area or a limitation on the number of available crewmen. The high level represents a high demand operation or operations conducted over a large operational area. Allow early waiting, AEW, refers to the ability of a vehicle to arrive at a pickup or delivery location prior to the service window opening. The low level allows waiting and represents more stable operations where security is not an issue or the actual sites have the physical space to accommodate the vehicles. The high level does not allow waiting and might represent a higher threat environment where it is undesirable to have lines of vehicles waiting on the road. This factor impacts, among other things, departure scheduling for vehicles leaving their depots. Demand Distribution is the last scheduling factor and represents the expected spread in hours of all demands throughout an operational day. The low level represents stable operations and the demands are spread out over a 12 hour time period. The high level models more intensified operations where demands are clustered in a tighter three hour time horizon. This represents typical operational scheduling where entire formation movements are in coordination with each other.

The routing related factors describe the size and scope of the operational area. Theater of war size is the actual operational box designated by the combatant commander for the contingency operation. Box size is not related to the expected
operational tempo, only to the available operational area. The low level of 200 km represents operations such as RESTORE HOPE and JUST CAUSE. The high level of 1000 km represents the operational area of larger scaled operations such as DESERT STORM or IRAQI FREEDOM. The number of demands is related to the operational intensity and the expected number of demands in an operational period. The operational period for this test is 24 hours. The low level represents small intensity type or humanitarian type operations. The high level represents higher intensity or initial employment operations. Max distance between nodes is the largest distance in kilometers between any two nodes. The low level models a tighter distribution network or an area with a well established network infrastructure. The high level models a larger distribution area or an area with a limited network structure. The number of depots represents the available or authorized locations in the theater of war to establish vehicle depots. There is always at least one of these depots located outside the theater of war to represent an intermediate staging base. The low level represents an austere operating area containing limited facilities and the high level models a more robust operating environment. The number of supply points is the available set of locations containing supplies demanded by the customers. The low level represents a high threat operating area limiting the number of available locations, while the high level models a larger operation or low threat robust operation area. The number of theater points models the theater depot and supply points available to support operations inside the designated theater of war. These points might represent establishing an intermediate staging base or existing pre-established bases. The factor levels represent the percentage of all available
bases which are physically located outside the theater of war's area of operation. Allow refueling is the last routing factor. This factor impacts on the route length of a given vehicle before it is forced to return to the depot. Allowing a vehicle to refuel at either a supply point or designated refuel point enables the vehicle to cover larger distances. The low level allows for refuels, while the high level limits the vehicle's travel distance to its fuel efficiency.

Chapter V covered the tabu search factors used in this experimentation. Tabu tenure is the tenure scheme used during the ATS search process. ATS uses the single tenure at the low level, while at the high level both the solution and move tabu tenures are in operation. The objective is to determine if there is any significant advantage to implementing a dual tabu tenure on the search process. Initial solution refers to the method used during the pre-processing phase to generate the initial solution. The goal is to see how the ATS process is affected by the quality of the initial solution. The greedy solution considers none of the LPDPTW's associated routing and scheduling constraints and generally constructs a poor infeasible initial solution. The insertion procedure does not guarantee an initial feasible solution but it does consider several of the LPDDTW's side constraints during its route construction. These considerations result in a better initial solution than the greedy approach. LRP literature indicates that a poor initial solution hampers the search process, while tabu search literature reports that the search process overcomes any initial solution construction. The low level represents an insertion type initial solution construction, while the high level is a simple greedy scheme based on distance between supply points and demands. The combined neighborhood
factor simply describes whether the ATS uses a neighborhood scheme consisting of a combination of insert and swap moves or just a single function neighborhood that targets a specific aspect of the current solution. This factor provides information concerning the contribution of a combined neighborhood scheme to the ATS search process. Elite list utilization represents an additional advanced tabu search technique. Elite lists allow the search process to return to previously visited promising regions for additional exploration. This factor in the experiment provides information concerning the contribution of returning to these regions during the ATS search process. Adaptive objective penalty weights represent the final factor, an advanced tabu search technique, considered in this research. The ability of ATS to control adjusting these penalty weights allows the search to strategically oscillate throughout the solution space. The objective is to determine if there is an impact on the search process of allowing ATS to leave and return to the feasible region.

### 6.2.2 Objective Responses

The first six responses (1a) directly relate to the problem's objective of minimizing the theater of operation's logistics footprint. The six responses are the selected number of depots, supply points and vehicles in both the best and near feasible solutions. These responses coupled with the 16 design factors provide an indication of the routing and scheduling characteristics that affect the problem.

The seventh response (1b) is the number of ATS iterations necessary to solve the problem. The ATS search process is event driven and terminates when a given set of conditions are achieved versus stopping after a fixed number of iterations or elapsed time.

The number of iterations necessary to find good solutions to a problem increases as the size and scope of the problem increases. This is directly related to the size of the neighborhoods and the time necessary to evaluate each neighborhood move. If changing the initial solution construction or neighborhood development schemes decreases the number of iterations necessary to solve the problem, then the change improves the ATS search process.

The next response (1c) is the number of conjugacy classes visited during the search process. As mentioned in Chapter II, conjugacy classes represent a complete partition of the symmetric group's solution space. The number of conjugacy classes increases as the number of depots, supply points, vehicles and demands increase. The number of conjugacy classes visited provides a measure of diversification during the ATS search process. As the solution space size increases, the number of visited conjugacy classes should increase as the ATS process increases its repartitioning or diversification to visit more areas of the solution space. Studying this response helps to determine which factors may affect the search process and counteract the problems associated with the size of the solution space or problem.

The ninth response (1d) is the average neighborhood size of the search. This response helps study the impact of utilizing combined and single neighborhood schemes on the search process. This response provides additional information concerning which factors affect the number of iterations necessary to solve the problem.

### 6.2.3 Two - Level Fractional Factorial Design

Table 6-1 contains a total of 16 factors considered in this experiment. A complete replicate of the $2^{16}$ factorial design requires 65,536 observations or design runs. Each observation represents a different combination of the 16 factors and their two-levels. The objective of the factorial design is to examine the factor effects to determine which variables are important. A full implementation of the $2^{16}$ factorial design requires almost four years of continuous computer running time, based on early test run solution times of approximately 30 minutes per design point. Obviously, implementing a full design is impractical; therefore, this research conducts a factor screening experiment to identify those factors that have large or significant effects.

This experiment implements a fractional factorial design as a screening experiment to weed out those factors that have little or no effect on the problem. This research utilizes Design Expert (7.0.1) to evaluate and select the appropriate 2-level factorial design for the experiment (Stat-Ease, 2005). A resolution IV design is a design structure where the main effects (16 factors) are not aliased with any other main effect or any two-factor interactions (Montgomery, 2005: 286). This design provides an indication of each of the 16 design factor's individual contribution to the problem. However, twofactor interactions may be aliased with other two-factor interactions under this design structure. Aliasing occurs when it is impossible to differentiate contributions between two factors (Montgomery, 2005: 284).

This research implements a $2_{\mathrm{IV}}^{16-11}$ resolution IV design as the initial screening design. The design structure (Appendix B) contains the initial 32 completely randomized
experiment combinations or design points with two replicates for a total of 64 problem instances. The replicate in the initial screening effort is intended to capture the variability associated with several problem elements. For example, the MDBN factor at the low level requires that the distance between any two nodes is less than 100KM. This does not imply that the distance between a given node A and node B is the same in all 64 generated problem instances, only that it is less than 100 KM . The replicate attempts to capture the significance of this variability in the MDBN factor on the problem. The design's alias structure (Appendix B) indicates that none of the main effects are aliased with each other or any two-factor interactions. However, all two-factor interactions are confounded and provide no clear means to demonstrate their contribution. For example, if the experimental results indicate that the aliased pair of two-factor interactions NumDds(NumSplyPts) and IntSoln(Tenure) are significant, there is no statistical method to indicate which of the two interactions is significant. However, this information is valuable in providing insight into the problem and provides a starting point for additional investigation.

The intent of the initial screening experiment is to identify one or more of the 16 factors as insignificant to the problem and any two level factors that are significant. The second phase of the experiment process in this research eliminates any insignificant factors from the design and conducts a randomized semi-fold over reduced Resolution IV design to separate aliased two factor interactions. The semi-fold design requires fewer additional design runs than a full fold over but only addresses one set of two factor
interactions. The intent is to continue this sequential experimentation until all significant two factor interactions are no longer aliased with other two factor interactions.

### 6.3 An Excel - VBA based LPDPTW Problem Generator

The design of experiment described in Section 6.1 requires the sequential generation of a large number of problem instances. This section describes the distribution problem generator developed in this research for constructing the randomized problem runs of the experiment. Without this generator, the systematic analysis called for by the design of experiment would become time prohibitive.

### 6.3.1 Motivation

The problem generator is an Excel - VBA based application designed to quickly and efficiently generate the required set of problems outlined in Section 6.1. The problem generator provides a mechanism to rapidly generate these problems while controlling the host of necessary supporting variables and assumptions not examined in the design of experiments. The goals of the distribution problem generator are as follows:

1) Develop a problem generator that generates problems which allow a robust and systematic analysis of the ATS process.
2) Develop a problem generator that allows rapid updating and parameter adjustment for any type distribution situation.
3) Develop a generator that is extendable beyond this research to allow users to generate their own problem instances for analysis without a requirement to understand the necessary Java ${ }^{\mathrm{TM}}$ structure or input formats.

### 6.3.2 The Problem Generator Components

The problem generator is comprised of three main components; a user interface, the problem generation worksheets, and the data worksheets. The generator allows a user to control the host of factors necessary to construct the LPDPTW. This control provides the developer the ability to determine which distribution factors are of interest and construct problems that allow analysis of these specific factors. This generator in conjunction with a proper statistical analysis allows the design of experiment to identify the subset of significant factors.

## User Interface

The user interface worksheet represents the heart of the distribution generator and provides the entry point for users to create a distribution problem that represents their particular scenario. The interface requires a user to define the main LPDPTW elements outlined in Section 5.1.3 such as the number of depots, number of supply points, demand distribution, etc. The interface then provides a mechanism for generating both the map and distance matrixes and the eight text files required for implementing the ATS.

The resolution IV design located in Appendix B provides the required setting levels for the user defined requirements. These design inputs ensure a systematic examination of the LPDPTW during the design of experiment phase. The MDBN and demand distribution represent two input requirements with a variable outcome in the problem generation. To ensure variability in the experiment, this research allows the problem generator to randomly select the $(\mathrm{x}, \mathrm{y})$ coordinates of the various network nodes while ensuring that the desired MDBN is not violated. The problem generator also
randomly determines the demand distribution based on a user defined window of operation. The window of operation was held constant at 0600 to 0900 for the tight distribution and 0600 to 1600 for the loose distribution during the construction of the DOE problems. Allowing the problem generator to construct these two parameters prevents the unintentional construction of a biased or skewed problem.

## Data Worksheets

The problem generator contains four data worksheets: depot, master supply, demand and vehicle. These four worksheets represent a user defined database that contains the basic information for each object type that the generation worksheets require to create the desired distribution network. The worksheets provide a user the opportunity to control problem characteristics of interest. A user can update these worksheets for every problem instance or just change an individual characteristic of a particular item to determine if there is any impact on the solution.

To control variability in the experiment, this experiment held all associated object characteristics constant throughout the generation of the set of DOE design problems. This insured that the only potential variability occurred in the factors directly examined during the experiment.

## Problem Generation Worksheets

Table 6-2 illustrates the eight problem generation worksheets contained in the LPDPTW problem generator. These worksheets create the necessary individual instances for the depots, supply points, demands, master supply points and vehicles as outlined in Section 5.1.3.1. These worksheets do not require user input or manipulation for ATS
implementation. They are simply the Excel versions of the eight text files that are required to implement the ATS algorithm.

## Table 6-2 Problem Generation Worksheets

| Generation Worksheets |
| :--- |
| Depot |
| Master Supplier |
| Supplier |
| Demander |
| Vehicle |
| Ground Distance Matrix |
| Air Distance Matrix |
| Sea Distance Matrix |

### 6.3.3 Conclusion

The distribution problem generator provides a necessary mechanism to systematically analyze the LPDPTW. The generator provides a method to control variability in the generated problem and reduce the potential impact of user bias or errors. The design of experiment outlined in Section 6.2 requires a sequential analysis of the LPDPTW. This means that there is no set number of test problems for the analysis. The set of problems continues to expand after each sequential analysis. Without this generator, it would be difficult to ensure a continuity of problem design during each sequential phase of the design of experiment

### 6.4 Determining Optimal / Lower Bound Solutions for the LPDPTW

This research utilizes three methods to develop optimal and lower bound solutions for a set of LPDPTW problems. The intent is to compare the effectiveness of the ATS
methodology by examining the best solutions found with ATS and comparing them to the optimal and lower bound solutions developed in this section. The first approach consisted of developing 15 problems that were small enough to allow total enumeration of all potential solutions. These problems were initially designed to test the accuracy and routing ability of the ATS methodology. The second approach entailed utilizing LINGO (Release 9.0) to program the LPDPTW mathematical formulation developed in Chapter IV. However, as the problem size increases, the exponentially increasing number of required sub-tour constraints makes the program impractical for larger problem sizes. The final approach presented in this section determines a feasible integer lower bound for the LPDPTW and provides the underlying distribution network. This approach is applied to a test set of 25 LPDPTW instances.

The lower bound approach decomposes the problem into a location and routing sub-problem and seeks a lower bound for each component. The lower bound has three terms derived from the structure of each of the two main components of the LPDPTW. The first two terms refer to the location costs of the depots and supply points and the second to the routing costs. The location cost terms are obtained by solving a Location Problem (LP) and the routing costs are obtained by solving a multi-depot shortest path routing problem with time windows. The first term is derived from the costs incurred when opening supply points, as well as the costs for satisfying demands from that point. The second term is derived from the costs incurred when opening depots, as well as the costs for supporting the vehicle fleet. The final term is derived from the costs of the edges that connect depot, supply and demand locations.

The first requirement is to determine the cost of opening supply points and processing demand requirements through these supply points. Fori $i \in I$, if plant $i$ is opened, there is obviously an incurred fixed cost of $f_{i}$ but, since the supply point now contains at least one associated non-empty route, there is an associated cost, $v_{i}$, for processing the demanded commodity $(p \in P)$ quantity, $\sigma_{p j}$. Let $x_{p i}=\sum_{i \in I} \sigma_{p i}$ represent the sum of all commodity demands processed at supply point $i \in I$. Then, $\tilde{f}_{i}=f_{i}+x_{p i} v_{i}$ is a lower bound on the cost for opening supply point $i$ and processing the demanded quantity. Let $\underline{Z}_{L P}$ be the optimal value to the following:

$$
\begin{array}{cl}
\underline{z}_{L P}=\operatorname{minimize} \sum_{i \in I} \tilde{f}_{i} y_{i} & \\
\text { subj. to } \sum_{i \in I} b_{p i} y_{i} \geq D_{p} & \forall p \in P \\
\sum_{i \in I} x_{p i} y_{i}-\sum_{j \in J} \sigma_{p j} \geq 0 & \forall p \in P \\
x_{p i} \leq b_{p i} & \forall p \in P, \forall i \in I \\
y_{i} \in\{0,1\}, & \forall i \in I \\
x_{p i} \geq 0, & \forall p \in P, \forall i \in I . \tag{6-6}
\end{array}
$$

Constraint (6-2) ensures that enough supply points are opened to meet the total commodity demanded. Constraint (6-3) ensures that all demands are satisfied and provides a link to opening the supply points. Constraint (6-4) ensures that support from a given supply point does not exceed its capacity. Therefore, $\underline{Z}_{L P}$ represents a valid lower bound on the supply point location costs. This term is strengthened by taking into
account that not all capacity of a given supply point may be consumed by the set of demands. The maximum capacity of a supply point $i \in I, \bar{b}_{p i}$, is the storage capacity of the supply point that can be drawn upon by the set of demands or its throughput capacity. This maximum capacity becomes:

$$
\begin{align*}
& \bar{b}_{p i}=\operatorname{maximize} \sum_{j \in J} \sigma_{p j} z_{j}  \tag{6-7}\\
& \text { subj. to } \sum_{j \in J} \sigma_{p j} z_{j} \leq s_{p i} \quad \forall i \in I  \tag{6-8}\\
& \sum_{j \in J} \sigma_{p j} z_{j} \leq m_{p i} \quad \forall i \in I  \tag{6-9}\\
& z_{j} \in\{0,1\}, j \in J . \tag{6-10}
\end{align*}
$$

Now, $\underline{Z}_{L P}$ can be strengthened to $\underline{Z}_{\bar{L} \bar{P}}$ by solving (6-1)-(6-6) after substituting the problem's $b_{p i}$ coefficients by $\bar{b}_{p i}$.

The second term is derived from the cost for opening the depots and supporting the fleet of vehicles. For $k \in K$, if depot $k$ is opened, there is an incurred fixed cost of $f_{k}$. As in the supply point case, opening a depot implies there is at least one vehicle assigned to the depot with an associated depot cost, $g_{v k}$ for supporting this vehicle. Let $n_{v k}$ represent the total number of vehicles $v \in V$ assigned to $\operatorname{depot} k \in K$. Then, $\tilde{f}_{k}=f_{k}+n_{v k} g_{v k}$ is a lower bound on the cost for opening depot $k$ and assigning a set of vehicles to the depot. Let $\underline{z}_{L P 2}$ be the optimal objective function value of the following problem:

$$
\begin{align*}
& \underline{Z}_{L P 2}=\operatorname{minimize} \sum_{k \in K} \tilde{f}_{k} y_{k}  \tag{6-11}\\
& \text { subj. to } \sum_{k \in K} \operatorname{mog}_{k} y_{k} \geq|V|  \tag{6-12}\\
& \sum_{k \in K} n_{v k} \geq|v| \quad \forall v \in V^{\prime}  \tag{6-13}\\
& y_{k} \in\{0,1\}, \quad \forall k \in K  \tag{6-14}\\
& n_{v k} \geq 0, \quad \forall v \in V, k \in K . \tag{6-15}
\end{align*}
$$

Therefore, $\underline{z}_{L P 2}$ becomes a valid lower bound on the depot location costs. This term is strengthened by taking into account that the optimal solution may not require all vehicles in the fleet. The routing solution provides additional information concerning the expected vehicle fleet size and type. Taking advantage of this information allows adjusting the total number of vehicles, $|\mathrm{V}|$ and the number of vehicles by type, $|v|$ in Equations (6-12) and (6-13), respectively. Now, $\underline{Z}_{L P 2}$ can be strengthened to $\underline{Z}_{\overline{L P} 2}$ by solving (6-11)-(6-15) after adjusting the right hand sides of Equations (6-12) and (6-13).

The final term for the LPDPTW lower bound addresses the routing requirement of the problem. This term seeks to minimize the associated vehicle routing costs of the imbedded PDPTW problem. The cost function takes into account the fixed cost, $f_{v}$, of assigning vehicle $v \in V$ to a route and its associated cost, $c_{i j v}$ of traveling between nodes $i$ and $j$. Assume for a moment that vehicles may be assigned to a pick-up point instead of a vehicle depot; the PDPTW may be decomposed into a VRPTW for one demand or supply point or a multi-depot VRPTW (MDVRPTW) if there is more than one available supply or pick-up point. Then, as depicted in Figure 6.1, a viable lower bound routing cost to
the PDPTW, $\underline{z}_{\text {PDPTW }}$, includes three terms; the cost of the imbedded VRPTW or MDVRPTW route (minus the routing cost between the last customer in the route and the supply point), the routing cost between the vehicle depot and supply point, and finally the routing cost between the last customer in the vehicle's route and the vehicle depot. Vehicle depot selection is based on the minimum routing cost between depot, supply point, and last customer in the route.


Figure 6-1 Bound on Routing Costs

The lower bound for the LPDPTW then becomes $L B=\underline{z}_{L P 1}+\underline{z}_{L P 2}+\underline{z}_{P D P T W}$. This integer lower bound represents a feasible solution to the LPDPTW and is the final method utilized in this research for determining a solution or lower bound to the LPDPTW test set. These solutions are used in Section 6.5.2 to evaluate the ATS methodology.

### 6.5 Design of Experiment Results

This section presents the results of the DOE conducted in this research. The results cover the objectives described in Section 6.1 and provide a measure of the quality of the ATS process by comparing its solutions to the solutions from Section 6.4.

### 6.5.1 Hypothesis Testing and Examination of the Factor Effects

This research utilized statistical hypothesis testing to determine if there were any significant factors for each of the examined responses. The hypothesis test utilized in this research assumed that the mean effect $\mu_{i j}$ for the set of design factors and response combinations were equal. This test may be stated formally as:

$$
\begin{aligned}
& H_{o}: \mu_{i j}=0 \\
& H_{1}: \mu_{i j} \neq 0
\end{aligned}
$$

This hypothesis testing utilized analysis of variance (ANOVA) with the F-test as the test statistic and the p -value as the rejection region with a level of significance, $\alpha$, value of 0.05 for evaluating the hypothesis for each response. A p -value $\leq \alpha$ allows for rejecting the null hypothesis and indicates that the factor is significant in explaining the associated response. A p-value $>\alpha$ prevents rejection of the null hypothesis and does not allow us to make any additional statements concerning the factor and response.

Appendix C contains the ANOVA calculations for the 10 responses presented in Section 6.1. Table 6-3 represents a consolidation of these ANOVA tables and provides the mean effect of each design factor with respect to the ten observations. The table contains both numeric and X values for each factor and response combination. The X in
the table indicates that the null hypothesis as described above was not rejected and that this factor is likely insignificant in terms of the associated response. A numeric value in the table indicates that the factor is significant and the numeric value represents the expected effect on the average response. Shading in the factor title box indicates that there were one or more significant two-factor interactions.

Table 6-3 Factor Effects for the LPDPTW Responses

| Factor | Feasible Solution |  |  |  | Near Feasible Solution |  |  | Iterations | Visited CC | $\begin{gathered} \hline \text { Neighbor } \\ \text { Size } \end{gathered}$ | Tenure Size |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Footprint | Num Depot | Num Supply | Num Vehicle | Num Depot | Num Supply | Num Vehicle |  |  |  |  |
| Avg Value | 15355.86 | 1.31 | 2.3 | 15.44 | 1.25 | 2.38 | 14.81 | 5576.67 | 84.03 | 676.25 | 53.83 |
| TWS | X | X | X | -2.94 | 0.12 | X | -3.00 | X | X | X | -8.22 |
| COT | X | X | X | X | X | X | X | X | X | X | X |
| AEW | X | X | X | X | X | X | X | X | X | X | X |
| DD | X | X | X | X | 0.31 | X | X | X | X | X | X |
| BoxSize | X | X | X | X | X | X | X | X | X | X | X |
| NumDds | 22349.72 | 0.25 | 0.88 | 22.00 | 0.37 | 0.78 | 21.62 | 4818.06 | 35.31 | 1267.94 | X |
| MDBN | 5225.66 | X | X | X | X | X | X | X | X | X | X |
| NumDepots | X | X | -0.43 | X | X | -0.28 | X | X | X | X | X |
| NumSplyPts | X | X | X | X | X | X | X | X | X | 120.94 | X |
| NumThpPts | X | X | X | X | X | X | X | X | X | X | X |
| Refuel | X | X | X | X | X | X | X | X | X | X | X |
| Tenure | X | X | X | X | 0.12 | X | X | X | X | X | X |
| IntSoln | X | X | X | X | X | X | X | X | X | X | X |
| CmbNeigh | X | X | X | X | X | X | X | X | X | 387.81 | X |
| ELU | -4032.03 | X | X | X | X | X | X | 9822.94 | 60.56 | X | X |
| OpWeights | X | X | X | X | X | X | X | X | X | X | X |

A review of the table indicates that there are seven design factors: crew operating tempo (COT), allow early waiting (AEW), theater operational area (BoxSize), number of theater supply points, allow refuel, tenure and initial solution construction (IntSoln) that are not significant for any response and offer potential candidates for elimination from further investigation. However, the ANOVA tables in Appendix C also indicate that there are several significant two factor interactions for eight of the eleven factors. Unfortunately, each of these two level factors is aliased with other two level factors and there is no clear means to indicate which factors are truly significant. A review of the alias structure for the significant two factor interactions indicates that three of the seven design factors AEW, BoxSize and IntSoln are not part of any of the significant two factor
interactions or their aliased two factor combinations. Therefore, this research removed these three factors and conducted a $2_{\mathrm{IV}}^{13-8}$ resolution IV semi-fold over design to garner additional insight into the significant main effects and the time window size (TWS) two factor interactions. This semi-fold over required an additional 32 runs to de-alias each of the TWS two factor interaction combinations from all other two factor interactions. The additional 32 design runs (labeled $65-96$ ) and updated design alias structure are contained in Appendix B.

Table 6-4 represents a consolidation of the ANOVA tables (Appendix C) for the updated $2_{\text {IV }}^{13-8}$ design run with respect to elements of the logistics footprint for both the feasible and near feasible solutions. The table contains the mean effect of each design factor and either a numeric or X value for each factor and response combination and the significant two factor interactions for each response.

Table 6-4 Factor Effects for LPDPTW Objectives

| Factor | Feasible Solution |  |  |  | Near Feasible Solution |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Footprint | Num Depot | Num Supply | Num Vehicle | Num Depot | Num Supply | Num Vehicle |
| Avg Value | 13843.71 | 1.25 | 2.33 | 14.17 | 1.21 | 2.35 | 13.93 |
| A - TWS | -1872.13 | X | X | -3.13 | X | X | -3.03 |
| B - COT | X | X | X | X | X | X | X |
| C - DD | X | X | X | X | X | X | X |
| D - NumDds | 19780.17 | 0.29 | 0.026 | 21.33 | 0.34 | 0.83 | 22.34 |
| E - MDBN | 6158.04 | 0.21 | X | 1.81 | X | X | 1.52 |
| F - NumDepots | X | X | X | X | X | X | X |
| G - NumSplyPts | X | X | X | X | X | X | X |
| H - NumThpPts | X | X | X | X | X | -0.25 | X |
| J - Refuel | X | X | X | X | X | X | X |
| K - Tenure | X | X | -0.25 | X | X | X | X |
| L - CmbNeigh | X | X | X | X | X | X | X |
| M - ELU | -1820.2 | X | X | X | X | X | X |
| N - OpWeights | X | X | X | X | X | X | X |
| AB | X | 0.18 | X | X | 0.22 | X | X |
| AC | X | X | X | -1.55 | X | 0.25 | -1.37 |
| AD | X | 0.16 | X | -1.69 | X | X | -1.72 |
| AE | X | -0.16 | X | X | X | X | X |
| AF | X | -0.17 | X | X | X | X | X |

It is clear that, for the LPDPTW's objective of minimizing the logistics footprint, several characteristics of the distribution network dominate the solution. The number of demands, or customers, is especially significant in determining the size of the logistics footprint. This relationship is expected since as the number of requirements increase in a given operational period, the number of vehicles, depots and supply points necessary to satisfy all associated LPDPTW constraints is likely to increase. This increase in vehicles and facilities directly results in a larger logistics footprint.

As the maximum distance between nodes (MDBN) increased in the network, so did the number of vehicles and support facilities (depots and supply points) necessary to meet demand time constraints. The time window size - demand distribution (TWS/DD) and time window size - maximum distance between nodes (TWS/MDBN) two factor interactions provide a glimpse of why increasing the distance between nodes increases the footprint size. As the time window size for pickup or delivery decreases from 360 minutes to 60 minutes, the demand distribution and distance between nodes becomes critical. If demands are distributed throughout the day, a vehicle has the opportunity to make a visit to a supply point and then visit multiple demand points despite the tight customer time window. However, if the demands are distributed within a tight time period, there is insufficient travel time available to visit multiple demand points in the same route. This results in adding additional vehicles to satisfy the customer time window constraints. It might be interesting to note that after observing the ATS process for the design problems, it became apparent that each design problem began with a more than adequate fleet size. In terms of the associated problem sizes (pickup and delivery
requirements), this assigned fleet size results in a problem with an unlimited fleet capacity. Placing a restriction on the number of available vehicles is likely to impact ATS and LPDPTW performance. For example, during construction of the insert initial solution, a new vehicle route is created once the current route is not able to support inserting a new pickup and delivery pair. The addition of vehicles continues until all requirement pairs are assigned by the heuristic. Limiting the number of available vehicles reduces the ability of the insert heuristic to quickly find valid insert points for all requirement pairs.

The time window size and crew operating tempo (TWS/COT) two factor interaction is significant in determining the required number of feasible depots. As the size of the pickup and delivery time window decreases, the available crew operating time becomes critical. Logically, reducing the crew or vehicle available time reduces the allowable route length and forces adding more depots closer to the pickup or delivery locations to satisfy LPDPTW constraints.

The option to allow vehicle refueling is insignificant for all responses. This result at first seems counter-intuitive since allowing refueling increases vehicle travel distance. However, it appears that many of the problem instances were constrained by customer time window requirements rather than allowed vehicle route travel distances. When a vehicle was able to conduct a refuel, the authorized doctrinal travel distances became the vehicle route distance binding constraint. It is possible that these doctrinal distance settings may have overshadowed advantages gained from allowing a vehicle to refuel.

Future research should include examining the impact or influence of eliminating these doctrinal distance requirements when vehicles are allowed to refuel.

It is interesting to note that the actual number of depots, supply points and theater through-points did not significantly influence the problem's logistics footprint objective. This outcome may stem from how the problems were initially constructed in Section 6.3. The associated cost differential between each element in the set of depots and supply points may have been too large. Since no distribution solution required more than three supply points, the set of available supply points may have possessed too much throughput and storage capability. Reducing the associated capability of each supply point or dramatically increasing the demand requirements will likely force the ATS process to select more supply points to achieve the LPDPTW constraints. Since no final solution required more than two depots, the associated MOG constraint and depot costs differential might have been too high for the set of depots. Lowering the MOG or MOG to depot cost ratio will likely force the ATS process to select more depots. Exploring the solution effects of adjusting the associated parameters of the depots, supply points and theater throughput points represent excellent future research opportunities.

The majority of the ATS factor elements had no apparent influence on the explored objective responses. The type of tabu memory structure or tenure does not provide a statistical difference in ATS objective value performance. This observation may stem from the ATS process utilizing an adaptive memory structure versus a memory structure with a fixed tenure size. The ATS process adjusted the tenure length based on observed conditions and appears to have worked well at preventing cycling in the search
process. On the other hand, the solution memory structure's objectives were to force the ATS algorithm to visit a larger section of the solution space and prevent cycling by preventing re-visits to a previously visited solution for a set number of iterations. Anecdotally, it appears to have forced the ATS process to visit more conjugacy classes, (1c) but it did not statistically improve the objective's response. The solution memory structure's parameter tuning may have been responsible for its performance. Experimenting with a different tuning combination may provide better results. The use of a combined neighborhood scheme statistically performs as well as the targeted neighborhood scheme for all responses. The combined neighborhood scheme does provide a direct advantage over the targeted scheme based on the number of tabu search parameters that the ATS process needs to track. However, this decreased bookkeeping requirement is not significant enough to impact the objective responses. Finally, the objective penalty weights do not provide a statistical difference in the objective responses. This outcome may have resulted from utilizing the same continuous set of penalty weights, between $1 / 2$ and 2 , for each penalty constraint.

The utilization of the elite list represents the only ATS element that provides a statistically significant impact on the LPDPTW objectives. There appears to be a clear advantage in the ability of ATS to reduce the logistics footprint by allowing the ATS process to conduct a restart from the best found feasible and near feasible solutions. This result seems logical since the first phase of the ATS search process is utilizing the select first improving move approach in selecting the next neighborhood move to implement. The elite list restart with an evaluation of the entire neighborhood allows the ATS process
to revisit a previously found solution but with a different set of tabu search parameters. These conditions appear to allow ATS to move in a different exploration direction to a region of the solution space with better solutions.

Table 6-5 represents a consolidation of the ANOVA tables (Appendix C) for the updated $2_{\mathrm{IV}}^{13-8}$ design run with respect to performance measures of the ATS process. The problem's number of demands significantly impacts many of the ATS performance measures. The number of ATS iterations, number of visited conjugacy classes and average neighborhood size increases as the number of demands in the problem increase. As the number of demands increase, the overall size of the problem and the resulting solution space, $S_{n}$, increases. This larger problem size provides a greater opportunity to improve upon the initial solution which results in a larger number of required iterations.

Table 6-5 Factor Effects for Tabu Search Parameters

| Factor | Iterations | Visited CC | Neighbor <br> Size | Tenure <br> Size |
| :--- | :---: | :---: | :---: | :---: |
| Avg Value | 7317.83 | 89.44 | 672.68 | 54.65 |
| A - TWS | X | X | X | -8.63 |
| B - COT | X | -27.83 | X | X |
| C - DD | X | X | X | 6.21 |
| D - NumDds | 4632.01 | 41.25 | 1354.62 | X |
| E - MDBN | X | X | X | X |
| F - NumDepots | X | X | 87.59 | X |
| G - NumSplyPts | X | X | 133.27 | -5.63 |
| H - NumThpPts | X | X | X | X |
| J - Refuel | X | X | X | X |
| K - Tenure | X | X | X | X |
| L - CmbNeigh | X | X | 425.61 | X |
| M - ELU | 8020.39 | 49.45 | X | X |
| N - OpWeights | 1314.97 | 25.04 | X | 5.29 |
| AB | X | X | X | X |
| AC | X | X | X | -5.82 |
| AD | X | X | X | X |
| AE | X | 25.38 | 47.16 | X |
| AF | X | X | X | -5.67 |

The larger solution space possesses a correspondingly larger symmetric group size which provides a larger number of potential conjugacy classes to visit during the search process. The increased neighborhood size is another direct result of increasing the problem size. The increased number of demands provides significantly more potential swap and neighborhood moves for examination.

The average neighborhood size of the ATS process is significantly affected by three of the LPDPTW problem characteristics and only one tabu search factor. Three of the LPDPTW characteristics, number of demands, number of depots, and number of supply points, are directly related to the size of the LPDPTW problem and the corresponding solution space. Increasing the number of these three entities increases the number of available swap and insert moves. Table 6-5 also indicates that the time window size and maximum distance between nodes (TWS/MDBN) two factor interaction significantly affects the average neighborhood size. This increase in neighborhood size is the result of increasing the time window size and the distance between nodes. This interaction provides a larger number of potential solutions as the ATS conducts its search process. The combined neighborhood size is the last factor that significantly affects the average neighborhood size. This increase in size is logical since the combined neighborhood represents a collection of the various targeted neighborhoods. However, this increase in neighborhood size also requires a longer evaluation of all available moves as compared to the targeted neighborhoods without a significant improvement in the logistics footprint.

The ATS factors of elite list and objective weights significantly affect the number of ATS iterations and number of visited conjugacy classes. The increased number of iterations resulting from utilizing an elite list is logical since implementing an elite list scheme results in several restarts during the ATS process. Each of these restarts continues until the ATS surpasses the iterations since good solution threshold, which increases the total number of ATS iterations. However, as demonstrated previously, this increased number of iterations provides a statistically significant better logistics footprint. Implementing the objective function weighting scheme provides a strategic oscillation affect to the ATS process. It appears that ATS implementation of this strategic oscillation scheme provides a radical oscillation along the search path that requires the ATS process to implement more iterations without a statistical improvement in the logistic footprint. This oscillation appears to also force the ATS search process into new conjugacy classes along its search path.

The use of a greedy or insertion based initial solution do not statistically impact any of the responses studied in this research. This outcome appears to coincide with the accepted belief that tabu search is able to overcome any provided start point. In terms of the ATS process, it appears that the phase one implementation of selecting the first improving solution from a neighborhood allowed the search process to quickly move across the solution space much like a steepest ascent/descent approach to find a feasible solution. Despite the comparatively poor start point of the greedy initial solution, ATS was able to traverse the solution space quickly and thus overcome the advantage provided by a better start with the insert heuristic. It was apparent from observing data output
during the solution process that ATS was able to quickly find an improving move during the first phase of the operation. With the greedy initial solution, ATS implemented multiple iterations in rapid succession traversing poor but improving solutions until it found a feasible region in the solution space. As the objective function value improves, ATS is required to evaluate more potential moves in the neighborhood scheme before finding the first improving move. Starting with an insertion based initial solution quickly puts the ATS process in a feasible region where it is required to evaluate more moves in the neighborhood. It is at this point that the ATS with a greedy initial solution appears to catch and match the performance achieved by the ATS with an initial insert solution.

As discussed in Section 6.1, once a controlled experiment is implemented over a set of evaluated parameter levels, the choice of the process settings should be clear. The ATS process should be implemented utilizing an elite list search scheme. This increases the computational time for the process but does provide a statistically significant improvement in the objective function value. The tabu tenure scheme should be set to single. Both tenure schemes appear robust enough for the LPDPTW but the single tenure offers a lower computational requirement. Finally, ATS can implement either the combined or targeted neighborhood scheme with or without objective penalty weights since they all appear robust enough to find acceptable solutions to the problem.

### 6.5.2 Comparing ATS Solutions to Known LPDPTW Optimal Solutions

The following section measures the quality of the ATS solution by examining a number of smaller LPDPTW problem instances. These problems were small enough to allow for enumeration of all feasible solutions or for practical implementation in the

LINGO solver. All problems in this test set contained three depots, three supply points, 10 vehicles and between 5 to 15 demands. The primary performance measure is based on the ATS solution's percentage from the respective optimal solution or lower bound:
$\%$ dist $_{i}=\frac{\left(\text { actualvalue }_{i}-\text { solutionvalue }_{i}\right)}{\text { solutiondvalue }_{i}}$,
where $j \in\{$ Depots, Supplypoints, Numbervehicles, Footprint\}
Twenty-five of the problems in the test section were small enough to determine the optimal solution as described in Section 6.4. Appendix E provides a summary of each test problem and its \%dist from the optimal solution as calculated by Equation (6.16). The results show that ATS finds the optimal solution for 22 ( $88 \%$ ) of the 25 small test problems. Of course, ATS is a metaheuristic with no guarantees of finding the optimal solution only excellent solutions. Table 6.6 summarizes the results for the three problems where ATS failed to find an optimal solution. The results show that ATS still

Table 6-6 Summary of Sub-optimal ATS Solutions

| Test <br> Problem | Depots |  | Supply |  | Vehicles |  | FootPrint |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Optimal | ATS | Optimal | ATS | Optimal | ATS | Optimal | ATS | \%dist |
| T104 | 2 | 2 | 2 | 2 | 2 | 2 | 3030 | 3105 | 2.48 |
| T105 | 2 | 2 | 2 | 2 | 2 | 2 | 2961 | 2996 | 1.18 |
| T122 | 2 | 2 | 2 | 2 | 2 | 2 | 3273 | 3328 | 1.68 |

finds excellent solutions even when it did not find the optimal solution. ATS found both the optimal number and correct set of depots, supply points and vehicles in all problems. The ATS average $\% d i s t$ from the optimal objective function value (logistics footprint) is $1.78 \%$ for the three problems in Table 6-6. The solution difference in all three cases was in the vehicle routing between demand locations. The largest solution discrepancy
(2.48\%) occurred in T104, where ATS found the correct routing sequence only in reverse order.

### 6.5.3 Comparing ATS Solutions to Lower Bounds for the LPDPTW

This section measures the quality of the ATS solutions by examining the number of depots, supply points, vehicles and logistics footprint as measured by the objective function value for the best feasible ATS solution with respect to the lower bound developed in Section 6.4. Equation (6-16) serves as the primary performance measure for evaluating the differences between the ATS solution and the lower bound.

The first step in the evaluation compares results of the lower bound method developed in Section 6.4 against known optimal solutions. Table $6-7$ provides the average and standard deviation for \%dist of the lower bound with respect to the optimal

Table 6-7 Lower Bound Comparison to Known Optimal

|  | Logistics | LB Selected Number of |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Footprint | Depots | Suppliers | Vehicles |
| Ave \%dist | -5.33 | -70.83 | -75.00 | -8.33 |
| SD \%dist | 8.81 | 55.00 | 53.16 | 28.23 |

solutions generated in Section 6.5.3. As Table 6-7 indicates, the lower bound method generates an average objective function value (logistics footprint) 5.33 percent lower than the average optimal objective function value with a standard deviation of 8.81 percent The 5.33 percent includes the three problems (T104, T105, and T122) for which ATS was not able to find the optimal objective function value. The lower bound objective function value was also poor for these three problems. Removing these three problems from the evaluation reduces the lower bound's average logistics footprint \%dist from
5.33 percent to -2.61 percent. The lower bound methodology performed extremely poorly with respect to the number of required depots and suppliers. The main reason is the low number of depots and suppliers in the problem. A review of the data indicates that in most cases (23 of 25), the lower bound method selected only one depot or supplier. The depots and suppliers in the 25 test problems were not constrained (MOG, throughput, etc.) enough to require the lower bound methodology to select more than one depot or supplier. However, the optimal solution opens more than one of these nodes in 16 of the 25 test problems to reduce the travel distance and vehicle variable cost. These insights into the lower bound methodology led to the construction of 25 larger test problems to evaluate ATS performance.

The test set includes 25 problem instances with either 25 or 50 demand locations. Appendix E summarizes these test problems and ATS \%dist from the determined lower bound. Table 6-8 provides the average and standard deviation for

## Table 6-8 ATS Comparison to Lower Bound Solutions

|  | Logistics | ATS Selected Number of |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Footprint | Depots | Suppliers | Vehicles |
| Ave \%dist | 2.63 | 0.00 | 20.00 | 7.50 |
| SD \%dist | 2.34 | 0.00 | 42.16 | 16.87 |

\%dist with respect to the lower bound for the 25 problems. The average ATS objective function value (logistics footprint) $\%$ dist of 2.63 percent is in line with the above lower bound evaluation. ATS matched the lower bound solution in three of the test cases. These three instances represent optimal solutions, since the ATS solutions are feasible, and the lower bound methodology provides the lowest selection of depots, suppliers,
vehicles and logistics footprint. The lower bound method does not guarantee that the bound is a feasible solution. ATS may be finding the optimal solution although its solution does not match the lower bound. ATS proved that it is capable of finding good solutions for the LPDPTW test problems in this section and Section 6.5.2. In addition, ATS demonstrated in these two sections that it found the optimal solution for 50 percent of the LPDPTW test problems.

### 6.6 Conclusion

This chapter discusses conducting an experimental design to address how various factors of the LPDPTW and ATS affected ATS performance and LPDPTW solution parameters. The design of experiment clearly demonstrated the impact of the number of demands in the problem on both ATS and solution performance. It also provides evidence of the advantages of implementing the ATS with its elite list scheme. The chapter also compared the quality of the ATS solutions by examining the optimal values for a set of small problems and the generated lower bounds for a set of larger problems. The next chapter addresses the final objective question from Section 6.1 of how does the ATS process perform on a large LPDPTW and TDP problem outside the design space.

## VII Application of the ATS

This chapter provides an example application of the ATS algorithm to larger versions of the LPDPTW and several theater distribution based problems (TDP). The intent of this chapter is to explore ATS performance on LPDPTW problem sizes outside the design of experiment region explored in Chapter VI and on several TDP based scenarios.

### 7.1 LPDPTW Problem Instances

The following section explores the ATS performance on two large LPDPTW. The first LPDPTW instance consists of 90 individual demand locations or customers and the second problem has 180 demand locations. Both problem instances contain a set of four potential depot locations and eight potential supply locations. These problems respectively contain 9 and 18 times as many customer locations as both the largest problem in the experimental design of Chapter VI and the largest theater distribution problem examined by Crino (Crino et al., 2004). The LPDPTW explored in this section represents a fully connected network graph. This results in an underlying distribution network consisting of, respectively, 10,404 and 36,864 connecting arcs. These connected networks are 26 and 92 times the size of the connected network Crino created for the theater distribution problem (Crino et al. 2004).

### 7.1.1 LPDPTW - 1 Details

LPDPTW-1 represents a fully connected bi-directional graph with 102 nodes and 10,404 bi-directional arcs. The problem includes 90 customer (demand) locations, four supporting vehicle depot locations and eight supply point locations randomly distributed
inside a 100 km by 100 km box. In general terms, each customer possesses a multicommodity demand and both a time definite demand requirement and a required time window for their delivery. Each supply point in the distribution network possesses its own time window of operation and both a storage and throughput capability. The addition of the four vehicle depots, with associated MOG requirements completes the set of nodes. The specific requirements and capabilities of these 102 nodes are included in Appendix D. The problem also contains a fixed size vehicle fleet ( 26 vehicles) consisting of two generic vehicle model types (Veh-1, Veh-4). The specific capabilities of these vehicles are included in Appendix D.

### 7.1.2 ATS Results for LPDPTW - 1

ATS utilizing both penalty weights and the elite list operating on a COMPAQ nc6000 with an Intel® Pentium® 41600 MHz processor and 512 MB of RAM is used to solve the problem. ATS performed a total of 25,770 iterations before reaching termination conditions. The best feasible solution displayed in Table 7.1 was obtained at

Table 7-1 LPDPTW-1 Objective Values and Footprint

| Criterion | Feasible Solution |  | Near Feasible Solution |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Value | Node / Vehicle | Value | Node |
| Total Depot Fixed Cost | 10 | (0) | 10 | (0) |
| Total Depot Variable Cost | 140 |  | 140 |  |
| Total Supply Point Fixed Cost | 2,200 | (0)(1)(6) | 2,200 | (0)(1)(6) |
| Total Supply Point Variable Cost | 1,080 |  | 1,080 |  |
| Total Vehicle Fixed Cost | 1,130 | (Veh 1-1)(Veh 4-13) | 1,130 | (Veh 1-1)(Veh 4-13) |
| Total Vehicle Variable Cost | 18,967 |  | 19,127 |  |
| TDD Violation | 0 |  | 0 |  |
| Demand Shortfall Penalty | 0 |  | 0 |  |
| Route Length Violation Penalty | 0 |  | 0 |  |
| Time Window Violation Penalty | 0 |  | 220 |  |
| Storage Violation Penalty | 0 |  | 0 |  |
| MOG Violation Penalty | 0 |  | 0 |  |
| Total Objective Value | 23,527 |  | 23,907 |  |

iteration 12,601. ATS produced a near feasible initial solution with a value of 831,972 , utilizing three depots, eight supply points and 23 vehicles. This poor solution was the result of trying to draw more supplies than available from one supply point. This led to a delivery failure for five separate demands resulting in large penalties. Figure 7.1 shows the ATS progress with respect to minimizing the logistics footprint (objective function value). ATS discovered a feasible solution, with a logistics footprint value of 53,506 at iteration 564 in 3 minutes and 34 seconds.

Figure 7.1 clearly demonstrates that ATS continues to oscillate between feasible and near feasible solutions as it seeks improving solutions to the problem. By iteration


Figure 7-1 LPDPTW-1 ATS Objective Value Progress

772, ATS has made a $36 \%$ improvement on the first feasible solution discovered at iteration 564, and by iteration 1,148, ATS has improved upon the first feasible solution by $87.3 \%$ and is within $0.3 \%$ of the best solution found at iteration 12,601 .

At iteration 1,391, the ATS process met conditions to begin phase II or elite list implementation. The ATS neighborhood manager switched from selecting the first improving move in the selected neighborhood scheme to selecting the best move after evaluating all moves in the neighborhood. At this point, ATS appears to stall in terms of improving upon the best solution found at iteration 1,148 until it initiates the $5^{\text {th }}$ restart from the near feasible elite list. Figure 7.2 illustrates the ATS progress in terms of minimizing the objective function value (logistics footprint) until finding the best solution for the problem at iteration 12,601, about four hours into the solution process.


Figure 7-2 LPDPTW-1 ATS Progress (Elite Restart)

Figure 7.2 visually demonstrates how the ATS process quickly moves across the poor solution quality regions of the solution space to find areas of good solutions. It is in these regions (iteration 11,000 to 12,500 ) that ATS tends to oscillates between feasible and near feasible solutions seeking improving solutions. ATS conducts five additional elite list restarts after iteration 12,601 but is not able to improve upon the solution. However, over the course of next 12,000 iterations, the ATS process is able to work itself back to this objective function value several times. ATS achieves termination conditions in just over seven hours.

### 7.1.3 LPDPTW - 2 Details

LPDPTW -2 is very similar in construction to LPDPTW - 1. LPDPTW-2 represents a fully connected bi-directional graph consisting of 192 nodes and 36,864 bidirectional arcs. This problem has 180 customer nodes which is twice the number of customer locations of LPDPTW-1 and contains over 3.5 times the number of connecting arcs. The 180 customer locations, four supporting vehicle depot locations and eight supply point locations are randomly distributed inside a 100 km by 100 km box. The specific information concerning the requirements and capabilities of all 192 nodes are contained in Appendix D. The problem also contains a fixed size vehicle fleet (36 vehicles) with two generic vehicle model types (Veh-1, Veh-4). The specific capabilities of these vehicles are presented in Appendix D.

### 7.1.4 ATS Results for LPDPTW - 2

ATS utilized the same settings as described in Section 7.1.2. Table 7.2 provides the best objective function values found by ATS and the underlying distribution network
for the problem. ATS' performance in solving LPDPTW-2 was similar to its performance in solving the smaller version of the problem.

Table 7-2 LPDPTW-2 Objective Values

| Criterion | Feasible Solution |  | Near Feasible Solution |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Value | Node / Vehicle | Value | Node |
| Total Depot Fixed Cost | 2,010 | $(0)(1)$ | 2,010 | $(0)(1)$ |
| Total Depot Variable Cost | 3,510 |  | 3,510 |  |
| Total Supply Point Fixed Cost | 300 | $\mathbf{( 0 ) ( 1 ) ( 2 )}$ | 300 | $(0)(1)(2)$ |
| Total Supply Point Variable Cost | 1,188 |  | 1,188 |  |
| Total Vehicle Fixed Cost | 730 | $($ Veh 1-5)(Veh 4-23) | 730 | (Veh 1-5)(Veh 4-23) |
| Total Vehicle Variable Cost | 16,495 |  | 16,575 |  |
| TDD Violation | 0 |  | 0 |  |
| Demand Shortfall Penalty | 0 |  | 0 |  |
| Route Length Violation Penalty | 0 |  | 0 |  |
| Time Window Violation Penalty | 0 |  | 22 |  |
| Storage Violation Penalty | 0 |  | 0 |  |
| MOG Violation Penalty | 0 |  | 04,335 |  |
| Total Objective Value |  |  |  |  |
| 24,233 |  |  |  |  |

ATS produced a near feasible initial solution with a value of 161,652 , utilizing all four depots, eight supply points and 36 vehicles. The initial solution possessed a minor TDD violation and significant time window violations. It is interesting to note that the problem's additional 10 vehicles in the fleet, as compared to LPDPTW-1, allowed for construction of a better initial solution. ATS found a feasible solution by iteration 296, approximately 17 minutes after starting the search process and was able to find the best objective function value at iteration 1,756.

### 7.1.5 OPLOG Planner Results for the LPDPTW

As a means for comparison, the algorithms from OPLOG planner were utilized with the ATS objective function to determine the distribution network for the two LPDPTW problems. As noted in Chapter III, OPLOG planner represents the planning tool utilized by many military logistics planners. The OPLOG planner and ATS results

Table 7-3 OPLOG and ATS Objective Comparison

| Criterion | LPDPTW - 1 |  | LPDPTW - 2 |  |
| :--- | :---: | :---: | :---: | :---: |
|  | ATS | OPLOG | ATS | OPLOG |
| Total Depot Fixed Cost | 10 | 10 | 2,010 | 10 |
| Total Depot Variable Cost | 140 | 140 | 3,510 | 3,510 |
| Total Supply Point Fixed Cost | 2,200 | 2,200 | 300 | 300 |
| Total Supply Point Variable Cost | 1,080 | 1,080 | 1,188 | 1,188 |
| Total Vehicle Fixed Cost | 1,130 | 13,230 | 730 | 1,030 |
| Total Vehicle Variable Cost | 18,967 | 84,565 | 16,495 | 131,135 |
| TDD Violation | 0 | $?$ | 0 | $?$ |
| Demand Shortfall Penalty | 0 | $?$ | 0 | $?$ |
| Route Length Violation Penalty | 0 | 0 | 0 | 0 |
| Time Window Violation Penalty | 0 | $?$ | 0 | $?$ |
| Storage Violation Penalty | 0 | 0 | 0 | 0 |
| MOG ViolationPenalty | 0 | 0 | 0 | 0 |
| Total Objective Value |  |  |  |  |

are given in Table 7.3. The shaded regions in the OPLOG columns represent network information provided by ATS. OPLOG planner's algorithms are based on an established hierarchal distribution system and are not designed to select either the depots or supply points necessary for the network. OPLOG planner's set of algorithms requires this information as a necessary start condition to determine the required number of vehicle assets for the mission. Therefore, the depots, supply points and assignment of demands to supply points were pre-selected by ATS for OPLOG planner. In both problems, ATS outperformed OPLOG planner both in terms of the required number of vehicles and their associated operating costs. OPLOG planner's solution for LPDPTW-1 actually required more vehicles than were initially available in the fleet.

OPLOG planner's set of algorithms are not designed to construct the routing and scheduling scheme necessary to ensure time definite delivery of the requirements. The set of demands for OPLOG planner were scheduled based on chronological order to
determine vehicle operating costs and route lengths. However, comparisons for any of the time window and TDD requirements are not realistic. OPLOG planner's scheduling and routing shortcomings represent a major issue for planners in developing a distribution network that ensures time definite delivery. ATS is able to construct this critical routing and scheduling component of the distribution system, and as indicated in Table 7.3 ATS developed a routing and scheduling plan to achieve all time window and TDD requirements.

### 7.2 TDP Problem Instances

ATS was applied to a set of four different theater distribution scenarios. These scenarios were based on six unit of action scenario vignettes presented in TRADOC PAM 525-3-90 (July 2002) and represent operationally realistic problem instances for theater distribution support of the future combat system equipped unit of action. TRADOC originally designed these vignettes to test new tactical concepts and organizational design principles, and they provide a good platform to examine the ATS methodology. The intent is to investigate the efficacy of the ATS methodology and demonstrate its ability to capture the feel of realistic problems.

The following elements capture the key problem characteristics of each scenario: the number of vehicles, number of depots, number of supply points, number of demand locations and total number of multi-commodity demands. Table 7.4 displays the four scenarios and their key elements. The in/out for depots and supply points refers to the number of nodes located inside and outside the area of operation. The vehicle column refers to the total number of available ground or air transportation assets and may include

Table 7-4 Scenario Problem Parameters

| Scenario | Depots <br> (In/Out) | Supply <br> Points <br> (In/Out) | Demand <br> Locations | Number of <br> Demands | Vehicles <br> (Ground/Air) | Scenario Emphasis |
| :--- | :---: | :---: | :---: | :---: | :---: | :--- |
| TDP 1 | $4 / 0$ | $8 / 0$ | 20 | 100 | $20 / 60$ | Army vs AF Air Spt |
| TDP 2A | $2 / 2$ | $6 / 2$ | 20 | 100 | $50 / 0$ | Traditional Ground Spt |
| TDP 2B | $2 / 2$ | $6 / 2$ | 20 | 100 | $50 / 40$ | Limited Airfields |
| TDP 2C | $2 / 2$ | $6 / 2$ | 20 | 100 | $50 / 40$ | Aerial Delivery |
| TDP 3 | $3 / 1$ | $3 / 1$ | 8 | 200 | $20 / 60$ | Remote Location Spt |
| TDP 4 | $0 / 7$ | $2 / 2$ | 7 | 300 | $0 / 150$ | UA Attack Support |

several vehicle model types. For example, scenario TDP 1 includes two vehicle types based on the M871 and palletized loads system and three aircraft types based on the Army CH47 and Air Force C-130 and C-17. Scenario TDP 2 contains three versions (A, $B, C)$ that represent the same underlying problem requirement but each of the three entries represents different distribution concepts. The intent of TDP 2(A, B, C) is to demonstrate ATS' ability to conduct what-if drills and capture changes in the underlying distribution system.

### 7.2.1 An Example of Theater Distribution

TDP 2(A, B, C) provides an excellent problem set to illustrate the ATS methodology. TDP $2(\mathrm{~A}, \mathrm{~B}, \mathrm{C})$ actually represents three individual problems that differ only in the distribution policy or availability of additional distribution vehicles.

### 7.2.1.1 Scenario TDP 2 Details

The three TDP 2 problems represent a mid-intensity small scale contingency type operation conducted in a highly compartmentalized area of operation. The southern regions of Afghanistan or the areas of Trans-Caucasus provide an excellent representation of the intended difficult terrain in the area of operation. Logistics planners
are required to determine the support structure and routing requirements necessary to sustain the units operating in the region.

Figure 7.3 provides a graphical representation of TDP 2. The problem's area of operation is approximately 30,000 square miles in size and currently contains one notional Unit of Employment (UE) with three Unit of Actions (UA) in the field. The area


Figure 7-3 TDP 2 Graphical Representation
of operation is completely surrounded by the theater of operation and contains a single no fly zone. The presence of the no fly zone and the region's difficult terrain represent two reasons why the ATS methodology utilizes associated ground and air distance matrixes versus relying on straight line distances. The problem contains 20 UA demand locations. However, C-130 type aircraft are capable of landing at only $50 \%$ of these locations. The problem also contains four potential depot locations and eight potential supply storage
and distribution points. The entire distribution network is represented by a fully connected ( 32 nodes) ground and air graph containing 2,048 arcs. This problem contains three times the number of locations and approximately five times as many arcs as Crino's (Crino et al., 2004) theater distribution network. Each supply point and demand location possesses a time window of operation. This $100 \%$ time window requirement places an additional burden on the routing and scheduling aspect of the problem. The problem has 100 multi-commodity (six, commodities) configured load demands every 24 hours uniformly distributed across the 20 demand locations. Table 7.5 provides an example set of 100 requirements (demands) for this problem.

Table 7-5 Example TDP 2(A, B, C) Demand Requirements

| Dmd Pnt ID Ltr | Demand |  | Delivery Window |  | TDD Required | Location Coord |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Type } \\ \text { (Class) } \end{gathered}$ | Quantity (STONS) | $\begin{gathered} \hline \text { Open } \\ \text { (MINS) } \end{gathered}$ | $\begin{gathered} \text { Close } \\ \text { (MINS) } \end{gathered}$ |  | (x) | (y) |
| 0 | $\begin{gathered} \hline \hline \text { I } \\ \text { II } \\ \text { V } \\ \text { IX } \\ \hline \end{gathered}$ | $\begin{gathered} 4.8 \\ 3.5 \\ 10 \\ 2 \end{gathered}$ | 412 | 772 | true | 85 | 108 |
| 1 | $\begin{gathered} \hline \text { I } \\ \text { II } \\ \text { III(P) } \\ \text { V } \\ \text { IX } \end{gathered}$ | $\begin{gathered} \hline 4.8 \\ 3.5 \\ 0.5 \\ 8 \\ 2 \\ \hline \end{gathered}$ | 459 | 819 | true | 37 | 32 |
| 2 | $\begin{gathered} \hline \text { I } \\ \text { II } \\ \text { III(P) } \\ \text { V } \\ \text { IX } \end{gathered}$ | $\begin{gathered} 4 \\ 2.5 \\ 1.5 \\ 11.5 \\ 2.5 \end{gathered}$ | 481 | 841 | true | 43 | 147 |
| 3 | $\begin{gathered} \hline \text { II } \\ \text { III(P) } \\ \text { IV } \\ \text { V } \\ \text { IX } \end{gathered}$ | $\begin{gathered} \hline 2.5 \\ 2.5 \\ 4 \\ 10 \\ 3 \end{gathered}$ | 488 | 848 | true | 8 | 30 |

### 7.2.1.2 ATS Results for TDP 2

The TDP 2 problem was solved four times under the following conditions:

1. OPLOG planner algorithms with traditional hierarchal support relationships
2. TDP 2A - ATS method with only ground distribution assets available
3. TDP 2B - ATS method with a mix of air/ground assets available
4. TDP 2C - ATS method with air/ground assets and the Army aerial delivery system available

These four conditions allow for a direct comparison between the distribution system created by the current solution method (OPLOG planner) and the resulting ATS changes in the distribution system from changing the distribution concept or the addition of new equipment. Table 7.6 contains the ATS objective function values for TDP 2 under the four evaluated conditions. Condition 4, in Table 7.6 contains two entries. The first value

Table 7-6 TDP 2 Objective Function Values

| Criterion | TDP 2 Conditions |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | $4 *$ |
| Total Depot Fixed Cost | 1,000 | 1,000 | 1,000 | $1000 / 0$ |
| Total Depot Variable Cost | 250 | 230 | 110 | $290 / 0$ |
| Total Supply Point Fixed Cost | 6,000 | 3,000 | 1,500 | $1000 / 0$ |
| Total Supply Point Variable Cost | 6,850 | 3,760 | 3,040 | $4,500 / 0$ |
| Total Vehicle Fixed Cost | 25 | 23 | 20 | $29 / 0$ |
| Total Vehicle Variable Cost | 13,106 | 22,942 | 9,624 | $70,353 / 0$ |
| TDD Violation | 0 | 0 | 0 | 0 |
| Demand Shortfall Penalty | 208,800 | 0 | 0 | 0 |
| Route Length Violation Penalty | 0 | 0 | 0 | 0 |
| Time Window Violation Penalty | 0 | 0 | 0 | 0 |
| Storage Violation Penalty | 290,000 | 0 | 0 | 0 |
| MOG ViolationPenalty | 0 | 0 | 0 | 0 |
| Total Objective Function Value | 526,031 | 30,955 | 15,294 | $77,172 / 0$ |

represents the logistics footprint cost for the theater of operation and the second value is the logistics footprint cost for the area of operation.

Condition 1 was solved using OPLOG planner algorithms. Condition 1 represents logistics distribution under the current hierarchal support doctrine utilizing OPLOG planner algorithms to determine the distribution system. The hierarchical support concept implies that each UA possesses a dedicated supporting element and each of these supporting elements is supported by a higher supply source. The problem contained a fixed fleet of 50 Palletized Load System (PLS) type ground vehicles, with the assumption that all vehicles were available at time zero. The resulting OPLOG planner network was evaluated using the ATS objective function. The OPLOG planner distribution network opened four supply distribution points and two vehicle depots. Demand distribution required the utilization of 25 PLS type vehicles. The established network achieved all time window and time definite delivery requirements. However, as indicated in Table 7.6, the solution violated supply point constraints. The storage violation and demand shortfall penalty is the result of throughput violations at two of the four supply distribution points. These violations are an example of the planning inefficiencies associated with the current distribution methodology and OPLOG planner. OPLOG planner's inability to construct a routing and scheduling plan forces logistics planners to make manual adjustments to correct for discrepancies. Since there is a throughput violation, a logistics planner is now required to manually juggle demands by reallocating shortfalls to other distribution points. This reallocation results in a logistics footprint increase for the supply distribution points and vehicle routes that may result in violations
of other penalties. Several OPLOG planner attempts, over the course of 60 minutes, were made to select different supply point and route combinations but each result contained one or more ATS objective function violations.

Conditions $2-4$ were solved using the ATS methodology. Condition 2 (TDP 2A) utilizes the same problem characteristics as those in Condition 1 with one exception. Distribution support is changed from a hierarchal support concept to a LPDPTW support condition. The major difference is that a requestor no longer has a dedicated support element. The only requirement is the timely delivery of the requested items. ATS performed a total of 22,276 iterations. The best objective function value presented in Table 7.6 was found at iteration 1,081 in approximately 20 minutes. The ATS solution opened five supply points and the same two vehicle depots as Condition 1. Demand distribution required 23 PLS type vehicles but a much different routing scheme as compared to Condition 1. The selection of a different combination of supply points and a much longer routing scheme provides a feasible solution and a likely lower bound logistics footprint solution for a hierarchal distribution solution (Condition 1). The ATS approach dominated the manual planning method both in terms of solution feasibility and solution time.

Condition 3 (TDP 2B) utilizes the same problem characteristics as those in Condition 1 with two exceptions. The first is the LPDPTW support condition and the second exception is the inclusion of 40 C -130 type aircraft to the fleet with the assumption that all aircraft are available at time zero. These exceptions allow exploring the impact of allowing air delivery on the associated distribution network. ATS
performed a total of 24,296 iterations. The best objective function value presented in Table 7.6 was found at iteration 20,745 in approximately 461 minutes. However, ATS found a feasible objective function solution value of 15,649 (within $2.4 \%$ of best solution found) in 91 minutes and a feasible objective function solution value of 19,220 in approximately 24 minutes. The major differences in these two solutions were the choice of delivery vehicles and their routes. ATS opened four supply points and all four vehicle depots to establish the distribution network. The major difference from Conditions 1 and 2 is that ATS opened the two depots and supply points in the theater of operation. The addition of aircraft to the problem made opening these nodes feasible. The major advantage in opening these theater nodes is the direct reduction of the required logistics footprint in the area of operation. The solution provides a direct measurement of the benefits associated with adding aircraft consideration to the planning process, an item missing from the current OPLOG planning process. The distribution of supplies utilized nine aircraft assigned to the two depots in the theater of operation and 11 PLS vehicles assigned to the two depots in the area of operation. These 11 ground vehicles were still necessary since only $50 \%$ of the demand locations possessed the ability for aircraft landings. However, their associated logistics footprint cost is significantly better than the best logistics footprint of Condition 2 (TDP 2A). There is a clear benefit associated with allowing logistics support for the nine aircraft so the can reside outside the area of operation.

Condition 4 (TDP 2C) utilizes the same problem characteristics as Condition 3 (TDP 2B) with one exception. The problem now allows the use of the Army's
experimental aerial delivery system. This system is designed to deliver air dropped cargo with precision accuracy to a requested location. This additional capability now makes it possible to support all demand locations with aircraft and determine the potential effectiveness of the new delivery system in terms of a reduced logistics footprint in the area of operation. The objective function value in Table 7.6 provides a value for both the logistics footprint in the theater of operation and the area of operation. As indicated in Table 7.6, there is no required logistics footprint in the area of operation. ATS opened both depots and supply points in the theater area of operation and utilized 29 C -130 type aircraft for the distribution. This number of aircraft more than doubled the required number of aircraft of Condition 2 (TDP 2A) but the benefit lies in moving all support requirements outside the area of operation. There is still an associated logistics footprint requirement for supporting the demands but these requirements no longer concern the UE commander. For example, the vehicle variable cost for supporting the $29 \mathrm{C}-130$ s is 70,353. This variable cost is much higher than that of any other evaluated Condition and assumes that the assigned depots can support the operational sustainment requirements of the C-130s.

### 7.2.1.3 TDP 2 Concluding Thoughts

The objective of TDP 2 was to show how ATS adjusts the distribution network based on changes in delivery capabilities or distribution assets. ATS was able to reduce the logistics presence in the area of operation each time an additional capability or an asset was added to the problem. In each case, the ATS solution dominated OPLOG planner in terms of the objective function value. In terms of solution time, ATS was able
to find reasonable feasible solutions in a quicker time than the manual OPLOG planning method. ATS also constructs the routing and scheduling scheme to support the distribution concepts which is a capability missing from current planning tools.

### 7.2.2 An Additional Theater Distribution Example

TDP 4 provides an opportunity to illustrate the ATS heuristic in support of a simulation exercise. TDP 4 is based on an actual TRADOC Analysis Center (TRAC) scenario and simulation exercise to evaluate the employment capabilities of the future Unit of Employment (UE) equipped with the future combat system.

### 7.2.2.1 Scenario TDP 4 Details

TDP 4 represents a high-intensity small scale contingency type operation conducted in a highly compartmentalized area of operation. The problem consists of a deployment and sustainment phase. Logistics planners are required to determine the support structure and routing requirements necessary to deploy the force from staging bases in Turkey to Tbilisi and then sustain combat operations in the area of operation. The original simulation exercise assumed the UE had completed its deployment into the area of operation and did not construct the necessary distribution network in Turkey to deploy the force. Figure 7.4 provides a graphical representation of the theater of operation and its sub-designated area of operation.


Figure 7-4 TDP 4 Theater of Operation

The theater of operation is approximately 100,000 square miles in size and consists of two potential intermediate staging bases (ISB), two additional supply points in the area of operation and seven potential vehicle depots in Turkey to support the aircraft fleet available in the theater of operation. The first phase of the operation assumes that the UE deploys from the United States to one or both ISBs and logistics planners have 96 hours to deploy the force into Tbilisi. The ATS requirement for Phase I is to minimize the theater of operation's logistics footprint.

The second phase spans an operational period of 120 hours and covers the UE combat action from Tbilisi to Baku. Sustainment requirements during this phase of the operation are tied directly to operational timelines and consist of either a supply delivery (SRO) or supply and maintenance delivery (MSO) requirement. A maintenance delivery requirement represents a two part requirement. The first part requires delivery of


Figure 7-5 TDP 4 Sustainment Timeline
maintenance personnel and assets to a designated location at a designated time and the second part requires picking up these individuals at the end of the sustainment phase. Figure 7.5 provides a graphical representation of the six sustainment periods for each associated Unit of Action. Each sustainment period possesses a time definite delivery requirement and location to begin operations. The ATS requirement for Phase II is to minimize the area of operation's logistics footprint.

Table 7.7 provides a comparison of the ATS objective function values for Phase I and Phase II of the scenario and the simulation sustainment plan. A Phase I comparison between ATS and the simulation is not possible since the simulation started with the UE already deployed in Tbilisi. ATS was able to construct a feasible support structure to achieve the 96 hour deployment requirement. The underlying distribution network

Table 7-7 TDP 4 Objective Function Values

| Criterion | TDP 4 Phase |  | Simulation |
| :--- | :---: | :---: | :---: |
|  | I | II |  |
| Total Depot Fixed Cost | 5,000 | 1,000 | 1,000 |
| Total Depot Variable Cost | 600 | 120 | 900 |
| Total Supply Point Fixed Cost | 1,000 | 1,000 | 1,000 |
| Total Supply Point Variable Cost | 3,000 | 528 | 570 |
| Total Vehicle Fixed Cost | 3,000 | 500 | 900 |
| Total Vehicle Variable Cost | 423,400 | 87,456 | 194,040 |
| TDD Violation | 0 | 0 | 0 |
| Demand Shortfall Penalty | 0 | 0 | 0 |
| Route Length Violation Penalty | 0 | 0 | 0 |
| Time Window Violation Penalty | 0 | 0 | 0 |
| Storage Violation Penalty | 0 | 0 | 0 |
| MOG ViolationPenalty | 0 | 0 | 0 |
| Total Objective Value |  |  |  |

established five depots to support a mix of $60 \mathrm{C}-130 / \mathrm{C}-17$ type aircraft and opened Armenia ISB. This ISB was selected to provide better (lower logistics footprint) support during Phase II of the operation.

As Table 7.7 indicates, ATS was able to generate a smaller logistics footprint (objective function value) than the simulation. The ATS solution did not utilize any of the available locations inside the area of operation. The simulation deployed three

HEMTT-LHS equipped companies with a total of 90 vehicles to support the Phase II sustainment mission. ATS was able to support the sustainment mission without the use of ground vehicles. As Table 7.7 indicates, the ATS sustainment support from the Armenia ISB provides a better supply point and vehicle variable cost (smaller logistics footprint) than the simulation's support from the Tbilisi supply point. The simulation's large variable cost is directly associated with the deployment of the 90 vehicles and traveling the longer required support distances.

### 7.2.3 Other Theater Distribution Instances

TDP scenarios 1 and 3 were run using the ATS settings described in Section 7.1. Table 7.8 presents the best found ATS objective function values (logistics footprint) for TDP scenarios 1 and 3 .

Table 7-8 ATS Objective Function Values

| Criterion | TDP Scenario |  |
| :--- | :---: | :---: |
|  | 1 | 3 |
| Total Depot Fixed Cost | 4 | 950 |
| Total Depot Variable Cost | 34 | 270 |
| Total Supply Point Fixed Cost | 1,200 | 1,100 |
| Total Supply Point Variable Cost | 1,200 | 528 |
| Total Vehicle Fixed Cost | 34 | 42 |
| Total Vehicle Variable Cost | 42,570 | 121,125 |
| TDD Violation | 0 | 0 |
| Demand Shortfall Penalty | 0 | 0 |
| Route Length Violation Penalty | 0 | 0 |
| Time Window Violation Penalty | 0 | 0 |
| Storage Violation Penalty | 0 | 0 |
| MOG ViolationPenalty | 0 | 0 |
| Total Objective Value | 45,042 | 124,015 |

### 7.2.3.1 Scenario TDP 1

TDP 1 has 100 multi-commodity (five commodities) demands in a 24 hour period uniformly distributed across 20 demand locations. The distribution fleet consists of 20 M871 and PLS type vehicles and 30 each CH-47 and C-130 type aircraft, which were assumed available at time zero. The area of operation is represented by a $40,000 \mathrm{~km}^{2}$ box containing four potential depots and eight potential supply points. All depots and supply points were capable of supporting both ground and air vehicles. The scenario's objective was to determine the impact on the distribution network of utilizing a mix of available ground vehicles and aircraft.

ATS executed a total of 20,770 iterations prior to achieving termination conditions. The best objective function solution, indicated in Table 7.8, was found at iteration 15,042 in approximately 166 minutes. However, ATS did find an objective function solution $(45,982)$ within the first 20 minutes that was within $2 \%$ of the best objective function solution found. The distribution network had all four depots and three of the eight supply points. ATS selected all available ground vehicles and five of the available C-130s to satisfy the demands. The selection of the ground vehicles over aircraft is logical since all distribution nodes were located inside the area of operation and most demand time windows were achievable with the speed of the ground vehicles. However, for ATS to meet all time window requirements required opening all depots. ATS did not select any CH-47 type vehicles since the C-130 dominates the CH-47 in all problem characteristics (speed, load size, distance) with the exception of loading and unloading time.

### 7.2.3.2 Scenario TDP 3

TDP 3 has 200 multi-commodity demands in a 24 hour period randomly distributed across 8 demand locations. These demands are scattered throughout the 24 hour period for several of the demand locations. This forces multiple visits to the demand locations at different times of the day. The distribution fleet consists of 20 PLS type vehicles, $15 \mathrm{CH}-47$ and $45 \mathrm{C}-130$ type aircraft, which were assumed available at time zero. The area of operation is represented by a $90,000 \mathrm{~km}^{2}$ box containing four potential depots and four potential supply points. All depots and supply points were capable of supporting both ground and air vehicles.

ATS executed a total of 40,908 iterations prior to achieving termination conditions. The best objective function solution, indicated in Table 7.8, was found at iteration 4,203 in approximately 93 minutes. However, ATS did find an objective function solution $(125,065)$ within the first 24 minutes that was within $<1 \%$ of the best objective function solution found. The distribution network used all four depots and three of the eight supply points. ATS selected all available ground vehicles and 31 of the available C-130s to satisfy the demands. Unlike TDP 1, the size of the operational area prevented all but one ground vehicle from making more than one delivery to a location. The travel time and tight time windows prevented multiple round trips. This forced ATS to select more C-130 aircraft to satisfy time window constraints. The higher C-130 operating cost dramatically increases the required supporting infrastructure in the area of operation. There were sufficient C-130s in the fleet to keep ATS from selecting the CH47 for any missions.

### 7.3 Conclusion

This chapter discussed the application of ATS to five problem (2-LPDPTW, 3TDP) instances. ATS offers good solutions in a reasonable amount of time and provides a level of detail that allows analysts to evaluate the performance of what-if scenarios and their solutions. In applicable cases, ATS dominates the current planning algorithms in all aspects.

The following chapter provides concluding remarks and describes key contributions of this research and suggests potential future research.

## VIII Concluding Remarks

This chapter outlines the major contributions of this research and discusses future avenues of research for the LPDPTW and outlines potential improvements to the ATS methodology.

### 8.1 Major Contributions

This section discusses the contributions produced by this research. The discussion starts with contributions to the operational research community and concludes with a discussion of potential military contributions.

### 8.1.1 Operations Research Contributions

This research effectively merges two routing and scheduling problem classes and presents the first mathematical programming formulation of the location pickup and delivery problem with time windows (LPDPTW). This formulation represents a more general case than previously presented individually for either the location routing problem (LRP) or pickup and delivery problem with time windows (PDPTW). The formulation supported development of the ATS methodology for quickly finding excellent solutions to the LPDPTW.

This research presents the first ATS metaheuristic methodology for seeking excellent solutions to the LPDPTW and theater distribution problem. The research demonstrated the effectiveness of the ATS methodology in solving the LPDPTW and larger theater distribution problems. The results for the theater distribution problems represent a significant improvement over the current methodology used in the field.

This research presents the first ever solution technique that simultaneous solves both the location of depots and the routing of vehicles for the underlining location routing problem. Traditional LRP solution methods seek to either first provide a solution to the location of the depots and then solve the corresponding routing problem (locate first, route second) or first solve the routing problem and then determine the depot locations (route first, locate second). The ATS methodology simultaneously considers both location and routing moves as it seeks to improve the overall solution cost. The ATS methodology also considers both vehicle and customer allocation to depots simultaneously with the location and routing aspects of the LRP. This allocation is generally executed as a separate phase from the location and routing in the LRP class problems.

To the best of my knowledge, the underlining pickup and delivery problem solved by the ATS methodology represents the most general PDPTW case solved in the literature. The incorporation of multiple depots, multiple commodities and nonhomogeneous vehicles to the PDPTW extends the research beyond the current solution techniques.

This research demonstrates the effectiveness of group theory as a foundation for tabu search in seeking solutions for the LPDPTW. The research shows that group theory provides an excellent solution structure for the new LPDPTW combinatorial class problem. It demonstrates the effectiveness of conjugation and template based moves for the LPDPTW and develops effective conjugacy class hash functions.

### 8.1.2 Military Contributions

The ATS methodology developed in this research provides an analytic foundation for determining the distribution network and necessary vehicle routing and scheduling requirements to ensure the time definite delivery of demands, as opposed to the current spreadsheet methods used in the field. The ATS methodology provides military analysts a tool for rapidly conducting what-if drills for planning operations.

This research defined and created a general LPDPTW (theater distribution) problem generator. This generator was instrumental in creating the data set necessary for conducting a statistical analysis of the ATS in this research. The generator allows researchers to construct benchmark data sets when analyzing their own what-if drills.

### 8.2 Avenues for Future Research and Enhancements

There are clear inefficiencies in the Java ${ }^{\text {TM }}$ code of the ATS algorithm. While the code finds excellent solutions in a reasonable amount of time, correcting these inefficiencies will enhance the heuristic's performance. For example, the objective function evaluation reconstructs the entire solution evaluation for each ATS considered move. As the size of the move neighborhood increases, ATS spends an increasing amount of time during an iteration evaluating the list of potential moves. Strengthening the code to only calculate the actual changes occurring during a move will dramatically decrease the iteration evaluation time. An in-depth analysis by an expert programmer will likely uncover more inefficiencies than the above examples and their elimination will reduce overall ATS solution times.

This research is directly related to Lambert's Strategic Airlift Problem (Lambert, 2004). Lambert's tabu search approach provides solutions that address movement of requirements from CONUS to an APOD in the theater. Typically these APODs reside outside the area of operation and smaller assets ( $\mathrm{C}-130 \mathrm{~s}$ and $\mathrm{C}-17 \mathrm{~s}$ ) move these requirements to their final destinations in the theater. Lambert's strategic modeling tool needs to be combined with the intra-theater capabilities of the model in this research. Combining these two tools could result in efficiency gains from linking the entire distribution network from start to finish. This research utilized text files to import all demand (data). A future enhancement should include the ability to read the data from a time phased force deployment list (TPFDL). This ability provides a better opportunity to support intra-theater airlift modeling.

The heart of the ATS methodology is its ability to minimize the logistics footprint for the theater of operation. The value of the ATS solution is based on the accuracy of the associated footprint costs for each depot, supply point, and vehicle entities in the problem. This research utilized several assumptions concerning the values of these entities. Future research should focus on developing a better or more accurate cost for each entity. This information will allow the ATS methodology to provide a more accurate distribution network.

The LPDPTW addressed in this research is both static and deterministic. Future research needs to look at incorporating a dynamic and stochastic element to the problem. This research fixed the size and shape of the operational box prior to solving the problem. The inclusion of a dynamic operating environment allows planners to explore the impact
of both opening and closing supply points and vehicle depots during the progress of the operation. It also provides the opportunity to realistically capture the impact of a unit establishing a lodgment area and expanding outward. All demands in this research were assumed to be known in advance of solving the problem. Including a stochastic element for both the size of the demand and its actual delivery location provides an opportunity to incorporate more realistic operations in the model.

This research utilized conjugacy class information in building the neighborhoods and tracking the search exploration. Future research needs to evaluate the information gained from exploring a conjugacy class to determine if this information can further support exploration. Their may be benefits provided by the search information from a conjugacy class on selecting the next neighborhood scheme or implementing a diversification or intensification strategy.

### 8.3 Summary

This research presented an effective solution methodology for solving the LPDPTW with application to theater distribution. The ATS methodology incorporates an efficient representation of the LPDPTW to capture sufficient details for constructing the distribution network and routing and scheduling of assigned vehicles to ensure the time definite delivery of requirements. The research highlights the benefits of conducting statistical analysis on metaheuristics and opens several avenues for future research.

## Appendix A Theater Distribution Problem Entity Characterization

General LPDPTW and theater distribution modeling requirements are grouped by modes, nodes, and routes, as previously described in Chapter IV. Modes, or transport vehicles, are means of transporting cargo and personnel between nodes. The requirements are specific enough to account for the air, ground, and water modes. The network nodes are categorized as depots, supply points, and customers. Depots represent vehicle terminals and supply points are locations that distribute cargo. The customers are locations that receive requested demands.

Tables A-1 to A-5 provide a brief description of the modeling requirements and
ATS Java ${ }^{\text {TM }}$ elements grouped by mode, node, and route.

## Mode: Vehicles

| Theater Distribution (LPDPTW) Modeling Characteristics |  |
| :--- | :--- |
| Vehicle Characteristics (Constraints) | ATS Java Element |
| Owned by US services, US contracted, or host nation | vehicleIdentificationLetter <br> vehicleProblemIdentificationLetter |
| Represents multiple nonhomogeneous types | vehicleType |
| Represents ground, air, water mode of travel | vehicleClassificationLetter |
| Possesses fixed assignment cost | vehicleFixedCost |
| Possesses variable operating cost | vehicleVariableCost |
| Constrained by the commodity of cargo it can deliver | vehicleCargoType |
| Constrained by commodity based load capacities | vehicleCommodityCapacity |
| Possesses load and unload times at all locations | vehicleLoadTime <br> vehicleUnloadTime |
| Possess average cruising speed | vehicleSpeed |
| Constrained by route length distance | vehicleRouteLength |
| May be limited to certain routes it can travel | vehicleClassificationLetter |
| Possesses daily service (maintenance) times | vehicleServiceTime |
| Constrained by crew rest times or duty days | vehicleOperatingTime |
| May be allowed single or multiple trips per planning horizon | vehicelTripsPerDay |
| May be allowed refuel at non-depot locations | supplyVehicleSupportCapability |

## Node: Vehicle Depots

| Theater Distribution (LPDPTW) Modeling Characteristics |  |
| :--- | :--- |
| Depot Characteristics (Constraints) | ATS Java Element |
| Represents multiple terminal types (in and out of AO) | depotIdentificationLetter |
| depotProblemIdentificationLetter |  |
| Represents ground, air, water or multiple terminals | depotClassificationLetter |
| Possesses assigned vehicles by type and quantity | vehicleCurrentDepotAssignment |
| Possesses open or closed status | depotOpenStatus |
| Possesses fixed opening cost | depotFixedCost |
| Possesses variable operating cost | depotVariableCost |
| Possesses limitation on number of assigned vehicles | depotCapacity |

## Node: Supply Points

| Theater Distribution (LPDPTW) Modeling Characteristics |  |
| :--- | :--- |
| Supply Point Characteristics (Constraints) |  |
| ATS Java Element |  |
| Represents multiple logistics processing points | supplyIdentificationLetter <br> supplyPointIdentification <br> supplyProblemIdentificationLetter |
| Possesses limitation on supported vehicle types | supplyPointType |
| Possesses storage constraints by commodity (Upper \& OH) | supplyCargoCapacity <br> supplyCommodityCapacity |
| Possesses limited vehicle processing capability (MOG) | supplyVehicleTypeThroughput |
| Possesses limited commodity throughput capability | supplyPointThroughput |
| Possesses fixed establishing cost | supplyFixedCost |
| Possesses variable operating cost | supplyVariableCost |
| May posses limitation on type of supported vehicle | supplySupportedVehicleType |
| May posses service time window | supplyOpenWindowTime <br> supplyCloseWindowTime |
| May posses ability to resupply distribution vehicles | supplyVehicleSupportCapability |

## Node: Customer

| Theater Distribution (LPDPTW) Modeling Characteristics |  |
| :--- | :--- |
| Customer Characteristics (Constraints) | ATS Java Element |
| Represents specific request for a given customer | demandIdentificationLetter <br> demandPointIdentificationLetter <br> demandProblemIdentificationLetter |
| Represents specific commodity type | demandCargoType |
| Represents requested commodity amount | demandCargoDemand |
| May posses time definite delivery (TDD) requirement | demandTDDRequirement |
| May posses time delivery window | demandOpenDeliveryWindow <br> demandCloseDeliveryWindow |
| May posses limit on number of vehicles supported (MOG) | demandVehicleCapacityRequirement |

## Arcs: Vehicle Routes

| Theater Distribution (LPDPTW) Modeling Characteristics |  |
| :--- | :--- |
| Routes Characteristics | ATS Java Element |
|  | problemDistanceMatrix |
| Planners designate routes connecting network nodes | problemAirDistanceMatrix |
|  | problemSeaDistanceMatrix |

## Appendix B Resolution IV Fractional Factor Design

| $\mathrm{DP}=$ Design Point | (Displayed in Standard Order) |
| :--- | :--- |
| $\mathrm{A}=\mathrm{TWS}$ | (Time Window Size) |
| $\mathrm{B}=\mathrm{COT}$ | (Crew Operational Tempo) |
| $\mathrm{C}=\mathrm{AEW}$ | (Allow Early Waiting) |
| $\mathrm{D}=\mathrm{DD}$ | (Demand Distribution) |
| $\mathrm{E}=\mathrm{BoxSize}$ | (Theater of War Size) |
| $\mathrm{F}=$ NumDds | (Number Demands) |
| $\mathrm{G}=$ MDBN | (Max Distance Between Nodes) |
| $\mathrm{H}=$ NumDepots | (Number of Depots) |
| $\mathrm{I}=$ NumSplyPts | (Number of Supply Points) |
| $\mathrm{J}=$ NumThPts | (Number of Theater Points) |
| $\mathrm{K}=$ Refuel | (Allow Refuel) |
| $\mathrm{L}=$ Tenure | (Tabu Tenure) |
| $\mathrm{M}=$ IntSoln | (Initial Solution) |
| $\mathrm{N}=\mathrm{CmbNeigh}$ | (Combined Neighborhood) |
| $\mathrm{O}=\mathrm{ELU}$ | (Elite List Utilization) |
| $\mathrm{P}=\mathrm{OPWgts}$ | (Adaptive Obj Penalty Weights) |


| $\mathbf{D P}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{I}$ | $\mathbf{J}$ | $\mathbf{K}$ | $\mathbf{L}$ | $\mathbf{M}$ | $\mathbf{N}$ | $\mathbf{O}$ | $\mathbf{P}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\mathbf{2}$ | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\mathbf{3}$ | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 |
| $\mathbf{4}$ | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 |
| $\mathbf{5}$ | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| $\mathbf{6}$ | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| $\mathbf{7}$ | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 |
| $\mathbf{8}$ | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 |
| $\mathbf{9}$ | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 |
| $\mathbf{1 0}$ | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 |
| $\mathbf{1 1}$ | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 |
| $\mathbf{1 2}$ | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 |
| $\mathbf{1 3}$ | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| $\mathbf{1 4}$ | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| $\mathbf{1 5}$ | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| $\mathbf{1 6}$ | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| $\mathbf{1 7}$ | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| $\mathbf{1 8}$ | -1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| $\mathbf{1 9}$ | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| $\mathbf{2 0}$ | 1 | -1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| $\mathbf{2 1}$ | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 |
| $\mathbf{2 2}$ | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 |
| $\mathbf{2 3}$ | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 |
| $\mathbf{2 4}$ | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 |


| DP | A | B | C | D | E | F | G | H | I | J | K | L | M | N | 0 | P |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 |
| 26 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 |
| 27 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| 28 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| 29 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 |
| 30 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 |
| 31 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 32 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 33 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 34 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 35 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 36 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 37 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| 38 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| 39 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 40 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 41 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 |
| 42 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 |
| 43 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 44 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 45 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 46 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 47 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |
| 48 | 1 | 1 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |
| 49 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |
| 50 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |
| 51 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 52 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 53 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 54 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 55 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 |
| 56 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 |
| 57 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 58 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 59 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| 60 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| 61 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 62 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 63 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 64 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Initial Screning Factorial Effects Aliases (Three level interactions assumed insigincant)
Main Effects: [A through P] are all clear of main effects and two level interactions
Aliased Two Factor Interactions (three level intearactions assumed insignificant):

$$
\begin{aligned}
& {[\mathrm{AB}]=\mathrm{AB}+\mathrm{CF}+\mathrm{DG}+\mathrm{EK}+\mathrm{HJ}-\mathrm{LM}+\mathrm{NO}+\mathrm{PQ}} \\
& {[\mathrm{AC}]=\mathrm{AC}+\mathrm{BF}+\mathrm{DH}+\mathrm{EL}+\mathrm{GJ}-\mathrm{KM}+\mathrm{NP}+\mathrm{OQ}} \\
& {[\mathrm{AD}]=\mathrm{AD}+\mathrm{BG}+\mathrm{CH}+\mathrm{EN}+\mathrm{FJ}+\mathrm{KO}+\mathrm{LP}-\mathrm{MQ}} \\
& {[\mathrm{AE}]=\mathrm{AE}+\mathrm{BK}+\mathrm{CL}+\mathrm{DN}-\mathrm{FM}+\mathrm{GO}+\mathrm{HP}+\mathrm{JQ}} \\
& {[\mathrm{AF}]=\mathrm{AF}+\mathrm{BC}+\mathrm{DJ}-\mathrm{EM}+\mathrm{GH}+\mathrm{KL}+\mathrm{NQ}+\mathrm{OP}} \\
& {[\mathrm{AG}]=\mathrm{AG}+\mathrm{BD}+\mathrm{CJ}+\mathrm{EO}+\mathrm{FH}+\mathrm{KN}+\mathrm{LQ}-\mathrm{MP}} \\
& {[\mathrm{AH}]=\mathrm{AH}+\mathrm{BJ}+\mathrm{CD}+\mathrm{EP}+\mathrm{FG}+\mathrm{KQ}+\mathrm{LN}-\mathrm{MO}} \\
& {[\mathrm{AJ}]=\mathrm{AJ}+\mathrm{BH}+\mathrm{CG}+\mathrm{DF}+\mathrm{EQ}+\mathrm{KP}+\mathrm{LO}-\mathrm{MN}} \\
& {[\mathrm{AK}]=\mathrm{AK}+\mathrm{BE}-\mathrm{CM}+\mathrm{DO}+\mathrm{FL}+\mathrm{GN}+\mathrm{HQ}+\mathrm{JP}} \\
& {[\mathrm{AL}]=\mathrm{AL}-\mathrm{BM}+\mathrm{CE}+\mathrm{DP}+\mathrm{FK}+\mathrm{GQ}+\mathrm{HN}+\mathrm{JO}} \\
& {[\mathrm{AM}]=\mathrm{AM}-\mathrm{BL}-\mathrm{CK}-\mathrm{DQ}-\mathrm{EF}-\mathrm{GP}-\mathrm{HO}-\mathrm{JN}} \\
& {[\mathrm{AN}]=\mathrm{AN}+\mathrm{BO}+\mathrm{CP}+\mathrm{DE}+\mathrm{FQ}+\mathrm{GK}+\mathrm{HL}-\mathrm{JM}} \\
& {[\mathrm{AO}]=\mathrm{AO}+\mathrm{BN}+\mathrm{CQ}+\mathrm{DK}+\mathrm{EG}+\mathrm{FP}-\mathrm{HM}+\mathrm{JL}} \\
& {[\mathrm{AP}]=\mathrm{AP}+\mathrm{BQ}+\mathrm{CN}+\mathrm{DL}+\mathrm{EH}+\mathrm{FO}-\mathrm{GM}+\mathrm{JK}} \\
& {[\mathrm{AQ}]=\mathrm{AQ}+\mathrm{BP}+\mathrm{CO}-\mathrm{DM}+\mathrm{EJ}+\mathrm{FN}+\mathrm{GL}+\mathrm{HK}}
\end{aligned}
$$

Sequential Reduced Factorial Design

| $\mathrm{DP}=$ Design Point | (Displayed in Standard Order) |
| :--- | :--- |
| $\mathrm{A}=\mathrm{TWS}$ | (Time Window Size) |
| $\mathrm{B}=\mathrm{COT}$ | (Crew Operational Tempo) |
| $\mathrm{C}=\mathrm{DD}$ | (Demand Distribution) |
| $\mathrm{D}=$ NumDds | (Number Demands) |
| $\mathrm{E}=$ MDBN | (Max Distance Between Nodes) |
| $\mathrm{F}=$ NumDepots | (Number of Depots) |
| $\mathrm{G}=$ NumSplyPts | (Number of Supply Points) |
| $\mathrm{H}=$ NumThPts | (Number of Theater Points) |
| $\mathrm{I}=$ Refuel | (Allow Refuel) |
| $\mathrm{J}=$ IntSoln | (Initial Solution) |
| $\mathrm{K}=\mathrm{CmbNeigh}$ | (Combined Neighborhood) |
| $\mathrm{L}=$ ELU | (Elite List Utilization) |
| $\mathrm{M}=$ OPWgts | (Adaptive Obj Penalty Weights) |


| $\mathbf{D P}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{I}$ | $\mathbf{J}$ | $\mathbf{K}$ | $\mathbf{L}$ | $\mathbf{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{6 5}$ | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | $\{1\}$ | $\{-1\}$ | $\{-1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{6 6}$ | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | $\{1\}$ | $\{-1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{6 7}$ | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | $\{1\}$ | $\{-1\}$ | $\{1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{6 8}$ | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{6 9}$ | 1 | -1 | $\{1\}$ | -1 | 1 | 1 | 1 | -1 | -1 | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{7 0}$ | -1 | -1 | $\{1\}$ | 1 | -1 | -1 | 1 | 1 | $\{1\}$ | -1 | $\{1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{7 1}$ | 1 | 1 | $\{1\}$ | 1 | -1 | 1 | -1 | 1 | -1 | $\{1\}$ | -1 | $\{1\}$ | $\{-1\}$ |
| $\mathbf{7 2}$ | -1 | 1 | $\{1\}$ | -1 | 1 | -1 | -1 | -1 | $\{1\}$ | $\{-1\}$ | -1 | $\{1\}$ | $\{1\}$ |
| $\mathbf{7 3}$ | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{7 4}$ | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | $\{1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{7 5}$ | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | $\{1\}$ | $\{1\}$ | -1 | $\{1\}$ | $\{-1\}$ |
| $\mathbf{7 6}$ | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | $\{1\}$ | $\{1\}$ |
| $\mathbf{7 7}$ | 1 | -1 | $\{1\}$ | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | $\{1\}$ | $\{1\}$ |
| $\mathbf{7 8}$ | -1 | -1 | $\{1\}$ | -1 | -1 | 1 | -1 | -1 | $\{1\}$ | $\{1\}$ | -1 | $\{1\}$ | $\{-1\}\}$ |
| $\mathbf{7 9}$ | 1 | 1 | $\{1\}$ | -1 | -1 | -1 | 1 | -1 | -1 | -1 | $\{1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{8 0}$ | -1 | 1 | $\{1\}$ | 1 | 1 | 1 | 1 | 1 | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{8 1}$ | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | $\{1\}$ | -1 | -1 | $\{1\}$ | $\{1\}$ |
| $\mathbf{8 2}$ | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | $\{1\}$ | -1 | $\{1\}$ | $\{-1\}\}$ |
| $\mathbf{8 3}$ | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | $\{1\}$ | -1 | $\{1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{8 4}$ | -1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{8 5}$ | 1 | -1 | $\{1\}$ | -1 | 1 | 1 | 1 | -1 | $\{-1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{8 6}$ | -1 | -1 | $\{1\}$ | 1 | -1 | -1 | 1 | 1 | $\{1\}$ | $\{-1\}$ | $\{1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{8 7}$ | 1 | 1 | $\{1\}$ | 1 | -1 | 1 | -1 | 1 | $\{-1\}$ | $\{1\}$ | $\{-1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{8 8}$ | -1 | 1 | $\{1\}$ | -1 | 1 | -1 | -1 | -1 | $\{1\}$ | $\{-1\}$ | $\{-1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{8 9}$ | 1 | -1 | $\{-1\}$ | -1 | -1 | -1 | -1 | 1 | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{9 0}$ | -1 | -1 | $\{-1\}$ | 1 | 1 | 1 | -1 | -1 | $\{-1\}\{-1\}$ | $\{1\}$ | $\{1\}$ | $\{-1\}$ |  |


| $\mathbf{D P}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{I}$ | $\mathbf{J}$ | $\mathbf{K}$ | $\mathbf{L}$ | $\mathbf{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{9 1}$ | 1 | 1 | $\{-1\}$ | 1 | 1 | -1 | 1 | -1 | $\{1\}$ | $\{1\}$ | $\{-1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{9 2}$ | -1 | 1 | $\{-1\}$ | -1 | -1 | 1 | 1 | 1 | $\{-1\}$ | $\{-1\}$ | $\{-1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{9 3}$ | 1 | -1 | $\{1\}$ | 1 | 1 | -1 | -1 | 1 | $\{-1\}$ | $\{-1\}$ | $\{-1\}$ | $\{1\}$ | $\{1\}$ |
| $\mathbf{9 4}$ | -1 | -1 | $\{1\}$ | -1 | -1 | 1 | -1 | -1 | $\{1\}$ | $\{1\}$ | $\{-1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{9 5}$ | 1 | 1 | $\{1\}$ | -1 | -1 | -1 | 1 | -1 | $\{-1\}$ | $\{-1\}$ | $\{1\}$ | $\{1\}$ | $\{-1\}$ |
| $\mathbf{9 6}$ | -1 | 1 | $\{1\}$ | 1 | 1 | 1 | 1 | 1 | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |

Main Effects: [A through P] are all clear of main effects and two level interactions
Two Factor Interactions: TWS (A) two factor interactions are clear of all other two factor interactions

## Aliased Two Factor Interactions:

$$
\begin{aligned}
{[\mathrm{BC}]=} & \mathrm{BC}+\mathrm{DF}+\mathrm{JN}+\mathrm{KM}-\mathrm{ABJ}-\mathrm{ACN}-\mathrm{AEM}-\mathrm{AFL} \\
{[\mathrm{BD}]=} & \mathrm{BD}+\mathrm{CF}+\mathrm{EG}+\mathrm{HK}+\mathrm{LN} \\
{[\mathrm{BE}]=} & \mathrm{BE}+\mathrm{DG}+\mathrm{HL}+\mathrm{JM}+\mathrm{KN}-\mathrm{ABK}-\mathrm{ACM}-\mathrm{ADH}-\mathrm{AEN}-\mathrm{AGL} \\
{[\mathrm{BF}]=} & \mathrm{BF}+\mathrm{CD}+\mathrm{HM}+\mathrm{JL}-\mathrm{ACL}-\mathrm{ADJ}-\mathrm{AFN}-\mathrm{AGM} \\
{[\mathrm{BG}]=} & \mathrm{BG}+\mathrm{DE}+\mathrm{HN}+\mathrm{KL}+\mathrm{BCL}+\mathrm{BDJ}+\mathrm{BFN}+\mathrm{BGM}+\mathrm{CDN}+\mathrm{CEH}+\mathrm{CFJ}+ \\
& \mathrm{CGK}+\mathrm{DEM}+\mathrm{DFL}+\mathrm{EFK}+\mathrm{EGJ}+\mathrm{FGH}+\mathrm{HJK}+\mathrm{HMN}+\mathrm{JLN}+\mathrm{KLM} \\
{[\mathrm{BH}]=} & \mathrm{BH}+\mathrm{DK}+\mathrm{EL}+\mathrm{FM}+\mathrm{GN} \\
{[\mathrm{BJ}]=} & \mathrm{BJ}+\mathrm{CN}+\mathrm{EM}+\mathrm{FL}-\mathrm{ABC}-\mathrm{ADF}-\mathrm{AJN}-\mathrm{AKM} \\
{[\mathrm{BK}]=} & \mathrm{BK}+\mathrm{CM}+\mathrm{DH}+\mathrm{EN}+\mathrm{GL}-\mathrm{ABE}-\mathrm{ADG}-\mathrm{AHL}-\mathrm{AJM}-\mathrm{AKN} \\
{[\mathrm{BL}]=} & \mathrm{BL}+\mathrm{DN}+\mathrm{EH}+\mathrm{FJ}+\mathrm{GK} \\
{[\mathrm{BM}]=} & \mathrm{BM}+\mathrm{CK}+\mathrm{EJ}+\mathrm{FH}-\mathrm{ACE}-\mathrm{AFG}-\mathrm{AJK}-\mathrm{AMN} \\
{[\mathrm{BN}]=} & \mathrm{BN}+\mathrm{CJ}+\mathrm{DL}+\mathrm{EK}+\mathrm{GH}+\mathrm{BCE}+\mathrm{BFG}+\mathrm{BJK}+\mathrm{BMN}+\mathrm{CDG}+\mathrm{CHL}+ \\
& \mathrm{CJM}+\mathrm{CKN}+\mathrm{DEF}+\mathrm{DHJ}+\mathrm{DLM}+\mathrm{EJN}+\mathrm{EKM}+\mathrm{FHN}+\mathrm{FKL}+\mathrm{GHM}+ \\
& \mathrm{GJL} \\
{[\mathrm{CE}]=} & \mathrm{CE}+\mathrm{FG}+\mathrm{JK}+\mathrm{MN}-\mathrm{ABM}-\mathrm{ACK}-\mathrm{AEJ}-\mathrm{AFH} \\
{[\mathrm{CG}]=} & \mathrm{CG}+\mathrm{EF}+\mathrm{HJ}+\mathrm{LM}-\mathrm{ACH}-\mathrm{ADM}-\mathrm{AFK}-\mathrm{AGJ}
\end{aligned}
$$

$$
\begin{aligned}
& {[\mathrm{CH}]=\mathrm{CH}+\mathrm{DM}+\mathrm{FK}+\mathrm{GJ}-\mathrm{ACG}-\mathrm{AEF}-\mathrm{AHJ}-\mathrm{ALM}} \\
& {[\mathrm{CL}]=\mathrm{CL}+\mathrm{DJ}+\mathrm{FN}+\mathrm{GM}-\mathrm{ABF}-\mathrm{ACD}-\mathrm{AHM}-\mathrm{AJL}}
\end{aligned}
$$

## Appendix C ANOVA Tables for Design of Experiments

Initial ANOVA Tables for Factorial Design

|  | Analysis of Variance for Logistics Footprint |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 17 | $7.91 \mathrm{E}+09$ | $4.65 \mathrm{E}+08$ | $1.48 \mathrm{E}+01$ | $<0.0001$ |
| A-TWS | 1 | $4.56 \mathrm{E}+07$ | $4.56 \mathrm{E}+07$ | $1.45 \mathrm{E}+00$ | 0.2350 |
| B-COT | 1 | $1.32 \mathrm{E}+07$ | $1.32 \mathrm{E}+07$ | $4.20 \mathrm{E}-01$ | 0.5201 |
| C-AEW | 1 | $2.98 \mathrm{E}+06$ | $2.98 \mathrm{E}+06$ | $9.47 \mathrm{E}-02$ | 0.7596 |
| D-DD | 1 | $4.03 \mathrm{E}+07$ | $4.03 \mathrm{E}+07$ | $1.28 \mathrm{E}+00$ | 0.2638 |
| E-BoxSize | 1 | $1.77 \mathrm{E}+06$ | $1.77 \mathrm{E}+06$ | $5.62 \mathrm{E}-02$ | 0.8136 |
| F-NumDds | 1 | $6.75 \mathrm{E}+09$ | $6.75 \mathrm{E}+09$ | $2.14 \mathrm{E}+02$ | $<0.0001$ |
| G-MDBN | 1 | $6.15 \mathrm{E}+08$ | $6.15 \mathrm{E}+08$ | $1.95 \mathrm{E}+01$ | $<0.0001$ |
| H-NumDepots | 1 | $2.54 \mathrm{E}+07$ | $2.54 \mathrm{E}+07$ | $8.06 \mathrm{E}-01$ | 0.3740 |
| J-NumSplyPts | 1 | $1.00 \mathrm{E}+07$ | $1.00 \mathrm{E}+07$ | $3.18 \mathrm{E}-01$ | 0.5756 |
| K-NumThPts | 1 | $1.10 \mathrm{E}+07$ | $1.10 \mathrm{E}+07$ | $3.49 \mathrm{E}-01$ | 0.5576 |
| L-Refuel | 1 | $1.88 \mathrm{E}+07$ | $1.88 \mathrm{E}+07$ | $5.96 \mathrm{E}-01$ | 0.4439 |
| M-Tenure | 1 | $5.57 \mathrm{E}+06$ | $5.57 \mathrm{E}+06$ | $1.77 \mathrm{E}-01$ | 0.6762 |
| N-IntSoln | 1 | $7.23 \mathrm{E}+06$ | $7.23 \mathrm{E}+06$ | $2.30 \mathrm{E}-01$ | 0.6341 |
| O-CmbNeigh | 1 | $4.06 \mathrm{E}+07$ | $4.06 \mathrm{E}+07$ | $1.29 \mathrm{E}+00$ | 0.2623 |
| P-ELU | 1 | $7.95 \mathrm{E}+07$ | $7.95 \mathrm{E}+07$ | $2.52 \mathrm{E}+00$ | 0.1189 |
| Q-OPWgts | 1 | $1.96 \mathrm{E}+06$ | $1.96 \mathrm{E}+06$ | $6.24 \mathrm{E}-02$ | 0.8039 |
| AD | 1 | $2.35 \mathrm{E}+08$ | $2.35 \mathrm{E}+08$ | $7.47 \mathrm{E}+00$ | 0.0089 |
| Error | 46 | $1.45 \mathrm{E}+09$ | $3.15 \mathrm{E}+07$ |  |  |
| Total |  |  |  |  |  |


| Analysis of Variance for Feasible Number of Supply |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 16 | 18.75 | 1.17 | 3.17 | 0.0011 |
| A-TWS | 1 | 0.14 | 0.14 | 0.38 | 0.5402 |
| B-COT | 1 | 0.39 | 0.39 | 1.06 | 0.3090 |
| C-AEW | 1 | 0.14 | 0.14 | 0.38 | 0.5402 |
| D-DD | 1 | 0.02 | 0.02 | 0.04 | 0.8379 |
| E-BoxSize | 1 | 0.14 | 0.14 | 0.38 | 0.5402 |
| F-NumDds | 1 | 13.14 | 13.14 | 35.58 | $<0.0001$ |
| G-MDBN | 1 | 0.39 | 0.39 | 1.06 | 0.3090 |
| H-NumDepots | 1 | 1.89 | 1.89 | 5.12 | 0.0283 |
| J-NumSplyPts | 1 | 0.77 | 0.77 | 2.07 | 0.1566 |
| K-NumThPts | 1 | 0.39 | 0.39 | 1.06 | 0.3090 |
| L-Refuel | 1 | 0.02 | 0.02 | 0.04 | 0.8379 |
| M-Tenure | 1 | 0.77 | 0.77 | 2.07 | 0.1566 |
| N-IntSoln | 1 | 0.02 | 0.02 | 0.04 | 0.8379 |
| O-CmbNeigh | 1 | 0.14 | 0.14 | 0.38 | 0.5402 |
| P-ELU | 1 | 0.02 | 0.02 | 0.04 | 0.8379 |
| Q-OPWgts | 1 | 0.39 | 0.39 | 1.06 | 0.3090 |
| Error | 47 | 17.36 | 0.37 |  |  |
| Total | 63 | 36.11 |  |  |  |


| Analysis of Variance for Feasible Number of Vehicles |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 16 | 8152.50 | 509.53 | 39.23 | $<0.0001$ |
| A-TWS | 1 | 138.06 | 138.06 | 10.63 | 0.0021 |
| B-COT | 1 | 18.06 | 18.06 | 1.39 | 0.2442 |
| C-AEW | 1 | 6.25 | 6.25 | 0.48 | 0.4913 |
| D-DD | 1 | 0.00 | 0.00 | 0.00 | 1.0000 |
| E-BoxSize | 1 | 20.25 | 20.25 | 1.56 | 0.2180 |
| F-NumDds | 1 | 7744.00 | 7744.00 | 596.24 | $<0.0001$ |
| G-MDBN | 1 | 42.25 | 42.25 | 3.25 | 0.0777 |
| H-NumDepots | 1 | 14.06 | 14.06 | 1.08 | 0.3034 |
| J-NumSplyPts | 1 | 45.56 | 45.56 | 3.51 | 0.0673 |
| K-NumThPts | 1 | 20.25 | 20.25 | 1.56 | 0.2180 |
| L-Refuel | 1 | 45.56 | 45.56 | 3.51 | 0.0673 |
| M-Tenure | 1 | 1.56 | 1.56 | 0.12 | 0.7303 |
| N-IntSoln | 1 | 10.56 | 10.56 | 0.81 | 0.3718 |
| O-CmbNeigh | 1 | 33.06 | 33.06 | 2.55 | 0.1173 |
| P-ELU | 1 | 9.00 | 9.00 | 0.69 | 0.4094 |
| Q-OPWgts | 1 | 4.00 | 4.00 | 0.31 | 0.5816 |
| Error | 47 | 610.44 | 12.99 |  |  |
| Total | 63 | 8762.94 |  |  |  |


| Analysis of Variance for Near Feasible Number of Depots |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 18 | 8.13 | 0.45 | 7.22 | $<0.0001$ |
| A-TWS | 1 | 0.25 | 0.25 | 4.00 | 0.0516 |
| B-COT | 1 | 0.25 | 0.25 | 4.00 | 0.0516 |
| C-AEW | 1 | 2.25 | 2.25 | 36.00 | $<0.0001$ |
| D-DD | 1 | 0.06 | 0.06 | 1.00 | 0.3227 |
| E-BoxSize | 1 | 0.25 | 0.25 | 4.00 | 0.0516 |
| F-NumDds | 1 | 2.25 | 2.25 | 36.00 | $<0.0001$ |
| G-MDBN | 1 | 0.06 | 0.06 | 1.00 | 0.3227 |
| H-NumDepots | 1 | 0.06 | 0.06 | 1.00 | 0.3227 |
| J-NumSplyPts | 1 | 0.06 | 0.06 | 1.00 | 0.3227 |
| K-NumThPts | 1 | 0.00 | 0.00 | 0.00 | 1.0000 |
| L-Refuel | 1 | 0.00 | 0.00 | 0.00 | 1.0000 |
| M-Tenure | 1 | 0.25 | 0.25 | 4.00 | 0.0516 |
| N-IntSoln | 1 | 0.06 | 0.06 | 1.00 | 0.3227 |
| O-CmbNeigh | 1 | 0.06 | 0.06 | 1.00 | 0.3227 |
| P-ELU | 1 | 0.06 | 0.06 | 1.00 | 0.3227 |
| Q-OPWgts | 1 | 0.06 | 0.06 | 1.00 | 0.3227 |
| AB | 1 | 1.56 | 1.56 | 25.00 | $<0.0001$ |
| AD | 1 | 0.56 | 0.56 | 9.00 | 0.0044 |
| Error | 45 | 2.81 | 0.06 |  |  |
| Total | 63 | 10.94 |  |  |  |


| Analysis of Variance for Near Feasible Number of Supplys |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 18 | 16.91 | 0.94 | 2.67 | 0.0039 |
| A-TWS | 1 | 0.02 | 0.02 | 0.04 | 0.8340 |
| B-COT | 1 | 0.77 | 0.77 | 2.18 | 0.1471 |
| C-AEW | 1 | 0.02 | 0.02 | 0.04 | 0.8340 |
| D-DD | 1 | 0.02 | 0.02 | 0.04 | 0.8340 |
| E-BoxSize | 1 | 0.02 | 0.02 | 0.04 | 0.8340 |
| F-NumDds | 1 | 9.77 | 9.77 | 27.76 | $<0.0001$ |
| G-MDBN | 1 | 0.14 | 0.14 | 0.40 | 0.5304 |
| H-NumDepots | 1 | 1.27 | 1.27 | 3.60 | 0.0643 |
| J-NumSplyPts | 1 | 0.39 | 0.39 | 1.11 | 0.2976 |
| K-NumThPts | 1 | 0.77 | 0.77 | 2.18 | 0.1471 |
| L-Refuel | 1 | 0.14 | 0.14 | 0.40 | 0.5304 |
| M-Tenure | 1 | 0.14 | 0.14 | 0.40 | 0.5304 |
| N-IntSoln | 1 | 0.14 | 0.14 | 0.40 | 0.5304 |
| O-CmbNeigh | 1 | 0.02 | 0.02 | 0.04 | 0.8340 |
| P-ELU | 1 | 0.02 | 0.02 | 0.04 | 0.8340 |
| Q-OPWgts | 1 | 0.14 | 0.14 | 0.40 | 0.5304 |
| AC | 1 | 1.89 | 1.89 | 5.38 | 0.0250 |
| AD | 1 | 1.27 | 1.27 | 3.60 | 0.0643 |
| Error | 45 | 15.83 | 0.35 |  |  |
| Total | 63 | 32.73 |  |  |  |


| Analysis of Variance for Near Feasible Number of Vehicles |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 17 | 7779.75 | 457.63 | 68.35 | $<0.0001$ |
| A-TWS | 1 | 144.00 | 144.00 | 21.51 | $<0.0001$ |
| B-COT | 1 | 14.06 | 14.06 | 2.10 | 0.1541 |
| C-AEW | 1 | 0.56 | 0.56 | 0.08 | 0.7732 |
| D-DD | 1 | 0.25 | 0.25 | 0.04 | 0.8476 |
| E-BoxSize | 1 | 25.00 | 25.00 | 3.73 | 0.0595 |
| F-NumDds | 1 | 7482.25 | 7482.25 | 1117.48 | $<0.0001$ |
| G-MDBN | 1 | 39.06 | 39.06 | 5.83 | 0.0198 |
| H-NumDepots | 1 | 0.06 | 0.06 | 0.01 | 0.9235 |
| J-NumSplyPts | 1 | 12.25 | 12.25 | 1.83 | 0.1828 |
| K-NumThPts | 1 | 5.06 | 5.06 | 0.76 | 0.3891 |
| L-Refuel | 1 | 7.56 | 7.56 | 1.13 | 0.2934 |
| M-Tenure | 1 | 4.00 | 4.00 | 0.60 | 0.4435 |
| N-IntSoln | 1 | 1.00 | 1.00 | 0.15 | 0.7009 |
| O-CmbNeigh | 1 | 14.06 | 14.06 | 2.10 | 0.1541 |
| P-ELU | 1 | 0.06 | 0.06 | 0.01 | 0.9235 |
| Q-OPWgts | 1 | 0.25 | 0.25 | 0.04 | 0.8476 |
| $\quad$ AF | 1 | 30.25 | 30.25 | 4.52 | 0.0389 |
| Error | 46 | 308.00 | 6.70 |  |  |
| Total | 63 | 8087.75 |  |  |  |


| Analysis of Variance for Number of Iterations |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 17 | $2.29 \mathrm{E}+09$ | $1.35 \mathrm{E}+08$ | 11.92 | $<0.0001$ |
| A-TWS | 1 | $1.47 \mathrm{E}+07$ | $1.47 \mathrm{E}+07$ | 1.30 | 0.2608 |
| B-COT | 1 | $1.46 \mathrm{E}+07$ | $1.46 \mathrm{E}+07$ | 1.29 | 0.2615 |
| C-AEW | 1 | $7.19 \mathrm{E}+06$ | $7.19 \mathrm{E}+06$ | 0.64 | 0.4295 |
| D-DD | 1 | $2.49 \mathrm{E}+07$ | $2.49 \mathrm{E}+07$ | 2.20 | 0.1448 |
| E-BoxSize | 1 | $1.19 \mathrm{E}+06$ | $1.19 \mathrm{E}+06$ | 0.10 | 0.7475 |
| F-NumDds | 1 | $3.71 \mathrm{E}+08$ | $3.71 \mathrm{E}+08$ | 32.81 | $<0.0001$ |
| G-MDBN | 1 | $7.31 \mathrm{E}+03$ | $7.31 \mathrm{E}+03$ | 0.00 | 0.9798 |
| H-NumDepots | 1 | $5.46 \mathrm{E}+06$ | $5.46 \mathrm{E}+06$ | 0.48 | 0.4908 |
| J-NumSplyPts | 1 | $4.38 \mathrm{E}+05$ | $4.38 \mathrm{E}+05$ | 0.04 | 0.8450 |
| K-NumThPts | 1 | $1.15 \mathrm{E}+06$ | $1.15 \mathrm{E}+06$ | 0.10 | 0.7510 |
| L-Refuel | 1 | $1.86 \mathrm{E}+05$ | $1.86 \mathrm{E}+05$ | 0.02 | 0.8986 |
| M-Tenure | 1 | $1.37 \mathrm{E}+06$ | $1.37 \mathrm{E}+06$ | 0.12 | 0.7294 |
| N-IntSoln | 1 | $5.32 \mathrm{E}+06$ | $5.32 \mathrm{E}+06$ | 0.47 | 0.4967 |
| O-CmbNeigh | 1 | $4.27 \mathrm{E}+05$ | $4.27 \mathrm{E}+05$ | 0.04 | 0.8469 |
| P-ELU | 1 | $1.54 \mathrm{E}+09$ | $1.54 \mathrm{E}+09$ | 136.36 | $<0.0001$ |
| Q-OPWgts | 1 | $2.33 \mathrm{E}+07$ | $2.33 \mathrm{E}+07$ | 2.06 | 0.1582 |
| $\quad$ AO | 1 | $2.79 \mathrm{E}+08$ | $2.79 \mathrm{E}+08$ | 24.68 | $<0.0001$ |
| Error | 46 | $5.21 \mathrm{E}+08$ | $1.13 \mathrm{E}+07$ |  |  |
| Total | 63 | $2.82 \mathrm{E}+09$ |  |  |  |


| Analysis of Variance for Conjugacy Classes |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 16 | 118571.50 | 7410.72 | 2.85 | 0.0027 |
| A-TWS | 1 | 6320.25 | 6320.25 | 2.43 | 0.1256 |
| B-COT | 1 | 11556.25 | 11556.25 | 4.45 | 0.0404 |
| C-AEW | 1 | 315.06 | 315.06 | 0.12 | 0.7293 |
| D-DD | 1 | 1350.56 | 1350.56 | 0.52 | 0.4746 |
| E-BoxSize | 1 | 3335.06 | 3335.06 | 1.28 | 0.2631 |
| F-NumDds | 1 | 19951.56 | 19951.56 | 7.67 | 0.0080 |
| G-MDBN | 1 | 2425.56 | 2425.56 | 0.93 | 0.3390 |
| H-NumDepots | 1 | 81.00 | 81.00 | 0.03 | 0.8606 |
| J-NumSplyPts | 1 | 5402.25 | 5402.25 | 2.08 | 0.1561 |
| K-NumThPts | 1 | 4590.06 | 4590.06 | 1.77 | 0.1903 |
| L-Refuel | 1 | 600.25 | 600.25 | 0.23 | 0.6331 |
| M-Tenure | 1 | 6.25 | 6.25 | 0.00 | 0.9611 |
| N-IntSoln | 1 | 49.00 | 49.00 | 0.02 | 0.8914 |
| O-CmbNeigh | 1 | 90.25 | 90.25 | 0.03 | 0.8530 |
| P-ELU | 1 | 58685.06 | 58685.06 | 22.57 | $<0.0001$ |
| Q-OPWgts | 1 | 3813.06 | 3813.06 | 1.47 | 0.2319 |
| Error | 47 | 122181.50 | 2599.61 |  |  |
| Total | 63 | 240753.00 |  |  |  |
|  |  |  |  |  |  |


| Source |  |  |  |  | DF |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Analysis of Variance for Neighborhood Size |  |  |  |  |  |
| Model | 17 | $3.07 \mathrm{E}+07$ | $1.81 \mathrm{E}+06$ | 106.57 | $<0.0001$ |
| A-TWS | 1 | $1.64 \mathrm{E}+04$ | $1.64 \mathrm{E}+04$ | 0.97 | 0.3310 |
| B-COT | 1 | $6.38 \mathrm{E}+02$ | $6.38 \mathrm{E}+02$ | 0.04 | 0.8472 |
| C-AEW | 1 | $3.48 \mathrm{E}+03$ | $3.48 \mathrm{E}+03$ | 0.21 | 0.6528 |
| D-DD | 1 | $2.50 \mathrm{E}+01$ | $2.50 \mathrm{E}+01$ | 0.00 | 0.9696 |
| E-BoxSize | 1 | $1.96 \mathrm{E}+02$ | $1.96 \mathrm{E}+02$ | 0.01 | 0.9149 |
| F-NumDds | 1 | $2.57 \mathrm{E}+07$ | $2.57 \mathrm{E}+07$ | 1515.64 | $<0.0001$ |
| G-MDBN | 1 | $1.17 \mathrm{E}+03$ | $1.17 \mathrm{E}+03$ | 0.07 | 0.7938 |
| H-NumDepots | 1 | $9.52 \mathrm{E}+04$ | $9.52 \mathrm{E}+04$ | 5.61 | 0.0221 |
| J-NumSplyPts | 1 | $2.34 \mathrm{E}+05$ | $2.34 \mathrm{E}+05$ | 13.79 | 0.0006 |
| K-NumThPts | 1 | $1.49 \mathrm{E}+04$ | $1.49 \mathrm{E}+04$ | 0.88 | 0.3529 |
| L-Refuel | 1 | $4.00 \mathrm{E}+04$ | $4.00 \mathrm{E}+04$ | 2.36 | 0.1316 |
| M-Tenure | 1 | $1.11 \mathrm{E}+04$ | $1.11 \mathrm{E}+04$ | 0.65 | 0.4233 |
| N-IntSoln | 1 | $6.01 \mathrm{E}+03$ | $6.01 \mathrm{E}+03$ | 0.35 | 0.5548 |
| O-CmbNeigh | 1 | $2.41 \mathrm{E}+06$ | $2.41 \mathrm{E}+06$ | 141.79 | $<0.0001$ |
| P-ELU | 1 | $1.66 \mathrm{E}+04$ | $1.66 \mathrm{E}+04$ | 0.98 | 0.3272 |
| Q-OPWgts | 1 | $6.81 \mathrm{E}+01$ | $6.81 \mathrm{E}+01$ | 0.00 | 0.9498 |
| $\quad$ AP | 1 | $2.18 \mathrm{E}+06$ | $2.18 \mathrm{E}+06$ | 128.32 | $<0.0001$ |
| Error | 46 | $7.81 \mathrm{E}+05$ | $1.70 \mathrm{E}+04$ |  |  |
| Total | 63 | $3.15 \mathrm{E}+07$ |  |  |  |


| Analysis of Variance for Average Tenure Size |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 16 | 5351.25 | 334.45 | 1.30 | 0.2354 |
| A-TWS | 1 | 1080.77 | 1080.77 | 4.21 | 0.0458 |
| B-COT | 1 | 13.14 | 13.14 | 0.05 | 0.8220 |
| C-AEW | 1 | 523.27 | 523.27 | 2.04 | 0.1600 |
| D-DD | 1 | 415.14 | 415.14 | 1.62 | 0.2098 |
| E-BoxSize | 1 | 74.39 | 74.39 | 0.29 | 0.5929 |
| F-NumDds | 1 | 192.52 | 192.52 | 0.75 | 0.3909 |
| G-MDBN | 1 | 276.39 | 276.39 | 1.08 | 0.3048 |
| H-NumDepots | 1 | 40.64 | 40.64 | 0.16 | 0.6925 |
| J-NumSplyPts | 1 | 631.27 | 631.27 | 2.46 | 0.1236 |
| K-NumThPts | 1 | 570.02 | 570.02 | 2.22 | 0.1429 |
| L-Refuel | 1 | 31.64 | 31.64 | 0.12 | 0.7271 |
| M-Tenure | 1 | 0.14 | 0.14 | 0.00 | 0.9814 |
| N-IntSoln | 1 | 682.52 | 682.52 | 2.66 | 0.1097 |
| O-CmbNeigh | 1 | 34.52 | 34.52 | 0.13 | 0.7155 |
| P-ELU | 1 | 141.02 | 141.02 | 0.55 | 0.4623 |
| Q-OPWgts | 1 | 643.89 | 643.89 | 2.51 | 0.1200 |
| Error | 47 | $1.21 \mathrm{E}+04$ | 256.73 |  |  |
| Total | 63 | $1.74 \mathrm{E}+04$ |  |  |  |

## ANOVA Tables for Reduced Factorial Design (Semi-foldover)

| Source |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Analysis of Variance for Logistics Footprint |  |  |  |  |  |
| Model | DF | SS | MS | F Value | P Value |
| A-TWS | 13 | $1.07 \mathrm{E}+10$ | $7.12 \mathrm{E}+08$ | 29.08 | $<0.0001$ |
| B-COT | 1 | $8.41 \mathrm{E}+07$ | $8.41 \mathrm{E}+07$ | 3.44 | 0.0467 |
| D-DD | 1 | $2.06 \mathrm{E}+06$ | $2.06 \mathrm{E}+06$ | 0.08 | 0.7724 |
| F-NumDds | 1 | $5.16 \mathrm{E}+07$ | $5.16 \mathrm{E}+07$ | 2.11 | 0.1505 |
| G-MDBN | 1 | $9.39 \mathrm{E}+09$ | $9.39 \mathrm{E}+09$ | 383.83 | $<0.0001$ |
| H-NumDepots | 1 | $9.10 \mathrm{E}+08$ | $9.10 \mathrm{E}+08$ | 37.20 | $<0.0001$ |
| J-NumSplyPts | 1 | $2.78 \mathrm{E}+07$ | $2.78 \mathrm{E}+07$ | 1.14 | 0.2900 |
| K-NumThPts | 1 | $1.16 \mathrm{E}+06$ | $1.16 \mathrm{E}+06$ | 0.05 | 0.8281 |
| L-Refuel | 1 | $3.30 \mathrm{E}+05$ | $3.30 \mathrm{E}+05$ | 0.01 | 0.9078 |
| M-Tenure | 1 | 4920641.51 | 4920641.51 | 0.20 | 0.6550 |
| O-CmbNeigh | 1 | $1.66 \mathrm{E}+06$ | $1.66 \mathrm{E}+06$ | 0.07 | 0.7951 |
| P-ELU | 1 | $1.88 \mathrm{E}+07$ | $1.88 \mathrm{E}+07$ | 0.77 | 0.3832 |
| Q-OPWgts | 1 | $7.95 \mathrm{E}+07$ | $7.95 \mathrm{E}+07$ | 3.25 | 0.0456 |
| Error | 1 | 4803465.38 | 4803465.38 | 0.20 | 0.6589 |
| Total | 82 | $1.93 \mathrm{E}+09$ | $2.45 \mathrm{E}+07$ |  |  |
|  | 95 | $1.26 \mathrm{E}+10$ |  |  |  |
| Source |  |  |  |  |  |


| Analysis of Variance for Feasible Number of Supply Points |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source |  | DF | SS | MS | F Value |
| Model | 13 | 24.89 | 1.91 | 6.35 | $<0.00001$ |
| A-TWS | 1 | 0.38 | 0.38 | 1.24 | 0.2682 |
| B-COT | 1 | 0.17 | 0.17 | 0.55 | 0.4595 |
| D-DD | 1 | 0.17 | 0.17 | 0.55 | 0.4595 |
| F-NumDds | 1 | 20.17 | 20.17 | 66.84 | $<0.0001$ |
| G-MDBN | 1 | 0.17 | 0.17 | 0.55 | 0.4595 |
| H-NumDepots | 1 | 0.67 | 0.67 | 2.21 | 0.1410 |
| J-NumSplyPts | 1 | 0.67 | 0.67 | 2.21 | 0.1410 |
| K-NumThPts | 1 | 0.67 | 0.67 | 2.21 | 0.1410 |
| L-Refuel | 1 | 0.00 | 0.00 | 0.00 | 1.0000 |
| M-Tenure | 1 | 1.50 | 1.50 | 4.97 | 0.0285 |
| O-CmbNeigh | 1 | 0.17 | 0.17 | 0.55 | 0.4595 |
| P-ELU | 1 | 0.02 | 0.02 | 0.05 | 0.8206 |
| Q-OPWgts | 1 | 0.17 | 0.17 | 0.55 | 0.4595 |
| Error | 82 | 24.44 | 0.30 |  |  |
| Total | 95 | 49.33 |  |  |  |


| Analysis of Variance for Feasible Number of Vehicles |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 15 | 11477.01 | 765.13 | 78.73 | $<0.0001$ |
| A-TWS | 1 | 234.38 | 234.38 | 24.12 | $<0.0001$ |
| B-COT | 1 | 8.17 | 8.17 | 0.84 | 0.3621 |
| D-DD | 1 | 2.67 | 2.67 | 0.27 | 0.6019 |
| F-NumDds | 1 | 10922.67 | 10922.67 | 1123.85 | $<0.0001$ |
| G-MDBN | 1 | 57.04 | 57.04 | 5.87 | 0.0177 |
| H-NumDepots | 1 | 20.17 | 20.17 | 2.07 | 0.1537 |
| J-NumSplyPts | 1 | 9.38 | 9.38 | 0.96 | 0.3290 |
| K-NumThPts | 1 | 1.04 | 1.04 | 0.11 | 0.7442 |
| L-Refuel | 1 | 30.88 | 30.88 | 3.18 | 0.0785 |
| M-Tenure | 1 | 0.38 | 0.38 | 0.04 | 0.8448 |
| O-CmbNeigh | 1 | 22.01 | 22.01 | 2.26 | 0.1364 |
| P-ELU | 1 | 9.00 | 9.00 | 0.93 | 0.3388 |
| Q-OPWgts | 1 | 8.17 | 8.17 | 0.84 | 0.3621 |
| AC | 1 | 57.42 | 57.42 | 5.91 | 0.0173 |
| AD | 1 | 68.88 | 68.88 | 7.09 | 0.0094 |
| Error | 80 | 767.80 | 9.72 |  |  |
| Total | 95 | 12244.81 |  |  |  |


| Analysis of Variance for Near Feasible Number of Depots |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 17 | 7.44 | 0.44 | 4.02 | $<0.0001$ |
| A-TWS | 1 | 0.38 | 0.38 | 3.45 | 0.0672 |
| B-COT | 1 | 0.00 | 0.00 | 0.00 | 1.0000 |
| D-DD | 1 | 0.17 | 0.17 | 1.53 | 0.2195 |
| F-NumDds | 1 | 3.38 | 3.38 | 31.03 | $<0.0001$ |
| G-MDBN | 1 | 0.38 | 0.38 | 3.45 | 0.0672 |
| H-NumDepots | 1 | 0.04 | 0.04 | 0.38 | 0.5378 |
| J-NumSplyPts | 1 | 0.00 | 0.00 | 0.00 | 1.0000 |
| K-NumThPts | 1 | 0.04 | 0.04 | 0.38 | 0.5378 |
| L-Refuel | 1 | 0.04 | 0.04 | 0.38 | 0.5378 |
| M-Tenure | 1 | 0.17 | 0.17 | 1.53 | 0.2195 |
| O-CmbNeigh | 1 | 0.02 | 0.02 | 0.19 | 0.6629 |
| P-ELU | 1 | 0.06 | 0.06 | 0.57 | 0.4507 |
| Q-OPWgts | 1 | 0.02 | 0.02 | 0.19 | 0.6629 |
| AB | 1 | 1.02 | 1.02 | 9.39 | 0.0030 |
| AD | 1 | 0.19 | 0.19 | 1.72 | 0.1931 |
| BD | 1 | 0.04 | 0.04 | 0.38 | 0.5378 |
| ABD | 1 | 1.50 | 1.50 | 13.79 | 0.0004 |
| Error | 78 | 8.38 | 0.11 |  |  |
| Total | 95 | 15.81 |  |  |  |


| Analysis of Variance for Near Feasible Number of Supply Points |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 14 | 22.10 | 1.58 | 5.78 | $<0.0001$ |
| A-TWS | 1 | 0.04 | 0.04 | 0.15 | 0.6971 |
| B-COT | 1 | 0.38 | 0.38 | 1.37 | 0.2448 |
| D-DD | 1 | 0.04 | 0.04 | 0.15 | 0.6971 |
| F-NumDds | 1 | 16.67 | 16.67 | 61.03 | $<0.0001$ |
| G-MDBN | 1 | 0.17 | 0.17 | 0.61 | 0.4370 |
| H-NumDepots | 1 | 0.67 | 0.67 | 2.44 | 0.1221 |
| J-NumSplyPts | 1 | 0.38 | 0.38 | 1.37 | 0.2448 |
| K-NumThPts | 1 | 1.50 | 1.50 | 5.49 | 0.0216 |
| L-Refuel | 1 | 0.63 | 0.63 | 2.31 | 0.1327 |
| M-Tenure | 1 | 0.38 | 0.38 | 1.37 | 0.2448 |
| O-CmbNeigh | 1 | 0.04 | 0.04 | 0.15 | 0.6971 |
| P-ELU | 1 | 0.02 | 0.02 | 0.06 | 0.8116 |
| Q-OPWgts | 1 | 0.17 | 0.17 | 0.61 | 0.4370 |
| AC | 1 | 1.51 | 1.51 | 5.51 | 0.0214 |
| Error | 81 | 21.85 | 0.27 |  |  |
| Total | 95 | 43.95 |  |  |  |


| Analysis of Variance for Near Feasible Number of Vehicles |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 14 | 22.10 | 1.58 | 5.78 | $<0.0001$ |
| A-TWS | 1 | 0.04 | 0.04 | 0.15 | 0.6971 |
| B-COT | 1 | 0.38 | 0.38 | 1.37 | 0.2448 |
| D-DD | 1 | 0.04 | 0.04 | 0.15 | 0.6971 |
| F-NumDds | 1 | 16.67 | 16.67 | 61.03 | $<0.0001$ |
| G-MDBN | 1 | 0.17 | 0.17 | 0.61 | 0.4370 |
| H-NumDepots | 1 | 0.67 | 0.67 | 2.44 | 0.1221 |
| J-NumSplyPts | 1 | 0.38 | 0.38 | 1.37 | 0.2448 |
| K-NumThPts | 1 | 1.50 | 1.50 | 5.49 | 0.0216 |
| L-Refuel | 1 | 0.63 | 0.63 | 2.31 | 0.1327 |
| M-Tenure | 1 | 0.38 | 0.38 | 1.37 | 0.2448 |
| O-CmbNeigh | 1 | 0.04 | 0.04 | 0.15 | 0.6971 |
| P-ELU | 1 | 0.02 | 0.02 | 0.06 | 0.8116 |
| Q-OPWgts | 1 | 0.17 | 0.17 | 0.61 | 0.4370 |
| $\quad$ AC | 1 | 1.51 | 1.51 | 5.51 | 0.0214 |
| Error | 81 | 21.85 | 0.27 |  |  |
| Total | 95 | 43.95 |  |  |  |


| Analysis of Variance for Near Feasible Number of Vehicles |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 17 | 11154.15 | 656.13 | 110.84 | $<0.0001$ |
| A-TWS | 1 | 196.02 | 196.02 | 33.11 | $<0.0001$ |
| B-COT | 1 | 5.51 | 5.51 | 0.93 | 0.3377 |
| D-DD | 1 | 0.09 | 0.09 | 0.02 | 0.9002 |
| F-NumDds | 1 | 10647.09 | 10647.09 | 1798.56 | $<0.0001$ |
| G-MDBN | 1 | 49.59 | 49.59 | 8.38 | 0.0049 |
| H-NumDepots | 1 | 1.26 | 1.26 | 0.21 | 0.6458 |
| J-NumSplyPts | 1 | 0.26 | 0.26 | 0.04 | 0.8344 |
| K-NumThPts | 1 | 0.26 | 0.26 | 0.04 | 0.8344 |
| L-Refuel | 1 | 14.08 | 14.08 | 2.38 | 0.1271 |
| M-Tenure | 1 | 5.51 | 5.51 | 0.93 | 0.3377 |
| O-CmbNeigh | 1 | 7.52 | 7.52 | 1.27 | 0.2632 |
| P-ELU | 1 | 0.06 | 0.06 | 0.01 | 0.9184 |
| Q-OPWgts | 1 | 0.00 | 0.00 | 0.00 | 1.0000 |
| AB | 1 | 0.08 | 0.08 | 0.01 | 0.9059 |
| AC | 1 | 40.33 | 40.33 | 6.81 | 0.0109 |
| BD | 1 | 63.02 | 63.02 | 10.65 | 0.0016 |
| AM | 1 | 25.52 | 25.52 | 4.31 | 0.0412 |
| Error | 78 | 455.82 | 5.92 |  |  |
| Total | 95 | 11609.97 |  |  |  |


| Analysis of Variance for Number of Iterations |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 13 | $2.63 \mathrm{E}+09$ | $2.03 \mathrm{E}+08$ | 14.20 | $<0.0001$ |
| A-TWS | 1 | $3.38 \mathrm{E}+07$ | $3.38 \mathrm{E}+07$ | 2.37 | 0.1276 |
| B-COT | 1 | $1.72 \mathrm{E}+07$ | $1.72 \mathrm{E}+07$ | 1.21 | 0.2749 |
| D-DD | 1 | $3.64 \mathrm{E}+07$ | $3.64 \mathrm{E}+07$ | 2.55 | 0.1141 |
| F-NumDds | 1 | $9.64 \mathrm{E}+08$ | $9.64 \mathrm{E}+08$ | 67.59 | $<0.0001$ |
| G-MDBN | 1 | $5.79 \mathrm{E}+06$ | $5.79 \mathrm{E}+06$ | 0.41 | 0.5257 |
| H-NumDepots | 1 | 0.38 | 0.38 | 0.00 | 0.9999 |
| J-NumSplyPts | 1 | $2.29 \mathrm{E}+06$ | $2.29 \mathrm{E}+06$ | 0.16 | 0.6895 |
| K-NumThPts | 1 | $1.01 \mathrm{E}+06$ | $1.01 \mathrm{E}+06$ | 0.07 | 0.7908 |
| L-Refuel | 1 | 30.37 | 30.37 | 0.00 | 0.9988 |
| M-Tenure | 1 | 4056.00 | 4056.00 | 0.00 | 0.9866 |
| O-CmbNeigh | 1 | $1.25 \mathrm{E}+06$ | $1.25 \mathrm{E}+06$ | 0.09 | 0.7675 |
| P-ELU | 1 | $1.54 \mathrm{E}+09$ | $1.54 \mathrm{E}+09$ | 108.24 | $<0.0001$ |
| Q-OPWgts | 1 | $2.82 \mathrm{E}+07$ | $2.82 \mathrm{E}+07$ | 1.98 | 0.1634 |
| Error | 84 | $1.16 \mathrm{E}+09$ | $1.43 \mathrm{E}+07$ |  |  |
| Total | 95 | $3.79 \mathrm{E}+09$ |  |  |  |


| Analysis of Variance for Number of Conjugacy Classes |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 14 | 172110.44 | 12293.60 | 5.28 | $<0.0001$ |
| A-TWS | 1 | 6501.04 | 6501.04 | 2.79 | 0.0988 |
| B-COT | 1 | 18592.67 | 18592.67 | 7.98 | 0.0060 |
| D-DD | 1 | 240.67 | 240.67 | 0.10 | 0.7488 |
| F-NumDds | 1 | 40837.50 | 40837.50 | 17.52 | $<0.0001$ |
| G-MDBN | 1 | 2340.37 | 2340.37 | 1.00 | 0.3193 |
| H-NumDepots | 1 | 96.00 | 96.00 | 0.04 | 0.8397 |
| J-NumSplyPts | 1 | 6970.04 | 6970.04 | 2.99 | 0.0876 |
| K-NumThPts | 1 | 522.67 | 522.67 | 0.22 | 0.6371 |
| L-Refuel | 1 | 737.04 | 737.04 | 0.32 | 0.5754 |
| M-Tenure | 1 | 931.92 | 931.92 | 0.40 | 0.5289 |
| O-CmbNeigh | 1 | 247.04 | 247.04 | 0.11 | 0.7456 |
| P-ELU | 1 | 58685.06 | 58685.06 | 25.18 | $<0.0001$ |
| Q-OPWgts | 1 | 15050.04 | 15050.04 | 6.46 | 0.0130 |
| $\quad$ AE | 1 | 15462.13 | 15462.13 | 6.64 | 0.0118 |
| Error | 81 | 186426.43 | 2330.33 |  |  |
| Total | 95 | 358536.88 |  |  |  |


|  | Analysis of Variance for Average Tenure Size |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 16 | $1.13 \mathrm{E}+04$ | $7.09 \mathrm{E}+02$ | 3.98 | $<0.0001$ |
| A-TWS | 1 | $1.79 \mathrm{E}+03$ | $1.79 \mathrm{E}+03$ | 10.02 | 0.0022 |
| B-COT | 1 | $5.10 \mathrm{E}+01$ | $5.10 \mathrm{E}+01$ | 0.29 | 0.5941 |
| D-DD | 1 | $9.25 \mathrm{E}+02$ | $9.25 \mathrm{E}+02$ | 5.19 | 0.0255 |
| F-NumDds | 1 | $3.23 \mathrm{E}+02$ | $3.23 \mathrm{E}+02$ | 1.81 | 0.1824 |
| G-MDBN | 1 | $1.50 \mathrm{E}+00$ | $1.50 \mathrm{E}+00$ | 0.01 | 0.9271 |
| H-NumDepots | 1 | $2.04 \mathrm{E}+02$ | $2.04 \mathrm{E}+02$ | 1.15 | 0.2878 |
| J-NumSplyPts | 1 | $7.59 \mathrm{E}+02$ | $7.59 \mathrm{E}+02$ | 4.26 | 0.0423 |
| K-NumThPts | 1 | $4.51 \mathrm{E}+02$ | $4.51 \mathrm{E}+02$ | 2.53 | 0.1158 |
| L-Refuel | 1 | 7.92 | 7.92 | 0.04 | 0.8336 |
| M-Tenure | 1 | $3.04 \mathrm{E}+01$ | $3.04 \mathrm{E}+01$ | 0.17 | 0.6809 |
| O-CmbNeigh | 1 | $5.04 \mathrm{E}+00$ | $5.04 \mathrm{E}+00$ | 0.03 | 0.8669 |
| P-ELU | 1 | $1.41 \mathrm{E}+02$ | $1.41 \mathrm{E}+02$ | 0.79 | 0.3765 |
| Q-OPWgts | 1 | 1290.67 | 1290.67 | 7.24 | 0.0087 |
| AC | 1 | 812.63 | 812.63 | 4.56 | 0.0359 |
| AF | 1 | 770.67 | 770.67 | 4.32 | 0.0409 |
| BL | 1 | $3.63 \mathrm{E}+03$ | $3.63 \mathrm{E}+03$ | 20.35 | $<0.0001$ |
| Error | 79 | $1.39 \mathrm{E}+04$ | $1.78 \mathrm{E}+02$ |  |  |
| Total | 95 | $2.52 \mathrm{E}+04$ |  |  |  |


| Analysis of Variance for Average Neighborhood Size |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Source | DF | SS | MS | F Value | P Value |
| Model | 15 | $4.75 \mathrm{E}+07$ | $3.17 \mathrm{E}+06$ | 260.01 | $<0.0001$ |
| A-TWS | 1 | $2.78 \mathrm{E}+03$ | $2.78 \mathrm{E}+03$ | 0.23 | 0.6345 |
| B-COT | 1 | $1.81 \mathrm{E}+04$ | $1.81 \mathrm{E}+04$ | 1.48 | 0.2270 |
| D-DD | 1 | $8.61 \mathrm{E}+03$ | $8.61 \mathrm{E}+03$ | 0.71 | 0.4032 |
| F-NumDds | 1 | $3.91 \mathrm{E}+07$ | $3.91 \mathrm{E}+07$ | 3212.38 | $<0.0001$ |
| G-MDBN | 1 | $2.70 \mathrm{E}+04$ | $2.70 \mathrm{E}+04$ | 2.22 | 0.1403 |
| H-NumDepots | 1 | $1.23 \mathrm{E}+05$ | $1.23 \mathrm{E}+05$ | 10.07 | 0.0021 |
| J-NumSplyPts | 1 | $3.79 \mathrm{E}+05$ | $3.79 \mathrm{E}+05$ | 31.09 | $<0.0001$ |
| K-NumThPts | 1 | $3.41 \mathrm{E}+04$ | $3.41 \mathrm{E}+04$ | 2.80 | 0.0984 |
| L-Refuel | 1 | 20584.08 | 20584.08 | 1.69 | 0.1975 |
| M-Tenure | 1 | $2.93 \mathrm{E}+04$ | $2.93 \mathrm{E}+04$ | 2.40 | 0.1250 |
| O-CmbNeigh | 1 | $3.86 \mathrm{E}+06$ | $3.86 \mathrm{E}+06$ | 317.12 | $<0.0001$ |
| P-ELU | 1 | $1.66 \mathrm{E}+04$ | $1.66 \mathrm{E}+04$ | 1.37 | 0.2461 |
| Q-OPWgts | 1 | 31.51 | 31.51 | 0.00 | 0.9596 |
| BE | 1 | $3.07 \mathrm{E}+05$ | $3.07 \mathrm{E}+05$ | 25.16 | $<0.0001$ |
| BN | 1 | $3.24 \mathrm{E}+06$ | $3.24 \mathrm{E}+06$ | 265.63 | $<0.0001$ |
| Error | 80 | $9.63 \mathrm{E}+05$ | $1.22 \mathrm{E}+04$ |  |  |
| Total | 95 | $4.85 \mathrm{E}+07$ |  |  |  |

Appendix D LPDPTW 1 \& 2 Problem Data

| LPDPTW 1 ( 90 Demands) |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Dmd Pnt } \\ \text { ID Ltr } \end{gathered}$ | Map ID | $\begin{gathered} \hline \text { Demand } \\ \text { Cargo } \\ \text { Type } \\ \hline \hline \end{gathered}$ | $\begin{gathered} \hline \text { Demand } \\ \text { Cargo } \\ \text { ID } \\ \hline \end{gathered}$ | Amount Cargo Demand | Vehicle <br> Capacity <br> Req | Vehicle <br> Support <br> Capability | Open <br> Window | Close <br> Window | TDD <br> Required |  |  |
| 0 | 12 | 1 | 0 | 12 | 2 | 1 | 750 | 1110 | true | 4 | 29 |
| 1 | 13 | 1 | 0 | 12 | 2 | 1 | 1030 | 1390 | true | 15 | 23 |
| 2 | 14 | 1 | 0 | 12 | 2 | 1 | 840 | 1200 | true | 15 | 78 |
| 3 | 15 | 1 | 0 | 12 | 2 | 1 | 1195 | 1555 | true | 34 | 28 |
| 4 | 16 | 1 | 0 | 12 | 2 | 1 | 613 | 973 | true | 48 | 8 |
| 5 | 17 | 1 | 0 | 12 | 2 | 1 | 1025 | 1385 | true | 52 | 22 |
| 6 | 18 | 1 | 0 | 12 | 2 | 1 | 528 | 888 | true | 64 | 38 |
| 7 | 19 | 1 | 0 | 12 | 2 | 1 | 388 | 748 | true | 29 | 34 |
| 8 | 20 | 1 | 0 | 12 | 2 | 1 | 939 | 1299 | true | 20 | 52 |
| 9 | 21 | 1 | 0 | 12 | 2 | 1 | 726 | 1086 | true | 75 | 6 |
| 10 | 22 | 1 | 0 | 12 | 2 | 1 | 1160 | 1520 | true | 25 | 32 |
| 11 | 23 | 1 | 0 | 12 | 2 | 1 | 1086 | 1446 | true | 62 | 42 |
| 12 | 24 | 1 | 0 | 12 | 2 | 1 | 1087 | 1447 | true | 62 | 22 |
| 13 | 25 | 1 | 0 | 12 | 2 | 1 | 984 | 1344 | true | 27 | 20 |
| 14 | 26 | 1 | 0 | 12 | 2 | 1 | 824 | 1184 | true | 52 | 42 |
| 15 | 27 | 1 | 0 | 12 | 2 | 1 | 507 | 867 | true | 33 | 2 |
| 16 | 28 | 1 | 0 | 12 | 2 | 1 | 555 | 915 | true | 41 | 76 |
| 17 | 29 | 1 | 0 | 12 | 2 | 1 | 814 | 1174 | true | 61 | 2 |
| 18 | 30 | 1 | 0 | 12 | 2 | 1 | 938 | 1298 | true | 44 | 5 |
| 19 | 31 | 1 | 0 | 12 | 2 | 1 | 1124 | 1484 | true | 58 | 57 |
| 20 | 32 | 1 | 0 | 12 | 2 | 1 | 904 | 1264 | true | 36 | 38 |
| 21 | 33 | 1 | 0 | 12 | 2 | 1 | 1059 | 1419 | true | 52 | 24 |
| 22 | 34 | 1 | 0 | 12 | 2 | 1 | 1110 | 1470 | true | 67 | 42 |
| 23 | 35 | 1 | 0 | 12 | 2 | 1 | 942 | 1302 | true | 37 | 13 |
| 24 | 36 | 1 | 0 | 12 | 2 | 1 | 790 | 1150 | true | 19 | 71 |
| 25 | 37 | 1 | 0 | 12 | 2 | 1 | 710 | 1070 | true | 24 | 54 |
| 26 | 38 | 1 | 0 | 12 | 2 | 1 | 535 | 895 | true | 18 | 44 |
| 27 | 39 | 1 | 0 | 12 | 2 | 1 | 870 | 1230 | true | 77 | 54 |
| 28 | 40 | 1 | 0 | 12 | 2 | 1 | 408 | 768 | true | 16 | 21 |
| 29 | 41 | 1 | 0 | 12 | 2 | 1 | 779 | 1139 | true | 5 | 8 |
| 30 | 42 | 1 | 0 | 12 | 2 | 1 | 583 | 943 | true | 30 | 7 |
| 31 | 43 | 1 | 0 | 12 | 2 | 1 | 1083 | 1443 | true | 51 | 78 |
| 32 | 44 | 1 | 0 | 12 | 2 | 1 | 1170 | 1530 | true | 75 | 16 |
| 33 | 45 | 1 | 0 | 12 | 2 | 1 | 605 | 965 | true | 61 | 10 |
| 34 | 46 | 1 | 0 | 12 | 2 | 1 | 1198 | 1558 | true | 67 | 6 |
| 35 | 47 | 1 | 0 | 12 | 2 | 1 | 590 | 950 | true | 48 | 11 |
| 36 | 48 | 1 | 0 | 12 | 2 | 1 | 384 | 744 | true | 36 | 7 |
| 37 | 49 | 1 | 0 | 12 | 2 | 1 | 369 | 729 | true | 27 | 18 |
| 38 | 50 | 1 | 0 | 12 | 2 | 1 | 725 | 1085 | true | 32 | 32 |
| 39 | 51 | 1 | 0 | 12 | 2 | 1 | 394 | 754 | true | 67 | 67 |
| 40 | 52 | 1 | 0 | 12 | 2 | 1 | 1050 | 1410 | true | 48 | 35 |
| 41 | 53 | 1 | 0 | 12 | 2 | 1 | 1069 | 1429 | true | 21 | 8 |
| 42 | 54 | 1 | 0 | 12 | 2 | 1 | 827 | 1187 | true | 48 | 27 |
| 43 | 55 | 1 | 0 | 12 | 2 | 1 | 673 | 1033 | true | 2 | 14 |
| 44 | 56 | 1 | 0 | 12 | 2 | 1 | 992 | 1352 | true | 67 | 17 |
| 45 | 57 | 1 | 0 | 12 | 2 | 1 | 446 | 806 | true | 55 | 78 |


| LPDPTW 1 ( 90 Demands) |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dmd Pnt <br> ID Ltr | Map ID | $\begin{gathered} \hline \text { Demand } \\ \text { Cargo } \\ \text { Type } \\ \hline \hline \end{gathered}$ | Demand Cargo ID | Amount Cargo Demand | Vehicle <br> Capacity <br> Req | Vehicle <br> Support <br> Capability | Open <br> Window | Close <br> Window | TDD <br> Required | Loca (x) |  |
| 46 | 58 | 1 | 0 | 12 | 2 | 1 | 777 | 1137 | true | 2 | 6 |
| 47 | 59 | 1 | 0 | 12 | 2 | 1 | 591 | 951 | true | 7 | 40 |
| 48 | 60 | 1 | 0 | 12 | 2 | 1 | 1082 | 1442 | true | 73 | 48 |
| 49 | 61 | 1 | 0 | 12 | 2 | 1 | 802 | 1162 | true | 77 | 47 |
| 50 | 62 | 1 | 0 | 12 | 2 | 1 | 919 | 1279 | true | 44 | 35 |
| 51 | 63 | 1 | 0 | 12 | 2 | 1 | 717 | 1077 | true | 28 | 31 |
| 52 | 64 | 1 | 0 | 12 | 2 | 1 | 704 | 1064 | true | 65 | 20 |
| 53 | 65 | 1 | 0 | 12 | 2 | 1 | 1150 | 1510 | true | 16 | 3 |
| 54 | 66 | 1 | 0 | 12 | 2 | 1 | 449 | 809 | true | 66 | 65 |
| 55 | 67 | 1 | 0 | 12 | 2 | 1 | 778 | 1138 | true | 60 | 13 |
| 56 | 68 | 1 | 0 | 12 | 2 | 1 | 955 | 1315 | true | 1 | 12 |
| 57 | 69 | 1 | 0 | 12 | 2 | 1 | 494 | 854 | true | 36 | 14 |
| 58 | 70 | 1 | 0 | 12 | 2 | 1 | 850 | 1210 | true | 9 | 39 |
| 59 | 71 | 1 | 0 | 12 | 2 | 1 | 789 | 1149 | true | 43 | 26 |
| 60 | 72 | 1 | 0 | 12 | 2 | 1 | 851 | 1211 | true | 40 | 75 |
| 61 | 73 | 1 | 0 | 12 | 2 | 1 | 639 | 999 | true | 40 | 45 |
| 62 | 74 | 1 | 0 | 12 | 2 | 1 | 563 | 923 | true | 50 | 51 |
| 63 | 75 | 1 | 0 | 12 | 2 | 1 | 810 | 1170 | true | 32 | 78 |
| 64 | 76 | 1 | 0 | 12 | 2 | 1 | 843 | 1203 | true | 23 | 73 |
| 65 | 77 | 1 | 0 | 12 | 2 | 1 | 591 | 951 | true | 12 | 70 |
| 66 | 78 | 1 | 0 | 12 | 2 | 1 | 1086 | 1446 | true | 30 | 45 |
| 67 | 79 | 1 | 0 | 12 | 2 | 1 | 1099 | 1459 | true | 26 | 59 |
| 68 | 80 | 1 | 0 | 12 | 2 | 1 | 1198 | 1558 | true | 29 | 68 |
| 69 | 81 | 1 | 0 | 12 | 2 | 1 | 931 | 1291 | true | 25 | 28 |
| 70 | 82 | 1 | 0 | 12 | 2 | 1 | 1168 | 1528 | true | 26 | 27 |
| 71 | 83 | 1 | 0 | 12 | 2 | 1 | 608 | 968 | true | 53 | 20 |
| 72 | 84 | 1 | 0 | 12 | 2 | 1 | 942 | 1302 | true | 50 | 43 |
| 73 | 85 | 1 | 0 | 12 | 2 | 1 | 897 | 1257 | true | 67 | 71 |
| 74 | 86 | 1 | 0 | 12 | 2 | 1 | 728 | 1088 | true | 70 | 55 |
| 75 | 87 | 1 | 0 | 12 | 2 | 1 | 1128 | 1488 | true | 45 | 27 |
| 76 | 88 | 1 | 0 | 12 | 2 | 1 | 754 | 1114 | true | 49 | 25 |
| 77 | 89 | 1 | 0 | 12 | 2 | 1 | 388 | 748 | true | 59 | 46 |
| 78 | 90 | 1 | 0 | 12 | 2 | 1 | 859 | 1219 | true | 10 | 69 |
| 79 | 91 | 1 | 0 | 12 | 2 | 1 | 781 | 1141 | true | 30 | 65 |
| 80 | 92 | 1 | 0 | 12 | 2 | 1 | 886 | 1246 | true | 79 | 58 |
| 81 | 93 | 1 | 0 | 12 | 2 | 1 | 1066 | 1426 | true | 58 | 74 |
| 82 | 94 | 1 | 0 | 12 | 2 | 1 | 1157 | 1517 | true | 10 | 32 |
| 83 | 95 | 1 | 0 | 12 | 2 | 1 | 733 | 1093 | true | 61 | 34 |
| 84 | 96 | 1 | 0 | 12 | 2 | 1 | 736 | 1096 | true | 10 | 43 |
| 85 | 97 | 1 | 0 | 12 | 2 | 1 | 1186 | 1546 | true | 62 | 7 |
| 86 | 98 | 1 | 0 | 12 | 2 | 1 | 1000 | 1360 | true | 73 | 25 |
| 87 | 99 | 1 | 0 | 12 | 2 | 1 | 737 | 1097 | true | 66 | 32 |
| 88 | 100 | 1 | 0 | 12 | 2 | 1 | 369 | 729 | true | 15 | 56 |
| 89 | 101 | 1 | 0 | 12 | 2 | 1 | 432 | 792 | true | 73 | 23 |


| LPDPTW 2 (180 Demands) |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dmd Pnt |  | Demand Cargo | $\begin{gathered} \hline \text { Demand } \\ \text { Cargo } \end{gathered}$ | Amount Cargo | Vehicle <br> Capacity | Vehicle <br> Support | Open |  | TDD | Location Coord |  |
| ID Ltr | Map ID | Type | ID | Demand | Req | Capability | Window | Window | Required | (x) | (y) |
| 0 | 12 | 1 | 0 | 6 | 2 | 1 | 120 | 580 | true | 42 | 14 |
| 1 | 13 | 1 | 0 | 6 | 2 | 1 | 120 | 580 | true | 21 | 0 |
| 2 | 14 | 1 | 0 | 6 | 2 | 1 | 122 | 582 | true | 77 | 25 |
| 3 | 15 | 1 | 0 | 6 | 2 | 1 | 124 | 584 | true | 28 | 22 |
| 4 | 16 | 1 | 0 | 6 | 2 | 1 | 131 | 591 | true | 17 | 65 |
| 5 | 17 | 1 | 0 | 6 | 2 | 1 | 131 | 591 | true | 56 | 0 |
| 6 | 18 | 1 | 0 | 6 | 2 | 1 | 152 | 612 | true | 51 | 60 |
| 7 | 19 | 1 | 0 | 6 | 2 | 1 | 153 | 613 | true | 24 | 42 |
| 8 | 20 | 1 | 0 | 6 | 2 | 1 | 160 | 620 | true | 39 | 79 |
| 9 | 21 | 1 | 0 | 6 | 2 | 1 | 165 | 625 | true | 46 | 56 |
| 10 | 22 | 1 | 0 | 6 | 2 | 1 | 178 | 638 | true | 67 | 43 |
| 11 | 23 | 1 | 0 | 6 | 2 | 1 | 182 | 642 | true | 67 | 15 |
| 12 | 24 | 1 | 0 | 6 | 2 | 1 | 187 | 647 | true | 63 | 4 |
| 13 | 25 | 1 | 0 | 6 | 2 | 1 | 189 | 649 | true | 47 | 7 |
| 14 | 26 | 1 | 0 | 6 | 2 | 1 | 191 | 651 | true | 7 | 33 |
| 15 | 27 | 1 | 0 | 6 | 2 | 1 | 193 | 653 | true | 32 | 54 |
| 16 | 28 | 1 | 0 | 6 | 2 | 1 | 202 | 662 | true | 24 | 62 |
| 17 | 29 | 1 | 0 | 6 | 2 | 1 | 209 | 669 | true | 74 | 52 |
| 18 | 30 | 1 | 0 | 6 | 2 | 1 | 213 | 673 | true | 51 | 57 |
| 19 | 31 | 1 | 0 | 6 | 2 | 1 | 214 | 674 | true | 57 | 40 |
| 20 | 32 | 1 | 0 | 6 | 2 | 1 | 217 | 677 | true | 77 | 60 |
| 21 | 33 | 1 | 0 | 6 | 2 | 1 | 225 | 685 | true | 62 | 32 |
| 22 | 34 | 1 | 0 | 6 | 2 | 1 | 229 | 689 | true | 5 | 6 |
| 23 | 35 | 1 | 0 | 6 | 2 | 1 | 232 | 692 | true | 64 | 30 |
| 24 | 36 | 1 | 0 | 6 | 2 | 1 | 237 | 697 | true | 52 | 13 |
| 25 | 37 | 1 | 0 | 6 | 2 | 1 | 239 | 699 | true | 40 | 33 |
| 26 | 38 | 1 | 0 | 6 | 2 | 1 | 245 | 705 | true | 71 | 77 |
| 27 | 39 | 1 | 0 | 6 | 2 | 1 | 252 | 712 | true | 68 | 67 |
| 28 | 40 | 1 | 0 | 6 | 2 | 1 | 255 | 715 | true | 65 | 41 |
| 29 | 41 | 1 | 0 | 6 | 2 | 1 | 258 | 718 | true | 53 | 7 |
| 30 | 42 | 1 | 0 | 6 | 2 | 1 | 261 | 721 | true | 17 | 18 |
| 31 | 43 | 1 | 0 | 6 | 2 | 1 | 265 | 725 | true | 41 | 70 |
| 32 | 44 | 1 | 0 | 6 | 2 | 1 | 272 | 732 | true | 10 | 58 |
| 33 | 45 | 1 | 0 | 6 | 2 | 1 | 275 | 735 | true | 56 | 51 |
| 34 | 46 | 1 | 0 | 6 | 2 | 1 | 294 | 754 | true | 7 | 9 |
| 35 | 47 | 1 | 0 | 6 | 2 | 1 | 306 | 766 | true | 10 | 76 |
| 36 | 48 | 1 | 0 | 6 | 2 | 1 | 315 | 775 | true | 64 | 6 |
| 37 | 49 | 1 | 0 | 6 | 2 | 1 | 316 | 776 | true | 57 | 23 |
| 38 | 50 | 1 | 0 | 6 | 2 | 1 | 316 | 776 | true | 50 | 71 |
| 39 | 51 | 1 | 0 | 6 | 2 | 1 | 321 | 781 | true | 52 | 66 |
| 40 | 52 | 1 | 0 | 6 | 2 | 1 | 324 | 784 | true | 74 | 27 |
| 41 | 53 | 1 | 0 | 6 | 2 | 1 | 326 | 786 | true | 75 | 20 |
| 42 | 54 | 1 | 0 | 6 | 2 | 1 | 333 | 793 | true | 68 | 23 |
| 43 | 55 | 1 | 0 | 6 | 2 | 1 | 338 | 798 | true | 56 | 40 |
| 44 | 56 | 1 | 0 | 6 | 2 | 1 | 339 | 799 | true | 53 | 35 |
| 45 | 57 | 1 | 0 | 6 | 2 | 1 | 340 | 800 | true | 3 | 76 |
| 46 | 58 | 1 | 0 | 6 | 2 | 1 | 340 | 800 | true | 54 | 65 |
| 47 | 59 | 1 | 0 | 6 | 2 | 1 | 345 | 805 | true | 34 | 45 |
| 48 | 60 | 1 | 0 | 6 | 2 | 1 | 345 | 805 | true | 35 | 24 |
| 49 | 61 | 1 | 0 | 6 | 2 | 1 | 347 | 807 | true | 10 | 54 |


| LPDPTW 2 (180 Demands) |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dmd Pnt |  | Demand Cargo | Demand Cargo | Amount Cargo | Vehicle <br> Capacity | Vehicle <br> Support | Open | Close | TDD | Location Coord |  |
| ID Ltr | Map ID | Type | ID | Demand | Req | Capability | Window | Window | Required | (x) | (y) |
| 50 | 62 | 1 | 0 | 6 | 2 | 1 | 360 | 820 | true | 59 | 43 |
| 51 | 63 | 1 | 0 | 6 | 2 | 1 | 360 | 820 | true | 53 | 29 |
| 52 | 64 | 1 | 0 | 6 | 2 | 1 | 362 | 822 | true | 53 | 32 |
| 53 | 65 | 1 | 0 | 6 | 2 | 1 | 364 | 824 | true | 9 | 5 |
| 54 | 66 | 1 | 0 | 6 | 2 | 1 | 367 | 827 | true | 1 | 35 |
| 55 | 67 | 1 | 0 | 6 | 2 | 1 | 373 | 833 | true | 73 | 67 |
| 56 | 68 | 1 | 0 | 6 | 2 | 1 | 373 | 833 | true | 35 | 71 |
| 57 | 69 | 1 | 0 | 6 | 2 | 1 | 376 | 836 | true | 72 | 5 |
| 58 | 70 | 1 | 0 | 6 | 2 | 1 | 376 | 836 | true | 8 | 10 |
| 59 | 71 | 1 | 0 | 6 | 2 | 1 | 377 | 837 | true | 16 | 38 |
| 60 | 72 | 1 | 0 | 6 | 2 | 1 | 384 | 844 | true | 77 | 5 |
| 61 | 73 | 1 | 0 | 6 | 2 | 1 | 388 | 848 | true | 7 | 66 |
| 62 | 74 | 1 | 0 | 6 | 2 | 1 | 388 | 848 | true | 66 | 52 |
| 63 | 75 | 1 | 0 | 6 | 2 | 1 | 390 | 850 | true | 3 | 40 |
| 64 | 76 | 1 | 0 | 6 | 2 | 1 | 391 | 851 | true | 56 | 21 |
| 65 | 77 | 1 | 0 | 6 | 2 | 1 | 393 | 853 | true | 15 | 21 |
| 66 | 78 | 1 | 0 | 6 | 2 | 1 | 412 | 872 | true | 79 | 50 |
| 67 | 79 | 1 | 0 | 6 | 2 | 1 | 416 | 876 | true | 75 | 43 |
| 68 | 80 | 1 | 0 | 6 | 2 | 1 | 425 | 885 | true | 7 | 16 |
| 69 | 81 | 1 | 0 | 6 | 2 | 1 | 427 | 887 | true | 66 | 56 |
| 70 | 82 | 1 | 0 | 6 | 2 | 1 | 442 | 902 | true | 34 | 2 |
| 71 | 83 | 1 | 0 | 6 | 2 | 1 | 445 | 905 | true | 75 | 40 |
| 72 | 84 | 1 | 0 | 6 | 2 | 1 | 449 | 909 | true | 8 | 53 |
| 73 | 85 | 1 | 0 | 6 | 2 | 1 | 456 | 916 | true | 24 | 71 |
| 74 | 86 | 1 | 0 | 6 | 2 | 1 | 469 | 929 | true | 44 | 49 |
| 75 | 87 | 1 | 0 | 6 | 2 | 1 | 482 | 942 | true | 37 | 52 |
| 76 | 88 | 1 | 0 | 6 | 2 | 1 | 500 | 960 | true | 51 | 20 |
| 77 | 89 | 1 | 0 | 6 | 2 | 1 | 501 | 961 | true | 16 | 68 |
| 78 | 90 | 1 | 0 | 6 | 2 | 1 | 513 | 973 | true | 77 | 49 |
| 79 | 91 | 1 | 0 | 6 | 2 | 1 | 517 | 977 | true | 66 | 65 |
| 80 | 92 | 1 | 0 | 6 | 2 | 1 | 517 | 977 | true | 52 | 35 |
| 81 | 93 | 1 | 0 | 6 | 2 | 1 | 521 | 981 | true | 22 | 55 |
| 82 | 94 | 1 | 0 | 6 | 2 | 1 | 538 | 998 | true | 59 | 60 |
| 83 | 95 | 1 | 0 | 6 | 2 | 1 | 542 | 1002 | true | 58 | 49 |
| 84 | 96 | 1 | 0 | 6 | 2 | 1 | 549 | 1009 | true | 5 | 36 |
| 85 | 97 | 1 | 0 | 6 | 2 | 1 | 553 | 1013 | true | 60 | 59 |
| 86 | 98 | 1 | 0 | 6 | 2 | 1 | 556 | 1016 | true | 53 | 16 |
| 87 | 99 | 1 | 0 | 6 | 2 | 1 | 558 | 1018 | true | 67 | 53 |
| 88 | 100 | 1 | 0 | 6 | 2 | 1 | 559 | 1019 | true | 61 | 43 |
| 89 | 101 | 1 | 0 | 6 | 2 | 1 | 559 | 1019 | true | 15 | 22 |
| 90 | 102 | 1 | 0 | 6 | 2 | 1 | 562 | 1022 | true | 38 | 23 |


| LPDPTW 2 (180 Demands) |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dmd Pnt |  | Demand Cargo | $\begin{gathered} \hline \text { Demand } \\ \text { Cargo } \end{gathered}$ | Amount Cargo | Vehicle <br> Capacity | Vehicle <br> Support | Open | Close | TDD | Location Coord |  |
| ID Ltr | Map ID | Type | ID | Demand | Req | Capability | Window | Window | Required | (x) | (y) |
| 91 | 103 | 1 | 0 | 6 | 2 | , | 571 | 1031 | true | 5 | 65 |
| 92 | 104 | 1 | 0 | 6 | 2 | 1 | 579 | 1039 | true | 47 | 31 |
| 93 | 105 | 1 | 0 | 6 | 2 | 1 | 582 | 1042 | true | 26 | 35 |
| 94 | 106 | 1 | 0 | 6 | 2 | 1 | 583 | 1043 | true | 27 | 33 |
| 95 | 107 | 1 | 0 | 6 | 2 | 1 | 584 | 1044 | true | 3 | 29 |
| 96 | 108 | 1 | 0 | 6 | 2 | 1 | 585 | 1045 | true | 50 | 34 |
| 97 | 109 | 1 | 0 | 6 | 2 | 1 | 591 | 1051 | true | 53 | 56 |
| 98 | 110 | 1 | 0 | 6 | 2 | 1 | 596 | 1056 | true | 34 | 53 |
| 99 | 111 | 1 | 0 | 6 | 2 | 1 | 597 | 1057 | true | 9 | 9 |
| 100 | 112 | 1 | 0 | 6 | 2 | 1 | 613 | 1073 | true | 76 | 11 |
| 101 | 113 | 1 | 0 | 6 | 2 | 1 | 623 | 1083 | true | 64 | 76 |
| 102 | 114 | 1 | 0 | 6 | 2 | 1 | 623 | 1083 | true | 54 | 28 |
| 103 | 115 | 1 | 0 | 6 | 2 | 1 | 632 | 1092 | true | 24 | 38 |
| 104 | 116 | 1 | 0 | 6 | 2 | 1 | 632 | 1092 | true | 74 | 72 |
| 105 | 117 | 1 | 0 | 6 | 2 | 1 | 634 | 1094 | true | 20 | 54 |
| 106 | 118 | 1 | 0 | 6 | 2 | 1 | 639 | 1099 | true | 62 | 57 |
| 107 | 119 | 1 | 0 | 6 | 2 | 1 | 642 | 1102 | true | 14 | 67 |
| 108 | 120 | 1 | 0 | 6 | 2 | 1 | 645 | 1105 | true | 12 | 18 |
| 109 | 121 | 1 | 0 | 6 | 2 | 1 | 655 | 1115 | true | 4 | 1 |
| 110 | 122 | 1 | 0 | 6 | 2 | 1 | 658 | 1118 | true | 41 | 35 |
| 111 | 123 | 1 | 0 | 6 | 2 | 1 | 660 | 1120 | true | 9 | 24 |
| 112 | 124 | 1 | 0 | 6 | 2 | 1 | 663 | 1123 | true | 31 | 65 |
| 113 | 125 | 1 | 0 | 6 | 2 | 1 | 665 | 1125 | true | 29 | 48 |
| 114 | 126 | 1 | 0 | 6 | 2 | 1 | 666 | 1126 | true | 4 | 30 |
| 115 | 127 | 1 | 0 | 6 | 2 | 1 | 668 | 1128 | true | 29 | 49 |
| 116 | 128 | 1 | 0 | 6 | 2 | 1 | 671 | 1131 | true | 53 | 9 |
| 117 | 129 | 1 | 0 | 6 | 2 | 1 | 673 | 1133 | true | 10 | 45 |
| 118 | 130 | 1 | 0 | 6 | 2 | 1 | 679 | 1139 | true | 65 | 28 |
| 119 | 131 | 1 | 0 | 6 | 2 | 1 | 681 | 1141 | true | 44 | 48 |
| 120 | 132 | 1 | 0 | 6 | 2 | 1 | 687 | 1147 | true | 9 | 38 |
| 121 | 133 | 1 | 0 | 6 | 2 | 1 | 689 | 1149 | true | 48 | 60 |
| 122 | 134 | 1 | 0 | 6 | 2 | 1 | 711 | 1171 | true | 22 | 35 |
| 123 | 135 | 1 | 0 | 6 | 2 | 1 | 714 | 1174 | true | 4 | 31 |
| 124 | 136 | 1 | 0 | 6 | 2 | 1 | 714 | 1174 | true | 67 | 76 |
| 125 | 137 | 1 | 0 | 6 | 2 | 1 | 718 | 1178 | true | 61 | 20 |
| 126 | 138 | 1 | 0 | 6 | 2 | 1 | 720 | 1180 | true | 34 | 44 |
| 127 | 139 | 1 | 0 | 6 | 2 | 1 | 724 | 1184 | true | 31 | 18 |
| 128 | 140 | 1 | 0 | 6 | 2 | 1 | 733 | 1193 | true | 61 | 33 |
| 129 | 141 | 1 | 0 | 6 | 2 | 1 | 736 | 1196 | true | 34 | 71 |
| 130 | 142 | 1 | 0 | 6 | 2 | 1 | 744 | 1204 | true | 53 | 10 |
| 131 | 143 | 1 | 0 | 6 | 2 | 1 | 756 | 1216 | true | 67 | 25 |
| 132 | 144 | 1 | 0 | 6 | 2 | 1 | 763 | 1223 | true | 20 | 10 |
| 133 | 145 | 1 | 0 | 6 | 2 | 1 | 768 | 1228 | true | 73 | 42 |
| 134 | 146 | 1 | 0 | 6 | 2 | 1 | 772 | 1232 | true | 37 | 64 |
| 135 | 147 | 1 | 0 | 6 | 2 | 1 | 775 | 1235 | true | 33 | 68 |


| LPDPTW 2 (180 Demands) |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dmd Pnt |  | Demand Cargo | Demand Cargo | Amount Cargo | Vehicle Capacity | Vehicle Support | Open | Close | TDD | Location Coord |  |
| ID Ltr | Map ID | Type | ID | Demand | Req | Capability | Window | Window | Required | (x) | (y) |
| 136 | 148 | 1 | 0 | 6 | 2 | 1 | 776 | 1236 | true | 51 | 52 |
| 137 | 149 | 1 | 0 | 6 | 2 | 1 | 787 | 1247 | true | 78 | 11 |
| 138 | 150 | 1 | 0 | 6 | 2 | 1 | 792 | 1252 | true | 30 | 15 |
| 139 | 151 | 1 | 0 | 6 | 2 | 1 | 792 | 1252 | true | 16 | 64 |
| 140 | 152 | 1 | 0 | 6 | 2 | 1 | 795 | 1255 | true | 30 | 58 |
| 141 | 153 | 1 | 0 | 6 | 2 | 1 | 800 | 1260 | true | 75 | 50 |
| 142 | 154 | 1 | 0 | 6 | 2 | 1 | 816 | 1276 | true | 20 | 7 |
| 143 | 155 | 1 | 0 | 6 | 2 | 1 | 824 | 1284 | true | 67 | 12 |
| 144 | 156 | 1 | 0 | 6 | 2 | 1 | 830 | 1290 | true | 47 | 17 |
| 145 | 157 | 1 | 0 | 6 | 2 | 1 | 857 | 1317 | true | 49 | 34 |
| 146 | 158 | 1 | 0 | 6 | 2 | 1 | 863 | 1323 | true | 78 | 24 |
| 147 | 159 | 1 | 0 | 6 | 2 | 1 | 865 | 1325 | true | 23 | 35 |
| 148 | 160 | 1 | 0 | 6 | 2 | 1 | 867 | 1327 | true | 7 | 60 |
| 149 | 161 | 1 | 0 | 6 | 2 | 1 | 869 | 1329 | true | 5 | 42 |
| 150 | 162 | 1 | 0 | 6 | 2 | 1 | 870 | 1330 | true | 21 | 21 |
| 151 | 163 | 1 | 0 | 6 | 2 | 1 | 875 | 1335 | true | 72 | 52 |
| 152 | 164 | 1 | 0 | 6 | 2 | 1 | 883 | 1343 | true | 29 | 27 |
| 153 | 165 | 1 | 0 | 6 | 2 | 1 | 888 | 1348 | true | 22 | 34 |
| 154 | 166 | 1 | 0 | 6 | 2 | 1 | 889 | 1349 | true | 26 | 21 |
| 155 | 167 | 1 | 0 | 6 | 2 | 1 | 894 | 1354 | true | 27 | 61 |
| 156 | 168 | 1 | 0 | 6 | 2 | 1 | 909 | 1369 | true | 68 | 72 |
| 157 | 169 | 1 | 0 | 6 | 2 | 1 | 910 | 1370 | true | 9 | 31 |
| 158 | 170 | 1 | 0 | 6 | 2 | 1 | 918 | 1378 | true | 68 | 57 |
| 159 | 171 | 1 | 0 | 6 | 2 | 1 | 921 | 1381 | true | 14 | 66 |
| 160 | 172 | 1 | 0 | 6 | 2 | 1 | 924 | 1384 | true | 23 | 25 |
| 161 | 173 | 1 | 0 | 6 | 2 | 1 | 929 | 1389 | true | 62 | 39 |
| 162 | 174 | 1 | 0 | 6 | 2 | 1 | 932 | 1392 | true | 25 | 77 |
| 163 | 175 | 1 | 0 | 6 | 2 | 1 | 934 | 1394 | true | 19 | 18 |
| 164 | 176 | 1 | 0 | 6 | 2 | 1 | 935 | 1395 | true | 50 | 0 |
| 165 | 177 | 1 | 0 | 6 | 2 | 1 | 951 | 1411 | true | 42 | 70 |
| 166 | 178 | 1 | 0 | 6 | 2 | 1 | 962 | 1422 | true | 14 | 53 |
| 167 | 179 | 1 | 0 | 6 | 2 | 1 | 968 | 1428 | true | 44 | 55 |
| 168 | 180 | 1 | 0 | 6 | 2 | 1 | 968 | 1428 | true | 9 | 37 |
| 169 | 181 | 1 | 0 | 6 | 2 | 1 | 975 | 1435 | true | 65 | 21 |
| 170 | 182 | 1 | 0 | 6 | 2 | 1 | 980 | 1440 | true | 1 | 18 |
| 171 | 183 | 1 | 0 | 6 | 2 | 1 | 982 | 1442 | true | 10 | 5 |
| 172 | 184 | 1 | 0 | 6 | 2 | 1 | 988 | 1448 | true | 78 | 79 |
| 173 | 185 | 1 | 0 | 6 | 2 | 1 | 991 | 1451 | true | 56 | 72 |
| 174 | 186 | 1 | 0 | 6 | 2 | 1 | 993 | 1453 | true | 30 | 20 |
| 175 | 187 | 1 | 0 | 6 | 2 | 1 | 997 | 1457 | true | 42 | 36 |
| 176 | 188 | 1 | 0 | 6 | 2 | 1 | 997 | 1457 | true | 4 | 32 |
| 177 | 189 | 1 | 0 | 6 | 2 | 1 | 997 | 1457 | true | 42 | 77 |
| 178 | 190 | 1 | 0 | 6 | 2 | 1 | 1009 | 1469 | true | 45 | 31 |
| 179 | 191 | 1 | 0 | 6 | 2 | 1 | 1016 | 1476 | true | 19 | 6 |

Appendix E ATS Comparison to Optimal and Lower Bound Solutions

| Optimal Test Problem Set |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Problem | Optimal | ATS | \%dist <br> Delta | ATS Selected Number of |  |  |
|  |  |  | Depots | Supplies | Vehs |  |
| 100 | 1205 | 1205 | 0.00 | 1 | 1 | 1 |
| 101 | 1255 | 1255 | 0.00 | 1 | 1 | 1 |
| 102 | 1537 | 1537 | 0.00 | 1 | 1 | 2 |
| 103 | 1724 | 1724 | 0.00 | 2 | 2 | 2 |
| 104 | 3030 | 3105 | 2.48 | 2 | 2 | 2 |
| 105 | 6260 | 6260 | 0.00 | 2 | 2 | 3 |
| 106 | 2187 | 2187 | 0.00 | 2 | 2 | 2 |
| 107 | 3273 | 3328 | 1.68 | 2 | 2 | 2 |
| 108 | 1255 | 1255 | 0.00 | 1 | 1 | 1 |
| 109 | 1920 | 1920 | 0.00 | 1 | 1 | 2 |
| 110 | 915 | 915 | 0.00 | 1 | 1 | 1 |
| 111 | 1365 | 1365 | 0.00 | 1 | 1 | 1 |
| 112 | 2090 | 2090 | 0.00 | 2 | 2 | 2 |
| 113 | 2540 | 2540 | 0.00 | 2 | 2 | 2 |
| 114 | 3560 | 3560 | 0.00 | 3 | 3 | 3 |
| 115 | 3455 | 3455 | 0.00 | 1 | 1 | 3 |
| 116 | 6120 | 6120 | 0.00 | 2 | 2 | 4 |
| 117 | 6150 | 6150 | 0.00 | 2 | 2 | 4 |
| 118 | 4740 | 4740 | 0.00 | 2 | 2 | 2 |
| 119 | 5905 | 5905 | 0.00 | 2 | 2 | 3 |
| 120 | 6120 | 6120 | 0.00 | 2 | 2 | 3 |
| 121 | 5090 | 5090 | 0.00 | 2 | 2 | 2 |
| 122 | 2961 | 2996 | 1.18 | 2 | 2 | 2 |
| 123 | 4410 | 4410 | 0.00 | 1 | 2 | 2 |
| 124 | 2915 | 2915 | 0.00 | 1 | 1 | 1 |
|  |  |  |  |  |  |  |

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## Vita

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He is currently assigned as a mathematics instructor at the United States Military Academy at West Point.


