Modeling and Control Design of Quadcopter Motor

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Abstract

This paper discussed the modeling and control design of quadcopter motor. Modeling was performed using the Laplace transform method. The quadcopter mathematical model was used as the load torque of brushless dc motor to control the stability of altitude vertical take-off motion. Motion stability was influenced by the body frame mass, propeller speed, and wind disturbance during hovering. Hence, the quadcopter motion experienced oscillation to reach a steady state. The tuning of Ziegler-Nichols PID and fuzzy gain scheduling PID were suggested to overcome the stability of altitude vertical take-off motion. The simulation results showed that fuzzy gain scheduling PID reduced the overshoot 0.1% smaller than the Ziegler-Nichols tuning.

Keywords
Brushless dc Motor, Quadcopter, Ziegler-Nichols, Fuzzy Gain Scheduling PID.

1. Introduction

The modeling and control design of the quadcopter motor was conducted using the Laplace Transform method. The mathematical model of quadcopter was used as the load torque of brushless dc motor to control the stability of altitude vertical take-off motion. The quadcopter was moved by four brushless dc motor (BLDC) and propeller. The propeller was controlled by clockwise and counterclockwise rotation to create thrust force so that the motor speed can maintain stability by concerning the body frame. The body frame (quadcopter) was designed using Solidwork, and the design was used as the load torque of the brushless dc motor.

The vertical take-off motion is the basic principle of lift setting by combining the propeller motor speed where the propeller motor speed must be higher than the body frame mass. The stability is influenced by body frame mass, Hence, the motion experiences oscillation to reach a steady state.

The stability of vertical take-off motion was obtained by adding PID control. PID was used as the parameter setting for *Kp*, *Ki*, and *Kd* based on the output system. However, parameter setting has a gain limit so that the gain should be tuned manually. This research proposed Ziegler-Nichols PID tuning and fuzzy gain scheduling PID to overcome the stability of vertical take-off quadcopter motion. The control design was modeled based on the motor and quadcopter mathematical model. Then, the control design was validated using Matlab Simulink.

2. Related Research

Four brushless dc motor was modeled systematically. Wind disturbance affected the stability of quadcopter motion. The PID controller was suggested and quadcopter was designed by using Solidwork. The simulation results showed that the quadcopter model reduced noise disturbance in the lift force [1]. The fuzzy self-tuning PID control was used to control the altitude quadcopter. PID control had limitations in the search for parameter values. This paper suggested self-tuning PID control the gain parameter of *Kp*, *Ki*, and *Kd*. The simulation results showed that the parameters were obtained accurately during the searching [2].

The performance of the two brushless dc motor control techniques was compared. PID self-tuning control and references adaptive control (MRAC) models with the PID compensator. Control was reached at different speeds, and load interference. The simulation showed that the design of two controls had an excellent performance on the system [3]. The mathematical model was used to set the plan in different settings processes. Dynamic modeling was suggested in this research by implementing different control performance of PI and fuzzy logic. The simulation showed that the gain scheduling fuzzy PI logic gave a better performance than conventional control [4].

This paper suggested fuzzy logic based on the PID control system for the altitude quad-rotor stability. Fuzzy logic was used to set the PID parameter values efficiently. The simulation results explained that fuzzy logic based on the PID had better stability and rise time compared to conventional control [5]. The control design was used to control the vertical take-off. The altitude quadcopter control was obtained using the Euler-Newton equation. PID controller was proposed to control vertical take-off quadcopter. The simulation was conducted using Matlab Simulink [6]. This paper discussed the development of a fuzzy reinforcement schedule scheme from PID control. Fuzzy control was used to determine the parameter value based on the

error signal and the initial error signal difference. The simulation results showed that the fuzzy gain scheduling PID control gave a quicker response than the Ziegler Nichols and Kitamori's PID control [7].

2.1 Theories

Dc motor converts electrical energy into mechanical energy. Dc motor is classified into two: brushless dc and brushed dc. Each motor has six essential parts: rotor (armature), stator, commutator, field magnet, and brushed. The magnetic field is created by the permanent magnet, whereas the overall electrical current creates other magnetic fields in the motor winding [8]. The mathematical model of the dc motor consists of R resistance and L armature inductor. Meanwhile, the back-EMF coefficient is represented by e [9].
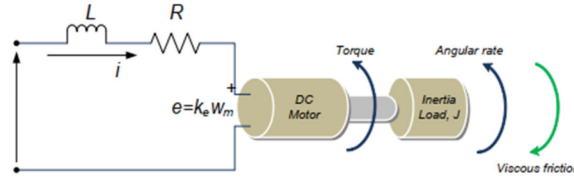

Figure 1. The equivalent circuit of a brushless dc motor

2.2 Modeling System

The modeling system was used to find the load torque and inertia speed in the brushless dc motor. In addition to modeling the system, the characteristics of a brushless dc motor can be calculated with the following equation [9].
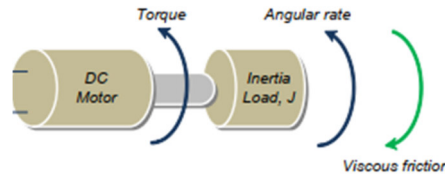

Figure 2. The inertia of DC brushless motor

By ignoring mechanical losses, overall electromagnetic power becomes kinetic energy [11].

$$P_e = T_e \Omega \tag{1}$$

Where:
T_e = Electromagnetic torque
Ω = Rotational angle speed
P   = Number of pole
The spring equation at torque is as follows.

$$T(t) = K\theta(t) \tag{2}$$

The torsion viscous damper equation is as follows.

$$T(t) = B\frac{d\theta(t)}{dt} \tag{3}$$

The electromagnetic torque equation is as follows.

$$T_e = 2p\psi_m i_A = K_T i \tag{4}$$

Where:
K_T = Constant torque coefficient
i   = Phase current
So the overall system modeling of electromagnetic torque and the equation of motion is explained in the equation (5).

$$T_e - T_L = j\frac{d\Omega}{dt} + B_V\Omega \tag{5}$$

The equation of inertia (acceleration).

$$T(t) = j\frac{d^2\theta(t)}{dt^2} \tag{6}$$

Where:
T_L = Load torque
j   = Rotor inertia moment
B, B_V = Friction coefficient

2.3 Kirchhoff's Voltage Law

Kirchhoff's voltage law is applied in a close circuit equal to zero. Or the sum of each R and L equals zero [10].
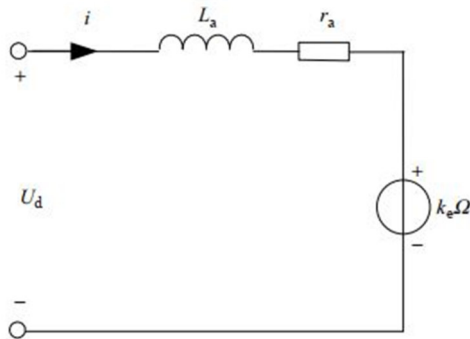

Figure 3. Brushless dc motor equivalent circuit

$$V = R_a . I_a + L\frac{di}{dt} + E \tag{7}$$

$$E. \phi. \omega = K_b. \phi. \omega \tag{8}$$

$$\omega = \frac{d\theta}{dt} \tag{9}$$

$$E = K_b. \omega \tag{10}$$

$$E = K_b . \frac{d\theta}{dt} \tag{11}$$

$$T. \phi. I_a = \phi. K_T. I_a = K_T. I_a \tag{12}$$

$$T = K_T. I_a \tag{13}$$

$$T = j\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \tag{14}$$

Where:

R_a	= Armature resistance
L_a	= Armature inductance
i_a	= Armature current
E_a	= Back EMF induction
V, U_d	= Voltage
T	= Torque
θ	= Angular displacement
j	= Rotor inertia moment
K_T	= Constant torque coefficient
$K_b K_e$	= Back EMF coefficient
ω	= Angular velocity
Ω	= Rotor speed
T_L	= Load torque
T_e	= Electromagnetic torque

2.4 The Laplace Transform

The Laplace transform was applied to the brushless DC motor electric circuit and quadcopter mechanics so the equation can be calculated as follows.

Laplace brushless dc motor transformation in the equation (7—14).

$$V(s) = R_a. I_a(s) + L_a(s)I_a(s) + E(s) \tag{15}$$

$$V(s) - E(s) = I_a(s)[R_a + L_a(s)] \tag{16}$$

$$E(s) = K_b(s)\theta(s) \tag{17}$$

$$T(s) = K_T I_a(s) \tag{18}$$

$$T(s) = (s)^2 j\theta(s) + B(s)\theta(s) \tag{19}$$

$$T(s) = s\theta(s)[(s)j + B] \tag{20}$$

$$I_a(s) = \frac{V(s) - E(s)}{R_a + L_a s} \tag{21}$$

$$E(s) = K_b \Omega(s) \tag{22}$$

Laplace system modeling transformation in equation (2, 3, and 6).

$$T(s) = [j(s)^2 + B(s) + K]\theta \tag{23}$$

$$\omega = \frac{d\theta}{dt} = \theta rad/s \tag{24}$$

$$T(s) = [Js + B]\Omega(s) \tag{25}$$

$$\Omega(s) = \frac{T(s)}{js+B} = \Omega(s) = \frac{T(s)}{js+C_D} \tag{26}$$

2.5 Quadcopter Motor Equation

The quadcopter motor equation is used to produce the following torque [8].
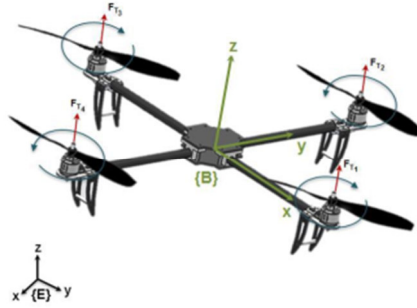

Figure 4. Quadcopter

Where:
E   = Fixed reference frame (ground)
B   = Body reference frame
F_{Ti} = Thrust force of the propeller

$$Q = K_q I \tag{27}$$

$$V = R_a I + K_e \omega \tag{28}$$

The voltage equation for the quadcopter motor power is explained in equation (29).

$$P = I.V \tag{29}$$

$$P = \frac{Q}{K_q} V \tag{30}$$

Where:
Q   = Motor torque
V   = Voltage across the motor
I   = Motor current
ω = Rotor angular speed
K_q = Torque-current relation
R_a = Motor armature resistance
K_e = Back EMF coefficient

2.6 Thrust Force

Thrust force is produced by the speed of the propeller on the motor, where the thrust force is positioned at rest on the ground. The power at the propeller is also calculated as rpm to calculate thrust. So the equation is explained as follows [8].

$$T = \frac{\pi}{4} D^2 \rho \, V \, \Delta V \tag{31}$$

Where:
T   = Thrust
D   = Diameter propeller (m)
V   = Velocity of air at propeller (m/s)
ΔV = Velocity of air accelerated by propeller (m/s)
ρ = Air density (1.225 kg/m^3)
P   = Motor power (watt)

Propeller air speed $V = \frac{1}{2}\Delta V$ is used in the quadcopter [8].

$$T = \frac{\pi}{8} D^2 \rho \, V \, \Delta V \tag{32}$$

Power on the propeller motor as follows [8].

$$P = \frac{T\Delta V}{2} => \Delta V = \frac{2P}{T} \tag{33}$$

$$T = \left(\frac{\pi}{2} D^2 \rho P^2\right)^{1/3} \tag{34}$$

The equation uses Newton second law as follows [8].

$$m = \frac{F}{a} = \frac{T}{g} = \frac{\left[\frac{\pi}{2} D^2 \rho P^2\right]^{1/3}}{g} \tag{35}$$

$$T = m. g \tag{36}$$

Where:

F   = Force (N)
T   = Thrust (N)
m   = Mass (kg. m/s^2)
g   = Gravitation (9.81 m/s^2)
D   = Propeller diameter (m)
ρ = Air density (1.225 kg/m^3)
P   = Power (watt)
a   = Object's speed

Using Newton Second Law; thrust force, gravity, and drag force are obtained in below equation [8].

$$F = m. a \tag{37}$$

$$F_{thrust} - F_{gravity} - T_{air\ drag} = m\dot{v} \tag{38}$$

Thrust is a force measured in Newton. In the motor datasheet, thrust is explained as thrust in mg. The propeller is multiplied by gravity as follows [8].

$$T = p. g \tag{39}$$

2.7 Drag Force

Drag force is determined based on the dynamic model of the motor, where I represents the air velocity, and V represents the angular speed of the motor. So that the thrust can be calculated with the angular velocity equation as follows [8].

$$F_D = \frac{1}{2}\rho V^2 A C_D \tag{40}$$

$$T = \frac{1}{2}\rho C_D A V^2 \tag{41}$$

Where:

ρ = 1.225 (Air density, Kg/m^3)
A   = πr^2 (Area swept by the propeller, m^2)
r   = 0.0635 (Radius of the propeller blade, m)
C_D = 1.3 (Coefficient of drag)
V   = (Velocity of the air by propeller blade, m/s^2)
A   = $\frac{1}{2}(MTM)^2 + 3 \times \pi \times r_{propeller}^2$
MTM   = Motor to distance motor
R_{prop} = Radius of the propeller

3. Method

This research method used Proportional-Integral-Derivative (PID), PID tuning based on Ziegler-Nichols, and Fuzzy gain scheduling PID. Modeling brushless dc motor with quadcopter load torque is explained as follows.

3.1 Proportional-Integral-Derivative

The combination of proportional-integral-derivative control was applied to the stability of the vertical take-off quadcopter movement. The PID control was used to obtain the parameter values of *Kp, Ki*, and *Kd* based on the altitude output error. If there is a change in altitude output, the sensor will read the change then compared to the altitude input. The difference in altitude input and output is the error value. Furthermore, the controller will respond to changes by adjusting the altitude output and the same altitude input. The PID control equation is explained as follows [12].

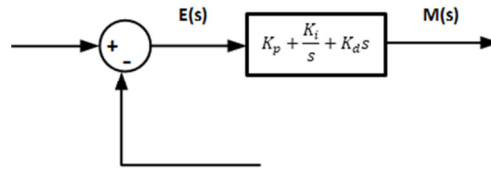$$\frac{M(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d.s \qquad (42)$$

Figure 5. Proportional Integral Derivative

3.2 Ziegler-Nichols Tuning

Ziegler-Nichols was used as the parameter tuning based on the K_u (ultimate gain) and T_u (ultimate period) step frequency response method to obtain the *Kp, Ki,* and *Kd*. PID control had a gain limitation, so it should be manually tuned. Ziegler-Nichols was designed to give a better response system if there is a change in the stability of the quadcopter movement, load disturbance, and correction of small error values. The Ziegler-Nichols tuning equation is explained as follows. Where $Ku = 8$ and $Tu = 2\pi\sqrt{3} = 3.63$ [13].

TABLE I
Ziegler-Nichols Tuning Parameters

Control	K	T_i	T_d	T_p
P	$0.5\,K_u$	-	-	T_u
PI	$0.4\,K_u$	$0.4\,K_u$	-	$1.4\,T_u$
PID	$0.6\,K_u$	$0.5\,K_u/T_u$	$0.125\,K_uT_u/8$	$0.85\,T_u$

$$K_p = 0.6 \times K_u = 4.8 \qquad (43)$$
$$K_i = 0.5 \times K_u/T_u = 1.10 \qquad (44)$$
$$K_d = 0.125 \times K_u \times T_u/8 = 0.45 \qquad (45)$$

3.3 Fuzzy Gain Scheduling PID

Fuzzy gain scheduling PID was tuned based on the Ziegler-Nichols parameter with *Kp* and *Kd* parameters. Where *Kp* and *Kd* was assumed in each range $[K_{p.min}, K_{p.max}]$ and $[K_{d.min}, K_{d.max}]$.. Next, *Kp* and *Kd* were normalized into the range of 0 and 1. The PID parameter was determined based on the current error *e(k)* and initial error difference $\Delta e(k)$. Hence, *Kp, Kd,* and α are calculated using the below equations [7].

$$K'_p = (K_p - K_{p.min})/(K_{p.max} - K_{p.min}) \qquad (46)$$
$$K'_d = (K_d - K_{d.min})/(K_{d.max} - K_{d.min}) \qquad (47)$$
$$T_i = \alpha T_d \qquad (48)$$
$$K_i = K_p/(\alpha T_d) = K_p^2/(\alpha K_d) \qquad (49)$$
$$K_p = (K_{p.max} - K_{p.min})K'_p + K_{p.min} \qquad (50)$$
$$K_d = (K_{d.max} - K_{d.min})K'_d + K_{d.min} \qquad (51)$$
$$K_i = K_p^2/(\alpha K_d) \qquad (52)$$
$$K_{p.min} = 1.76\,K_u, \ K_{p.max} = 2.48\,K_u \qquad (53)$$
$$K_{d.min} = 0.29\,K_uT_u, \ K_{d.max} = 0.58\,K_uT_u \qquad (54)$$

3.4 Fuzzification

Fuzzification converts numeric input into languages with the membership function. There are two inputs to control error (e) and change in error value (du / dt) of the signal [-1 1], linguistic input NB (Negative Big), NM (Negative Medium), NS (Negative Small), Z (Zero), PS (Positive Small), PM (Positive Medium), PB (Positive Big) [7].
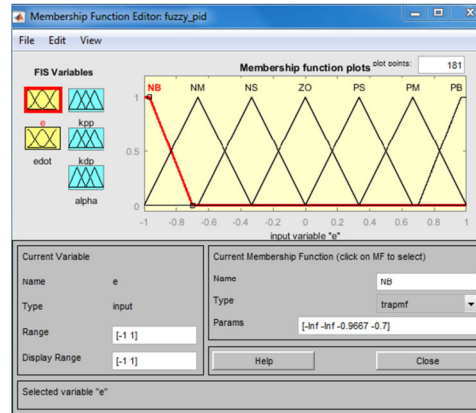
Figure 6. Input fuzzification of e and (du/dt)

Output fuzzification is assumed within the range of [0 1] and membership function parameters *Kp, Ki,* and *Kd* are set with small and big.
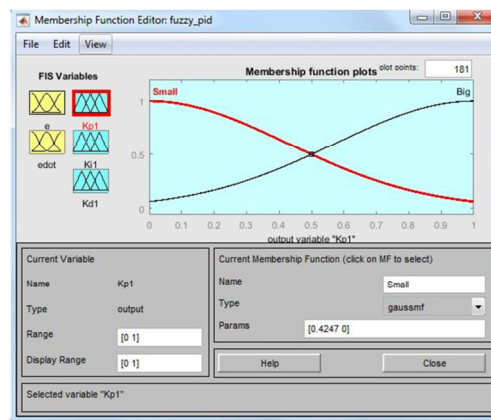

Figure 7. Output fuzzification of K_p, K_i and K_d

The rule base was designed based on the characteristics of a brushless dc motor. Where input linguistic error (e) 7 and input error (du/dt) 7 so that the overall input of (e) and (du/dt) are 7x7 = 49 rules.

TABLE II
Fuzzy Rule Kp

Fuzzy Rule		du/dt						
Kp		NB	NM	NS	Z	PS	PM	PB
	NB	B	B	B	B	B	B	B
	NM	S	B	B	B	B	B	B
	NS	S	S	B	B	B	S	S
e	Z	S	S	S	B	S	S	S
	PS	S	S	B	B	B	S	S
	PM	S	B	B	B	B	B	S
	PB	B	B	B	B	B	B	B

TABLE III
Fuzzy Rule Ki

Fuzzy Rule		du/dt						
Ki		NB	NM	NS	Z	PS	PM	PB
	NB	S	S	S	S	S	S	S
	NM	B	B	S	S	S	B	B
	NS	B	B	B	S	B	B	B
e	Z	B	B	B	B	B	B	B
	PS	B	B	B	S	B	B	B
	PM	B	B	S	S	S	B	B
	PB	S	S	S	S	S	S	S

TABLE IV
Fuzzy Rule Kd

Fuzzy Rule Kd		du/dt						
		NB	NM	NS	Z	PS	PM	PB
e	NB	2	2	2	2	2	2	2
	NM	3	3	2	2	2	3	3
	NS	4	3	3	2	3	3	4
	Z	5	4	3	3	3	4	5
	PS	4	3	3	2	3	3	4
	PM	3	3	2	2	2	3	3
	PB	2	2	2	2	2	2	2

Defuzzification was used to convert linguistic variables into exact numeric values. The center of gravity (COG) method was applied to defuzzification.

4. Hasil

Figure 8 presents the motor modeling and control design with quadcopter load torque.
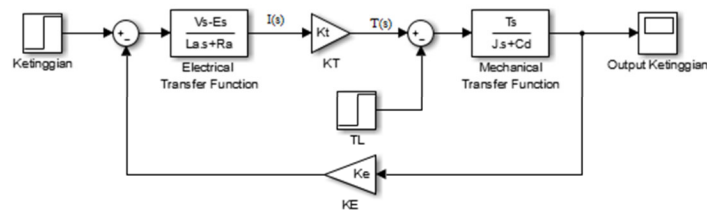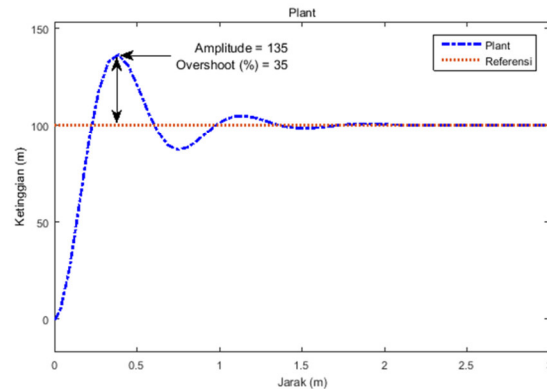

Figure 8. Control modeling of quadcopter motor

Figure 9. Stability response of quadcopter movement

The altitude response of the quadcopter at 100 m was oscillated to achieve the expected target. This occurrence was influenced by the load and lift force. Where the peak amplitude was 135, and the overshoot was 35%, so the oscillation of the height response affected the stability of the quadcopter's movement to achieve a steady-state

Then the quadcopter movement was controlled using proportional-integral-derivative and PID tuning based on Ziegler-Nichols. Figure 11 displays the simulation results.
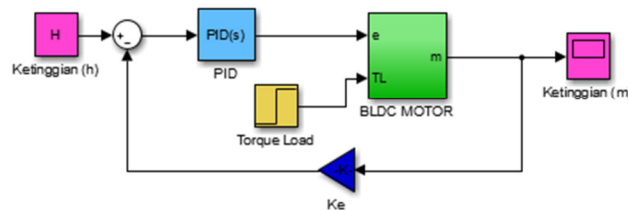

Figure 10. PID tuning based on Ziegler-Nichols

Bruhsless dc motor was used to regulate stability motion of quadcopter by two electrical and mechanical equations. Electrical equations were current, voltage, inductance and power. Mechanical equations were thrust force and drag force of quadcopter that it was used as load torque of brushless dc motor. Then, two equations were modeled by Laplace transform method to easy control process.

Stability control of quadcopter motion used PID controller to tune *Kp, Ki,* and *Kd* parameter based on the value of altitude output error. If the altitude input changed, then, the stability of motion undergone the changing of altitude output. The change was read by a sensor and feedback to altitude input. If the output altitude was not equal to the altitude input to reach stability. Then error was taken place and PID controller sets to input and output equal to obtained the small error.
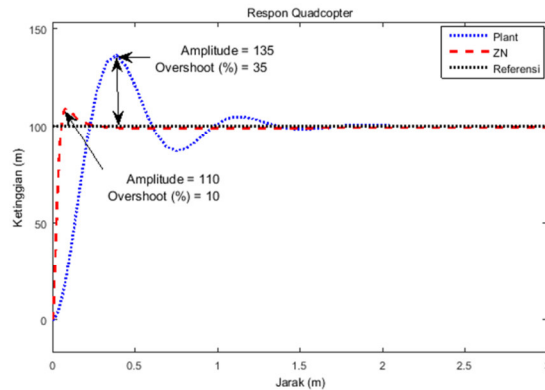

Figure 11. Response to quadcopter movement stability with Ziegler-Nichols PID tuning

PID tuning calculation based on Ziegler-Nichols with K_u (ultimate gain) and T_u (ultimate period) obtained parameter values *Kp* = 4.8, *Ki* = 1.10, and *Kd* = 0.45. The quadcopter response experienced an overshoot of 10% and a peak amplitude of 110. Whereas the system plan experienced an amplitude of 135 and an overshoot of 35%. Ziegler-Nichols PID tuning could reduce the oscillation faster in quadcopter movement stability to achieve the desired set-point response.

Figure 13 shows the quadcopter motion stability that was controlled using fuzzy gain scheduling PID.
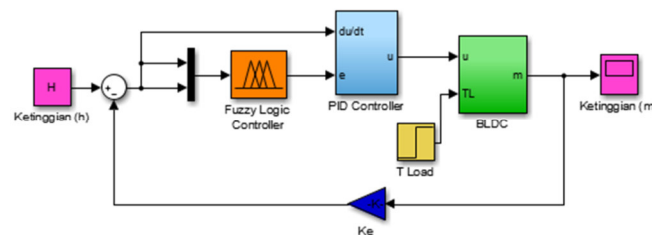

Figure 12. PID control modeling, Ziegler-Nichols and fuzzy gain scheduling PID

PID controller had the parameter gain limitation, then the parameter should be tuned manually. Whereas tuning Ziegler-Nichols (ZN) lacked selection of parameter *Kp, Ki, Kd,* and had an excessive overshoot in time response. Consequently, PID and ZN need accuracy parameter tuning to reach the target. Fuzzy gain scheduling PID controller was utilized to design a new scheme rule-based gain scheduling PID controller. The new scheme of the fuzzy rule was used to determine *Kp, Ki, Kd* parameter based on the changing of altitude output error.

An approach of parameter value was assumed to range $K_p = [K_{p.min}, K_{p.max}]$ and $K_d = [K_{d.min}, K_{d.max}]$. An adaptation of parameter value was assumed by experiment and would be given in equation (53), (54). Then, *Kp* and *Kd* were normalized into the range between zero and one. In the scheme of fuzzy gain scheduling, PID parameter was determined based on current error $e(k)$ and its first difference $\Delta e(k)$ with representing membership function in figure 6. *Ki* was determined by referencing a parameter value of Kd. Where *Ki* was presented in equation (48), and integral gain was obtained by equation (49).

Variable linguistic (NB, NS, NM, Z, PS, PM, PB) were chosen based on characteristic brushless dc motor by quadcopter load to reach the target of stability of motion. The rule fuzzy could be derived at Tables II, III, and IV.

- If $e(k)$ is PB, and $\Delta e(k)$ is Z, then *Kp* is big, *Kd* is small, and $\alpha = 2$.
- If $e(k)$ is Z, and $\Delta e(k)$ in NB, then *Kp* is small, *Kd* is big, and $\alpha = 5$.

To produce a big control signal, the PID controller should have big gain *Kp*, big gain *Ki*, and small gain *Ki*. PID controller that was given big or small gain at *Ki* would produce strong stability of motion. *Ki* was

determined by a new scheme with comparison tuning Ziegler-Nichols rule. In the tuning Ziegler-Nichols rule, *Ki* was always taken four times as large as Kd. That α was equal to 4. In the proposed scheme, α took a value less than 4, (say 2) to generate the strong Ki. The α value was determined by the singleton membership function. If α controller was small, it would be produced small overshoot, and PID controller should have small gain *Kp*, big gain *Kd*, and small gain *Ki* with the fuzzy rule.
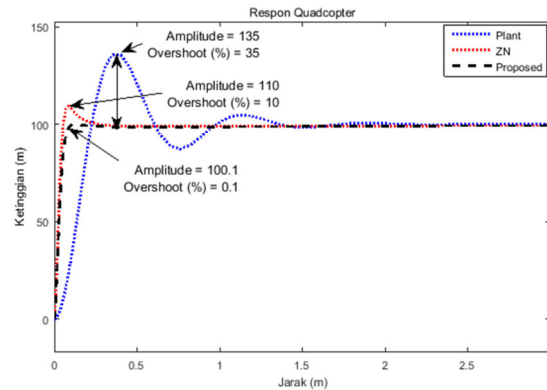

Figure13. Stability response of quadcopter motion using PID control, ZN and Fuzzy gain scheduling PID

Fuzzy gain scheduling calculation results PID with K_u (ultimate gain) and T_u (ultimate period) obtained parameter values K (p.min) = 1.76, K (p.max) = 2.48, K (d.min) = 0.29, K (d.max) = 0.58. The responses showed that the altitude of the quadcopter had an overshoot of 0.1% and a peak amplitude of 100.1. Whereas the response plan experienced an amplitude of 135 and an overshoot of 35%. Fuzzy gain PID scheduling could reduce oscillation by 0.1% compared to Ziegler-Nichols PID tuning in quadcopter stability to achieve steady-state

5. Conclusion

The modeling and control design of the quadcopter motor was performed using the Laplace transform method. The quadcopter mathematical model was used as the load torque of brushless dc motor to control the stability motion in altitude vertical take-off. The motion stability was influenced by body frame mass, propeller speed, and wind disturbance during hovering. Hence, the quadcopter motion experienced oscillation to reach a steady state. Ziegler-Nichols PID and fuzzy gain scheduling PID tuning were suggested to overcome the motion stability in altitude vertical take-off. The simulation results showed that fuzzy gain scheduling PID reduced the overshoot 0.1% smaller than Ziegler-Nichols tuning.

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Nomenclature

T_e	= Electromagnetic torque
Ω	= Rotational angle speed
P	= Number of pole
K_T	= Constant torque coefficient
i	= Phase current
T_L	= Load torque
j	= Rotor inertia moment
B, B_v	= Friction coefficient
R_a	= Armature resistance
L_a	= Armature inductance
i_a	= Armature current
E_a	= Back EMF induction
V, U_d	= Voltage
T	= Torque
θ	= Angular displacement
j	= Rotor inertia moment
K_T	= Constant torque coefficient
$K_b K_e$	= Back EMF coefficient
ω	= Angular velocity
Ω	= Rotor speed
T_L	= Load torque
T_e	= Electromagnetic torque
E	= Fixed reference frame (ground)
B	= Body reference frame
F_{Ti}	= Thrust force of the propeller
Q	= Motor torque
V	= Voltage across the motor
I	= Motor current
ω	= Rotor angular speed
K_q	= Torque-current relation
R_a	= Motor armature resistance
K_e	= Back EMF coefficient
D	= Diameter propeller (m)
ΔV	= Velocity of air accelerated by propeller (m/s)
ρ	= Air density (1.225 kg/m^3)
P	= Motor power (watt)
F	= Force (N)
T	= Thrust (N)
m	= Mass (kg. m/s^2)
g	= Gravitation (9.81 m/s^2)
D	= Propeller diameter (m)
P	= Power (watt)
a	= Object's speed
ρ	= 1.225 (Air density, Kg/m^3)
A	= πr^2 (Area swept by the propeller, m^2)
r	= 0.0635 (Radius of the propeller blade, m)
C_D	= 1.3 (Coefficient of drag)
V	= (Velocity of the air by propeller blade, m/s^2)

A $= \frac{1}{2}(MTM)^2 + 3 \times \pi \times r^2_{propeller}$

MTM $=$ Motor to distance motor

R_{prop} $=$ Radius of the propeller