

# CONCEPT AND CALCULATION OF THE LIMIT TRANSVERSE SIZE OF CAPILLARIES

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## ABSTRACT

Porous medium are products of processing in food, agricultural, chemical and many other industries. Calculations of processes with wet porous medium are based on capillary properties of the liquid in a pore space. The capillary properties of liquids in porous media are established in pore models in the form of thin tubes of circular or slit transverse sections. The intensity of the processes occurring in it depends on the nature of the filling of the pore space with liquid. Filling with liquid and the formation of a capillary layer is possible only in small pores. However, there is no analytical justification for the transverse pore size, more than which it cannot be filled with liquid by capillary forces. With this in mind, the concept of the limiting transverse size of a capillary for a liquid under conditions of complete wetting is introduced. The limiting size calculation is based on two conditions: the shape of the axial section of the meniscus surface has the appearance of a semicircle and its extremum point is located at the level of the free surface of the fluid supplying the capillary. A capillary column cannot form in larger pores. The absence of formulas for calculating capillaries of the limiting sizes can introduce a significant error into the analytical calculation of the moisture content in the capillary layer of a liquid in porous media and moisture transfer processes. The aim of the study was to obtain formulas for calculating the limiting (largest) sizes of capillaries of a circular, flat slit section and annular transverse sections with complete wetting of their walls. For the conditions above, it was identified that the limiting distance between the walls was independent from annular capillary diameter. The formulas for the limiting transverse sizes of the flat slit and annular capillaries turned out to be the same under the assumptions above. This indicates a weak dependence of the limiting size of a slit capillary on the curvature of its transverse section. Examples of calculations of capillaries of the limiting sizes are performed.

## 1. Introduction

There is a wide variety of practical problems where the capillary properties of porous medium are shown [1,2,3,4]. Calculations of the processes associated with wetness porous medium are based on taking into account liquid capillary properties in pore space [5,6,7]. Capillarity is the effect of the liquid adhesiveness to the contacting material, caused by surface tension and contact angle wetting. Capillarity in porous medium with communicating pores represents the most important practice cases [2,8,9]. In these cases, the meniscus leads to a capillary lift up of the liquid level with the formation of a capillary layer in the porous medium [10,11,12]. It is usually assumed that in the capillary layer, starting from the level of the free surface of the wetting liquid supporting the porous layer, pores are completely filled with it. However, the effect of a drastic decrease in moisture in the capillary layer can be observed even at this level [13]. This phenomenon is called «clogged air» in pores [14], which can occur in large communicating pores. The calculation of capillaries of such sizes is important in determining moisture and calculating moisture transfer processes in a porous medium. They cannot be calculated by known methods, since there is no numerical definition of the concept of a capillary as a tube in which a capillary column of liquid may appear. In this regard, the analytical solution to the problem of obtaining the calculation formulas of such capillaries is relevant.

The analytical properties of porous medium and moisture transfer processes are considered on pore models in the form of circular and slit of small transverse sections [7,15]. Such models are assigned a priori to the concept of a capillary. For example, the height of the rise of the capillary column is calculated by the formula of Jurin [15,16]. For a capillary of circular transverse section and zero contact angle of liquid wetting, corresponding

to the concept of complete wetting, the formula will have the following form

$$h_c = \frac{4\sigma}{(\rho - \rho_0)gd_t} \approx \frac{4\sigma}{\rho g d_t} \quad (1)$$

where

$h_c$  is the height of the rising column of capillary fluid with complete wetting of the walls of the capillary, m;  
 $\sigma$  is the surface tension of the liquid, N/m;  
 $\rho$  and  $\rho_0$  – density of liquid and air, kg/m<sup>3</sup>;  
 $g$  – the acceleration of gravity, m/s<sup>2</sup>;  
 $r$  and  $d_t = 2r_c$  – the diameter and radius of the capillary of circular transverse section, m. Due to  $\rho \gg \rho_0$ , the air density is usually neglected.

The formula (1) was derived based on the static problem of the balance of capillary forces of the liquid surface tension and its mass forces in this column. Since the formula is used for models of small-radius capillaries and a significant height of the capillary column, then the weight of the liquid in the meniscus is ignored [16,17]. In this formula, the height of the capillary column corresponds to the distance between the mirror level of the free surface of the liquid wetting the capillary and the lower point of concave meniscus. For the application of formula (1), the assigned value of the transverse size of the capillary is not specified, although it is clear that there is a maximum, at which the capillary column of a particular liquid cannot form in it. In the future, we will call such pores caverns. However, there are no formulas to calculate the transverse size of the tube, relating it to capillaries or cavities. Without determining the limiting (maximum) size of the capillary, it is impossible to correctly calculate the moisture content of the porous medium in the capillary layer of the porous medium and to calculate the processes occurring in it. It is important to note

that the limiting size depends not only on the liquid properties, but also on the transverse shape of the capillary tube. The basis of its calculation should be the meniscus parameters.

Shape of the meniscus surface changes depending on the transverse size and shape of the tube. At a certain significant tube transverse size, a practically flat surface of the liquid around its axis appears. The arc shape of the profile of the axial section of the meniscus at the walls of the tube smoothly fits into the straight profile of a flat surface [18]. The same transition is observed between the raised curvilinear edge of the liquid near the wall of a large vessel and a flat surface far from its wall. Starting with a certain small transverse size, the flat surface of the liquid disappears, and the meniscus extends to the entire cross section of the capillary. The shape of the meniscus surface in the axial section will be described by a complex curve [7,15]. Under further decrease in the transverse size of the capillary, the axial section of the meniscus will give a line approaching the arc of a circle. With an increase in the transverse size of the tube, the deviation of the profile of the meniscus arc from the circular arc becomes larger. And conversely, the smaller the transverse size of the capillary, the closer to the circumference of the arc is the axial section of the meniscus surface.

In publications, there is no estimate of the limiting capillary size. The critical width of a flat capillary, considered in [7], is discussed from the point of view of the existence of a non-spherical meniscus, that is, the conditions for the appearance of a flat axial surface are actually accepted. Therefore, the critical width in [7] is much larger than the limiting capillary size.

The aim of the study was to obtain the calculation formulas of the limiting (largest) transverse sizes for three model sections of capillaries under conditions of complete wetting.

**2. Materials and methods**

The problem under consideration relates to porous medium with an open pore system. They are widespread in production processes. These are food raw materials and products, soils, bulk materials in pharmaceuticals, porous medium in chemical and oil and gas industries, construction materials and materials in many other industries. Physical models of a porous medium are considered using examples of a tube of round transverse section, a flat and an annular slit, and their both ends is open. The capillaries are immersed with their lower end in a liquid with a free surface. The contact angle wetting is taken equal to zero (complete wetting). Complete wetting means smooth coupling of the meniscus surface with the surface of the capillary wall. The transverse curvature of the slit capillary is modeled by a tube of annular transverse section. The curvature of its walls varies by changing the size of the inner or outer diameter of the annulus. The static problem of force equilibrium in a meniscus fluid in a capillary of the limiting transverse size is considered.

**3. Results and discussion**

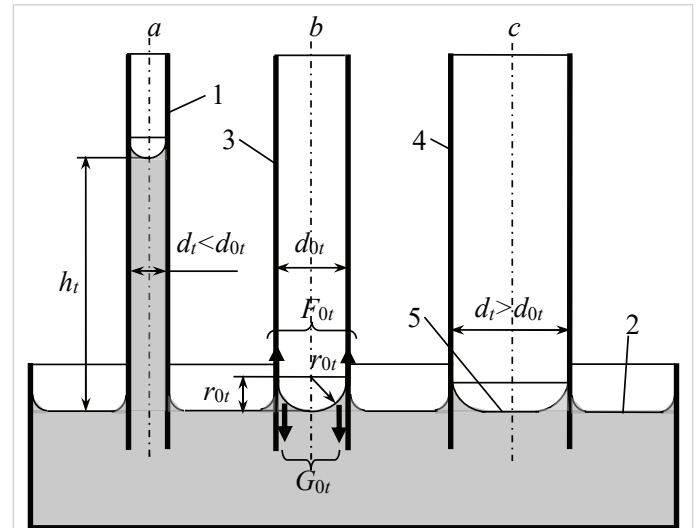
Let us determine the limiting sizes of model capillaries of round, flat slit and annular transverse sections under the assumption of complete wetting of the capillary walls.

In the figures and in the formulas below, subscripts  $0i$  will indicate the capillary corresponds to the limiting (maximum) size (index 0) for  $i$ -th of transverse section shape. Index  $i$  takes the form:  $i = t$  – for a tubular capillary with a round transverse section;  $i = f$  – for a capillary in the form of a flat slit;  $i = a$  – for a capillary with an annular transverse section.

At a zero angle of wetting to the material, the meniscus surface in the longitudinal plane of the capillary column has the form of a semicircle (Figure 1a, Figure 1b). A tube of annular transverse section with a diameter  $d_t$  equal to (tube 3) or smaller (tube 1) of the limiting  $d_{0t}$  refers to capillaries ( $d_t \leq d_{0t}$ ) for this fluid. The size

of capillary 3 by the condition  $d_t = d_{0t}$  means that the capillary column is absent ( $h_t = 0$ ) and the lower point of the hemisphere of the meniscus surface is located at the level of the free surface 2 of the liquid supplying capillary 3 (Figure 1b). Tube 4 (Figure 1c) of a limiting (largest) transverse size ( $d_t > d_{0t}$ ) is not a capillary, since a column of liquid cannot appear in it ( $h_t = 0$ ) and the meniscus surface will turn out to be non-spherical.

To quantify the limiting transverse size  $d_{0t}$  of the pore model relating it to the capillary, it is necessary to take into account the weight  $G_{0t}$  of the liquid in the meniscus of height  $h_{0t}$  and the surface tension force of the meniscus  $F_{0t}$ . When determining the limiting (largest) diameter of the capillary  $d_t = d_{0t}$ , the height of the capillary column of the liquid is  $h_{ct} = 0$ .



**Figure 1.** Illustration of the concept of a capillary with a round transverse section ( $i = t$ ) with complete wetting by the liquid of its inner surface:  $F_{0t}$  – capillary surface forces;  $G_{0t}$  – gravitational forces of the weight of the liquid in the meniscus;  $d_t$  – tube diameters;  $d_{0t} = 2r_{0t}$  – the limiting diameter and radius of the capillary;  $h_t$  – the height of the capillary column above the free surface of the liquid in the vessel without of the meniscus height; 1 – a cylindrical capillary having  $d_t > d_{0t}$ ; 2 – level of the free (open) surface of the liquid in the vessel; 3 – a cylindrical capillary of limiting (largest) diameter  $d_{0t}$ ; 4 – diameter  $d_t > d_{0t}$ ; 5 – the flat part of the surface in the tube 4, indicating the existence of the inequality  $d_t \gg d_{0t}$  in it

For the  $i$ -th form of the capillary of the limiting transverse size and the contact angle taken equal to zero, the equality

$$\sum F_i = 0.$$

Two forces act on the meniscus liquid, and by (2) we have

$$F_{0t} - G_{0t} = 0, \tag{2}$$

where  $F_{0t}$  – the capillary force of the surface tension  $\sigma$  in the meniscus acting along the perimeter of the wetting of the capillary  $L_{0t}$

$$F_{0t} = \sigma L_{0t}, \tag{3}$$

and  $G_{0t}$  – the weight of the liquid of the meniscus volume  $V_{0t}$

$$G_{0t} = \rho g V_{0t}. \tag{4}$$

Consider a tubular capillary of round transverse section ( $i = t$ ) with a diameter  $d_{0t}$ . Its axial section corresponds to fig. 1b. The meniscus surface is taken spherical. Then the liquid volume in the meniscus will be the difference between the cylinder volume  $V_{0c}$  of diameter  $d_{0t}$  and height  $h_{0t} = d_{0t}/2$  and half the hollow volume of the sphere  $V_{0s}$

$$V_{0t} = V_{0c} - \frac{1}{2}V_{0s} = \frac{\pi d_{0t}^2}{4} \frac{d_{0t}}{2} - \frac{1}{2} \frac{4}{3} \pi \frac{d_{0t}^3}{8} = \frac{\pi}{24} d_{0t}^3. \tag{5}$$

The wetting perimeter of the capillary wall for (3) is

$$L_{0t} = \pi d_{0t} \quad (6)$$

Therefore, by (2)-(6)

$$\sigma \pi d_{0t} - \frac{1}{24} \rho g \pi d_{0t}^3 = 0,$$

where the limiting diameter will be determined by the formula

$$d_{0t} = \sqrt{\frac{24\sigma}{\rho g}}. \quad (7)$$

For water with a surface tension of  $\sigma = 72.8 \times 10^{-3}$  N/m and density  $\rho = 1000$  kg/m<sup>3</sup>, the limiting diameter of a capillary of round transverse-section according to (7) will be

$$d_{0t} = \sqrt{\frac{24 \times 73 \times 10^{-3}}{1000 \times 9.81}} \approx 13.36 \times 10^{-3} \text{ m} \quad (8)$$

This means that if there were pores larger than 13.36 mm in a porous medium, then the pore layer with capillary water (capillary layer) would be with voids already at the level of free surface of water.

The axial section of the flat slit capillary corresponds to Fig. 1b, but with the notation  $i = f$  and the distance between the walls as  $b_{0f}$  instead of  $d_{0t}$ . It has two walls, each of length  $l$ . Therefore  $L_{0f} = 2l$  and by (3)

$$F_{0f} = \sigma_s 2l. \quad (9)$$

The meniscus volume  $V_{0f}$  with height  $h_{0f} = 0.5b_{0f}$  is defined as the difference between the volume of the rectangular parallelepiped  $V_{p0f}$  and half the volume of the cylinder  $V_{t0f}$

$$V_{p0f} - \frac{1}{2} V_{t0f} = b_{0f} l \frac{b_{0f}}{2} - \frac{1}{2} \frac{\pi b_{0f}^2}{4} l = \left(1 - \frac{\pi}{4}\right) \frac{b_{0f}^2}{2} l. \quad (10)$$

Then

$$G_{0f} = \rho g \left(1 - \frac{\pi}{4}\right) \frac{b_{0f}^2}{2} l \quad (11)$$

and by (2), (9) – (11)

$$\sigma 2l - \rho g \left(1 - \frac{\pi}{4}\right) \frac{b_{0f}^2}{2} l = 0,$$

where from

$$b_{0f} = 2 \sqrt{\frac{\sigma}{\rho g (1 - \pi/4)}} \quad (12)$$

As we see from (12), the limiting distance between the walls of a flat capillary does not depend on its length.

For water, this limiting distance will be

$$b_{0f} = 2 \sqrt{\frac{73 \times 10^{-3}}{1000 \times 9.81 (1 - \pi/4)}} \approx 11.77 \times 10^{-3} \text{ m}. \quad (13)$$

This means that the capillary layer will appear immediately above the level of the free liquid surface in the material with slit pores of a less distance.

Consider a capillary in the form of a tube of annular transverse section (Figure 2). For it,  $i = a$  and the ratio of the large diameter  $D$  to the smaller diameter  $d$  of the inner walls of the capillary is denoted as

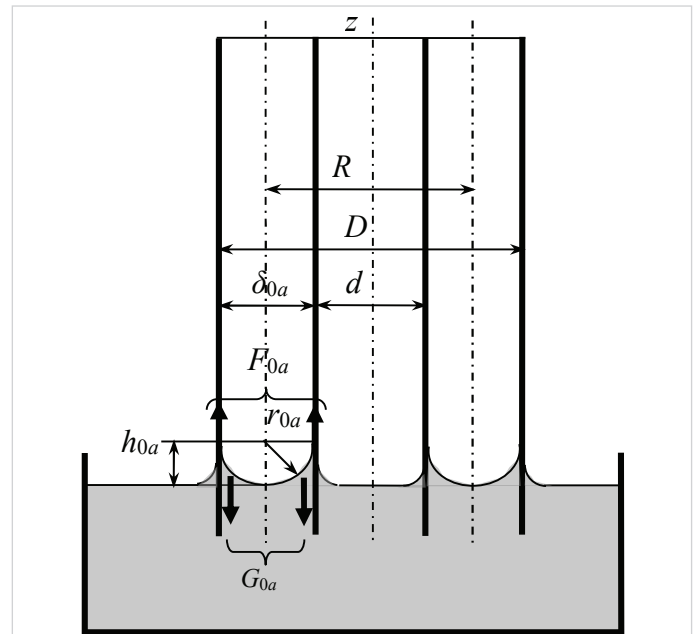
$$c = \frac{D}{d}. \quad (14)$$

Then we have for the concave meniscus

$$F_{0a} = \sigma \pi (D + d) = \sigma \pi (c + 1) d \quad (15)$$

and

$$G_{0r} = \rho g V_{0r} \quad (16)$$



**Figure 2.** Model of an annular capillary:  $z$  – axis of the annular capillary;  $D$  and  $d$  – diameters of the far and near walls of the annular capillary;  $R$  – the average radius of the torus annulus;  $\delta_{0a}$  – the limiting distance between the walls

Making the previous assumption about the same height of raising the upper line of the meniscus edge on both cylindrical walls, we calculate the volume of the annular meniscus  $V_{0a}$  as the difference between the volume  $V_{0ac}$  of the annular cylinder of height  $h_{0a} = 0.5\delta_{0a}$  and half the volume of the torus  $V_t$

$$V_{0a} = V_{0ac} - \frac{V_t}{2}. \quad (17)$$

The distance between the cylindrical walls of the capillary is

$$\delta_{0a} = \frac{1}{2} (D - d) = \frac{d}{2} (c - 1). \quad (18)$$

The torus radii are:

– average radius of the torus annular

$$R = \frac{1}{2} \frac{D + d}{2} = \frac{d}{4} (c + 1), \quad (19)$$

– the radius of the circle forming the surface of the torus,

$$r_{0a} = \frac{\delta_{0a}}{2} = \frac{d_{0a}}{4} (c - 1). \quad (20)$$

The volume of the of the annulus meniscus  $V_{0a}$  is equal to the difference between the volume of the annular cylinder

$$V_{0ac} = \left(\frac{\pi D^2}{4} - \frac{\pi d^2}{4}\right) \frac{\delta_{0r}}{2} = \frac{\pi d^3}{16} (c^2 - 1)(c - 1), \quad (21)$$

and half the volume of the torus inscribed in it

$$\begin{aligned} \frac{1}{2} V_t &= \frac{1}{2} 2\pi^2 R r_{0a}^2 = \pi^2 \frac{d}{4} (c + 1) \left[\frac{d}{4} (c - 1)\right]^2 = \\ &= \frac{\pi^2}{64} d^3 (c^2 - 1)(c - 1), \end{aligned} \quad (22)$$

In view of (17), (19)-(22), we obtain

$$\begin{aligned} V_{0a} &= \frac{\pi}{16} d^3 (c^2 - 1)(c - 1) - \frac{\pi^2}{64} d^3 (c^2 - 1)(c - 1) = \\ &= \frac{\pi}{16} d^3 (c^2 - 1)(c - 1) \left(1 - \frac{\pi}{4}\right). \end{aligned} \quad (23)$$

Then by (16) and (23)

$$G_{0a} = \rho g \frac{\pi}{16} d^3 (c^2 - 1)(c - 1) \left(1 - \frac{\pi}{4}\right), \quad (24)$$

by (2) and (15) for

$$F_{0a} - G_{0a} = 0$$

we have

$$\sigma \pi (c + 1) d - \rho g \frac{\pi}{16} d^3 (c^2 - 1)(c - 1) \left(1 - \frac{\pi}{4}\right) = 0.$$

The positive root of this incomplete quadratic equation will be

$$d = \sqrt{\frac{16\sigma}{\rho g} \frac{1}{(c - 1)^2 \left(1 - \frac{\pi}{4}\right)}} = \frac{4}{c - 1} \sqrt{\frac{\sigma}{\rho g} \frac{1}{\left(1 - \frac{\pi}{4}\right)}}. \quad (25)$$

We define the formula, i. e. the limiting distance  $\delta_{0a}$  between the walls of the capillary

$$\delta_{0a} = d \frac{c - 1}{2} = 2 \sqrt{\frac{\sigma}{\rho g} \frac{1}{\left(1 - \frac{\pi}{4}\right)}}. \quad (26)$$

Therefore, for the annular transverse section of the capillary, the limiting distance between the walls  $\delta_{0a}$  does not depend on the small or large diameters. It is equal to the limit wall spacing of the flat slit capillary  $\delta_{0a} = b_{0r}$ . The resulting equality of formulas can be associated with the assumption that the menisci have the same height on the walls of the small and large cylinders of the capillary. We also note that the same formulas for the flat slit (12) and annular (26) capillaries differ from the formula for a capillary of round transverse section (7) only by a constant coefficient.

Of the three parameters that make up formula (14), only one can be arbitrarily specified. The other two are determined by the available ratios:

- if  $c$  is given, calculate  $d$  by (25) and then  $D$  from (14);
- if  $d$  is given, then we determine from (25)

$$c = 1 + \frac{4}{d} \sqrt{\frac{\sigma}{\rho g \left(1 - \frac{\pi}{4}\right)}}$$

and then  $D$  from (14) either by

$$D = d + 2\delta_{0a}, \quad (27)$$

which is the same thing.

For a capillary having  $c=2$ , with water under normal conditions, we obtain from (25)

$$d = \frac{4}{2 - 1} \sqrt{\frac{73 \times 10^{-3}}{1000 \times 9,81 \left(1 - \frac{\pi}{4}\right)}} = 23.55 \times 10^{-3} \text{ m.} \quad (28)$$

and by (26)

$$\delta_{0r} = 2 \sqrt{\frac{73 \times 10^{-3}}{1000 \times 9,81 \left(1 - \frac{\pi}{4}\right)}} = 11.78 \times 10^{-3} \text{ m.,} \quad (29)$$

We also note that the distances between the walls of the flat and annular slits have the same values  $\delta_{0a} = b_{0r} = 11.78$  mm. Therefore, for the model under consideration, the curvature of the slit between the cylindrical walls did not lead to a change in the limit wall spacing.

For an annular transverse section with the smallest diameter of the inner wall of the capillary  $d \approx 0$  and the accepted  $c=2$ , we obtain from (27)

$$D = 0 + 2 \times 11.78 = 23.56 \text{ mm,} \quad (30)$$

which is almost twice as much as the limit diameter of a capillary with a annular transverse section according to (8), equal to 13.36 mm. This increase in size is due to the presence of an additional wall in the annular transverse section of the capillary with a meniscus on it. For the same reason, the formula for the

annular transverse section (30) at  $d = 0$  does not go over into the formula for the variant of the round transverse section (8).

For comparison, we will find the relationship between the limiting distance  $\delta_{0a}$  according to (26) and the diameter  $d_{0t}$  according to (7) and the limiting volumes of menisci of these capillaries  $V_{0a}$  and  $V_{0t}$

$$\frac{\delta_{0a}}{d_{0t}} = \frac{2 \sqrt{\frac{\sigma}{\rho g} \frac{1}{\left(1 - \frac{\pi}{4}\right)}}}{\sqrt{\frac{24\sigma}{\rho g}}} = \sqrt{\frac{4}{24 \left(1 - \frac{\pi}{4}\right)}} = 0,88. \quad (31)$$

and

$$\begin{aligned} \frac{V_{0a}}{V_{0t}} &= \frac{\frac{\pi}{16} \left(1 - \frac{\pi}{4}\right) d^3 (c^2 - 1)(c - 1)}{\frac{\pi}{24} d_{0t}^3} = \\ &= 0,322 (c^2 - 1)(c - 1) \frac{d^3}{d_{0t}^3} \end{aligned} \quad (32)$$

For  $c=2$ , we obtain from (32) for water under normal conditions

$$\frac{V_{0a}}{V_{0t}} = 0,322 (2^2 - 1)(2 - 1) \left(\frac{23,55 \times 10^{-3}}{13,36 \times 10^{-3}}\right)^3 = 5,29.$$

Hence, in the annular meniscus, the limit size is  $\delta_{0a} = 0,88 d_{0t}$ , but the volume of liquid in it at  $c=2$  is 5.29 times larger in comparison with the volume of liquid in the meniscus of the round transverse section.

Let us evaluate the influence of the volume of liquid in the meniscus in the full volume of the capillary column of a round transverse section capillary

In this case, taking into account (2), we have

$$F_t - (G_{mt} + G_{ht}) = 0. \quad (33)$$

where  $G_{ht}$  – the weight of the liquid in the capillary column.

For the considered capillary, formula (33) has the form

$$\sigma \pi d_t - \rho g (V_{mt} + V_{ht}) = 0. \quad (34)$$

From here

$$V_{ht} = S_t h_{ct} = \frac{\pi d_t^2}{4} h_{ct}. \quad (35)$$

and by analogy with (5)

$$V_{mt} = \frac{\pi}{24} d_t^3. \quad (36)$$

In (34) – (36) it is indicated:  $V_{mt}$  – the volume of liquid in the meniscus,  $V_{ht}$  – the volume of liquid in the capillary column excluding the meniscus,  $S_t$  – the cross section of the capillary,  $h_c = h_{ct}$  – the height of the liquid column in the capillary from the mirror of the liquid free surface to the lower point meniscus.

Then, taking into account the Jurin formula according to (1) will be

$$\sigma \pi d_t - \left( \rho g \frac{\pi}{24} d_t^3 + \rho g \frac{\pi d_t^2}{4} h_{ct} \right) = 0,$$

and the height of the column of capillary fluid will be determined by the formula

$$h_{ct} = \frac{4 \left( \sigma \pi d_t - \rho g \frac{\pi}{24} d_t^3 \right)}{\rho g \pi d_t^2} = \frac{4\sigma}{\rho g d_t} - \frac{d_t}{6}. \quad (37)$$

For the most common food porous media with pore sizes  $d_t \leq 10^{-3}$  m and water in the capillaries, the first term in (37), determined by the capillary force, is approximately 180 times larger than the second, determined by the weight of the water in the meniscus, which is less than 0.6%. Therefore, in practical cases, the height of

the capillary column of liquid can be determined without taking into account the weight of the liquid in the meniscus according to the Jurin formula (1).

Thus, for pore models in the form of tubes of round, flat slit, and annular transverse sections, formulas are obtained for determining their limit transverse dimensions on condition complete wetting. Such sizes allow us to consider them capillaries. The capillary column will be absent in the pores of the limiting and larger sizes, and wetness in the capillary layer of the porous medium will be less than unity already at the level of the free surface of the supply liquid.

#### 4. Conclusion

A quantitative concept of the limiting transverse size of a capillary is introduced. It is based on the fulfillment of two conditions: the axial sectional shape of the meniscus surface in the form of an arc of a circle and the position of its extremum point at the level of the free surface of the liquid supplying the capillary. Considering the mutual influence of the properties of both the liquid and the wall material in contact with the forma-

tion of the meniscus, the limiting size of the capillary will vary for different wetting liquids and unchanged other factors. The calculation formulas of the limiting sizes for the capillaries of three forms of cross sections: tubular round, flat slit and annular, were obtained at a zero contact angle wetting. For flat slit and annular capillaries, they turned out to be the same and differ from the formula for a capillary of circular transverse section only by a constant coefficient. The capillary with a round transverse section has the largest limiting size, and the volume of capillary liquid in the meniscus is maximum in the annular capillary. The formulas of the limiting sizes of the flat slit and annular capillaries turned out to be the same for the assumptions made. It indicates a weak dependence of the slit capillary on its curvature in the transverse section.

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