# MRAS-Type Speed and Flux Estimator with Additional Adaptation Mechanism for the Induction Motor Drive

Grzegorz Tarchala, Teresa Orlowska-Kowalska, Mateusz Dybkowski
Wroclaw University of Technology, Institute of Electrical Machines, Drives and Measurements, Wroclaw, POLAND,
email: grzegorz.tarchala@pwr.wroc.pl, teresa.orlowska-kowalska@pwr.wroc.pl, mateusz.dybkowski@pwr.wroc.pl

*Abstract*—**This paper deals with a novel Model Reference Adaptive System (MRAS) estimator of the rotor flux and speed reconstruction for induction motor drive. The proposed estimator is based directly on the mathematical model of the induction motor. In comparison with existing methods the additional variable is introduced, which has to reduce partly the estimator sensitivity to the rotor time constant changes. Chosen simulation tests of the proposed solution are presented.**

***Keywords— Induction motor, sensorless drives, state estimators, MRAS techniques***

## I. Introduction

In order to avoid placing flux sensors inside the machine, almost all modern induction motor (IM) drives require stator or rotor flux estimation. This information is employed in a control structure for flux amplitude stabilization. Also so called sensorless drives gain still growing popularity, due to cost minimization, cabling reducing and the elimination of the speed sensor [1].

Among many different types of the estimators the Model Reference Adaptive System (MRAS) type ones are characterized by their simplicity, due to the PI regulator used in the speed estimation mechanism. Simultaneously the stator or rotor flux vectors are estimated. Also among MRAS estimators many different kinds can be distinguished, starting with the classical approach, named MRAS$^{F}$ [2], [3], full order adaptive observer NFO$^{A}$ [4], and having on the other hand MRAS$^{CC}$ estimator [5].

Since every algorithmic estimation method is sensitive to motor parameters mismatch, significant attention has been focused on motor parameters estimation. A large group of them were designed to estimate rotor time constant, especially intended for using in the Indirect Field Oriented Control of the IM drive. However, the motor speed and rotor resistance cannot be estimated simultaneously, when the rotor flux amplitude is kept constant [6]. Special efforts have to be made in order to make this process possible [7].

In this paper an another method of reducing the influence of the rotor time constant improper identification is presented. Speed estimation method is similar to the one included in the MRAS$^{CC}$ estimator. An auxiliary variable is added to retune partly the rotor time constant used in this observer. This approach is slightly different than the one that can be found in the classical Sliding Mode Observer [8]. However, sign functions are replaced by PI regulators, thus the problematic filtration problem is omitted.

The mathematical model of the induction motor is presented first. Then the short description of the tested control structure (Direct Field Oriented Control, DFOC) and the novel MRAS type flux and speed estimator are described. The proposed solution is verified with simulation tests.

## II. Induction Motor Mathematical Model

The induction motor mathematical model will be introduced in this section. This model, derived with the classical simplifying assumptions, like: machine symmetry, magnetizing linearity, etc., will be written in the stationary reference frame, using per unit system [1] as follows:

$$\mathbf{u_s} = r_s \mathbf{i_s} + T_N \frac{d}{dt}\boldsymbol{\psi}_\mathbf{s} \tag{1}$$

$$\mathbf{0} = r_r \mathbf{i_r} + T_N \frac{d}{dt}\boldsymbol{\psi}_\mathbf{r} - j\omega_m \boldsymbol{\psi}_\mathbf{r} \tag{2}$$

$$\boldsymbol{\psi}_\mathbf{s} = x_s \mathbf{i_s} + x_m \mathbf{i_r} \tag{3}$$

$$\boldsymbol{\psi}_\mathbf{r} = x_r \mathbf{i_r} + x_m \mathbf{i_s} \tag{4}$$

$$\frac{d\omega_m}{dt} = \frac{1}{T_M}(m_e - m_L) \tag{5}$$

$$m_e = \operatorname{Im}\left(\boldsymbol{\psi}_\mathbf{s}^{*} \mathbf{i_s}\right) = \psi_{s\alpha} i_{s\beta} - \psi_{s\beta} i_{s\alpha} \tag{6}$$

where, respectively:
$\mathbf{u_s}=u_{sa}+ju_{sb}$, $\mathbf{i_s}=i_{sa}+ji_{sb}$ – stator voltage and current vectors,
$\mathbf{i_r}=i_{ra}+ji_{rb}$ – rotor current vector,
$\boldsymbol{\psi}_\mathbf{s}=\psi_{s\alpha}+j\psi_{s\beta}$, $\boldsymbol{\psi}_\mathbf{r}=\psi_{r\alpha}+j\psi_{r\beta}$ – stator and rotor flux vectors,
$r_s$, $x_s=x_m+x_{s\sigma}$ – stator winding resistance and reactance,
$r_r$, $x_r=x_m+x_{r\sigma}$ – rotor winding resistance and reactance,
$x_{s\sigma}$, $x_{r\sigma}$, $x_m$, – stator and rotor leakage reactances, magnetizing reactance,
$T_M$ – mechanical time constant,
$\omega_m$, $m_e$, $m_L$ – motor speed, torque and load torque,
$\sigma=1-x_m^2/(x_s x_r)$, $T_N=1/(2\pi f_{sN})$, $f_{sN}$ – motor nominal frequency.

The term $T_N$ is the effect of the per unit system introducing – base values are presented in the Appendix.

## III. MRAS-TYPE FLUX AND SPEED OBSERVER – MATHEMATICAL BASICS

The proposed MRAS estimator is based directly on the IM mathematical model (1) - (6). The estimation algorithm will be designed to force the stator current vector components estimation errors to zero, thus the current estimation is essential. In order to minimize the influence of the rotor time constant mismatch on the estimation process, the additional factor $\mu$ is added. After some algebraic transformations stator current vector estimation becomes:

$$T_N \frac{d\hat{\mathbf{i}}_{\mathbf{s}}}{dt} = \frac{1}{x_s\sigma}\left(\mathbf{u}_{\mathbf{s}} - r_s\hat{\mathbf{i}}_{\mathbf{s}} - \frac{x_m}{x_r}\left(\eta\hat{\mathbf{i}}_{\mathbf{s}} - (\tau_r + \hat{\mu})\hat{\boldsymbol{\psi}}_{\mathbf{r}} - j\hat{\omega}_m\hat{\boldsymbol{\psi}}_{\mathbf{r}}\right)\right) . \quad (7)$$

Rotor flux vector is calculated as below:

$$T_N \frac{d\hat{\boldsymbol{\psi}}_{\mathbf{r}}}{dt} = \eta\hat{\mathbf{i}}_{\mathbf{s}} - (\tau_r + \hat{\mu})\hat{\boldsymbol{\psi}}_{\mathbf{r}} + j\hat{\omega}_m\hat{\boldsymbol{\psi}}_{\mathbf{r}}, \quad (8)$$

where: $\tau_r = r_r/x_r$, $\eta = x_m \cdot r_r/x_r$.

If the following functions are chosen:

$$\begin{aligned} s_\omega &= e_{s\beta}\hat{\psi}_{r\alpha} - e_{s\alpha}\hat{\psi}_{r\beta} \\ s_\mu &= e_{s\beta}\hat{\psi}_{r\beta} + e_{s\alpha}\hat{\psi}_{r\alpha} \end{aligned} \quad (9)$$

where: $e_{s\alpha} = \left(\hat{i}_{s\alpha} - i_{s\alpha}\right), e_{s\beta} = \left(\hat{i}_{r\beta} - i_{s\beta}\right),$

a positive Lyapunov function candidate can be:

$$L = \frac{1}{2}\mathbf{s}^T\mathbf{s}, \quad \mathbf{s} = \begin{bmatrix} s_\omega & s_\mu \end{bmatrix}^T . \quad (10)$$

and its derivative is:

$$\dot{L} = \dot{\mathbf{s}}^T\mathbf{s} = \left(\dot{s}_\omega s_\omega + \dot{s}_\mu s_\mu\right) \quad (11)$$

The derivative of $\mathbf{s}$ is divided into two parts: one dependent and one independent on the estimated variables: $\hat{\omega}_m, \hat{\mu}$:

$$\dot{\mathbf{s}} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} g_1 & g_2 \\ g_3 & g_4 \end{bmatrix}\begin{bmatrix} \hat{\omega}_m \\ \hat{\mu} \end{bmatrix} \quad (12)$$

where:

$$g_1 = \frac{1}{T_N}\left(-\frac{1}{x_s\sigma}\frac{x_m}{x_r}\left(\hat{\psi}_{r\alpha}^2 + \hat{\psi}_{r\beta}^2\right) - e_{s\beta}\hat{\psi}_{r\beta} - e_{s\alpha}\hat{\psi}_{r\alpha}\right),$$

$$g_2 = \frac{1}{T_N}\left(-e_{s\beta}\hat{\psi}_{r\alpha} + e_{s\alpha}\hat{\psi}_{r\beta}\right),$$

$$g_3 = \frac{1}{T_N}\left(e_{s\beta}\hat{\psi}_{r\beta} - e_{s\alpha}\hat{\psi}_{r\alpha}\right),$$

$$g_4 = \frac{1}{T_N}\left(\frac{1}{x_s\sigma}\frac{x_m}{x_r}\left(\hat{\psi}_{r\alpha}^2 + \hat{\psi}_{r\beta}^2\right) - e_{s\beta}\hat{\psi}_{r\beta} - e_{s\alpha}\hat{\psi}_{r\alpha}\right).$$

Then the derivative of the Lyapunov function is calculated:

$$\dot{L} = f_1 s_\omega + f_2 s_\mu + \left(g_1 s_\omega + g_3 s_\mu\right)\hat{\omega}_m + \left(g_2 s_\omega + g_4 s_\mu\right)\hat{\mu} . \quad (13)$$

Let the estimation process be described as:

$$\begin{bmatrix} \hat{\omega} \\ \hat{\mu} \end{bmatrix} = \begin{bmatrix} -K_\omega h_\omega(s_\omega, s_\mu) \\ -K_\mu h_\mu(s_\omega, s_\mu) \end{bmatrix} . \quad (14)$$

The choice of $h(s_\omega, s_\mu)$ functions determines the type of the estimator. If the sign function is chosen the estimator becomes the Sliding-Mode Observer of the form slightly different than the one in [8]. The sign function can be approximated by saturation or sigmoid function, as it is shown in Fig.1a [9]. Another solution is to choose the linear function from Fig. 1b (or its odd power). The estimator becomes then of the MRAS-type. The *h* function is as follows:

$$\begin{bmatrix} h_\omega(s_\omega, s_\mu) \\ h_\mu(s_\omega, s_\mu) \end{bmatrix} = \begin{bmatrix} g_1 s_\omega + g_3 s_\mu \\ g_2 s_\omega + g_4 s_\mu \end{bmatrix} \quad (15)$$

Then (13) becomes:

$$\dot{L} = f_1 s_\omega + f_2 s_\mu - K_\omega\left(g_1 s_\omega + g_3 s_\mu\right)^2 - K_\mu\left(g_2 s_\omega + g_4 s_\mu\right)^2 \quad (16)$$

Large enough observer gains $K_\omega$ and $K_\mu$ assure negative value of the derivative of the Lyapunov function (16) and in consequence zero convergence of the estimation error of the stator current vector components in (9).

Since the steady-state speed estimation error appears if (14) is applied, the integration part is added in order to eliminate this phenomenon:

$$\begin{bmatrix} \hat{\omega} \\ \hat{\mu} \end{bmatrix} = \begin{bmatrix} -\left(K_\omega + \frac{K_{\omega i}}{s}\right)\left(g_1 s_\omega + g_3 s_\mu\right) \\ -\left(K_\mu + \frac{K_{\mu i}}{s}\right)\left(g_2 s_\omega + g_4 s_\mu\right) \end{bmatrix} . \quad (17)$$

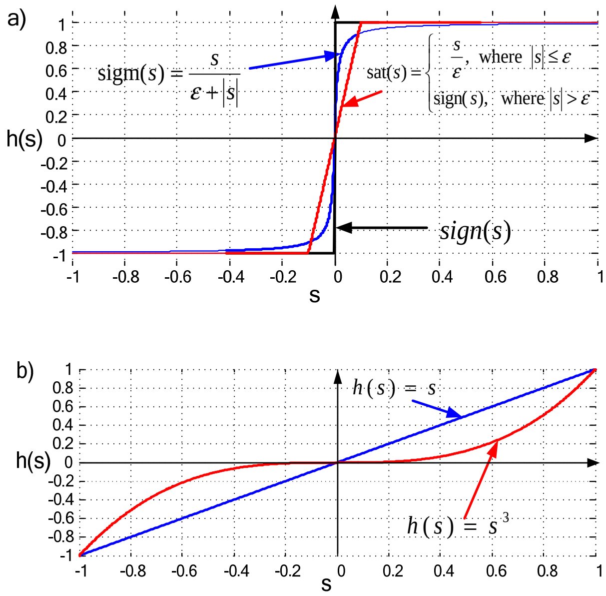



Figure 1. Diffrent forms of *h(s)* functions: a) sign function and its continuous approximations: saturation and sigmoid functions, b) linear function and its odd (third) power.

The estimation algorithm can be simplified, taking into consideration that: $g_1 s_\omega + g_3 s_\mu \approx g_1 s_\omega$, $g_2 s_\omega + g_4 s_\mu \approx g_4 s_\mu$, and $g_1<0$, $g_4>0$. Then:

$$\begin{bmatrix}\hat{\omega}\\ \hat{\mu}\end{bmatrix} = \begin{bmatrix}\left(K_\omega + \dfrac{K_{\omega i}}{s}\right)s_\omega \\ -\left(K_\mu + \dfrac{K_{\mu i}}{s}\right)s_\mu\end{bmatrix}. \qquad (18)$$

If the auxiliary variable $\mu$ is omitted the estimation algorithm becomes identical than the one in [5]. The general diagram of the proposed solution is presented in Fig. 2. The induction motor is the reference model itself. The described estimator is the adaptive model.

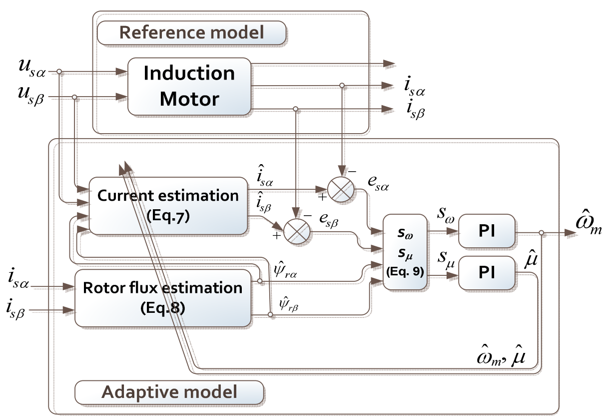



Figure 2. General scheme of the proposed MRAS-type estimator.

## IV. Sensorless Control Structure

The proposed observer was tested within the speed sensorless DFOC structure, shown in Fig. 3. Estimated speed, rotor flux amplitude and its angle are employed in the control algorithm. Four PI regulators with anti-windup mechanisms are applied to control motor speed, rotor amplitude and stator vector components, respectively. The rotor flux angle is used to perform reference frame transformations, as it was shown in the figure. The stator voltage vector is estimated using transistor switching signals and the DC bus voltage. The reference voltage vector components are modified by signals from the decoupling and dead-time compensator blocks [1].

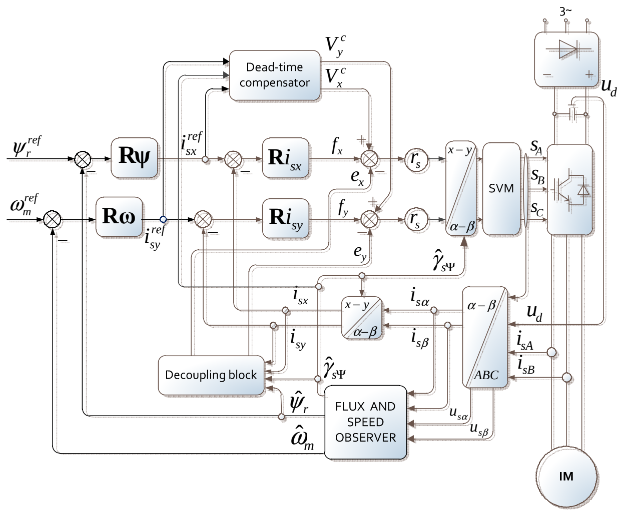



Figure 3. Direct field oriented sensorless control structure of the IM drive.

## V. Simulation Results

The proposed solution was tested under simulation. The tests were performed using Matlab-Simulink software, employing the Euler's integration method with constant sampling time ( 1e-5 s).

First, the estimator was tested in an open-loop manner, i.e. when the control feedback signals were taken from the well-known current flux simulator [1], and signals reconstructed by MRAS estimator were taken only for the comparison.

Simulation results for speed reverse operation can be seen in Fig. 4. The flux is stabilized on the nominal level, the rated external torque is applied after 0.5 s. It can be noticed, that transient of estimation errors (during start-up or reverse) of the motor speed, rotor flux and stator current components are practically negligible (Fig. 4 b, d, e).

As was mentioned in the previous section, the steady-state error can occur (especially for the rotor flux estimation) when the integration factor is not included in the speed and the additional variable ($\mu$) estimation algorithms. This situation is shown in Fig. 5.

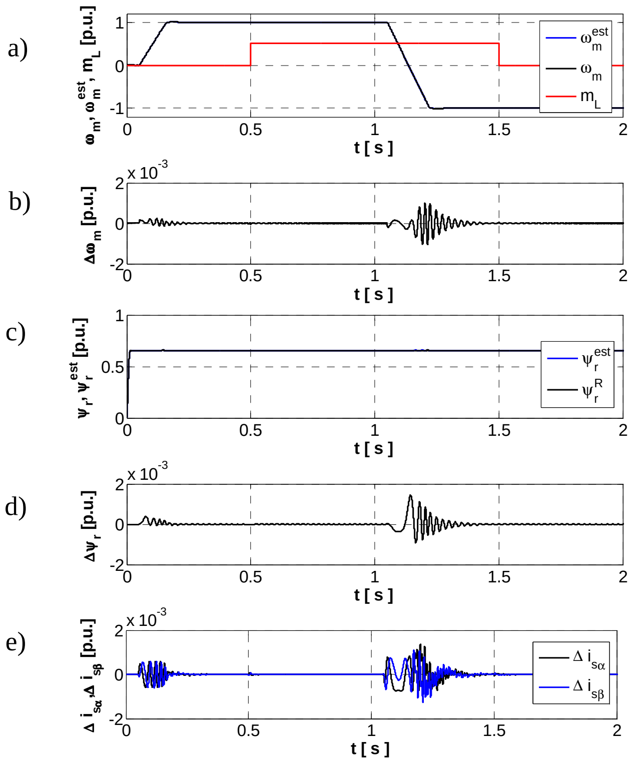



Figure 4. Simulation results of the MRAS estimator in an open-loop operation: a) estimated and real speed, load torque, b) speed estimation error, c) estimated and real rotor flux amplitudes, d) rotor flux amplitude estimation error, e) estimation error of the stator current components

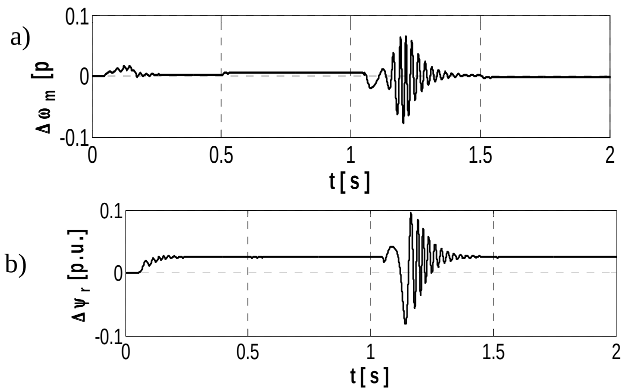



Figure 5. Steady-state estimation errors visible in a) speed and b) rotor flux amplitude when the integration part is not included.

Next, tests of the field-oriented sensorless control structure with the proposed observer were performed. All control feedback signals (rotor flux and speed) are taken from the proposed MRAS estimator. The reference speed, flux and load torque are similar as previously. The speed and flux estimation errors are shown in Fig. 6. The errors reach slightly larger values on transients, however they still can be omitted.

Introducing the auxiliary variable, $\mu$, one can expect increased insensitivity over rotor time constant changes. Appropriate tests were performed, and their results can be seen in Fig. 7 – Fig. 9. First figure illustrates the situation when the inverse of the rotor time constant ($\tau_r$) used in the observer decreases after 0.5s and the load torque is zero. All estimation errors can be found disregarded when the observer gain $K_\mu$ is non-zero (black lines).

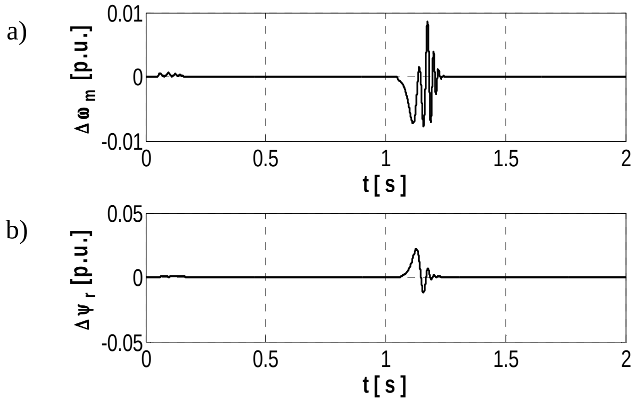
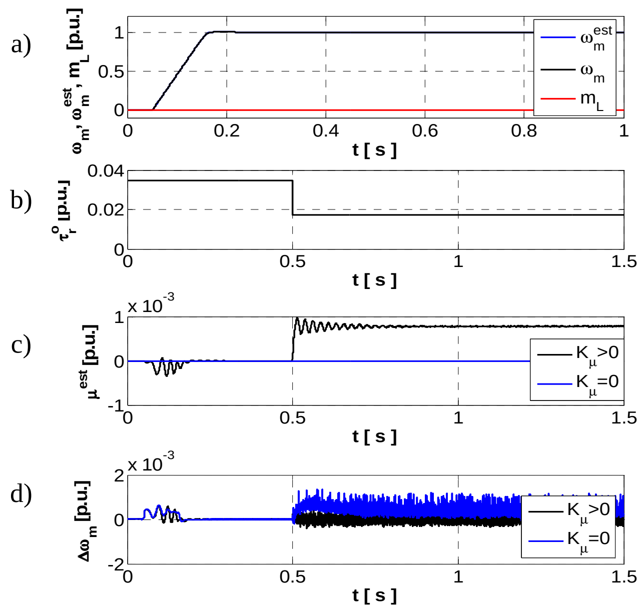



Figure 6. Estimation errors of a) speed and b) rotor flux amplitude during speed sensorless operation, the estimator works within the control structure.

Similar tests were performed for nominal torque operation and shown in Fig. 8. Speed estimation error becomes much larger than previously, and its value is almost independent on the observer gain $K_\mu$ value. Rotor flux and stator current estimation errors are reduced significantly.

In Fig. 9 the opposite situation is presented – the inverse of the rotor time constant used in the observer increases. The speed estimation process becomes incorrect – quite large oscillations can be seen in Fig. 9a. However, this situation is much more improbable – when rotor resistance grows with temperature, the inverse of the rotor time constant of the IM grows, too – thus the value used in the observer is lower in a real drive system.



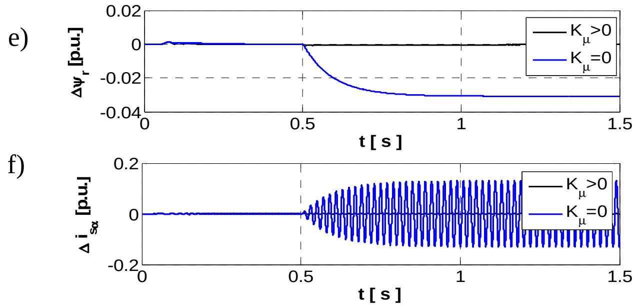



Figure 7. Simulation results of sensorless control structure with the proposed estimator for negative rotor time constant mismatch, $\tau_{r,o}=0.5\tau_{r,R}$, and load torque $m_L= 0$: a) real and estimated speed, b) rotor time constant used in the observer, c) the additional variable $\mu$, d) speed estimation error, e) rotor flux amplitude estimation error, f) stator current $\alpha$ component estimation error

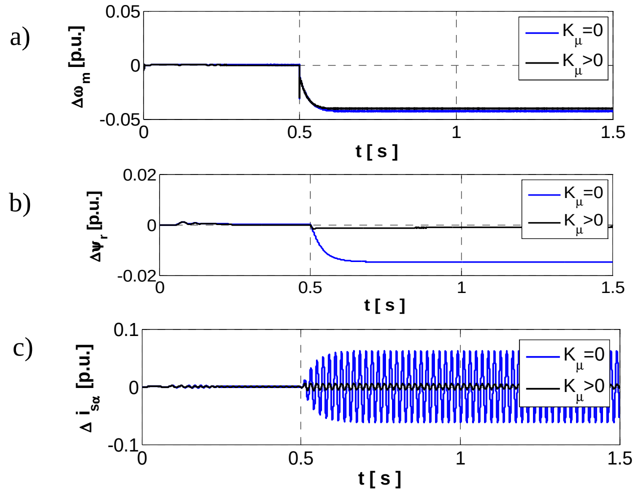



Figure 8. Simulation results of rotor time constant negative mismatch tests, $\tau_r = 0.5\tau_{r,R}$,operation with full load torque, $m_L = m_N$

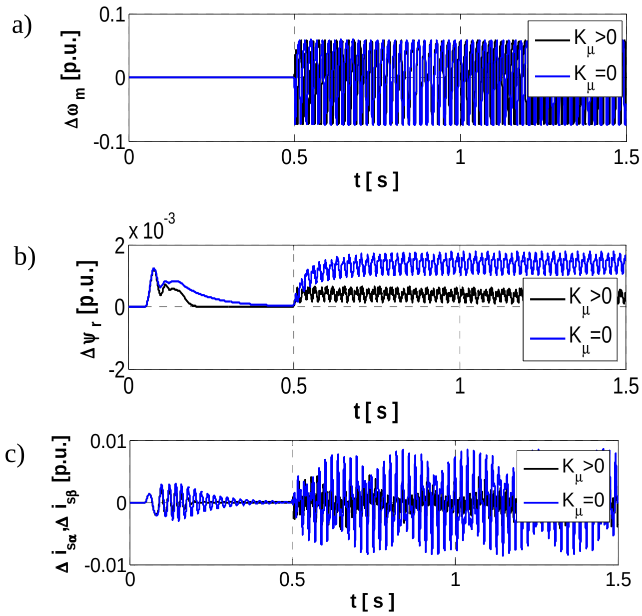



Figure 9. Positive rotor time constant mismatch influence on sensorless IM drive performance, $\tau_{r,o} = 1.25\tau_r$

The presented results indicate clearly that the proposed solution can be successfully applied in sensorless IM drives, wherever the rotor time constant is rising with temperature. An easy application is assured due to the simple PI mechanisms used in the estimation process.

## VI. Conclusions

In the paper speed and flux estimator for sensorless induction motor drives is proposed. The estimator uses the Model Reference Adaptive System theory to calculate motor state variables. The adaptive model consists of two differential equations which allow estimating the stator current and the rotor flux vectors. The induction motor constitutes the reference model. Two signals being a combination of estimation errors of the stator current vector and rotor flux vector components are used as adaptive signals. First signal is the estimated motor speed, the second one is the additional variable acting as a rotor time constant tuning factor.

The simulation tests were performed in order to validate the presented mathematical assumptions. The estimator performance was examined within the field-oriented speed sensorless control structure. The rotor time constant mismatch tests were also performed. The proposed solution, i.e. the auxiliary variable introduction, remarkably reduces the rotor flux and stator current vector estimation errors, for no load as well as for the nominal torque operation. However, in comparison with the base solution (without $s_\mu$ function) speed estimation is almost the same, the effect is insignificant. The estimator is especially sensitive to rotor time constant change when the value used in the estimator is larger than the real one. However, due to temperature rising with operating time of the IM, the real time constant becomes gradually higher then the initial value used in the estimator.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## APPENDIX

Motor rated data

| | | | |
|---|---|---|---|
| $P_N = 1.1$ | [kW] | $n_N = 1380$ | [rpm] |
| $U_N = 230/400$ | [V] | $f_N = 50$ | [Hz] |
| $I_N = 5.0/2.9$ | [A] | $p_b = 2$ | |

Parameters of the tested IM

| $R_s$ | $R_r$ | $X_s$ | $X_r$ | $X_m$ | |
|---|---|---|---|---|---|
| 5.9 | 4.5 | 131.1 | 131.1 | 123.3 | [Ω] |
| 0.07 | 0.06 | 1.725 | 1.725 | 1.62 | [p.u.] |

Per unit system calculation methodology (reference values):

$U_b = \sqrt{2}\, U_{Nf}$, $I_b = \sqrt{2}\, I_N$, $Z_b = U_b/I_b$, $\omega_b = 2\pi f_{sN}$, $\Psi_b = U_b/\omega_b$, $S_b = (3/2) U_b I_b$, $M_b = S_b p_b/\omega_b$,

Parameters in per unit system:

$r_s = R_s/Z_b$, $r_r = R_r/Z_b$, $x_s = l_s = X_s/Z_b$, $x_r = l_r = X_r/Z_b$, $x_m = l_m = X_m/Z_b$

## THE AUTHORS

**Grzegorz Tarchala** received the MSc degree from Electrical Engineering Faculty, Wroclaw University of Technology, Wroclaw, Poland in 2009. Since 2009, he has been a PhD student in the Institute of Electrical Machines, Drives and Measurements. His main field of interest is the induction motor drive control and state variable estimation, control theory applications in electric drives and digital signal processors.

**Teresa Orlowska-Kowalska** received the Ph.D. and D.Sc. degrees from Wroclaw University of Technology, Wroclaw, Poland, in 1976 and 1990, respectively. Since 1993, she has been a Professor of electrical engineering and the Chair of the Electric Drives Control, Institute of Electric Machines, Drives, and Measurements, Wroclaw University of Technology. She is the author and coauthor of close to 400 journal papers and conference proceedings, two textbooks, and fourteen chapters in monographs. Her research interests include the mathematical modeling and microprocessor control of electric drives and power electronic systems, the application of modern control methods to electric drives, the state estimation of induction motors using state observers, Kalman filters, and neural networks, and, in the last few years, neural networks and fuzzy-logic techniques applied to electric drives control. Prof. Orlowska-Kowalska is a member of the Electrical Engineering Committee of the Polish Academy of Science (since 1996), the European Power Electronics Association, and international steering committees of a few well-known European conferences. Since 1997, she has been a Reviewer of the IEEE Transactions on Industrial Electronics and the IEEE Transactions on Industry Applications. Since 2004, she has been an Associate Editor for the IEEE Transactions on Industrial Electronics.

**Mateusz Dybkowski** received the MSc and PhD degrees from the Electrical Engineering Faculty, Wroclaw University of Technology, Wroclaw, Poland, in 2004 and 2008, respectively. Since 2008, he has been a member of the academic staff with the Electric Drives Control, Institute of Electric Machines, Drives and Measurements, Wroclaw University of Technology. He is an author and co-author of close to 50 scientific papers. His main fields of interest are the induction motor drive control and state variable estimation, control theory applications in electric drives, digital signal processors, and field-programmable gate array applications.