Higher Harmonics in Output Voltage of Inverters

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Abstract — **The article presents the basic mathematical and programming tools for analyzing the higher harmonics of inverters. Harmonic spectrum of the output voltage in a simple and cascade connection of an inverter models are analyzed using Matlab/SimPower Systems. The influence of the cascade connection on the output voltage harmonic spectrum is observed. There is also analyzed the total harmonic distortion at different ways of switching.**

Keywords — converter circuits, simulations, harmonics

I. INTRODUCTION

The output voltage of an inverter has in general non-sinusoidal shape. The required AC output quantity - frequency and voltage – is created by a sequence of "segments" properly cut out from the input variable quantity, which is a DC-voltage. The required output quantities, AC voltage amplitude and frequency, are created either from rectangular pulses or by the pulse-width-modulation (PWM).

Power source with a non-sinusoidal voltage supplied to an electric equipment brings some undesirable effects. For example, it can cause additional losses in the windings and ferromagnetic circuits of transformers. In AC motors the additional losses are higher and operating characteristics of motors are worse. In photovoltaic power sources, the use of inverters must be carefully considered, because a wide range of harmonics can be generated. These would greatly decline the quality of produced and transmitted electric energy.

II. BRIDGE CONNECTION OF INVERTERS

Basic schema of a bridge inverter is shown in Fig. 1. The operation principle of the inverter is described in [1]. The positive voltage at the load is obtained when the transistors V1 and V4 are switched on, the negative voltage is obtained when the transistors V2 and V3 are switched on. The zero voltage at the load occurs when all transistors are turned off. For inductive loads, the current inertia of inductance is looped through the anti-parallel diodes.

Individual semiconductor switches can lead the current maximally for a half of the period. It means that for a fundamental output frequency f = 50 Hz (T = 2π) the maximum value of the conduction angle of switches is π. This kind of inverter is called "an inverter with 180 degrees conduction angle". The conduction angle of switches may be reduced. The inverter can have a 150 degrees conduction angle or 120 degrees conduction angle or even less. Reduced conduction angle is needed only for two of the switches. 120 degrees inverter can be considered as the best one [1].

The content of higher harmonics in the output voltage u_z depends highly on the conduction angle Ψ. In [1] it is derived the Fourier series of the voltage u_z shown in Fig. 2. The voltage shown in this figure does not contain a DC-component. Only odd harmonics have non-zero value. The harmonic spectrum of the voltage at the load in dependence on the conduction angle Ψ can be described in the form (1)

$$u_z(\omega_z t) = \frac{4U}{\pi}\sum_{k=1}^{\infty}\frac{1}{2k-1}\left[(2k-1)\frac{\pi-\psi}{2}\right]\sin[(2k-1)\omega_z t] \qquad (1)$$

where U is the value of DC voltage, $\omega_z=2\pi f_z$, and harmonic order $n = 2k-1$.

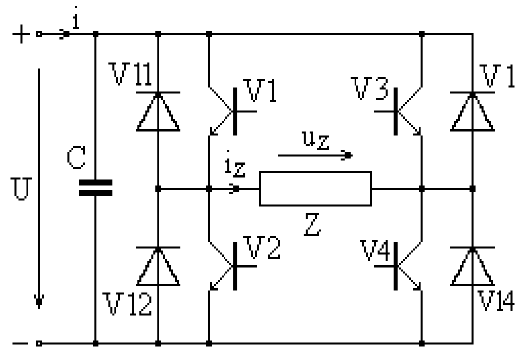

Fig.1. Single-phase voltage inverter

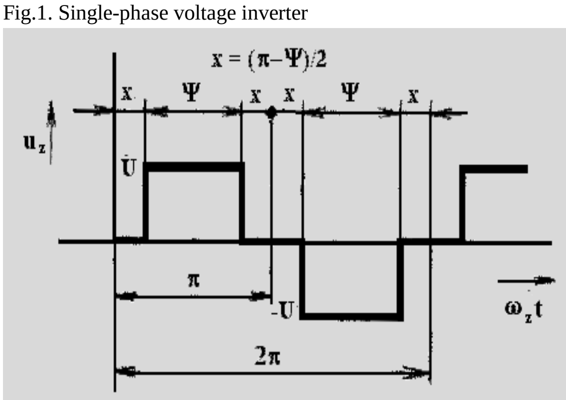

Fig. 2. Output voltage of the single-phase bridge inverter with the conduction angle of switches Ψ

Fig. 3 shows the course of the 1st and several higher harmonic waveforms for one period obtained by the equation (1) when the conduction angle Ψ = 180 degrees.

The superposition of the 1st, 3rd, 5th, 7th, 9th and 11 harmonic is approaching the rectangular shape of the output voltage from the inverter.

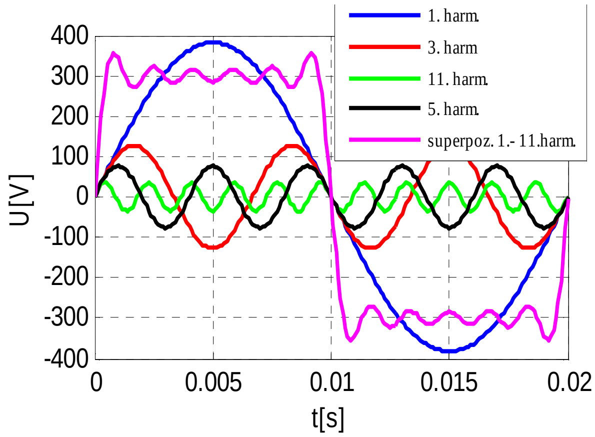

Fig. 3. Waveform of calculated higher harmonics for voltage waveform shown in Fig. 2.

If the DC input voltage of the inverter is U = 300 V and the conduction angle is Ψ = 180 degrees, then the effective value of the basic/first harmonic for the output voltage according to the (2) is U_{ef1} = 270,1 V [2].

$$U_{ef1} = \frac{4U}{\pi\sqrt{2}} \tag{2}$$

If the values of the first harmonic U_1 and also of the higher harmonics U_n are known, it is possible to calculate the total harmonic distortion *THD*

$$THD = \frac{\sqrt{\sum_{n=2,3,...}^{\infty} U_n^2}}{U_1} \tag{3}$$

For the case described above, when considering first eleven odd harmonics the *THD* is 0,44 (44 %). When considering further higher harmonics the *THD* converges to the value of 48 %. If *THD* is smaller, the course of voltage is nearer to the fundamental harmonic.

In the *Simulink/SimPowerSystems* it is possible to perform the same analysis by using the block for FFT analysis. In Fig. 4 it is shown the output of this block with the percentage values of the first 100 harmonics in the inverter output voltage. The amplitude of the first harmonic obtained by FFT analysis is U = 381,9 V. This value corresponds exactly to the amplitude of first harmonic in Fig. 3. The effective voltage of the first harmonic is equivalent to the result obtained by (2) U_{ef1} = 381,9 V / √2 = 270 V. The otal harmonic distortion *THD* is 48,31 %. This value corresponds to the calculation according (3). From the known value of the first harmonic, it is possible to calculate further values of harmonic amplitudes as well. For example, the amplitude of the third harmonic has the value of about 33 % of the first harmonic. Thus the third harmonic has absolute amplitude of about 126 V (see also the waveform of the 3rd harmonic in Fig. 3).

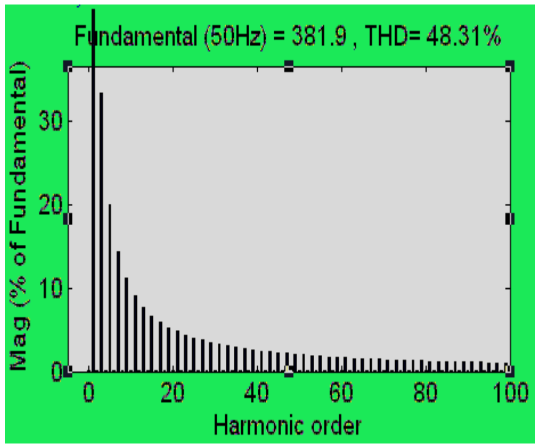

Fig.4. Spectrum of higher harmonics for 180 degrees inverter obtained by FFT Analysis

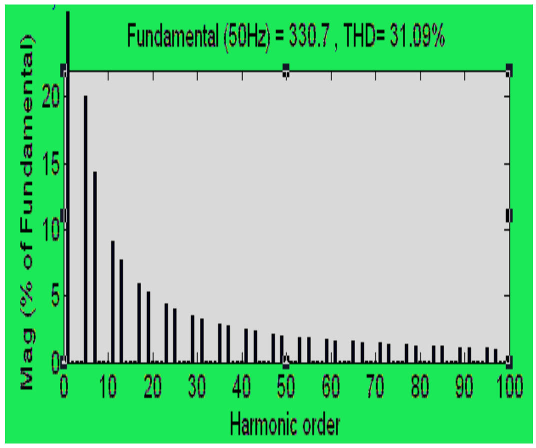

Fig. 5. Spectrum of higher harmonics for 120 degrees inverter obtained by FFT Analysis

Using the same method it is possible to analyze waveforms and harmonic spectra for other conduction angles. In Fig. 5 and 6 there are shown the results of analysis for Ψ = 120 degrees. It can be seen that the third harmonic and its multiples were eliminated. However, reducing the conduction angle Ψ of the inverter for the same DC input voltage U = 300 V gives reduced output voltage *RMS* value.

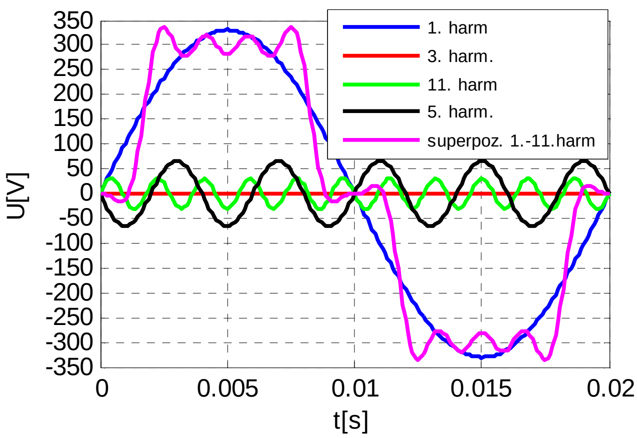

Fig. 6. Waveform of higher harmonics obtained by equation (1) for 120 degrees inverter

III. MULTI-LEVEL CASCADE INVERTERS

The advantages of multilevel inverters are lower voltage strain of the power semiconductor devices and more levels of the output voltage. This has a positive effect on the harmonic spectrum of the output voltage. The disadvantages are difficult connection of the converter, complicated switching control algorithms and variation of potentials in the middle of the capacitive divider [3].

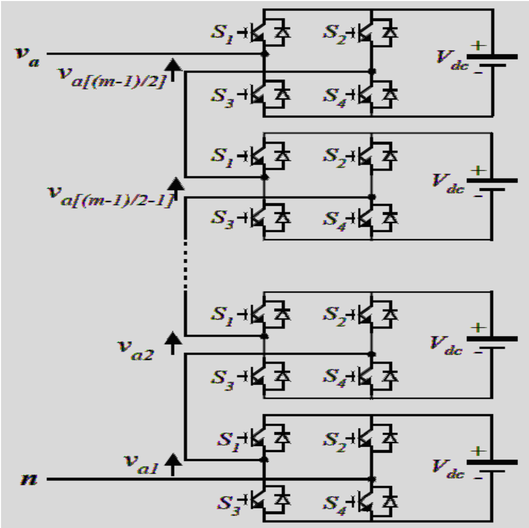

Fig.7. Structure of single-phase cascade multilevel inverter

Fig. 7 shows the structure of the single-phase cascade multilevel inverter [4]. Each parallel source - for example the photovoltaic module - is connected to a single-phase fully controlled bridge inverter. Each inverter can generate three different levels of output voltages, namely +Vdc, 0 and -Vdc. The voltage +Vdc is obtained when the transistors S1 and S4 are switched on, the voltage -Vdc is obtained when the transistors S2 and S3 are switched on and the output voltage is zero when all four switching transistors are switched on.

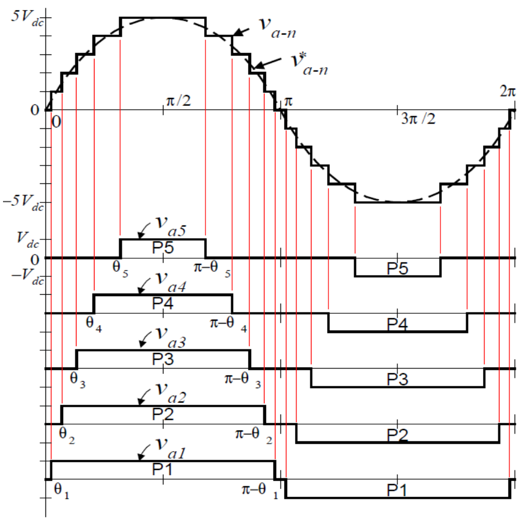

Fig. 8. Assembling the output voltage in multilevel inverters

AC inverters of each level are connected in series. Total output voltage of the whole inverter is the sum of voltages of each sub-inverter (Fig. 8). The duty cycle of each voltage level is yet another. The conduction angles Ψ1, ..., Ψ5 can be chosen in such a way that the THD is minimized.

The influence of various conduction angles on the higher harmonics content was investigated on a computer model of a 5-level inverter created in the Matlab/Simulink.

In the first case the input voltage of each level was set to 80 V.

In the second case the input voltage of each level was set to 85 V. The output voltage waveforms for both cases are shown in Fig. 9. The effective value of the output voltage was the same in both cases. Relations between the change in the inverter input voltage and conduction angle in order to keep constant rms value of output voltage are investigated in [6].

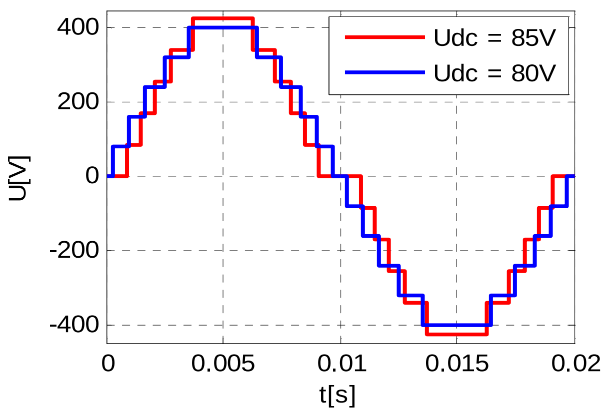

Fig. 9. Output voltage waveforms of multilevel inverters

Fig. 10 shows the harmonic spectrum of the output voltage of the bridge inverter with larger conduction angles and lower input voltage 80 V. On the other hand, Fig. 11 shows the output harmonic spectrum of the same inverter, but with smaller conduction angles and with higher input voltage 85 V.

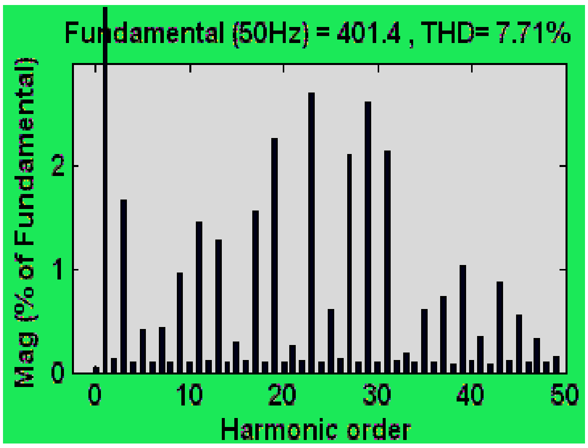

Fig. 10. Spectrum of higher harmonics in a multilevel inverter with input voltage 80 V

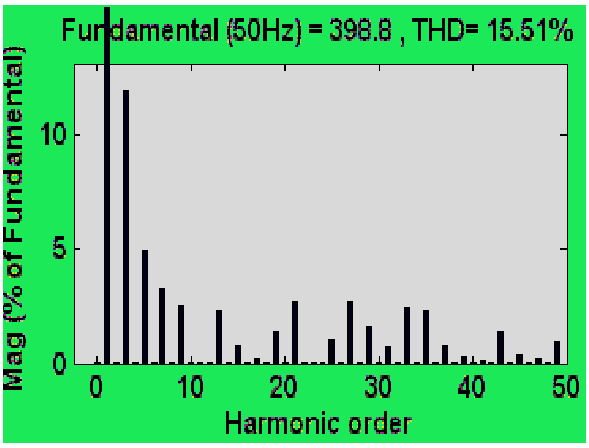

Fig. 11. Spectrum of higher harmonics in a multilevel inverter with input voltage 85 V

IV. CONCLUSION

The aim of this article was to introduce the basic techniques used for analysis of the harmonic content of output voltage in the single-phase voltage inverters. Various methods for the higher harmonics elimination in frequency spectrum of the generated voltage were introduced. The elimination of higher harmonics is necessary in order to decrease the additional power losses in electric machines and devices and also for improving the operation characteristics of electric motors. Created computer model of the inverter will represent a subsystem in computer model of electrical vehicle drive system, which is currently under development.

V. REFERENCES

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