

GFEM AND LSFEM IN THE SOLUTION OF THE TRANSIENT BIDIMENSIONAL CONVECTION-DIFFUSION EQUATION

E. C. Romão^a,
J. B. Aparecido^b,
J. B. Campos-Silva^b,
and L. F. M. de Moura^a

^aUniversidade Estadual de Campinas
Faculdade de Engenharia Mecânica
Departamento de Engenharia Térmica e
Fluidos
Rua Mendeleev, 200
Cidade Universitária "Zeferino Vaz"
Distrito de Barão Geraldo
CEP:13083-860 Campinas SP Brasil
estaner23@yahoo.com.br

^bUniversidade Estadual Paulista
Departamento de Engenharia Mecânica
Av. Brasil Centro, 56
CEP - 15385-000 - Ilha Solteira SP Brasil

ABSTRACT

Convection dominated flows result in a hyperbolic system of equations which leads to ill-conditioned matrices and oscillatory approximations when using the classical Galerkin Finite Element Method (GFEM). In this paper, the Least Square Finite Method (LSFEM) is introduced in the study of transient bidimensional convection-diffusion problems. The differentiated equation of second order which describes the convective-diffusive phenomenon is transformed into an equivalent system of partial differentiated equations of first order which is discretized by the formulation of the LSFEM resulting in a defined algebraic, symmetrical and positive system. The performance of the method is verified by the solution of a test- problem.

Keywords: Finite Element Method, Least Squares, Galerkin, Convection-Diffusion, Transients Problems.

NOMENCLATURE

k	thermal conductivity
x, y	space coordinates
t	time coordinate
f	source
N_j	interpolation function
N_{node}	number of nodes in each finite element
q_x	heat flux along the x-axis
q_y	heat flux along the y-axis
\hat{q}_x^e	heat flux approximation along x-axis
\hat{q}_y^e	heat flux approximation along t-axis
u	temperature
\hat{u}_j^e	temperature approximation in the finite element
v	weight function
v_i^e	weight function in the element
R_j	residual equations
\Re	real numbers

Greek symbols

Ω	two-dimensional domain
Ω^e	two-dimensional domain in the element
α	coefficient of Crank-Nicholson Method

Subscripts

node identification of a node

1. INTRODUCTION

From the 50s on, some researchers such as Turner et al. (1956), Clough (1960) and Argyris (1963) started to use the Least Square Finite Element Method in its patterned format and the Galerkin Method to solve dominant diffusive problems and they had excellent results.

However most part of the Mechanics of Fluids problems and Heat and Mass Transfer are classified as dominant convective. Several authors presented papers in the last decades demonstrating that the application of the Galerkin Method in the solution of dominant convective problems in transient regime generate numerical oscillations, for example, Camprub et al. (2000), Romão (2004) e Romão et al. (2008), fact that could be avoided only with a large refinement of the mesh what would be computationally and financially expensive for bi and three-dimensional problems.

In this paper, the application of the Least Square Finite Method in the numerical solution of dominant convective phenomenon in transient regimes on a bidimensional domain will be presented. The application of the LSFEM always yield to an algebraic system of equations symmetrical and

positive defined that can be solved, for example, by conjugate gradient methods and the solutions are free of oscillations, even in problems with the convective phenomenon dominant.

Firstly, the time discretization of the partial differential equations which model the transient convective-diffusive phenomenon will be presented followed by the spatial discretization. For the time discretization, the Crank-Nicolson scheme has been used for the spatial discretization, the Galerkin Finite Element Method (GFEM) and the Least Square Finite Element Method (LSFEM) are applied, in order to compare the solutions.

2. EQUATION MODEL

It is introduced here a numerical study of the partial differential equation that models a general transient bidimensional convective-diffusive phenomenon defined in the $\Theta = \Xi \otimes \Omega \subset \mathfrak{R}^3$ domain in which Ξ and Ω are closed and bounded domains. The governing equation is:

$$\psi \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left(k \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial u}{\partial y} \right) + A_x \frac{\partial u}{\partial x} + A_y \frac{\partial u}{\partial y} + Bu + f = 0 \quad (1)$$

in which is admitted $k = \text{constant} \neq 0$, $u = u(x,y)$, $A_x = A_x(x,y)$, $A_y = A_y(x,y)$, $B = B(x,y)$, $\Psi = \Psi(x,y)$ and $f = f(x,y)$ with $x, y, t \in \mathfrak{R}$ with boundary conditions of the first and second type as well the initial condition.

3. TIME DISCRETIZATION

As presented in Romão et al. (2004), the bidimensional equation which will be evaluated in this paper is a parabolic equation, that is, an equation of first order in time and of second order in space. The method used for the discretization in time of equation (1), is a method from the family of α approach, in which the moderated average from the derivated in relation to the time of the dependent variable is approached by two consecutive steps in time by a linear interpolation of the values of the variable for the two steps

$$\frac{\partial u}{\partial t} \approx \frac{\{u\}^{s+1} - \{u\}^s}{\Delta t^{s+1}} = (1-\alpha) \cdot \{u\}^s + (\alpha) \cdot \{u\}^{s+1}, \quad \text{for } 0 \leq \alpha \leq 1, \quad (2)$$

in which $\{ \ }^s$ refers to the value of the variable in the step "s" and $\Delta t^{s+1} = t^{s+1} - t^s$ is the (s + 1) step in time. In this paper the Crank-Nicolson scheme in which $\alpha = 1/2$ will be used (Chung, 2002).

4. SPATIAL DISCRETIZATION

In this item the spatial discretization of Eq.(1) will be presented by the formulations of Galerkin and the Least Square Finite Element Methods.

4.1 Galerkin Method

Initially, the discretization must be made approaching the u function described in Eq. (1) by a function \hat{u} , as follows

$$\tilde{u} \approx \hat{u}^e = \sum_{j=1}^{N_{nodes}} N_j \hat{u}_j^e \quad (3)$$

and N_{nodes} is the number of nodes in each element, N_j are the interpolation functions and \hat{u}_j^e are the nodal values of u in the element. By the described approach in Eq. (3) and using Eq. (1), a residual is defined as

$$R = \left\{ \frac{\psi}{\Delta t} \tilde{u} + \alpha \left[\frac{\partial}{\partial x} \left(k \frac{\partial \tilde{u}}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial \tilde{u}}{\partial y} \right) + A_x \frac{\partial \tilde{u}}{\partial x} + A_y \frac{\partial \tilde{u}}{\partial y} + B\tilde{u} \right] \right\}^{s+1} + f + \left\{ -\frac{\psi}{\Delta t} \tilde{u} + (1-\alpha) \times \left[\frac{\partial}{\partial x} \left(k \frac{\partial \tilde{u}}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial \tilde{u}}{\partial y} \right) + A_x \frac{\partial \tilde{u}}{\partial x} + A_y \frac{\partial \tilde{u}}{\partial y} + B\tilde{u} \right] \right\}^s \quad (4)$$

Now, it is possible to start introducing the approach by the Galerkin Finite Method in which there is the need to define the variational formulation of the general problem (1), as follows. It must be found $\hat{u}^e \in V^e$, $V^e \in C^2(\Omega)$ in a way that

$$\int_{\Omega^e} R v_i^e d\Omega = 0, \forall v_i^e \in V^e, i = 1, 2, \dots, N_{nodes} \quad (5)$$

in which $\Omega \subset \mathfrak{R}^2$ is a limited and closed domain.

Equation (5) will be integrated for any $v_i^e = N_i$, $i = 1, 2, \dots, N_{nodes}$, that is, in the Galerkin Finite Method the weight function is equal to the interpolation function. Using Eq. (3), effecting the proper integrations and some algebra we have the following matricial linear system

$$[K] \{ \hat{u}^{s+1, e} \} = \{ F \} - [H] \{ \hat{u}^{s, e} \} \quad (6)$$

in which

$$K_{ij} = -\int_{\Omega^e} \alpha k \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} d\Omega - \int_{\Omega^e} \alpha k \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} d\Omega + \int_{\Omega^e} \alpha A_x N_i \frac{\partial N_j}{\partial x} d\Omega + \int_{\Omega^e} \alpha A_y N_i \frac{\partial N_j}{\partial y} d\Omega + \int_{\Omega^e} \left(\frac{\psi}{\Delta t} + \alpha B \right) N_i N_j d\Omega \quad (7a)$$

$$H_{ij} = -\int_{\Omega^e} (1-\alpha) k \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} d\Omega - \int_{\Omega^e} (1-\alpha) k \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} d\Omega + \int_{\Omega^e} (1-\alpha) A_x N_i \frac{\partial N_j}{\partial x} d\Omega + \int_{\Omega^e} (1-\alpha) A_y N_i \frac{\partial N_j}{\partial y} d\Omega + \int_{\Omega^e} \left(\frac{-\psi}{\Delta t} + (1-\alpha) B \right) N_i N_j d\Omega \quad (7b)$$

$$F_i = -\int_{\Omega^e} f N_i d\Omega \quad (7c)$$

with $i, j = 1, 2, \dots, Nnodes$. Further details about the formulation described before can be found in Romão (2004).

4.2 Least Square Finite Element Method

The approach by the Least Square Method has as initial characteristic the addition of two extra variables in the problem by the following equations

$$q_x = -k \frac{\partial u}{\partial x}, \quad q_y = -k \frac{\partial u}{\partial y} \quad (8a-b)$$

then the problem is not represented by the partial differentiated equation of second order anymore, but it is a system of three partial differential equations of first order. After replacing the Eqs. (8a-b) in the Eq. (1) and time discretization we have

$$\psi \frac{\tilde{u}^{s+1} - \tilde{u}^s}{\Delta t} = \alpha \left[\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} - A_x \frac{\partial \tilde{u}}{\partial x} - A_y \frac{\partial \tilde{u}}{\partial y} - B \tilde{u} \right]^{s+1} - f + (1-\alpha) \left[\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} - A_x \frac{\partial \tilde{u}}{\partial x} - A_y \frac{\partial \tilde{u}}{\partial y} - B \tilde{u} \right]^s \quad (9a)$$

$$\tilde{q}_x + k \frac{\partial \tilde{u}}{\partial x} = 0, \quad \tilde{q}_y + k \frac{\partial \tilde{u}}{\partial y} = 0, \quad (9b-c)$$

Now, we effect the approximation in each element of the functions \tilde{u} , \tilde{q}_x e \tilde{q}_y by the functions \hat{u}^e , \hat{q}_x^e e \hat{q}_y^e in the following way:

$$\tilde{u} \cong \hat{u}^e = \sum_{i=1}^{Nnode} N_i \hat{u}_i^e; \quad \tilde{q}_x \cong \hat{q}_x^e = \sum_{i=1}^{Nnode} N_i \hat{q}_{x_i}^e; \quad \tilde{q}_y \cong \hat{q}_y^e = \sum_{i=1}^{Nnode} N_i \hat{q}_{y_i}^e, \quad (10a-c)$$

for any step in time and with $Nnode$ being the number of nodes in each element.

After the spatial approximation be defined, we can write the residual equations of Eqs. (10 a-c) as follows

$$R_1(x, y) = \left\{ \frac{\psi}{\Delta t} \hat{u} + \alpha \left[-\frac{\partial \hat{q}_x}{\partial x} - \frac{\partial \hat{q}_y}{\partial y} + A_x \frac{\partial \hat{u}}{\partial x} + A_y \frac{\partial \hat{u}}{\partial y} + B \hat{u} \right] \right\}^{s+1} + f + \left\{ -\frac{\psi}{\Delta t} \hat{u} + (1-\alpha) \left[-\frac{\partial \hat{q}_x}{\partial x} - \frac{\partial \hat{q}_y}{\partial y} + A_x \frac{\partial \hat{u}}{\partial x} + A_y \frac{\partial \hat{u}}{\partial y} + B \hat{u} \right] \right\}^s \quad (11a)$$

$$R_2(x, y) = \hat{q}_x + k \frac{\partial \hat{u}}{\partial x}, \quad R_3(x, y) = \hat{q}_y + k \frac{\partial \hat{u}}{\partial y} \quad (11b-c)$$

The basic idea of the Least Square Finite Element Method is determining $\hat{u}^e \in V^e$ as minimizer of the functional (Jiang, 1998)

$$I(R_1, R_2, R_3) = \int_{\Omega^e} R_1^2(x, y) d\Omega + \int_{\Omega^e} R_2^2(x, y) d\Omega + \int_{\Omega^e} R_3^2(x, y) d\Omega \quad (12)$$

that is, the first variation of the functional of \hat{u} , \hat{q}_x e \hat{q}_y results in

$$\int_{\Omega^e} (\delta R_1) R_1 d\Omega + \int_{\Omega^e} (\delta R_2) R_2 d\Omega + \int_{\Omega^e} (\delta R_3) R_3 d\Omega = 0 \quad (13)$$

Making the appropriate algebraism and necessary integrations, the Eq. (13) generates the following matricial linear system.

$$\begin{bmatrix} A & B & C \\ B^T & D & E \\ C^T & E^T & G \end{bmatrix} \begin{Bmatrix} \hat{u}^{e,s+1} \\ \hat{q}_x^{e,s+1} \\ \hat{q}_y^{e,s+1} \end{Bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} - \begin{bmatrix} H & I & J \\ M & K & L \\ P & S & O \end{bmatrix} \begin{Bmatrix} \hat{u}^{e,s} \\ \hat{q}_x^{e,s} \\ \hat{q}_y^{e,s} \end{Bmatrix} \quad (14)$$

in which

$$A_{ij} = \int_{\Omega^e} \left\{ \left[\alpha A_x \frac{\partial N_i}{\partial x} + \alpha A_y \frac{\partial N_i}{\partial y} + \left(\frac{\psi}{\Delta t} + \alpha B \right) N_i \right] \times \left[\alpha A_x \frac{\partial N_j}{\partial x} + \alpha A_y \frac{\partial N_j}{\partial y} + \left(\frac{\psi}{\Delta t} + \alpha B \right) N_j \right] + k^2 \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + k^2 \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right\} d\Omega \quad (15a)$$

$$B_{ij} = \int_{\Omega^e} \left\{ \left[\alpha A_x \frac{\partial N_i}{\partial x} + \alpha A_y \frac{\partial N_i}{\partial y} + \left(\frac{\psi}{\Delta t} + \alpha B \right) N_i \right] \times \left[-\alpha \frac{\partial N_j}{\partial x} \right] + k \frac{\partial N_i}{\partial x} N_j \right\} d\Omega \quad (15b)$$

$$C_{ij} = \int_{\Omega^e} \left\{ \left[\alpha A_x \frac{\partial N_i}{\partial x} + \alpha A_y \frac{\partial N_i}{\partial y} + \left(\frac{\psi}{\Delta t} + \alpha B \right) N_i \right] \times \left[-\alpha \frac{\partial N_j}{\partial y} \right] + k \frac{\partial N_i}{\partial y} N_j \right\} d\Omega$$

$$D_{ij} = \int_{\Omega^e} \left[\alpha^2 \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + N_i N_j \right] d\Omega \quad (15d)$$

$$E_{ij} = \int_{\Omega^e} \alpha^2 \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial y} d\Omega \quad (15e)$$

$$G_{ij} = \int_{\Omega^e} \left[\alpha^2 \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + N_i N_j \right] d\Omega \quad (15f)$$

$$H_{ij} = \int_{\Omega^e} \left\{ \left[\alpha A_x \frac{\partial N_i}{\partial x} + \alpha A_y \frac{\partial N_i}{\partial y} + \left(\frac{\psi}{\Delta t} + \alpha B \right) N_i \right] \times \left[(1-\alpha) A_x \frac{\partial N_j}{\partial x} + (1-\alpha) A_y \frac{\partial N_j}{\partial y} + \left(-\frac{\psi}{\Delta t} + (1-\alpha) B \right) N_j \right] \right\} d\Omega \quad (15g)$$

$$I_{ij} = \int_{\Omega^e} \left[\alpha A_x \frac{\partial N_i}{\partial x} + \alpha A_y \frac{\partial N_i}{\partial y} + \left(\frac{\psi}{\Delta t} + \alpha B \right) N_i \right] \times \left[- (1-\alpha) \frac{\partial N_j}{\partial x} \right] d\Omega \quad (15h)$$

$$J_{ij} = \int_{\Omega^e} \left[\alpha A_x \frac{\partial N_i}{\partial x} + \alpha A_y \frac{\partial N_i}{\partial y} + \left(\frac{\psi}{\Delta t} + \alpha B \right) N_i \right] \times \left[- (1-\alpha) \frac{\partial N_j}{\partial y} \right] d\Omega \quad (15i)$$

$$M_{ij} = \int_{\Omega^e} \left[-\alpha \frac{\partial N_i}{\partial x} \right] \times \left[(1-\alpha) A_x \frac{\partial N_j}{\partial x} + (1-\alpha) \times A_y \frac{\partial N_j}{\partial y} + \left(-\frac{\psi}{\Delta t} + (1-\alpha) B \right) N_j \right] d\Omega \quad (15j)$$

$$K_{ij} = \int_{\Omega^e} (1-\alpha) \alpha \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} d\Omega \quad (15k)$$

$$L_{ij} = \int_{\Omega^e} (1-\alpha) \alpha \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial y} d\Omega \quad (15l)$$

$$P_{ij} = \int_{\Omega^e} \left[-\alpha \frac{\partial N_i}{\partial y} \right] \times \left[(1-\alpha) A_x \frac{\partial N_j}{\partial x} + (1-\alpha) \times A_y \frac{\partial N_j}{\partial y} + \left(-\frac{\psi}{\Delta t} + (1-\alpha) B \right) N_j \right] d\Omega \quad (15m)$$

$$S_{ij} = \int_{\Omega^e} \alpha (1-\alpha) \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial x} d\Omega \quad (15n)$$

$$O_{ij} = \int_{\Omega^e} (1-\alpha) \alpha \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} d\Omega \quad (15o)$$

$$F_{1i} = \int_{\Omega^e} (-f) \times \left[\alpha A_x \frac{\partial N_i}{\partial x} + \alpha A_y \frac{\partial N_i}{\partial y} + \left(\frac{\psi}{\Delta t} + \alpha B \right) N_i \right] d\Omega \quad (15p)$$

$$F_{2i} = \int_{\Omega^e} (-f) \times \left[-\alpha A_x \frac{\partial N_i}{\partial x} \right] d\Omega \quad (15q)$$

$$F_{3i} = \int_{\Omega^e} (-f) \times \left[-\alpha A_y \frac{\partial N_i}{\partial y} \right] d\Omega \quad (15r)$$

with $i, j = 1, \dots, Nnodes$.

5. NUMERICAL APPLICATIONS

All the coefficients involved in the matricial systems (6) and (14), respectively from the GFEM and the LSFEM are functions of the space coordinates x and y . In this paper the Gaussian Quadrature ruler (Reddy, 1993) will be used to calculate the integrals found in the matricial system, for that it is necessary to rewrite the integrals in

terms of a reference coordinates ξ and η ($-1 \leq \xi, \eta \leq 1$), as described in Romão (2004). The interpolations functions in reference coordinates for quadrangular and triangular elements can be found in Dhatt and Touzot (1984).

Right after, two numerical applications will be presented. The first one is about a problem totally diffuse and the second one is about a diffusive-convective problem with convective dominant terms, with the objective of analyzing the performance of the Galerkin and the Least Square Methods for the mesh previously determined.

Regarding the boundary conditions, the first application presents a case with three boundary conditions of the prescribed function type and one of the prescribed flux ones (isolation) (Figure 1), while the second application of this chapter presents a case with two boundary conditions of the prescribed function type and two of the prescribed flux ones (isolation).

5.1- Pure Transient Diffusion

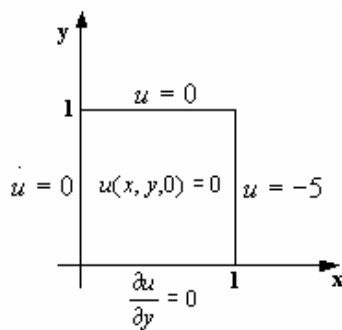


Figure 1. Cartesian Geometry with boundary conditions of the first and second type.

Governing Equation:

$$\frac{\partial u(x, y, t)}{\partial t} = \frac{\partial^2 u(x, y, t)}{\partial x^2} + \frac{\partial^2 u(x, y, t)}{\partial y^2}$$

In this situation, the numerical solution of a completely diffusive case in transient regime is presented for the instant $t = 1$. The mesh used in this application is uniform and contains 625 quadrilaterals of 8 nodes, all the elements being of size $h = 0,04$. For the time discretization the steps $\Delta t = 10^{-2}$ and $\alpha = 0,5$ were used. The Figures show the results for the first case analyzed. The values computed are the functions and their first derivatives.

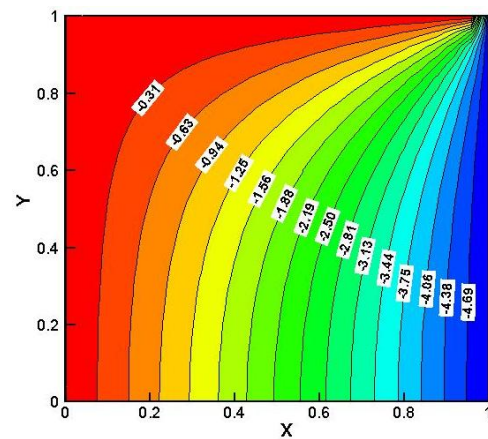


Figure 2. Numerical solutions of $u(x,y,t)$ by GFEM.

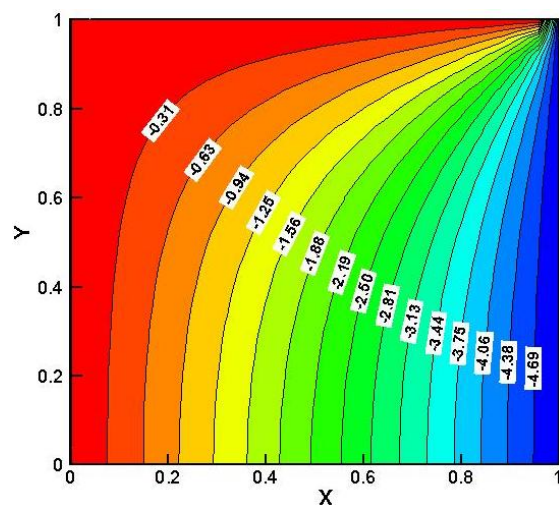


Figure 3. Numerical solutions of $u(x,y,t)$ by LSFEM.

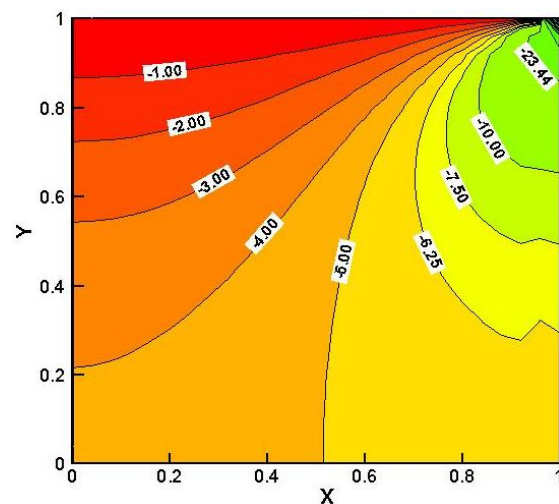


Figure 4. Numerical solutions of $\frac{\partial u}{\partial x}$ by GFEM.

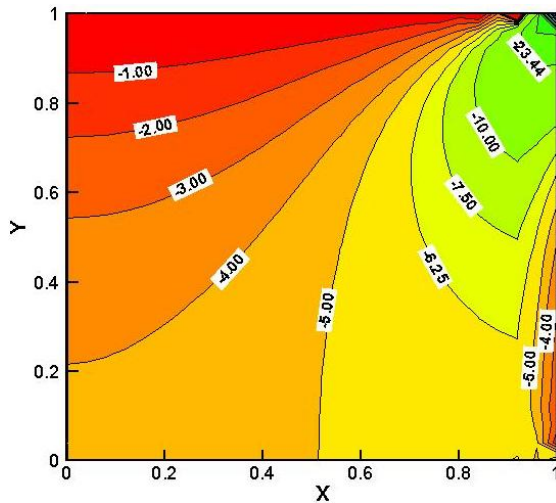


Figure 5. Numerical solutions of $\frac{\partial u}{\partial x}$ by LSFEM.

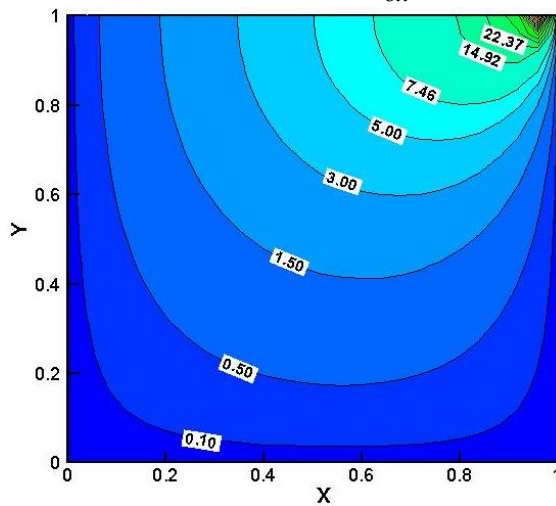


Figure 6. Numerical solutions of the $\frac{\partial u}{\partial y}$ by GFEM.

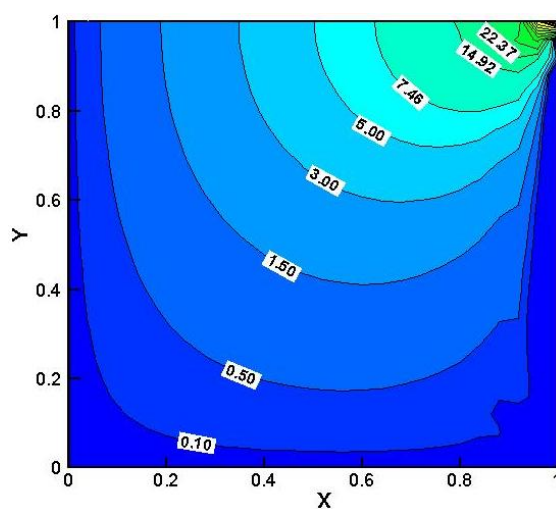


Figure 7. Numerical solutions of the $\frac{\partial u}{\partial y}$ by LSFEM.

Analyzing Figures 2 – 7, we can notice that the GFEM presented good results in all the solutions what did not happen with the LSFEM which presented some numerical oscillations in the solution of $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ in the proximities of the $x = 1$ side.

As expected the GFEM presented good results for a purely diffusive problem. The oscillations of the LSFEM may due to the gross mesh.

The initial condition is the prescribed function type according to Fig. 8.

5.2 Transient Convection-Diffusion

Governing Equation:

$$\frac{\partial u}{\partial t} + 10 \frac{\partial u}{\partial x} + 10 \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

with $u = u(x,y,t)$.

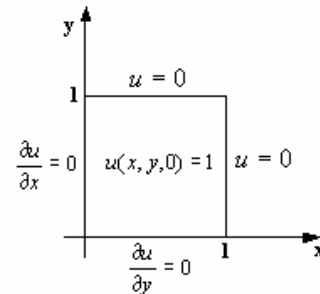


Figure 8. Cartesian Geometry with boundary conditions of the first and second type.

The mesh used in this application contains 1250 triangular elements of six nodes, and all the elements are of identical size $h = 0,04\sqrt{2}$. Figures 9 – 14 present numerical solutions for the instant $t = 0,5$. In the time discretization, $\Delta t = 10^{-2}$ and $\alpha = 0,5$ was adopted.

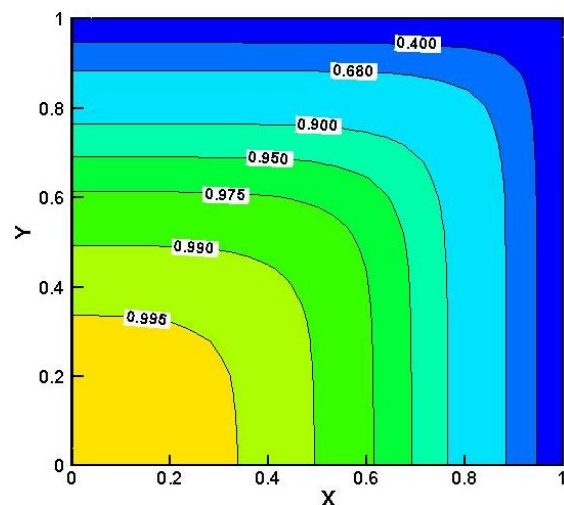


Figure 9. Numerical solution of $u(x,y,t)$ by GFEM.

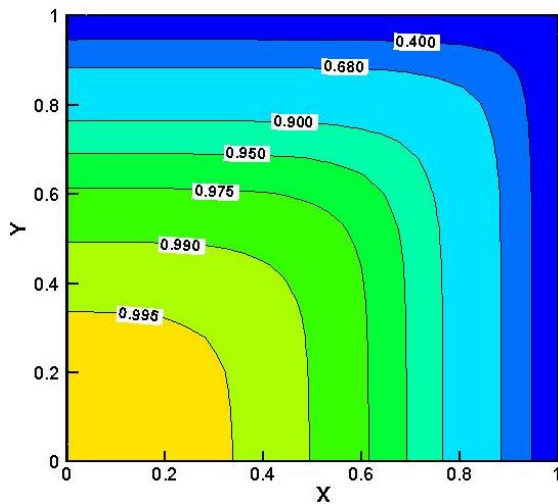


Figure 10. Numerical solution of $u(x,y,t)$ by LSFEM.

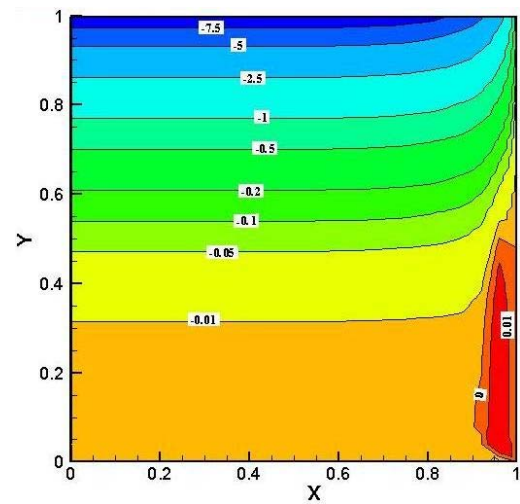


Figure 13. Numerical solutions of $\frac{\partial u}{\partial y}$ by GFEM.

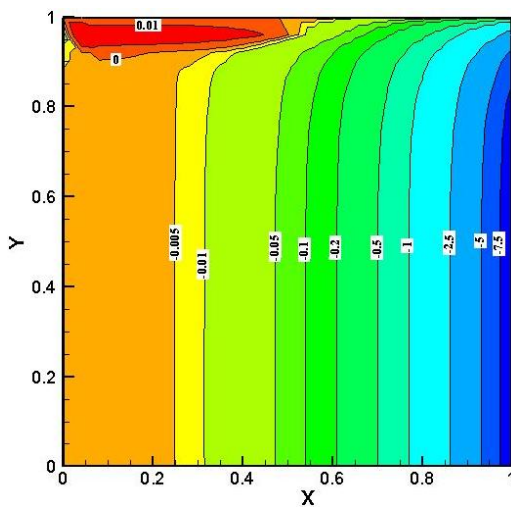


Figure 11. Numerical solutions of $\frac{\partial u}{\partial x}$ by GFEM.

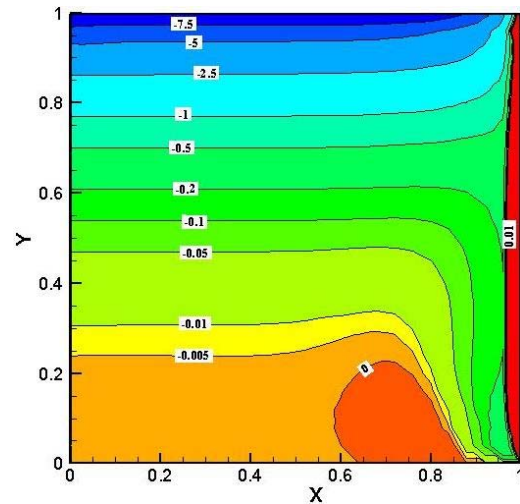


Figure 14. Numerical solutions of $\frac{\partial u}{\partial y}$ by LSFEM.

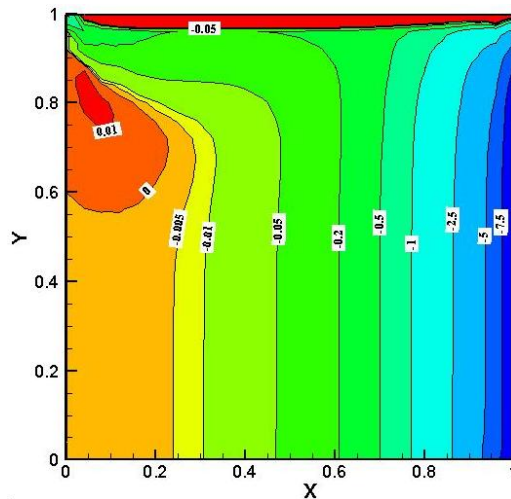


Figure 12. Numerical solutions of $\frac{\partial u}{\partial x}$ by LSFEM.

The conclusion from analyzing the Figures from 9 to 14 is that the GFEM and the LSFEM presented good results in all solutions, but for more convective problems the GFEM may present solutions with oscillations. Some difference appears in the derivative calculation. For the LSFEM the derivative are calculated directly as part of the solution, while for the GFEM the calculation of the derivatives require a post-processing.

6. CONCLUSIONS

The LSFEM always results in symmetrical and positive defined algebraic system, which is interesting from the view point of solution methods and revealed to be a good tool in the solution of transient convective-diffusive problems. When the LSFEM was applied to the dominant convective problem proposed, it presented satisfactory results, decreasing the oscillations in the areas next to $x = 1$ e

$y = 1$ while in the GFEM the oscillations were more expressive.

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