

LAMINAR FORCED CONVECTION IN HORIZONTAL CHANNEL WITH HEAT GENERATION PLATES COOLED BY WATER

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ABSTRACT

This paper describes the study of the behavior of two-dimensional and parallel horizontal flat plates with uniform internal heat generation subjected to laminar forced convection of water. The governing equations are solved numerically using the finite volume technique with Power-Law interpolating scheme and the SIMPLE algorithm. After the simulations the temperature and velocity fields were obtained for various plates spacing and fluid inlet velocities, as well as the Nusselt number values.

Keywords: forced convection, numerical methods

NOMENCLATURE

D	diameter, m
H	distance between plates, m
h	heat transfer coefficient, $W/(m^2 \cdot K)$
k	thermal conductivity, $W/(m \cdot K)$
L	length of the channel, m
N	dimensionless pressure head
Nu	Nusselt number
P	dimensionless pressure
p	pressure, N/m^2
Pr	Prandtl number
q'''	internal heat generation of the plates, W/m^3
R	conductivity ratio
Re	Reynolds number
T	Temperature, K
t	plate thickness, m
U, V	dimensionless velocities
u, v	velocities, m/s
X, Y	dimensionless distances
x	horizontal distance, m
y	vertical distance, m

Greek symbols

α	thermal diffusivity, m^2/s
θ	dimensionless temperature
μ	absolute viscosity, $Pa \cdot s$
ν	fluid kinematic viscosity, m^2/s

ρ	density, kg/m^3
ϕ	variable in Eq. (12)

Subscripts

b	bulk
f	fluid
h	hydraulic
max	maximum
o	inlet
s	surface
w	wall

Superscripts

k	iteration
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INTRODUCTION

The technological development in recent years has achieved large advances in the area of electronic equipment. Therewith the study of heat removal generated by electronic equipments becomes extremely important in its design and operation. This importance is due to the fact that the highest temperature of the package must not exceed an upper limit.

There are several ways to extract heat generated from electronic devices (Incropera (1988) and Çengel

(2007) which includes the natural and forced convection, using mainly air as coolant. The use of water in forced convection is suitable for electronic components with high heat fluxes. Bejan (1996) pursued to optimize the heat exchange in channels arranged side by side in a predetermined space. Leung et al. (2000) build up the numerical results of steady-state forced convection of laminar flow of air in horizontal printed circuit boards (PCB). They employed the configuration in which the upper plate was insulated (no heat flux) while the bottom one was taken to simulate the PCB. Silva and Ganzarolli (2002) determined the optimal spacing of plates with uniform internal generation of heat, cooled by air in laminar flow. They looked at various distances between the plates while maintaining the total pressure difference fixed in order to check which spacing is the maximum heat removal of the arrangement without exceeding the maximum temperature limit. Bhowmik et al. (2005) experimentally investigated convective heat transfer in a vertical rectangular channel cooled by water, covering the three convection modes (laminar, forced, natural and mixed convection) from a linear array of flush-mounted heat sources. It was obtained correlations for relations using Nusselt number, Reynolds number and Grashof number. The work of Hamouche and Bessaih (2009) numerically investigated the two-dimensional laminar mixed convection to air in a horizontal channel with two identical protruding heat sources using the finite volume method and SIMPLER algorithm. Boutina and Bessaih (2011) studied the laminar mixed convection air cooling of two identical heat sources mounted in an inclined channel, obtaining correlations of Nusselt numbers of two components. The influence of Reynolds number and inclination angle on heat transfer was analysed, as well as the influence of the dimensions of the heat sources and the distance between them.

This work has a main goal to obtain the Nusselt numbers of an arrangement of horizontal parallel plates, with constant heat generation, for a range of values of distances between boards, inlet velocities and conductivity of the plates. The fluid is water and the flow regime is laminar. The governing equations were approximated numerically using the technique of finite volume (Patankar, 1980).

PROBLEM FORMULATION

The geometry analyzed in this work is shown in Fig. 1. The dotted region corresponds to the domain used at work. The gray part represents the plate area. The thermophysical properties of the fluid are assumed to be constant. The governing equations in a dimensionless form for steady-state laminar, incompressible and two-dimensional fluid flow are:

Mass Conservation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

Momentum Conservation

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re_L} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \tag{2}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re_L} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) \tag{3}$$

Energy Conservation

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re_L Pr} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \tag{4}$$

The energy equation for the solid region corresponds to:

$$R \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + 1 = 0 \tag{5}$$

And the dimensionless parameters in the above equations are defined by:

$$\begin{aligned} X &= \frac{x}{L}; & Y &= \frac{y}{L}; & Re_L &= \frac{u_o L}{\nu}; \\ U &= \frac{u}{u_o}; & V &= \frac{v}{u_o}; & P &= \frac{p}{\rho u_o^2}; \\ Pr &= \frac{\nu}{\alpha}; & R &= \frac{k_w}{k_f}; & \theta &= \frac{(T - T_o) k_f}{q'' L^2}; \end{aligned} \tag{6}$$

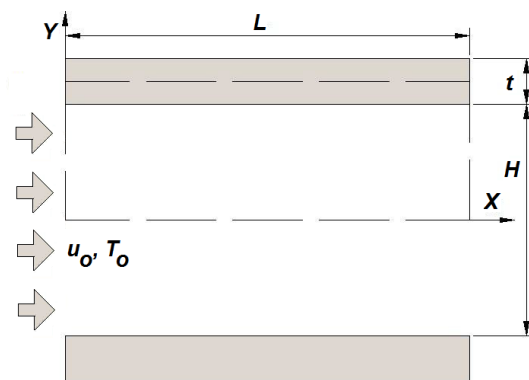


Figure 1. Geometry and Computational Domain.

The boundary conditions are:

$$\left. \begin{aligned} X=0 \\ 0 \leq Y < \frac{H}{2L} \end{aligned} \right\} U = 1; \quad V = \theta = 0$$

$$\left. \begin{array}{l} X=0 \\ \frac{H}{2L} \leq Y < \frac{H+t}{2L} \end{array} \right\} U = V = \theta = 0$$

$$\left. \begin{array}{l} X=1 \\ 0 \leq Y < \frac{H}{2L} \end{array} \right\} \frac{\partial U}{\partial X} = \frac{\partial V}{\partial X} = \frac{\partial \theta}{\partial X} = 0$$

$$\left. \begin{array}{l} X=1 \\ \frac{H}{2L} \leq Y < \frac{H+t}{2L} \end{array} \right\} U = V = \frac{\partial \theta}{\partial X} = 0$$

$$\left. \begin{array}{l} 0 \leq X < 1 \\ Y=0 \end{array} \right\} \frac{\partial U}{\partial Y} = \frac{\partial \theta}{\partial Y} = V = 0$$

$$\left. \begin{array}{l} 0 \leq X < 1 \\ Y = \frac{H+t}{2L} \end{array} \right\} U = V = \frac{\partial \theta}{\partial Y} = 0 \quad (7)$$

The local Nusselt number is defined as:

$$Nu_x = \frac{h \cdot D_h}{k_f} = \frac{q'' \cdot t \cdot D_h}{k_f} \quad (8)$$

Where D_h is hydraulic diameter (equal to $2H$ for infinite parallel plates (Bejan (1995))) and the subscript s denotes the surface of the plate, and b the bulk temperature of the fluid, which can be expressed as follows:

$$T_b = \frac{\int uTdy}{\int udy} \quad (9)$$

Finally, the dimensionless form of the Nusselt number:

$$Nu_X = \frac{2 \cdot (t/L) \cdot (H/L)}{(\theta_{X,s} - \theta_b)} \quad (10)$$

The average Nusselt number is calculated as follows:

$$\overline{Nu}_X = \frac{\int Nu_X dX}{\int dX} \quad (11)$$

NUMERICAL PROCEDURE

The governing equations are solved using the finite volume method developed by Patankar (1980), with staggered grid for velocity field while the scalar quantities were stored in the center of these volumes. The convective terms of momentum and energy are approximated by Power-Law scheme. The SIMPLE algorithm is applied to solve the coupling of pressure-

velocity. The Tri-Diagonal Matrix Algorithm (TDMA) method is employed to figure out the discretized set of equations and the programming language of the code is FORTRAN. The adopted convergence criterion is:

$$\left| \frac{\phi^{k+1} - \phi^k}{\phi^k} \right| \leq 10^{-6} \quad (12)$$

Where ϕ performs U, V, θ and the maximum residual in the continuity equation and k the current iteration.

Grid Independence Study

Two uniform grids were analyzed in this work: 54×402 ($Y \times X$) and 106×802 ($Y \times X$). The problem choosed was to obtain the maximum temperature of the channel and the pressure at the centerline of the channel ($Y = 0, X = 1$) for $H/L = 0.25, Re_L = 400, Pr = 7$ and $R = 10$. The results are summarized at Table 1, where it is clear that the results for 54×402 points provides good results compared with the 106×802 with less computational effort.

Table 1. Results for the two grid points evaluated.

Grid	θ_{max}	P	Time (h)
54 x 402	1.30E-3	0.722	1 hour
106 x 802	1.29E-3	0.722	10 hours

The mesh 54×402 has 21708 control volumes, 400 in the horizontal direction and 52 the vertical one. The length of each control volume, both in X and Y direction, is 0.0025. This is the value used for obtaining the others H/L values in the present work.

Code Validation

The numerical code was compared with the numerical results of Silva and Ganzarolli (2002), which obtained maximum dimensionless temperature profiles as a function of a dimensionless pressure head N defined as:

$$N = \left(\frac{\Delta p L^2}{Pr \mu \alpha} \right)^{1/4} \quad (13)$$

Results are shown in Fig. 2 for the dimensionless distance between the plates equal to $H/L = 0.05$. The plate thickness adopted was $t/L = 0.006$ and $R = 30$. It was varied the Reynolds number to obtain the different pressure values. There is good agreement between the results of the present work and the ones of Silva and Ganzarolli (2002).

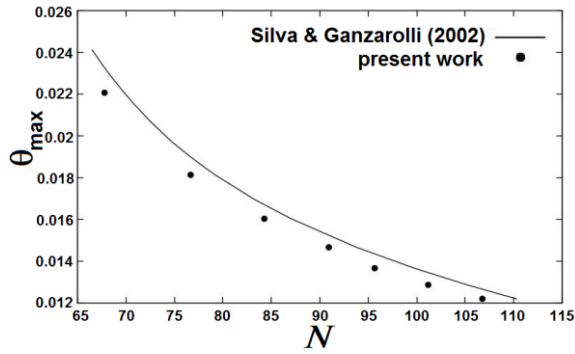


Figure 2. Comparison between the numerical results of maximum temperature in function of N .

RESULTS AND DISCUSSION

A numerical study has been carried out on laminar forced convection over a horizontal channel. Four distances between the boards (H/L) were chosen, namely: 0.05, 0.10, 0.15 and 0.2. The $(H/L)Re_L$ values used were equal to: 100, 200, 300, 400, 500, 600, 700 and 800. Each plate spacing was calculated the Local Nusselt Number. After integrating the Local Nusselt Number, the Average Nusselt Number was attained. The thickness of the plate adopted (t/L) was equal to 0.01. Two values of R were specified: 10 and 20. The Prandtl Number is 7, which corresponds to water.

The conjugate problem (solid/fluid region) is adopted to solve the governing equations, and the temperature profile from energy equation is obtained separately from mass and momentum equations, after the determination of the velocity and pressure fields.

Figure 3 shows the Local Nusselt Number along the plate surface in the horizontal direction (X) for the various H/L distances and $R = 10$. It is clear the effect of the velocity in the convective heat transfer. The increase of the Reynolds Number corresponds to the growing of the Local Nusselt Number. It is the indication of the decrease of the temperature with growing of mass flow inside the channel. This effect is also observed in the Average Nusselt Number (Table 2). Figure 4 presents the Local Nusselt Number for $R = 20$, with the objective to analyze the effect of the rise of the conductivity in heat transfer. The results denote that the augment of k_w leads to less thermal resistance of conduction, which corresponds a lower Nusselt Number. However, to change the value of R from 10 to 20 does not correspond to the values of Nusselt Number decay from half of the value.

Table 2 presents the results of Average Nusselt Number for the several H/L values ($R = 10$) and Tab. 3 exhibits the results for $R = 20$. In both cases the raising of the mass flow it provides the increase of Nusselt Number, and the same behavior is observed when increasing the distance between the plates. As explained, the increases in R factor indicates a diminution in Average Nusselt Number.

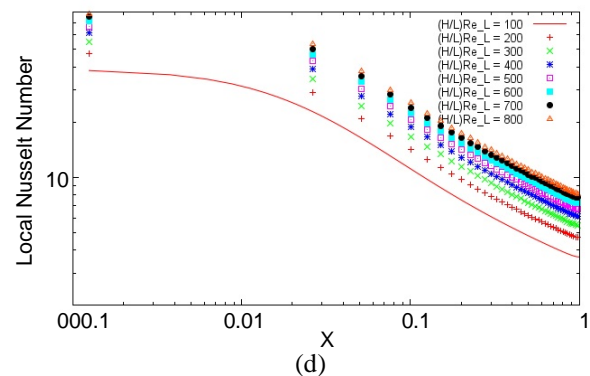
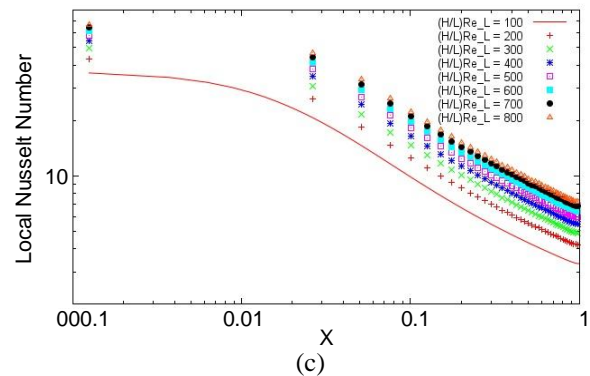
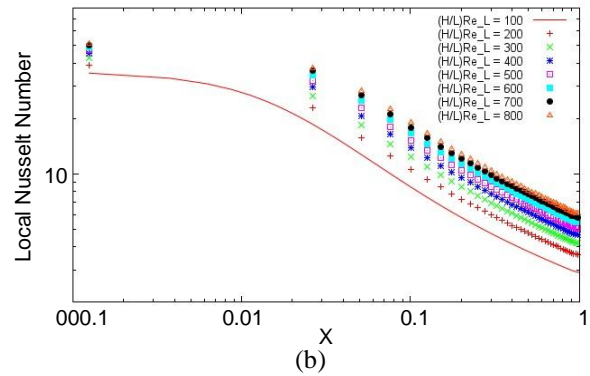
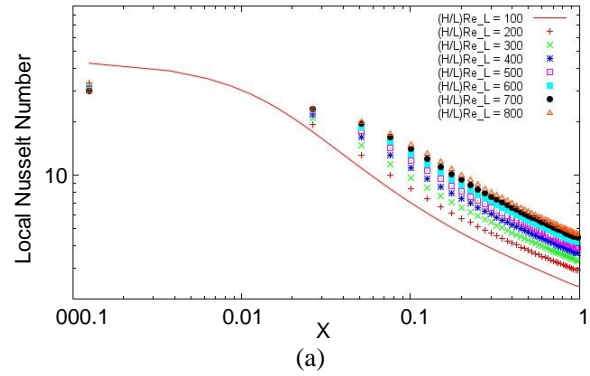


Figure 3. Local Nusselt Number ($R = 10$): (a) $H/L = 0.05$, (b) $H/L = 0.10$, (c) $H/L = 0.15$, (d) $H/L = 0.20$.

As examples of the decay of temperature for the data obtained the value for temperature at $X = 1$ and $Y = 0$ was equal to $1.6 \cdot 10^{-6}$ ($R = 10$, $(H/L)Re_L = 100$ and $H/L = 0.2$). At the same position ($X = 1$ and $Y = 0$) the

value of the temperature was equal to $2.2 \cdot 10^{-19}$ ($R = 10$, $(H/L)Re_L = 800$ and $H/L = 0.2$). The same behavior is observed when increased the value of R .

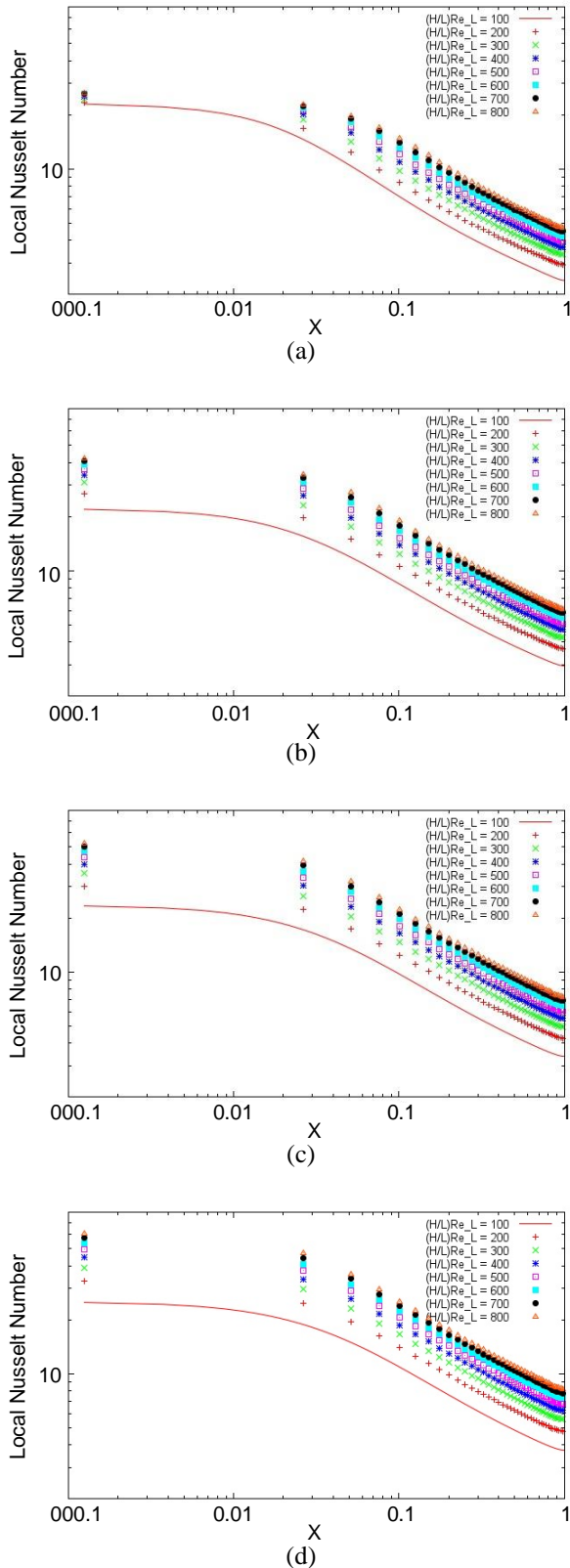


Figure 4. Local Nusselt Number ($R = 20$): (a) $H/L = 0.05$, (b) $H/L = 0.10$, (c) $H/L = 0.15$, (d) $H/L = 0.20$.

Table 2. Average Nusselt Number in function of H/L ($R = 10$).

$(H/L)Re_L$	H/L			
	0.05	0.10	0.15	0.20
100	4.4322	5.1694	5.8856	6.5164
200	5.1685	6.4316	7.4819	8.3784
300	5.8350	7.4508	8.7448	9.8378
400	6.4122	8.3199	9.8180	11.0759
500	6.9236	9.0862	10.7655	12.1692
600	7.3850	9.7764	11.6212	13.1580
700	7.8072	10.4076	12.4061	14.0662
800	8.1979	11.9917	13.1337	14.9101

Table 3. Average Nusselt Number in function of H/L ($R = 20$).

$(H/L)Re_L$	H/L			
	0.05	0.10	0.15	0.20
100	4.1987	4.9747	5.6762	6.3040
200	5.0202	6.2451	7.2612	8.1478
300	5.7157	7.2624	8.5105	9.5902
400	6.3139	8.1316	9.5734	10.8149
500	6.8424	8.9008	10.5139	11.8978
600	7.3182	9.5961	11.3655	12.8789
700	7.7528	10.2342	12.1488	13.7820
800	8.1541	10.8262	12.8773	14.6230

CONCLUSIONS

It was obtained numerically values of the Nusselt Number (local and average) for a laminar forced convection of water in a two-dimensional horizontal channel composed by flat plates with internal heat generation. Results shown that the increase in Reynolds Number, for a fixed plates spacing, denotes a growth of Nusselt Number. The same effect occurs at the rise of plates spacing, for a fixed Reynolds Number. It was noticed that the increment of factor R corresponds to a reduction of the Nusselt Number.

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