GEOMETRIC EVALUATION OF T AND H-SHAPED CAVITIES INSERTED IN A SOLID WITH HEAT GENERATION APPLYING CONSTRUCTAL DESIGN

F. B. Teixeira[a], M. S. Pereira[a], B. C. Feijó[a], L. A. O. Rocha[b], L. A. Isoldi[a], and E. D. Santos[a]

[a]Universidade Federal do Rio Grande
Programa de Pós-Graduação em Engenharia Oceânica (PPGEO)
Av. Itália, km 8, Rio Grande, RS, Brasil
fbrancoteixeira@gmail.com
[b]Universidade Federal do Rio Grande do Sul
Departamento de Engenharia Mecânica
Rua Sarmento Leite, 425,CEP 90050-170, Porto Alegre, RS, Brasil

Received: May 25, 2017
Revised: June 19, 2017
Accepted: July 17, 2017

ABSTRACT

In this work, the influence of geometry on the behavior of the temperature field in a square plate with T and H-shaped cavities is studied. The ratio between the cavity area and the plate area will be kept constant and its geometry will be varied in order to find the optimum geometry (the one that results in the temperature field with the lowest maximum temperature). The cavity will occupy 10% of the area of the plate and will be varied from the T-shaped configuration to the H-shaped one. According to the Constructal Design principles, the degrees of freedom of the problem and its restrictions will be defined. The height of the initial T was selected as H_1, where H_1/L_1 is one of the degrees of freedom for the problem. The second degree of freedom is the ratio H_2/L_2, the ratio of height by the width of the first bifurcation, and the other geometric ratio (H_3/L_3) is the ratio of height by the width of the second bifurcation and is a function of H_1. For the simulations, a code based on the Finite Element Method (FEM) was used to solve the energy conservation equation. The results showed that it is possible to minimize the maximum excess temperature by 54.4% when an H-shaped geometry with irregular legs is used compared with the T-shaped cavity. In order to reach the optimum geometry, H_1/L_1 was reduced by 68.37%, and H_2/L_2 was increased in 64.71% when compared to the initially proposed T-shaped cavity.

Keywords: Constructal design, T-shaped cavity, H-shaped cavity, numerical study, FEM

NOMENCLATURE

A_0 plate area, m²
A_1 cavity area, m²
H_0 plate height, m
H_1, H_2, H_3, H_4, H_5, H_6 cavity dimensions on the vertical direction, m
H_1/L_1 aspect ratio of the cavity main section
H_2/L_2, H_3/L_3 aspect ratios of the cavity's bifurcations
i vertical step
k thermal conductivity, W/(m.K)
L_0 plate length, m
L_1, L_2, L_3, L_4, L_5 cavity dimensions on the horizontal direction, m
q''' rate of energy generation per volume unit, W/m³
T temperature, K
x, y cartesian coordinates, m

Greek symbols

φ cavity fraction (A_1/A_0)

Superscripts

f final
j current

Subscripts

m once minimized
max maximum
mm twice minimized
o once optimized
oo twice optimized

INTRODUCTION

With the increased energy requirements of embedded systems and the consequent increase in heat generation, new cooling solutions are needed. Due to this, it is currently attempted to reduce the dimensions of these systems while maintaining the same rate of heat transfer. Problems of diffusion of heat are verified daily, a notorious example is the carcass of the modern cell phones that heats perceptibly with the use of applications and games more and more demanding. In fact, in many cell phones, this process of heat dissipation through the housing is even a design issue to avoid damaging the electronics and the battery. Therefore, the geometries of the components and sinks are of great importance for the proper operation of the equipment, since the shape exerts great importance in the processes of heat transfer. Constructal Design is a geometric evaluation method based on the principle of constraints and objectives and a physical principle of maximizing access to flow. This principle is called the Constructal Law defined by Bejan (2000): "For a finite-size system to persist in time (to survive), it must evolve its form and structure to provide better access to the currents flowing through it." Bejan and Lorente (2008), present several applications of the Constructal Law for geometric evaluation in heat transfer problems, Lorenzini and Rocha (2009) studied the geometric optimization of TY-shaped cavities inscribed in a conductive wall and compared with C-shaped cavities. Rocha et. al. (2010) studied the convection cooling in C-shaped cavities. Xie et al. (2010) applied the Constructal Theory in the evaluation of T-shaped cavities inserted in a trapezoidal solid with external adiabatic walls. Hajmohammadi et al. (2013) reanalyzed various cavity shapes (T, Y, T-Y and H) and confronted the use of multiple, simpler I-shaped cavities (by simplifying the manufacturing process) by keeping the cavity volume constant.

In this work, the geometry of a cavity inserted in a square solid with uniform heat generation will be numerically studied. The outer surfaces of the solid are thermally insulated and the cavity acts as the sink of the internally generated energy in the solid. The geometry of the cavity will be varied between a T-shape (initial shape) to an irregular H-shape, using the Constructal Design method by Bejan et al. (2012) in order to find the design that presents the best performance. According to Biserni et al. (2007), the optimum configuration in H-shaped cavity has better performance than the optimum T-shaped configuration, but this work intends to analyze the performance gain when maintaining the thickness of the legs of the T and H--shapes and only varying the lengths in several pre-established widths. There are 50 vertical steps relative to the length of the legs of T and H-shapes (H_1 and H_3) and 15 horizontal steps relative to the width of the upper stem of the shape T (L_2). For the construction of the geometries, the PDE tool of the MATLAB software by Mathworks (2000) was used. The cavity area is maintained at 10% of the total area of the plate and only its geometry is varied. For the present study, H_1/L_1 and H_2/L_2 were selected as degrees of freedom. The dimension H_3 is plotted against H_1, keeping H_2, L_1, L_2, L_3 and L_4 constant for each horizontal step. With the changed H_2/L_2 ratio (horizontal step), a further 50 vertical steps are performed and so on until the 15 horizontal steps are completed. The work will follow equation methods similar to Biserni et. al. (2007) which also numerically analyzes the H geometry, and will use the same Constructal method applied in Biserni et. al. (2004) and Rocha et al. (2005). The Constructal principle shows that geometry is malleable and is deduced from the principle of global performance maximization also subject to global constraints.

PROBLEM DESCRIPTION

The system under study consists of a two-dimensional, energy diffusion problem in a dimensionless square plate of 1.0×1.0 side (area A_0 = 1.00) in which a T-shaped cavity occupying 10% of (A_1 = 0.10). In this way, the ratio A_1/A_0 (φ = 0.1) will be kept fixed. The construction of the initial form did not follow any specific method, since the intention was to maintain φ = 0.1, but some parameters were imposed as L_1, L_2, L_3 and H_4 (H_1 + H_2), being H_3 placed as a function of H_1 so that the 10% restriction of the plate area was achieved. In this way, the process of modifying the geometry between the two different shapes (T and H) was started. Therefore, for the thickness dimensions between the legs of the H and the lower leg of the T, a ratio of 1:2 was selected. Dimensions H_3, L_2 and L_4 (horizontal step) were fixed, and dimension H_1 (vertical step) was determined using a spreadsheet tool to obtain 10 % of the board area. In this work, as mentioned before, the initial T-shaped geometry will be varied to an H-shaped for each horizontal step. Fig. 1 (a) and (b) exemplify the initial and final variations, respectively, of the vertical step for a given horizontal step. Fig. 1 (c) and (d) show the initial and final variations respectively of the horizontal step for a given vertical step.

Also in Fig. 1, the boundary conditions imposed on the problem can be seen, where it was considered that the external walls (cross-hatching) are thermally isolated, whereas a minimum prescribed temperature is imposed on the cavity surfaces.

For the problem in question, in each horizontal step (H_2/L_2), H_1/L_1 was varied by keeping the area constant by adjusting the H_3/L_3 ratio as a function of H_1. The initial dimensions selected are in Tab. 1 and set so that the area is maintained at 10% of the board area with a minimum precision of 10^{-7}. This value is selected based on the finding of a technical incapacity of the PDETOOL tool. It was noted during the execution of the work that the software presented errors by varying values in the sixth decimal place as a function of the way in which the tool constructs the geometry and imposes the boundary conditions.

Table 1. Initial plate and cavity dimensions.

Variable	Dimension	Variable	Dimension
H_0	1.0000	H_3	0.4350
L_0	1.0000	L_3	0.0300
H_1	0.8738	L_4	0.3800
L_1	0.0600	H_5	0.1000
H_2	0.0262	L_5	0.6000
L_2	0.8200	H_6	0.4650

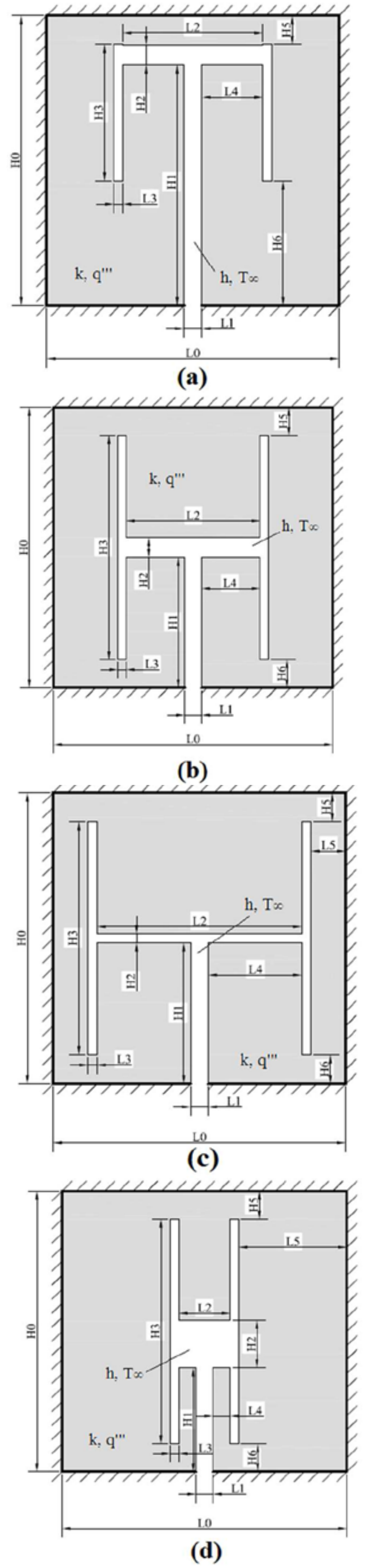


Figure 1. Description of the problem domain with (a) vertical step 1 and horizontal step 7, (b) vertical step 50 and horizontal step 7, (c) vertical step 50 and horizontal step 1, and (d) vertical step 50 and horizontal step 15.

In addition to these initial dimensions, some final dimensions were imposed as constraints. The final value of L^f_4 must be equal to the dimension of L_1, that is, $L_4 = 0.06$ and the final dimension of H_6 must equal the size of H_5, $H^f_6 = 0.1$.

MATHEMATICAL MODELING AND NUMERICAL PROCEDURE

The equations for the cavity area and the relation as a function of H_1 can be seen below.

$$A_1 = H_1L_1 + H_2L_2 + 2H_3L_3 \tag{1}$$

$$H_3^i = H_3^0 + \left|H_1^0 - H_1^{0+i}\right| \tag{2}$$

where *i* represents the vertical step.

Starting from the existing boundary conditions and constraints, the energy equation for the present problem in the steady state and two-dimensional domain is given by:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+q'''=0 \tag{3}$$

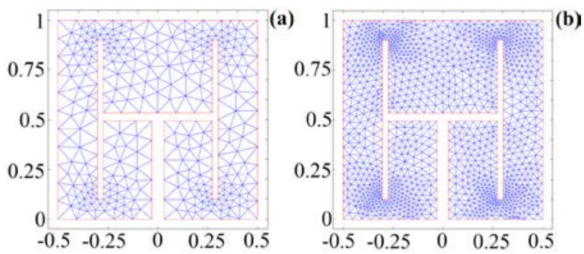
PDETOOL is a tool of commercial software MATLAB and it employs the Finite Element Method (FEM) for mesh creation and problem solving. The mesh generation and refining process is automated by the program, requiring only clicks on the appropriate mesh initialization and refining options. The mesh generated is an unstructured mesh with triangular elements as can be seen in Figs. 2 (a) - (f).

Table 2 shows the mesh independence test for the shape shown in Fig. 2.

Table 2. Analysis of mesh independence.

Elements	T_{max}	$\lvert(T^J_{max} - T^{J+1}_{max})/ T^J_{max}\rvert$
603	0.0345540	2.70×10^{-2}
2,412	0.0354871	3.95×10^{-3}
9,648	0.0356274	1.51×10^{-3}
38,592	0.0356811	5.81×10^{-4}
154,368	0.0357018	2.26×10^{-4}
617,472	0.0357099	-

For the work in question, the mesh will be considered independent when $|(T^J_{max} - T^{J+1}_{max})/ T^J_{max}| < 5.0 \times 10^{-4}$, so with 4 refinements the most critical shape (H) presented convergence in the required precision.



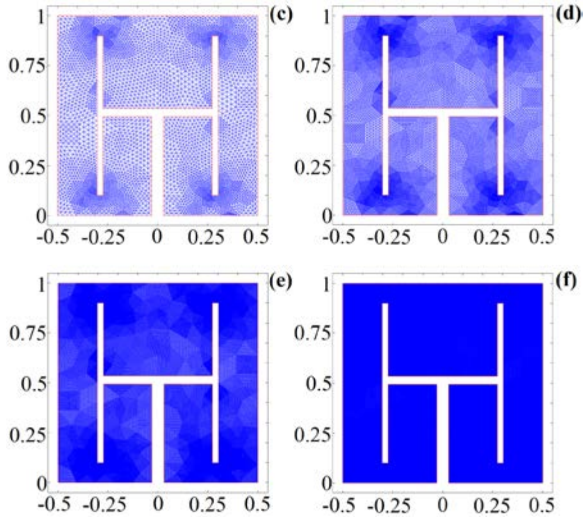


Figure 2. Refining process of the mesh: (a) mesh generated; (b) first refinement; (c) second refinement; (d) third refinement; (e) fourth refinement; (f) fifth refinement.

The optimization process is divided into two steps, as shown in Fig. 3. In the first step, the geometry is optimized through an exhaustive search process by the variation of degree of freedom H_1/L_1. The lowest magnitude for the maximum temperature obtained in the domain will be the maximum temperature once minimized $(T_{max})_m$, while the corresponding optimum geometry will be the once optimized H_1/L_1 ratio, $(H_1/L_1)_o$. In a later step, H_1/L_1 is varied for different H_2/L_2 values. The lowest value for the maximum temperature obtained will be minimized twice $(T_{max})_{mm}$ and the corresponding optimal geometry will be $(H_1/L_1)_{oo}$ (twice optimized) and $(H_2/L_2)_o$ (once optimized).

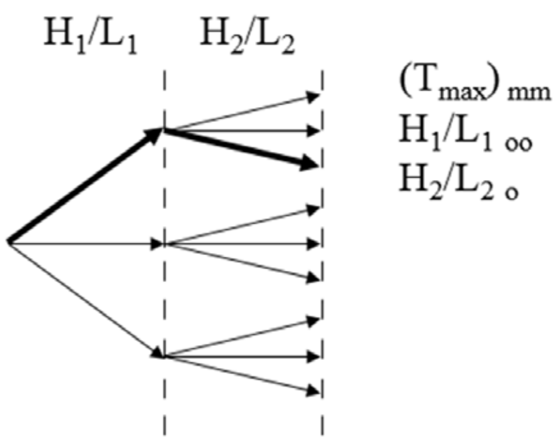


Figure 3. Diagram illustrating the process of evaluation of the cavity geometry.

RESULTS AND DISCUSSION

In the simulations, the values of the coefficient of thermal conductivity "k" and the rate of energy generation per volume unit q''' were kept constant in order to analyze the effects of the geometry variation exclusively. The values will be treated in a dimensionless way since the interest of the study is to analyze the variation presented between the geometries. In this way, the temperature field was determined by the solution of the problem, and the initial value of H_1 was then reduced in steps of 0.0073 (vertical step) for each horizontal step whose determined value was 0.02286. The values for the vertical and horizontal steps were determined in a way that allowed a fixed value of steps (50 vertical and 15 horizontal) respecting the restrictions imposed for the dimensions of L_4, H_5, L_5 and H_6. For each step, a new ratio of H_1/L_1 and a field of temperatures were obtained. Figure 4 shows the behavior of the maximum temperature taking into account this relation. The values to the right represent the initial form T and the reduction of this relation indicates the transformation to the form H. The final values to the left for each step, indicates the fully developed form respecting the restriction of H_6. In general, it can be seen that the tendency of T_{max} to be minimized with the reduction of the H_1/L_1 ratio, that is, with the formation of an H, which confirms the previous studies regarding the best performance of H-shaped geometries in relation to the T-shaped seen in Biserni et. al. (2007). Furthermore, since the H_3/L_3 ratio is a function of H_1/L_1, since the loss of the central branch area of the cavity due to the decrease in H_1 is replaced by the increase of the area of the two branches bifurcated for H_3. It can be observed that in the best cases, the best geometry is the one that has intermediate values of H_1/L_1, resulting in no gains for the minimization of T_{max} by extending the legs of the H until the end of the domain.

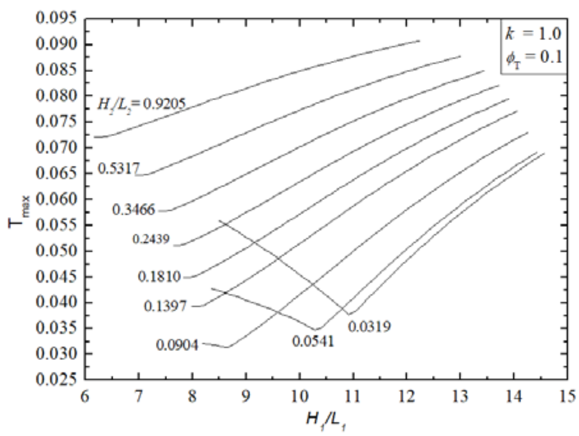


Figure 4. Effect of the H_1/L_1 ratio on T_{max} for different H_2/L_2 ratios.

Analyzing the behavior of T_{max} once minimized, $(T_{max})_m$, as a function of the relation H_2/L_2 (Fig. 5a), it is possible to find the $(T_{max})_{mm}$ (twice minimized) where it was observed a reduction of 54.43% for the value of the maximum temperature in the plate compared to the form T initially proposed, and a reduction of 56.93% when compared to the worst case of the same horizontal step. It is also observed that H_2/L_2 ratios higher than 0.1367 present a much lower performance in the minimization of T_{max} and are even worse than the initial T form. Finally, we observed the behavior of $(H_1/L_1)_o$ as a function of H_2/L_2 (Fig. 5b).

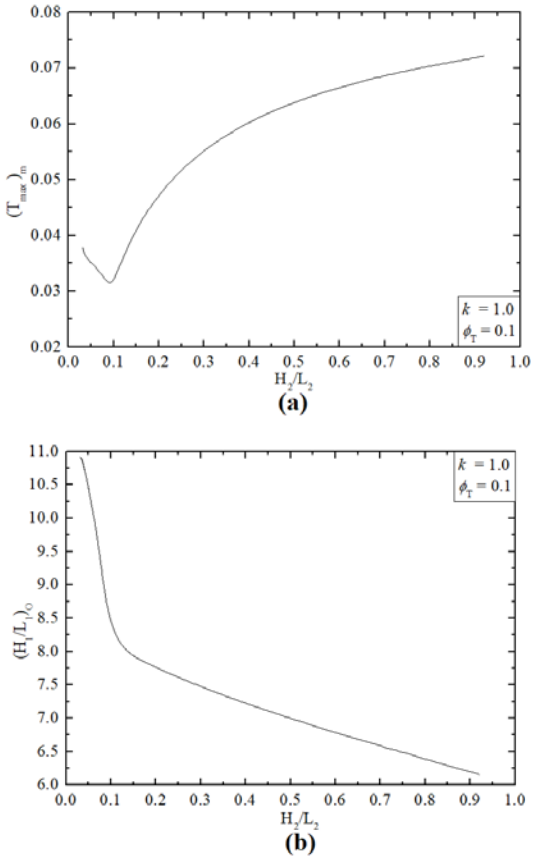


Figure 5. Effect of H_2/L_2 on: (a) $(T_{max})_m$ (b) $(H_1/L_1)_o$.

Figures 6 - 8 illustrate the temperature fields for the cases with initial T-shaped configurations (first vertical step) where the ratio H_2/L_2 is varied in the following horizontal steps: horizontal step 1, horizontal step 8 (optimal shape for this configuration), horizontal step 50, respectively. Figures 9 – 11 depict the optimal shapes for the ratios H_2/L_2 presented in Figs. 6 to 8, respectively. More precisely, these temperature fields represent the ones obtained with the following parameters: horizontal step 1 and vertical step 30 (Fig. 9), the twice optimized shape with horizontal step 8 and vertical step 46 (Fig. 10) and horizontal step 15 and vertical step 50 (Fig. 11). Table 3 shows the ratios H_1/L_1, H_2/L_2 and the maximum temperature reached in the domain for the cases presented in Figs. 6 - 11.

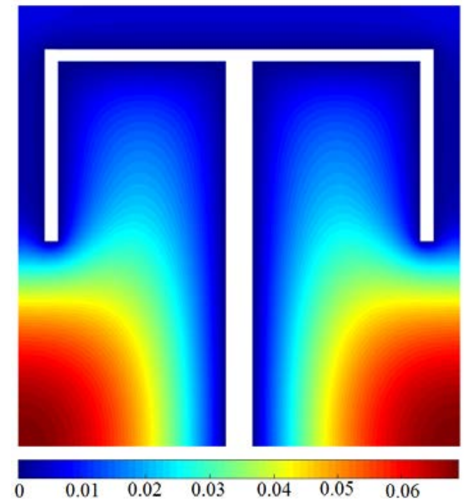


Figure 6. Temperature field obtained for the T-shaped cavity with H_1/L_1 = 14.5636 and H_2/L_2 = 0.0319.

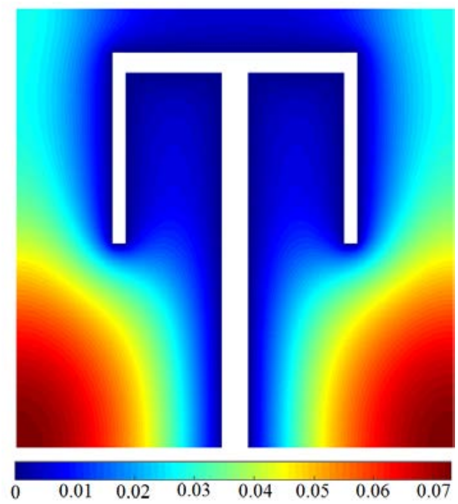


Figure 7. Temperature field obtained for the T-shaped cavity with H_1/L_1 = 14.2463 and H_2/L_2 = 0.0904.

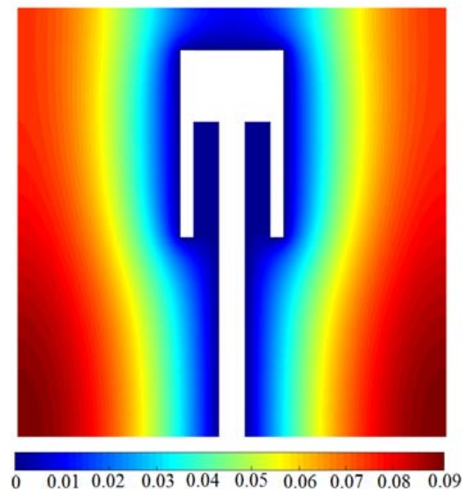


Figure 8. Temperature field obtained for the T-

shaped cavity with H_1/L_1 = 12.2375 and H_2/L_2 = 0.9205.

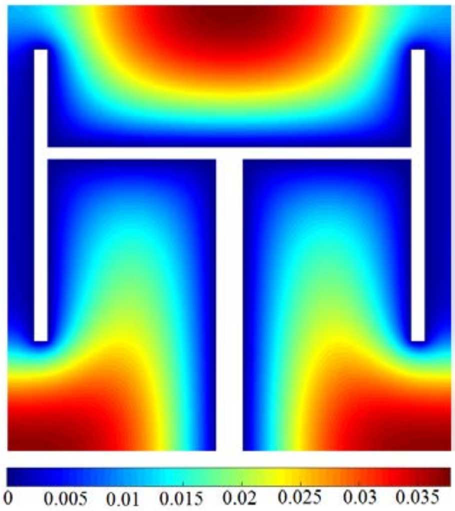


Figure 9. Temperature field obtained for the H-shaped cavity with $(H_1/L_1)_o$ = 10.9136 and H_2/L_2 = 0.0319.

A decrease in T_{max} is observed as H_1/L_1 reduces for all cases and the T-shape becomes H-shape. Analyzing Figs. 6 and 9 it is notable the changing of the distribution with two points of maximum for the T-shape and three points of maximum for H-shape. Similarly, comparing Figs. 7 and 10, the initial T-shape also presents two points of maximum, whereas the H-shape shows five maximum points. Therefore, it is noted that the T_{max} tends to fall with the increase of the maximum points. This can be explained by the principle of optimal distribution of imperfections. The cases shown in Figs. 8 and 11, although they also have a varied form between a T and H, due to the high ratio of H_2/L_2 have somehow only two large zones of maximum temperature.

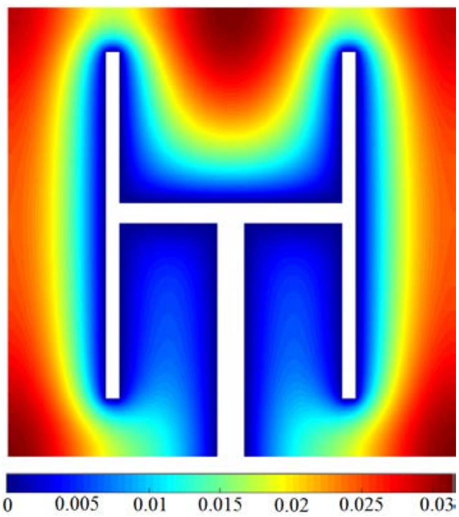


Figure 10. Temperature field obtained for the H-shaped cavity with $(H_1/L_1)_{oo}$ = 8.6496 and $(H_2/L_2)_o$ = 0.0904.

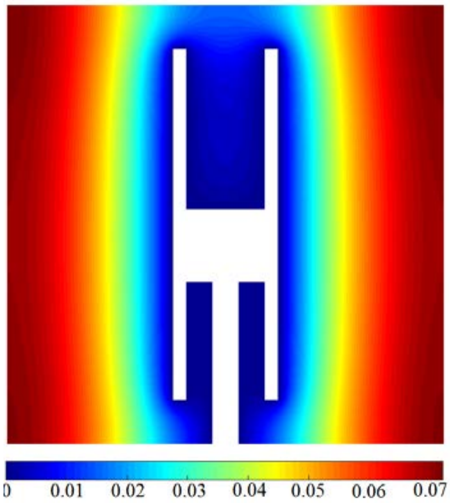


Figure 11. Temperature field obtained for the H-shaped cavity with $(H_1/L_1)_o$ = 6.1542 and H_2/L_2 = 0.9205.

Table 3 compiles the results obtained for T_{max} shown in Figs. 6 – 11.

Table 3. Maximum temperatures obtained for the configurations presented in Figs. 6 – 11.

H_1/L_1	H_2/L_2	Shape	Figure	T_{max}
14.5636	0.0319	T	Fig. 6	0.068923
14.2463	0.0904	T	Fig. 7	0.07299
12.2375	0.9205	T	Fig. 8	0.090714
10.9136	0.0319	H	Fig. 9	0.037755
8.6496	**0.0904**	**H**	**Fig. 10**	**0.03141**
6.1542	0.9205	H	Fig. 11	0.07212

CONCLUSIONS

A numerical study was carried out to evaluate the geometry of a cavity inserted in a solid with internal energy generation using the Constructal Design method. Geometric optimization is performed using the exhaustive search method. More precisely, an H-shaped cavity has been evaluated which can be simplified to a T-shaped geometry, thus enabling a comparison between the cavities and showing how the geometry can evolve in a flow system. The analyzed system has two restrictions (areas of the solid and cavity) and two degrees of freedom were studied (H_1/L_1 and H_2/L_2). For each new geometry the finite element method was used to solve the energy conservation equation for the heat conduction problem.

For this study, the analysis of only one degree of freedom (H_1/L_1) reiterates the best performance of H-shaped cavities versus the T-shaped for the minimization of the maximum plate temperature for the same φ, which is according to Rocha et al. (2005) and Biserni et al. (2007). This occurs even when the H-shape is vertically asymmetrical, that is, it presents the upper legs with a size different from the lower ones. The analysis of the second degree of freedom (H_2/L_2) leads us to conclude that the more symmetrical and centralized H-shape, respecting the previously imposed restrictions, presented the best performance. Using the proposed relationships between the geometric areas and the leg thicknesses of the T and H shapes, a minimum T_{max} of 54.43% was found to be lower than the initially established shape, with a reduction of 68.37% on H_1/L_1 and an increase of 64.71% on H_2/L_2. In later works it would be pertinent to study a third and fourth degree of freedom, referring to the thickness of the legs of the T and H (L_1 and L_3), arriving at new relations of H_1/L_1 and H_3/L_3, in order to study more emphatically the questions of vertical and horizontal symmetry of the geometries.

REFERENCES

Bejan, A., 2000, *Shape and Structure, from Engineering to Nature*, Cambridge University, Cambridge, UK.

Bejan, A., Lorente, S., and Lee, J., 2008, Unifying Construtal Theory of Tree Roots, Canopies and Forests, Journal of Theorical Biology, Vol. 254, pp. 529-40.

Bejan, A., and Zane, J. P., 2012, *Design in Nature: How the Constructal Law Governs Evolution in Biology, Physics, Technology, and Social Organizations*, 1st Edition, Anchor Books.

Biserni, C., Rocha, L. A. O., and Bejan, A., 2004, Inverted Fins: Geometric Optimization of the Intrusion into a Conducting Wall, International Journal of Heat and Mass Transfer, Vol. 47, pp. 2577-2586.

Biserni, C., Rocha, L. A. O., Stanescu, G., and Lorenzini, E., 2007, Constructal H-Shaped Cavities According to Bejan’s Theory, International Journal of Heat and Mass Transfer, Vol. 50, pp. 2132-2138.

Hajmohammadi, M. R., Poozesh, S., Campo, A., and Nourazar, S. S., 2013, Valuable Reconsideration in the Constructal Design of Cavities, Energy Conversion and Management, Vol. 66, pp. 33-40.

Lorenzini, G., and Rocha, L. A. O., 2009, Geometric Optimization of T-Y-Shaped Cavity According to Constructal Design, International Journal of Heat and Mass Transfer, Vol. 55, pp. 4683-4688.

Mathworks, 2000, MATLAB *User’s Guide*, Version 6.0.088, Release 12, The Mathworks Inc.

Rocha, L. A. O., Lorenzini, E., and Biserni, C., 2005, Geometric Optimization of Shapes on the Basis of Bejan’s Constructal Theory, International Communications in Heat and Mass Transfer, Vol. 32, pp. 1281-1288.

Rocha, L. A. O., Lorenzini, E., Biserni, C., and Cho, Y., 2010, Constructal Design of a Cavity Cooled by Convection, International Journal of Design & Nature and Ecodynamics, Vol. 5, No. 3, pp. 212-220.

Xie, Z., Chen, L., and Sun, F., 2010, Geometry Optimization of T-Shaped Cavities According to Constructal Theory, Mathematical and Computer Modelling, Vol. 52, pp. 1538-1546.