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NATURAL CONVECTION IN RECTANGULAR CAVITIES WITH DIFFERENT ASPECT RATIOS

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ABSTRACT

Natural convection in closed cavities has been extensively studied in recent decades. This spontaneous method of heat transfer has a wide range of applications in engineering. In the present work, natural convection was numerically analyzed in a rectangular cavity heated on one of the sides and cooled on the opposite side. Temperatures of the heated wall and of the cooled wall were assumed to be constant. The objective of these studies was to determine the effects of the aspect ratio and the Rayleigh number on flow behavior and heat transfer in the cavity. In the simulations, the Rayleigh number drastically influenced the flow profile and heat transfer inside de cavity, as well as the thickness of the thermal boundary layer. It was also verified that the Nusselt number is strongly dependent on the L/D (Length/Height) ratio, and that this dimensionless variable increases with the increase of the W/L. The simulation of natural convection problems in the CFD Studio satisfactorily described the studied situations.

Keywords: heat transfer, CFD, fluid flow

NOMENCLATURE

L	Length of the cavity, m
H	Height of the cavity, m
Nu_L	Nusselt number, adm
Ra_L	Rayleigh number, adm
Pr	Prandt number, adm
T	Temperature, K
t	time, s
u	Velocity in the x-direction, m/s
v	Velocity in the y-direction, m/s
V	Velocity of the fluid, m/s
x, y	Coordinates defined in figure 1
P	Pressure, N/m²
x*, y*	dimensionless Coordinates
u*	Velocity in the dimensionless x-direction
v*	Velocity in the dimensionless y-direction
P*	dimensionless pressure
k	Thermal conductivity, W/(m.K)
h	Convective heat transfer coefficient, W/(m².K)
	Rate of heat generation per unit volume, W/m³
c_p	Specific heat, J/(kg.K)

Greek symbols

θ	dimensionless temperature
μ	Dynamic viscosity, (N.s)/m²
ν	Kinematic viscosity, m^2/s
ρ	Specific mass, kg/m^3
β	Thermal expansion coefficient, 1/K
α	Thermal diffusivity, m²/s

Subscripts

1	hot wall
0	cold wall

INTRODUCTION

According to Incropera and Dewitt (2002), convection is the term utilized to describe the transfer of energy between a surface and a fluid in movement over this surface. Although the diffusion mechanism (random motion of fluid molecules) contributes to this transfer, the dominant contribution is generally defined as the global movement or majority of the fluid particles. Situations in which the fluid is not forced to pass over the surface, but still there is a convection current within the fluid, are called free or natural convection and are originated when a force acts on a fluid in which there are gradients of specific mass. The net effect is a buoyant force which induces free convection currents. In the most common case, the specific mass gradient is due to the temperature gradient, and force is due to the gravitational field.

According to Baïri (2007) and Oliveira (2003), natural convection in closed cavities has been extensively studied in recent decades. This spontaneous mode of heat transfer has many applications in engineering. The electronics industry, for example, takes advantage of natural convection for assembly of components in a confined environment, which is very common. The agricultural sector utilizes this phenomenon for drying applications and storage.

Many geometries, more or less complex, have been studied by theoretical, numerical or experimental approximations. However, due to the difficulties in developing experimental studies to determine the parameters related to convection, other methods have emerged as important tools for solving these problems.

With the increase in computing power and development of numerical algorithms implemented with systems of differential equations which correlate the thermal and dynamic aspects of the flow and allow for application of particular initial and boundary conditions, the numerical solution of these problems has become feasible both technically and economically.

Oliveira (2003) cited some studies that focus on the problem of natural convection in cavities. Hasnaoui et al. (1998) analyzed natural convection in a cavity heated at its base. Ganzarolli and Milanez (1995) numerically analyzed permanent natural convection in a cavity heated at the base and cooled on the sides. Valencia and Frederick (1989) numerically studied the natural convection of air in square cavities with side walls half active and half isolated for Rayleigh numbers of 103 to 107. Chinnokotla et al. (1996) performed a parametric study of fluid flow and energy transport in L-shaped cavities with asymmetrically heated surfaces. Selamet and Arpaci (1992) studied natural convection in a vertical slit with a narrow upper section and analyzed the effect of sudden changes in temperature of the hot wall on the flow and energy transport. Chu et al. (1976) conducted a numerical and experimental study on the phenomenon of two-dimensional laminar natural convection in rectangular cavities to examine the effects of heater dimensions and location, aspect ratio, and boundary conditions on this cavity. November and Nansteel (1987) studied natural convective flow in a square cavity with one cooled vertical wall and a heated base. Poulikakos (1985) obtained numerical results for natural convection in a cavity heated and cooled along a single wall. In their numerical studies, Aydin et al. (1999) considered the case of a square cavity heated on one side and cooled at the top, and studied the influence of the Rayleigh number on the flow field and energy transport.

In the present study, natural convection is analyzed numerically in a rectangular cavity heated on one side and cooled on the opposite side. The temperatures of the heated wall and cold wall are assumed to be constant. The main objective of this study is to determine the effects of the aspect ratio and Rayleigh number on flow behavior and heat transfer in the cavity.

THEORY

Geometry and Boundary Conditions

The geometries studied and the coordinate system of the proposed problem are shown in fig. 1. The dimension of the cavity in the z direction is assumed to be infinitely long. The cavity is heated on its right side and cooled on the opposite side (left), while the top and bottom are maintained isolated. Temperatures of the hot (right) and cold (left) walls are listed as constants, i.e., T1 and T0 respectively, while the top and bottom are adiabatic.

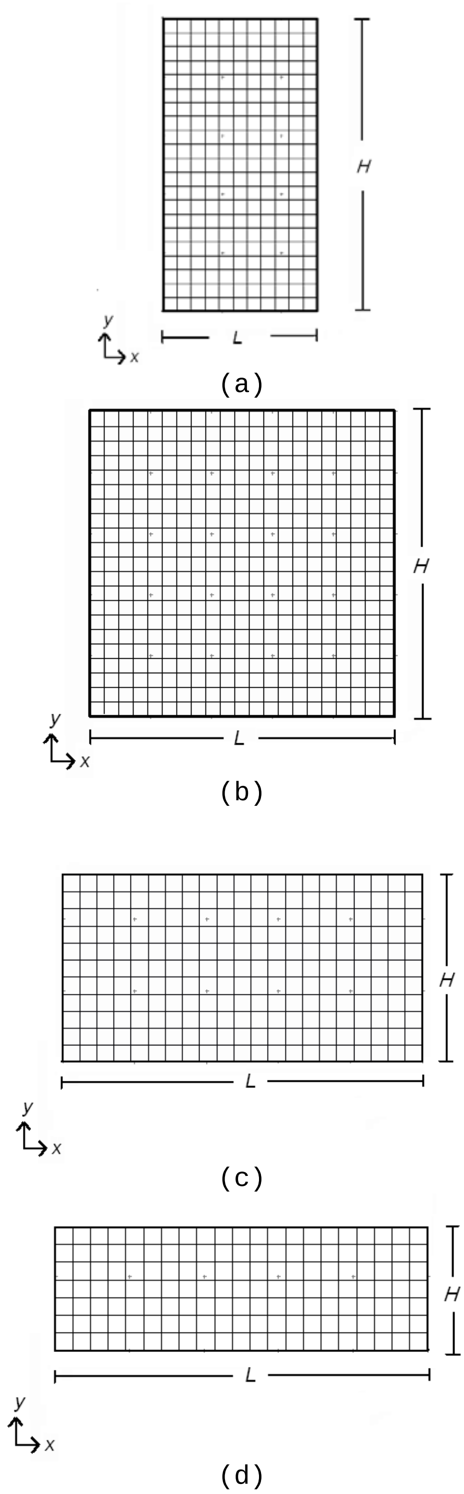


Figure 1. Mesh used to solve problems.

(a) $\frac{L}{H} = 0.5$, (b) $\frac{L}{H} = 1$, (c) $\frac{L}{H} = 2$ and (d) $\frac{L}{H} = 3$

The equations governing this problem are those of Navier-Stokes along with the energy equation. The Navier-Stokes equations are applied to incompressible flows and Newtonian fluids, including the continuity equation and the equations of conservation of momentum on the x and y axes, according to equations.

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\dot{q}}{\rho c_p} \quad (4)$$

The equation for conservation of energy is derived from the 1st Law of Thermodynamics and can be written as in equation 4.

From the equations mentioned above, a dimensionless model was developed for determining the parameters characterizing natural convection. It was considered to be at steady state and the differential pressure gradient in relation to the x dimension is approximated by the static gradient ($-\rho_\infty g$). The parameters assessed were the Nusselt number and Rayleigh number. Therefore, the Prandt number is considered to be constant according to the following equations:

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=\frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)+\theta.Ra.\Pr \quad (5)$$

$$Nu_l=\frac{h.l}{k} \quad (6)$$

$$Ra_l=\frac{g.\beta.(T_1-T_0).l^3}{\nu.\alpha} \quad (7)$$

where l is the characteristic length that was considered as being the ratio L/D.

$$\Pr=\frac{\alpha}{\nu}=1 \quad (8)$$

For determination of the Nusselt number and Rayleigh number the following parameters were utilized:

ρ= 1
cp= 1
k= 1
μ= 1
β= Varying from 1 to 100000
$\dot{q}$ = 0
g= Varying from 1 to 10

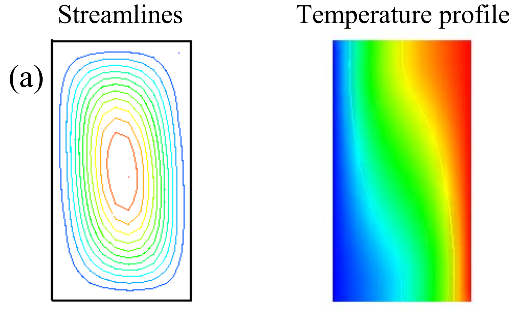
Therefore, the effect of Rayleigh number ranging from 10^4 to 10^6 was studied for ratios of length to width of the cavity of 0.5, 1, 2 and 3 and the Nusselt number for natural convection.

NUMERICAL METHOD

To model the problem, the computational program Modeling and Simulation of Numerical Problems CFD Studio was used. The CFD Studio is a software platform (Windows and Unix) that allows the numeric simulation of 2D problems involving fluid flow and heat transfer. It is designed for educational purposes and can be used in heat transfer courses (conduction and convection) and courses on Computational Fluid Mechanics. The program operates for incompressible flow, Newtonian fluid with and without heat transfer, utilizing the Boussinesq approximation in rectangular coordinates (x, y).

RESULTS AND DISCUSSION

Figure 2 shows the influence of the Rayleigh number equal to 10,000 on the flow and heat transfer in the cavities studied. In figure 3 the streamlines and isotherms are encountered for the Rayleigh number equal to 1,000,000. Fluid movement was characterized by a recirculation where the fluid rises along the hot wall (right) and descends along the cold wall (left).



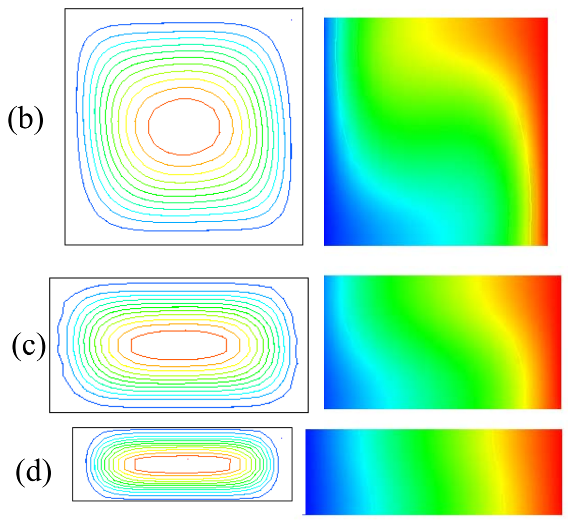


Figure 2. Streamlines and temperature profiles inside the cavities for a Rayleigh number equal to 1x10^4 with the following ratios:

(a) $\frac{L}{H}=0.5$, (b) $\frac{L}{H}=1$, (c) $\frac{L}{H}=2$ and (d) $\frac{L}{H}=3$

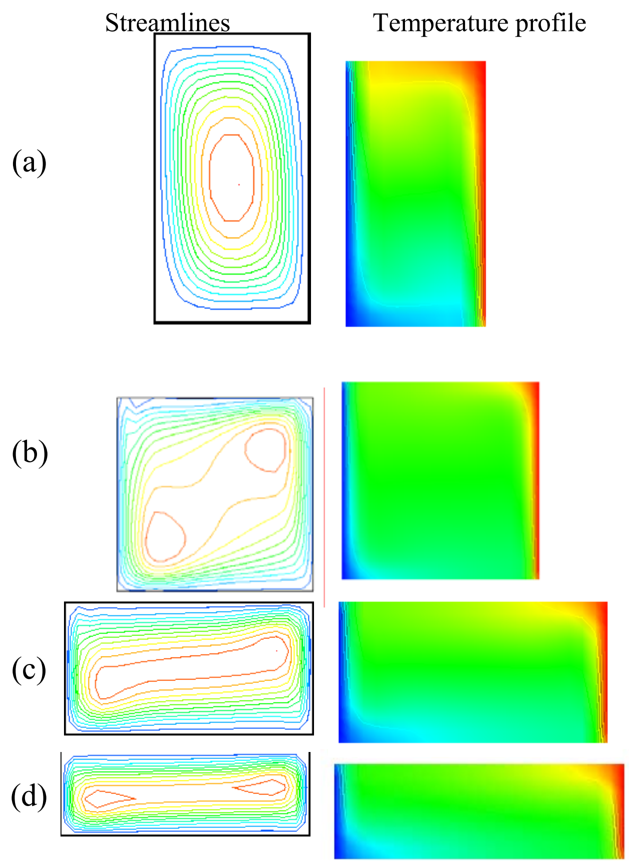


Figure 3. Streamlines and temperature profiles inside the cavities for a Rayleigh number equal to 1x10^6 with the following ratios:

(a) $\frac{L}{H}=0.5$, (b) $\frac{L}{H}=1$, (c) $\frac{L}{H}=2$ and (d) $\frac{L}{H}=3$

Based on the above figures it can be observed that in all cases studied only one convection cell was formed for the predetermined Rayleigh values. For $Ra_l \le 10^4$ the flow induced by buoyancy was deficient and heat transfer principally occurred via conduction through the fluid. For $Ra_l \le 10^4$, flow through the cell was intensified and became concentrated in thin boundary layers close to the sidewalls. It was also observed that the center region presents no movement although additional streamlines development near the vertices due to the transition from laminar to turbulent flow near to the walls.

From the graphs of temperature the formation of a thermal boundary layer along the hot and cold walls is verified, which becomes denser as the Rayleigh number increases. The temperature gradient along the heater vertical wall is maximized at the top and decreases from top to bottom.

From data of the Nusselt number as a function of the Rayleigh number the graphs shown in figures 4-7 were obtained and the points of the L/D ratio between 0.5 and 3 were plotted. Thus, the equations were obtained that demonstrate the behavior of the Nusselt number as a function of Rayleigh number.

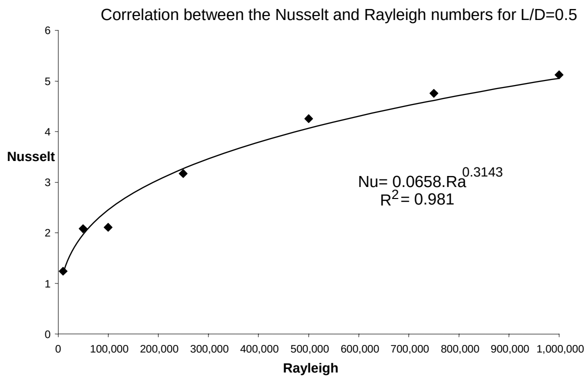


Figure 4. Correlation between the Nusselt and Rayleigh numbers for L/D=0.5.

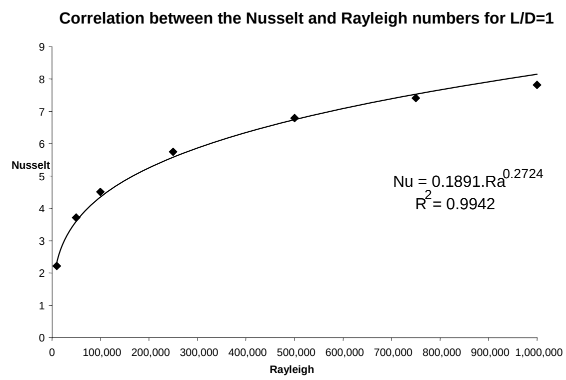


Figure 5. Correlation between the Nusselt and Rayleigh numbers for L/D=1.

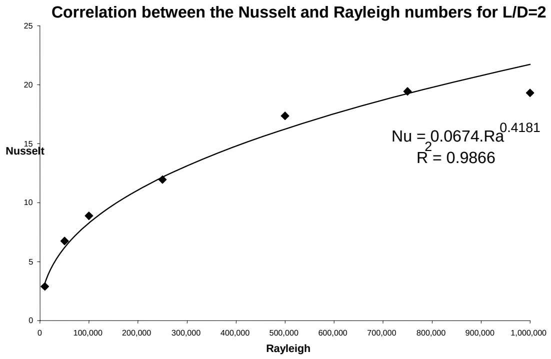


Figure 6. Correlation between the Nusselt and Rayleigh numbers for L/D=2.

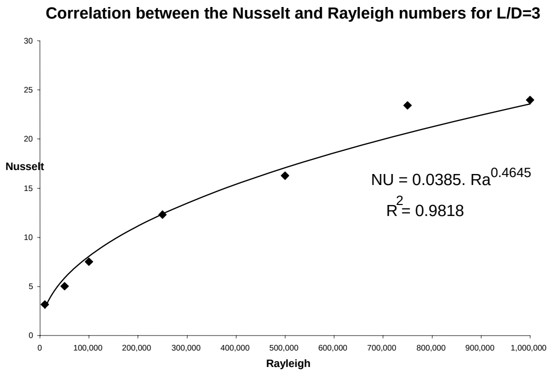


Figure 7. Correlation between the Nusselt and Rayleigh numbers for L/D=3.

As can be observed in the figures above, all equations obtained presented a good fit with a correlation coefficient greater than 0.98.

$$\overline{Nu_l} = 0.22\left(\frac{\Pr}{0.2+\Pr}Ra_l\right)^{0.28}\left(\frac{L}{D}\right)^{-1/4} \quad \left[\begin{array}{l} 2 < \frac{L}{D} < 10 \\ \Pr < 10^5 \\ 10^3 < Ra_l < 10^{10} \end{array}\right] \qquad (9)$$

$$\overline{Nu_l} = 0.18\left(\frac{\Pr}{0.2+\Pr}Ra_l\right)^{0.29} \quad \left[\begin{array}{l} 1 < \frac{L}{D} < 2 \\ 10^{-3} < \Pr < 10^5 \\ 10^3 < \frac{Ra_l \Pr}{0.2+\Pr} \end{array}\right] \qquad (10)$$

As cited by Incropera and Dewitt (2002), some correlations for the Nusselt number to ratios of $1 < \frac{L}{D} < 10$ were suggested, in accordance with equations 9 and 10.

$$\overline{Nu_l} = 0.1891\left(Ra_l\right)^{0.2724} \qquad (11)$$

$$\overline{Nu_l} = 0.22\left(\frac{1}{0.2+1}Ra_l\right)^{0.28}\left(1\right)^{-1/4}$$
$$\overline{Nu_l} = 0.209\left(Ra_l\right)^{0.28} \qquad (12)$$

When comparing the equation obtained for the ratio L/D=1 (equation 11) with equation 9, the equation adjusted for the cited case is obtained, as shown in equation 12.

As can be observed, the simulated equation was very close to the equation cited by Incropera and Dewitt (2002).

CONCLUSIONS

For the simulations, the Rayleigh number dramatically influenced the flow profile and heat transfer within the cavity, as well as the thermal boundary layer thickness. It was also found that the Nusselt number strongly depends on the L/D ratio, and that this dimensionless variable increases with the increasing dimensionless ratio L/D. Simulation of natural convection problems in the CFD Studio satisfactorily described the situations examined.

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