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APPROXIMATION OF NUMERICAL INTEGRATION APPLIED TO *Araucaria angustifolia* STEM TAPER MODELS

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Abstract

Functions describing stem shape allow to determine dimension and volume class, with direct application in technical and economic activities in the forest. The aim of this study was to evaluate the accuracy of the Trapezoidal and Simpson 1/3 rules in the approximation of numerical integration applied to Kozak, Lee *et al*. and Sharma and Zhang models. The evaluated data were diameter and height of sixty *Araucaria angustifolia* (Bertol.) Kuntze individuals from a planted forest located in Caçador, State of Santa Catarina, Brazil. Analysis of results showed the high efficiency of the two volume and assortments determination methods compared to the exact value of the numerical integration. Kozak and Lee *et al.* models showed better results compared to Sharma and Zhang models. The first, when estimating volume of trunk with bark, applying the Trapezoidal rule, showed an error smaller than 10^{-4}, through eight integration intervals, equidistant between sections with relative length equal to 20% of the total height. The Simpson 1/3 rule resulted in greater accuracy with an error smaller than 10^{-6}, though with complex mathematical structure using six equidistant integration intervals.
Keywords: Taper; stem profile; individual volume.

Resumo

Aproximação da integral numérica de modelos de forma do tronco para Araucaria angustifólia. Funções que descrevem a forma do tronco de árvores permitem determinar classes de dimensão e volume com aplicação direta no planejamento das atividades técnicas e econômicas na floresta. Neste sentido o presente estudo avaliou a acurácia da regra dos Trapézios e de Simpson 1/3 na aproximação da integral numérica aplicada aos modelos de Kozak, Lee *et al*. e Sharma e Zhang. Os dados avaliados incluem diâmetros e alturas relativas de sessenta árvores de *Araucaria angustifolia* (Bertol.) Kuntze provenientes de plantios florestais situados em Caçador, SC. A análise dos resultados mostrou alta eficiência dos dois métodos de determinação do volume e dos sortimentos quando comparados com o valor exato da integral numérica. Os modelos de Kozak e Lee *et al.* obtiveram resultados superiores quando confrontados com Sharma e Zhang, sendo estes, quando estimado o volume do tronco com casca, aplicando a regra do Trapézio, mostrou um erro inferior a 10^{-4} mediante a oito intervalos de integração equidistantes entre secções de comprimento relativo igual a 20% da altura total. A regra de Simpson 1/3 propiciou maior acurácia com erro inferior a 10^{-6}, mas com estrutura matemática complexa utilizando seis intervalos de integração equidistantes.
Palavras-chave: Afilamento; perfil do fuste; volume individual.

INTRODUCTION

Taper models are mathematical expressions describing the diameter decrease rate over the profile of plant stems, whose integration allows the tridimensional reconstruction of the solid of revolution associated to the geometric form of the stem or of parts of it.

These models are widely used in volume estimation, simulation and optimization of assortments, between any positions along the stem, with direct application in the definition of silviculture technical activities, in harvesting decisions and in economic planning of the forest company.

Some mathematical models, like polynomials, are easy to integrate and allow to directly define partial and total volumes of the timber (KOZAK *et al.*, 1969; GOULDING; MURRAY, 1976; MAX; BURKHART, 1976), however more complex models may be more accurate, but they are not analytically integrable (KOZAK, 1988; LEE *et al.*, 2003; SHARMA; ZHANG, 2004), demanding the use of numerical integration techniques (THOMAS *et al.*, 2010).

Numerical integration technique is normally used to determine a defined integral, whose function is not available or does not have an analytic solution. Solution of this integral is obtained by approximation of a defined integral of the type:

$$I = \int_a^b f(x)dx \qquad (1)$$

through a sum of the type:

$$I = \int_a^b f(x)dx \cong \sum_{i=1}^{n} W_i f(x_i)\Delta x \qquad (2)$$

where: $f(x_i)$ = values of the function f(x);

$\Delta x = x_{i+1} - x_i$;

$w_{i\,=}$ is a numeric weighting value also known as weight function.

The numeric solution of a simple integral is possible by means of the Newton-Cotes formula, which applies equally spaced values of f(x); or by the gaussian quadrature formula, which uses different spaces determined by a certain property of the orthogonal polynomials.

Within the formula of Newton-Cotes, Trapezoidal and Simpson 1/3 rules are frequently determined starting from an interpolating polynomial, whose approximation is possible because the polynomial is easily integrable (POLYANIN; MANZHIROV, 2007).

The Trapezoidal rule consists in approximating the value of the continuous f(x) function, in the interval [a, b], with a first order function, which is the same as approximating any curve to a line. The area below f(x) is equivalent to the integral of this function, approximated by the area of the trapezium whose width is equal to the interval (b – a), and whose average height is equal to [f(a)+f(b)]/2. Calculating the difference $\Delta x = b - a,$ the formula to calculate integral may be written as:

$$\int_a^b f(x)dx \cong \frac{h}{2}\left[f(a) + f(b)\right] \qquad (3)$$

This way the composite Trapezoidal rule may be written as:

$$\int_a^b f(x)dx \cong \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right] \qquad (4)$$

The Simpson's rule is an approximation method of the continuous function in the interval [a,b] by a second order function, which corresponds to approximating any curve to a parabola. The area below the function f(x) corresponds to the integral of this function, which has the form:

$$\int_a^b f(x)dx \cong \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right], \text{ where } h = \Delta x = x_2 - x_1 = x_1 - x_0 \qquad (5)$$

This formula is known as Simpson's 1/3 rule, due to the factor that multiplies h. In this case, at least three values of $f(x_i)$ are necessary to calculate the integral by the Simpson's rule. In the notation, x_0=a, x_2=b, and x_1 is the point which is equidistant from x_0 and x_2. For n Δx intervals, it may be written:

$$\int_a^b f(x)dx \cong \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right] \qquad (6)$$

where: n=even number of integration intervals or equivalent, Simpson's 1/3 rule can be applied only for an uneven number of points x_i, $f(x_i)$.

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In view of the above, this study had the objective to assess accuracy of Trapezoidal and Simpson's 1/3 rules in the approximation of numerical integration applied to stem taper models of *Araucaria angustifolia* (Bertol.) Kuntze trees.

MATERIAL AND METHODS

Data collection

The study was conducted using rigorously collected volumetric data from plantations of *Araucaria angustifolia* of the National Forest of Caçador, SC, located in a municipality with the same name, whose climate is classified as Cfb in the Köppen system, with average annual temperature of 16.5 °C and average annual rainfall close to 1600mm (PANDOLFO *et al*., 2002).

A total of 60 trees had their diameter with bark measured at 0.1, 0,3, and 1.3 meters from the ground, defined as $h_{0.1}$, $h_{0.3}$, $h_{1.3}$ and, starting from these positions, were measured meter by meter (h_i), until reaching the total height of the trees.

A group of 2/3 of the sampled trees was randomly separated, representing all the diameter dimensions found, to adjust the tamper equations, while the remaining group was kept to validate accuracy of the Trapezoidal and Simpson's 1/3 rules. Biometrical characteristics of the assessed trees were summarized in table 1.

Table 1. Biometrical characteristics of araucaria trees.
Tabela 1. Características biométricas das árvores de araucária.

Data	Variable	Frequency	Mean	Median	Minimum	Maximum	Std. Deviation
	d		23.3	23.5	10.4	40.8	6.9
Adjust	h	40	15.7	16.1	7.8	20.3	2.6
	hc		14.1	14.8	4.1	18.7	3.1
	d		23.5	23.8	13.2	38.0	7.2
Validation	h	20	15.2	14.9	10.2	18.8	2.3
	hc		13.7	13.6	9.0	17.2	2.5

d: diameter at breast height, in cm: h: total height, in m; hc: commercial height, in m.

Stem taper models and statistical criteria

In the statistical analysis of the regression models performed in the study (Table 2), the adjusted coefficient of determination (R^2aj.), the root mean squared error (Sxy) and the Akaike information criterion were considered. Estimate precision of the respective diameters was assessed by tests based of the stratified residuals by relative height class (hi/h) resumed in table 3, according to the methodology presented by Figuereido-Filho *et al*. (1996), Scolforo *et al*. (1998) and Souza (2009).

Table 2. Adjusted stem taper models.
Tabela 2. Modelos de forma do tronco ajustados.

Author	Models	
Kozak (1988)	$d_i = \beta_0 d^{\beta_1} \beta_2^{d} \left(\frac{1-\sqrt{h_i/h}}{1-\sqrt{p}} \right)^{\beta_3 \left(\frac{h_i}{h}\right)^2 + \beta_4 \ln\left(\frac{h_i}{h} + 0{,}001\right) + \beta_5 \sqrt{\frac{h_i}{h}} + \beta_6 e^{\left(\frac{h_i}{h}\right)} + \beta_7 \left(\frac{d}{h}\right)} + \varepsilon$	(7)
Lee *et al.* (2003)	$d_i = \beta_1 d^{\beta_2} \left(1 - \frac{h_i}{h}\right)^{\beta_3 \left(\frac{h_i}{h}\right)^2 + \beta_4 \left(\frac{h_i}{h}\right) + \beta_5} + \varepsilon$	(8)
Sharma and Zhang (2004)	$\left(\frac{d_i}{d}\right)^2 = \beta_0 \left(\frac{h_i}{h_{1,3}}\right)^{2 - \left[\beta_1 + \beta_2 \left(\frac{h_i}{h}\right) + \beta_3 \left(\frac{h_i}{h}\right)^2\right]} \left(\frac{h - h_i}{h - h_{1,3}}\right) + \varepsilon$	(9)

h: total height; h_i : relative height at position i. over the stem; d : diameter at breast height; d_i : relative diameter at position I over the stem; β_0,..,β_7: are parameters of the model; ε : residual error; ln : natural logarithm; p : point of inflexion considered at 1.3h. *All the models were adjusted by the NLIN procedure, through the method of Maquardt in the statistical system SAS V 9.1 (SAS Institute Inc., 2004).

Table 3. Statistics to evaluate the precision of relative estimated diameters.
Tabela 3. Estatísticas para avaliar a precisão das estimativas dos diâmetros relativos.

Statistics	Formula
Deviation (D)	$\sum_{i=1}^{n} \left(y_j - \hat{y}_j\right) / N$
Squared Sum of Relative Residual (SSRR)	$\sum_{i=1}^{n} \left[\left(y_j - \hat{y}_j\right)/y_j\right]^2$
Residual Percentage (RP)	$\sum_{i=1}^{n} \left(y_j - \hat{y}_j / y_j\right).100 \Big/ N$

$y_j, \hat{y}_j$: observed and estimated diameter for the i^th plant in the j^th position on the stem; N : number of observations. *small values of D, SSRR and RP are preferable.

Volume of the stem was obtained integrating the basal areas g_i between the limits inferior height h_i and superior height h_2 desired, being the integral written as follows:

$$V = \int_{h_1}^{h_2} g_i \delta h \qquad (10)$$

simplifying (10) we obtain:

$$V = K\int_{h_1}^{h_2} d_i^2 \delta h \qquad (11)$$

where: $K = \frac{\pi}{40000}$; h_2 = height in the upper position of the section; h_1= height in the lower position of the section.

The area obtained by the $f(d_i)$ over the i-th heights in the stem was obtained by the two numerical integration methods; by the Trapezoidal and Simpson's 1/3 rule, for a constant interval Δx, consisting in determination of the weight function w_i value.

Approximation of volumes was possible in each section of the stem by equations (4) and (6). The constant $K = \left[\frac{\pi}{40000}\right]$ multiplies value of the weight $w_i = \left[\frac{(h_2 - h_1)}{n}\right]$ by summation of f(di) of each term of the stem taper model, squared, in the following way:

Trapezoidal rule

$$V \cong K\left\{\frac{1}{2} w_i \left[f(di_0)^2 + 2f(di_1)^2 + 2f(di_2)^2 + \ldots + 2f(di_{n-2})^2 + 2f(di_{n-1})^2 + f(di_n)^2\right]\right\} \quad (12)$$

Simpson's 1/3 rule

$$V \cong K\left\{\frac{1}{3} w_i \left[f(di_0)^2 + 4f(di_1)^2 + 2f(di_2)^2 + \ldots + 2f(di_{n-2})^2 + 4f(di_{n-1})^2 + f(di_n)^2\right]\right\} \quad (13)$$

To assess effectiveness of each rule, results were compared to the exact volume of the solid of revolution obtained in trees by the f(di) in relative positions corresponding to ($h_{0,1}$-$h_{0,2h}$; $h_{0,2h}$-$h_{0,4h}$; $h_{0,4h}$-$h_{0,6h}$; $h_{0,6h}$-$h_{0,8h}$ and $h_{0,8h}$-h), applying four, six and eight equidistant integration intervals between the sections. This procedure was possible through the development of an algorithm in Visual Basic. Statistical analysis was performed in the statistical system SAS V.9.1 (SAS Institute Inc., 2004) and simulation of numerical integration methodologies was performed with the program MAPLE 13.0 (MAPLE Inc., 2009).

RESULTS AND DISCUSSIONS

The three models assessed to describe stem profile of araucaria trees showed accuracy higher than 98.0%, maximum error between 0.85 and 0.97 m in the estimates, and all the coefficients were significant (p<0,0001), according to table 4. Statistical comparison indicated superiority of Kozak models (Equation 7) and Lee *et al*. (Equation 8) compared to Sharma and Zhang models (Equation 9).

Table 4. Stem taper models statistics (standard error in parenthesis).
Tabela 4. Estatísticas de ajustes dos modelos de forma do tronco (erro padrão em parêntesis).

Eq.	Estimated coefficients								Statistics		
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	R²aj.	Syx	AIC
7	1.1790 (0.0999)	0.9234 (0.0378)	1.0032 (0.0015)	-0.8602 (0.1478)	0.1378 (0.0322)	-2.3251 (0.2936)	1.2946 (0.1635)	0.0741 (0.0080)	98.8	0.85	-207.5
8	-	1.4000 (0.0314)	0.9373 (0.0068)	1.7367 (0.0722)	-2.6575 (0.1013)	1.4873 (0.0372)	-	-	98.6	0.90	-138.6
9	0.9800 (0.0068)	2.1002 (0.0047)	-0.3675 (0.0375)	0.1498 (0.0405)	-	-	-	-	98.4	0.97	-42.6

Eq.: equation; $\hat{\beta}_0 ... \hat{\beta}_7$: Estimated coefficients.

Analysis of statistics of table 4 allows to infer on behavior of the model in relation to the average, it does not guarantee its performance, but its capacity to maintain integrity of predictions. Among the three models, no one presented statistical superiority compared to the others in terms of the adjusted coefficient of determination and of standard error. Statistical differences were highlighted just by the Akaike criterion.

Analysis of prediction accuracy of relative diameters over the stem (d_i), resumed in table 5, showed that in the base position of the stems ($0.0 < hi/h \leq 0.2$) and ($0.2 < hi/h \leq 0.4$), Kozak model (Equation 7) showed the best performance in terms of the criteria used to assess accuracy of predictions with sum of ($\Sigma = 3$). Another important performance was detected by the Lee *et al*. model (Equation 8), in the position ($0.0 < hi/h \leq 0.2$), with the greatest sum in terms of accuracy criterions ($\Sigma = 9$), indicating great instability in prediction of diameters of this portion of the stems. It is worth to highlight that stability of predictions in this portion favors its application and quantification of the assortments in trees of great dimension, being the region that concentrates the greatest wood volume and value of timber.

In the intermediate upper position of the stem ($0.4 < hi/h \leq 0.8$), the Lee et al model (Equation 8) gave the best results, with the lowest sum of indexes ($\Sigma = 4$). In this region the Sharma and Zhang model (Equation 9) reached the greatest bias estimating diameters over all positions in the stem, with general sum of indexes $\Sigma = 37$ according to table 5, resulting inadequate to describe stem profile of araucaria trees in this study.

Table 5. Residual analysis of stem taper models.
Tabela 5. Análise residual das equações de forma de tronco.

Relative height Interval	Statistics	n. obs.	[7]	[8]	[9]
0.0 < hi/h ≤ 0.2	D	175	0.0379(1)	-0.1404(3)	0.0934(2)
	SSRR		0.1288(1)	0.2029(3)	0.1307(2)
	RP		0.0675(1)	-0.8774(3)	0.3908(2)
	Σ		**3**	9	6
0.2 < hi/h ≤ 0.4	D	126	-0.0097(1)	0.1385(2)	-0.1884(3)
	SSRR		0.1306(1)	0.1330(2)	0.1658(3)
	RP		-0.1723(1)	0.5458(2)	-0.7503(3)
	Σ		**3**	6	9
0.4 < hi/h ≤ 0.6	D	124	-0.2132(3)	0.0132(1)	-0.0297(2)
	SSRR		0.2112(3)	0.1826(1)	0.2036(2)
	RP		-1.5376(3)	-0.1315(2)	0.0622(1)
	Σ		9	**4**	5
0.6 < hi/h ≤ 0.8	D	123	0.1199(2)	-0.0091(1)	0.4994(3)
	SSRR		0.2790(1)	0.3296(2)	0.6415(3)
	RP		0.7956(2)	0.3312(1)	4.0512(3)
	Σ		5	**4**	9
0.8 < hi/h ≤ 1.0	D	124	0.0411(1)	0.2027(3)	-0.0791(2)
	SSRR		6.9364(2)	5.5235(1)	8.9938(3)
	RP		-3.9775(2)	1.2364(1)	-5.4873(3)
	Σ		**5**	5	8
Geral	Σ		**25**	28	37

[7, 8 and 9] : stem taper equations; n. obs.: number of observations; Between brackets the equations Ranking.

In relative positions higher than 60% of the total stem height, punctuations of Kozak (Equation 7) and Lee *et al*. models (Equation 8) were similar, with general sum of the indexes respectively of $\Sigma = 25$ and $\Sigma = 28$ (Table 5). This analysis showed the importance of residuals based tests in different positions along the stem to properly choose the equation. This way, flexibility allied to good performance of Kozak (Equation 7) model, in terms of residual criterions, were highlighted, mainly in the estimate of the inferior and superior section of the stems, justifying its application for quantification of tree volumes and determination of araucaria wood assortments.

Validation test

During the validation phase, considering the group of twenty trees randomly selected for this purpose, and all the positions over the stems simultaneously taken, the Lee *et al*. (Equation 8) model resulted in the lowest sum of scores $\Sigma = 26$, followed by Kozak (Equation 7) with $\Sigma = 28$ and Sharma and Zang (Equation 9) with $\Sigma = 36$; whose distribution by D, SSRR and RP criterions, according to classes of hi/h, are reported in figure 1.

The deviations (D) criterion, used to validate predictions, pointed out that the three models have general tendency to overestimate diameter values smaller than 60% of the total height. Sharma and Zang model (Equation 9) has the bigger tendency to underestimate with (D=0.3476) cm between positions $0.6 < hi/h \leq 0.8$.

When verified with the squared sum of relative residual (SSRR) criterion, all models had similar tendencies up to 60% of the total height. Starting from this position, the greatest values of the bias were found for the Sharma and Zang (Equation 9) model, with (SSRR = 3.3073), followed by Lee *et al*. (Equation 8) with (SSRR = 2.3975), presented in figure 1. The residual percentage (RP) revealed a smaller variation tendency in the Lee *et al*. (Equation 8) model, between the relative positions considered.

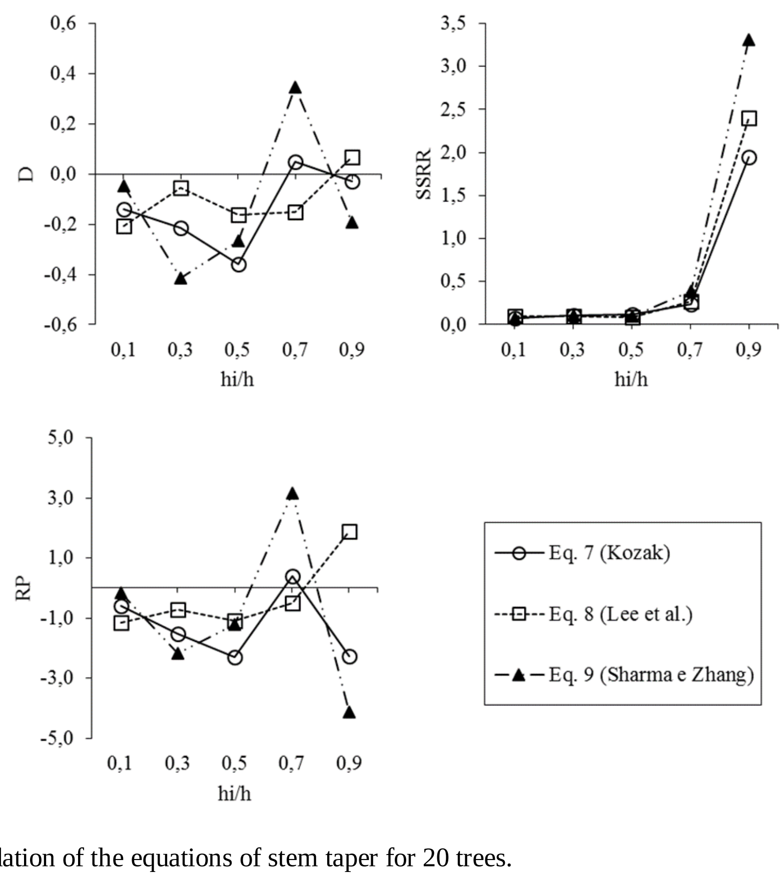

Figure 1. Validation of the equations of stem taper for 20 trees.
Figura 1. Validação das equações de forma de tronco para as 20 árvores.

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It is possible to observe that one model was better in determinate positions than in others, making a clear selection difficult in terms of stability of predictions in the relative positions, which justified the analysis of the Trapezium and Simpson's 1/3 rules efficiency for the stem taper models defined in equations 7 and 8.

In the procedure shown here, with a 32.2 cm diameter and 17.8 m height tree, using the Kozak (Equation 7), shape and volumetric size of sections was calculated by the numerical integration technique and rotation of f(x) $\rightarrow$(di) function around the x$\rightarrow$(hi) axis, repeated for all the relative positions of 20% of the total height, thus generating volume of the solid of revolution in these portions, for each tree (Figure 2).

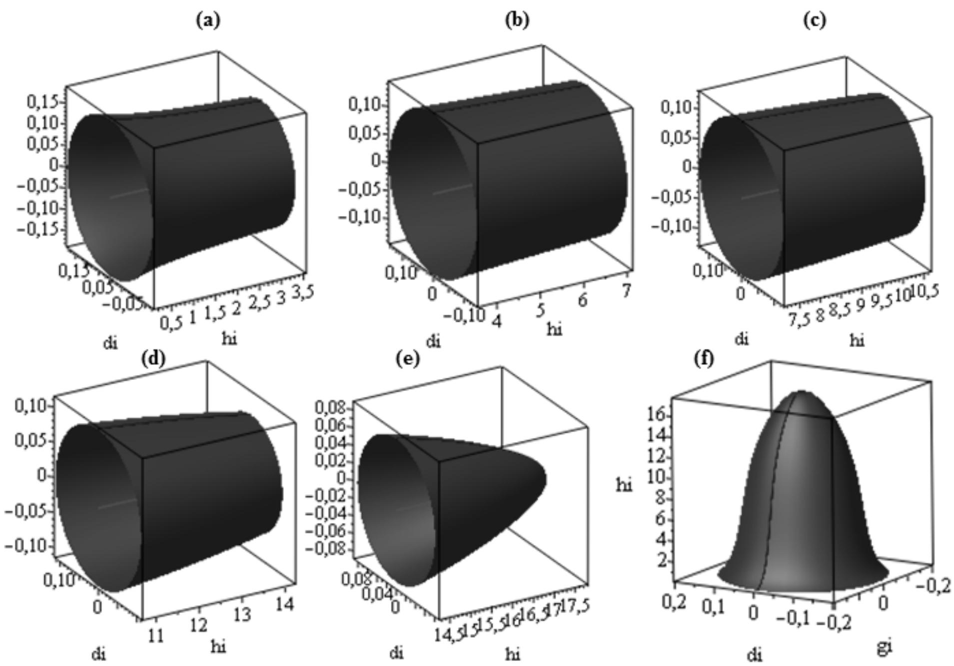

Figure 2. Solid of revolution volume obtained by Kozak (Equation 7) function in 20% height relative positions: (Árv.1 - d = 32,2 cm, h = 17,8 m); (a) portion between 0,1 ≤ hi ≤ 3,56 - v = 0,2732 m³, (b) portion between 3,56 ≤ hi ≤ 7,12 - v = 0,2061 m³, (c) portion between 7,12 ≤ hi ≤ 10 68 - v = 0,1635 m³, (d) portion between 10,68 ≤ hi ≤ 14,24 - v = 0,1089 m³, (e) portion between 14,24 ≤ hi ≤ 17,8 - v = 0,0354 m³; (f) Profile total of the tree - v = 0,7871 m³.

Figura 2. Volume do sólido de revolução obtido pela função de Kozak (Equação 7) em posições relativas de 20% da altura: (Árv. 1 - d = 32,2 cm; h = 17,8 m); (a) porção entre 0,1 ≤ hi ≤ 3,56 – v = 0,2732 m³; (b) porção entre 3,56 ≤ hi ≤ 7,12 – v = 0,2061 m³; (c) porção entre 7,12 ≤ hi ≤ 10,68- v = 0,1635 m³; (d) porção entre 10,68 ≤ hi ≤ 14,24 – v = 0,1089 m³; (e) porção entre 14,24 ≤ hi ≤ 17,8 – v = 0,0354 m³; (f) Perfil total da árvore - v = 0,7871 m³.

Difference between exact volume of the solid of rotation and volume calculated by the algorithm, developed to approximate the value obtained from numerical integration, gave the error made using Trapezoidal and Simpson's 1/3 rules. These values in tree 1 represented in figure 2 reached an absolute error lower than 10^{-3} estimating the total tree volume, with four equidistant integration intervals, applying the Trapezoidal rule. Same with the Simpson's 1/3 rule, where absolute error was lower than 10^{-4}, considering all the relative positions along the stem too.

The same process was applied to all trees of the validation group and also varying the number of integration intervals between sections, allowing to compare, in table 6, maximum, mean and minimum values of the error found during volume estimation.

Results indicated that increasing the number of integration intervals between sections caused improvements in trees volume predictions. However, for the two tested rules, the model originated increasing the number of integration intervals increases its mathematical complexity in prediction, requiring computational tools to apply them in practice. Considering numerical integration rules separately, it was determined that, for the same number of integration intervals, Simpson's 1/3 rule reached greatest accuracy compared to Trapezoidal rule.

Table 6. Accuracy of trapezoidal and Simpson 1/3 rules in partial volume estimative by numeric integral approximation of stem form models.

Tabela 6. Acurácia das regras do Trapézio e Simpson 1/3 na estimativa dos volumes parciais pela aproximação da integral numérica dos modelos de forma do tronco.

Integration Intervals			**$h_{0.1}$ - $h_{0.h}$**		**$h_{0.2h}$ - $h_{0.4h}$**		**$h_{0.4h}$ - $h_{0.6h}$**		**$h_{0.6h}$ - $h_{0.8h}$**		**$h_{0.8h-h}$**	
			(6)	**(8)**	**(6)**	**(8)**	**(6)**	**(8)**	**(6)**	**(8)**	**(6)**	**(8)**
Error m^3w Trapezoidal	Eq.7	Min.	-1.3E-03	-6.9E-04	-7.3E-05	-4.1E-05	3.2E-06	1.8E-06	7.4E-06	4.2E-06	-1.6E-07	3.6E-07
		Méd.	**-4.2E-04**	**-2.3E-04**	**-2.7E-05**	**-1.5E-05**	**1.7E-05**	**9.7E-06**	**3.1E-05**	**1.7E-05**	**4.6E-06**	**3.0E-06**
		Máx.	-7.3E-05	-4.0E-05	-6.5E-06	-3.6E-06	4.2E-05	2.3E-05	6.9E-05	3.9E-05	9.3E-06	5.8E-06
	Eq.8	Min.	-4.1E-04	-2.3E-04	-1.3E-04	-7.4E-05	1.6E-06	9.0E-07	9.7E-06	5.5E-06	-1.5E-04	-8.7E-05
		Méd.	**-1.6E-04**	**-8.8E-05**	**-5.1E-05**	**-2.9E-05**	**7.9E-06**	**4.4E-06**	**4.8E-05**	**2.7E-05**	**-5.8E-05**	**-3.4E-05**
		Máx.	-2.9E-05	-1.7E-05	-1.0E-05	-5.8E-06	2.0E-05	1.1E-05	1.2E-04	7.0E-05	-1.2E-05	-6.8E-06
Error m^3w Simpson 1/3	Eq.7	Min.	-5.8E-06	2.3E-06	-6.4E-07	-2.1E-07	-1.1E-07	-3.6E-08	-3.2E-07	-1.0E-07	-1.6E-07	3.6E-07
		Méd.	**5.5E-06**	**1.1E-05**	**-2.3E-07**	**-7.5E-08**	**-3.7E-08**	**-1.2E-08**	**-1.2E-07**	**-3.8E-08**	**4.6E-06**	**3.0E-06**
		Máx.	8.9E-06	2.3E-05	-5.1E-08	-1.6E-08	-6.2E-09	-2.0E-09	-2.2E-08	-7.0E-09	9.3E-06	5.8E-06
	Eq.8	Min.	-1.1E-06	-3.6E-07	-2.5E-07	-7.8E-08	-3.3E-08	-1.1E-08	-5.3E-07	-1.7E-07	-1.7E-05	-8.8E-06
		Méd.	**-4.3E-07**	**-1.4E-07**	**-9.5E-08**	**-3.0E-08**	**-1.3E-08**	**-4.2E-09**	**-2.1E-07**	**-6.5E-08**	**-6.5E-06**	**-3.4E-06**
		Máx.	-7.7E-08	-2.4E-08	-1.9E-08	-6.1E-09	-2.6E-09	-8.4E-10	-4.1E-08	-1.3E-08	-1.3E-06	-6.9E-07

Integration Intervals : number of integration intervals (6) and (8) equidistant between sections in the relative positions corresponding to ($h_{0.1}$-$h_{0.2h}$; $h_{0.2h}$-$h_{0.4h}$; $h_{0.4h}$-$h_{0.6h}$; $h_{0.6h}$-$h_{0.8h}$ e $h_{0.8h}$-h);

Mean errors resulting from the Kozak (Equation 7) model and Lee *et al*. (Equation 8) for a same common rule, as presented in table 6, result from flexibility and quality of the adjustment of each function to better describe a certain position in the stem, with now underestimated and then overestimated values. However, in all situations, presenting very small errors.

It is important to highlight that accuracy of each numerical integration technique was directly associated to the level of adjustment and precision of the chosen refining functions, which also depend on natural variability of data; in other words, variations registered between diameters over the stem depend on species, age, location, forest management, and other factors. Another fact to be considered is that, with the increase of section lengths, the number of integration intervals must be increased to obtain small errors.

CONCLUSIONS

According to analysis and discussion of the results, the following conclusions were reached:

- Flexibility and efficiency of Kozak (1988) and Lee *et al*. (2003) models in prediction of diameters (di) with bark over the entire stem of araucaria trees from forest plantations, encourage their use in volume predictions and formation of wood assortments.
- Use of Trapezoidal rule to determine stem volume of araucaria trees, applying eight equidistant integration intervals, in sections with length equal to 20% of total height, and use of Simpson's 1/3 rule, with six equidistant integration intervals, created errors smaller than 10^{-4} and 10^{-6}, respectively. Simpson's 1/3 calculation rule increases complexity of its solution increasing the number of integration intervals.
- On a practical point of view, Trapezoidal and Simpson's 1/3 rules may be used to calculate volume of the stem sections with not analytically solvable models. The methodology was efficient and may be transferred and used to calculate wood assortments of other forest species.

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