STEM TAPER EQUATION WITH EXTENSIVE APPLICABILITY TO SEVERAL AGE CLASSES OF *Pinus taeda* L.

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Abstract

The aim of this study was to develop stem taper equations for trees of *Pinus taeda* L. inside and outside bark with extensive applicability to several age classes. Data were obtained from 631 trees of *P. taeda*, ranging from 4 to 31 years of age, in different stands distributed in several counties of the Midwest region of the state of Santa Catarina, Brazil. The taper models of Biging (1984), Schöpfer (1966), and Kozak *et al.* (1969) were tested. These models were fitted in their simplest formulation as well as by varying their coefficient linear functions of stand age with the procedure PROC NLIN in the software SAS® University Edition. There were no significant differences in accuracy and precision in the estimates between the simplest formulation of the Schöpfer (1966) model and the variation of its coefficient *b* linear function of age, and thus, the simplest formulation was chosen. Among all taper models tested, the Schöpfer (1966) model presented greater accuracy and precision to estimate diameters inside and outside bark, throughout the stem of *Pinus taeda*, in the Midwest region of the state of Santa Catarina, Brazil.

Keywords: Polynomial models, profile models, fitting of simultaneous equations.

Resumo

Equação de afilamento de aplicabilidade extensiva a diversas classes de idade para Pinus taeda *L.* Este estudo teve como objetivo desenvolver equações de afilamento do tronco de *Pinus taeda* L. com e sem casca, de aplicabilidade extensiva, a diversas classes de idade. Os dados foram provenientes de 631 árvores de *Pinus taeda*, com idades entre 4 e 31 anos, de diversos povoamentos distribuídos em vários municípios da região Meio-Oeste do estado de Santa Catarina. Foram testados os modelos de afilamento de Biging (1984), Kozak *et al.* (1969) e Schöpfer (1966), os quais foram ajustados em suas formulações simples e com seus coeficientes função linear da idade das árvores, por meio do procedimento PROC NLIN do aplicativo computacional SAS® *University Edition*. Não houve diferenças expressivas de precisão e exatidão entre as estimativas do modelo de Schöpfer (1966) simples e com os coeficientes função linear da idade, selecionando-se, então, sua formulação simples. Dentre todos os modelos de afilamento testados, o modelo de Schöpfer (1966) apresentou maior precisão e exatidão para estimativas de diâmetros com e sem casca ao longo do fuste de *Pinus taeda*, na região Meio-Oeste de Santa Catarina.

Palavras-chave: Modelos polinomiais, modelos de perfil, ajuste de equações simultâneas.

INTRODUCTION

The volume of timber obtained from a tree depends, ultimately, on its future application, i.e., when it is intended to produce lumber, the usable volume of timber is different to the volume when it is intended to be used as firewood (BATISTA *et al.*, 2014). Thus, the timber volume of a tree depends on its class of use or its timber assortment.

When timber assortment volumes of a forest stand are available, it is possible to plan the optimized sale of the timber, aiming for a greater valuation in the consumer market, and thus maximizing profits. Taper models, which can estimate timber volumes for any assortment, enable production planning, transport logistics, and marketing to be undertaken based on the timber assortment, with the number of logs destined to each specific product measured (COSTA *et al.*, 2016), as well as being an important tool in information management for forestry companies (EISFELD *et al.*, 2008). The shape of a tree or log cannot be determined only by measuring, as the diameter or height; one can only establish indices that represent the shape. Calculation of the shape is possible by measuring stem diameters at different heights and its expression is given by processes involving form quotients, form factors, and taper functions.

Usually, the development of a taper model is restricted to a single species, age class, management type, study region, or even diameter, height, and form classes (EISFELD *et al.*, 2008; SOUZA *et al.*, 2008; MIGUEL *et al.*, 2011; FAVALESSA *et al.*, 2012a; FAVALESSA *et al.*, 2012b; SOUZA *et al.*, 2012; KOHLER *et al.*, 2013; LANSSANOVA *et al.*, 2013; TÉO *et al.*, 2013; DAVID *et al.*, 2014; FIGUEIREDO FILHO *et al.*, 2014a; FIGUEIREDO FILHO *et al.*, 2015; COSTA *et al.*, 2016; KOHLER *et al.*, 2016; TERRA *et al.*, 2018). According to Figueiredo Filho *et al.* (2014b), the use of these equations is recommended for datasets experiencing the same conditions.

To develop equations with greater applicability, Queiroz *et al.* (2008), using the model identity test, developed a taper equation for Bracatinga (*Mimosa scabrella* Bentham) having different age classes. Other authors have developed similar assessments to Queiroz *et al.* (2008) that recommend the adjustment of taper functions by age class for different forest species (KOHLER *et al.*, 2013; FIGUEIREDO FILHO *et al.*, 2015; TERRA *et al.*, 2018). Tomé *et al.* (2007) attempted to adjust the volume and taper equations for *Eucalyptus globulus* Labill., in which the parameters were expressed as linear combinations of planting age, planting density, site index, and variables that were indicative of rotation, coppice, and climatically homogeneous regions of Portugal to ensure the extensive applicability of the developed equations.

Thus, the objective of the present study was to develop stem taper equations for trees of *Pinus taeda* L. inside and outside bark that would have extensive applicability for several age classes in the Midwest region of the state of Santa Catarina, Brazil.

MATERIAL AND METHODS

The present study was undertaken in stands of *P. taeda* belonging to the company Juliana Florestal Ltd., which is associated with FRAME Madeiras Especiais Ltd. and is based in Caçador, state of Santa Catarina, Brazil. In addition to Caçador, the study area included the municipalities of Calmon, Lebon Régis, Macieira, Rio das Antas, Santa Cecília, and Timbó Grande, which are all located in the Midwest region of the state of Santa Catarina, Brazil.

Based on the Köppen classification, the study region has a Cfb climate type, that is, a wet subtropical zone, oceanic climate, without a dry season and with temperate summers. The average temperature of the hottest month is 19.7 °C and the coldest month is 11.5 °C, and the annual precipitation is 1,736 mm (ALVARES *et al.*, 2013). The original vegetation of the study region is a Mixed Ombrophilous Forest, mainly in its formation of Montana Mixed Ombrophilous Forest (IBGE, 2007). The main study area contains soils that are Haplic Cambisols with widely varying depths and drainage, ranging from sharp to imperfect and clayey or very clayey Bruno Nitisol, which are moderately acidic (IBGE, 2007).

Data were obtained from 631 *P. taeda* trees, with ages between 4 and 31 years, from several stands distributed in the study area. Of this total, bark thickness data were collected from 519 trees, which were cut to perform the cubage procedure. The remaining 112 trees had their outside bark diameters measured only along the stem, using a Criterion RD 1000 and TruPulse 200B in combination, which allowed the measurements to be obtained in a non-destructive manner without felling the trees. Such equipment was used to reduce the number of felled trees, and thus reduce the costs of data collection for the volume of *P. taeda* trees sampled.

For the 519 *P. taeda* trees that were felled, diameter measurements were taken of bark and bark thickness at 0.5%, 1%, 5%, 10%, 15%, 20%, 25%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, and 95% of the total height of the tree. For the 112 standing trees, outside bark diameters were measured in the same positions relative to the total height of the tree, up to at least 70%, and the stump height considered was 0.1 m.

The distribution of the 631 *P. taeda* trees, cubed at different ages, as well as the values of mean diameter ($\bar{d}$), maximum diameter (d_{max}), minimum diameter (d_{min}), mean height ($\bar{h}$), maximum height (h_{max}) and minimum height (h_{min}) are shown in Table 1.

Two sets of data were obtained as the following: 1) 10,489 section-level observations to develop the stem taper equation outside bark diameter and 2) 8,823 section-level observations to develop the inside bark diameter stem taper equation. The adjustment of several other taper models was first made in their simplest formulations to verify the data and to determine the candidate models that obtained the best performance (Table 2).

Adjustments to the models were made with the PROC NLIN procedure in the software program SAS® University Edition. As shown in Table 2, all taper models were adjusted in their simplest formulations and with one of their coefficients as a linear function of age by adjusting the simultaneous equations (Equations 1, 2, and 3) for coefficients a, b and c as a linear function of age for the Biging model (1984), respectively. This procedure was also followed for the Kozak *et al.* (1969) and Schöpfer (1966) models.

Table 1. Diameter at breast height, total height, and number of trees *Pinus taeda* L. measured by age class in the Midwest region of the state of Santa Catarina, Brazil.

Tabela 1. Diâmetro à altura do peito, altura total e número de árvores de *Pinus taeda* L., cubadas por classe de idade, da região Meio Oeste de Santa Catarina.

Age	**n**	$\bar{d}$	d_{max}	d_{min}	$\bar{h}$	h_{max}	h_{min}
4	30	8.60	14.50	3.90	5.28	7.40	3.10
5	63	12.80	21.00	3.70	7.65	10.20	3.37
6	89	13.43	23.00	2.90	8.37	12.70	4.08
7	60	15.19	23.10	3.90	9.77	12.80	5.10
8	30	16.25	26.00	4.00	12.41	16.25	5.17
9	30	19.66	32.00	7.70	13.98	16.80	7.90
10	59	19.69	32.50	6.00	14.71	17.80	8.30
11	30	19.60	31.70	8.20	14.81	18.80	9.22
12	39	23.53	35.50	12.50	18.57	22.60	14.90
14	29	23.95	36.92	13.37	18.88	21.70	15.80
16	23	22.99	37.56	15.60	19.45	22.30	16.20
18	37	28.07	48.06	15.92	23.38	30.20	17.30
21	65	35.00	49.97	21.65	28.98	33.00	21.50
26	13	26.98	43.90	17.50	27.69	29.50	25.00
27	12	29.31	38.83	21.96	28.87	32.80	25.40
30	12	35.83	51.80	22.60	29.23	32.00	27.30
31	10	32.60	44.20	25.10	31.08	35.00	25.00

Table 2. Taper models tested to estimate stem diameters inside and outside bark of *Pinus taeda* L. trees.

Tabela 2. Modelos de afilamento testados para estimar os diâmetros com e sem casca de árvores de *Pinus taeda* L.

Author	**Model**
Biging (1984)	$d_i = d\left[a + b\ \ln\left(1 - \left(\frac{h_i}{h}\right)^{\frac{1}{c}}\left(1 - e^{\frac{-a}{b}}\right)\right)\right]$
Kozak *et al.* (1969)	$d_i = d\ \left[a + b\left(\frac{h_i}{h}\right) + c\left(\frac{h_i}{h}\right)^2\right]^{\frac{1}{2}}$
Schöpfer (1966)	$d_i = d\ \left[a + b\left(\frac{h_i}{h}\right) + c\left(\frac{h_i}{h}\right)^2 + f\left(\frac{h_i}{h}\right)^3 + g\left(\frac{h_i}{h}\right)^4 + m\left(\frac{h_i}{h}\right)^5\right]$

d: diameter at breast height outside bark (cm); h: total height (m); d_i: diameter, at height h_i of the stem of the tree, inside and outside bark (cm); h_i: height at position i of the stem of the tree (m); a, b, c, f, g, m: coefficients to be estimated; ln: natural logarithm; $and\ e$: 2.718281829...

$$d_i = d\left[a + b\ \ln\left(1 - \left(\frac{h_i}{h}\right)^{\frac{1}{c}}\left(1 - e^{\frac{-a}{b}}\right)\right)\right] \text{ in which, } a = a_0 + a_1 t \tag{1}$$

$$d_i = d\left[a + b\ \ln\left(1 - \left(\frac{h_i}{h}\right)^{\frac{1}{c}}\left(1 - e^{\frac{-a}{b}}\right)\right)\right] \text{ in which, } b = b_0 + b_1 t \tag{2}$$

$$d_i = d\left[a + b\ \ln\left(1 - \left(\frac{h_i}{h}\right)^{\frac{1}{c}}\left(1 - e^{\frac{-a}{b}}\right)\right)\right] \text{ in which, } c = c_0 + c_1 t \tag{3}$$

In which: d_i, d, h_i, h, a, b, c: have been defined above; $a_0, a_1, b_0, b_1, c_0, c_1$: coefficients to be estimated; and t: age (year).

During the first phase, the best performance model was selected based on the following criteria: adjusted coefficient of determination (R^2_{aj}); relative standard error ($syx\%$); mean of differences (MD); and analysis of studentized residuals. A graphical analysis of the studentized residuals was performed on the estimated dependent variable ($\hat{Y}_i$), which was aimed at the uniform distribution of residuals with the absence of patterns and heteroscedasticity. Once heteroscedasticity of the model resulting from the adjustment by the ordinary least squares method was detected, the weighted least squares method was adjusted.

The weights tested included various combinations of explanatory variables of the model, based on the protocol described by Parresol (1993), which assumes that the variance of the residuals is an exponential function of multiple explanatory variables of the model, as well as their combinations. To verify whether the studentized residuals had normal distribution or not, a graphical representation of these increasingly ordered values on the theoretical quantiles of the normal distribution was constructed. Once the non-normal distribution of the studentized residuals was verified, the weighted least squares method was adjusted, with weights assigned to the studentized residuals exceeding the interval ± 2, based on the method described by Huber (1964) and recommended by Myers (1986).

The best performance model selected during this first phase, both in its simplest form and with variations of its linear age function coefficients, was submitted to validation using PRESS residuals ($e_{i,-i}$). The calculation of $e_{i,-i}$ was performed considering the set of data in which the observation "i" was taken from the sample, leaving "$n-1$" observations, which were used to estimate the coefficients of the models (Equation 4). This procedure continued until all observations were removed one by one, and thus the model was adjusted "n" times (MYERS, 1986).

(4) $$e_{i,-i} = Y_i - \hat{Y}_{i,-i}$$

In which: Y_i: value of the dependent variable for observation i; and $\hat{Y}_{i,-i}$: dependent variable estimated by the model when adjusted without observation i.

PRESS residuals are the residuals obtained by the estimation $\hat{Y}_{i,-i}$ and are independent of Y_i. Thus, the estimation Y_i was not used simultaneously in the adjustment and evaluation of the model, being, therefore, a true validation test (MYERS, 1986). For the final selection of the model, via the PRESS residuals, the following validation statistics were calculated: modeling efficiency (EM); average of the PRESS differences (MD_{PRESS}); average of the PRESS absolute differences (MAD_{PRESS}); and the 5% and 95% percentile values of the PRESS residuals.

The efficiency of modeling is a statistic that is analogous with the coefficient of determination and expresses the proportion of variance explained by the model (Equation 5). The average of the PRESS differences indicates the bias of the model (Equation 6), whereas the average absolute PRESS differences indicates the accuracy (Equation 7). The 5% and 95% percentile values express the magnitude and symmetry of the distribution of PRESS residuals. Finally, the model for best performance with its coefficients linear function of age was compared to its simplest form by means of the values of MD_{PRESS} and MAD_{PRESS} by age class for the different measurements taken from the tree height, i.e., 0%, 0.5%, 1%, 5%, 10%, 15%, 20%, 25%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, and 95%.

(5) $$EM = 1 - \left[\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_{i,-1}\right)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}\right]$$

(6) $$MD_{PRESS} = \frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_{i,-1}\right)}{n}$$

(7) $$MAD_{PRESS} = \frac{\sum_{i=1}^{n}\left|Y_i - \hat{Y}_{i,-1}\right|}{n}$$

In which: $Y_i, \hat{Y}_{i,-i}, n$: have been defined previously.

RESULTS

During the adjustment and selection phase of candidate models (Table 2), one observation with very high studentized error was found in all taper models inside and outside bark. Thus, this observation was considered atypical (an outlier) and was eliminated from the dataset utilized in the present study.

Among the models tested, the Schöpfer (1966) model displayed better adjustment (R^2_{aj}) and lower error ($syx\%$) when estimating the diameters outside bark along the stems of trees of *P. taeda*. Biging model (1984), however, displayed the lowest bias (MD) (Table 3). It was only possible to adjust the Kozak *et al.* (1969) model in its simplest formulation because the model did not converge with its coefficients linear function of age (Table 3).

To estimate the stem diameter outside bark, there was an improvement, although it was almost insignificant, in the R^2_{aj} and $syx\%$ criteria when the adjustments to the Schöpfer (1966) and Biging (1984) models were compared between varying their coefficients according to age with their simplest formulations (Table 3). Schöpfer (1966) and Biging (1984) models with coefficients linear function of age displayed greater bias (MD) when compared to their simplest formulations, apart from the Biging model (1984) varying the coefficient c depending on age.

To estimate the stem diameter inside bark, the Schöpfer (1966) model presented a small advantage over the Biging (1984) model for all selection criteria evaluated (Table 3). In particular, the Schöpfer (1966) model was better when coefficient b was varied, with better adjustment (R^2_{aj}) and smaller error ($syx\%$), when estimating the diameters inside bark in trees of $P.\ taeda$ and when varying the coefficient m, which displayed the smallest bias (MD) (Table 3). The worst performance was found in the Kozak *et al.* (1969) model, which fit only in its simplest formulation. There was a small advantage in the adjustment (R^2_{aj}) and precision ($syx\%$) of the Schöpfer (1966) and Biging (1984) models with their coefficients as a linear function of age. However, this advantage was not observed when analyzing the bias (MD) of the models (Table 3).

Table 3. Selection statistics of the stem taper models in *Pinus taeda* L. trees outside and inside bark.
Tabela 3. Estatísticas de seleção dos modelos de afilamento do tronco com e sem casca para árvores de *Pinus taeda* L.

Model			R^2_{aj}	$syx\%$	MD **(cm)**
Diameter outside bark	Biging (1984)	Simple	0.9765	11.04	-0.03024
		a	0.9765	11.03	-0.03252
		b	0.9798	10.22	0.08097
		c	0.9775	10.78	0.00608
	Kozak *et al.* (1969)	Simple	0.9724	11.95	0.13207
		a		No convergence	
		b		No convergence	
		c		No convergence	
	Schöpfer (1966)	Simple	0.9814	9.82	0.14307
		a	0.9814	9.81	0.15216
		b	0.9827	9.46	0.20722
		c	0.9824	9.55	0.18940
		f	0.9820	9.65	0.17607
		g	0.9818	9.71	0.16750
		m	0.9816	9.75	0.16182
Diameter inside bark	Biging (1984)	Simple	0.9775	11.28	-0.05764
		a	0.9779	11.17	-0.01113
		b	0.9800	10.63	0.04730
		c	0.9789	10.92	0.04273
	Kozak *et al.* (1969)	Simple	0.9719	12.61	-0.06940
		a		No convergence	
		b		No convergence	
		c		No convergence	
	Schöpfer (1966)	Simple	0.9796	10.73	-0.02030
		a	0.9804	10.53	0.05042
		b	0.9817	10.17	0.06770
		c	0.9811	10.34	0.04086
		f	0.9806	10.48	0.02369
		g	0.9803	10.57	0.01287
		m	0.9801	10.62	0.00573

In which: a, b, c, f, g, m: model coefficients linear function of age.

The distribution of the studentized residuals was very similar to the results obtained from the Schöpfer (1966) and Biging (1984) models for all formulations tested to estimate stem diameters of *P. taeda* inside and outside bark. Taking this into consideration, together with the selection criteria (Table 3), we chose to present only the results from the analysis of studentized residuals of the Schöpfer (1966) model (Figure 1), which was selected for the validation phase. When analyzing the dispersion of residuals of the Schöpfer (1966) model on the estimated stem diameters outside bark, there were a few negative diameter values estimated (Figure 1). This occurred for all variants of the Schöpfer (1966) model up to the total height of the tree ($h_i/h = 1$), in which the diameter observed was equal to zero. Moreover, there was a higher frequency of negative values of studentized residuals for the highest values of estimated diameters outside bark ($d'_{cc} > 35\ cm$), which might indicate a tendency of overestimation when applying this model. Finally, the ordered studentized residuals coincided with the theoretical quantiles of the normal distribution for all variants of the Schöpfer (1966) model after application of the Huber method (1964).

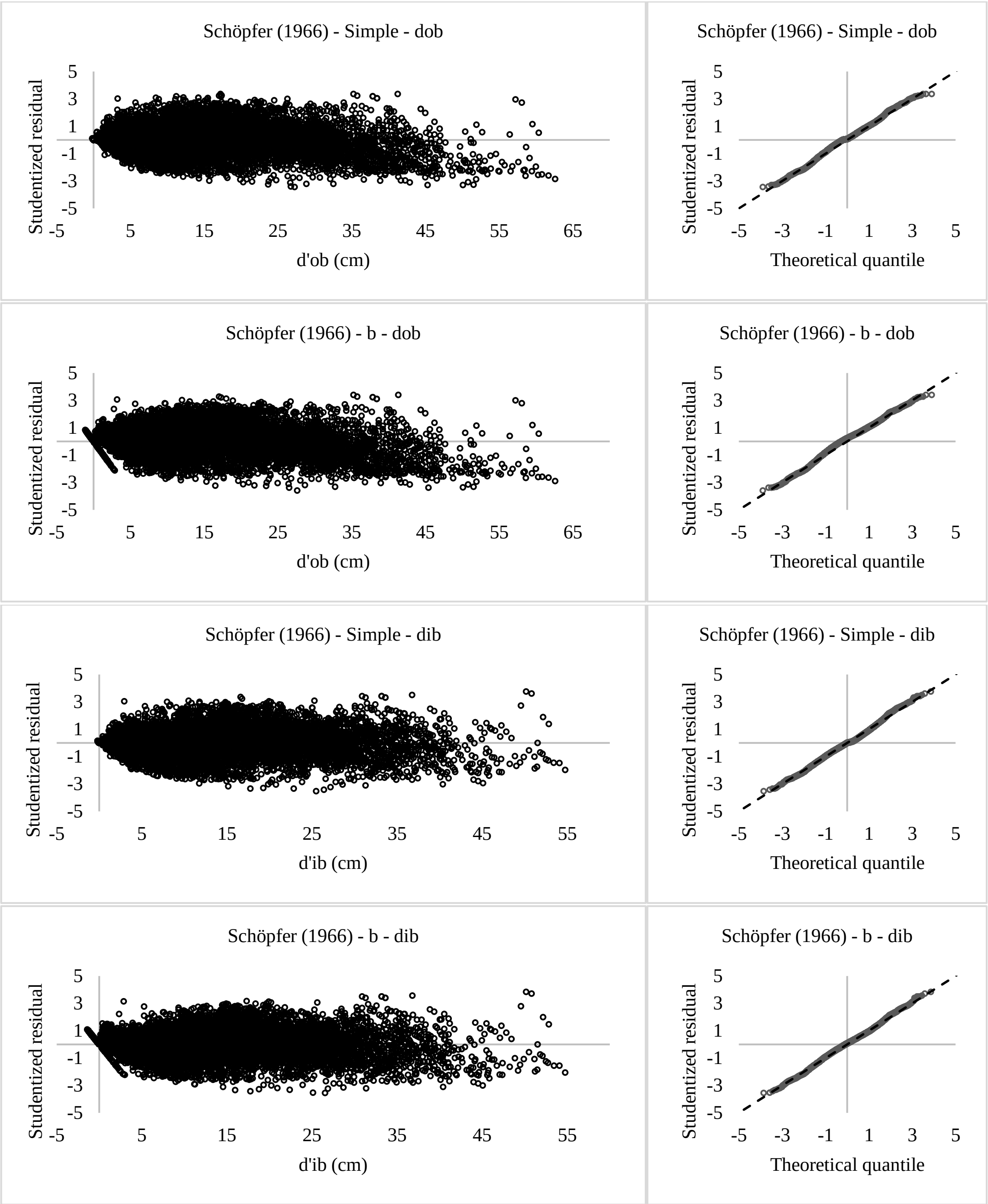

Figure 1. Residual analysis of the Schöpfer model (1966) in its simplest formulation and with variation in the coefficient *b* linear function of age to estimate stem diameters inside and outside bark of trees of *Pinus taeda* L.

Figura 1. Análise de resíduos do modelo de Schöpfer (1966) simples e com o coeficiente *b* função linear da idade para a estimativa de diâmetros com e sem casca de *Pinus taeda* L.

The dispersion of the studentized residuals on the estimated stem diameters inside bark was uniform. The studentized residuals presented normal distribution for all formulations of the Schöpfer (1966) model, after application of the Huber (1964) method (Figure 1). Similar to the stem diameter outside bark, there were negative values for the estimated stem diameters inside bark when applying the Schöpfer (1966) model varying the coefficient *b* according to age. However, the distribution of residuals was uniform (Figure 1).

Validation statistics indicated a greater modeling efficiency and accuracy of the estimated stem diameter inside and outside bark for the Schöpfer (1966) model with the coefficient *b* linear function of age. However, a lower bias was presented by the simple Schöpfer model (1966) and varying the coefficient m in the estimation of stem diameters outside and inside bark, respectively. The results for EM and MAD_{PRESS} were very similar for all variants of the Schöpfer (1966) model (Table 4).

Table 4. Statistics to validate the Schöpfer model (1966) to estimate stem diameters inside and outside bark of trees of *Pinus taeda* L.

Tabela 4. Estatísticas de validação do modelo de Schöpfer (1966) para estimar diâmetros com e sem casca de árvores de *Pinus taeda* L.

Model		Diameter outside bark				
		EM	MD_{PRESS}	MAD_{PRESS}	$P5$	$P95$
Schöpfer (1966)	Simple	0.9814	0.14252	1.08780	-2.34214	2.38289
	a	0.9814	0.15484	1.08934	-2.34649	2.38756
	b	0.9827	0.21294	1.08056	-2.23017	2.26697
	c	0.9824	0.19381	1.08827	-2.27679	2.28266
	f	0.9820	0.17909	1.09501	-2.31032	2.32247
	g	0.9818	0.16946	1.09749	-2.32558	2.34710
	m	0.9817	0.16300	1.09800	-2.33129	2.36514
Model		Diameter inside bark				
		EM	EM	MD_{PRESS}	MAD_{PRESS}	$P5$
Schöpfer (1966)	Simple	0.9796	-0.02388	1.05628	-2.26170	2.29914
	a	0.9804	0.05067	1.03924	-2.21405	2.31348
	b	0.9817	0.06923	1.02280	-2.14048	2.19222
	c	0.9811	0.04135	1.04074	-2.18212	2.19512
	f	0.9806	0.02315	1.05237	-2.22561	2.24037
	g	0.9803	0.01155	1.05798	-2.23874	2.25343
	m	0.9801	0.00379	1.06064	-2.24633	2.26424

In which: a, b, c, f, g, m: model coefficients linear function of age.

There were no clear trends in bias (MD_{PRESS}) for the two formulations of the Schöpfer (1966) model that were tested to estimate stem diameter outside bark for the different age classes. For several age classes, varying coefficient b in the Schöpfer (1966) model presented greater bias than that in its simplest formulation (Figure 2). As to the accuracy (MAD_{PRESS}) by age class, there were differences found in the results between the two formulations of the Schöpfer (1966) model tested. The behavior of the simple Schöpfer model (1966) and varying the coefficient b model was very similar with respect to accuracy and precision analyzed by relative height class (Figure 2). The behavior of the two formulations of the model was also very similar, for both the statistics MD_{PRESS} and MAD_{PRESS}, from 0% to 40% of the height of the tree. There was a slight superiority for the Schöpfer (1966) model varying the coefficient b for the positions of 50% to 90% and better performance of the simple Schöpfer (1966) model for positions of 95% and 100% (Figure 2).

Regarding the estimates of stem diameter inside bark, the Schöpfer (1966) model with variation of the coefficient b linear function of age presented lower bias for 11 of the 17 age classes analyzed (Figure 2). There were no significant differences in accuracy between the simple Schöpfer (1966) model and the model that varied the coefficient b for estimates of stem diameter inside bark in *P. taeda* at different ages (Figure 2). The behavior of the two formulations of the model for both the statistics MD_{PRESS} and MAD_{PRESS} was very similar from 0% to 30% of the height of the tree. There was a slight superiority of the Schöpfer (1966) model that varied coefficient b for the positions of 40% to 90% and better performance of the simple Schöpfer (1966) model for the positions of 95% and 100% (Figure 2).

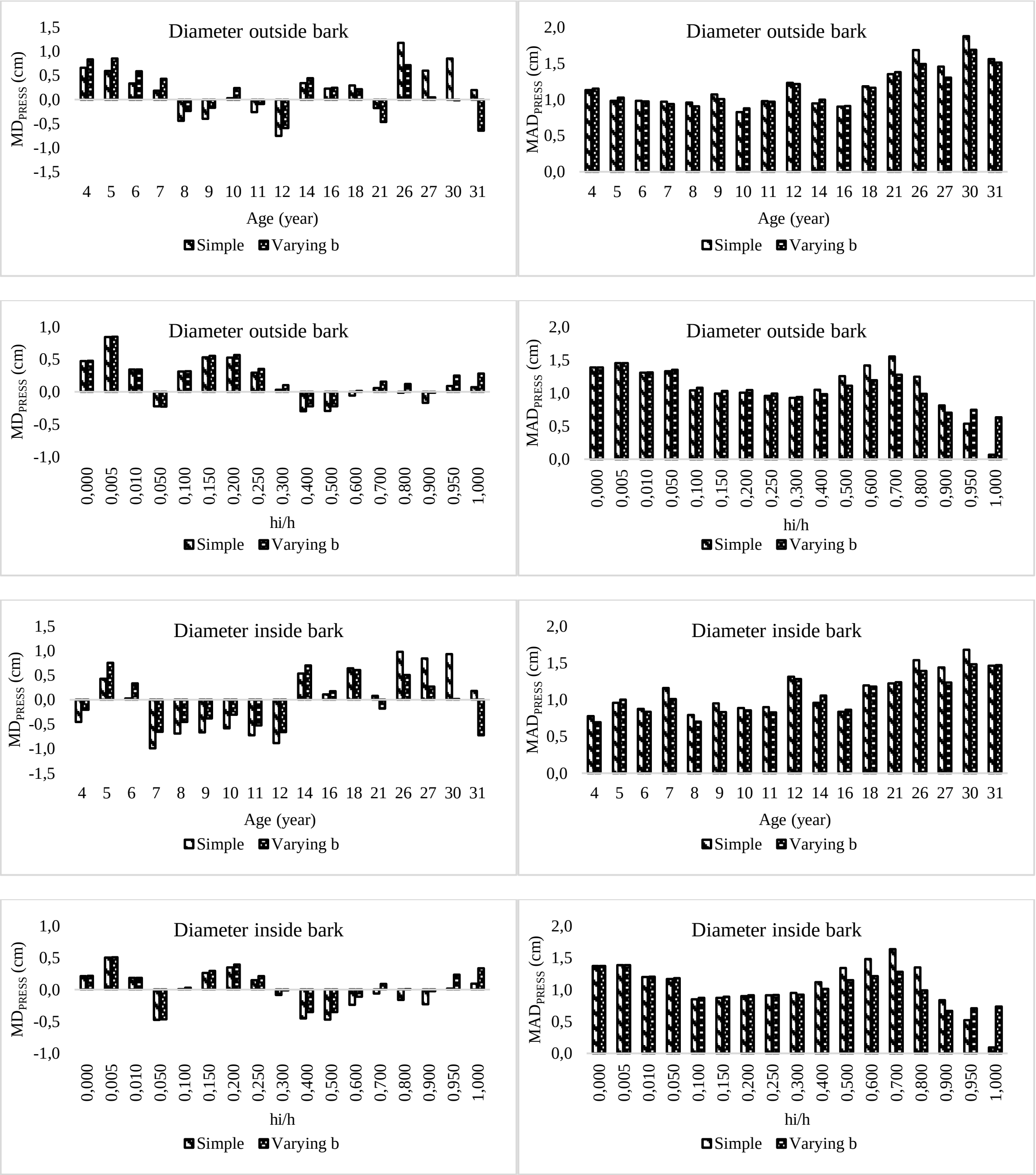

Figure 2. Means of PRESS residual and absolute PRESS residual of the Schöpfer (1966) model in its simplest formulation and with variation in the coefficient *b* linear function of age, by age and relative height class when estimating the stem diameter inside and outside bark of *Pinus taeda* L. trees.

Figura 2. Média das diferenças PRESS e média das diferenças absolutas PRESS do modelo de Schöpfer (1966) simples e com o coeficiente *b* função linear da idade, por classe de idade e altura relativa, para a estimativa de diâmetros com e sem casca de *Pinus taeda* L.

Based on the selection criteria and validation of the Schöpfer (1966) model, there was no advantage in selecting the more complex formulation to estimate the diameter of *P. taeda* stems inside and outside bark. Thus, the selected equations were derived from the simple Schöpfer (1966) model (Equations 8 and 9).

(8) $$d_{iob} = d\left[1.20813 - 3.58707\left(\frac{h_i}{h}\right) + 14.87828\left(\frac{h_i}{h}\right)^2 - 32.19722\left(\frac{h_i}{h}\right)^3 + 30.44277\left(\frac{h_i}{h}\right)^4 - 10.74801\left(\frac{h_i}{h}\right)^5\right]$$

$$(9) \qquad d_{iib} = d\left[1.05712 - 2.73771\left(\frac{h_i}{h}\right) + 11.71706\left(\frac{h_i}{h}\right)^2 - 25.43124\left(\frac{h_i}{h}\right)^3 + 23.40369\left(\frac{h_i}{h}\right)^4 - 8.01307\left(\frac{h_i}{h}\right)^5\right]$$

In which: d_{iob}: diameter outside bark at height h_i of the tree stem (cm); d_{iic}: diameter inside bark at height h_i of the tree stem (cm); and d, h_i, h: have been defined previously.

The profiles of the average stem diameter inside and outside bark, generated respectively by Equations 8 and 9, are presented in Figure 3. These profiles represent the shape of *P. taeda* stems, aged 4 to 31 years, in the Midwest region of the state of Santa Catarina, Brazil.

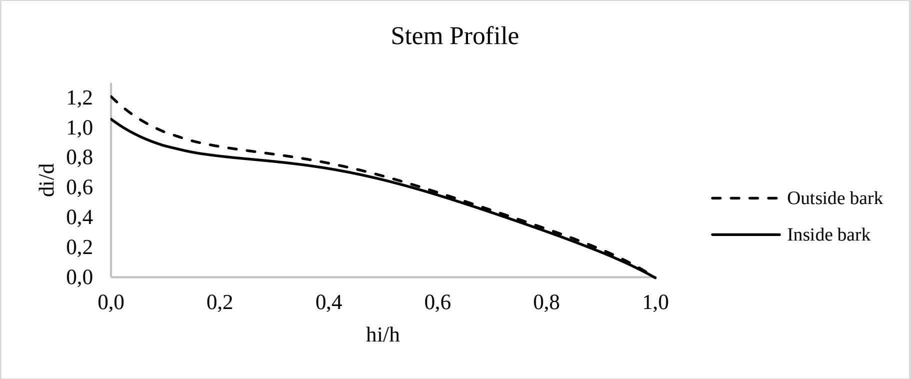

Figure 3. Profiles of stem diameters inside and outside bark produced by the Schöpfer (1966) model for trees of *Pinus taeda* L. in the Midwest region of Santa Catarina State, Brazil.

Figura 3. Perfil do fuste com e sem casca, gerado pelo modelo de Schöpfer (1966) para árvores de *Pinus taeda* L., na região Meio Oeste de Santa Catarina.

DISCUSSION

The values of R_{aj}^2 and $syx\%$ for the stem taper equations (Equations 8 and 9) selected in the present study are very similar to those found by Kohler *et al.* (2013) for the Schöpfer (1966) model adjusted for *P. taeda* that were 10 to 23 years of age from the state of Santa Catarina. The values of $syx\%$ in the stem taper functions selected in the present study are slightly higher than those founded by Téo *et al.* (2013) and Kohler *et al.* (2013) for the Schöpfer (1966) model adjusted by age class for *P. elliottii* Engelm. and *P. taeda*, respectively. Except for MD_{PRESS} for the stem diameter outside bark (Equation 8) in the 26 years age class, the statistics MD and MD_{PRESS} of the stem taper functions (Equations 8 and 9) did not exceed the limit ± 1, which according to Souza *et al.* (2008) is desirable.

Several authors who have undertaken studies with tree taper modeling of different aged trees have recommended the use of a taper equation for each age class owing to better statistical performance of these equations when compared to a general equation (KOHLER *et al.*, 2013; FIGUEIREDO FILHO *et al.*, 2015; KOHLER *et al.*, 2016; TERRA *et al.*, 2018). However, there is an inconvenience associated with using several taper equations simultaneously that was avoided in the modeling approach used in the present study.

Similar to the modeling used in the present study, Tomé *et al.* (2007) tested several taper models in their simplest formulation and with coefficients as a linear function of climatic and management regimes variables for estimates of diameters inside and outside bark in *E. globulus* trees from Portugal. At the end of the analysis, these authors recommended the Biging (1984) model in its simplest formulation be utilized. The Schöpfer (1966) model, however, was not tested by Tomé *et al.* (2007).

The stem taper equation generated by Schöpfer (1966) model varying coefficient *b* based on age allowed the representation of different stem profiles according to age. This characteristic is extremely desirable, given that the shape of the stem varies according to age (KOHLER *et al.*, 2013; TÉO *et al.*, 2013; FIGUEIREDO FILHO *et al.*, 2014a, 2015; KOHLER *et al.*, 2016). Results from the Schöpfer (1966) model adjusted with the coefficient *b* linear function of age showed that stem profiles presented lower tapering with higher ages, from approximately 15% of height, which is in agreement with the results found by Téo *et al.* (2013) in a study of *P. elliottii* stems in the state of Santa Catarina.

Nevertheless, as only one of the coefficients was a linear function of age each time, the changes were concentrated in only a portion of the stem profile and the remainder remained unchanged according to age. This resulted in lower precision and accuracy of the estimates of diameters inside and outside bark in the uppermost portion of the stem (95% and 100%) (Figure 2) and estimates of negative diameters for the lower ages and positive for the higher ages at 100% of height, which is evidenced by the pattern close to the origin of the studentized residuals graph (Figure 1).

CONCLUSIONS

- Among all the taper models tested, the Schöpfer (1966) model presented higher precision and accuracy of estimates for stem diameters inside and outside bark of trees of *P. taeda* from the Midwest region of the state of Santa Catarina, Brazil.
- The adjusted Schöpfer (1966) model with coefficients linear function of age did not present any pronounced improvement in the precision and accuracy of the estimates of stem diameters inside and outside bark when compared to its simplest formulation.

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