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Double extractor induction motor: Variational calculation using the Hamilton-Jacobi-Bellman formalism

Motor de inducción de doble extractor: Cálculo variacional usando el formalismo Hamilton-Jacobi-Bellman

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Abstract

This contribution presents optimal control over a double extractor induction motor using formalism through variational model. The criterion is subject to non-stationary equations of a reduced order (Dynamics equations of a reduced order model (DSIM)). As is well know, in this model the state variables are the rotor flow and motor speed in a circuit mechanical process. For non-stationary and stationary states, based on the theory of optimal control, this limit provides a high expensive function given as a weighted contribution of a DSIM theory. To order to acquire a lowest energy rotor flow path, the idea is to minimize the function to a dynamic of two equations of the motor speed and rotor flow. This problem is solved using with the Hamilton-Jacobi-Bellman equation and a time dependent solution for the rotor flow is determined analytically.

Keywords: dynamic programmic; optimal control theory; cost function; subderivates.

Resumen

Esta contribución presenta un control óptimo sobre un motor de inducción de doble extractor usando formalismo a través del modelo variacional. El criterio está sujeto a ecuaciones no-estacionarias de orden reducido (Ecuaciones Dinámicas de un Modelo de Orden Reducido (DSIM)). Como es bien sabido, en este modelo las variables de estado son el flujo del rotor y la velocidad del motor en un proceso mecánico de circuito. Para estados no-estacionario y estacionarios, basados en la teoría del control óptimo, este límite proporciona una función costosa dada como una contribución ponderada de una teoría DSIM. Para adquirir una ruta de flujo de rotor de energía más baja, la idea es minimizar la función a una dinámica de dos ecuaciones de la velocidad del motor y el flujo del rotor. Este problema se resuelve utilizando la ecuación de Hamilton-Jacobi-Bellman y se determina analíticamente una solución dependiente del tiempo para el flujo del rotor.

Palabras clave: programación dinámica; teoría optima de control; función de costo; subderivadas.

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1. Introduction

Variational problems have stimulated, in the last decade, theoretical and analytical research in several topics in mathematical physics. The variational model has disadvantages when the method is applied for analysis in high order plants. However, unrestricted design may not work properly in practice. Due to the limited variables and the restricted control could cause serious performance deterioration. The controllers must be synthesized to achieve the desired one. A great attempt was made in non-linear minimization and development on this system attracting a big attention of the scientific community. The objective is to design a controller for a type of systems with limitations on states and control. The model gives a practical model in which the designer can resume the limited controllers to get the goals of the synthesis below is the Jacobi-Bellman equation [6].

Let a systems be simulated by the equation:

$$\dot{x} = a(x(t), u(t), t) \tag{1}$$

The measure to be minimized is:

$$J = h(x(t_f), t_f) + \int_{t_0}^{T} g(x(\tau), u(\tau), \tau) d\tau \tag{2}$$

h and g are specified functions, t_0 and t_f are bounded and τ is a phantom integration variable now it is considered longer problems and subdividing the interval is:

$$J^*(x(t), t) = \int_{t}^{t+\Delta t} g(\tau) d\tau + \int_{t+\Delta t}^{t_f} g(\tau) d\tau + h(x(t_f), t_f) \tag{3}$$

The principle of optimality is:

$$J^*(x(t), t) = \min \{+ \int_{t}^{t+\Delta t} g(\tau) d\tau + J^*(x(t + \Delta t), t + \Delta t\} \tag{4}$$

Where $J^*(x(t + \Delta t), t + \Delta t$ is the minimum process cost for a time interval $t + \Delta t \leq \tau \leq t_f$ with an initial state x(t + Δt). Assuming that the second partial derivative of J^* exists and that it is limited $J^*(x(t + \Delta t), t + \Delta t$ can be expanded, $t + \Delta t$ in Taylor series around one point $(x\ (t), t)$ is obtained.

$$J^*(x(t), t) = \min\left\{+ \int_{t}^{t+\Delta t} g(\tau) d\tau + J^*(x(t), t) + \left[\frac{\partial J^*}{\partial x}(x(t), t)\right] \Delta t + \left[\frac{\partial J^*}{\partial x}(x(t), t)\right]^T [x(t + \Delta t - x(t)]\right\} + o(\Delta t) \tag{5}$$

Now for little ones Δt

$$J^* = (x(t), t) = \min \{g(x(t), u(t), t)\Delta t + J^*(x(t), t) + J_t{}^*(x(t), t)\Delta t + J_x{}^*(x(t), t)[a(x(t), u(t), t]\Delta t + O(\Delta t)\} \tag{6}$$

Where $O(\Delta t)$ denotes the terms contained $[\Delta t]^2$ of the highest order of Δt that arise from the approximation of the integral and the truncation produced from the expansion in Taylor series, now removing the terms $J^*(x\ (t), t)$ and $J_t{}^*(x(t), t)$.

Minimization is obtained:

$$J_t{}^*(x(t), t) \min \{g(x(t), u(t), t)\Delta t + J^*(x(t), t) + J_x{}^*(x(t), t)[a(x(t), u(t), t]\Delta t + o(\Delta t)\} = 0 \tag{7}$$

Dividing by Δt taking the limit when $\Delta t \to 0$ gives:

$$0 = J_t{}^*(x(t), t) + \min \{g(x(t), u(t), t) + J^*(x(t), t) + J_x{}^*(x(t), t)[a(x(t), u(t), t]\ \} \tag{8}$$

To find the limits for this differential equation let $t = t_f$ give:

$$J^*(x(t_f), t_f) = h(x(t_f), t_f) \tag{9}$$

The Hamiltonian is defined as:

$$(\mathrm{x(t), u(t\,)}, J_x{}^t, t) = \mathrm{g(x(t), u(t), t)} + J_x{}^*(x(t), t)[a(x(t), u(t), t] \tag{10}$$

$$(\mathrm{x(t), u(t\,)}, J_x{}^*, t) = \min(\mathrm{x(t), u(t)}, J_x{}^*, \mathrm{t}) \tag{11}$$

Since the minimization of the control will depend on $x, J_x{}^*$, and t using those definitions the Hamilton-Jacobi equation is obtained:

$$0 = J_t{}^*(x(t), t) + (\mathrm{x\ (t)}, u^*\mathrm{(t)}, J_x{}^*, \mathrm{t}) \tag{12}$$

This equation is the analog in continuous time to the Bellman's recurrence equation, however it will refer to (12) as the Hamilton-Jacobi- Bellman equation [7].

2. Application problem

The complete dynamic model of the DSIM is [1]:

$$\begin{cases} \dot{I}_{s(d,q)} = -(\gamma I + (\dot{\rho} + p\Omega)J)I_{s(d,q)} \\ \dot{\varphi}_{r(d,q)} = -(aI + \dot{\rho}J)\varphi_{r(d,q)} + bI \\ \dot{\Omega} = -\frac{K_l}{J_m} + \frac{Y}{J_m} \end{cases} \quad (13)$$

Where:

$$I_{s(d,q)} = \begin{pmatrix} I_{s1d} & I_{s2d} \\ I_{s1q} & I_{s2q} \end{pmatrix} = \begin{pmatrix} I_{sd} \\ I_{sq} \end{pmatrix}; \quad v_{s(d,q)} = \begin{pmatrix} v_{s1d} & v_{s2d} \\ v_{s1q} & v_{s2q} \end{pmatrix} = \begin{pmatrix} v_{sd} \\ v_{sq} \end{pmatrix}; \ \varphi_{r(d,q)} = \begin{pmatrix} \varphi_{rd} \\ \varphi_{rq} \end{pmatrix}; \ \varphi_1 = 1 - (\frac{M^2}{L_s L_r})$$

$$\varphi_2 = 1 - (\frac{M^2}{L_s L_{sp}}); \ \gamma = \frac{1}{\varphi_1 L_s + \varphi_2 L_{sp} +}(R_s + \frac{M^2}{L_r^2}R_r); \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \ J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; \ a = \frac{R_r}{R_b}; \ b = aM;$$

By notation $W_s = \dot{\rho}$ is the frequency of the motor, R_s and R_r are the stator and rotor resistors respectively $L_s \ y \ L_r$, are the inductances of the stator and rotor respectively. M is the magnetization, L_{sp} is the main cyclic inductance, J_m is the moment of inertia of the rotor, K_l is the constant torque load, $L_{s1d}, L_{s2d}, L_{s1q}$ y L_{s2q} are the direct and quadrature current of stator 1 and stator 2 respectively. $v_{s1d}, v_{s2d}, v_{s1q}$ y v_{s2q} are respectively the voltages of each stator on the dq axes, p is the number of poles, γ is the electromagnetic torque [2].

To eliminate the non-linear terms of (13), the order of the DSIM model of an integral proportional control (PI) is reduced to a simple one with proportional gain and the following is defined:

$$v_{s(d,q)} = \frac{\sigma_1 L_s + \sigma_2 L_{sp}}{\epsilon} \quad (13.1)$$

$$U = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} I_{s1d} & I_{s2d} \\ I_{s1q} & I_{s2q} \end{pmatrix} \quad (13.2)$$

Where $0 < \epsilon < 1$ and U is a system command, the usual form of reduced model is obtained as follows: the cost of the function can be defined as:

$$J = \int_0^T f(I_{sd}, I_{sq}, \varphi_r, \Omega)\mathrm{dt} \quad (14)$$

The index corresponding to the weighted sum is

$$f(I_{sd}, I_{sq}, \varphi_r, \Omega) = \beta_1 \omega_L + \beta_2 P_J + \beta_3 P_m \quad (15)$$

The factors $\beta_1, \beta_2, \beta_3$, are weighted and are used to scale the energy power while minimizing the cost function minimizes the stored magnetic energy and minimize of losses in the winding increasing the efficiency of the machine [3]. The power in the rotating frame (d-q) is:

$$P_a = \frac{3}{2}(v_{s(d,q)})^t I_{s(d,q)} \quad (16)$$

The system (13) has that the input power is given by:

$$P_a = \frac{3}{2}(\sigma_1 L_s + \sigma_2 L_{sp})((\dot{I}_{s(d,q)})^t (I_{s(d,q)})) + \gamma((\dot{I}_{s(d,q)})^t (I_{s(d,q)})) - 2\eta((\varphi_{r(d,q)})^t (I_{s(d,q)})) + \eta P\Omega(\varphi_{r(d,q)})^t J I_{s(d,q)} \quad (17)$$

The relationship between the stator and rotor current is given by:

$$I_{r(d,q)} = \frac{1}{L_r}(\varphi_{r(d,q)} - L_s I_{s(d,q))}) \quad (18)$$

The active power is:

$$P_a = \frac{\partial}{\partial t}W + P_j + P_m \quad (19)$$

The equation (17) is defined as the derivative of the stored magnetic field is given as follows:

$$\frac{\partial W}{\partial t} = \frac{3}{2}(\left(\frac{\sigma_1 L_s + \sigma_2 L_{sp}}{2}\right)(u_1^2 + u_2^2) + \frac{1}{2L_r}\varphi_r^2) \quad (20)$$

Joule's losses are given by:

$$P_J = \frac{3}{2}[(R_s(\dot{I}_{s(d,q)})^t I_{s(d,q)} + R_r(\dot{I}_{s(d,q)})^t (I_{r(d,q)})] \quad (21)$$

By using equation (13) and equation (21), Power losses can be as:

$$P_J = \frac{3}{2}((R_s + R_r\left(\frac{M}{L_r}\right)^2 (u_1^2 + u_2^2) - \frac{3}{2}\frac{R_r}{L_r^2}\varphi_r^2) \quad (22)$$

In this document, the operations of the mechanical process are restricted to Acceleration modes to limit the transitional study. Then the engine speed can be expressed as follows:

$$\Omega^* = c_0 t + c_1 \quad (23)$$

With $c_0 > 0$.

2.1 Determination of the Hamilton-Jacobi-Bellman equation

To increase the readability of subsequent equations it will be denoted $x_1 = \varphi_r$ and $x_2 = \Omega$.
The dynamics of the system and the cost of the function are defined as:

$$J_r = \frac{\beta_2}{L_r}\left(\varphi_r{}^2(0) - \varphi_r{}^2(T) + \int_0^T (r_1 u_1{}^2 + r_2 u_2{}^2 + q_1 x_1{}^2 + q_2 \varphi_r u_2 x_2\right) dt \qquad (24)$$

And

$$\begin{cases} \dot{x}_1 = a x_1 + b u_1 \\ \dot{x}_2 = \frac{K_l}{J_m} x_2 + \frac{c u_2 x_1}{J_m} \end{cases} \qquad (25)$$

Therefore, the function $v\,(T, x_1)$, which necessarily satisfies the boundary condition $v\,(T, x_1), = k\,(x_1(t))$, where $k\,(x_1(t))$ is the final state:

$$k(x_1(t)) = \frac{\beta_2}{L_r}\,(\varphi^2{}_r(0) - \varphi^2{}_r(T)) = \frac{\beta_2}{L_r}(x^2{}_1(0) - x^2{}_1(T)) \qquad (26)$$

The optimal co-state $\lambda^*(t)$ corresponds to the gradient of the cost function to be optimized:

$$\lambda^*(t) = \frac{\partial V(T, x_1{}^*(t))}{\partial x_1{}^*} \qquad (27)$$

Where $\frac{\partial V(T, x_1{}^*(t))}{\partial x_1{}^*}$ is continuously differentiable with respect to x_1 and $x_1 = x_1{}^*$ along the admissible path $u_1(.)$ and $u_1{}^*(.)$ corresponding sub-optimal cost function is:

$$(\mathrm{v}(\mathrm{t}, x_1{}^*)) = k(x_1(t)) + \int_t^T L(x(t), u(t), t)dt \qquad (28)$$

It is given as:

$$\mathrm{v}(\mathrm{t}, x_1{}^*) = \frac{\beta_2}{L_r}(x^2{}_1(0) - x^2{}_1(T)) + \int_0^T (r_1 u_1{}^2 + r_2 u_2{}^2 + q_1 x_1{}^2 + q_2 \varphi_r u_2 x_2) dt \qquad (29)$$

According to the following differential equation:

$$\frac{v(t,x_1)}{dx_1} = \frac{v(t,x_1)}{dx_1} x_1 + \frac{v(t,x_1)}{dx_1} = \lambda(-a x_1 + b u_1)$$
$$\frac{\partial v(t, x_1)}{\partial t} = L(x_1(t), u_1(t), t\,) \qquad (30)$$

Thus:

$$\frac{\partial v(t, x_1)}{\partial t} = -\lambda(-a x_1 + b u_1) - L(x_1(t), u_1(t), t) = -\,\mathrm{H}(\mathrm{t}, x_1, u_1, \lambda) \qquad (31)$$

This is an equation for the optimal cost function, which is called (HJB) [4] [5]. So, we found an optimum for the variable $u^*(t, x_1)$, with the subsequent state variable $x_1{}^*(t)$, considering of the following equation.

$$v(t, \Phi_r{}^*) = min \frac{x}{L_r}\left(\Phi_r{}^2(0) - \Phi_r{}^2(T)\right) + \int_t^T (r_1 u_1{}^2 + r_2 u_2{}^2 + q_1 \Phi_r{}^2 + q_2 \Phi_r u_2 \Omega)\,\partial\tau \qquad (32)$$

Which transfers the initial state $\Phi_r(0) = \Phi_{r0}$ and a final state $x_1(0) = \Phi_r(T)$ with the limit:

$$x_1 = -a\dot{x}_1 + b u_1 \qquad (33)$$

This determines the HJB equation:

$$H(t, x_1, u_1) = r_1 u_1{}^2 + r_1 \frac{1}{x^2}(At + B)^2 + q_1 x_1{}^2 + q_2(At + B)(c_0 t + c_1) + \frac{\partial v(t, x_1)}{\partial x_1} - (a x_1 + u_1) \qquad (34)$$

The model described in (34) was simulated numerically and the results are shown below.

4. Conclusions

We present an optimal control over a double stator induction motor using an energy minimization model via a DSIM model. For to obtain a minimum energy rotor flow path, the process is to minimize this limited function to a no-static of two equations set of the rotor flow and motor speed. The optimum control system is analyzed by the HJB equation and the rotor flow solution is determined in a mathematical manner which varies over time.

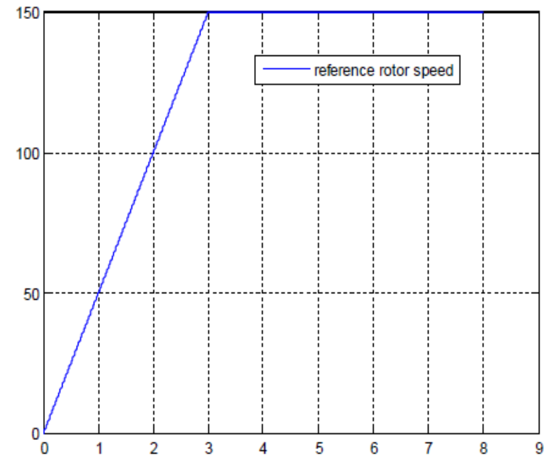

Figure 1. Rotor speed reference

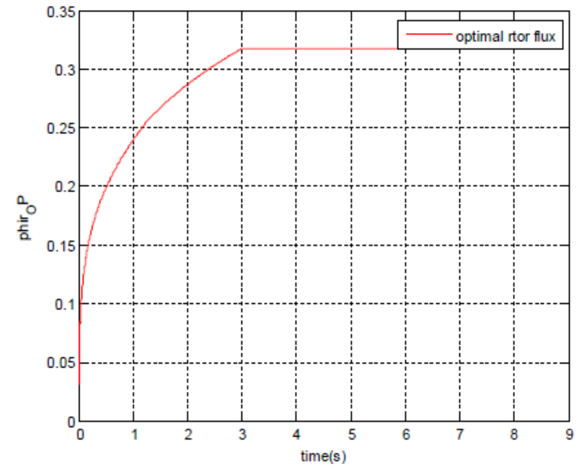

Figure 2. Optimum rotor flow

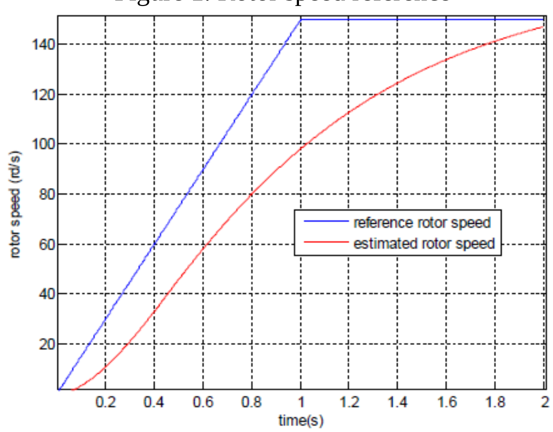

Figure 3. Engine reference speed estimated speed

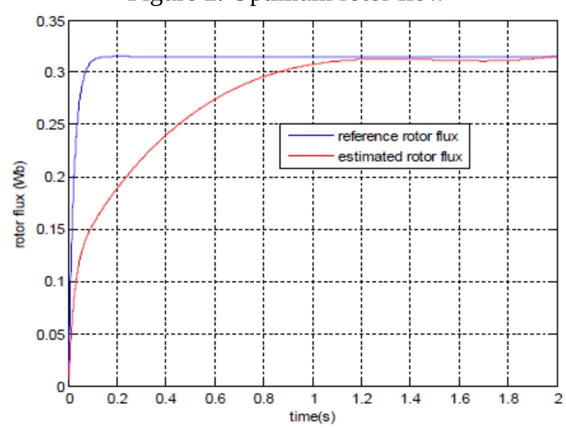

Figure 4. Reference flow and estimated flow

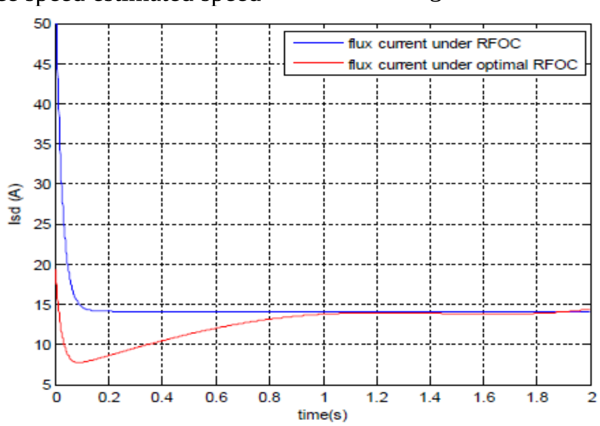

Figure 5. Reference current and current under control

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