Vietnam Journal of Science and Technology ***58*** *(3) (2020)*

doi:10.15625/2525-2518/58/3/14394

CREATING FATIGUE CURVE FOR STEEL MACHINE ELEMENTS USING FATIGUE TEST METHOD WITH GRADUALLY INCREASING STRESS AMPLITUDE

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Received: 4 September 2019; Accepted for publication: 30 March 2020

Abstract. Fatigue curve presents the relation between stress (mean stress, maximum stress or stress amplitude) and the number of stress cycles till a machine element is completely broken. This curve is served as the important basis of design as well as lifetime prediction for machine elements. In order to create a fatigue curve, the traditional fatigue test method is applied to specimens using a cyclic stress with constant amplitude. However, this method has disadvantages such as the experimental results could not be used because the specimens break before reaching the expected stress amplitude, or the tests may be stopped before the specimens break because of limitation of time. To overcome this hurdle of the traditional method, an experimental method using cyclic stress with gradually increasing amplitude was proposed to build the fatigue curve for steel machine elements. A comparison of the estimated fatigue curve and experimental data was performed showing that the fatigue curve of machine elements bearing the cyclic stress with constant amplitude can be created by applying the fatigue test method with gradually increasing stress amplitude.

Keywords: fatigue curve, fatigue test, stress, lifetime, gradually increasing amplitude.

Classification numbers: 2.9.1, 5.5.1, 5.4.6.

1. INTRODUCTION

Fatigue fracture was found around the middle of the 19th century and is considered as a norm in design of machines as well as machine elements. It is a material failure that occurs as a result of excessive cyclic loading [1, 2]. Under cyclic loading, micro cracks such as defects on the element surface induced by producing process grow gradually in each cycle until the critical crack length to be reached and the element is broken. Reality shows that 90 % of machine elements are broken by fatigue cracks [3], therefore it is necessary to calculate to prevent it or predict fatigue lifetime of the machine elements. This task was based on the fatigue curve which shows the relation between stress (mean stress, maximum stress or stress amplitude) and number of stress cycles till a machine element has completely broken. The fatigue curve is also named as S-N curve where S is usually the stress amplitude and N is the number of cycles to failure. The fatigue process determining the lifetime was known to be described by the Paris' law, where the cracks extends from the initial cracks to the critical cracks [4]. The equivalent lengths of the cracks were calculated by using the distribution of the initial strength [4,5]. The unknown parameters in the Paris' law are obtained by fitting the equation describing the S-N curve to the results of fatigue tests.

In order to create the S-N curve, fatigue tests are used where cyclic stress with a constant stress amplitude is applied to specimens until failure occurs [6]. This traditional experimental method is called normal fatigue test [7]. The normal fatigue tests have a disadvantage that in the case of the cycle number exceeds an expected time limit presented by Z in Fig. 1(a), the tests may have to be stopped before failure. In addition, it is difficult to evaluate fatigue behavior with the normal fatigue test when the specimens break before reaching the expected stress amplitude as indicated with X or Y. This situation can be seen at high stress amplitude level or when the defects on the specimens are inhomogeneous.

This paper presents an experimental method named as ramping fatigue test [7]. It was first used by Huy *et. al.* to estimate the fatigue lifetime [7] and by Ikeda *et.al.* to investigate the fatigue behavior under inert environment [8] of silicon specimens with the size at micro-scale. It is used here to build the fatigue curve of steel machine elements, where cyclic stress with gradually increasing amplitude as shown in Fig. 1(b) is applied to specimens. In this method, the stress amplitude σ linearly increases with the number of cycles N and $\Delta\sigma$ is the increment of the stress amplitude per cycle. This method avoids the situations in the normal fatigue tests, in which the specimens break before reaching the expected stress amplitude, because the stress amplitude gradually increases in all the test periods. The small stress amplitude at the beginning gives fatigue degradation of strength and the large stress at the final stage makes sure all the specimens break within a planned period of time. Therefore, the ramping test method avoids the disadvantages of the normal fatigue test.

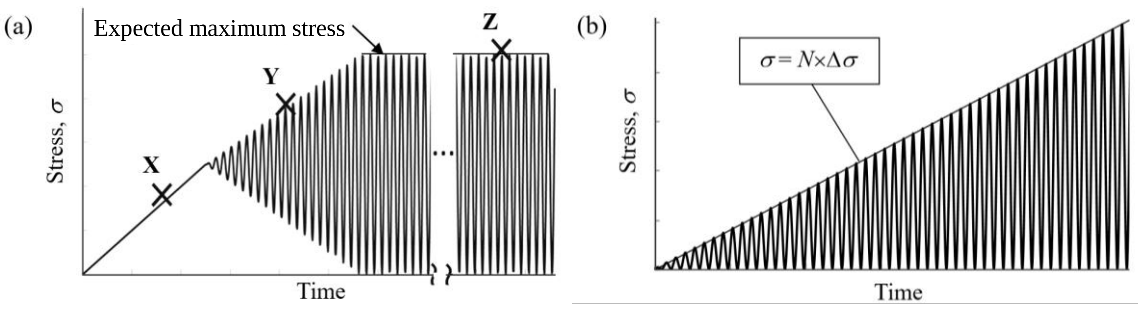


Figure 1. Imagine of the stress histories in the (a) normal and (b) ramping fatigue tests.

2. STATISTICAL ANALYSIS OF TRADITIONAL EXPERIMENTAL METHOD

It is commonly accepted that the strength of materials such as steel is well described by the Weibull distribution [9, 10]. In this paper, it is assumed that the initiation of fracture distributes on the machined surfaces of machine elements or specimens and cracks open in mode I. Therefore, the cumulative fracture probability F of the machined surfaces of a specimen with the nonuniform stress distribution is defined in general form as [9]

$$F = 1 - \exp\left[-\int_{A_0}\left(\frac{\sigma_{\rm a}}{\sigma_0}\right)^m \frac{dA}{A_0}\right], \tag{1}$$

where m denotes the Weibull modulus which represents the data scatter, σ_0 denotes the scale parameter related to the average strength of the infinitesimal volume dV, and σ_a is the applied stress. The symbol V_0 indicates the volume of the entire machined part of specimens. For the case of flat specimens, the stress distribution in the thickness direction is homogeneous, Eq. (1) can be rewritten as

$$F = 1 - \exp\left[-\int_{A_0}\left(\frac{\sigma_{\rm a}}{\sigma_0}\right)^m \frac{dA}{A_0}\right], \tag{2}$$

where the symbol A_0 indicates the volume of the entire machined surfaces of specimens.

If the distribution of the stress σ_a applied to the specimens with arbitrary shape is obtained by the tests then Eqs. (1) and (2) can be used. For the calculation in this study, Eq. (2) is rewritten in the discretized form as

$$F = 1 - \exp\left[-\sum_e\left(\frac{A_e}{A_0}\right)\left(\frac{\sigma_e}{\sigma_0}\right)^m\right], \tag{3}$$

where A_e is the area and σ_e is the average stress in each surface element as illustrated in Fig. 2. It is imagined that a specimen's machined surface is composed of the small elements, where the stress σ_e in each element is uniform. The stress σ_e is assumed by the linear elastic deformation to be correlated to the maximum stress σ in the specimen by the ratio $k_e = \sigma_e/\sigma$. The stress distribution on the sepcimens and therefore the ratio k_e can be estimated by finite element method (FEM). Besides, the ratio of the area of the surface element A_e to the area A_0 is notated as γ. By replacing the stress σ_e and the area A_e in Eq. (3) with the notations k_e and γ, respectively, it obtains a function of a variable σ as

$$F = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m \sum_e \gamma k_e^m\right]. \tag{4}$$

It was known that both the static strength as well as the fatigue lifetime are correlated to the same initial defects, which are engendered by the machined process. The defects were described as equivalent cracks on the machined surfaces as depicted in Fig. 2. The cyclic loads applied on the specimens in the fatigue tests were sinusoidal, in which the stress amplitude is smaller than the static strength. This leads to that the specimens broke after a number of load cycles N which is called as the fatigue lifetime of the specimens. In the applying load process, equivalent cracks in an element propagate from their initial length a_{0e} to the critical length a_c. The extension rate of the equivalent crack under cyclic loading, named the crack growth rate da/dN, is formulated by Paris' law [1-3] in the form normalized by fracture toughness K_{Ic} as

$$\frac{da}{dN} = C\left(\frac{\Delta K}{K_{\rm Ic}}\right)^n, \tag{5}$$

where C, n are the unknown parameters in Paris' law, which need to be determined from experiments, and ΔK is the amplitude of the stress intensity factor. Since σ_e is the stress amplitude applied in each element, ΔK for an element under mode I is defined as [2]

$$\Delta K = \beta\sigma_e\sqrt{\pi a_e}\ , \tag{6}$$

where a_e is the equivalent crack length at the cycle N, β is the dimensionless constant of a correction factor reflecting the geometry of both the cracks and the structures. It is expected that the stress intensity factor at the tip of the critical crack is equal to the toughness K_{Ic}, therefore the equivalent length of the critical crack is formulated as

$$a_c = \left(\frac{K_{Ic}}{\beta \sigma_e \sqrt{\pi}} \right)^2 . \tag{7}$$

By substituting Eq. (6) into Eq. (5) and then integrating it corresponding a from a_{0e} to a_c and N from zero to the number of cycles N that the critical crack length is reached, the initial crack length a_{0e} is obtained as

$$a_{0e} = \left(\frac{K_{Ic}}{\beta \sigma k_e \sqrt{\pi}} \right)^2 \left[1 + \frac{C(n-2)}{2} \left(\frac{\beta \sigma k_e \sqrt{\pi}}{K_{Ic}} \right)^2 N \right]^{2/(2-n)} \tag{8}$$

By using the correlation of stress to crack length $\sigma = K_{Ic}/\beta(\pi a)^{1/2}$, the cumulative probability F in Eq. (2) is rewritten in terms of crack length as

$$F = 1 - \exp\left[-\sum_e \gamma \left(\frac{a_{0e}}{a_{\sigma 0}} \right)^{-m/2} \right], \tag{9}$$

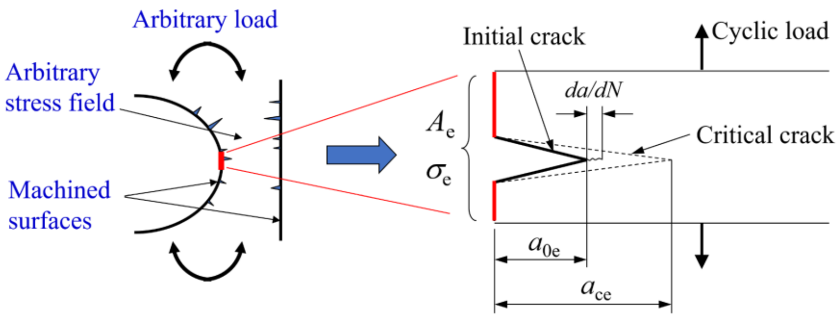


Figure 2. Schematic of the fatigue extension of equivalent cracks starting from initial defects.

where $a_{\sigma 0} = (K_{Ic}/\beta \sigma_0 \pi^{1/2})^2$ is a constant. Therefore, the cumulative fracture probability F of the entire machined surfaces is formulated as a function of both the maximum stress σ in the specimen and the number of cycle N as

$$F = 1 - \exp\left\{ -\left(\frac{\sigma}{\sigma_0} \right)^m \sum_e \gamma k_e^m \left[1 + \frac{C(n-2)}{2} \left(\frac{\beta \sigma k_e \sqrt{\pi}}{K_{Ic}} \right)^2 N \right]^{m/(n-2)} \right\} \tag{10}$$

Fatigue behavior of the arbitrarily-shaped specimens can be estimated at arbitrary applied load levels by using this equation. It means that the *S*-*N* curve showing the relation between σ and N is formulated by Eq. (10). By fitting Eq. (10) to fatigue test data then the values of C, n are obtained, and therefore *S*-*N* curve can be drawn.

3. STATISTICAL ANALYSIS OF RAMPING EXPERIMENTAL METHOD

As described in Fig. 1(b), the maximum value of applied stress σ at the cycle number N in the ramping tests was $\sigma = N\Delta\sigma$, where $\Delta\sigma$ is the ramping increment per cycle.

It means that the applied stress in each element as $\sigma_e = k_e\sigma = k_e N\Delta\sigma$. Therefore

$$\Delta K = \beta\sigma_e\sqrt{\pi a_e} = \beta k_e N\Delta\sigma\sqrt{\pi a_e} \;. \tag{11}$$

By substituting ΔK from Eq. (11) into Eq. (5), then Eq. (5) is rewritten as

$$\frac{da}{a_e^{\frac{n}{2}}} = C\left(\frac{\beta k_e N\Delta\sigma\sqrt{\pi}}{K_{Ic}}\right)^n dN \;. \tag{12}$$

Integrating Eq. (12) with respect to the crack length in the element from the initial crack length a_{0e} to the critical length $a_{ce} = (K_{Ic}/\beta\sigma_e\pi^{1/2})^2$ corresponding to the number of cycles from 0 to N, the initial crack length in each elements is obtained as

$$a_{0e} = \left[\frac{C(n-2)}{2(n+1)}\left(\frac{\beta k_e\sigma\sqrt{\pi}}{K_{Ic}}\right)^n \frac{\sigma}{\Delta\sigma} + \left(\frac{K_{Ic}}{\beta k_e\sigma\sqrt{\pi}}\right)^{2-n}\right]^{2/(2-n)} \tag{13}$$

By the same way as mentioned in the traditional method, by combining Eqs. (9) and (13), the cumulative fracture probability F in ramping fatigue tests is formulated as the function of the increasing maximum stress σ as

$$F = 1 - \exp\left\{-\left(\frac{\sigma}{\sigma_0}\right)^m \sum_e \gamma k_e^m \left[1 + \frac{C(n-2)}{2(n+1)}\left(\frac{\beta\sigma k_e\sqrt{\pi}}{K_{Ic}}\right)^2 \frac{\sigma}{\Delta\sigma}\right]^{m/(n-2)}\right\} \tag{14}$$

when the ramping increment $\Delta\sigma$ comes to infinity, then Eq. (14) becomes identical to Eq. (4) showing the static strength distribution. By fitting Eq. (14) to fatigue test data obtained by the new experimental method, the values of the parameters C and n could be obtained. Using those values of C and n for Eq. (10), S-N curve can be drawn.

4. DISCUSSION

In order to see the validation of this theory, the tensile specimens made of carbon steel sheet with the thickness of 5 mm as shown in Fig. 3 was used for the tests here. The carbon steel sheet has the chemical component as 98.4 % of Ferris, 0.4 % of Carbon, 0.221 % of Silicon, 0.568 % of Manganese, etc. The specimens were machined by CNC milling machine without any annealing. The specimen shape was designed to avoid stress concentration on the testing part with the length of 50 mm and the width of 5 mm, where the flare parts are designed by a set of arcs with different radii. Stress distribution on the specimens was estimated by FEM on ANSYS Workbench software as shown in Fig. 4, where quadrangular mesh with the size of 1 mm was used.

The specimens are tested on the machine designed and producted by ourself as shown in Fig. 5, where the load was measured by the loadcell PST-KELI (capacity: 1.2 ton, output: 2.0 ± 0.003 mV/V, accuracy class: OIML R60 C3). The diagram of this machine is presented in Fig. 6 in order to explain the working principle. All the elements are set on the static frame (5) made of shaped steel bars by welding. At the start position, the specimen-holder table (15) is at the left limit (presented as the position A), both the motors (3,12) are on the stop state. By pressing the start button in the software on the computer (2), the computer sends a command to the control box (1) to control the operation of the two motors (3,12). The stepper motor (15) makes the movement of the specimen-holder table (15) to the intended position on the right side along the screw shaft (13), which helps increasing the deformation of the specimen and therefore increases the applied stress on the specimen. When the AC motor (12) rotates, the crank-and-rocker mechanism consisting of the AC motor (12), the eccentric (11), the connecting rod (10) and the shaking rod (7) creates the shake of the shaking rod (7), which pulls the vertical rod and the loadcell (8) moving up and down cyclically. Therefore, the specimens will be excited by a tensile or bending cyclic load depending on the setup of the experiment as shown in Fig. 6(a) for tensile test and Fig. 6(b) for bending test. The loadcell will measure the load applied to the specimen and send it to the computer. All the details of this machine will be published in another paper.

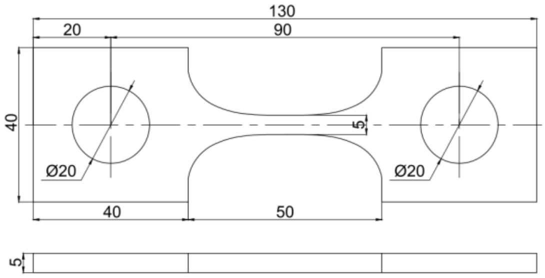


Figure 3. Specimen design.

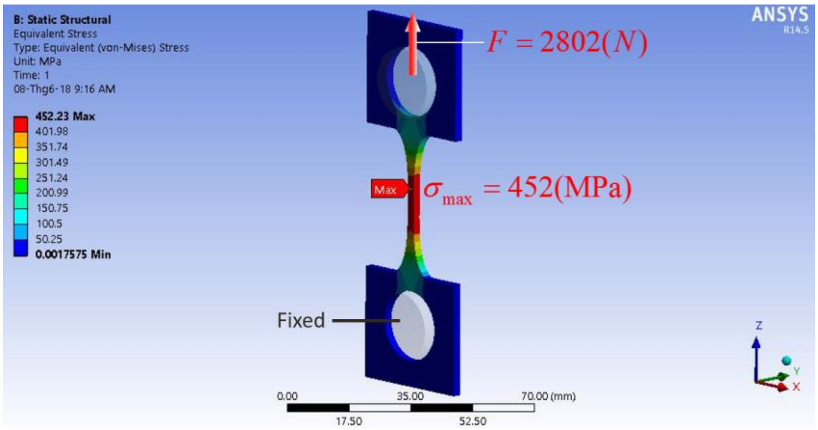


Figure 4. Stress distribution on specimen.

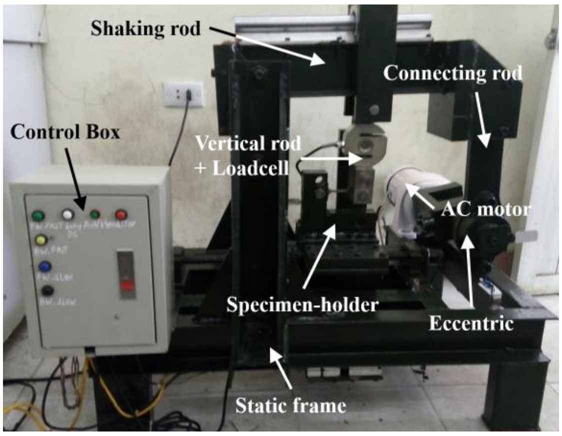
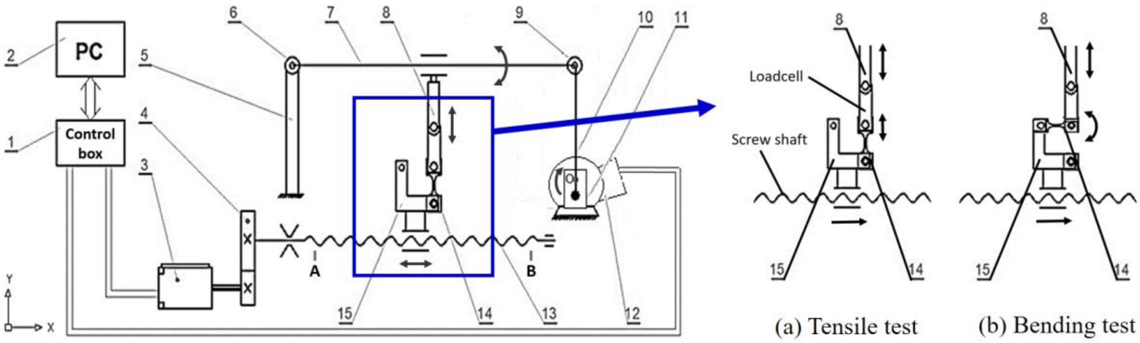


Figure 5. Fatigue test machine.



1. Control Box
2. Personal computer
3. Stepper motor
4. Spur gears
5. Static frame
6. Bearings (rotation joint)
7. Shaking rod
8. Vertical rod + Loadcell
9. Bearings (rotation joint)
10. Connecting rod
11. Eccentric
12. AC motor
13. Ball screw
14. Specimen
15. Specimen-holder table

Figure 6. Machine diagram.

By using tensile test with monotonically increasing load at the rate of 42 N/s, static strength of the speimen was evaluated as 0.452 GPa (evaluated by FEM as shown in Fig. 4) corresponding to the applied load of 2802 N. Fig. 7 shows the specimen before and after static test, where the broken region was tighten showing plastic deformation. However, the plastic deformation was not seen in the specimens tested with normal and ramping fatigue tests as shown in Figs. 8 and 9, i.e., fatigue cracks extend in the region of elastic deformation. It means that Paris' law could be used to describe fatigue crack extension for the cracks on these fatigue specimens, and therefore the above equations can be used to create S-N curve for the steel specimens.

Up to this point, only one specimen was tested with the normal fatigue test and one specimen was tested with the ramping fatigue test as shown in Figs. 8 and 9, respectively. These experimental data are used to confirm the agreement of the theory. The normal fatigue test was performed with the load amplitude of 2100 N correspoding to the stress amplitude of 0.339 GPa, then the specimen was broken after approximately $10^{4.57}$ cycles. The ramping fatigue test was performed with the ramping increment $\Delta\sigma$ = 2525 Pa/cycle, and the specimen was broken after $10^{5.15}$ cycles, i.e., at the stress amplitude σ = 0.355 GPa.

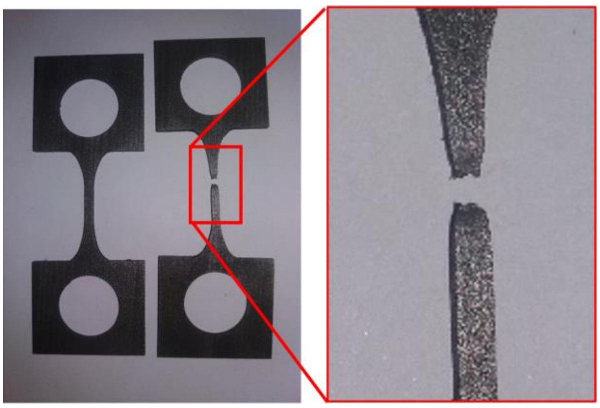


Figure 7. Specimen after static test.

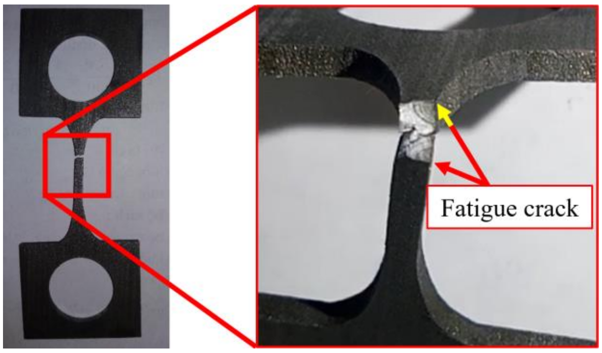


Figure 8. Specimen after normal fatigue test.

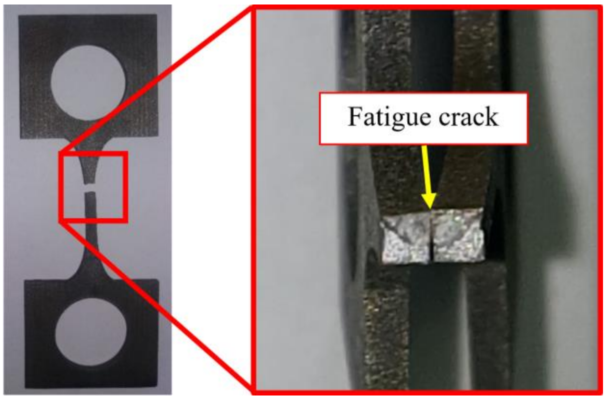


Figure 9. Specimen after ramping fatigue test.

For creation of the S-N curve of the specimens in this study, because of the limitation of the number of specimens, some parameters are referred from the other studies for carbon steel as m = 56.027 [10], β = 1.12 [2], K_{Ic} = 60.10^6 Pa$\sqrt{m}$ [11], n = 4 [11]. From the static test, the average strength σ_0 is 0.452 GPa. By fitting Eq. (14) to the ramping test datum as shown in Fig. 10, the value of C was obtained as 1.58 × 10^{-7} m/cycle.

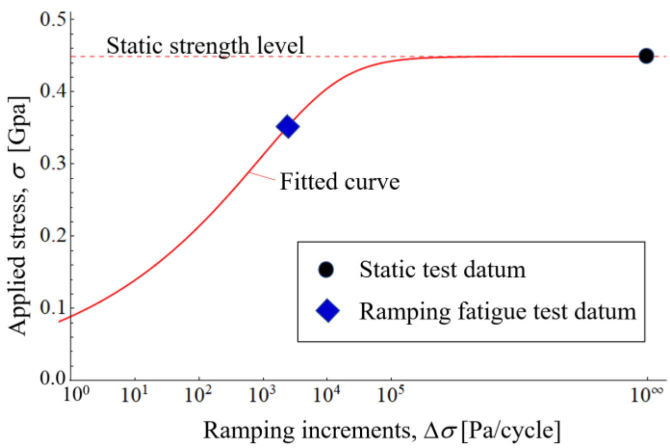


Figure 10. Ramping fatigue test result.

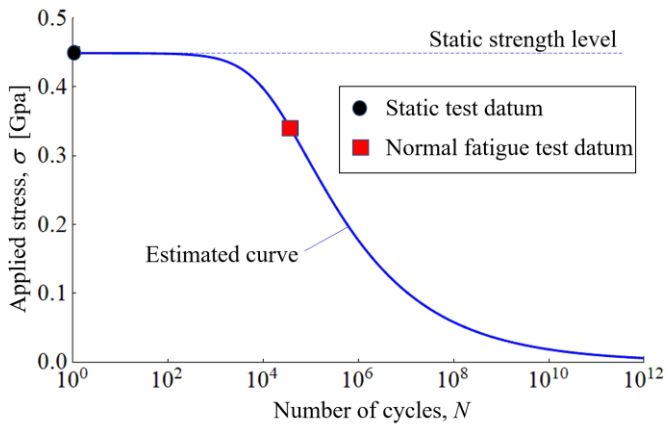


Figure 11. Estimated S-N curve in comparison with normal fatigue test datum.

Using the above values of the parameters for Eq. (10), the S-N curve is plotted at level of the cumulative probability F as 50 % as shown in Fig. 11. The cumulative probability F was selected to be 50 % since the probability density is the highest. When the values of the parameters F, m, σ_0, β, K_{Ic}, C, n were known, Eq. (10) becomes an explicit equation of N and σ and therefore the S-N curve is plotted easily. In this study, the fatigue limit is ignored because its level depends on many factors such as surface conditions, corrosion, temperature, residual stresses, etc. The normal fatigue test datum shown by the lozenge symbol in Fig. 11 mostly lies in the estimated S-N curve, where the difference in logarithmic scale is 2.79 %. This first result showed the posibility of using the new fatigue test method with gradually increasing stress amplitude to create the S-N curve for steel machine elements.

5. CONCLUSION

This paper presented the ramping fatigue test, which is an improved fatigue test method with gradually increasing stress amplitude for steel machine elements in order to circumvent the problems of the traditional fatigue test with constant stress amplitude. The ramping test method helps to obtain experimental data in an intended time limit. This method was formulated with Paris' law to draw the fatigue behavior in connection with the static strength distribution. Values of the parameters in Paris' law obtained from the ramping test were used to plot the stress-lifetime curve, which is traditionally established by using the fatigue tests with constant stress amplitude. The estimated stress-lifetime curve was compared to the experimental data obtained from the carbon steel specimens. Though the number of specimens is only one for each kind of tests, but the fatigue test data mostly lie in the estimated curve. It is necessary to increase the number of experimental data in order to consolidate the conclusion as well as estimate the time saved by the ramping fatigue test method. However, the obtained result showed the possibility that the fatigue lifetime of machine elements under constant stress amplitude can be predicted by applying the ramping fatigue test method.

Acknowledgements. This research is funded by PHENIKAA University under grant number 04.2019.02.

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