Operator-based robust nonlinear vibration control for flexible plate with piezoelectric actuator

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Summary

This dissertation discusses the operator based nonlinear vibration control problems for a flexible plate using piezoelectric actuator with hysteresis nonlinearity. By using operator theory, bounded input bounded output (BIBO) stability of the designed nonlinear vibration control systems is guaranteed and the desired vibration control performance is ensured by the proposed control schemes.

With the development of smart materials, many kinds of actuators and of sensors are made of them. Recently, vibration control using smart materials has been a key technology in vibration suppression techniques. Among them, the piezoelectric elements can be used as both actuators and sensors due to the piezoelectric effect. However, the piezoelectric actuator has hysteresis nonlinearity. When using the piezoelectric actuator, the control performance is affected by hysteresis nonlinearity. In this dissertation, to address the hysteresis nonlinearity of the piezoelectric actuator, Prandtl-Ishlinskii(P-I) hysteresis model is considered. The model of flexible plate is considered by theory of thin plates. Based on operator theory, nonlinear vibration control schemes are proposed in this dissertation.

First, for the plate with a free vibration and perturbations case, operator based controllers are designed to guarantee the robust stability of the nonlinear control system. Simultaneously, for ensuring the desired vibration control performance, operator based compensation method is given. In the designed compensator, the desired compensation performances of tracking and of perturbations are obtained by increasing the number of designed n-times feedback loops. The effectiveness of the proposed design scheme is verified by numerical simulation and experimental results.

Second, for the plate with a forced vibration case, operator based nonlinear vibration control scheme is given. At the step of designing the controller to satisfy stability, a controller including characteristics of Proportional-Integral-Differential

(PID) controller is designed. The designed controller can be controlled by only one design parameter without adjusting PID parameters. After that, for compensating the forced vibration to improve the vibration control performance, the compensator is given by designed operators. Both numerical simulation results and experimental results are shown to verify the effectiveness of the proposed control scheme.

Third, operator based some vibration control approaches are introduced. For improving vibration control performance, the time-varying unimodular function based robust right coprime factorization approach is considered. The system mismatching compensation approach is designed for plate with a forced vibration. Operator based estimation structure is considered in unknown input nonlinearity compensation approach for reducing the effect of unknown input nonlinearity to improving the vibration control performance.

In summary, this dissertation proposes operator based nonlinear vibration control schemes for a flexible plate using piezoelectric actuator. By using the designed controllers based on the concept of Lipschitz operator and robust right coprime factorization condition, the nonlinear vibration control systems are BIBO stable and the desired vibration control performances are realized.

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Chapter 1

Introduction

1.1 Background

The developments in the field of smart materials have motivated many researchers to work in the field of smart structures. The smart structure can be defined as the structure that can sense external disturbance and respond to that with active control in real time to maintain the mission requirements. There are some smart materials such as the piezoelectric material, shape memory alloy (SMA), electrorheological (ER) fluid and magnetorheological (MR) fluid, which are applied widely as actuators. Also, these actuators can sense and respond to environmental changes to realize desired goals by the characteristics of materials. Among them, the piezoelectric element is a smart element that has the nature of converting electrical energy into mechanical energy, and vice versa. Characteristics of a piezoelectric element are that, it has a fine resolution, a large resistance of the weight of a load and a fast response, and it does not generate a magnetic field. However, hysteresis nonlinearity behavior exists widely in smart materials and the performances of actuators are affected by the hysteresis nonlinearity [1],-,[26]. For addressing the hysteresis nonlinearity, many kinds of hysteresis models have been proposed [5].

The vibration control has been a key technology in many engineering fields.

The vibration control methods mainly falls into two categories: passive vibration control and active vibration control. From the viewpoint of energy saving, passive vibration control is desirable. However, active vibration control is necessary to suppress vibration more efficiently. Recently, vibration control using smart actuators attracts attention as one of the vibration suppression techniques. However, hysteresis nonlinearity of smart actuators effect the performances of actuators and let may become the vibration control systems using smart actuators are unstable. Therefore, the vibration control using smart materials is one of the interesting subjects. The study of the nonlinear vibration control of a flexible plate using smart actuators with hysteresis nonlinearity is a challenging topic in the control field. In addition, suppression of the vibration and compensation of the nonlinearity are important issues for obtaining the desired vibration control performance of a flexible plate [27],-,[29].

Contrary to linear system theory, it is difficult to find a general control theory to all nonlinear systems [32],-,[33]. The developments in the field of design of nonlinear control system have motivated many researchers to work in this field. Some nonlinear control approaches guarantee the stability of nonlinear system by using Lyapunov method with the condition of system states being observable. However, for nonlinear system, it is difficult to observe every state. Without considering the knowledge of system state, bounded input bounded output (BIBO) stability is easier than Lyapunov stability to be realized to guarantee the robust stabilization for nonlinear systems.

In recent years, nonlinear control method has been proposed based on operator theory. The robust right coprime factorization approach is used in this method [34],-,[40]. In addition, the bounded input bounded output (BIBO) stability of the nonlinear system can be guaranteed by this method. The output tracking problem of nonlinear systems has been considered in by extending the design scheme.

More recently, an applicable condition for the robust right coprime factorization has been proposed, and the robust stabilization of the nonlinear control system can be obtained by using the condition in practical applications, such as nonlinear vibration control design of flexible arm [72], nonlinear control for peltier actuated process [75] and so on. By using the approaches of the above literatures, nonlinear plants can be described by two factors of right coprime factorization of these plants. As a result, it brings advantage to apply the above methods to nonlinear vibration control using smart actuators [69],-,[98].

1.2 Motivation

In the field of vibration control using smart actuators, many studies have been made to improve vibration control performance. Among them, one of the important issues is to guarantee the stability of the nonlinear control system and improve the control performance by compensating for the nonlinearity.

Considering nonlinear vibration control using piezoelectric actuator with hysteresis nonlinearity, the stabilization and desired control performance of the nonlinear system should be realized. The operator based robust right coprime factorization method is proved to be effective for the control and design of the nonlinear system. In this dissertation, for solving the above problems, operator-based nonlinear vibration control schemes are provided.

1.3 Contribution

This dissertation provides the nonlinear vibration control schemes for a flexible plate using piezoelectric actuator with hysteresis nonlinearity, which can be applied for the design of the vibration control system using smart actuators with hysteresis nonlinearity for the purpose of realizing the desired vibration control performance and guaranteeing the robust stability of the designed control systems. The main contributions of this study are summarized as follows.

(1) **The plate with a free vibration and perturbations**

For the plate with a free vibration and perturbations, operator-based n-times feedback loops are designed. Simultaneously, the desired compensation performance of tracking and of perturbations are obtained by increasing the number of designed n-times feedback loops. In this case, we confirmed that the stability of the control system could be guaranteed even when there with perturbations. The numerical simulation and experimental results are shown to verify the effectiveness of the control scheme.

(2) **The plate with a forced vibration**

For the plate with a forced vibration, a controller including characteristics of Proportional-Integral-Differential (PID) controller is designed to improve vibration control performance. And the designed controller can be controlled by only one design parameter without adjusting PID parameters. The vibration control performance of designed control scheme is confirmed by numerical simulation and experiment.

(3) **Operator-based some new vibration control approaches**

Operator based some new vibration control approaches are introduced. The time-varying unimodular function based robust right coprime factorization is designed to realize the vibration control performance. In this case, the condition of output tends to zero is given by the inverse of the time-varying unimodular function tends to zero. And, the operator based system mismatching compensation method is given. The vibration suppression condition is given in system mismatching compensation approach. Operator based estimation structure is considered in unknown input nonlinearity compensation approach. The vibration control performances of the designed control schemes are confirmed.

1.4 Organization of the dissertation

This dissertation is organized as follows.

In Chapter 2, some mathematical preliminaries including the basic definitions and notations are provided for the nonlinear control design in this dissertation. For addressing the hysteresis nonlinearity of piezoelectric actuator, the P-I hysteresis model is used. The model of flexible plate is given by theory of thin plates. Based on these theories and problem setup, nonlinear vibration control schemes are stated in this dissertation.

In Chapter 3, operator based nonlinear control scheme is designed for plate with a free vibration and perturbations. Based on the dynamic model of the flexible plate, operator-based controllers are designed to guarantee the robust stability of the nonlinear control system. In addition, operator-based compensation method is given to ensure the desired vibration control performance of the flexible plate with a free vibration and sudden perturbations. Both of numerical simulation and experimental results are shown to verify the effectiveness of the proposed control scheme.

In Chapter 4, for considering plate with a forced vibration, the nonlinear control scheme is designed based on operator theory. A controller with characteristics of Proportional-Integral-Differential (PID) controller is considered. After that, for compensating the hysteresis nonlinearity and improving forced vibration vibration control performance, the compensator is designed by operator B and D_{PI}. The effectiveness of the designed control scheme is confirmed by numerical simulation and experimental results.

In Chapter 5, operator based some vibration control approaches are designed. The time-varying unimodular function based robust right coprime factorization approach is considered to realize the vibration control performance. The operator based system mismatching compensation approach is designed for plate with a forced vibration. Operator based unknown input nonlinearity compensation is designed to improve the vibration control performance. Both of numerical simulation and experiment results are shown to verify the effectiveness of the proposed design scheme.

In Chapter 6, the proposed control schemes in this dissertation are summarized. That is, the nonlinear vibration control systems of a flexible plate using piezoelectric actuator with hysteresis nonlinearity are BIBO stable and the desired control performances are realized by proposed nonlinear control schemes.

Chapter 2

Preliminaries and problem setup

2.1 Introduction

Coprime factorization for nonlinear feedback control systems has been one of the approach for nonlinear systems analysis, design, stabilization and control, and has been consistently pursued with tremendous effort by many researchers in the field. In this chapter, the mathematical preliminaries and problems setup are provided to serve the theoretical basis for the research and the following chapters in this dissertation.

In Section 2.2, definitions of spaces such as normed linear space, Banach space, extended linear space are introduced firstly. Then, definitions of operators are described. Based on these definitions, the right coprime factorization are introduced.

In Section 2.3, for addressing the hysteresis nonlinearity of piezoelectric actuator, model of piezoelectric actuator is introduced by P-I hysteresis model.

In Section 2.4, a dynamics model of flexible plate with piezoelectric actuator is introduced, and its model based on theory of thin plates.

In Section 2.5, the experimental devices on this dissertation is introduced.

In Section 2.6, the problem setup of this dissertation is given. That is, realization issues of operator based robust nonlinear vibration control systems is studied.

2.2 Preliminaries

In this section, some fundamental definitions on spaces and operators for operator theory are introduced.

2.2.1 Definitions of spaces

In mathematics, two basic spaces include a linear spaces (also called a vector spaces) and a topological spaces. The linear spaces are of algebraic nature. On linear spaces, there are real linear spaces (over the field of real numbers), complex linear spaces (over the field of complex numbers), and more generally, linear spaces over any field. In this dissertation, the used space is based on linear spaces.

Normed linear spaces

A space V of time functions, V is said to be a vector space if it is closed under addition and scalar multiplication. The space V is said to be normed if each element v in V is endowed with norm $\| \cdot \|_V$, satisfying the follow three conditions:

(1) $\| v \| \geq 0$; and $\| v \| = 0$ if and only if $v = 0$;

(2) $\| av \| = | a | \| v \|$, for any scalar a;

(3) $\| v_1 + v_2 \| \leq \| v_1 \| + \| v_2 \|$;

Banach space

In mathematics, a Banach space is a complete normed vector space. This means that a Banach space is a vector space V over the real or complex numbers with a norm $\| \cdot \|$ such that every Cauchy sequence (with respect to the metric $d(b_1, b_2) = \| b_1 - b_2 \|$) in V has a limit in V.

Extended linear space

let V be a Banach space of real-valued measurable functions which is defined on the time domain $[0,\ \infty)$. If the following linear vector space (usually not complete) of real-valued measurable functions $f(t)$ which is defined on the time domain $[0,\ \infty)$:

$$V_n^e = \{f(t) :\} \parallel f_T \parallel_{V_n} < \infty, \forall T \in [0,\ \infty)\} \tag{2.1}$$

where f_T is the truncation of $f(t)$ by T which is defined as

$$f_T(t) = \begin{cases} f(t), & t \leq T \\ 0, & t > T \end{cases}$$

V_n^e is called an extended Banach space associated with V_n.

2.2.2 Definitions of operators

Let U and Y be linear spaces defined in the field of real numbers, and let U_s and Y_s be two normed linear spaces, called the stable subspaces of U and Y, respectively, defined suitably by two normed linear spaces under certain norm denoted $U_s = \{u \in U :\parallel u \parallel < \infty\}$ and $Y_s = \{y \in Y :\parallel y \parallel < \infty\}$.

Operator

An operator F: $U \rightarrow S$ is generally a mapping that acts on the elements of input space U to produce other elements of the output space S. The operator F can be expressed as $y(t) = F(u)(t)$ where $u(t)$ is the element of U and $y(t)$ is the element of S.

Linear and nonlinear operator

Let $S : U \rightarrow Y$ be an operator mapping from input space U to the output space Y, and denote by $\mathcal{D}(S)$ and $\mathcal{R}(S)$, respectively, the domain and range of S. If the operator $S : \mathcal{D}(S) \to Y$ satisfies Addition Rule and Multiplication Rule

$$S : av_1 + bv_2 \to aS(v_1) + bS(v_2)$$

for all v_1, $v_2 \in \mathcal{D}(S)$ and all $a, b \in$ R, then S is said to be linear operator; otherwise, it is called to be nonlinear operator.Since linearity is a special case of nonlinearity, in what follows "nonlinear" will always mean "not necessarily linear" unless otherwise indicated.

Bounded input bounded output (BIBO) stability

Let S be a nonlinear operator that acts on its domain $\mathcal{D}(S) \subseteq U$ and range $\mathcal{R}(S) \subseteq Y$. S is said to be input-output stable, if $S(U) \subseteq Y$. Another crucial definition is bounded input bounded output (BIBO) stability. From the viewpoint of signal processing, the BIBO is form of stability for linear signals and systems with taking inputs. If S maps all input functions from U_s into the output space Y_s, such that $S(U_s) \subseteq Y_s$, then operator S is said to be bounded input bounded output (BIBO) stable or simply stable. That is, the output will be bounded for every input to the system. Otherwise, S is said to be unstable, when S maps some inputs from U_s to $Y^e \backslash Y_s$ (if not empty). For any stable operators defined here and later, in this dissertation they always mean BIBO stable.

Invertible

An operator M is called invertible if there exists an operator F such that

$$M \circ F = F \circ M = I$$

where I denotes the identity operator, F is said to be the inverse of M expressed in the form of M^{-1}, in which $\circ$ denotes the operation defined in the operator theory which can be simple presented as $M \circ F$.

Unimodular operator

$\mathcal{F}(U,Y)$ is the set of stable operators from U to Y. Then, $\mathcal{F}(U,Y)$ contains a subset defined by

$$\sum(U,Y) = \{L : L \in \mathcal{S}(U,Y),\\ L \text{ is invertible with } L^{-1} \in \mathcal{F}(U,Y)\}.$$

Elements of $\sum(U,Y)$ are said to be unimodular operators.

Lipschitz operator

Let $\mathcal{L}(U,Y)$ be the family of all nonlinear operators mapping form U into Y. Let O be a subset of U and $\mathcal{K}(O,Y)$ be the family of operators A in $\mathcal{L}(U,Y)$ with $\mathcal{D}(A) = D$. A semi-norm on $\mathcal{K}(O,Y)$ is denoted by

$$\|S\| := \sup_{\substack{x_1,x_2 \in O \\ x_1 \neq x_2}} \frac{\|S(x_1) - S(x_2)\|_Y}{\|x_1 - x_2\|_U}$$

And, it is called the Lipschitz semi-norm of the operator S on U_s.

Note that, in general, $\| S \| = 0$ does not necessarily imply $S = 0$. In fact, $\| S \| = 0$ if and only if S is a constant-operator (need not be zero) that maps all elements from U_s to the same element in Y_s.

For any fixed $x_0 \in U_s$, the number

$$\| S \|_{Lip} := \| S(x_0) \|_{Y_s} + \sup_{\substack{x_1,x_2 \in U_s \\ x_1 \neq x_2}} \frac{\| S(x_1) - S(x_2) \|_{Y_s}}{\| x_1 - x_2 \|_{X_s}} \tag{2.2}$$

defines a norm for all $S \in Lip(U_s, Y_s)$. Then, $\| S \|_{Lip}$ is called the Lipschitz norm of S defined by $x_0 \in U_s$. It is worth reminding that, it amounts to showing that $\| S \|_{Lip} = 0$ implies $S = 0$, which called zero operator. It is also evident that a Lipschitz operator is both bounded and continuous on its domain.

Generalized Lipschitz operator

Let $\mathcal{L}(X, Y)$ denote the family of two normed linear operators over the complex numbers from X to Y. Let $\mathcal{N}(X, Y)$ be the family of all nonlinear operators mapping from X into Y, which are two Banach spaces. Obviously, $\mathcal{L}(X, Y) \subseteq \mathcal{N}(X, Y)$. In the case that $X = Y$, we use the notation $\mathcal{L}(X)$ and $\mathcal{N}(X)$, respectively, instead of $\mathcal{L}(X, Y)$ and $\mathcal{N}(X, Y)$ for simplicity.

Let X_u and Y_u be two extended linear spaces, which are associated with two given Banach spaces X and Y of real-valued measurable functions defined on the time domain $[0, \infty)$, respectively. Let U be a subset of X_u. If there exists a constant L such that

$$\| [S(x_1)]_T - [S(x_2)]_T \|_{Y_u} \leq L \| [x_1]_T - [x_2]_T \|_{X_u} \tag{2.3}$$

for all $x_1, x_2 \in U$ and for all $T \in [0; \infty)$. The nonlinear operator $S : U \longrightarrow Y_u$ is called a generalized Lipschitz operator on U, and its actual norm can be given by

$$\begin{aligned} \| S \|_{Lip} =& \| S(x_0) \|_{Y_u} + \| S \| = \| S(x_0) \|_{Y_u} \\ &+ \sup_{T \in [0,\infty)} \sup_{\substack{x_1, x_2 \in U \\ x_1 \neq x_2}} \frac{\| [S(x_1)]_T - [S(x_2)]_T \|_{Y_u}}{\| [x_1]_T - [x_2]_T \|_{X_u}} \end{aligned} \tag{2.4}$$

for any fixed $x_0 \in U$.

Note that the least such constants L shown in Eq. (2.3) is given by

$$\| S \| := \sup_{T \in [0,\infty)} \sup_{\substack{u_1, u_2 \in D^e \\ u_1 \neq u_2}} \frac{\| [Q(u_1)]_T - [Q(u_2)]_T \|}{\| [u_1]_T - [u_2]_T \|} \tag{2.5}$$

which is a semi-norm for general nonlinear operators.

2.2.3 Operator-based right coprime factorization

A nonlinear plant with uncertainties is denoted as operator $P + \Delta P : U \rightarrow Y$. Where P denotes the nominal plant, ΔP denote uncertainties of the system. U and Y are the input and output space of the system.

Right factorization

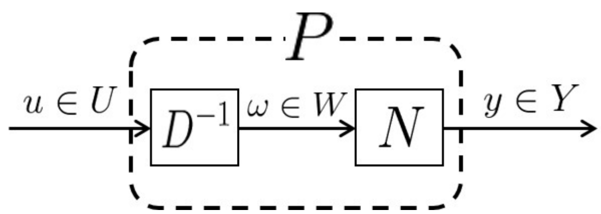

Figure 2.1: Right factorization of a nominal plant

The nominal plant operator $P : U \to Y$ is shown in Fig. 2.1. The given plant operator $P : U \to Y$ is said to have a right factorization, if there exist a linear space W and two stable operators $N : W \to Y$ and $D : E \to U$ such that D is invertible and $P = ND^{-1}$. Such a factorization of P is denoted by (N, D) and space W is called a quasi-state space of P.

Right coprime factorization

Let (N, D) be a right factorization of P. If there exist two stable operators $A : Y \to U$ and $B : U \to U$ that satisfying the following Bezout identity, the P is said to have a right coprime factorization, or factorization is said to be coprime.

$$AN + BD = L, \text{ for some } L \in \mathcal{U}(W,\ U),$$

where B is invertible and L is an unimodular operator. The right coprime factorization of a nonlinear feedback system is shown in Fig. 2.2.

Robust condition

Generally speaking, if the system with uncertainty remains stable, the system is said to have robust stability property. As to the nonlinear feedback control

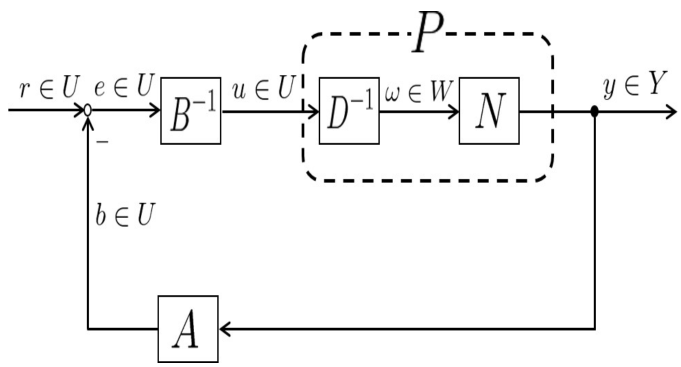

Figure 2.2: A nonlinear feedback system

systems with unknown bounded uncertainty, a robust condition about right coprime factorization was derived. The plant with uncertainty denoted as operator $P + \Delta P$, if the following Bezout identity is satisfied,

$$A(N + \Delta N) + BD = \tilde{L} \tag{2.6}$$

the BIBO stability of the nonlinear system with uncertainty can be guaranteed. Where, $\tilde{L}$ is an unimodular operator. For obtaining this condition, referred that if

$$A(N + \Delta N) = AN \tag{2.7}$$

under the condition of satisfaction of $\mathcal{R}(\Delta N) \subseteq \mathbf{N}(A)$, where $\mathbf{N}(A)$ is the null set defined by

$$\mathbf{N}(A) = \{x : \ x \in \mathcal{D}(A) \text{ and } A(x + y) = A(y) \text{ for all } y \in \mathcal{D}(A)\}$$

Based on the proposed sufficient condition, the fact that

$$A(N + \Delta N) + BD = AN + BD = L$$

is obtained, which guarantee the robust stability of the nonlinear systems with unknown bounded perturbations.

However, the nonlinear systems with unknown bounded uncertainty is crucial to realize. Therefore, a generalized sufficient condition is proposed in [65] which compared with [33] in order to improve and extend the condition.

Lemma 2.1 *Let D_e be a linear subspace of the extended linear space U_e associated with a given Banach space U, moreover denoted* $(A(N + \Delta N) - AN)M^{-1} \in Lip(D_e)$. *Denote the Bezout identity of the nominal system and the exact plant respected to ΔN in the form of $AN + BD = L$, $A(N + \Delta N) + BD = \tilde{L}$, respectively. If*

$$\| (A(N + \Delta N) - AN)M^{-1} \|_{Lip} < 1$$

the system is stable, where $\| \cdot \|_{Lip}$ denotes Lipschitz operator norm.

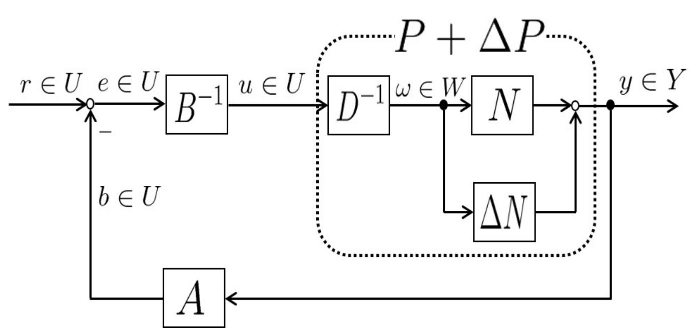

Figure 2.3: A nonlinear system with uncertainty

2.3 Model of piezoelectric actuator

There are many hysteresis models have been proposed, such as Preisach model, Prandtl-Ishlinskii model, Maxwell-Slip model, Krasnosel'skii-Pokrovskii model and so on. Among them, the Prandtl-Ishlinskii model is a subclass of the Preisach model. The Prandtl-Ishlinskii model has the same level of expression as the Preisach model, and the description is concise than the Preisach model. Further, the Prandtl-Ishlinskii model can be expressed based on both operators, called Play hysteresis operator or Stop hysteresis operator. In this dissertation, the Play hysteresis operator based Prandtl-Ishlinskii model is considered to express the hysteresis nonlinearity of the piezoelectric actuator.

Play Hysteresis Operator

The Play hysteresis operator is defined as follows.

$$\begin{aligned} F_h(u(0); u_1^*) &= f_h(u(0), u_1^*) \\ F_h(u(t); u_1^*) &= f_h(u(t), F_h(u(t_i); u_1^*)) \qquad (2.8) \\ &(\ t_i < t \leq t_{i+1}, \ 0 \leq i \leq N-1 \ ) \\ f_h(u, q) &= \max(u - h, \min(u + h, q)) \qquad (2.9) \\ &(\ 0 = t_0 < t_1 < \cdots < t_N = t_E, [0, t_E] \ ) \end{aligned}$$

where $u(t)$ is the input, u_1^* is the initial value, and h is the threshold value of Play hysteresis operator $F_h(u)(t)$. The $u(t)$ in $t_i < t \leq t_i + 1$ is piecewise monotonous with respect to t. By defining $F_h(u)(t)$ as Eq. (2.10), it is possible to substitute Eqs. (2.8) and (2.9) of the Play hysteresis operator by Eq. (2.11).

$$F_h(u)(t) = F_h(u(t); u_1^*) \qquad (2.10)$$

$$F_h(u)(t) = \begin{cases} u(t) + h, & u(t) + h \leq F_h(u(t_i)) \\ F_h(u)(t_i), & -h < u(t) - F_h(u)(t_i) < h \\ u(t) - h, & u(t) - h \geq F_h(u)(t_i) \end{cases} \qquad (2.11)$$

Prandtl-Ishlinskii Model

Based on Play hysteresis operator, the P-I hysteresis model is represented as following equation.

$$\begin{aligned} \mathrm{PI}(u)(t) &= \int_0^H p(h)F_h(u)(t)dh \\ &= D_{PI}(u)(t) + \Delta_{PI}(u)(t) \end{aligned} \tag{2.12}$$

where D_{PI} is an invertible and linearly controllable part, Δ_{PI} is its remaining part. And can be expressed as Eqs. (2.13) and (2.13).

$$D_{PI}(u)(t) = \int_0^{h_x} p(h)dh \cdot u(t) \tag{2.13}$$

$$\begin{aligned} \Delta_{PI}(u)(t) &= -\int_0^{h_x} S_n h p(h)dh \\ &\quad + \int_{h_x}^H p(h)[F_h(u)(t_i) - u(t)]dh \end{aligned} \tag{2.14}$$

$$S_n = \begin{cases} 1, & \text{if} \quad u(t) - F_h(u)(t_i) \geq 0 \\ -1, & \text{if} \quad u(t) - F_h(u)(t_i) < 0 \end{cases} \tag{2.15}$$

where h_x is the maximum number that satisfies $h \in [0, h_x]$ and $h \leq |u(t) - F_h(u)(t_i)|$ when $h = [h, h_x]$. And, $p(h)$ is $p(h) \geq 0$, the unknown density function and satisfies

$$\int_0^\infty hp(h)\mathrm{d}h < \infty \tag{2.16}$$

When H exists and $h > H$, the density function $p(h)$ satisfies condition of $p(h) = 0, (h > H)$.

2.4 Model of flexible plate

The controlled object as shown in Fig. 2.4. In Fig. 2.4, $\mathrm{p}_1 - \mathrm{p}_3$ denote piezoelectric actuators, and a, b represent length of ϵ, η directions, respectively. Where l

denotes the distance from flexible plate to the center of the servomotor, and α is an angle between a x-axis and ϵ-axis. Then the flexible plate is vibrated by the reciprocating movement $\omega_f(t)$ of the servo-motor. The activated piezoelectric actuators will induce moments in the flexible plate, and these moments can be described as following equation.

$$\frac{\partial^2 m_\epsilon}{\partial \epsilon^2} = M_p[\delta'(\epsilon - \epsilon_{1p_i}) - \delta'(\epsilon - \epsilon_{2p_i})][H(\eta - \eta_{1p_i}) - H(\eta - \eta_{2p_i})] \quad (2.17)$$

$$\frac{\partial^2 m_\eta}{\partial \eta^2} = M_p[H(\epsilon - \epsilon_{1p_i}) - H(\epsilon - \epsilon_{2p_i})][\delta'(\eta - \eta_{1p_i}) - \delta'(\eta - \eta_{2p_i})] \quad (2.18)$$

where M_p, $H(\cdot)$, $\delta'(\cdot)$ are the moments from piezoelectric actuators, Heaviside function, and differentiation of a delta function, respectively.

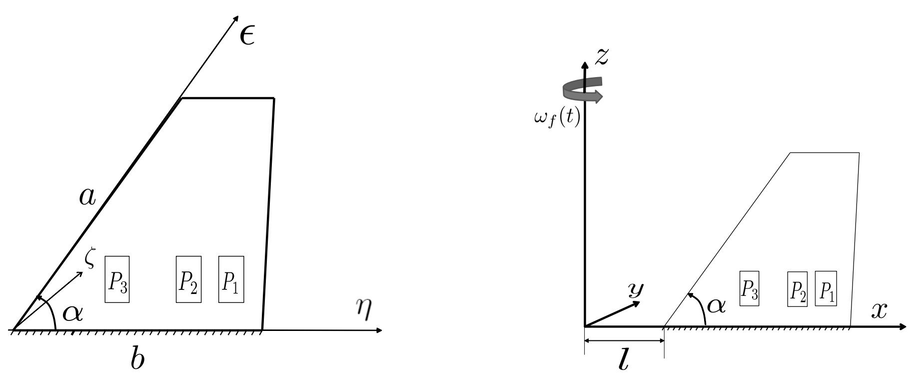

Figure 2.4: Controlled object.

The equation of motion for the flexible plate that is considered piezoelectric actuators and external force is represented as the following equation.

$$D_s \nabla^4 \omega + \rho t_s \frac{\partial^2 \omega}{\partial t^2} + \bar{c}_s \frac{\partial \omega}{\partial t} = F_d(t) + \frac{\partial^2 m_\epsilon}{\partial \epsilon^2} + \frac{\partial^2 m_\eta}{\partial \eta^2} \quad (2.19)$$

where $F_d(t)$ denotes a external force, and parameters of the flexible plate are shown in Table 2.1.

Table 2.1: Parameters of the flexible plate.

Definition	Value	Units
Young's Modulus	$E = 2.94 \times 10^9$	N/m^2
Poisson's Ratio	$\nu = 0.38$	–
Density	$\rho = 1430$	kg/m^3
Length of ϵ Direction	$a = 0.31$	m
Length of η Direction	$b = 0.27$	m
Thickness	$t_s = 2 \times 10^{-3}$	m
Bending Stiffness	$D_s = \frac{Et_s^3}{12(1-\nu^2)}$	$N \cdot m$
Damping Ratio	$\bar{c}_s = 0.6$	–

A displacement $\omega(t)$ of the flexible plate is given as follows using eigenfunctions $\phi_m(\epsilon)$ and $\psi_n(\eta)$ in ϵ and η directions.

$$\omega(t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \phi_m(\epsilon)\psi_n(\eta) f(t) \tag{2.20}$$

where m and n are vibration order mode of ϵ and η directions, respectively. According to the theory of thin plate, eigenfunctions $\phi_m(\epsilon)$ and $\psi_n(\eta)$ are described as follows.

$$\begin{aligned}
\phi_m(\epsilon) &= \cosh \frac{\gamma_\epsilon}{a}\epsilon - \cos \frac{\gamma_\epsilon}{a}\epsilon - \sinh \frac{\gamma_\epsilon}{a}\epsilon + \sin \frac{\gamma_\epsilon}{a}\epsilon \\
\psi_1(\eta) &= 1 \\
\psi_2(\eta) &= \sqrt{3}(1 - 2\eta/b) \\
\psi_n(\eta) &= \cosh \frac{\gamma_\eta}{b}\eta + \cos \frac{\gamma_\eta}{b}\eta - \sinh \frac{\gamma_\eta}{b}\eta - \sin \frac{\gamma_\eta}{b}\eta, (n > 2)
\end{aligned}$$

and where $\gamma_\epsilon = (2m - 1)\pi/2,\ \ \gamma_\eta = (2n - 3)\pi/2$. From orthogonality of eigenfunctions, Eq. (2.21) can be obtained by substituting Eq. (2.20) to Eq. (2.19).

$$k_1 \frac{d^2 f(t)}{dt^2} + k_2 \frac{df(t)}{dt} + k_3 f(t) = k_4 M_p(t) + k_5 F_d(t) \tag{2.21}$$

where $f(t)$ is displacement in a mode coordinates system of the flexible plate, and

$$
\begin{aligned}
k_1 &= \rho t_s \int_0^a \phi_m^2(\epsilon)d\epsilon \int_0^b \psi_n^2(\eta)d\eta \\
k_2 &= \bar{c}_s \int_0^a \phi_m^2(\epsilon)d\epsilon \int_0^b \psi_n^2(\eta)d\eta \\
k_3 &= D_s\Big(2\int_0^a \frac{d^2\phi_m(\epsilon)}{d\epsilon^2}\phi_m(\epsilon)d\epsilon \int_0^b \frac{d^2\psi_n(\eta)}{d\eta^2}\psi_n(\eta)d\eta \\
&\quad + \int_0^a \frac{d^4\phi_m(\epsilon)}{d\epsilon^4}\phi_m(\epsilon)d\epsilon \int_0^b \psi_n^2(\eta)d\eta \\
&\quad + \int_0^a \phi_m^2(\epsilon)d\epsilon \int_0^b \frac{d^4\psi_n(\eta)}{d\eta^4}\psi_n(\eta)d\eta\Big) \\
k_4 &= \left(\frac{d\phi_m(\epsilon_{2p_i})}{d\epsilon} - \frac{d\phi_m(\epsilon_{1p_i})}{d\epsilon}\right)\int_{\eta_{1p_i}}^{\eta_{2p_i}} \psi_n(\eta)d\eta \\
&\quad + \left(\frac{d\psi_n(\eta_{2p_i})}{d\eta} - \frac{d\psi_n(\eta_{1p_i})}{d\eta}\right)\int_{\epsilon_{1p_i}}^{\epsilon_{2p_i}} \phi_m(\epsilon)d\epsilon \\
k_5 &= \rho t_s\Big(\int_0^a \phi_m(\epsilon)d\epsilon \int_0^b \psi_n(\eta)\eta d\eta \\
&\quad + \int_0^a \phi_m(\epsilon)\epsilon\cos(\alpha)d\epsilon \int_0^b \psi_n(\eta)d\eta \\
&\quad + l\int_0^a \phi_m(\epsilon)d\epsilon \int_0^b \psi_n(\eta)d\eta\Big)
\end{aligned}
$$

Then we obtain

$$
f(t) = \mathcal{L}^{-1}\Big(\frac{1}{k_1 s^2 + k_2 s + k_3}\Big) * (k_4 M_p(t) + k_5 F_d(t)) \tag{2.22}
$$

and Eq. (2.20) can be represented as

$$
\begin{aligned}
\omega(t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Big[J_{mn}\int_0^t e^{-\alpha_{mn}(t-\tau)} \\
\cdot \sin\beta_{mn}(t-\tau)(k_4 M_p(\tau) + k_5 F_d(\tau))d\tau\Big]
\end{aligned} \tag{2.23}
$$

where

$$J_{mn} = \frac{\phi_m(\epsilon)\psi_n(\eta)}{k_1\sqrt{\frac{k_3}{k_1} - \frac{k_2^2}{4k_1^2}}}, \qquad \alpha_{mn} = \frac{k_2}{2k_1}, \qquad \beta_{mn} = \sqrt{\frac{k_3}{k_1} - \frac{k_2^2}{4k_1^2}}$$

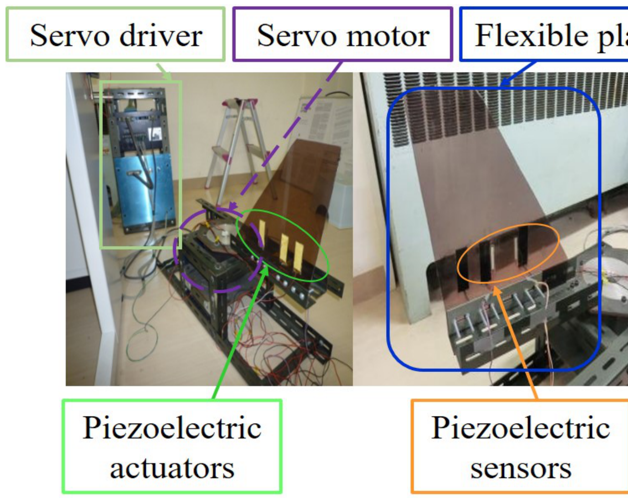

Figure 2.5: Experimental devices.

2.5 Experimental devices

The experimental devices and detailed explanation of the experimental system are shown in Figs. 2.5 and 2.6, respectively. The piezoelectric actuators are attached on the one side of the flexible plate, and the piezoelectric sensors are attached on the other side opposite to the actuators. The vibration of the flexible plate is generated by servo-motor. In experiments, controllers are designed in Visual C++ 6.0. The Analog-to-Digital (A/D) and Digital-to-Analog (D/A) conversion are performed by the PCI board. After that, the control input is transmitted to piezoelectric actuators by passing through a direct voltage amplifier from the PCI-3521 board. In this experiment, the output of the direct voltage amplifier is limited between +100V and -100V.

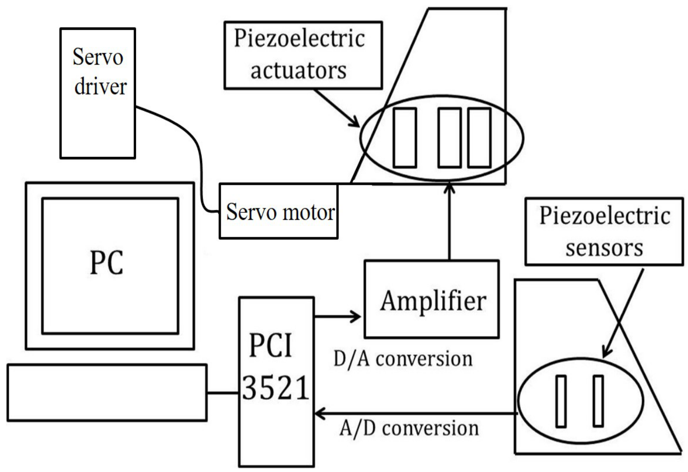

Figure 2.6: Detailed explanation of the experimental system.

2.6 Problem setup

The need of studying on vibration control of flexible plate with smart actuators has been motivated by the increasing complicated requirement of the practical problems in many engineering fields. However, the vibration control performance effected by hysteresis nonlinearity of smart actuator. In addition, how to design the nonlinear system to guarantee stability and to ensure desired control performance are difficult problems on vibration control using smart actuators.

Therefore, for dealing with these problems more effectively, robust right co-prime factorization approach is used to consider the vibration control on the flexible plate using piezoelectric actuator with hysteresis nonlinearity, which is the main objective of this dissertation. In details, the vibration control schemes of the plate

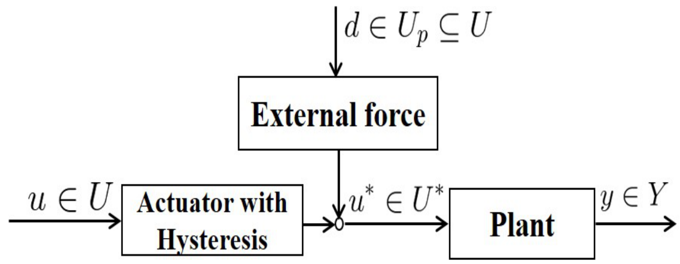

Figure 2.7: A plant with hystersis nonlinerarty.

with a free vibration and perturbations, and of the plate with a forced vibration are discussed respectively in the following chapters.

2.7 Conclusion

In this chapter, the mathematical preliminaries including the basic definitions and notations are introduced. Especially, the definitions of spaces and Lipschitz operators are introduced, which serve a foundation for the dissertation. The models of piezoelectric actuator and flexible plate are discussed. In addition, the experimental devices on this dissertation is introduced. After these preliminaries, the problems discussed in this dissertation are defined.

Chapter 3

Operator-based nonlinear control scheme for plate with a free vibration and perturbations

3.1 Introduction

The developments in the field of smart materials have motivated many researchers to work in the field of vibration control. Recently, smart actuators have been proposed such as piezoelectric element, shape memory alloy, magnetic fluid actuator, and so on. Among them, the piezoelectric elements can be used as both actuators and sensors due to the piezoelectric effect. However, the piezoelectric actuator has hysteresis nonlinearity. When using the piezoelectric element, if linearly approximated or ignored the hysteresis nonlinearity of the piezoelectric element, the control performance may deteriorate or the designed control system may become unstable. Therefore, when using the piezoelectric actuator, we need to consider the hysteresis nonlinearity.

In recent years, the nonlinear control method based on right coprime factorization approach has been proposed. Also, operator theory is one of the nonlinear control methods, and it can guarantee the Bounded Input Bounded Output (BIBO) stability of the nonlinear feedback control system by robust right coprime factorization. Considering the flexible plate with a free vibration and perturbations, operator-based nonlinear control scheme is shown in this chapter.

In Section 3.2, based on the dynamic model of the flexible plate, operator-based controllers are designed to guarantee the robust stability of the nonlinear control system. In addition, for ensuring the desired vibration control performance of the flexible plate with a free vibration and sudden perturbations, operator-based compensation method is given by the proposed design scheme. In the designed compensator, the desired compensation performances of tracking and of perturbations are obtained by increasing the number of designed n-times feedback loops.

In Section 3.3, based on the designed nonlinear control system, effectiveness of the proposed control scheme is discussed by numerical simulation results.

In Section 3.4, the designed nonlinear control scheme is performed by experiment. The experimental results are shown to confirm the control performance of the designed control scheme.

In Section 3.5, the conclusion of this chapter is given.

3.2 Operator-based nonlinear free vibration control of plate with sudden perturbations

In this section, the flexible plate with a free vibration and perturbations case is considered. The designed nonlinear control system is shown in Fig. 3.1. In Fig. 3.1, U, U^*, Y, and W are input space of the P-I hysteresis model, output spaces of the P-I hysteresis model and perturbations, output space of the original nonlinear plant, and quasi-state space, respectively. In this design scheme, the conditions of perturbations $d \in U_p$ and $U_p \subseteq U$ are considered. And, the target of this control is stabilizing the vibration at the flexible plate. Therefore, the target value of $r = 0$

is considered in this design scheme.

For guaranteeing the stability of the nonlinear system with hysteresis nonlinearity, the operator based controllers are designed. After that, design the compensator C to obtain tracking performance and desired perturbations compensation performance. In this design scheme, we consider the nominal vibration mode with a first-order mode, and uncertainties with second- and third-order modes. The plant with uncertainties is considered as the following equation.

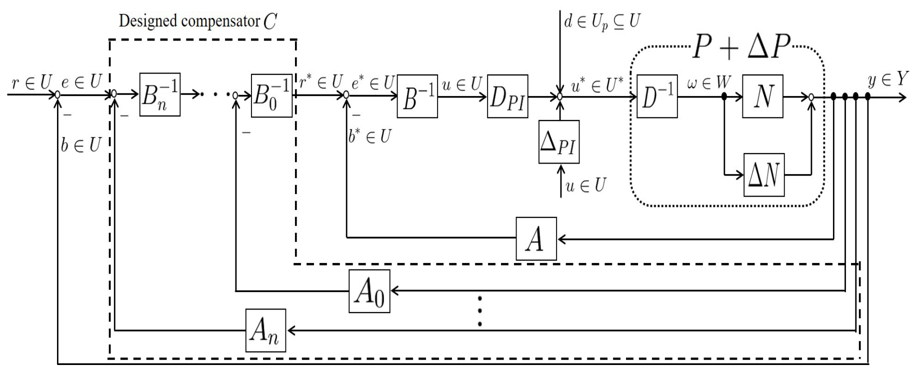

Figure 3.1: Proposed control system for plate with a free vibration and perturbations.

$$[P + \Delta P](u^*)(t) = (1 + \Delta)\left[J_{11}\int_0^t e^{-\alpha_{11}(t-\tau)}\sin\beta_{11}(t-\tau)u^*(\tau)d\tau\right] \tag{3.1}$$

where Δ denote uncertainties.

3.2.1 Controllers design for stability

The plant $[P + \Delta P]$ can be right factorized as follows.

$$[P + \Delta P](u^*)(t) = (N + \Delta N)D^{-1}(u^*)(t)$$

$$[N + \Delta N](\omega)(t) = (1 + \Delta) \cdot \left[J_{11}\int_0^t e^{-\alpha_{11}(t-\tau)}\sin\beta_{11}(t-\tau)\omega(\tau)d\tau\right] \tag{3.2}$$

$$D(\omega)(t) = I(\omega)(t) \tag{3.3}$$

where I is identity operator. Considering the P-I hysteresis model, the invertible and linearly controllable part D_{PI} is used to design the controller for stability of the nonlinear control system. And the remaining part Δ_{PI} is treated as a bounded uncertainty. The operator $\tilde{D}$ is treated as $\tilde{D} = D_{PI}^{-1}D$. And, when there exist two stable operators A and B that satisfy Bezout identity, the designed controllers can guarantee BIBO stability of the nonlinear system.

$$AN + B\tilde{D} = M \tag{3.4}$$

where M is an unimodular operator. The designed controllers A and B are shown in the following equations.

$$A(y)(t) = \frac{(1 - K_m)}{\beta_{11} J_{11}}(\ddot{y}(t) + 2\alpha_{11}\dot{y}(t) + (\alpha_{11}^2 + \beta_{11}^2)y(t)) \tag{3.5}$$

$$B(u)(t) = K_m D_{PI} u(t) \tag{3.6}$$

where K_m is a designed parameter. In addition, under the following equations are satisfied, robust stability of the designed nonlinear control system can be guaranteed.

$$A(N + \Delta N) + B\tilde{D} = \tilde{M} \tag{3.7}$$

$$\| [A(N + \Delta N) - AN]M^{-1} \|_{Lip} < 1 \tag{3.8}$$

where $\tilde{M}$ is an unimodular operator.

3.2.2 Compensation method for tracking and perturbations

Considering Fig. 3.1, the output $y(t)$ can be expressed by

$$\begin{aligned} y(t) &= (N + \Delta N)D^{-1}(D_{PI}B^{-1}(r^*(t) - b^*(t)) \\ &\quad +\Delta_{PI}(u)(t) + d(t)) \\ &= (N + \Delta N)(A(N + \Delta N) + B\tilde{D})^{-1} \\ &\quad \cdot(r^*(t) + BD_{PI}^{-1}(\Delta_{PI}(u)(t) + d(t))) \\ &= (N + \Delta N)\tilde{M}^{-1}(r^*(t) + BD_{PI}^{-1}\tilde{\Delta}) \end{aligned} \tag{3.9}$$

where $\tilde{\Delta} = \Delta_{PI}(u)(t) + d(t)$. In Eq. (3.9), if there only using controllers A and B, the output $y(t)$ can not track the target value $r^*(t)$. Therefore, compensator C is designed to compensate for the perturbations of the flexible plate and to ensure the desired tracking performance. From Fig. 3.1 and Eq. (3.9), the output $y(t)$ can be obtained by the following equations.

$$
\begin{aligned}
e(t) &= r(t) - b(t) \\
b(t) &= y(t) = (N + \Delta N)\tilde{M}^{-1}(C(e)(t) + BD_{PI}^{-1}\tilde{\Delta}) \\
r(t) &= e(t) + b(t) = e(t) + (N + \Delta N)\tilde{M}^{-1}(C(e)(t) + BD_{PI}^{-1}\tilde{\Delta}) \\
e(t) &= (I + (N + \Delta N)\tilde{M}^{-1}C)^{-1}(r(t) - (N + \Delta N)\tilde{M}^{-1}BD_{PI}^{-1}\tilde{\Delta}) \\
y(t) &= b(t) = r(t) - e(t) \\
&= r(t) - (I + (N + \Delta N)\tilde{M}^{-1}C)^{-1} \\
&\quad \cdot(r(t) - (N + \Delta N)\tilde{M}^{-1}BD_{PI}^{-1}\tilde{\Delta})
\end{aligned}
\tag{3.10}
$$

In Eq. (3.10), for obtaining the desired tracking performance, compensator C is designed to satisfy the following conditions.

1. The designed compensator C is stable.
2. In Eq. (3.10), when the condition of $e(t) \to 0$ can be satisfied by the designed compensator C, $(y(t) - r(t))$ can be made arbitrarily small.

For designing compensator C to satisfy the above conditions 1 and 2, operator-based design scheme is considered.

First, from Eqs. (3.7) and (3.9), we use the operators $[N + \Delta N]$ and $\tilde{M}$ design two stable operators A_0 and B_0 that satisfy Bezout identity.

$$A_0(N + \Delta N) + B_0\tilde{M} = M_0 \tag{3.11}$$

where M_0 is an unimodular operator.

Next, we use the operators $[N + \Delta N]$ and M_0 design each feedback loop of the n-times feedback loops to satisfy Bezout identity, respectively. The designed n-times feedback loops shown in the following equations.

$$A_1(N + \Delta N) + B_1 M_0 = M_1 \tag{3.12}$$

$$\vdots$$

$$A_n(N + \Delta N) + B_n M_{n-1} = M_n \tag{3.13}$$

where $M_0, M_1, \cdots, M_n$ are unimodular operators, A_1 to A_n and B_1 to B_n are designed stable operators, respectively. And, the designed operators are shown as follows.

$$A_0(y)(t) = \frac{s(t)}{J_{11}\beta_{11}}\left(\frac{a_0}{a_0 + |s(t)|}\right)^{c_0} \tag{3.14}$$

$$s(t) = (k_p y(t) + k_d \dot{y}(t)) \tag{3.15}$$

$$B_0 = k_0 \tag{3.16}$$

$$A_n = k^n A_0 \tag{3.17}$$

$$B_1 = \cdots = B_n = k M_t^{-1} < 1 \tag{3.18}$$

where a_0, c_0, k_p, k_d, k_0, and $0 < k < 1$ are designed parameters, respectively.M_t^{-1} is an designed unimodular operator. When each feedback loop of the designed n-times feedback loops satisfies Bezout identity, the following conditions can be obtained.

(a) When the designed operator B_0 and the designed unimodular operator M_t^{-1} satisfy conditions of $B_0\tilde{M} = I$ and $M_t^{-1}M_0 = I$, the following conditions can be obtained.

$$
\begin{aligned}
A_0(N+\Delta N)+B_0\tilde{M} &= (I+\alpha)=M_0 \\
A_1(N+\Delta N)+B_1M_0 &= k(I+\alpha)=kM_0=M_1 \\
A_2(N+\Delta N)+B_2M_1 &= k^2(I+\alpha)=k^2M_0=M_2 \\
&\vdots \\
A_n(N+\Delta N)+B_nM_{n-1} &= k^n(I+\alpha)=k^nM_0=M_n
\end{aligned}
$$

where I is identity operator and $\alpha = A_0(N+\Delta N)$. In this case, the unimodular operators $M_0, M_1, \cdots, M_n$ satisfy geometric progression with a geometric ratio of $0 < k < 1$. Therefore, the M_n can be made arbitrarily small.

(b) When the conditions of $B_0\tilde{M} = I$ and $M_t^{-1}M_0 = I$ can not be satisfied, the unimodular operator M_n also can be made arbitrarily small. Because the each feedback loop of the n-times feedback loops satisfy Bezout identity. Moreover, the designed parameters $0 < k < 1$ and $kM_t^{-1} < 1$. Therefore, when n $\to \infty$, conditions of $A_n(N+\Delta N) \to 0$ and $B_nM_{n-1} \to 0$ also can be obtained.

Thus, the condition of $M_n \to 0$ can be obtained by the designed compensator C. The equivalent system of Fig. 3.1 is shown in Fig. 3.2. Considering Fig. 3.2 and Eq. (3.10), the output $y(t)$ can be obtained by the following equations.

$$
\begin{aligned}
b(t) &= y(t)=(N+\Delta N)M_n^{-1}(e(t)+\Delta_p) \\
r(t) &= e(t)+b(t)=e(t)+(N+\Delta N)M_n^{-1}(e(t)+\Delta_p) \\
e(t) &= (I+(N+\Delta N)M_n^{-1})^{-1}(r(t)-(N+\Delta N)M_n^{-1}\Delta_p) \\
y(t) &= b(t)=r(t)-e(t) \\
&= r(t)-(I+(N+\Delta N)M_n^{-1})^{-1}(r(t)-(N+\Delta N)M_n^{-1}\Delta_p) \qquad (3.19)
\end{aligned}
$$

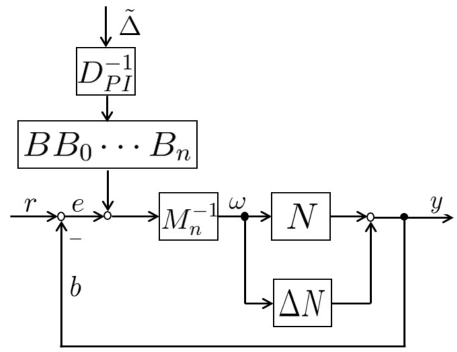

Figure 3.2: Equivalent system of Figure 3.1.

where

$$\Delta_p = B_n \cdots B_0 B D_{PI}^{-1} \tilde{\Delta} \tag{3.20}$$

From Eqs. (3.18), (3.19), and (3.20), the conditions of tracking and of perturbations compensation can be obtained as follows.

I) Condition of perturbations compensation:

When n $\to \infty$, conditions of $M_n \to 0$, $(I+(N+\Delta N)M_n^{-1})^{-1}(N+\Delta N)M_n^{-1} \to I$, and $BB_0 \cdots B_n \to 0$ can be obtained. Therefore, the Δ_p can be made arbitrarily small.

II) Condition of tracking:

In Eq. (3.19), when n $\to \infty$, $M_n \to 0$, and $\Delta_p \to 0$, inverse term $(I + (N + \Delta N)M_n^{-1})^{-1}(r)(t)$ of the right-hand side becomes arbitrarily small. Therefore, $(y(t) - r(t))$ can be made arbitrarily small.

3.3 Numerical simulations

In this section, effectiveness of the proposed design scheme will be discussed by numerical simulations. The simulation results show the vibration control performance of the flexible plate. In this simulations, only used the first-order mode of the flexible plate as the nominal plant. The second- and third-order modes of this simulation are treated as uncertainties. The sampling time is 0.01s. The flexible plate is added the moment $f_d(t) = 0.1\sin(2\pi f_0 t)$ at the bottom of it to make a vibration when $t \leq 5$s. Then $t > 5$s, we start to control the vibration of the flexible plate with free vibration. And, the perturbations $d(t) = 0.5 f_d(t)$ is added at $t \geq 8$s. Where $f_0 = 32.74/2\pi$[Hz] is eigenfrequency of the flexible plate. The density function $p(h) = 0.00038e^{-0.00086(h-1)^2}$ where $h \in [0, 30]$ of the P-I hysteresis model is used in this simulation. The parameters of designed controllers are shown in Table 3.1.

Table 3.1: Parameters of controllers

Parameter	Value	Units
K_m	0.53	–
a_0	0.25	–
c_0	2.4	–
k_p	45	–
k_d	18	–
k_0	0.8	–
k	0.9	–
M_t	1.1	–
Sampling Time	0.01	s
Simulation Time	15	s

Simulation results are shown in the following figures.

In Fig. 3.3, the simulation result without control shows a dashed line. At the full line in Fig. 3.3, we show the result of simulation only using controllers A and B case. The corresponding control input to piezoelectric actuators is shown in Fig. 3.4. From Figs. 3.3 and 3.4, when the flexible plate with a free vibration and sudden perturbations, we can confirm that the stability of the nonlinear system can

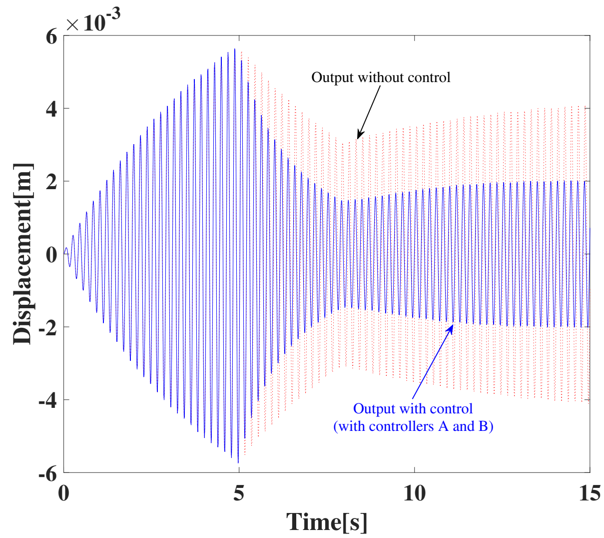

Figure 3.3: Outputs of the system with and without control (without considering compensator C; with a free vibration and sudden perturbations).

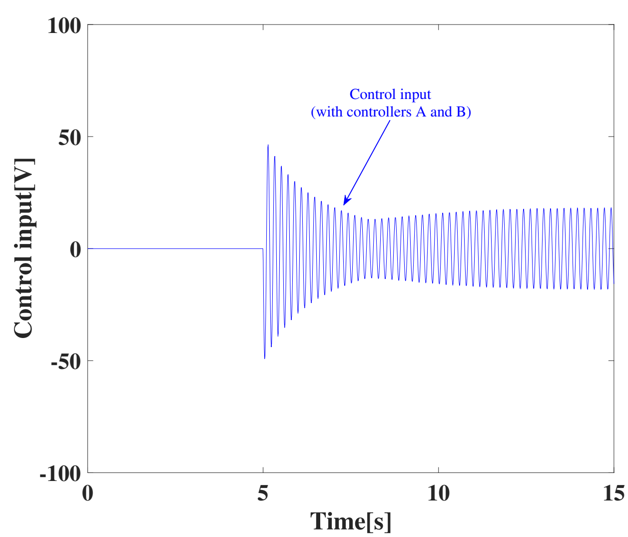

Figure 3.4: Control input of Figure 3.3 (without compensator C).

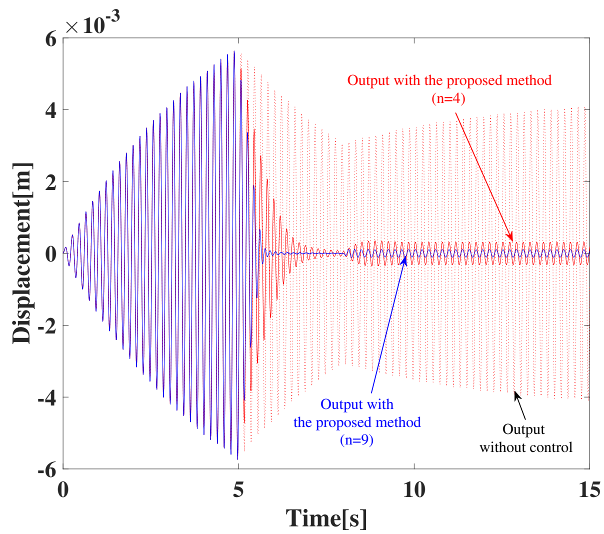

Figure 3.5: Comparison between the proposed method n = 4 and n = 9.

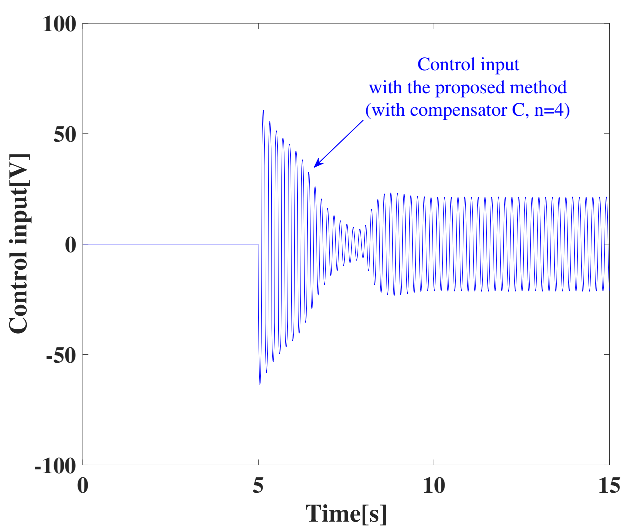

Figure 3.6: Control input of Figure 3.5 (with compensator C, n = 4).

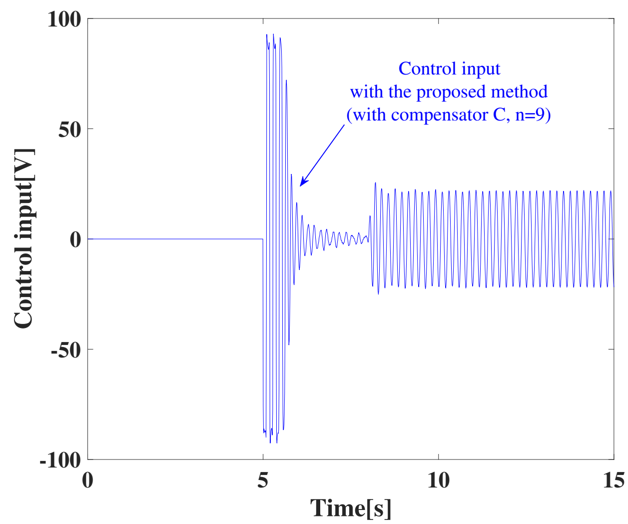

Figure 3.7: Control input of Figure 3.5 (with compensator C, n = 9).

be guaranteed by the designed controllers A and B.

Next, for confirming the tracking performance and perturbations compensation performance by the condition of $n \to \infty$, the results of $n = 4$ and of $n = 9$ are compared in without exceeding the maximum control input. The corresponding comparisons of the outputs are shown in Fig. 3.5. Corresponding control inputs of $n = 4$ and of $n = 9$ are shown in Figs. 3.6 and 3.7, respectively. From above results, we can observe the output with case of $n = 9$ is stabilized quickly than with case of $n = 4$ at the $5\text{s} < t < 8\text{s}$. And, at the $t \geq 8\text{s}$, the vibration with the case of $n = 9$ is suppressed well than with the case of $n = 4$. The effectiveness of the proposed design scheme is confirmed.

3.4 Experiments

In this experiment, the sampling time is 0.001s. The density function $p(h) = 0.00038e^{-0.00086(h-1)^2}$ where $h \in [0, 30]$ of the P-I hysteresis model is used in this experiment. In flexible plate with a free vibration case, the flexible plate is added a vibration at the $t \leq 5$s by reciprocating movement of servo-motor. Then $t > 5$s, we stopped the reciprocating movement of servo-motor to make a free vibration of the flexible plate. At the same time, we start to control the vibration of the flexible plate with a free vibration. In addition, for considering the perturbations, we started the reciprocating movement of servo-motor to make perturbations when $t > 8$s. The parameters of the designed controllers are shown in Table 3.2.

Table 3.2: Parameters of controllers

Parameter	Value	Units
K_m	0.75	–
a_0	0.165	–
c_0	3	–
k_p	40	–
k_d	3	–
k_0	0.8	–
k	0.9	–
M_t	1.25	–
Sampling Time	0.001	s
Experiment Time	15	s

In the flexible plate with a free vibration case, results of outputs with(blue line) and without(red line) control are shown in Fig. 3.8. In this case, the compensator C ($n = 5$) is used. The corresponding control input of this case is shown in Fig. 3.9. From Figs. 3.8 and 3.9, we can confirm the stability of the nonlinear system can be guaranteed by the proposed method. Also, we can observe that the output of the system with the proposed method is stabilized faster than without control.

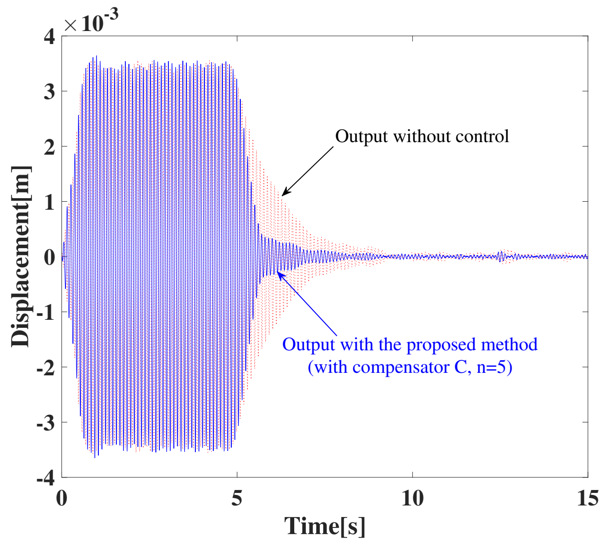

Figure 3.8: Outputs with and without control (with compensator C, n = 5).

In the flexible plate with a free vibration and sudden perturbations case, results of outputs with compensator C (n = 5) and (n = 8) are compared in without exceeding the maximum control input. The corresponding control inputs for the case in Fig. 3.10 are shown in Figs. 3.11 and 3.12, respectively. From Figs. 3.10, 3.11, and 3.12, when the flexible plate with a free vibration and sudden perturbations, we can confirm the output with case of n = 8 is stabilized quickly than case of n = 5, and the vibration is suppressed well than the case of n = 5. From the above results, effectiveness of the proposed design scheme is confirmed by experimental results.

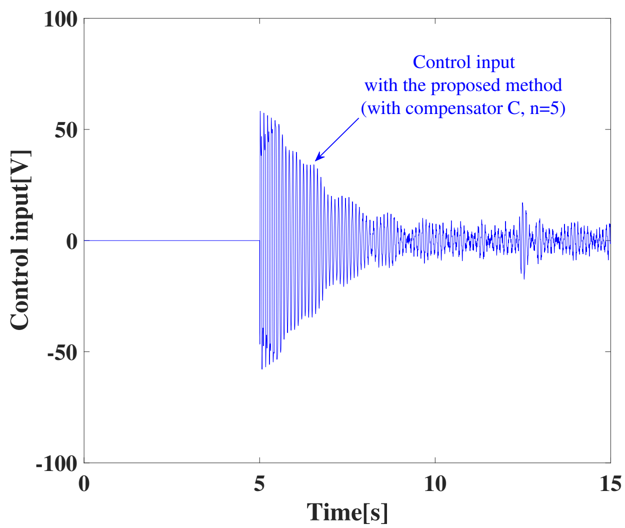

Figure 3.9: Control input of Figure 3.8 (with compensator C, n = 5).

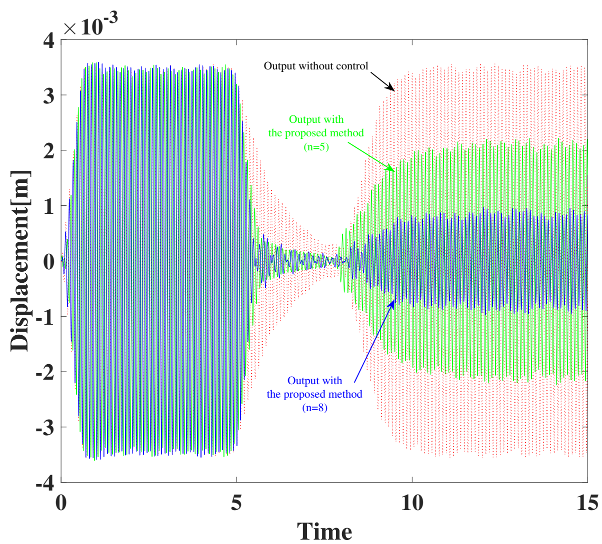

Figure 3.10: Comparison between the proposed method n = 5 and n = 8.

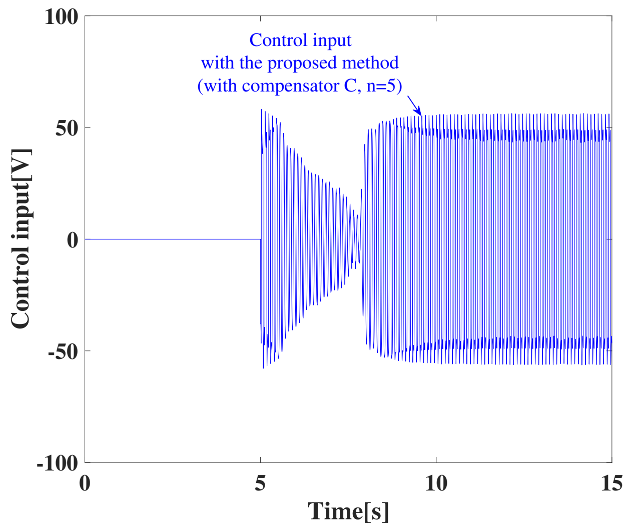

Figure 3.11: Control input of Figure 3.10 (with compensator C, n = 5).

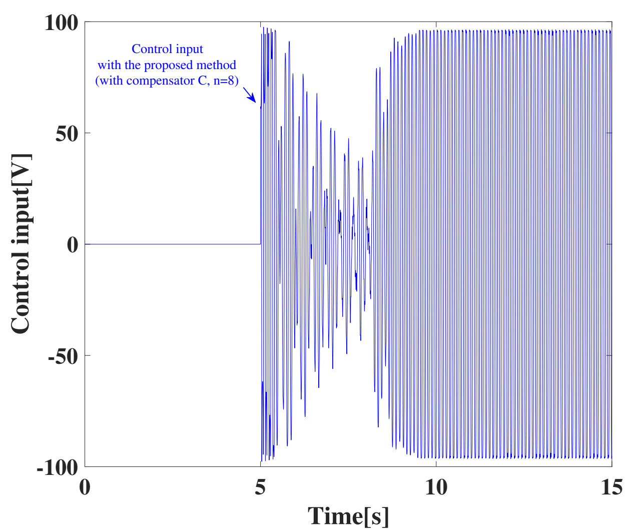

Figure 3.12: Control input of Figure 3.10 (with compensator C, n = 8).

3.5 Conclusion

In this chapter, for considering the flexible plate with a free vibration and sudden perturbations, a nonlinear vibration control scheme is given by using piezoelectric actuator with considering hysteresis nonlinearity. For guaranteeing robust stability of the nonlinear control system, the controllers are designed based on operator theory. At the same time, operator-based compensation method is designed to ensure the desired vibration control performance of the flexible plate with a free vibration and sudden perturbations. Also, in the proposed design scheme, the compensation conditions of tracking and of perturbations are given by increasing the number of designed n-times feedback loops. In the numerical simulation and experiment, when increasing the number of designed n-times feedback loops, we can be observed that vibration of the flexible plate can be suppressed more. Thus, the effectiveness of the proposed design scheme is verified by numerical simulation and experimental results.

Chapter 4

Operator-based nonlinear control scheme for plate with a forced vibration

4.1 Introduction

In this chapter, the operator-based nonlinear control scheme is considered for plate with a forced vibration. For expressing the hysteresis nonlinearity of the piezoelectric actuator, the P-I hysteresis model is used to describe it. Based on the dynamic model of the flexible plate, the control scheme is designed by operator-based robust right coprime factorization.

In Section 4.2, the designed control scheme is shown. At the step of designing the controller to satisfy stability, a controller including characteristics of Proportional-Integral-Differential (PID) controller is designed to satisfy Bezout equation. And that it can be controlled by only one design parameter without adjusting PID parameters. After that, for compensating the forced vibration to improve the vibration control performance, the designed operator B and D_{PI} is used to design the compensator.

In Section 4.3, numerical simulation is conducted the proposed control scheme, the simulation results are shown to confirm the effectiveness of the proposed control scheme.

In Section 4.4, the designed nonlinear control scheme is performed by experiment. The experimental results are shown to confirm the control performance of the designed control scheme.

In Section 4.5, the main contends of this chapter is summarized.

4.2 Operator-based control scheme using a controller with characteristics of PID controller

In this section, the flexible plate with a forced vibration case is considered. Operator theory can express a behavior of controlled object in time domain. The designed nonlinear control system is shown in Fig. 4.1. In Fig. 4.1, the control input u is inputs of piezoelectric actuators that stuck on the controlled object. The output y is the displacement at the piezoelectric sensor that is stuck on the opposite side of actuators, and the target of this control is stabilizing the vibration at the flexible plate. Therefore, the target value of $r = 0$ is considered in this design scheme. The d is considered the external forces. U is input space of the P-I hysteresis model (input space of piezoelectric actuators), U^* output spaces of the Prandtl-Ishlinskii hysteresis model (output space of piezoelectric actuators) and of the external forces. Let output space of the original plant and quasi-state space be Y and W. In this control scheme, we consider the nominal vibration mode with a first-order mode, and uncertainties with second- and third-order modes.

The plant with uncertainties is considered as the following equation.

$$[P + \Delta P](u^*)(t) = (1 + \Delta)\left[J_{11} \int_0^t e^{-\alpha_{11}(t-\tau)} \sin \beta_{11}(t - \tau) u^*(\tau) d\tau\right] \tag{4.1}$$

where Δ denote uncertainties.

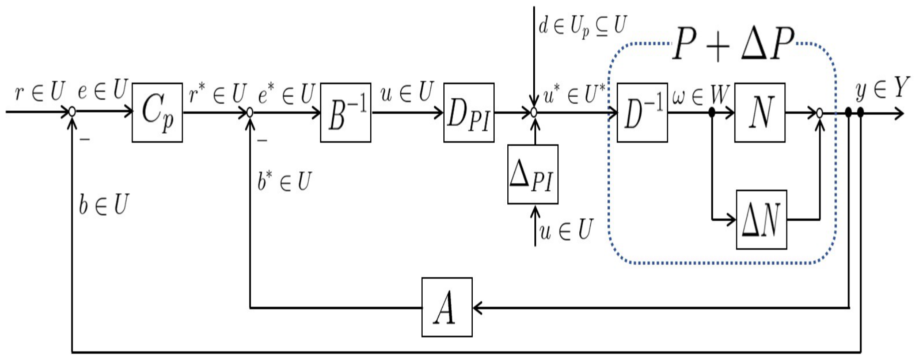

Figure 4.1: Proposed control system for plate with a forced vibration.

4.2.1 Controllers design for stability

From Eq. (4.1), the plant $[P + \Delta P]$ can be right factorized as follows.

$$[P + \Delta P](u^*)(t) = (N + \Delta N)D^{-1}(u^*)(t)$$

$$[N + \Delta N](\omega)(t) = (1 + \Delta) \cdot \left[J_{11} \int_0^t e^{-\alpha_{11}(t-\tau)} \sin \beta_{11}(t-\tau)\omega(\tau)d\tau \right] \tag{4.2}$$

$$D(\omega)(t) = I(\omega)(t) \tag{4.3}$$

where I is identity operator. In this control scheme, the $\tilde{D} = D_{PI}^{-1}D$ is considered by the invertible and linearly controllable part D_{PI} of the P-I hysteresis model. Also, the remaining part Δ_{PI} of the P-I hysteresis model is considered as a bounded uncertainty. For satisfying the following Bezout identity, controllers A and B are designed.

$$AN + B\tilde{D} = M \tag{4.4}$$

where M is an unimodular operator. If the designed controllers A and B that satisfy above Bezout identity, the designed controllers can guarantee BIBO stability of the nonlinear system. Also, the designed controllers A and B are shown in the following equations.

$$A(y)(t) = \frac{K_m}{\beta_{11}J_{11}}\int_0^t (\ddot{y}(\tau) + 2\alpha_{11}\dot{y}(\tau) + (\alpha_{11}^2 + \beta_{11}^2)y(\tau))d\tau \tag{4.5}$$

$$B(u)(t) = D_{PI}u(t) \tag{4.6}$$

where K_m is a designed parameter. In addition, under the following equations are satisfied, robust stability of the designed nonlinear control system can be guaranteed.

$$A(N + \Delta N) + B\tilde{D} = \tilde{M} \tag{4.7}$$

$$\| \, [A(N + \Delta N) - AN]M^{-1} \, \|_{Lip} < 1 \tag{4.8}$$

where $\tilde{M}$ is an unimodular operator.

4.2.2 Design a compensator

Considering Fig. 4.1, the output $y(t)$ can be expressed by

$$\begin{aligned} y(t) &= (N + \Delta N)D^{-1}(D_{PI}B^{-1}(r^*(t) - b^*(t)) \\ &\quad + \Delta_{PI}(u)(t) + d(t)) \\ &= (N + \Delta N)(A(N + \Delta N) + B\tilde{D})^{-1} \\ &\quad \cdot (r^*(t) + BD_{PI}^{-1}(\Delta_{PI}(u)(t) + d(t))) \\ &= (N + \Delta N)\tilde{M}^{-1}(r^*(t) + BD_{PI}^{-1}\tilde{\Delta}) \end{aligned} \tag{4.9}$$

where $\tilde{\Delta} = \Delta_{PI}(u)(t) + d(t)$. The equivalent system of Eq. (4.9) is shown in Fig. 4.2. From Eq. (4.9) and Fig. 4.2, the stability of the nonlinear control system can be satisfied by the designed controllers A and B. However, the hysteresis nonlinearity and the external forces affect vibration control performance. Therefore, for improving the vibration control performance, we need to compensate the hysteresis

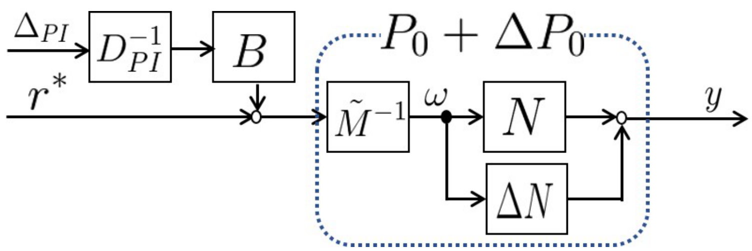

Figure 4.2: Equivalent system of Eq. (4.9).

nonlinearity and the external forces. The designed compensator is shown in Eq. (4.10).

$$C_p(e)(t) = BD_{PI}^{-1}k_P(e)(t) \tag{4.10}$$

where k_p is a designed parameter. Also, the operators B and D_{PI} are used to design the compensator C_p. From Figs. 4.1 and 4.2, the output $y(t)$ is represented in Eq. (4.11).

$$\begin{aligned} y(t) &= (N + \Delta N)(\tilde{M} + (N + \Delta N)BD_{PI}^{-1}k_p)^{-1} \\ &\quad \cdot(BD_{PI}^{-1}k_p(r)(t) + BD_{PI}^{-1}\Delta_{dis}) \\ &= (N + \Delta N)(\tilde{M} + (N + \Delta N)BD_{PI}^{-1}k_p)^{-1} \\ &\quad \cdot(BD_{PI}^{-1}\Delta_{dis}) \end{aligned} \tag{4.11}$$

where $\Delta_{dis} = \Delta_{PI}(u)(t) + d(t)$. From Eq. (4.11), the compensations of hysteresis nonlinearity and of the external forces are considered by adjusting the designed parameters K_m and k_p.

4.3 Numerical simulations

In this section, effectiveness of the proposed control scheme will be discussed by numerical simulations. The simulation results show the vibration control performance of the flexible plate. In this simulations, only used the first-order mode of the flexible plate as the nominal plant. The second- and third-order modes of this simulation are treated as uncertainties. The sampling time is 0.01s. The flexible plate is added the moment $f_d(t) = 0.08\sin(2\pi f_0 t)$ at the bottom of it to make a vibration when $t \le 5$s, where $f_0 = 32.74/2\pi$[Hz] is eigenfrequency of the flexible plate. Then $t > 5$s, we start to control the vibration of the flexible plate with free vibration. The density function $p(h) = 0.00038e^{-0.00086(h-1)^2}$, where $h \in [0, 30]$ of the P-I hysteresis model is used in this simulation. The designed parameters are shown in Table 4.1. In this simulation, we define with controllers A and B case as "proposed method 1", and we further define with controllers A, B and compensator case as "proposed method 2".

Table 4.1: Designed parameters

Parameter	Value	Units
K_m	2.5	–
k_p	40	–
Sampling Time	0.01	s
Simulation Time	15	s

Simulation results are shown in the following figures.

In Fig. 4.3, the result of using proposed method 1 case is shown. The corresponding control input to piezoelectric actuators is shown in Fig. 4.4. From Figs. 4.3 and 4.4, when the flexible plate with a forced vibration, we can confirm that the stability of the nonlinear system can be guaranteed by the proposed method 1.

Next, for confirming the hysteresis nonlinearity and the external forces compensation performances by the proposed method 2, the result of with proposed method 2 is shown in Fig. 4.5. Corresponding control inputs of Fig. 4.5 is

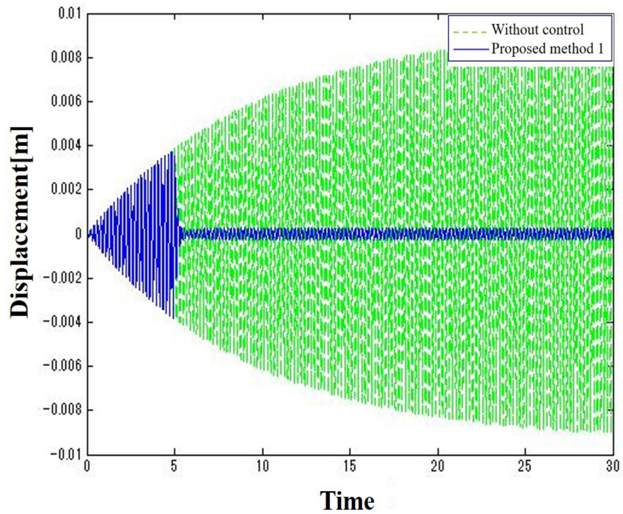

Figure 4.3: Outputs of the system with(without compensator C_p) and without control.

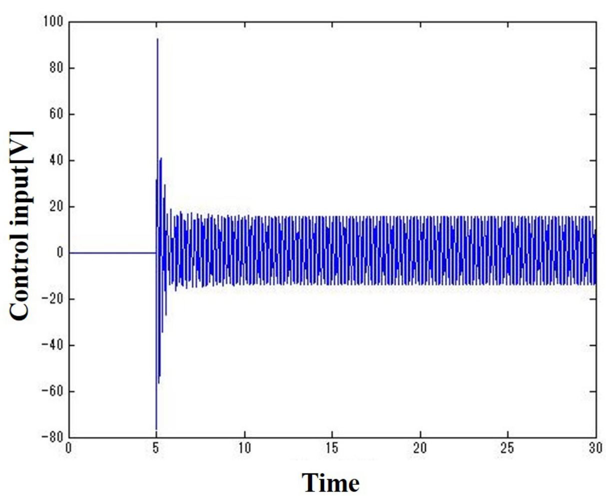

Figure 4.4: Control input of Figure 4.3.

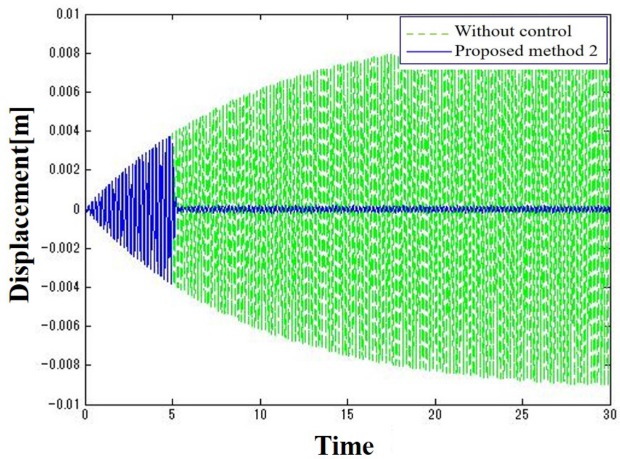

Figure 4.5: Outputs of the system with(with compensator C_p) and without control.

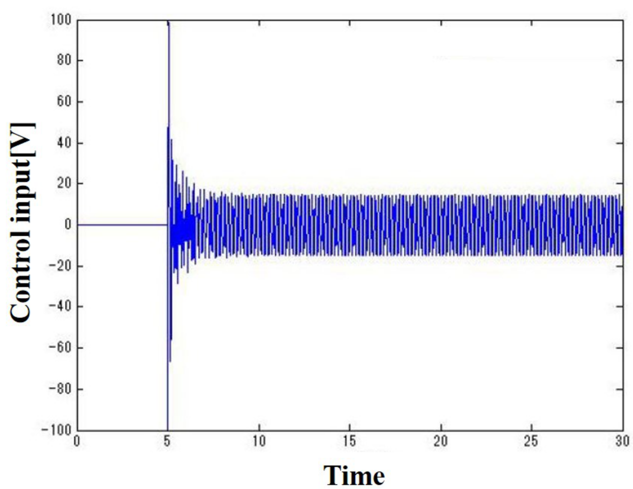

Figure 4.6: Control input of Figure 4.5.

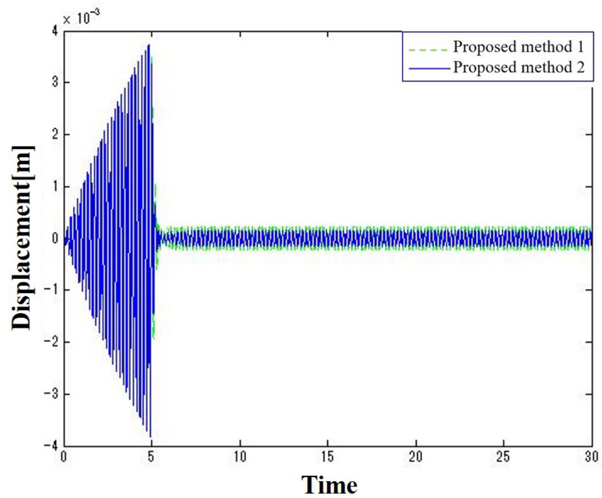

Figure 4.7: Comparison between with and without compensator C_p.

shown in Fig. 4.6. Also, results of comparison between proposed method 1 and proposed method 2 are shown in Fig. 4.7. From above results, we can confirm the vibration with proposed method 2 is suppressed more than proposed method 1. The effectiveness of the proposed design scheme is confirmed.

4.4 Experiments

The experimental devices and detailed explanation of the experimental system are introduced in Chapter 2. Based on the experimental devices, the designed control scheme is performed in this section. In flexible plate with a forced vibration case, the flexible plate is added a vibration at the $t \leq 5$s by reciprocating movement of servo-motor. At the same time, we start to control the vibration of the flexible plate with a free vibration. In this experiment, the sampling time is 0.001s. The density function $p(h) = 0.00038e^{-0.00086(h-1)^2}$ where $h \in [0, 30]$ of the P-I hysteresis model is used in this experiment. The parameters of the designed controllers are shown in Table 4.2. In this experiments, we define with controllers A and B case as "proposed method 1", and we further define with controllers A, B and compensator C_p case as "proposed method 2".

Table 4.2: Parameters of controllers

Parameter	Value	Units
K_m	0.025	–
k_p	0.005	–
Sampling Time	0.001	s
Experiment Time	15	s

The result of using proposed method 1 case is shown in Fig. 4.8. The corresponding control input to piezoelectric actuators is shown in Fig. 4.9. From Figs. 4.3 and 4.4, when the flexible plate with a forced vibration, we can observe that the stability of the nonlinear system can be guaranteed by the proposed method 1. Next, for confirming the compensation performance, result of proposed method 2 is shown in Fig. 4.10, and control input in this case is shown in Fig. 4.11. Also, results of comparison between proposed method 1 and proposed method 2 are shown in Fig. 4.12. Also, for confirming the effectiveness of the designed control scheme, the variance and standard deviation of proposed method 1 and proposed method 2 are shown Table 4.3.

From above results, we can confirm the vibration with compensator C_p case is suppressed more than without compensator C_p case. The effectiveness of the proposed design scheme is confirmed.

4.5 Conclusion

In this chapter, for considering the flexible plate with a forced vibration, a nonlinear forced vibration control scheme is given by using piezoelectric actuator with con-

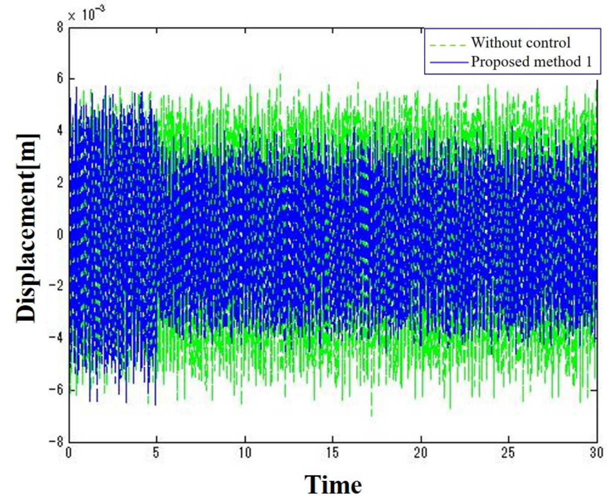

Figure 4.8: Outputs of the system with(without compensator C_p) and without control.

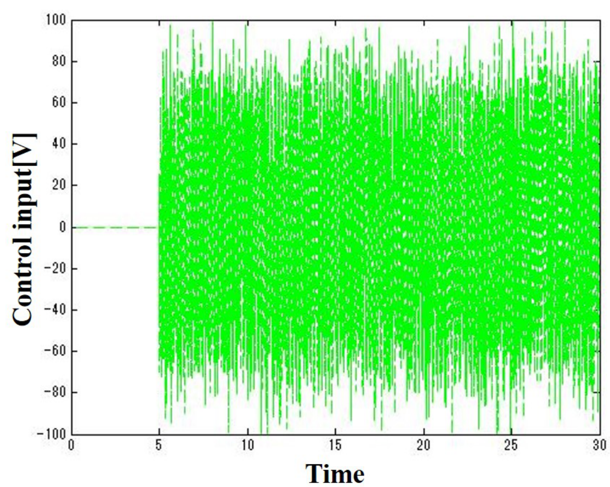

Figure 4.9: Control input of Figure 4.8.

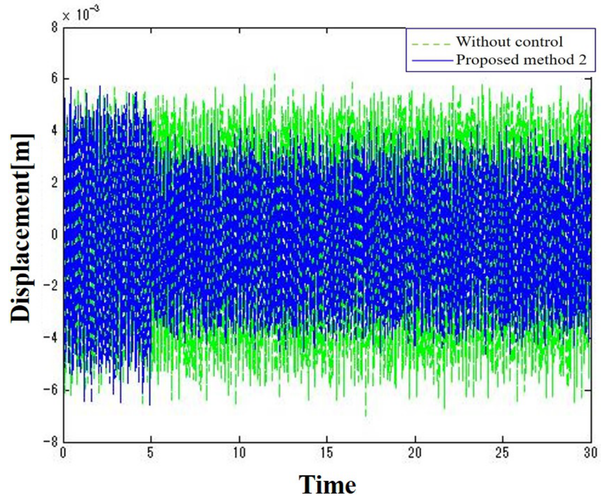

Figure 4.10: Outputs of the system with(with compensator C_p) and without control.

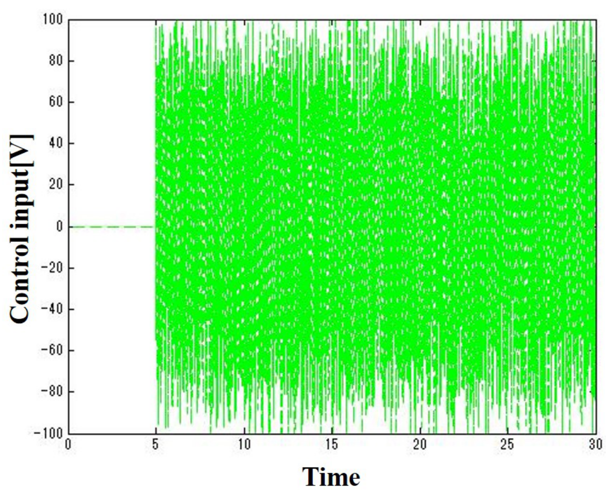

Figure 4.11: Control input of Figure 4.10.

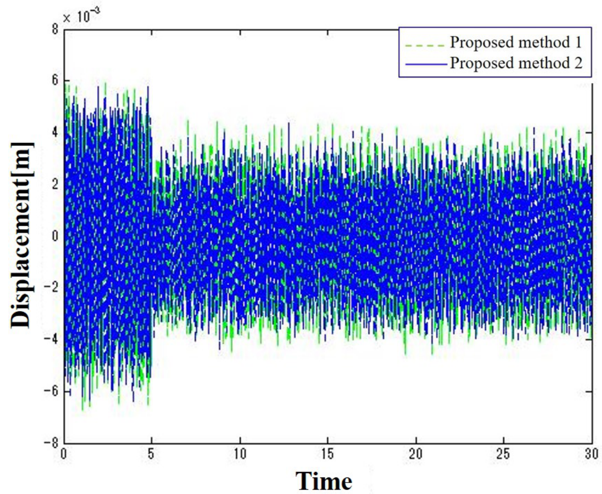

Figure 4.12: Comparison between with and without compensator C_p.

Table 4.3: Variance and Standard deviation

	Variance	Standard deviation
Without C_p	7.2448×10^{-6}	2.692×10^{-3}
With C_p	4.7661×10^{-6}	2.183×10^{-3}

sidering hysteresis nonlinearity. For guaranteeing robust stability of the nonlinear control system, the controllers are designed based on operator theory. At the same time, a controller including characteristics of Proportional-Integral-Differential (PID) controller is considered in this control scheme. And that it can be controlled by only one design parameter without adjusting PID parameters. For improving the forced vibration control performance, the compensator is designed. Then, the effectiveness of the proposed design scheme is verified by numerical simulation and experimental results.

Chapter 5

Operator-based vibration control approaches: Some new extensions

5.1 Introduction

In Chapter 4, for plate with a forced vibration case, operator based control scheme is discussed. In control scheme of Chapter 4, the controller including characteristics of Proportional-Integral-Differential (PID) controller is proposed. And, the compensation conditions of forced vibration and of hysteresis nonlinearity are considered by increasing the gain of designed compensator in outer loop. However, it is difficult to ensure the desired nonlinear vibration control performance. In this chapter, for improving the vibration control performance, operator based some new vibration control approaches are discussed.

In Section 5.2, for guaranteeing the robust stability of the noninear forced vibration control system, operator based controllers are designed. Simultaneously, for improving forced vibration control performance, the time-varying unimodular function is constructed by the designed controllers. If the inverse of the time-varying unimodular function tends to zero by the operator-based controllers and designed compensator, the output can be made arbitrarily small. Based on the designed nonlinear control system, effectiveness of the proposed control scheme is discussed by numerical simulations and experiments.

In Section 5.3, the system mismatching compensation method is considered. The designed nonlinear control system consists of three parts, the real control system, reference control system, and system mismatching compensation unit. With considering the effect of hysteresis nonlinearity from piezoelectric actuators, operator based controllers are designed to guarantee the stability of the nonlinear system. Simultaneously, for improving vibration control performance, the system mismatching compensation unit is given by the designed reference system and compensator. The simulation and experimental results are shown to confirm the control performance of the designed control scheme.

In Section 5.4, operator based unknown input nonlinearity compensation approach is discussed. With considering the effect of unknown input nonlinearity from the piezoelectric actuator, operator based controllers are designed to guarantee the robust stability of the nonlinear free vibration control system. Simultaneously, for ensuring the desired tracking performance and reducing the effect of unknown input nonlinearity, operator based tracking compensator and estimation structure are given, respectively. Finally, both simulation and experimental results are shown to verify the effectiveness of the designed control scheme.

In Section 5.5, the conclusion of this chapter is given.

5.2 Time-varying unimodular function based robust right coprime factorization approach

In this section, according to the obtained models, the nonlinear forced vibration control design scheme is proposed. First, in order to guarantee the stability of the nonlinear system with hysteresis nonlinearity, operator based nonlinear control system is designed. After that, the compensator is designed to guarantee the desired vibration control performance of the flexible plate. The designed nonlinear control system is shown in Fig. 5.1. In Fig. 5.1, $u(t)$ is control input to piezoelectric actuators, the output $y(t)$ denotes displacement of the flexible plate, the target value $r(t) = 0$ is the output displacement to become zero, and we consider the external force as a bounded disturbance $d(t)$.

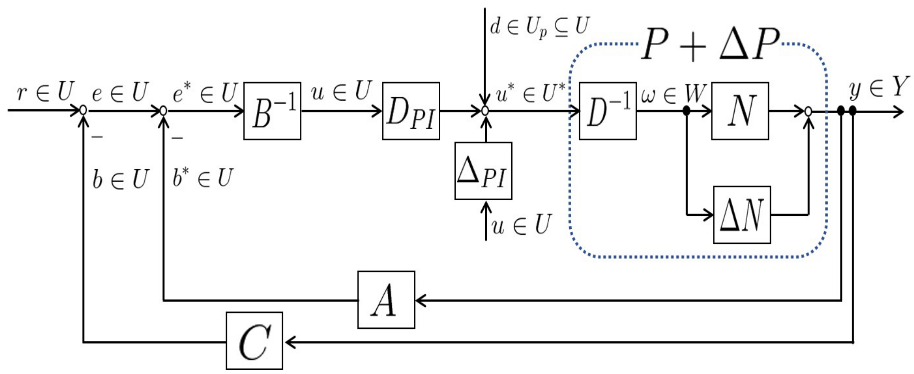

Figure 5.1: Proposed control system.

$$[P + \Delta P](u^*)(t) = (1 + \Delta)\left[J_{11}\int_0^t e^{-\alpha_{11}(t-\tau)} \sin\beta_{11}(t-\tau)u^*(\tau)d\tau\right] \qquad (5.1)$$

where Δ denote uncertainties.

5.2.1 Design scheme

Operator based controllers are designed in this Section. In this paper, we consider the nominal vibration mode with a first-order mode and the uncertainties with the second- and third-order modes. Therefore, the plant with uncertainties is considered as the following equation. The plant $[P + \Delta P]$ is right factorized by the following equations.

$$[P + \Delta P](u^*)(t) = (N + \Delta N)D^{-1}(u^*)(t) \tag{5.2}$$

$$[N + \Delta N](w)(t) = (1 + \Delta) \cdot J_{11} \int_0^t e^{-\alpha_{11}(t-\tau)} \cdot \sin \beta_{11}(t - \tau) \cdot \omega(\tau)d\tau \tag{5.3}$$

$$D(\omega)(t) = I(\omega)(t) \tag{5.4}$$

where I is identity operator, J_{11}, α_{11} and β_{11} are model parameters of the nominal plant, respectively. In Fig. 5.1, the invertible and linearly controllable part D_{PI} is used for the design of the controller and the remaining part Δ_{PI} treat as a bounded disturbance. The operator $\tilde{D}$ is treated as $\tilde{D} = D_{PI}^{-1}D$. And, when there exist two stable operators A and B that satisfy Bezout identity, the designed controllers can guarantee BIBO stability of the nonlinear system.

$$AN + B\tilde{D} = L \tag{5.5}$$

where L is an unimodular operator. In addition, if under the following conditions are satisfied, then robust stability of the designed nonlinear control system can be guaranteed.

$$A(N + \Delta N) + B\tilde{D} = \tilde{L} \tag{5.6}$$

$$\| \ [A(N + \Delta N) - AN]L^{-1} \ \|_{Lip} < 1 \tag{5.7}$$

where $\tilde{L}$ is an unimodular operator. The designed controllers A, B, and the compensator C are shown as follows.

$$A(y)(t) = \left(\frac{k_a}{4} + k_b Q^{-c_0}(t)\right)(2\dot{y}(t) + k_a y(t)) \tag{5.8}$$

$$B(u)(t) = D_{PI}(u)(t) \tag{5.9}$$

$$C(y)(t) = \frac{k_c s(t)}{|s(t)| + c_1} \tag{5.10}$$

where $s(t) = 2\dot{y}(t) + k_a y(t)$, k_a, k_b, c_0, k_c, and c_1 are designed parameters, respectively. And,

$$Q^{-c_0}(t) = \left(\frac{(\dot{y}(t) + k_a y(t))^2}{2} + \frac{\dot{y}^2(t)}{2}\right)^{-c_0}$$

where $0 < c_0 < 0.5$ is a designed parameter. When the designed controllers A and B satisfy conditions of Eqs. (5.5), (5.6), and (5.7), the output can be obtained as the following equation.

$$\begin{aligned} y(t) &= (N + \Delta N)D^{-1}(D_{PI}B^{-1}(e(t) - b_0(t)) + \Delta_{PI}(u)(t) + d(t)) \\ &= (N + \Delta N)(A(N + \Delta N) + B\tilde{D})^{-1}(e(t) + BD_{PI}^{-1}\Delta_{dis}(t)) \\ &= (N + \Delta N)\tilde{L}^{-1}(e(t) + BD_{PI}^{-1}\Delta_{dis}(t)) \end{aligned} \tag{5.11}$$

where $\Delta_{dis}(t) = d(t) + \Delta_{PI}(u)(t)$.

In Eq. (5.11), when the inverse of time-varying unimodular function $\tilde{L}^{-1} \to 0$ can be satisfied by the time-varying parameter $k_b Q^{-c_0}(t) \to \infty$, the output can be made arbitrarily small. And, only in the case of $A(y)(t) = 0$ the time-varying parameter $k_b Q^{-c_0}(t) = \infty$ because of $y(t) = 0$ and $\dot{y}(t) = 0$.

For obtaining the inverse of time-varying unimodular function $\tilde{L}^{-1} \to 0$ by the time-varying parameter $k_b Q^{-c_0}(t) \to \infty$, the differential of $Q(t)$ is considered in this paper. The $Q(t)$ and differential of the $Q(t)$ are shown in the following equations.

$$Q(t) = \left(\frac{(\dot{y}(t) + k_a y(t))^2}{2} + \frac{\dot{y}^2(t)}{2}\right) \tag{5.12}$$

$$\begin{aligned} \dot{Q}(t) &= \frac{\partial Q(t)}{\partial y(t)}\frac{dy(t)}{dt} + \frac{\partial Q(t)}{\partial \dot{y}(t)}\frac{d\dot{y}(t)}{dt} \\ &= (\dot{y}(t) + k_a y(t))(\ddot{y}(t) + k_a \dot{y}(t)) + \dot{y}(t)\ddot{y}(t) \\ &= k_a \dot{y}^2(t) + k_a^2 y(t)\dot{y}(t) + (2\dot{y}(t) + k_a y(t))\ddot{y}(t) \end{aligned} \tag{5.13}$$

Simultaneously, according to the obtained models and Eq. (5.11), the following equation is considered.

$$\ddot{y}(t) = r_0(y, \dot{y}, \Delta, d, \Delta_{PI}, t) + D_{PI}(u)(t) \tag{5.14}$$

where $u(t)$ is the control input, and we consider the function of $r_0(y, \dot{y}, \Delta, d, \Delta_{PI}, t)$ that included the uncertainties Δ, the disturbance $d(t)$, and the remaining part Δ_{PI}. And,

$$|r_0(y, \dot{y}, \Delta, d, \Delta_{PI}, t)| \leq r_p, (r_p > 0) \tag{5.15}$$

$$\begin{aligned} D_{PI}(u)(t) &= D_{PI}B^{-1}(e_0)(t) \\ &= -D_{PI}B^{-1}(A(y)(t) + C(y)(t)) \\ &= -I(A(y)(t) + C(y)(t)) \end{aligned} \tag{5.16}$$

where r_p is a positive number, and I denotes identity operator. From Eq. (5.15), we consider the function of $r_0(y, \dot{y}, \Delta, d, \Delta_{PI}, t)$ is bounded. Therefore, the condition of Eq. (5.17) can be obtained by substituting Eqs. (5.14), (5.15), and (5.16) to Eq. (5.13).

$$\begin{aligned} \dot{Q}(t) &= k_a\dot{y}^2(t) + k_a^2 y(t)\dot{y}(t) + s(t)\ddot{y}(t) \\ &= k_a\dot{y}^2(t) + k_a^2 y(t)\dot{y}(t) \\ &\quad + s(t)(r_0(y, \dot{y}, \Delta, d, \Delta_{PI}, t) + D_{PI}(u)(t)) \\ &\leq -\frac{k_a^3}{4}y^2(t) - k_b Q^{-c_0}(t)s^2(t) \\ &\quad -|s(t)|(\frac{k_c|s(t)|}{|s(t)| + c_1} - r_p) < 0 \end{aligned} \tag{5.17}$$

where $s(t) = 2\dot{y}(t) + k_a y(t)$, and the parameters k_c and c_1 are designed to satisfy condition of $\frac{k_c|s(t)|}{|s(t)|+c_1} > r_p$. From Eq. (5.13), the condition of $\dot{Q}(t) < 0$ can be obtained by the designed controllers A, B, and the compensator C. Therefore, condition of the $Q(t) \to 0$ can be guaranteed by the designed controllers. Thus, the output can be made arbitrarily small by the designed inverse of time-varying unimodular function $\tilde{L}^{-1} \to 0$.

5.2.2 Numerical simulations

In this section, effectiveness of the proposed design scheme will be discussed by numerical simulations. The designed parameters are shown in Table 5.1. In this simulation, only used first-mode of the flexible plate as the nominal plant. The second- and third-modes of this simulation are contained as uncertainties. There use density function $p(h) = 0.00032 \times e^{-0.00086(h-1)^2}$ where $h \in [0, 30]$.

Table 5.1: Parameters of controllers

Parameter	Value	Units
K_a	4	–
k_b	0.1	–
c_0	0.38	–
k_c	0.19	–
c_1	0.005	–
Sampling Time	0.01	s
Simulation Time	15	s

Simulation results of vibration control for the flexible plate are shown in the following figures. The control of flexible plate's vibration starts at 5s. The external force of the added vibration is $F(t) = 0.1\sin(2\pi f_0 t)$. Where $f_0 = 32.74/2\pi$[Hz] is the eigenfrequency of the flexible plate. In Fig. 5.2, outputs of the flexible plate with and without control are shown. In with control, there is not considered the compensator C. Corresponding control input in Fig. 5.2 is shown in Fig. 5.3. Next in Fig. 5.4, outputs of the flexible plate with(blue line) and without(green line) control are shown. In with control, the controllers A, B and compensator C

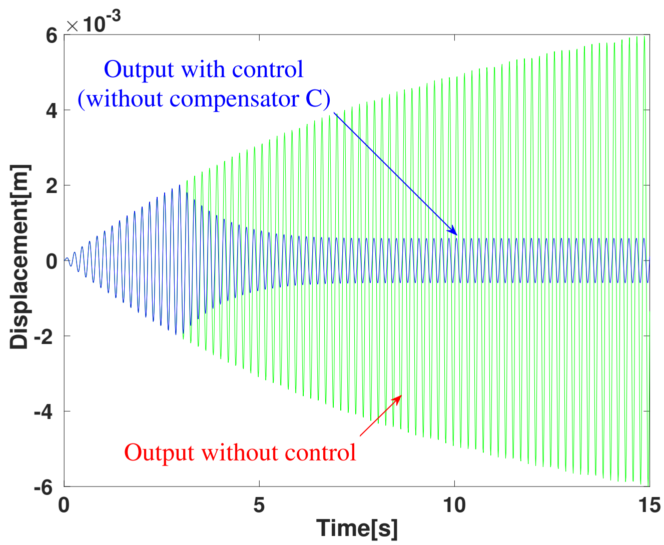

Figure 5.2: Outputs with operators A and B only case and without control case.

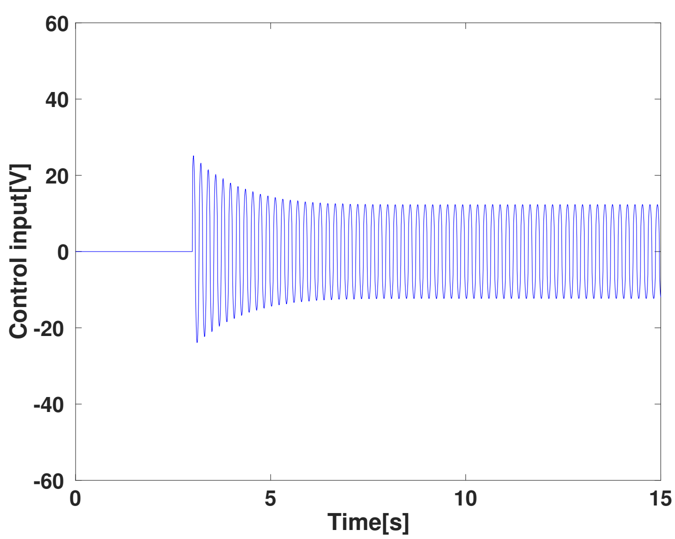

Figure 5.3: Control input (with operators A and B only case).

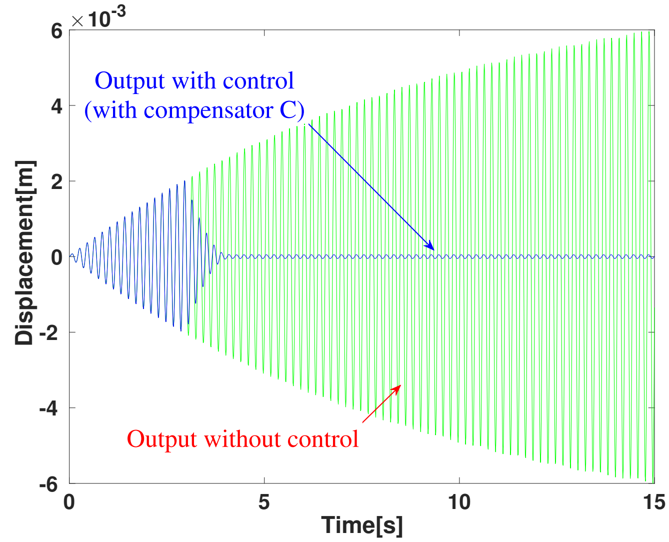

Figure 5.4: Outputs with compensator C case and without control case.

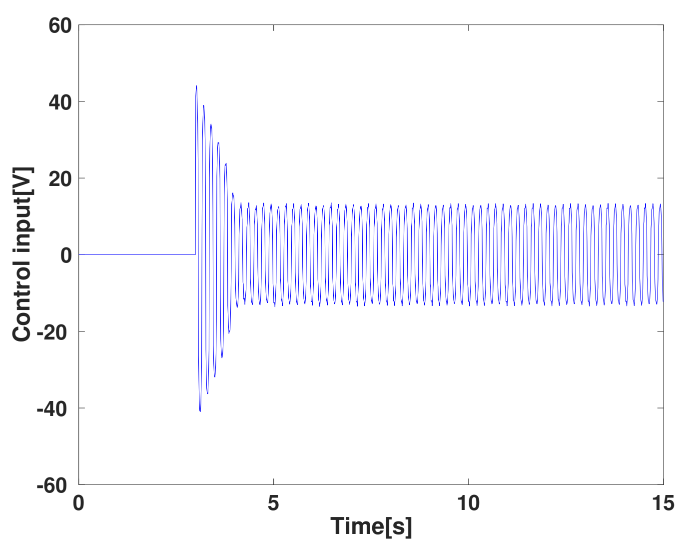

Figure 5.5: Control input (with compensator C case).

are considered in Fig. 5.4. Corresponding control input in Fig. 5.4 is shown in Fig. 5.5. From the above results, stability of the flexible plate is guaranteed in Fig. 5.2, and effectiveness of the proposed control scheme is confirmed in Fig. 5.4. Thus, effectiveness of the designed nonlinear forced vibration control system is confirmed.

5.2.3 Experiments

In this section, the proposed design scheme is confirmed by experiment. The used parameters of the designed controllers are shown in Tables 5.2. In this experiment, vibration generated at the flexible plate is 10Hz. If $t \geq 3$s, controllers are started to control the vibration of the flexible plate with forced vibration. The sampling time is 0.001s and the experiment time is 15s.

Table 5.2: Designed parameters

Parameter	Value	Units
K_a	4	–
k_b	0.1	–
c_0	0.42	–
k_c	0.1	–
c_1	0.039	–
Sampling Time	0.001	s
Experiment Time	15	s

First, outputs of the flexible plate with and without control are shown in Fig. 5.6. In with control, there is not considered the compensator C. The control input of with control without considering compensator C is shown in Fig. 5.7. Next in Fig. 5.8, outputs of the flexible plate with and without control are shown. In with control, there is considered the compensator C. The corresponding control input in Fig. 5.8 is shown in Fig. 5.9.

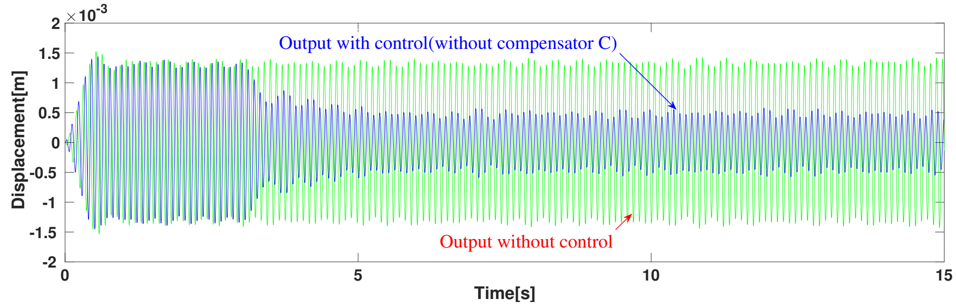

Figure 5.6: Outputs of the system with and without control(without considering compensator C).

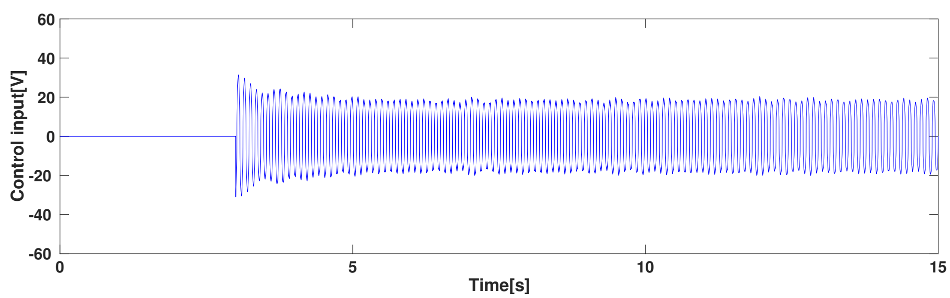

Figure 5.7: Control input(without considering compensator C).

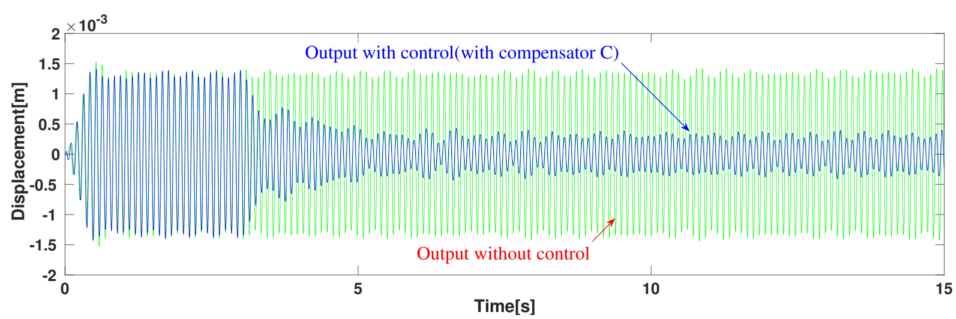

Figure 5.8: Outputs of the system with(blue line) and without(green line) control(with considering compensator C).

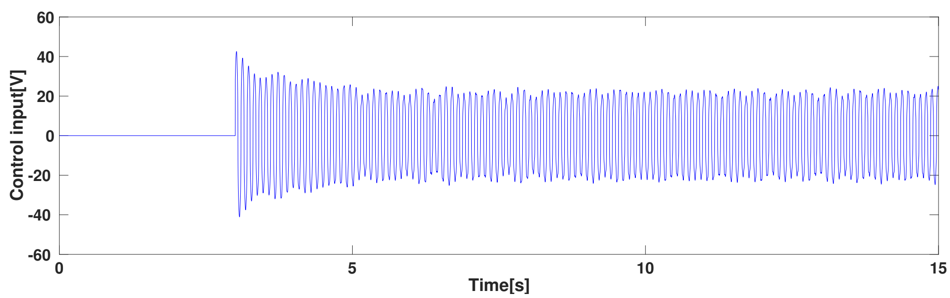

Figure 5.9: Control input(with considering compensator C).

From Fig. 5.6, we can confirm that vibration can be suppressed by operator based controllers A and B. Therefore, stability of the nonlinear control system based on operator theory is verified. The effectiveness of the with control with considering compensator C is confirmed in Fig. 5.8. Thus, effectiveness of the proposed control method is confirmed.

5.3 Operator-based system mismatching compensation approach

In this section, operator based robust nonlinear forced vibration control design scheme is proposed based on system mismatching compensation. First, in order to guarantee stability of the nonlinear system, operator based nonlinear control system is designed. After that, for improving the vibration control performance, the system mismatching compensation unit is considered by the designed reference system and compensator. The designed nonlinear control system is shown in Fig. 5.10. In Fig. 5.10, $u \in U$, $u_0 \in U$, $y \in Y$, $y_0 \in Y$, and $y_1 \in Y$ are the control input to piezoelectric actuators, control input of the reference system, displacement of the flexible plate, output of the reference system, and error between the output of the flexible plate and reference system, respectively. In this study, the conditions of disturbance $d \in U_p$ and $U_p \subseteq U$ are considered. And, the target of this control is stabilizing the vibration at the flexible plate. Therefore, the target value of $r = 0$ is considered.

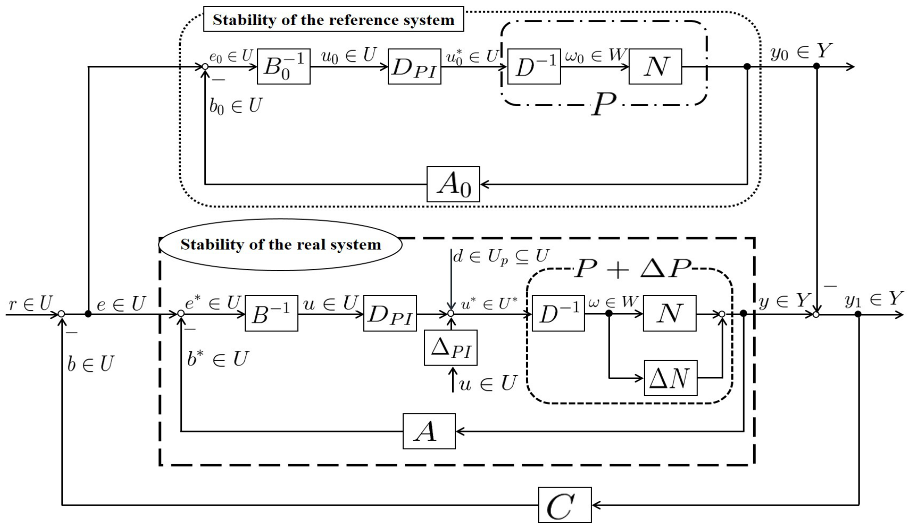

Figure 5.10: Designed nonlinear control system

5.3.1 Stability of the real system

Operator based controllers are designed in this section. In this study, we consider the nominal vibration mode with a first-order mode and the uncertainties Δ with the second- and third-order modes. Therefore, the plant with uncertainties is considered the following equation.

$$[P + \Delta P](u^*)(t) = (1 + \Delta)\left[J_{11}\int_0^t e^{-\alpha_{11}(t-\tau)} \sin\beta_{11}(t-\tau)u^*(\tau)d\tau\right] \tag{5.18}$$

Further, the plant with uncertainties can be right factorized by Eqs. (5.19) and (5.20)

$$[N+\Delta N](\omega)(t) = (1+\Delta)J_{11}\int_0^t e^{-\alpha_{11}(t-\tau)}\sin\beta_{11}(t-\tau)\omega(\tau)d\tau \tag{5.19}$$

$$D(\omega)(t) = I(\omega)(t) \tag{5.20}$$

where I is identity operator, α_{11} and β_{11} are parameters of the nominal plant, respectively. Also, the operator $\tilde{D}$ is considered as $\tilde{D} = D_{PI}^{-1}D$. Moreover, when there exist two stable operators A and B that satisfy Bezout equation, the designed controllers can guarantee BIBO stability of the nonlinear system.

$$AN + B\tilde{D}{=}M \tag{5.21}$$

where M is an unimodular operator. In addition, under the following conditions are satisfied, robust stability of the designed nonlinear control system can be guaranteed.

$$A(N+\Delta N) + B\tilde{D} = \tilde{M} \tag{5.22}$$

$$\| \, [A(N+\Delta N) - AN]M^{-1} \, \|_{Lip} < 1 \tag{5.23}$$

where $\tilde{M}$ denote an unimodular operator. The operators A and B are designed as follows.

$$A(y)(t) = \frac{(1-K_m)}{\beta_{11}J_{11}}(\ddot{y}(t) + 2\alpha_{11}\dot{y}(t) + (\alpha_{11}^2+\beta_{11}^2)y(t)) \tag{5.24}$$

$$B(u)(t) = K_m D_{PI} u(t) \tag{5.25}$$

where K_m is a designed parameter.

5.3.2 Design a system mismatching compensation unit

From Fig. 5.10 and Eq. (5.22), the output $y(t)$ can be expressed by

$$\begin{aligned} y(t)=&(N+\Delta N)D^{-1}(D_{PI}B^{-1}(e(t)-b^{*}(t)) \\ &+\Delta_{PI}(u)(t)+d(t)) \\ =&(N+\Delta N)(A(N+\Delta N)+B\tilde{D})^{-1} \\ &\cdot(e(t)+\tilde{\Delta}) \\ =&(N+\Delta N)\tilde{M}^{-1}(e(t)+\tilde{\Delta}) \end{aligned} \tag{5.26}$$

where $\tilde{\Delta} = BD_{PI}^{-1}(\Delta_{PI} + d)$. From Eq. (5.26), stability of the real system can be guaranteed by the designed operators A and B. However, due to the effect of $\tilde{\Delta}$, the desired vibration control performance cannot be obtained. Therefore, for improving the vibration control performance, the compensation of $\tilde{\Delta}$ term needed to consider. The system mismatching compensation unit is shown in Fig. 5.10. In Fig. 5.10, system mismatching compensation unit consists of the designed reference system and operator C. For compensating the $\tilde{\Delta}$, the following conditions are considered in this study.

1. For ensuring the BIBO stability of the real control system, the reference system should satisfy BIBO stability. If the reference system is BIBO stable, it can ensure that the input signal $e(t)$ is bounded.

2. The designed operator C is stable. And, the designed operator C satisfies the condition of $(I - CNM_0^{-1}) \to 0$.

First, for ensuring the above condition 1, operator based controllers are designed for the stability of the reference system. The designed operators A_0 and B_0 are shown as follows.

$$A_0(y_0)(t) = \frac{(1-K_0)}{\beta_{11}J_{11}}(\ddot{y}_0(t) + 2\alpha_{11}\dot{y}_0(t) + (\alpha_{11}^2 + \beta_{11}^2)y_0(t)) \tag{5.27}$$

$$B_0(u_0)(t) = K_0 D_{PI} u_0(t) \tag{5.28}$$

$$A_0 N + B_0 \tilde{D} = M_0 \tag{5.29}$$

Where M_0 is an unimodular operator, and K_0 is a designed parameter. Next, for obtaining the above conditions 2, the designed operator C is shown in Eq. (5.30).

$$C(y_1)(t) = \frac{(1-K_1)}{\beta_{11}J_{11}}(\ddot{y}_1(t) + 2\alpha_{11}\dot{y}_1(t) + K_2(\alpha_{11}^2 + \beta_{11}^2)y_1(t)) \tag{5.30}$$

Where K_1 and K_2 are designed parameters, respectively. From Fig. 5.10 and the designed operators, the output $y(t)$ can be represented as follows.

$$y_1(t) = y(t) - y_0(t) \tag{5.31}$$

$$y_0(t) = NM_0^{-1}(e)(t) \tag{5.32}$$

$$\begin{aligned} e(t) &= -C(y_1)(t) \\ &= -C(y)(t) + CNM_0^{-1}(e)(t) \\ &= -C(I - CNM_0^{-1})^{-1}(y)(t) \end{aligned} \tag{5.33}$$

From Eq. (5.31), the condition of $(y_1(t) + y_0(t)) \rightarrow 0$ is considered in this study. And, from Eqs. (5.26) and (5.33),

$$\begin{aligned} y(t) &= (N + \Delta N)\tilde{M}^{-1}(e(t) + \tilde{\Delta}) \\ &= (N + \Delta N)\tilde{M}^{-1}F^{-1}(I - CNM_0^{-1})(\tilde{\Delta}) \end{aligned} \tag{5.34}$$

where $F = (I - CNM_0^{-1} + (N + \Delta N)\tilde{M}^{-1}C)$.

Thus, from above conditions, condition of $y(t) \rightarrow 0$ can be obtained by the designed system mismatching compensation unit.

5.3.3 Numerical simulations

In this section, the effectiveness of the proposed design scheme will be discussed by simulation. The simulation results show the forced vibration control performance of the flexible plate. Parameters of the designed operators are shown in Tables 5.3. In this simulation, only used first-order mode of the flexible plate as the nominal plant. The second- and third-order modes of this simulation are treated as uncertainties. The density function $p(h) = 0.00038e^{-0.00086(h-1)^2}$ where $h \in [0, 30]$ is used in this simulation.

Table 5.3: Parameters of controllers.

Parameter	Value	Units
K_m	0.5	–
K_0	0.45	–
K_1	0.4	–
K_2	1	–
Sampling Time	0.01	s
Simulation Time	20	s

In Fig. 5.11, outputs of the flexible plate with and without control are shown. In with control, there is not considered the system mismatching compensation. The corresponding control input in Fig. 5.11 is shown in Fig. 5.12. Next in Fig. 5.13, outputs of the flexible plate with and without control are shown. The system mismatching compensation is considered in Fig. 5.13. The corresponding control input in Fig. 5.13 is shown in Fig. 5.14. From the above results, stability of the flexible plate is confirmed by Figs. 5.11 and 5.12. And, in Figs. 5.13, we can be observed that the vibration of the flexible plate with system mismatching compensation suppressed more than the result with operators A and B only. Therefore, effectiveness of the designed system mismatching compensation

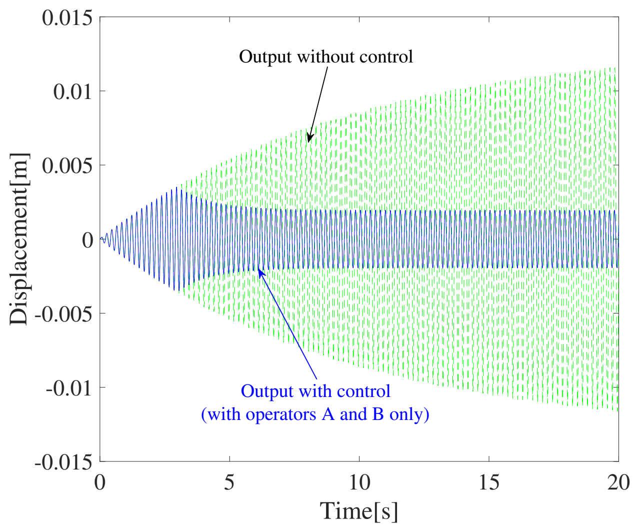

Figure 5.11: Outputs of the system with (with operators A and B only) and without control.

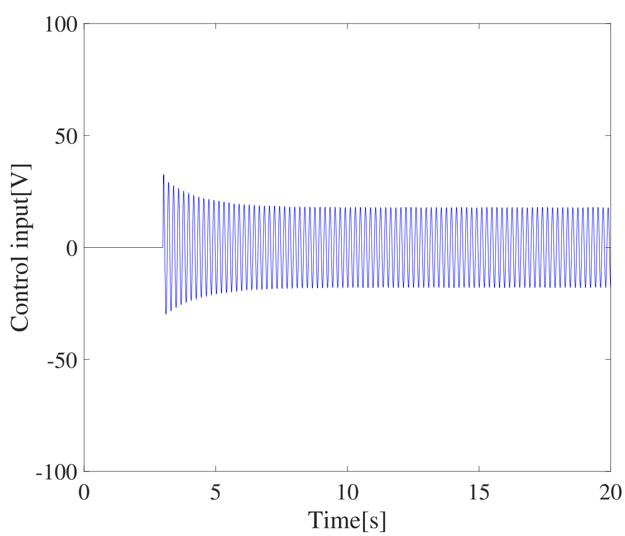

Figure 5.12: Corresponding control input in Figure 5.11.

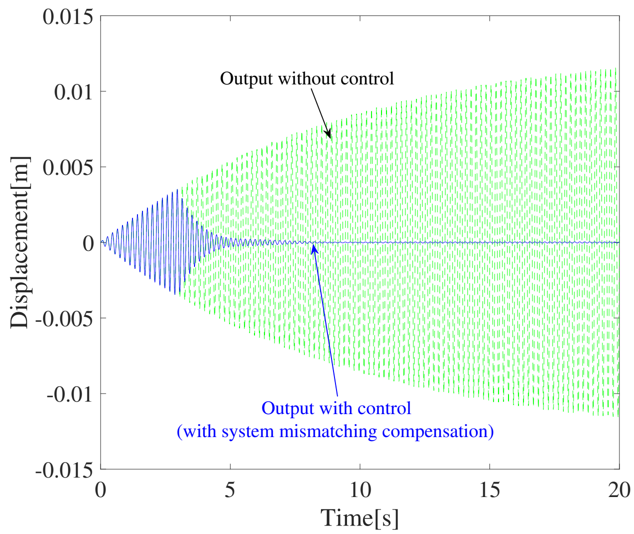

Figure 5.13: Outputs of the system with (with system mismatching compensation) and without control.

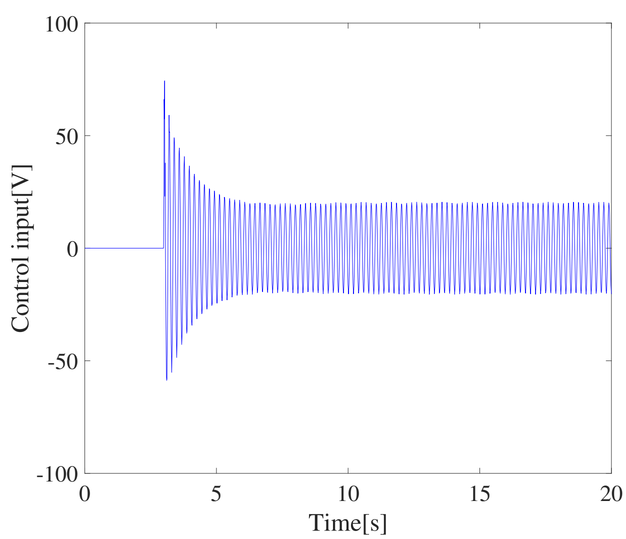

Figure 5.14: Corresponding control input in Figure 5.13.

can be confirmed by the Figs. 5.13 and 5.14. Thus, effectiveness of the proposed control design scheme is confirmed.

5.3.4 Experiments

In this section, the proposed design scheme is confirmed by experiment. The used parameters of the designed operators are shown in Tables 5.4. The frequency of the vibration is 10Hz, and the vibration of the flexible plate is generated by the servo-motor. The vibration control starts from $t \geq 3$s. The position of the piezoelectric sensor is the opposite side of the actuator. The actuator is stuck on the desired position. These positions obtain the strongest moment at the flexible plate. Although displacement of all points on the flexible plate may not become zero, we consider that all of the moments generated on the flexible plate become zero in this research, if the value of the sensor becomes zero.

Table 5.4: Parameters of controllers.

Parameter	Value	Units
K_m	0.93	–
K_0	0.936	–
K_1	0.9	–
K_2	2.3	–
Sampling Time	0.001	s
Simulation Time	15	s

First, outputs of the flexible plate with and without control are shown in Fig. 5.15. In with control, there is not considered the system mismatching compensation. The control input of Fig. 5.15 is shown in Fig. 5.16. Next in Fig. 5.17, output of the flexible plate with (with system mismatching compensation) and without control are shown. The corresponding control input in Fig. 5.17 is shown in Fig. 5.18. From Fig. 5.15, we can confirm that vibration can be suppressed by

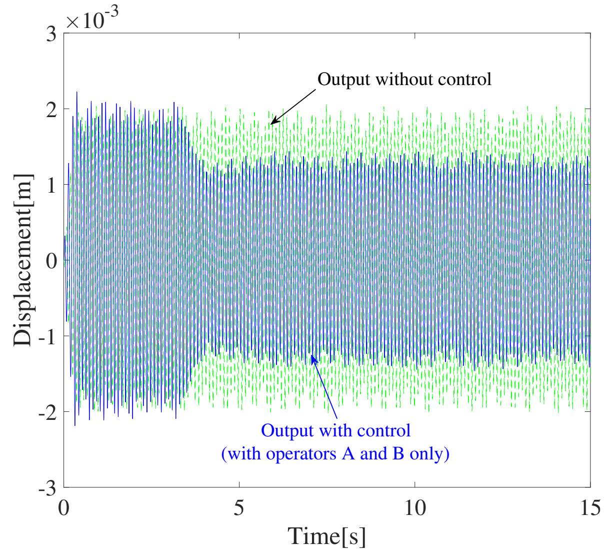

Figure 5.15: Outputs of the system with (with operators A and B only) and without control.

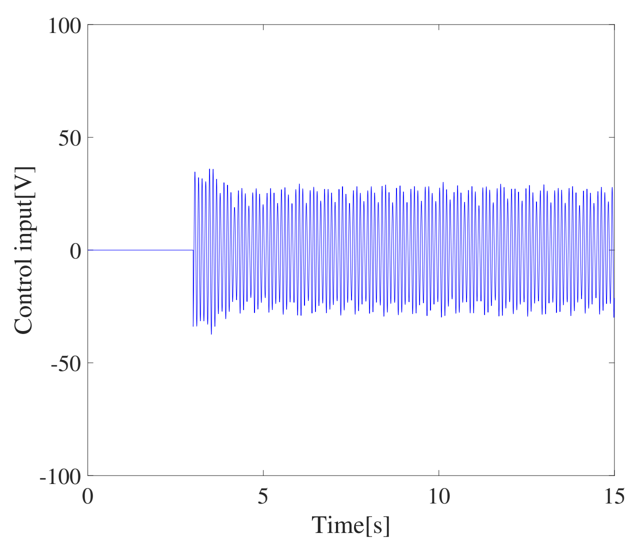

Figure 5.16: Control input of Figure 5.15.

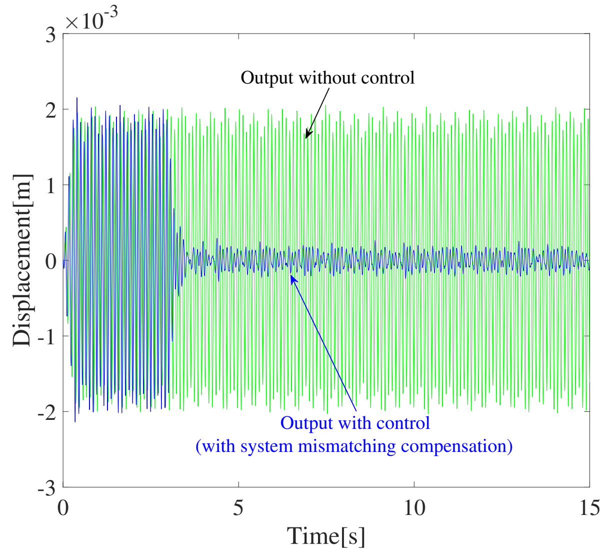

Figure 5.17: Outputs of the system with (with system mismatching compensation) and without control.

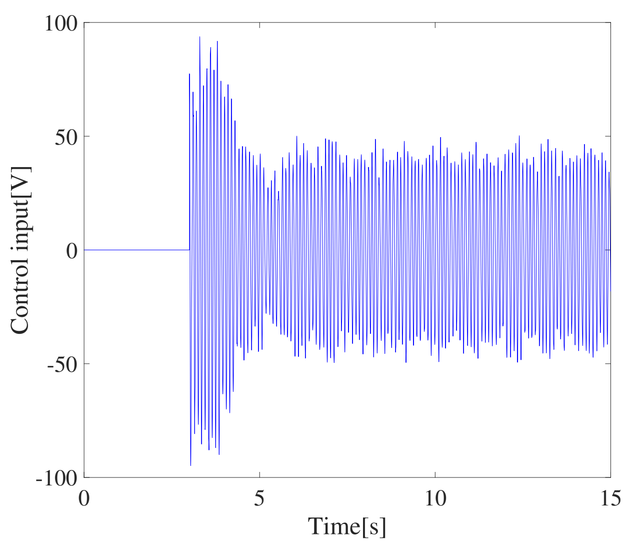

Figure 5.18: Control input of Figure 5.17.

operators A and B. Therefore, stability of the nonlinear control system based on operator theory is verified. The effectiveness of the with operators A, B, and system mismatching compensation is confirmed in Fig. 5.17. In result of with system mismatching compensation case, the condition output tends to zero is confirmed. Thus, effectiveness of the proposed control design scheme is confirmed.

5.4 Operator-based unknown input nonlinearity compensation approach

In this section, operator based unknown input nonlinearity compensation method is considered. First, in order to guarantee the stability of the nonlinear system with unknown input nonlinearity, operator based nonlinear control system is designed. After that, for ensuring the desired tracking performance and reducing the effect of unknown input nonlinearity, operator based tracking compensator and estimation structure are designed, respectively. The designed nonlinear control system is shown in Fig. 5.19. In this study, we consider the nominal vibration mode with a first-order mode and the uncertainties with the second- and third-order modes. Therefore, the plant with uncertainties is considered the following equation.

$$[P + \Delta P](u^*)(t) = (1 + \Delta)\left[J_{11}\int_0^t e^{-\alpha_{11}(t-\tau)} \sin\beta_{11}(t-\tau)u^*(\tau)d\tau\right] \quad (5.35)$$

where Δ and J_{11} are the bounded uncertainties of plant and the model parameter of plant, respectively.

5.4.1 Controllers Design for Stability

The plant $[P + \Delta P]$ can be right factorized by Eqs. (19) and (20).

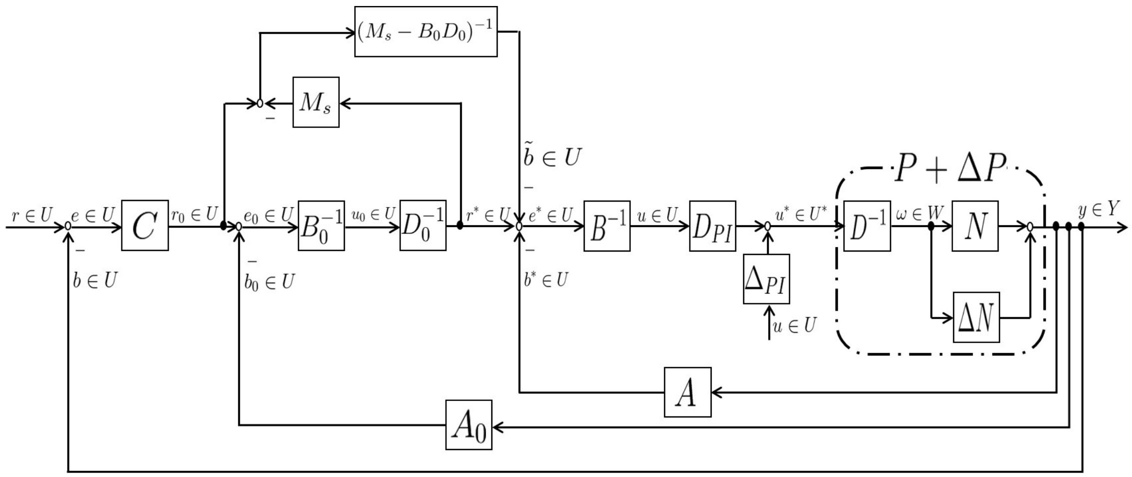

Figure 5.19: Proposed control system.

$$[P + \Delta P](u^*)(t) = (N + \Delta N)D^{-1}(u^*)(t) \tag{5.36}$$

$$[N + \Delta N](\omega)(t) = (1 + \Delta)J_{11} \int_0^t e^{-\alpha_{11}(t-\tau)} \cdot \sin \beta_{11}(t - \tau) \cdot \omega(\tau)d\tau \tag{5.37}$$

$$D(\omega)(t) = I(\omega)(t) \tag{5.38}$$

where I is identity operator, α_{11} and β_{11} are parameters of the nominal plant, respectively. From P-I hysteresis model, the invertible and linearly controllable part D_{PI} is used to design the controller and the remaining part Δ_{PI} treated as a bounded uncertainty. Also, the operator $\tilde{D}$ is considered as $\tilde{D} = D_{PI}^{-1}D$. For guaranteeing the BIBO stability of the nonlinear system, two stable operators A and B that satisfy following Bezout equation are designed.

$$AN + B\tilde{D} = I \tag{5.39}$$

where I is identity operator. In addition, under the following conditions are satisfied, robust stability of the designed nonlinear control system can be guaranteed.

$$A(N+\Delta N)+B\tilde{D}=\tilde{M} \tag{5.40}$$

$$\| [A(N+\Delta N)-AN]I^{-1} \|_{Lip}<1 \tag{5.41}$$

where $\tilde{M}$ denote an unimodular operator. The operators A and B are designed as follows.

$$A(y)(t)=\frac{(1-K_m)}{J_{11}\beta_{11}}(\ddot{y}(t)+2\alpha_{11}\dot{y}(t)+(\alpha_{11}^2+\beta_{11}^2)y(t)) \tag{5.42}$$

$$B(u)(t)=K_m K e^*(t) \tag{5.43}$$

$$K=\int_0^{h_x} p(h)dh$$

where K_m is a designed parameter.

5.4.2 Compensation for tracking and unknown input nonlinearity

Considering Fig. 5.19, the output $y(t)$ can be expressed by

$$\begin{aligned} y(t)=&(N+\Delta N)D^{-1}(D_{PI}B^{-1}(r^*(t)-b^*(t))+\Delta_{PI}(u)(t)) \\ =&(N+\Delta N)(A(N+\Delta N)+B\tilde{D})^{-1}(r^*(t)+\tilde{\Delta}(u)(t)) \\ =&(N+\Delta N)\tilde{M}^{-1}(r^*(t)+\tilde{\Delta}) \end{aligned} \tag{5.44}$$

where $\tilde{\Delta}=BD_{PI}^{-1}\Delta_{PI}(u)(t)$. From Eq. (5.44), stability of the nonlinear control system can be guaranteed by the designed operators A and B. However, the output $y(t)$ cannot be ensured to track the target value $r^*(t)$ and be sufficiently able to obtain damping performance of the plant. Simultaneously, due to the effect of unknown input nonlinearity term $\tilde{\Delta}$, desired vibration control performance cannot be obtained. Therefore, for improving the vibration control performance, the tracking compensator and compensation of the unknown input nonlinearity term $\tilde{\Delta}$ are needed to consider. The equivalent system of Fig. 5.19 is shown in Fig. 5.20. For ensuring the desired tracking performance and reducing the effect of unknown input nonlinearity, operator based tracking compensator and estimation structure are designed. In Fig. 5.20, operators $N_0 + \Delta N_0$ and D_0^{-1} are considered.

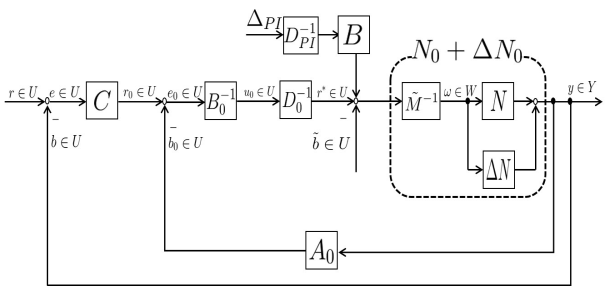

Figure 5.20: Equivalent system of Figure 5.19.

And, design operators D_0, A_0 and B_0 that satisfy Bezout equation. The designed operators are shown as follows.

$$A_0(y)(t) = S(y)(t) + k_0 A_0(y)(t-1) \tag{5.45}$$

$$S(y)(t) = \frac{(1-K_f)}{J_{11}\beta_{11}}(\ddot{y}(t) + 2\alpha_{11}\dot{y}(t) + \frac{(\alpha_{11}^2 + \beta_{11}^2)}{2} y(t))$$

$$B_0(u_0)(t) = K_f e_0(t) \tag{5.46}$$

$$D_0(r^*)(t) = B^{-1}\tilde{D}^{-1}(r^*)(t) \tag{5.47}$$

$$A_0 N_0 + B_0 D_0 = M_0 \tag{5.48}$$

$$A_0(N_0 + \Delta N_0) + B_0 D_0 = \tilde{M}_0 \tag{5.49}$$

$$\| [A(N_0 + \Delta N_0) - AN_0] M_0^{-1} \|_{Lip} < 1 \tag{5.50}$$

where $N_0 = NI^{-1}$, M_0 and $\tilde{M}_0$ are unimodular operators, K_f and $0 < k_0 < 1$ are designed parameters, respectively. Considering Fig. 5.20, the output $y(t)$ can be expressed by

$$\begin{aligned} y(t)=&(N_0+\Delta N_0)(\tilde{\Delta}_0+D_0^{-1}B_0^{-1}(-b_0(t) \\ &+C(r(t)-b(t)))) \\ =&(N_0+\Delta N_0)(M_0+(N_0+\Delta N_0)C)^{-1} \\ &\cdot(C(r)(t)+B_0D_0\tilde{\Delta}_0) \end{aligned} \tag{5.51}$$

and the designed operator C is shown in Eq. (32).

$$C(e)(t) = k_p e(t) \tag{5.52}$$

where $\tilde{\Delta}_0 = (\tilde{\Delta} - \tilde{b})$, and k_p is a designed parameter. Moreover, from Figs. 5.19 and 5.20, when the Bezout equation of Eq. (5.49) can be satisfied, the signal $\tilde{b}(t)$ can be expressed by

$$\begin{aligned} r^*(t)=&D_0^{-1}B_0^{-1}(r_0(t)-A_0(y)(t)) \\ B_0D_0(r^*)(t)=&r_0(t)-A_0(N_0+\Delta N_0)(r^*(t)+\tilde{\Delta}) \\ r_0(t)-\tilde{M}_0(r^*)(t)=&A_0(N_0+\Delta N_0)(\tilde{\Delta}) \\ =&(\tilde{M}_0-B_0D_0)(\tilde{\Delta}) \\ \tilde{b}(t)=&(M_s-B_0D_0)^{-1}(r_0(t)-M_s(r^*)(t)) \\ =&(M_s-B_0D_0)^{-1}(\tilde{M}_0-B_0D_0)(\tilde{\Delta}) \end{aligned} \tag{5.53}$$

where M_s is an designed unimodular operator. And,

$$\begin{aligned} M_s(r^*)(t)=&B_0D_0(r^*)(t)+S(r^*)(t) \qquad (5.54) \\ S(r^*)(t)=&I(r^*)(t)+R(r^*)(t) \\ R(r^*)(t)=&I(r^*)(t)+k_1R(r^*)(t-1) \end{aligned}$$

where k_1 is a designed parameter.

Thus, for ensuring the desired tracking performance and reducing the effect of unknown input nonlinearity, the following conditions are considered.

1. From Eq. (5.53), the unknown input nonlinearity term $(M_s - B_0D_0)^{-1}(\tilde{M}_0 - B_0D_0)(\tilde{\Delta})$ can be obtained by the operator based estimation structure. Therefore, when the condition of $(M_s - B_0D_0)^{-1}(\tilde{M}_0 - B_0D_0) \to I$ can be satisfied by the designed M_s, the effect of $\tilde{\Delta}$ can be made arbitrarily small.

2. From Eq. (5.49), when the condition of $(N_0 + \Delta N_0)C \gg M_0$ can be satisfied by the designed operators C, M_0 and $(N_0 + \Delta N_0)$, the tracking performance can be ensured.

5.4.3 Numerical simulation

In this section, the proposed design scheme is confirmed by simulation. The simulation results show the free vibration control performance of the flexible plate.

Parameter setting

Parameters of the piezoelectric actuator and the designed controllers are shown in Tables 5.5 and 5.6, respectively. In this simulation, only used first-order mode of the flexible plate as the nominal plant. The second- and third-order modes of this simulation are treated as uncertainties. The density function $p(h) = 0.00038e^{-0.00086(h-1)^2}$ where $h \in [0, 30]$ is used in this simulation.

Table 5.5: Parameters of the piezoelectric actuator.

Parameter	Value	Units
Young's Modulus	$E^p = 6.2 \times 10^{10}$	N/m^2
Length	$a_p = 50 \times 10^{-3}$	m
Width	$b_p = 20 \times 10^{-3}$	m
Thickness	$t_p = 0.5 \times 10^{-3}$	m
Piezo Constant	$d_{31} = -210 \times 10^{-12}$	m/V

Table 5.6: Parameters of the controllers.

Parameter	Value	Units
K_m	0.42	–
k_0	0.85	–
k_1	0.85	–
K_f	0.65	–
k_p	20	–
Sampling Time	0.01	s
Simulation Time	15	s

Simulation results

The simulation in MATLAB is performed. In this simulation, the flexible plate is added the moment $M_d(t) = 0.1\sin(f \cdot t)$ at the bottom of it to make a vibration when $t < 5s$. Where f is the eigenfrequency of the flexible plate and $f = 32.74/2\pi$. If $t \geq 5s$, we start to control the vibration of the flexible plate with free vibration.

In Fig. 5.21, output of the flexible plate with and without control are shown. In with control case, it does not consider the tracking and unknown input nonlinearity compensation. The blue line in Fig. 5.21 shows the output of the with control, the green line in Fig. 5.21 shows the output of the without control. Corresponding control input in Fig. 5.21 is shown in Fig. 5.22. Next in Fig. 5.23, output of the flexible plate with (blue line) operators A, B only and with (green line) considering tracking compensation are shown. The unknown input nonlinearity compensation was not considered in Fig. 5.23. Corresponding control input in Fig. 5.23 is shown in Fig. 5.24. From the above results, stability and effectiveness of tracking compensation are confirmed by Figs. 5.21, 5.22, 5.23, and 5.24. Furthermore, output of the flexible plate with (blue line) and without (green line) unknown input nonlinearity compensation are shown in Fig. 5.25. In Fig. 5.25, we can be observed that the displacement of the flexible plate with the unknown input

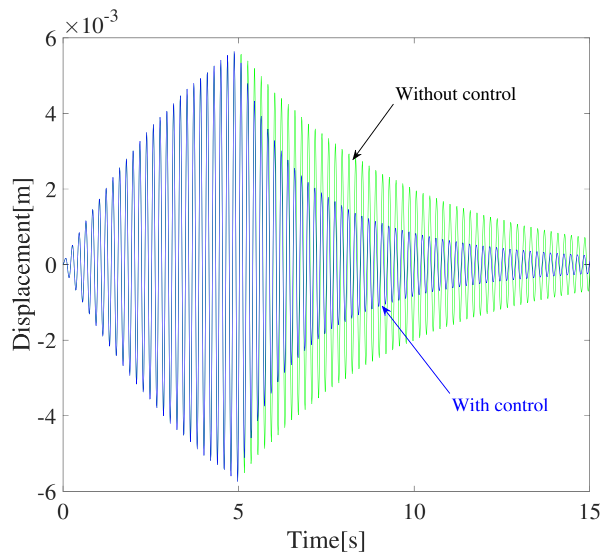

Figure 5.21: Output of the system with (blue line) and without (green line) control.

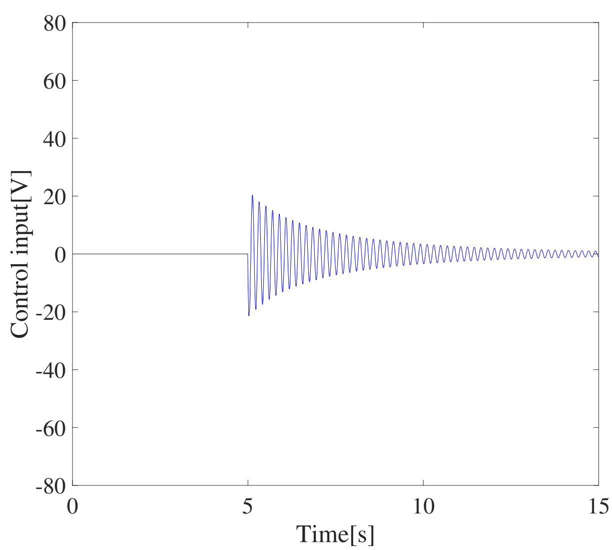

Figure 5.22: Corresponding control input in Figure 5.21.

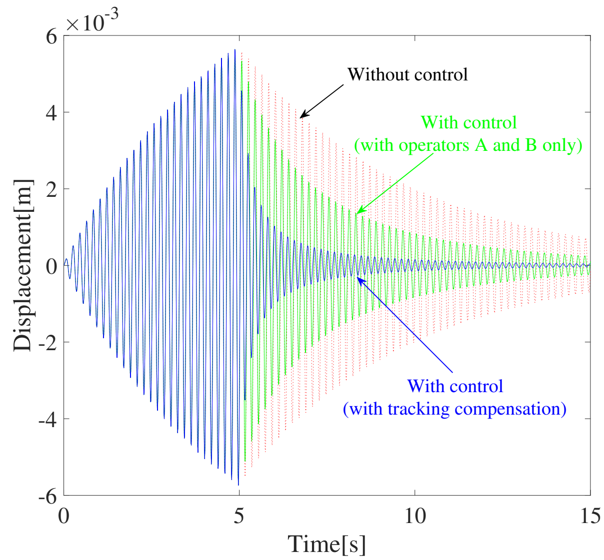

Figure 5.23: Output of the system with (green line) operators A, B only and with (blue line) considering tracking compensation (without unknown input nonlinearity compensation).

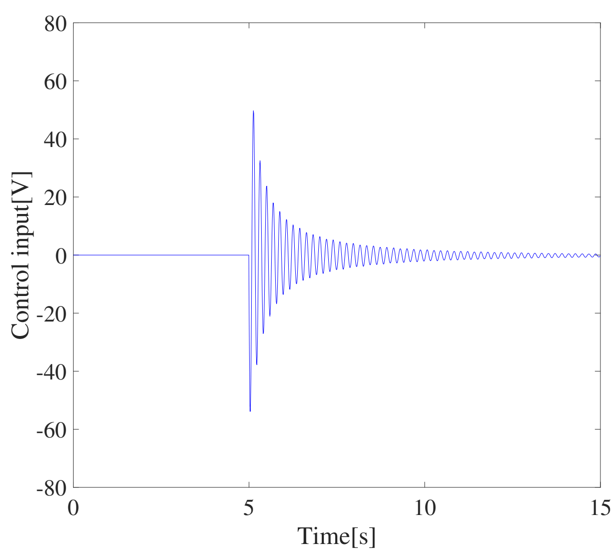

Figure 5.24: Corresponding control input in Figure 5.23.

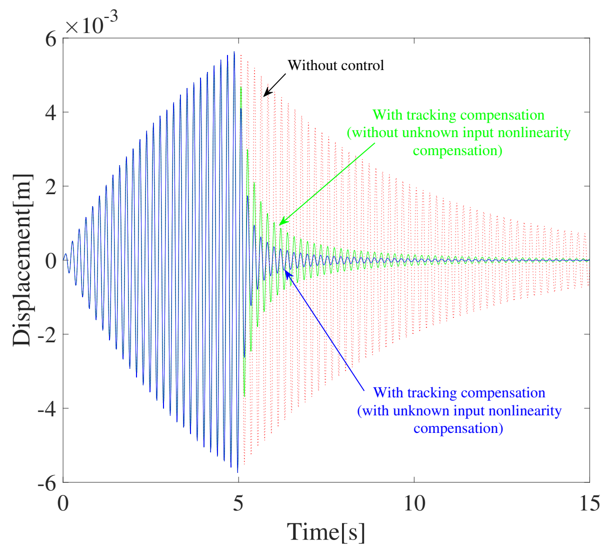

Figure 5.25: Output of the system with (blue line) and without (green line) unknown input nonlinearity compensation.

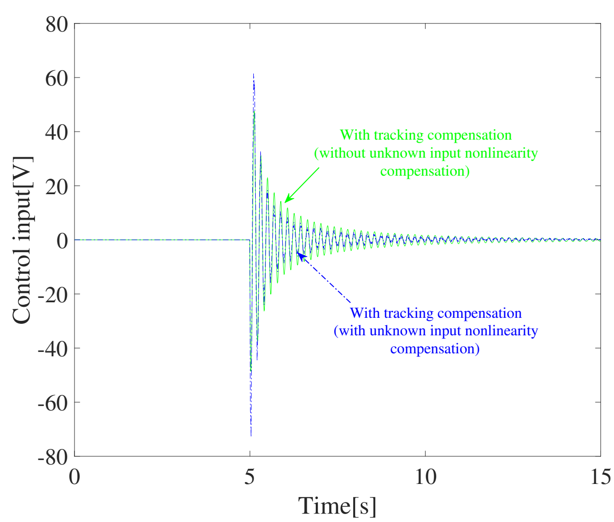

Figure 5.26: Corresponding control inputs in Figure 5.25.

Table 5.7: Parameters of the controllers.

Parameter	Value	Units
K_m	0.85	–
k_0	0.8	–
k_1	0.8	–
K_f	0.9	–
k_p	30	–
Sampling Time	0.001	s
Experiment Time	15	s

nonlinearity compensation reduce more than the result with tracking compensation only. Thus, effectiveness of the designed nonlinear control system is confirmed.

5.4.4 Experiment

In this section, the proposed design scheme is confirmed by experiment. The used parameters of the designed controllers are shown in Tables 5.7. The frequency of the vibration is 10Hz, and the vibration of the flexible plate is generated by the servo-motor. If $t \geq 5$s, controllers are started to control the free vibration of the flexible plate. In this experiment, the sampling time is 0.001s and the experiment time is 15s.

Outputs of the flexible plate with (blue line) tracking compensation and without (green line) control are shown in Fig. 12. The corresponding control inputs in Fig. 12 are shown in Fig. 13. In Fig. 5.27, compared with (blue line) tracking compensation and without (green line) control, the output with considering tracking compensation is stabilized faster than without control case. The stability and effectiveness of considering tracking compensation are confirmed by Figs. 5.27 and 5.28. Finally, outputs with (red line) unknown input nonlinearity compensation and without (green line) control are shown in Fig. 5.29. By the obtained

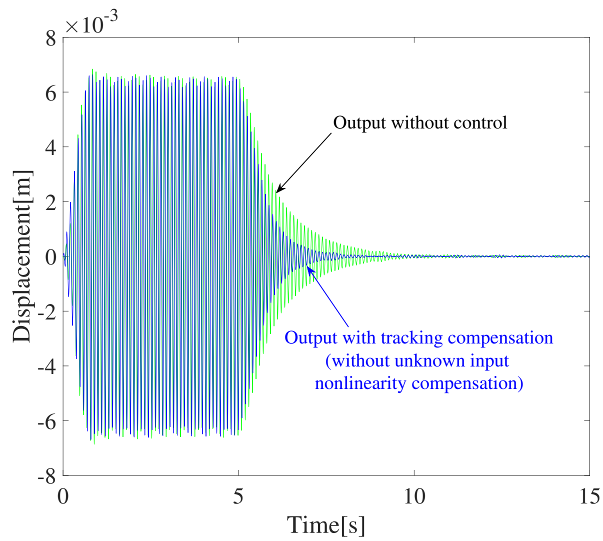

Figure 5.27: Outputs of the system with (blue line) tracking compensation and without (green line) control.

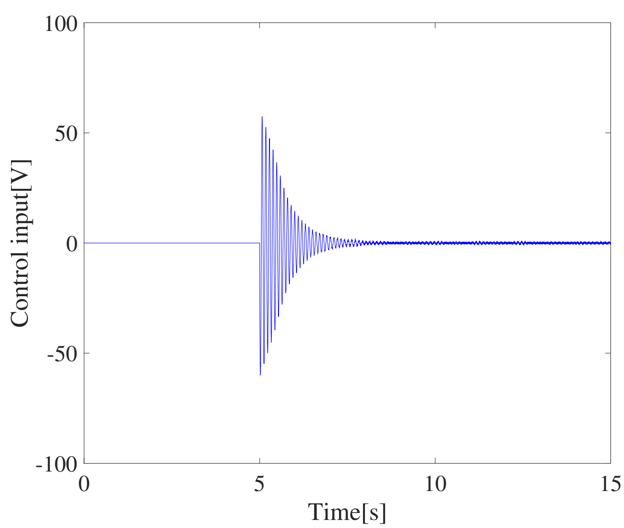

Figure 5.28: Corresponding control input in Figure 5.27.

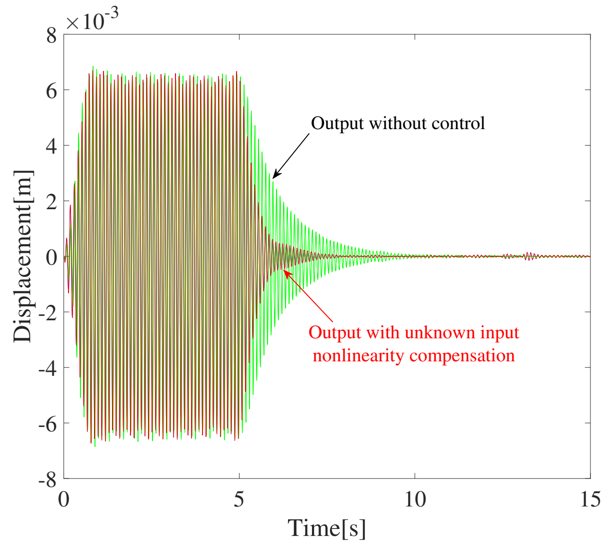

Figure 5.29: Outputs of the system with (red line) unknown input nonlinearity compensation and without (green line) control.

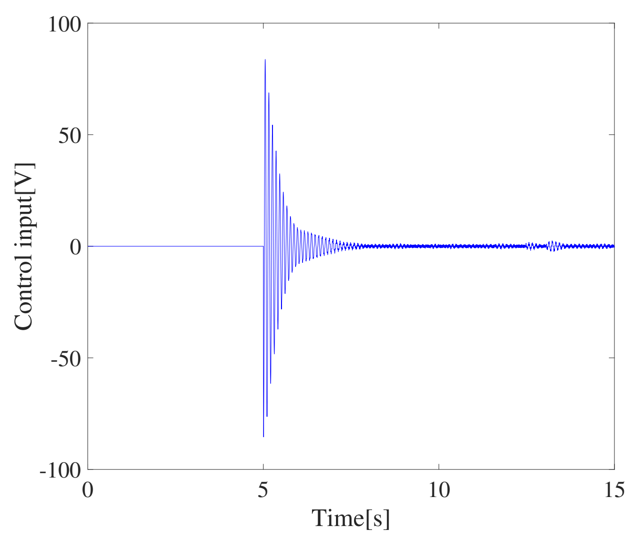

Figure 5.30: Corresponding control input in Figure 5.29.

experimental results, the vibration of the flexible plate is stabilized quickly using the unknown input nonlinearity compensation. Thus, the effectiveness of proposed control design scheme is confirmed.

5.5 Conclusion

In this chapter, operator based some new vibration control approaches are considered. In time-varying unimodular function based robust right coprime factorization approach, the condition of output tends to zero is obtained by the designed time-varying unimodular function. The condition of output tends to zero is also obtained by the operator based system mismatching compensation approach. In addition, operator based unknown input nonlinearity compensation approach is discussed. The effectiveness of the proposed design schemes are verified by numerical simulation and experimental results, respectively.

Chapter 6

Conclusions

This dissertation mainly discussed control schemes for nonlinear vibration control on flexible plate with hysteresis nonlinearity of piezoelectric actuator. With the considered models of piezoelectric actuator and of flexible plate, operator-based robust right coprime factorization approach is used to design control schemes. Based on numerical simulation and experimental results are proved the effectiveness of the proposed control schemes.

In Chapter 2, some basic definitions and notations are introduced for operator theory and right coprime factorization. The model of piezoelectric actuator is explained by P-I hysteresis model. Based on theory of thin plates, the model of flexible plate is considered. In addition, the experimental devices on this dissertation is introduced. The main study is discussed in this dissertation is also introduced in this chapter.

In Chapter 3, operator-based control scheme is proposed for plate with a free vibration and perturbations. For guaranteeing robust stability of the plate with perturbations case, operator-based controllers are designed. Simultaneously, operator-based n-times feedback loops are designed to compensate hysteresis nonlinearity of the piezoelectric actuator and to ensure the desired vibration control performance of the flexible plate with a free vibration and perturbations. In the designed control scheme, the desired compensation performances of tracking and of perturbations are obtained by increasing the number of designed n-times feedback loops. In addition, the effectiveness of designed control scheme is confirmed by numerical simulation and experimental results.

In Chapter 4, for considering plate with a forced vibration, operator-based nonlinear control scheme is proposed. In this Chapter, a controller with characteristics of Proportional-Integral-Differential (PID) controller is proposed. And the designed controller can be controlled by only one design parameter without adjusting PID parameters. After that, for compensating the forced vibration to improve the vibration control performance, the compensator is designed to improve the vibration control performance. The vibration control performance of designed control scheme is confirmed by numerical simulation and experiment.

In Chapter 5, operator based some vibration control approaches are discussed. In time-varying unimodular function based robust right coprime factorization approach, the time-varying unimodular function is constructed by the designed controllers. Simultaneously, the condition of output tends to zero is obtained by the designed time-varying unimodular function. In system mismatching compensation approach, with considering the effect of hysteresis nonlinearity from piezoelectric actuators, operator based controllers are designed to guarantee the stability of the nonlinear system. In addition, for improving vibration control performance, the system mismatching compensation unit is given by the designed reference system and compensator. For reducing the effect of unknown input nonlinearity to improving the vibration control performance, operator based estimation structure is considered in unknown input nonlinearity compensation approach. The numerical simulation and experimental results are shown to verify the effectiveness of the above proposed control schemes.

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Appendix A

Publications

Journal papers

1. **G. Jin**, M. Deng and S. Wakitani, Operator-based two loop nonlinear forced vibration control of a flexible plate, *Electrical Engineering in Japan*, Vol.203, No.2, pp.19-28, 2018.

2. **G. Jin** and M. Deng, Operator-based nonlinear free vibration control of a flexible plate with sudden perturbations, *Transactions of the Institute of Measurement and Control*, DOI: 10.1177/0142331219891352, 2019.

3. **G. Jin** and M. Deng, Operator-based robust nonlinear free vibration control of a flexible plate with unknown input nonlinearity, *IEEE/CAA Journal of Automatica Sinica*, Vol.7, No.2, pp.442-450, 2020.

Proceedings papers

1. **G. Jin** and M. Deng, Operator-based robust two loop nonlinear free vibration control of a flexible plate, *Proceedings of 2016 IEEE International Conference on Mechatronics and Automation*, pp.2240-2245, 2016.

2. **G. Jin** and M. Deng, Experimental study on operator-based robust two loop nonlinear free vibration control of a flexible plate, *Proceedings of 2016*

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3. **G. Jin** and M. Deng, Robust nonlinear forced vibration control of a flexible plate based on operator theory, *Proceedings of 2018 International Conference on Advanced Mechatronic Systems*, pp.36-39, 2018.

4. **G. Jin** and M. Deng, Operator-based robust nonlinear forced vibration control of a flexible plate, *Proceedings of 2019 IEEE 2nd International Conference on Renewable Energy and Power Engineering*, pp.68-72, 2019.

5. **G. Jin** and M. Deng, Time-varying unimodular function based robust right coprime factorization for nonlinear forced vibration control system, *Proceedings of 2020 IEEE International Conference on Networking, Sensing and Control*, 2020. (Accepted)

Other papers

1. **G. Jin** and M. Deng, Operator-based nonlinear vibration control for flexible plate with a forced vibration, *IEE Japan Technical Meeting on Control*, pp.35-38, 2015. (in Japanese)