Goodman's *Grue* — Relativized Pluralism and Paradigmatic Thought Experiment (*Whack*)

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Abstract:

In this paper we examine Nelson Goodman's *Grue* 'new riddle of induction' from the perspective of research into semantics of thought experiment. Since *Grue* may be considered a productive epistemic paradox TE, we can't agree on the skeptical resolution of the neologistic thought experiment to near-common sense of Goodman's entrenchment theory. Instead, we consider the thought experiment a prototype of a Kuhnian paradigmatic or revolutionary thought experiment, which may cover famous examples from history of science, whereof we will analyze one in more detail as a specimen with help of temporal predicate logic ('all swans are whack'). Next, we argue for a pluralistic interpretation of the paradox thought experiment as in line with Goodman's philosophical position of irrealism--pluralism of the paradox is not the problem, it is the resolution of the problem. Partly on basis of analogous analyses of WVO Quine's *Gavagai* thought experiment, we conclude by opting for Quinean ontological relativity, which does not exclude pluralism, but makes it hook upon (physical) reality. Modal logic, possible worlds semantics is proposed as alternate to classical logic as to deal with relativized pluralism and possible paradigm shifts in science.

Keywords: Grue; semantics; paradigmatic/revolutionary thought experiment; (elementary) temporal predicate logic (ETPL); modal logic; modus tollens; Whack; neologistic thought experiment; entrenchment theory; pluralism; possible worlds semantics (PWS); ontological relativity; Nelson Goodman; Carl Hempel; Thomas Kuhn; Willard V.O. Quine.

1. Identifications – Neologistic and Paradigmatic TE[1]

Goodman's 'new riddle' of *Grue* (e.g. Goodman 1983) may be understood in context of discussions of the problem of induction, referring to many contributions in the history of the problem from David Hume's habits and regularities, Carl Hempel's theory of confirmation to Rudolf Carnap's views on probability, cooperating with colleagues (e.g. Goodman, Schwartz, Scheffler 1970), however, omitting important theories at the same time, e.g., new philosophies of science as Thomas Kuhn's.

As many authors observe (e.g. Fain 1967, Stalker 1994, Elgin 1997, Cohnitz 2016) the literature on *Grue* is extensive, Goodman included who kept on updating the problem of induction with revisions of the original 1954 edition. One may discern a manifold of interpretations of the TE, usually according with different positions in epistemology and philosophy of science with regard to problem of induction (habit, regularity, prediction, disposition, learning, diverse forms of probability, simplicity, similarity, deduction, projectibility, conceptual spaces, pragmaticism, evolutionary epistemology, thought experiment, epistemic paradox, counterfactual, inductive inference) or problems specific to the linguistic approach of Goodman (entrenchment, disjunctive, (un-, non)projectible, qualitative, positional, locational predicates).

In 90s there appeared two collections with essays on Goodman's *Grue*, Stalker 1994 and Elgin 1997, anthologizing the previous decades. Many well-known contemporary philosophers have replied to *Grue*, e.g., Sydney Shoemaker (1975/2003b), Richard Swinburne (1968), Rudolf Carnap (e.g. 1937, 1945, 1947a, 1947b, 1950 referred to in Goodman's notes—1947a/b are included in Elgin 1997, 1-16, 21-22), Donald Davidson (1966, in Elgin 1997, 126-128), Willard Quine (1965, in Stalker 1994, 41- 56), Hilary Putnam (who has written a foreword to 1983 fourth edition to the *Grue* text, Goodman 1983, vii-xvi), Noam Chomsky (2006, discussing *Grue* in context of problem of acquisition of language). Next, a couple of philosophers who became well-known because of TE as *Color Scientist Mary* Frank Jackson (1975, in Stalker 1994, 79-96) and *The Violinist* Judith Jarvis Thomson (1966, in Elgin 87-108). We will refer to some of the analyses and interpretations in the footnotes as loci paralleli of positions in our research.[2]

We defend various positions in this paper going back to before the skeptical resolution of Goodman's entrenchment theory.

First, in line with the new philosophy of science of Thomas Kuhn and Karl Popper, analyzing *Grue* as a prototype of paradigmatic, revolutionary TE due to the disruptive shift at time *t* from one paradigm to another. After initial elementary temporal logical analyses of the neologistic predicates, we analyze *Grue* as a paradigmatic TE that falsifies an old theory and verifies a new theory, let's say, like a crucial (thought) experiment.

Second, rejecting the skeptical paradox we remain within (an undecidability interpretation of) Carl Hempel's pluralism of possible – only relatively verifiable – hypotheses until 'further notice of' time *t* when a possible paradigm shift may occur, which is consistent with the general outlook of Goodman's philosophy of pluralism, but contradicting the simplifying entrenchment theory as with regard to *Grue*.

Third, from a position of Quinean metaphilosophical gradualism we consider natural science as valuable as logic and philosophy – including empirical data from natural science (as wavelenghts and frequency measurements of light, e.g. Norton 2003/2006) and cross-cultural research into colors we arrive at a global cross-cultural color theory; beyond the general outlook of the problem of induction we are able to give some indications for a resolution of the limited problem of color perception.[3]

Fourth, in the one but last section we add two new (21st-century) postmodern conditions to the (20th-century) modernist (elimination) rules Goodman is looking for as resolution of the riddle, a global cross-cultural (and trans-historical) condition, whereof cross-cultural research into color perception is an example, and an environmental pragmaticist condition, safeguarding protection of human health and natural environment. Both pluralism and paradigmatic interpretations of *Grue* result from these considerations as with purpose of preventing unfortunate mistakes of the European scientific enterprise like they have been made in previous ages.

Fifth, finally, in the last section we concisely propose modal logic or PWS as alternate and successor to classical logic as to deal with relativized pluralism, global cross-culturalism and environmental pragmaticism, correct counter-intuitive results of standard logic and avoid one-dimensional short-term resolutions that may easily turn into, e.g., environmental disasters as caused by climate change.

Identification of the 'riddle' or 'paradox' of *Grue* as a TE precedes any logical analysis or epistemological interpretation. This identification is in no way controversial anymore since TE theory got more and more mainstream since the 60s and the very occurrence of *Grue* could be considered part of the second wave of TE that occurred after WWII (the first wave coincides with the rise of the experimental method in the sciences at the end of the 19th century as the design of experiments is quite often accompanied by TE preps). Quite a few philosophers discuss *Grue* as a kind of TE—e.g. Sorensen (2017, 1992a) discusses it as a 'epistemic paradox', when paradox is Sorensen's quintessential analysis of TE; Clatterbuck (2013) considers it a TE of a 'particular type of inductive inference'; Tittle (2005) lists it as an example of a TE in logic; Cohen (1979) analyzes it as a counterfactual-like TE.

Goodman's *Grue* TE or 'new riddle of induction' is often considered a paradox, e.g. an epistemic paradox. Epistemic paradoxes may be defined as according to Roy Sorensen in *Stanford Encyclopedia of Philosophy* (*SEP*)

> Epistemic paradoxes are riddles that turn on the concept of knowledge (*episteme* is Greek for knowledge). Typically, there are conflicting, well-credentialed answers to these questions (or pseudo-questions). Thus the riddle immediately informs us of an inconsistency. In the long run, the riddle goads and guides us into correcting at least one deep error – if not directly about knowledge, then about its kindred concepts such as justification, rational belief, and evidence. (Sorensen 2017)

Sorensen considers paradox analyses the quintessential approach to TE and holds TE are reducible to paradoxes.[4] The kind of epistemic paradox may still be considered part of our TE research since its goal doesn't seem to be about paralyzing or destructing the project of logic and reason altogether (in which latter case we exclude paradoxes from our TE definition which features on constructive pragmaticist devices designed to resolve predefined problems), but as Sorensen states, an epistemic paradox may in 'the long run' 'goad... and guide... us into correcting at least one deep error', e.g., correcting mistakes and inconsistencies that have been revealed by the paradox, and it may even be considered part of the process of error elimination which guides the progress of science as according to Karl Popper.

It is obviously Goodman's aim to try and resolve the epistemic riddle, but, alas, to no avail. Subsequent chapters of *Fact, Fiction, and Forecast* (Goodman 1954/55/65/73/83), in which book 'the new riddle of induction' appears, are mainly dedicated to designing 'rules of elimination', that reduce the complexity of the paradox, and which ideas are slightly changed from edition to edition as, e.g., from the 2nd (1965) to the 4th edition (1983), but in the end Goodman concludes that all he has offered is 'a study of some of the resources that a new approach offers us for dealing with a difficult problem' (Goodman 1983, 120) and that 'none of these speculations should be taken for a solution' (Goodman 1983, 123).

As we may see in the last part of this paper, there are some interpretations of Goodman's philosophy that may indicate the 'riddle' is stating an essential difficulty for, let's say, monism or realism in epistemology or philosophy of science, that is unresolvable if we don't leave these philosophical presuppositions for philosophical positions defended by Goodman in other parts of his oeuvre as irrealism and pluralism. This interpretation explains how Goodman could come up with so many innovative examples and extensions of the TE since *Grue* is, in fact, only a tiny part of a multiple series of TE that extends over many pages of *Fact, Fiction and Forecast*, including a rewritten Section IV.4 on conditionals, introducing neologisms like 'bleen', 'emerose', 'emeruby', 'sapphiruby', 'Eifferuby', 'grund', 'grare', 'gred', 'zig', 'bagleet', 'bagmarks', etc.[5] It seems no problem for Goodman to come up with ever new imaginative hypotheses that have 'equal confirmation' as the traditional, conventional hypotheses as if he is creating ever new versions of reality.

The use of neologisms may highlight another emphasis of Goodman's work, on choice of predicates and, generally speaking, language and symbolisms (as after, particularly, Ernst Cassirer), showing how wide the variety of predicates could be, and we may show how Goodman's 'entrenchment theory' – one alleged resolution of the TE – may relate to use of language first and only second to epistemology of these 'weird', 'outworldy' hypotheses (as according to the later Goodman). The resolution may be akin to skeptical paradoxes in Ludwig Wittgenstein's *Philosophical Investigations* (Wittgenstein 1953).

Because of the abundant use of neologisms we may baptize this new variety of TE 'neologistic TE.' The neologisms contribute to rhetorics and mystery of *Grue*, features that are often attributed to TE. In a strong TE multiple senses and connotations may ring true, it contains a manifold of condensed and possible meanings that can be explained in many ways. Part of mystery of *Grue* comes from magic of color shades in between green and blue, and 'grue' appears to be a linguistic technical term signifying possible cross-cultural and historical overlaps between green and blue (although Goodman never mentions this cross-cultural sense of 'grue').

In Section 2 we show the basic structure of the TE argument of *Grue* from the original text, the second lecture, printed as Chapter 3 in *Fact, Fiction, and Forecast* (Goodman 1954/55/65/73/83). The TE text remains the same over all editions of the book and includes, next to *Grue*, a *Bleen* TE.

In the deep analyses part (Section 3) we will, initially, propose a resolution which remains within the established presuppositions of realism and interpret the *Grue* paradox as 'undecidability'

since the paradox TE may remain eventually a pseudo-problem, that is, on careful reading it dissolves immediately because we simply can't know yet about hypotheses about a future time t, and no one will deny we can't decide on it already now, since history shows sometimes we have to include the occurrence of what Nassim Taleb calls the highly improbable. For this deep analysis we use an elementary form of temporal predicate logic (provisionally, ETPL) to disentangle the neologistic predicates (see Sections 3.1 – 3.3). The irrealist, imaginative 'otherworldly', 'weird' hypotheses can't be necessarily excluded beforehand, but we may trust it won't change our world completely (although, of course, it is not excluded it will predict an inescapable disaster and in case of e.g. discussion of climate change the disaster is likely to come about if we won't do anything to prevent it).

To falsify plain conventionalism of common sense of the entrenchment theory, we will, next, show a historical example, which the *Grue* TE seems to be inspired by and only absence of any reference to it may one want to think differently. We will demonstrate that Goodman's *Grue* can be considered a prototype or specimen of a very strong type of TE, which we call 'paradigmatic' or 'revolutionary TE' as after Thomas Kuhn's *The Structure of Scientific Revolutions*, that are indeed about to change the world of science, as they are about revolutionary paradigm shifts in history of science and philosophy:

> It is no accident that the emergence of Newtonian physics in the seventeenth century and of relativity and quantum mechanics in the twentieth century should have been both preceded by and accompanied by fundamental philosophical analyses of the contemporary research tradition.... Nor is it an accident that in both these periods the so-called thought experiment should have played so critical a role in the progress of research. As I have shown elsewhere, the analytical thought experimentation that bulks so large in the writings of Galileo, Einstein, Bohr, and others is perfectly calculated to expose the old paradigm to existing knowledge in the ways that isolate the root of the crisis with a clarity unattainable in the laboratory.... (Kuhn 1970, 88)

Not only discoveries in theoretical physics have brought about paradigm shifts (as from meso-level of Planet Earth to macro-level of cosmology and micro-level of particle physics), but also the historical overseas discoveries of new continents as America and Australia have substantially broadened the scope of sciences as biology, geology etc.[6] These paradigmatic, revolutionary new discoveries have contributed to what TE philosopher Tamar Gendler calls 'reconfiguration of conceptual commitments' (Gendler 1996/2000 *passim*).
E.g., the concept of 'bird' has been broadened by discovery of many new species as the black swan, which falsified medieval European universal statements as 'all swans are white.' We will discuss this example in Section 3.4 as a specimen of paradigm change that may be explained with help of Goodman's *Grue* TE structure.

The 'reconfiguration of conceptual commitments' has, of course, effected changes of laws in the field, if the former has not been changed by the latter instead, like in the field of theoretical physics. We will discuss criticisms of Goodman's notion of 'lawlikeness' in the Interpretations part of the paper, e.g., the criticism that colors (as colors of gem stones) are not traditionally considered

essential properties of entities but rather accidental properties (see Section 4.3). It is somehow abstruse that Goodman wants to discuss the differences between 'lawlike' and accidental statements with help of an example about possible change of an accidental property (as colors of emeralds).

Next, gemology is not considered a science that is vulnerable to change in the history of science, quite different from sciences as aforementioned physics and biology and if gemology has been modified as by changes in geology etc., Goodman does not draw attention to this possibility. Rest and continuity are among the cultural biases of science and background assumptions of induction, which are discussed in Section 4.4.

Furthermore, as recent research has confirmed (e.g. Berlin-Kay theory of basic color terms, Berlin and Kay 1969), not only connotations, but also denotations of color words like 'green' and 'blue' may differ from culture to culture, and 'grue' is, in fact, a scientific, linguistic cover term of both. This global cross-cultural interpretation is touched upon in Section 4.5.

Next to the interpretation in line of the new philosophy of science of Thomas Kuhn in Section 4.2, we will review Goodman's answer to the riddle, the so-called entrenchment theory or skeptical reduction of logic and theory to near-common sense of practice, in Section 4.1, and opt for a pursuit of the problem along the lines of Carl Hempel's tentativeness and Goodman's pluralism, albeit restrained by physical science to Quinean 'ontological relativity' (see Section 4.6).

In the last two Sections, 4.7 and 4.8, we emphasize the importance of global cross-culturalism and environmental pragmaticism (e.g. concern about human health and Earth's ecology) and add these conditions to an update of philosophy of science, visualized by an extension to one of Karl Popper's formulas about progress of science. In the last section we shortly indicate the outline of a modal logical, possible worlds interpretation, as different from Goodman's project of pluralism that could prefer mutually exclusive versions and visions over an 'actual world' among many more possible worlds.

2. Surface Analyses of *Grue* TE

The *Grue* TE text has been developed from three lectures delivered at the University of London, London, UK, in May of 1953 under the title 'Fact, Fiction, and Forecast.' Under the same title the text of the lectures has been printed, with ongoing revisions from the first British edition of 1954 to the fourth edition of 1983, when Goodman was still developing his ideas, particularly, about 'rules of elimination' to the criterion of lawlikeness. Our references are to the 1983 fourth edition, unless otherwise indicated.

The new riddle of induction is introduced in Chapter 3, Section 4, bearing the same name, 'The New Riddle of Induction.' It states its aim right in the beginning (and again in the end slightly revised)

> Confirmation of a hypothesis by an instance depends rather heavily upon features of the hypothesis other than its syntactical form. ... Only a statement that is lawlike—regardless of its truth or falsity or its scientific importance—is capable of receiving confirmation from an instance of it; accidental statements are not. Plainly, then, we must look for a way of distinguishing lawlike from accidental statements.
> ....

[P_1/H_1] Suppose that all emeralds examined before a certain time t are green.... At time t, then, our observations support the hypothesis that all emeralds are green; and this is in accord with our definition of confirmation. Our evidence statements assert that emerald a is green, that emerald b is green, and so on, and each confirms the general hypothesis that all emeralds are green. So far, so good.

[P_2/H_2] Now let me introduce another predicate less familiar than 'green'. It is the predicate 'grue' and it applies to all things examined before t just in case they are green but to other things just in case they are blue. Then at time t we have for each evidence statement asserting that a given emerald is green, a parallel evidence statement asserting that that emerald is grue. And the statements that emerald a is grue, that emerald b is grue, and so on, will each confirm the general hypothesis that all emeralds are grue.

[C_1] Thus according to our definition, the prediction that all emeralds subsequently examined will be green and the prediction that all will be grue are alike confirmed by evidence statements describing the same observations. (Goodman 1983, 72-74—bracketed letters added – H_1, H_2 hypotheses are P_1, P_2 premises to C_1 conclusion)

This is the basic argument.
Next it is generalized to any predicate – 'if we simply choose an appropriate predicate' or 'any prediction whatever', to any object, to 'indeed about anything else.' Goodman mentions an example of the latter in a footnote

Let 'emerose' apply to emeralds examined before time t, and to roses examined later (Goodman 1983, 74n10)

and the conclusion is generalized that without 'such criterion', i.e. a criterion for 'determining lawlikeness'

[C_2] We are left once again with the intolerable result that anything confirms anything. (Goodman 1983, 75 – bracketed letter added – C_2 conclusion 2)

The *Grue* TE next multiplies to a series when Goodman is adding one more neologism in his search for a criterion of lawlikeness, 'bleen' TE (p. 78-81), after which the section of the new riddle of induction is concluded

We have so far neither any answer nor any promising clue to an answer to the question what distinguishes lawlike or confirmable hypotheses from accidental or non-confirmable ones; and what may at first have seemed a minor technical difficulty has taken on the stature of a major obstacle to the development of a satisfactory theory of confirmation. It is this problem that I call the new riddle of induction. (Goodman 1983, 75)

In the next chapter, the third lecture of the 1953 London lectures, 'Prospects for a Theory of Projection', Goodman continues the search for a criterion of lawlikeness or 'projectibility' by adding more and more neologistic TE, as in the for the 3rd edition rewritten Section 4 of the last chapter, IV.4, 'Presumptive Projectibility,' focussing on conditionals, and in Section 5, 'Comparative Projectibility.'

As remarked, the neologistic TE are a new feature by which Goodman adds a new variety of TE to TE theory.

In next section on deep analyses we will see, however, that the neologistic predicates are easily reducible to existing predicates as by logical formalization. It is this feature of the TE that Goodman wants to resolve with help of the entrenchment theory, whereby he offers a sort of common sense resolution of the paradox. Goodman repeats this kind of skeptical reduction of the TE in *Ways of Worldmaking* (Goodman 1978 – please, see Section 4.1).

However, we hold that the paradox has already been resolved in many successful exemplifications in history of science and philosophy, notably in cases of paradigm change, and so that it may be considered a specimen of a revolutionary TE (see Section 3.4).

We can't agree on the skeptical reduction of the TE to (near-)common sense and argue that the paradox TE is consistent with Goodman's irrealism, in fact pluralism, so that we may conclude that the paradoxical ambiguity of the TE is not the problem, but, in fact, the resolution of the problem. Please, see Section 3.2 on 'undecidability', which analysis is prepared by application of temporal predicate logic to the neologistic predicates in Section 3.1.

Instead of reducing the paradox TE to near-common sense we acknowledge that there are a couple of reasons why *Grue* could be considered a pseudo-problem (see Section 3.3).

3. Deep Analyses – ETPL, Undecidability, Modus Tollens

We will show that Goodman's neologisms may be easily eliminated by use of an elementary temporal predicate logic (ETPL). We propose two undecidability analyses, the first one resulting in (Hempelian) tentativeness of multiple hypotheses, the second one reducing *Grue* to a pseudo-problem. Last but not least, we propose a new deep analysis of *Grue* TE as a prototype of a (Kuhnian) paradigmatic or revolutionary TE, which may fit in with wordings of Goodman's TE and could be easily applied to historical examples of (mini-) paradigm shifts as from 'all swans are white' to 'all swans are whack.'

3.1. Logical Elimination of Neologisms

The neologisms add to metaphysical—even possibly mysterious – appearance of the TE and may be easily eliminated by use of logic, as Rudolf Carnap already indicated in 1932/59 article on elimination of metaphysics through logical analysis of language. The logical elimination demonstrates it is not about irreducible metaphysics or logically or conceptually new predicates although they may still— possibly indirectly—refer to physically new, i.e., yet undiscovered, entities and features, in our example of Section 3.4, biological new species of the black swan (scientifically

baptized 'Cygnus atratus' in 1790 by John Latham).

Of course, the hypothesis 'all swans are white' should never have been seriously entertained as one may expect from experience with many more biological species that they may appear in a manifold of colors and that color is rather an extrinsic than an intrinsic biological feature, property or predicate, extrinsic in the sense that it doesn't or does hardly effect biological functioning of the organism as a whole or the species as a whole.

It may be different in gemology, and that is why the example of Goodman could be considered exceptional, but nothing in his theory excludes entertaining examples from different sciences as biology since the TE was designed to tell lawlike statements from accidental statements without any restriction to range of examples.

The formalization includes provisional temporal logic as after Carl Hempel (e.g. 1945a, 11), using a two-place predicate, of which one variable is an indication of time, e.g., E(x, t), E(x, >t).[7]

Hempel uses universal and existential quantification over *t*, we only use five time indicators, t (at *t*), <t (before *t*, until *t*), >t (after *t*, since *t*), ≤t (before and at *t*, up to and including, through *t*), ≥t (at *t* and after, from *t* on).

Goodman's *Grue* may presuppose a concept of linear time countable infinity to future. We add infinity to the past when we use this kind of temporal logic to formalize historical examples of revolutionary TE as 'all swans are whack.'

[1]

Suppose

E(x)	: x is an Emerald
GE(x)	: x is Green
GU(x)	: x is Grue
BU(x)	: x is Blue
BE(x)	: x is Bleen
t	: at *t*, particular, but unspecified point in time, *t*, presumably, (near) future
<t	: before *t* (until *t*)
≤t	: before and at *t* (up to and including, through *t*)
>t	: after *t* (since *t*)
≥t	: at *t* and after (from *t* on)
E(x, t)	: x is an Emerald at *t*
E(x, <t)	: x is an Emerald before *t*
/\	: conjunction
\/	: inclusive disjunction
=	: identity, is the same as

then

GE(x) = x is Green (before *t*, at *t*, and after *t*)   (1)

$$\text{GU}(x) = x \text{ is Green before } t \lor x \text{ is Blue at } t \text{ and after} \qquad (2)$$

The interpretations of *Grue* may vary with commentaries of secondary literature, but there are also ambiguities in the authentic TE text since usage may vary from passage to passage.

'Grue' is never unequivocally defined by Goodman, but only in context. Some more interpretations

$$\text{GU}(x) = x \text{ is Green before } t \lor x \text{ is not-Green at } t \text{ and after} \qquad (2^1)$$
$$\text{GU}(x) = x \text{ is Green before } t \lor x \text{ is Green or Blue at } t \text{ and after} \qquad (2^2)$$
$$\text{GU}(x) = x \text{ is Green before } t \lor x \text{ is not-Green or Blue at } t \text{ and after} \qquad (2^3)$$

In all cases (1) and (variety of) (2) are 'incompatible' (p. 74 – or 'conflicting', p. 99) hypotheses

(2^1) – is the strongest interpretation, when (1) and (2^1) are incompatible in sense of contradictory

(2) – is still a very strong interpretation, when (1) and (2) are incompatible[8]

(2^3) – is as strong as (2^1) since 'or' in not-Green or Blue is inclusively disjunctive $(\lor)$, so it may be as strong as just not-Green because Blue is only a variety of not-Green; it may help to pick out observed emeralds that are blue first as kind of intermediary ancillary step to arrive at not-Green

(2^2) – is the weakest interpretation and it seems Goodman sometimes implicitly includes it and sometimes emphatically excludes it; we use an analogue for our interpretation of *Whack* in [5'], [5''].

3.2. Undecidability – Tentativeness

First, we may summarize our surface analysis, next, we apply quantified predicate logic including time indicators (elementary temporal predicate logic, (E)TPL)[7], which we have already used to eliminate neologistic predicate grue in 3.1.

[2]

H_1 all emeralds are green

H_2 all emeralds are grue

C_1 all emeralds are green OR all emeralds are grue

$\qquad (H_1 = H_2) \lor (H_1 \lor H_2)$

C_1 says that two different hypotheses confirm the same evidence statements.

C_2 says that it is 'intolerable' that two different hypotheses confirm the same evidence statements. Missing premise is that the hypotheses are not only different but 'incompatible' (or 'conflicting' etc.), which explains step from C_1 to C_2.

Additionally *supposing* =/ not equal to, - negation, ─A─ incompatible, mutually exclusive, ----- (logical) inference (from premises to conclusion)

[3]

P_1/H_1

$\qquad$ emerald a is green

emerald b is green

....

----

all emeralds are green

P_2/H_2

emerald a is grue

emerald b is grue

....

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all emeralds are grue

C_1 all emeralds are green $\lor$ all emeralds are grue

$H_1 \lor H_2$

P_3 H_1 and H_2 are incompatible, conflicting

$H_1 \neq H_2$

$H_1 = - H_2$ (contradiction as after t)

~~$H_1 \land H_2$~~

C_2 C_1 is 'intolerable', contradictory, false

$- C_1$

P_3 is hidden from the main argument but it is in the text of the TE:

> .... Thus although we are well aware which of the two *incompatible* predictions is genuinely confirmed, they are equally well confirmed according to our present definition.... (Goodman 1983, 74, italics added)

P_3 is repeated in Goodman's theory of projection in Chapter 4, as 'conflicting' (e.g. p. 99) hypotheses. H_1 and H_2 are only contradictory after t on the strong interpretation of H_2 or grue (2^1).

Additionally *supposing* $\lor$x existential quantifier, $\rightarrow$ implication (if ... then ...)

$$\lor x \, (E(x) \rightarrow (GE(x, <t) \lor GE(x, \geq t))) \qquad\qquad (1), (3'')$$
$$\lor x \, (E(x) \rightarrow (GE(x, <t) \lor -GE(x, \geq t))) \qquad\qquad (2^1)$$

If we stay with (2) there is no logical contradiction and we need exact definitions of green and blue, e.g., with help of wavelengths. However, Goodman does not provide this kind of exact information, and the situation remains undecidable.

There is a strong similarity to WVO Quine's 1960 *Gavagai*, analyses of which TE may also yield result of (possible conclusion of) multiple hypotheses, with no obvious clue, so, undecidability

which one is correct, fitting, but in *Gavagai* no misleading unresolvable ambiguity, or fallacy appears. Quine's TE would amount to Quine's doctrine of indeterminacy of translation and related theory of ontological relativity.[9]

Goodman's *Grue* TE may result in conclusion of multiple hypotheses, no obvious decidability which hypothesis is correct, fitting, but in *Grue* there is a possible misleading unresolvable ambiguity, which may render the argument fallacious and its pluralism resolution irrealist.

'Undecidability' formalization including elementary temporal predicate logic (ETPL)

[4]

P_1/H_1 $\bigvee x\,(E(x) \rightarrow GE(x))$ (1′)

$\bigvee x\,(E(x) \rightarrow (GE(x, <t) \bigvee GE(x, >t)))$ (1″)

P_2/H_2 $\bigvee x\,(E(x) \rightarrow GU(x))$ (2′)

$\bigvee x\,(E(x) \rightarrow (GE(x, <t) \bigvee BU(x, >t)))$ (2″)

C_1 $\{\bigvee x\,(E(x) \rightarrow (GE(x, <t) \bigvee GE(x, >t)))\} \bigvee \{\bigvee x\,(E(x) \rightarrow (GE(x, <t) \bigvee BU(x, >t)))\}$ (3)

This analysis including temporal predicate logic may reveal a basic ambiguity in the *Grue* TE, on which interpretation the argument is fallacious. In H_1 at *t* itself the emerald(s) is (are) green—$GE(x, \geq t)$ – and in H_2 at *t* itself the emerald(s) are grue, that is not-green and/or blue—$BU(x, \geq t)$, or $-GE(x, \geq t)$. The informal wordings of the TE hide this ambiguity which is only revealed by the formal temporal logical formalization. C_1 is only valid on condition of this ambiguity where the emeralds are green at *t* in H_1 and not-green and/or blue at *t* in H_2.

P_3 explicitly states that H_1 and H_2 are conflicting or incompatible which makes C_2 'intolerable' or false since H_1 and H_2 are contradictory. However, since P_3 is not very obvious in the restricted text of the TE, it may be considered 'smuggled in' from discussions in the next lecture and sections, partly revised in later editions.

Because 'grue' is not defined in a very exact manner, we can't make sure whether it is blue, not-green, or not-green and/or blue, or green and/or not-green or blue. On the latter interpretation the pluralist interpretation and paradigmatic, revolutionary paradigm shift interpretation is valid, as we will see in the *Whack* analogue, which describes a historical paradigm shift in, let's say, swan theory of ornithology.

3.3. Undecidability – Pseudo-problem

3.3.1 After falsification of the riddle by successful exemplification in, e.g., *Whack* – see next section – the riddle may still point to a pseudo-problem emphasizing undecidability of hypotheses about possibly future events, in other words, the paradox and riddle remain unresolved for any other hypothesis (than paradigmatic examples as 'all swans are whack') because of the nonexcludable

occurrence of what Nassim Taleb calls the highly improbable.

Additionally, skeptics and sophists alike may not only want to invent the paradox, but also make the paradox come true in many trivial ways, playing tricks with reason and logic. This is a kind of abuse of TE and paradoxes, we exclude from our pragmaticist research that features on adequate resolutions of TE and paradoxes.

3.3.2 The strong interpretation (C_2: - C_1) may support and necessitate the near-common sense skeptical reduction that Goodman adheres to (e.g.) in his return to the riddle in *Ways of Worldmaking* (Goodman 1978) – 'Induction according to nonprojectible categories is not merely awkward but wrong ...'[10] (Please, see Section 4.1 on Goodman's entrenchment theory.)

Both sophist and skeptical non-resolutions we would like to avoid. Next to the scientific and logical alternates we propose in this paper, there have been developed many more approaches to find a criterion to distinguish lawlike from accidental hypotheses, e.g. by exploring concepts of simplicity (see e.g. Gilbert Harman's attempt in Section 4.4), by acknowledging natural science and global cross- cultural investigations as in the field of color theory (see e.g. wavelength theory of colors, BCT, Basic Color Terms, Berlin and Kay 1969 etc. in Section 4.5).

3.4. All Swans Are Whack (Falsification – (universal) modus tollens)

I can't breathe (George Floyd)*

From history of science we know that before discovery of black swans, the statement 'all swans are white' was considered universally true (see e.g. Hempel 1945b, Popper 1959/2002, Taleb 2007).

With discovery of the first black swan in the Swan River in Australia by Willem de Vlamingh in 1697 there was a kind of paradigm change in the field of swan theory, ornithology, biology, and aforementioned universal statement was falsified. It would take about one more century, until 1790, before the discovery was scientifically assimilated as with the description of, first, subgenus of Chenopis and, next, species Cygnus atratus (of genus Cygnus, swan) by John Latham, the 'Grandfather of Australian ornithology', who added many more Australian birds, such as the emu and Australian magpie.

We may now reformulate this mini-scientific revolution after structure and terminology of Nelson Goodman's *Grue* TE.

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* Antiracist motto 'I can't breathe' is derived from the last words of African American George Floyd, one of hundreds of unarmed Black victims of police brutality of last decades, which have become a slogan during protests in many major USA cities in May - June 2020 (see Cornel West 2013, Havercroft and Haven 2016). Possible racial connotations of (Whack as a mix of) Black and White in the sense of names of ethnic groups aren't discussed in this paper, but in chapter on TE analyses and interpretations of Martin Luther King Jr.s 'I Have a Dream' in Hertogh 2015b, which text has been selected for presentation at two American conferences in 2019 (see Hertogh 2019). It was also the most favored topic of the seminars 'Thought Experiments: Capita Selecta,' delivered at Chongqing University in the first term of 2013-2014. Obvious analogy to the historical swan example is that Black people were not considered part of same human category as White people until, let's say, abolition of slavery (in 1865 in USA).

Reformulation

[5']

all swans are white
all swans are whack

white: white (before 1697 and white in and after 1697)
whack: white before 1697 \/ white or black in and after 1697

Logical symbolization

[5'']

Suppose
S(x)   : x is a Swan
WI(x)   : x is White
WA(x)   : x is Whack
/\x   : universal quantifier
\/x   : existential quantifier
→   : (material) implication, if ... then ...
-                   : negation

Falsification as by (universal) modus tollens[11]

/\x (S(x) → WI(x))   (1)
\/x (S(x) → WA(x))   (2)

-/\x (S(x) → WI(x))   (3)

At this moment we don't need any possible worlds semantics nor any more neologisms than 'whack' to explain 'white' and 'whack', since the temporal dimension of the TE may be fully confined to the definition of the predicates.

At the introduction of ETPL we have assumed that the *Grue* TE presupposes countable linear time to the future, and that ETPL may add linear time development to the past. In the *Whack* example--more than in the imaginary examples of Goodman's *Grue*--there is revealed a further ambiguity of ontology versus epistemology or standard logic versus epistemic logic. *Whack* as formulated above is defined solely from perspective of the European western civilization--whack: white before 1697 \/ white or black in and after 1697. Ontologically, or realistically speaking, it should be

whack: white or black

without any differentiation to before, in and after 1697 because long before the European discovery of black swans there were already black swans in Australia (one must be an extreme eurocentric

skeptic to deny this, furthermore, for Australian Aborigines may have held 'all swans are black'). We may express it with help of a dyadic predicate letter introducing agent variables, constants (like elementary temporal logic uses time variables, constants), e.g. WI(x, E) as 'x is White for E', where E stands for European, western civilization, science etc. Above definition is in fact a shortened epistemic definition relative to perspective of western science. However, as it has no effect on validity and soundness of above argument, we may as well keep on using the shortened one-place predicate letters, or even reduce the predicates to atemporal, nonepistemic notions as 'white' meaning transtemporal, transspatial (though restricted to Planet Earth) white, and 'whack' meaning white or black, adding 'until further notice', which clause is again abstracted from in the formulas. Lastly, it should be noted that 'white or black' also abstracts from some intermediary colors of swans, e.g. partially black, such as the black-necked swan (native to South America).

Whack is a successful exemplification of (weak versions of) Goodman's riddle (strong versions may suppose--a slightly different definition of--whack is contradictory to white after t). It stops the infinite quantification to all possible predicates since only some of them are indeed confirmed and it is not possible to principally have all possible predicates confirmed like it is not possible to finitely confirm a universal statement as 'all swans are white', as according to Hempel.[12]

We don't have to return to conventionalism, common sense, or jump on to irrealism, but may stick to, e.g., Willard Quine's ontological relativity, since 'all swans are white' was falsified by an extension of Europe's medieval epistemology, quite literally and physically speaking, by extension of (knowledge of) Earth's landmasses in 17th century as to include Australia. In this way *Grue* could be considered a prototype of a revolutionary TE depicting a paradigm shift.

4. Interpretations
4.1. Entrenchment Theory

In the last of the London lectures Goodman proposes the entrenchment theory as answer to the riddle:

> The answer, I think, is that we must consult the record of past projections of the two predicates, Plainly 'green' as a veteran of earlier and many more projections than 'grue', has the most impressive biography. The predicate 'green', we may say, is much better entrenched than the predicate 'grue'. (Goodman 1983, 94)

Although Goodman proceeds by arguing that, e.g. '(inherited) entrenchment' and 'familiarity' are not the same, he eventually recapitulates his endeavor in *Ways of Worldmaking* by eliminating 'grue' and 'bleen' predicates as 'nonprojectible', 'trivial', 'awkward', belonging to a 'different world' and finally 'wrong'

> Induction according to nonprojectible categories is not merely awkward but wrong, whatever may be the outcome of the inductive conclusion drawn. Rightness of induction requires rightness of predicates projected, and that in turn may vary with practice. (Goodman 1978, 129)

On this interpretation the TE is kind of reductio ad absurdum, a skeptical paradox that concludes with reboot to practice, like Wittgenstein in *Philosophical Investigations* (Wittgenstein 1953) argues about following a rule.[13]

Nassim Taleb interprets Goodman's 'riddle of induction' in a similar way as if the final resolution of the riddle would be skeptical:

> The riddle of induction is another version of the narrative fallacy—you face an infinity of 'stories' that explain what you have seen. The severity of Goodman's riddle of induction is as follows: if there is no longer even a single unique way to 'generalize' from what you see, to make an inference about the unknown, then how should you operate? The answer, clearly, will be that you should employ 'common sense,' [...] but your common sense may not be so well developed with respect to some Extremistan variables. (Taleb 2007, 188)[14]

We argue in this paper that much in line of Goodman's imaginative development of the TE and his philosophical presuppositions, the riddle should not be resolved beyond the conclusion that pluralism of more than one applicable hypothesis is not the problem, but, in fact, the resolution of the problem. We should hold on to the tentativeness that Goodman demonstrates in the 1953 lectures, that are the hallmark of Hempel's 'until further notice' clause.[15]

Pluralism does not necessarily contradict the entrenchment theory since the latter may hold particularly for use of language in inductive hypotheses beyond mere conditions to syntactical form as urged for by, e.g., Karl Popper ('features of the hypothesis other than its syntactical form'— Goodman 1983, 72—see Section 2 of this paper). Goodman acknowledges the restriction to 'linguistic practices' in the very last section of the book, 'Survey and Speculations':

> If I am at all correct, then, the roots of inductive validity are to be found in our use of language. .... The suggestion I have been developing here is that such agreement with regularities in what has been observed is a function of our linguistic practices. Thus the line between valid and invalid predictions (or inductions or projections) is drawn upon the basis of how the world is and has been described and anticipated in words. (Goodman 1983, 120-121)

It seems Goodman replaces one type of nominalism (from theory and neologisms) by another (reboot to practice of entrenched names). We suggest to go back to just before the reboot, and retain scientific and logical innovations as part of a productive pluralism that may be restrained by ontological relativity only.

Chomsky proposes a resolution from evolutionary epistemology and theory of learning that may not resolve Goodman's nominalist problem, but could nevertheless serve to unravel the riddle better than Goodman's language approach:

... why every language-learner (in fact, every mouse, chimpanzee, etc.) uses green instead of grue as the basis for generalization. No doubt this is a simple consequence of certain properties of the sensory system, a conclusion that is quite uninteresting from Goodman's point of view, but not, for that reason, incorrect. (Chomsky 2006, 158)

4.2. Prototype of Revolutionary TE

It may not be possible to discover the new laws and hence new kind of lawlikeness before the arrival of a new paradigm that will be developed at and after *t* just because the laws (as with regard to gemology in Goodman's example) are part of another, completely new scientific paradigm that may hinge on another and completely new (scientific) world view. The paradigm shift will bring about changes in scientific principles and laws in many parts of reality and science, and it is likely that the first revolutionary, crucial experiments, the first development of the new paradigmatic laws, won't occur in the field of Goodman's investigation (as in this example gemology), but that this field of research will only be indirectly related to the field of the major paradigm shift (in this example it could be another theory of the history of the earth, geology, biology etc.), and, thus, it is impossible to discover a rule of relation between lawlike and accidental hypotheses. Goodman does not entertain the possibility of scientific revolutions and adheres to a strictly linear development of (western) science, but since the theory of revolutionary change of paradigm shifts is rather well accepted by the philosophical and scientific communities it should be taken into account. Anyway, Goodman could have made some remark on the possibility of paradigm shifts and how it relates to his TE and search for criterion of lawlikeness, and we couldn't find such a remark in *Fact Fiction and Forecast*, also not in the later editions, which have revised sections, but still do not mention any reference to scientific paradigms.[16]

Both Fred Wilson 1983 and Steve Fuller 2000 contrast Kuhn's scientific revolutions to Quine's and Goodman's possible conservatism, e.g.

.... Nevertheless, according to Quine and his Harvard colleague Nelson Goodman, if no foolproof method is available to determine which of two empirically equivalent theories should be pursued, then one should stick to the theory that has worked so far, rather than risking it on the theory that has not yet had a chance to work. This is what Goodman called 'entrenchment' and Quine 'conservatism'..... (Fuller 2000, 87)

4.3. 'Essential' versus 'Accidental' Property of Color

On introduction of the 'new riddle of induction' the example of lawlike statement is, e.g., 'all copper conducts electricity' versus an accidental statement as 'all man now in this room are third sons.' As previously remarked, it is somehow curious Goodman wants to discuss the difference between lawlike and accidental statements with help of an example about possible change of an accidental property (as colors of gems like emeralds). One may need to be an expert in gemology and related sciences as geology to possibly understand why color could be more essential or lawlike to gems than to (e.g.) birds, but again Goodman makes no effort to explain these complexities of the examples from gemology.

In the second conclusion of the *Grue* TE (C₂—see Section 2, surface analyses), Goodman generalizes with help of an example of 'emerose' to all possible entities and properties, so that one may assume that the choice of gemology is random rather than premeditated. Still, one could have expected more explanation (as about e.g. the very difference between essence and accident) and without such explanations one could consider the choice of an example of an accidental property for discussion of lawlikeness a basic category mistake on ground of which the TE may appear fallacious.

However, we need to add that, of course, the same criticisms may be raised against the choice of historical examples in logic like 'all swans are white' and 'all raven are black.'

Therefore, as in our discussion of Hilary Putnam's *Twin Earth*, we argue for an update of philosophical examples as to, e.g., 'water is H_2O' as an example of a lawlike statement and 'water is watery stuff' as an example of an accidental statement.[17]

4.4. Precautions against Overgeneralization

4.4.1. First, like more paradoxes in the field of confirmation theory and induction, e.g. Hempel's *Raven* and Quine's *Gavagai,* we may consider these TE, firstly, precautions against overgeneralization, jumping to conclusions. One should always be careful when making generalizations or conceiving of universal statements in science.[18] In terminology of Carl Hempel, our (empirical) hypotheses should remain relative, never become absolute, it is about 'relative verification'

> At any rate, the acceptance of a hypothesis on the basis of a sufficient body of confirming evidence will as a rule be tentative, and will hold only 'until further notice', i.e. with the provisio that if new and unfavourable evidence should turn up (in other words, if new observation reports should be accepted which disconfirm the hypothesis in question) the hypothesis will be abandoned again. (Hempel 1945b, 116)

4.4.2. Second, indeed the neologistic premises or hypotheses are, in fact, overgeneralizations making unwarranted and even imaginative claims about (possible state of affairs in) the future. It is obvious that we, in principle, can't make any statements about the future, surely not about an unidentified point t in the near or far future. To be sure we should remain 'undecided' about such a paradox TE; 'undecidability' is an interpretation philosophers, but foremost scientists, should arrive at in a first intuitional, instinctic reflex.[19] In words of Carl Hempel, in this situation we would prefer 'suspending judgement, awaiting the establishment of further relevant evidence.' (Hempel 1945b, 114)

4.4.3. Third, it is about neologistic overgeneralizations which run counter to background assumptions of daily reality and principles and constraints of physical reality as identity, rest, continuity, simplicity etc. by proposing hypotheses (sometimes even about different objects than the objects at hand) that are picturing the (near or far) future as changeable, discontinuous and complex.

We may stress that this need not be irrealist or unnaturalist as depending on which field of reality we are referring to. The example of gemology (gem stones as emeralds) is unfortunate in this respect as sudden changes could be possible here, but they are much less likely and lesser known from

past history to a general public (and we must say Goodman does not mention any real example of change of color of gem stones, e.g. as effected by chemical or geological transitions) than, for instance, evolutionary biology, ornithology, developmental psychology, fashion, ecology. In evolutionary biology, developmental psychology, ecology, scientists study, respectively, changes in species as after mutation over ages of time, changes in cognition and behavior in the development from infant to adult, and the effects of pollution as by CO_2 on possible change of the climate of Planet Earth.

Goodman's examples of gem stones could make more sense when studying, e.g., geological changes over the past eons.

The historical examples of 'all swans are white' and 'all raven are black' are instances from biology, ornithology wherein there is a wide variety of animals, birds over different climate zones and regions of Planet Earth, so that any restrictive universal statement on, e.g., color of animals or birds may easily be falsified by investigating another climate zone or region of the planet, and finding (new) species whose colors falsify the presumed universal statement. These are not examples of pluralism in biology and ornithology, but foremost of ontological relativity since different species may occur in different regions of the Earth, and what may hold for animals and birds living in one part of Earth's ontology does not hold for animals and birds in another part of the Earth, e.g., like Galileo's and Newton's gravitation theory may hold for daily life of Earth's meso-level and Einstein's relativity theory for   cosmological macrocosm and subatomic microcosm.

Quine's *Gavagai* is an example from behaviorist psychology of perception and linguistics akin to logical paradoxes about identity (and the like), which may indicate rest (Thales) or, contrarily, change (Parmenides—'one can't step twice in the same river'). Quine's *Gavagai* is akin to Goodman's *Grue* because it also questions identity, continuity, simplicity constraints.[20]

As Gilbert Harman's contribution on 'inductive simplicity' correctly emphasizes, there are good reasons to prefer maxims of simplicity over skepticism:

> In certain statistical learning problems, a policy of choosing simpler rules that account fairly well for data is likely to have less error on new cases than a policy of choosing complex rules that have less error on the data. The relevant kind of simplicity is not to be measured in terms of the number of parameters needed to specify a given member of a class of rules but might be measured in terms of the VC dimension of such a class. The rationale for using simplicity so measured can be extended to allow simplicity to decide among empirically equivalent hypotheses. The extended rationale provides reasons of simplicity to reject certain sorts of philosophical skepticism. (Harman 2003, 1[21])

4.5. Cross-cultural Color Interpretation

With help of natural sciences we may go beyond skeptical reductions to common sense. In ideal, scientific language of wavelengths (measured in nm, nanometer, 10^{-9} m, 1/billionth (AE) m. or 1/milliardth (BE) m.) 'green' as used in modern European languages represents range of ab. 520-570 nm., and blue ab. 440-490 nm. Research shows the distinction may be different in historical and non- European cultures and languages.

Wikipedia even states it may be about a particularly English lexical gap between green and blue:

> Many languages do not distinguish between what in English are described as 'blue' and 'green' and instead use a cover term spanning both. To describe this English lexical gap, linguists use the portmanteau word *grue*, from green and blue, which the philosopher Nelson Goodman coined in his 1955 *Fact, Fiction, and Forecast* to illustrate the 'new riddle of induction'. (*Wikipedia* Color term and basic color terms, Blue-green distinction in language – retrieved on February 16, 2018)

An example of Japanese color words evolution from a later study on color categories in thought and language

> A second observation about the Japanese nomenclature and the Berlin and Kay research concerns GREEN and BLUE terms and the historical separation of the GRUE category. Perhaps as late as 800 years ago, the modern Japanese term for blue, *ao*, included most of the green hues – that is colors which today would be called *midori* (GREEN). (Stanlaw 1997, 255)

Therefore, a global cross-cultural resolution applying ontological relativity (after e.g. Quine) may appear most appropriate in Goodman's particular example of grue, green-blue, too.

With regard to color words (colors of emeralds may need additional investigations), we may conclude with Clyde Hardin who bases his conclusion on research like 'Berlin-Kay theory of basic color terms' (BCT):

> ... It appears that a proper understanding of even the denotations – let alone the connotations – of a language's color terms requires a proper grasp of the relative contributions of biological, cultural, and environmental factors .... (Hardin 2013, 4)

4.6. Goodman's Pluralism and Quine's Ontological Relativity

When proposing a pluralist interpretation of *Grue* TE instead of a common sense interpretation we apply Goodman's later 1978 thoughts as expressed in *Ways of Worldmaking* to his early TE:

> The movement is from unique truth and a world fixed and found to a diversity of right and even conflicting versions or worlds in the making. (Goodman 1978, 10)

However, we assume that the conflict of Goodman's pluralism of competing hypotheses to explain one and the same set of observation reports, may be restrained and reduced to what WVO Quine calls 'ontological relativity.'

Both foundational shifts in theoretical physics, where Thomas Kuhn is referring to, and new discoveries of large areas of Planet Earth, may be considered extensions of daily and scientific ontology. If the new discoveries have not changed all of the scientific concepts and laws of science,

they may still have changed relative parts of the epistemology and ontology of science. These paradigm shifts can be philosophically comprehended under the denominator of Willard Quine's ontological relativity, rather than Nelson Goodman's 'irrealism', since the conflict between competing hypotheses may be relativized to different parts of our daily and scientific ontology.

Ontological relativity does not need to support pluralism in principle, but says there is not one (monistic) theory that could explain all (parts of) reality, but different parts of reality obey different laws, are described by different theories etc., e.g., relativity physics rules cosmology and particle physics (as quantum physics), and (an approximation of) Galilean-Newtonian gravitation still rules meso-level of Planet Earth, generally speaking. The laws that rule consciousness are different from laws that rule mechanics. Linguistic rules are different from mathematical laws (see Wittgenstein 1953). In Europe swans are usually white, and (since 1697 Europeans may know that) swans in Australia are usually black. Exactly like in the *Grue* TE of Goodman 'all emeralds are green' hold before *t* and possibly 'all emeralds are grue' may hold (at and) after *t*. We don't have to make universal statements that may hold for all time and all space, but should relativize our empirical statements to a particular time and space. Goodman tries to unnecessarily overgeneralize his hypotheses to, anyway, all time, an infinite progress of time to the future. If wished, the relativization of the hypotheses to particular time and space may be considered pragmaticism.

Possibly multiple hypotheses could be reduced to, reconstructed or unified by one unitary theory (as possibly mediated by information technology IT), but as they stand they may be sufficient and most suitable to explain the part of reality that they are said to govern.

Still, we do adhere to pluralism in the sense of Hempel's tentativeness and possibly Quine's ontological relativity, viz. that there may be more than one theory to explain the same phenomena and we won't know at this moment which one is most correct.

Quine's *Gavagai* TE doesn't only demonstrate inscrutability of reference and indeterminacy of translation, but also ontological relativity as explained in the 1969 essay with the same:

> It is thus meaningless within the theory to say which of the various possible models of our theory form is our real or intended model. Yet even here we can make sense still of there being many models. For we might be able to show that for each of the models, however unspecifiable, there is bound to be another which is a permutation or perhaps a diminution of the first. (Quine 1969, 54)

We guess that both the young Goodman and young Quine may have entertained Hempel's tentativeness since their theoretical positions are well-known as Goodman's pluralism, even irrealism, and Quine's ontological relativity. It is like going back to before the skeptical reduction of *Grue.*

Goodman seems to apply his pluralism to paradigm shifts, too, as in *Ways of Worldmaking* (Goodman 1978) about geocentrism and heliocentrism:

> Perhaps, though, we can reconcile ... by relativization to points or frames of reference rather than to systems or versions. A simpler example will be easier to handle here. The equally true conflicting

sentences concerning the daily motion of the earth and sun

(9) The earth rotates, while the sun is motionless
(10) The earth is motionless, while the sun revolves around it might be interpreted as amounting to
(11) The earth rotates relative to the sun
(12) The sun revolves relative to the earth, which are nonconflicting truths.
(Goodman 1978, 112-113)

Although Goodman comes back from this consideration by a rather artificial example, we guess that usually we can still try and regard two 'more comprehensive systems and versions that [seem to] conflict' as having 'their realms' 'within one world' – rather than in 'two different worlds.' (Goodman 1978, 116—bracketed words added)

4.7. Environmental Pragmaticism and Global Cross-culturalism (Progress of Science and Society)

Environmental pragmaticism relates (applications of) progress of science to progress of society and says, e.g., that the more damaging effects (the societal applications of) a theory (have) has on human health and ecology of Planet Earth, the less sound (they) it (are) is (environmentally pragmatically speaking). Global cross-culturalism wants to say that in a global digital world we should avoid referring to cultural specific concepts and images, valuing one culture over another, etc.; research shouldn't remain within one culture but derive data from more than just one culture. These are not political issues yet, but remain within the responsibility of philosophers and scientists, e.g., relating to how they present their theories and views. The former may be considered part of constructive principles of (philosophy of) sciences, the latter of (philosophy of) humanities. The subtheses could be included as conditions to nowadays updates of Goodman's criterion of lawlikeness, prospects of projectibility, Hempel's studies into the theory of confirmation, and Popper's error elimination as part of progress of science.

We return to these proposals elsewhere, but at this moment we will already present an adaptation of one of Popper's formulas.[22] Karl Popper is well-known for his work on falsifiability and growth of knowledge. Hypotheses that are developed for testing should use logical formalizations that are (in form at least) both verifiable and falsifiable, and progress of science can't proceed without error elimination. Popper proposes (e.g.) next formula to summarize his ideas about progress of science

Using 'P' for problem, 'TS' for tentative solutions and 'EE' for error-elimination we can describe the fundamental evolutionary sequence of events as follows:
....
$$P_1 \rightarrow TS \rightarrow EE \rightarrow P_2.\ \text{(Popper 1972/1979, 243)}$$

We want to add ecological and cross-cultural concerns to Popper's formula. If the TS, tentative solutions or theories, are not part of environmental or cross-cultural research, the possible negative effects on human health, natural environment and nowadays inclusive societies should be corrected as part of EE, error elimination. If we abbreviate environmental pragmaticism as EP and global cross-culturalism as GCC, we may add them to Popper's formula of growth of science like this

$$P_1 \rightarrow TS \rightarrow \ EE/EP\ GCC \rightarrow P_2$$

Please, note that EP and GCC involve effects of TS on society, so, by adding EP and GCC the formula now represents growth of knowledge in science and society since the possibly damaging effects of science on society are corrected in the phase of EE by focussing (/) on EP and GCC.

Nowadays, the additional epistemological condition of global cross-culturalism may require that one cannot generalize to all of mankind if there is no control research in at least one more culture; in this way one is able to double-check outcomes of humanities research on cultural biases. Nowadays, the additional epistemological condition of environmental pragmaticism may require to double-check if the possible societal applications of a new scientific theory will be hurting human health and natural environment, and, if so, the theory cannot be considered (environmental pragmaticist) sound.

4.8. Possible Worlds Semantics

Different from Goodman we will hold on to the tradition from modal logic (e.g. frame semantics, Kripke's PWS e.g. Kripke 1959, 1963, 1980) that there is an 'actual world' (e.g. w_0) or 'real world', e.g.

> The basis of the informal analysis which motivated these definitions [as (*universally*) *valid*, *satisfiable*] is that a proposition is necessary if and only if it is true in all 'possible worlds'. (It   is not necessary for our present purposes to analyze the concept of a 'possible world' any further.) Now let *A* be a formula with $P_1, ..., P_m$ as its propositional and predicate variables and $x_1, ..., x_n$ as its free individual variables. ... In modal logic ... we wish to know not only about the real world but about other conceivable worlds; *P* may be true in the real world but false in some imaginable one, and similarly for $P(x_1, ..., x_n)$. Thus we are led not to a single assignment but to a set **K** of assignments, all but one of which represent worlds   which are conceivable but not actual; the assignment representing the actual world is singled out as **G** .... (Kripke 1959, 3 – logical symbolism and vocabulary may change in later papers)

And we may define it as the meso-level of Planet Earth, that is Planet Earth as described or depicted as the 'outside', 'external' or 'natural world', that is intersubjective or objective and different from subjective inner worlds as Sartrean 'pour soi' and Nagelian 'what it is like to be an *X* for an *X*.'

But, still, the relevant (natural) laws are only actual 'until further notice' of change, falsification, and verification of new more relevant and adequate laws. Also if one won't like to

consider this 'actual world' objective, it may still be considered 'intersubjective' as kind of point or frame of reference from which we describe the other 'possible worlds' as to establish meaningful communication etc. It is not a claim about 'the only truth about the only world' (Goodman 1978, 5), but a claim about the intersubjectively considered most likely truth about what we here and now consider the most appropriate approximation of the 'actual world.' It does presuppose that 'physics itself is fragmentary and unstable' since it remains a set of relative hypotheses, that are liable to change by falsification and verification of--at this moment yet (completely) unknown--new relative hypotheses. These truths may be less certain, clear and distinct to the subjective mind than taste and feelings, they remain the best available intersubjective guess (at a particular time and place).

'How to interpret such terms as "real", "unreal", "fictive", and "possible"' is not 'a subsequent question' (Goodman 1978, 2), but this interpretation is part of the task of philosophy and logic, possibly it is not yet the task of the individual sciences. The pluralism of multiple actual worlds needs to be restrained until there is at least one world view that most adequately depicts what we then will call the 'real' or 'actual world' (at a particular time and place).

With regard to the two subtheses of environmental pragmaticism (EP) and global cross-culturalism (GCC), we have seen in Section 4.5 that the latter applies to color words (e.g. Berlin's and Kay's BCT, basic color terms). The different PW could be designated to scientifically identified regions of language communities, probably wide-spread over the globe, and attain a status of physically possible, linguistic worlds instead of only logically, conceptually or metaphysically possible worlds.

In conclusion, we may indicate a modal logical account of induction by a modification of Nelson Goodman's account of induction. Though Goodman's *Grue* TE starts off promising, suggesting a temporal logical reconstruction of induction theory, the old problems that could cause mistakes (as 'all swans are white''s painful refutation in 1697) return when Goodman goes on to eliminate 'grue' and 'bleen' predicates as 'nonprojectible', 'trivial', 'awkward', belonging to a 'different world' and finally 'wrong.'

Instead we can retain these possible worlds in a modal logical account of induction as e.g. the 'different world' of 'grue' is likely to occur in different cultures where color words refer to different parts of the color spectrum (in fact 'grue' denotes cover terms for both green and blue in many cultures), or we can--less awkwardly than Goodman suggests--adopt different frames of reference when describing motion (e.g. Albert Einstein's *Elevator* TE, Einstein, Infeld 1938), less awkwardly than Goodman guesses (see Goodman 1978, 128) translate our inductive arguments, e.g. gravity arguments, from the heliocentric system back to the geocentric system, and, use an approximation of Galilean-Newtonian gravity instead of Einsteinian relativity when talking about the meso-level of Planet Earth (as relativity may only apply to micro- and macrocosm).

To avoid mistakes as the historical 'all swans are white' we can probabilistically quantify over PW instead of over only evidence statements of the actual physical world of Planet Earth, that is in Goodman's terminology (but not in accordance with his eventual entrenchment theory), quantify over both 'green' and 'grue' predicates, worlds etc.

The example of the historical black swan returns in the title of Nassim Taleb's 2007 *The Black Swan*, explaining in the subtitle it is about the impact of the highly improbable. In a 21st-century global cross-cultural world induction can't be based anymore on only (Humean) habits and (English) color denotations and connotations--cultural biases should be eliminated, and probabilities (PW) need to be reestimated without local biases.[23]

Moreover, in the analyses part of this paper we have demonstrated that the metaphysical neologisms of *Grue* may be eliminated with help of an elementary temporal predicate logic, and that, next to the traditional epistemological interpretations of philosophy of science, *Grue* may be successfully analyzed and interpreted as a prototype of a Kuhnian paradigmatic, revolutionary thought experiment as with help of Karl Popper's procedure of falsification by means of modus tollens.

Endnotes:

1. Abbreviations used in this paper e.g.
    TE: thought experiment(s)
    PWS: possible worlds semantics
    H_n Hypothesis n, P_n Premise n, C_n Conclusion n
    All logical symbols can be made on a (f.i. QWERTY) keyboard, except symbols from set theory, intersection and empty set--see also notes 3, 9.

    Research question that started this paper in 2017 is how to analyze *Grue* as a TE, as part of research into semantics of TE. The results were not fully foreseen, e.g. that we would hit upon a prototype of a paradigmatic or revolutionary TE (in the sense of Thomas Kuhn, whose philosophy is not mentioned by Goodman at all). Next, as usual we connect the results of the TE analyses with presumptions and general outlook of the philosopher who conceived of the TE, and this procedure yields remarkable conclusions in this case, since we found that analyses of *Grue* may fit in with a philosophy of pluralism rather than with common sense, where the later Goodman reduces the TE to (in any case, the neologism part of the TE, as possibly after Wittgenstein, who also goes unmentioned by Goodman).
    Taxonomy of diverse types of TE are discussed in PhD diss. Hertogh 2015b, e.g. Paradox TE.
    Kuhnian revolutionary or paradigmatic TE are called Deconstructive TE, i.e. TE that destruct, falsify an old theory and construct, verify a new theory. James Brown (1991) may call this type of TE mystifyingly Platonic TE, but we stay with John Norton (2004a, b) in an empirical view of TE that avoids mysteries and magic instead of adding to it, although the rhetorical force of TE can't be denied. Neologistic TE--Goodman's *Grue* is a superb example--may be defined as TE whereof TE workings hinge on neologisms.

2. We hereby refer readers, who are not familiar with (one or some of) the approaches or problems mentioned, to the references and literature. Many approaches are valuable and may trigger worthwhile discussions. This paper, however, mainly focusses on aspects and points of views of the problem that may have been overlooked, partly caused by vaguenesses, ambiguities and possible inconsistencies in the text of *Grue* (see e.g. Sections 3.1, 3.2), possibly because of lack of communication between the old theory (e.g. Goodman, Quine) and the new philosophy of science (e.g. Popper, 'anti-inductivist[s]', see Goodman 1983, xxii, and Kuhn, not mentioned at all in Goodman 1983 – see Section 4.2).
    Quite a few authors point to vaguenesses, ambiguities etc. in the text of *Grue* (apart from the ambiguities the epistemic paradox may trigger itself), e.g. Fain 1967, Jackson 1975, Shoemaker 1975/2003b, Chomsky 2006. Particularly with regard to the definition of 'grue', authors acknowledge that there is more than just one interpretation of 'grue', and, next, that their definition may deviate from 'the concept Goodman actually had in mind', e.g. Fain 1967

I say 'less elliptically' somewhat hesitantly because a number of 'less elliptic' paraphrases of Goodman's casual definition of grue in *Fact, Fiction and Forecast* have appeared in the literature. On one interpretation, for example, grue objects are objects which are green prior to some time *t* and blue after *t*. The particular paraphrase used here is, I think, the closest to the text, although it may not adequately represent the concept Goodman actually had in mind. My analysis of the riddle of induction depends upon defining 'grue' exactly the way in which I do it. (Fain 1967, 61n3)

Shoemaker 1975/2003b observes in one of the first notes of his paper that there are 'various ways in which '''grue'-like'' could be defined.' Shoemaker explains that his definition is different from a standard one, which he calls the 'Barker-Achinstein definition' (referring to Barker and Achinstein 1960)

Sometimes Goodman's definition of 'grue' has been read differently from the way I read it; it has been read as meaning that for any time *t*, a thing *x* is grue at *t* if and only if either *t* is earlier than *T* and *x* is green at *t*, or *t* is *T* or later, and *x* is blue at *t*, where *T* is the time mentioned in Goodman's definition. Since the prevalence of this reading is due to an influential paper by Barker and Achinstein, let us call this, the Barker- Achinstein definition.... (Shoemaker 1975/2003b, 78)

Chomsky 2006 interpreting 'Gruebleen-like' 'languages' from point of view of universal grammar concludes that the notion is rather 'vague', 'unspecified':

He [Goodman] argues that ... one must be '... aware ...' ... of what in general is 'the difference between Gruebleen-like and English-like languages'. I think that this is a rather marginal issue, since much more deep-seated properties of 'English-like languages' have been formulated and investigated, but, since he brings up this example, it is well to point out that the difficulties to which he alludes are in large measure a consequence of the vagueness of the question he asks. ....As long as Goodman's vague notions 'English-like' and 'gruebleen-like' are left unspecified.... (Chomsky 2006, 157, 158)

For Jackson's comments on the definition of *Grue*, please, see note 3.

3. With regard to the problem of induction, generally speaking, we suggest to start again with the logical methods of John Stuart Mill (1843/1882) instead of the psychological theory of David Hume (1740). Mill is mentioned but not really discussed in Goodman 1983. In Hertogh 2013, 2015b we propose a couple of inductive analyses of TE, e.g. Thomas Nagel 1974, *What It Is Like to Be a Bat*. In this paper the provisional analyses of Quine's *Gavagai* remain inductive, too, but this account is peculiar because the TE wants to show at the same time that induction of direct ostension fails.

Carnap 'generic scheme' of modus ponens instantiation in Carnap (1966, 7, 17) shows how deduction may be transferred into induction – and the other way around – by summarizing scientific explanation similar to scientific prediction:

> *1. (x)(Px → Qx)*
> *2. Pa*
> *3. Qa*

This formula shows prediction, i.e., how conclusion 3 may be inferred from major 1 – general law, probabilistically formulated it runs fr (Q,P) = [e.g.] 0.8, where fr (Q,P) is the relative frequency of Q

with respect to P (e.g. 0.8, 80%) – and minor, instance 2 (modus ponens).

Conversely, it show explanation, 1 follows from 2 and 3, Pa, Qa, so, (x)(Px → Qx) – which accords with e.g. John Stuart Mill's inductive logic—it may be generalized like this

$$\text{'}Fa_1 \,\&... \,\& \,Fa_n\text{'} \text{ gives a good reason for } \text{'}Fa_{n+1}\text{'} \text{ (Jackson 1975, 113)}$$

where *Fx* is a property, *F* predicate letter and $a_1... a_n, a_{n+1}$ logical constants, escaping from possible formal complexities of material implication (→) by 'gives a good reason for.' Goodman's problem is how to formulate (elimination) rules that govern the latter inference.

(Using logical symbols as used in e.g. Kutschera 1978 for Carnap's formula--

/\x (Px → Qx), Pa, Qa (deduction, modus ponens) and Pa, Qa, /\x (Px → Qx) (induction), where /\x is universal quantifier and → is material implication, if ... then ....)

4. See Sorensen 1992, e.g.

> .... Crucial to the diagnosis is identification of the tricky thought experiments as *paradoxes*.
> A paradox is a small set of individually plausible yet jointly inconsistent propositions. In chapter 6 I extrapolate to the thesis that every thought experiment is reducible to such a set. ... (Sorensen 1992, 5)

5. The neologisms are contractions or portmanteaus (linguistic blendings) of two adjectives or nouns – 'bleen' (blue, green), 'emerose' (emerald, rose), 'emeruby' (emerald, ruby), 'sapphiruby' (sapphire, ruby), Eifferuby (Eiffel Tower, ruby), 'grund' (green, round), 'grare' (green, square), 'gred' (green, red) 'zig' (vs. bag, marbles in helter-shelter selection), 'bagleet' (bag, naval fleet), 'bagmarks' (bag, make of car).

6. These overseas explorations are considered part of European Age of Discovery. However, last decades there have been developed competing non-eurocentric and non-western theories in an international, global cross-cultural context as that, e.g., America was first discovered by navigations of Muslim Chinese explorer Zheng He (郑和 1371–1433), who may already have discovered the new world in 1421 as recorded by e.g. Gavin Menzies, and whose name is well-known among Chinese college students.

Next, it is well-known that Africa, Arabia, Mesoamerica and Asia have contributed to mathematical, scientific and technological developments to a high extent. China was a world leader in science and technology in the Ming Dynasty (1368 - 1644) as by the Four Great Inventions, papermaking, printing, compass and gunpowder.

With regard to explorations of outer space the achievements of USA, Europe and Russia are about to be countered by projected Chinese moon landing as scheduled for 2025.

7. (Elementary) temporal predicate logic may be abbreviated (E)TPL as after Valentin Goranko's use of acronyms in *SEP*. Our ETPL is inspired by Hempel:

> ... or the psychological hypothesis, "You can fool all of the people some of the time and some of the people all of the time, but you cannot fool all of the people all of the time", which may be symbolized by '(*x*)(*Et*)Fl(*x,t*). (*Ex*)(*t*)FL(*x,t*).-(*x*)(*t*)Fl(*x,t*)', (where 'Fl(*x,t*)' stands for "You can fool (person) *x* at time *t*"). (Hempel 1945a, 11 – saying is usually credited to Abraham Lincoln)

Frank Jackson (1975, resp. 115, 116, 118) uses similar 'semi-formal terms' (see also note 3) to explicate 'the three common kinds of ways of defining "grue"', $grue_1$, $grue_2$ and $grue_3$:

> A typical example of the first way is:

> D_1. *x* is grue iff *x* is green before *T* and blue thereafter.

where T is a chosen time in the future ... [n. 4--cf. Blackburn 1969, Kelley 1971]

On D_1, 'grue' is atemporal--an object is grue or not once and for all, it cannot be grue at one time and not grue at another--and in this respect differs from 'green'.

....

A typical example of the second way... of defining 'grue' is:

D_2. x is grue at t iff (x is green at t & $t < T$)
    or (x is blue at t & $t \geq T$).

'Grue' on this definition is like 'green' in being temporal: an object may be grue$_2$ at one time and not at another.

....

... the following definition is close to that he [Goodman] uses in *Fact, Fiction, and Forecast*:

D_3. x is grue at t iff (x is examined by T and x is green at t) or (x is not examined by T and x is blue at t).

As indicated by the 'at t' in D_3, this definition is for enduring objects. ...

Jackson 1975 and more literature demonstrate there are many possible 'hairsplitting' interpretations, distinguishing between temporal and enduring objects and colors, examined and unexamined objects etc. (as partly discussed in Goodman 1983). Because of the common sense nature of Goodman's entrenchment theory we suppose the paradox should be understood in a common sense way, too, and we are only suggesting ambiguities with regard to measurement at time t, and names of colors and objects, which may be supported by cross-cultural and trans-historical research.

'Hairsplitting' is used in Taleb 2007 to describe Quine's *Gavagai* TE, which TE is set in a cross-cultural setting beyond common sense.

8. Please, note the colors the predicates represent as blue and green are not complementary or opposite colors (blue and yellow are, and green and red), but actually quite similar, near in color scheme range. Analogously, when logically interpreting *Grue* we don't need to exclude anything but grue is not-green, $GU(x) = -GE(x)$. Actually, since blue and green are close to each other in color range, physically speaking wavelength, and grue may resemble a mix of green and blue as turquoise, our interpretation of *Grue* as referring to both green and shades of green that can't be considered green anymore, as turquoise and some shades of blue, $GU(x) = GE(x) \lor GU(x)$, is closer to ordinary and scientific usage than the later Goodman's skeptical elimination of any intermediates between green and blue. (Please, see Section 4.5 on cross-cultural color interpretation.)

9. The *Gavagai* TE appears in Quine's 1960 main treatise, *Word and Object*, in the end of the first section of Chapter II, 'Translation and Meaning', called 'First Steps of Radical Translation' (Quine 1960, 28-30).

    The *Gavagai* TE is about a field linguist and a native of an unknown, noncognate language, when the former is trying to guess what the latter may mean by the sound 'gavagai', uttered by the native speaker when she sees a rabbit—Does 'gavagai' refer to rabbit, rabbit-part, rabbit-stage, or possibly rabbithood etc. ...?

    Quine 1960, also quoted in Putnam 1974

> There can be no doubt that rival systems of analytical hypotheses can fit the totality of dispositions to speech behavior as well, and still specify mutually incompatible translations of countless sentences insusceptible of independent control.

It may be provisionally analyzed as failed induction because of multiple possible hypotheses, which however may support claims of ontological relativity, involving undecidable pluralism as consistent with Hempel's tentativeness of retaining all possible hypotheses 'until further notice.'

In *Word and Object* the TE is framed as part of discourse on explanation of analytical hypotheses and stimulus meaning, the latter being Quine's methodology to naturalize epistemology and theory of meaning, semantics, in a behaviorist way.

On logical analysis—as from the explanation in 'Ontological Relativity', the first John Dewey Lecture (Quine 1968a) – *Gavagai* appears to amount to failed induction from skepticism about direct ostension.

In our criticism, however, we may conclude that it is actually about hasty generalization.

[6]

Suppose

$H_{1,2,3}$		analytical hypotheses of field linguist
Gavagai		g
rabbit (thing/whole)		r (or rt)
rabbit part		rp
rabbit stage	rs	
(rabbithood		rh)
=		means, refers e.g. a = t : sign a refers to object/thing t

H_1:	g = r		(1)
H_2:	g = rp		(2)
H_3:	g = rs		(3)

----

g?

Although there may be more interpretations of Quine's TE, we guess Quine first introduces the *Gavagai* TE, concludes to the failure of radical translation, that is failure of direct ostension, and next infers to his doctrine of ontological relativity.

[7]

H_1:	g = r	(1')
H_2:	g = rp	(2')
H_3:	g = rs	(3')

$g = r \lor g = rp \lor g = rs$

[8]

H_1:	g = r	(1'')
H_2:	g = rp	(2'')
H_3:	g – rs	(3'')

~~$g = r \land g = rp \land g = rs$~~

Induction fails here (cf. [1] and [3] in main text), that is to say radical translation is not possible or

direct ostension is not possible. One may only conclude to disjunction or impossible conjunction of analytical hypotheses that are not mutually consistent (in case of conjunction—cf. [3] conjuncts stricken through cf. incompatible, mutually exclusive in set theory, probability theory--intersection of two sets or events is empty A ∩ B = ∅). [7] may show the inscrutibility of reference, (systematic) ambiguity, indeterminacy of (radical) translation, ontological relativity, since there is more than just one hypothesis about reference, (radical) translation, ontology of unknown language possible—this type of induction based on initial direct ostension cannot conclude yet to only one clear-cut hypothesis (theory). In fact, this kind of impossible situation occurs often as part of deferred ostension (e.g. referring to universal rabbithood by pointing to a particular rabbit cf. Becker 2012, 165). Quine wants to show with help of the *Gavagai* TE that direct ostension in case of (direct/radical) translation (i.e. translation of a yet completely unknown, remote language) is not possible.

We should note, however, that a single imaginary experiment or TE may not suffice to arrive at theoretical conclusions. According to scientific methodology, one needs much more experimental evidence, conduct more than just one experiment with more than just one respondent, repeat the experiments, conduct control experiments in different settings etc. to develop trustable and reliable theories. Concluding from the *Gavagai* TE that direct ostension or radical translation fails amounts to inductive fallacy of hasty generalization. Furthermore, the setting of the TE, Q&A in an open field, may indeed have a lot of possibly irrelevant information, experimental settings are quite often artificial, take place indoors, in a lab, etc. as to exclude irrelevant information. We may still use direct ostension in a classroom to teach students, whose mother tongue and culture are unrelated to the target language, new vocabulary.

In fact, in his 1968 lecture on ontological relativity, Quine gives many more examples, e.g. Japanese classifiers, to provide for more evidence for his doctrine. We may return to it elsewhere, possibly in another paper on inductive paradox TE.

10. Possibly Goodman wants to say here 'unprojectible' since in 1973 'Note to the Third Edition' Goodman distinguishes 'nonprojectible' from 'unprojectible', the former signifying something like neither projectible nor unprojectible, and the latter negation of projectible (Goodman 1983, xxi). However, we won't exclude the possibility that the use of these two or three terms may change from context to context (like 'grue'), cf. Goodman 1983, 100-101, Goodman 1978, 127-129.

11. See Hempel's logical analysis

> ...a purely universal hypothesis (symbolized by a formula consisting of one or more quantifiers followed by a sentential function containing no quantifiers) is falsifiable but not verifiable by an infinite universe of discourse. Thus, e.g., the hypothesis "(x)(Swan(x) → White(x))" is completely falsified by the observation report {Swan(a), - White(a)}; but no finite observation report can entail and thus verify the hypothesis in question. (Hempel 1945b, 113)

We follow the deductive logical analysis of falsification by Karl Popper as modus tollens:

$$p \rightarrow q$$
$$- q$$

$$- p$$

The Logic of Scientific Discovery (Popper 1959/2002), e.g.

> ... My proposal is based upon an asymmetry between verifiability and falsifiability; an asymmetry which results from the logical form of universal statements. ... For these are never

derivable from singular statements, but can be contradicted by singular statements. Consequently it is possible by means of purely deductive inferences (with the help of the modus tollens of classical logic) to argue from the truth of singular statements to the falsity of universal statements.... The falsifying mode of inference here referred to—the way in which the falsification of a conclusion entails the falsification of the system from which it is derived—is the modus tollens of classical logic.... (Popper 1959/2002, resp. 19, 55)

Modus tollens including quantifiers is sometimes called universal modus tollens (analogously to quantified modus ponens, which may be called universal modus ponens, akin to universal instantiation).

In formula [5″] the universal statement of the major (1, p → q) is contradicted by the example(s) of the minor (2, - q, WI(x) =/ WA(x)), and it results in negation, falsification of (1) in conclusion (3, - p): without quantifiers

$$S(x) \rightarrow WI(x) \qquad \text{if x is a swan, then it is white}$$
$$- WI(a) \qquad \text{a is not white (for it is black)}$$
$$\text{--------------------} \qquad \text{---}$$
$$- S(a) \qquad \text{a is not a swan}$$

Epistemic logic as indicated in Section 3.4 has been developed analogous to, e.g., Hempel′s temporal logic (Hempel 1945a--see note 7). Hintikka 1962 on knowledge and belief introduces e.g. an account with predicate letters as K and B for Know(s), resp., Believe(s), next to a modal account. Although many TE seem to transcend classical logic, standard logic may often do to analyze the deep structure of the TE argument, particularly because some complexities as temporal and epistemic interpretations may be abstracted from in logical symbolization of the core of the argument, e.g. because they only apply to analysis of some predicates (like in *Whack*).

12. As Hempel explains it is not possible to finitely confirm a universal statement as ′all swan are white′ – please, see end of Hempel′s quote in note 11

   ... but no finite observation report can entail and thus verify the hypothesis in question. (Hempel 1945b, 113)

13. Saul Kripke (1982) discovers a skeptical paradox in #202 of *Philosophical Investigations* as with regard to following of a rule is a practice.

   And hence also ′obeying a rule′ is a practice. And to think one is obeying a rule is not to obey a rule. Hence it is not possible to obey a rule ′privately′: otherwise thinking one was obeying a rule would be the same thing as obeying it. (Wittgenstein 1953, #202)

   Wittgensteinianly speaking we would remain with logico-philosophical treatise of *Tractatus* (Wittgenstein 1961) as an upstart to development of modal logic of PWS. For analyses of Goodman′s *Grue* we may only need an elementary temporal logic without recourse to PWS.

14. Nassim Taleb′s ′Black Swan′ is not only a metaphorical name for the highly improbable, but for the problem of induction generally speaking (Taleb 2007, 27). Taleb′s central distinction is between ′the Black Swan generating province of′ Extremistan as ′the province where the total can be conceivably impacted by a single observation′ and Mediocristan, ′the province dominated by the mediocre, with few extreme successes or failures′ (Taleb 2007, resp. 6, 308, 309).

15. The common sense interpretation of *Grue* as because of neologisms is even stronger in *Ways of Worldmaking* (Goodman 1978) than in the third and fourth edition of *Fact Fiction and Forecast*

(Goodman 1973, 1983). However, we may distinguish neologism resolution of *Grue* (skeptically reducing neologisms to 'awkward' usage) from multiple hypotheses resolution of *Grue* TE. Different from Goodman we don't reject validity and soundness of *Grue* because of neologisms (which can be logically eliminated on our interpretation), but because of ambiguity of before *t*, at *t* and after *t* (ambiguity of t, <t, ≤t, >t and ≥t), and because of partly related ambiguity and vagueness--that is lack of exact definitions--of grue, bleen etc. predicates.

With regard to Wittgenstein interpretation, we could add that logic still has a good use in removing and resolving ambiguities of ordinary language; first, we shouldn't take ordinary language at face value, as there may be a kind of conservatism baked in it (possibly related to prevalence of Nietzschean nihilism and skepticism in ordinary language); and, second, many nowadays scientific theories have scientific languages (consisting of mathematical, physical, chemical symbols as +, -, c: speed of light in vacuum 3×10^8 m/s, H: Hydrogen, O: Oxygen, H_2O: water) that can't be easily replaced by or translated into ordinary language (like, of course, mathematics can't be easily replaced by ordinary language, which is a fact that Wittgenstein does not deny in *Philosophical Investigations*). In other words, when reducing philosophy to computation, we could reduce it to many more scientific languages and entities (as chemical atoms, molecules etc.), too, which, in fact, belong to micro-level of Planet Earth instead of meso-level of Planet Earth; ordinary language and common sense only belong to the meso-level.

16. Goodman delivered the lectures in 1953. Kuhn's *Structure of Scientific Revolutions* appeared in 1962. The quite substantial revision of section on conditional hypotheses, 'Presumptive Projectibility', was added in 1983, that is nearly two decades after appearance of Kuhn's first edition.

17. Please, see e.g. Putnam 1973, 1975a. We identify, analyze and interpret Hilary Putnam's *Twin Earth* as a (modal) TE in chapter 7 of PhD Thesis 'Semantics of Thought Experiments' (Hertogh 2015b) and possibly forthcoming publication in *Integrated Science Vol II, Transdisciplinarity across the Different Disciplines* (edited by Nima Rezaei).

18. Part of philosophy and logic is dedicated to investigations into universality, which universal statements are possible under which conditions, e.g. modal logic studies concepts as 'necessary' and 'possible.' In Goodman's riddle of induction it is about empirical sciences which are usually opposed to deductive sciences as logic. Examples as 'copper conducts electricity', 'all emeralds are green' are treated as examples from empirical sciences, physics, gemology. Hempel's *Raven* paradox, 'all raven are black', could be considered as an example to investigate deductive logic, strengths and weaknesses of material implication. Hempel, however, discusses it as part of empirical science and arrives at a couple of testable empirical hypotheses.

19. Compare Charles Pierce's and Nicholas Rescher's pragmatism. Both Rescher 1976 on plausibilistic inference and Rescher 1978 on Peirce's theory of induction define nonanalytical plausibility with reference to Peirce:

> By plausibility, I mean the degree to which a theory ought to recommend itself to our belief independently of any kind of evidence other than our instinct urging us to regard it favorably. All the other races of animals certainly have such instincts; why refuse them to mankind?... Physicists certainly today continue largely to be influenced by such plausibilities in selecting which of several hypotheses they will first put to the test. (Peirce in c. 1910 as quoted partly in Rescher 1976 and wholly in Rescher 1978)

This description comes close to intuition, the mental faculty that is mentioned perhaps most often as informal explanation of the psychological mechanics of TE.

20. Supplementary surface and deep analyses of Hempel's *Black Raven* and Quine's *Gavagai* are possibly forthcoming.

21. VC dimension: Vapnik–Chervonenkis dimension, called after Russian scientists Vladimir Vapnik (1936) and Alexey Chervonenkis (1938-2014). It is a measure for richness or complexity of a set of hypotheses. See Harman 1994, Vapnik 2000.
Concisely explained e.g. – 'The point is that good inductive practice balances simplicity of rule against error in the data.' (Harman 2003, 3)

22. See e.g. Hertogh 2015b and 2016. In Hertogh 2015b the notions of environmental pragmaticism and global cross-culturalism are introduced as subtheses of progress of science and society view. In Hertogh 2016 a global cross-cultural interpretation is added to Descartes' Cogito, saying the Cogito ('I think, therefore, I am') does not only hold for white male European citizens of the 17th century, but for all mankind (which we assume is consistent with Descartes's intent).

23. See Hertogh 2015b, 68, where *Grue* is discussed as part of a section on critical thinking.

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