

Semi-Normed Difference Operator Triple Sequence Spaces Defined by a Double Orlicz-Functions

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Abstract

In this paper we introduce semi-normed difference operator triple sequence spaces by using a double Orlicz-functions, so we study their different properties like completeness, solidity, monotonicity, symmetricity etc.

Key words. Double Orlicz-functions, difference spaces , Triple sequences.

Introduction

Throughout this paper Ω^3 a symbol for the family of all complex or real triple sequences. Atriple sequences (complex or real) in [2][4] be a function from $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ (natural numbers) to (real or complex numbers $\mathbb{R}(\mathbb{C})$), and we denote $3c_0$ all null spaces, $3c$ the spaces of convergent, $3l_\infty$ be a linear spaces

Some new results of triple sequences spaces would studied by a double Orlicz functions using a function F where $F = (F_1(r), F_2(u))$, let $(x, y) = (x_{r,u,j}, y_{r,u,j})$ be a triple infinite array of elements.

Kizmaz [6] introduced single difference of single sequence space. We use this idea to define difference of triple sequence spaces as following:

$$\Omega(\Delta) = \{ (x_r) \in \Omega : (\Delta x_r) \in \Omega \},$$

for $\Omega = c, c_0, l_\infty$, where $\Delta x_r = x_r - x_{r+1}$ for all $r \in \mathbb{N}$.

The differences double sequence $(\Delta x, \Delta y)$ is defined by:

$$(\Delta x, \Delta y) = (\Delta x_{r,u}, \Delta y_{r,u})_{r,u=1}^\infty.$$

Let $\Omega^3(\Delta_{r,u,j})$ the difference triple sequence spaces defined by :

$$\Delta_{r,u,j}(x_{r,u,j}) = x_{r,u,j} - x_{r,u+1,j} - x_{r,u,j+1} + x_{r,u+1,j+1} - x_{1+r,u,1+j} - x_{1+r,1+u,j} + x_{1+r,u,1+j} - x_{1+r,u,1+j} + x_{1+r,1+u,1+j}$$

In this paper we introduced the vector-valued triple sequence spaces by a double Orlicz-functions F where $F(x, y) = (F_1(x), F_2(y))$ over semi-normed space (X^2, q) semi-normed by q , and construct some important properties of triple sequence spaces by a function F .

Maysoon [7] and Zainab [3] defined the N-function as follows:

Definition .1.1[3]. The double Orlicz functions is a function

$$F: [0, \infty) \times [0, \infty) \rightarrow [0, \infty) \times [0, \infty) \text{ such that } F(u, v) = (F_1(u), F_2(v))$$

$$F_1: [0, \infty) \rightarrow [0, \infty) \text{ and } F_2: [0, \infty) \rightarrow [0, \infty),$$

such that F_1, F_2 are Orlicz functions which are ,convex, non-decreasing ,even , continuous, and satisfies the four conditions :

$$i) F_1(0) = 0, F_2(0) = 0 \rightarrow F(0,0) = (F_1(0), F_2(0))$$

$$ii) F_1(u) > 0, F_2(v) > 0 \rightarrow F(u, v) = (F_1(u), F_2(v)) > (0,0) \text{ for } u > 0, v > 0, \text{ we mean that by } F(u, v) > (0,0) \text{ that } F_1(u) > 0, F_2(v) > 0$$

$$iii) F_1(u) \rightarrow \infty, F_2(v) \rightarrow \infty \text{ as } u, v \rightarrow \infty \text{ then}$$

$$F(u, v) = (F_1(u), F_2(v)) \rightarrow (\infty, \infty) \text{ as } (u, v) \rightarrow (\infty, \infty) \text{ we mean by}$$

$$F(u, v) \rightarrow (\infty, \infty), \text{ that } F_1(u) \rightarrow \infty, F_2(v) \rightarrow \infty.$$

Definition.2.1. A triple sequence spaces Ω^3 is called solidary if $(\alpha_{r,u,j}x_{r,u,j}, \beta_{r,u,j}y_{r,u,j}) \in \Omega^3$ whenever $(x_{r,u,j}, y_{r,u,j}) \in \Omega^3$ for all triple sequence $(\alpha_{r,u,j}, \beta_{r,u,j})$ of scalars with $|\alpha_{r,u,j}| \leq 1, |\beta_{r,u,j}| \leq 1$ and consequently $|(\alpha_{r,u,j}, \beta_{r,u,j})| \leq 1$, for all r, u and $j \in \mathbb{N}$.

Definition.3.1. A triple sequence space Ω^3 is called monotone if it consists of the canonical- pre-images of all its step space.

Definition.4.1. Atriple sequence spaces Ω^3 is called symmetric if $(x_{r,u,j}, y_{r,u,j}) \in \Omega^3$ implies $(x_{\pi(r),\pi(u),\pi(j)}, y_{\pi(r),\pi(u),\pi(j)}) \in \Omega^3$, where π is a permutation of \mathbb{N} .

Definition5.1. Let F be a double Orlicz-functions, $\varphi \geq 0$ is real numbers and $p = (p_{r,u,j})$ be a factorable triple sequence of positive real.

Definition6.1. Let F be a double Orlicz functions which satisfies Δ_2 –condition and let $0 < \delta < 1$, then for each $x \geq \delta, y \geq \delta$ we have $F(x, y) < k(x, y) \frac{1}{\delta} F(2)$ for some constant $k > 0$.

Now we define difference operator triple sequence spaces $3l_\infty, 3c, 3c_0$ on asemi-normed space (X^2, q) seminormed by q by F as following:

$$3c(\Delta_n^v, F, p, \varphi, q) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r,u,j \rightarrow \infty} (ruj)^{-\varphi} \left[\left(F_1 \left(\frac{q|\Delta_n^v x_{r,u,j} - l_1|}{\rho} \right) \right) \right. \right. \\ \left. \left. \left(F_2 \left(\frac{q|\Delta_n^v y_{r,u,j} - l_2|}{\rho} \right) \right) \right]^{p_{r,u,j}} = 0, \text{ for some } l_1, l_2 \in \mathbb{C}, \varphi \geq 0, \rho > 0 \right\}, \text{ where} \\ \left(F_1 \left(\frac{q|\Delta_n^v x_{r,u,j} - l_1|}{\rho} \right) \right) = 0 \text{ and } \left(F_2 \left(\frac{q|\Delta_n^v y_{r,u,j} - l_2|}{\rho} \right) \right) = 0.$$

$$3c_0(\Delta_n^v, F, p, \varphi, q) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r,u,j \rightarrow \infty} (ruj)^{-\varphi} \left[\left(F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho} \right) \right) \right. \right. \\ \left. \left. \left(F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho} \right) \right) \right]^{p_{r,u,j}} = 0 \text{ for some } \rho > 0, \varphi \geq 0 \right\}, \text{ where } \left(F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho} \right) \right) = 0, \\ \text{and } \left(F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho} \right) \right) = 0.$$

And :

$$3l_\infty(\Delta_n^v, F, p, \varphi, q) = \left\{ (x, y) \in \Omega^3 : \sup_{r,u,j} (ruj)^{-\varphi} \left[\left(F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho} \right) \right) \right. \right. \\ \left. \left. \left(F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho} \right) \right) \right]^{p_{r,u,j}} < \infty, \text{ for some } \rho > 0, l_1, l_2 \in \mathbb{C} \text{ and } \varphi \geq 0 \right\}, \text{ where } \left(F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho} \right) \right) < \infty, \text{ and} \\ \left(F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho} \right) \right) < \infty.$$

By meaning of Mursuleen[5],we can defined difference operator triple sequence spaces $3l_{\infty}, 3c, 3c_0$ on a semi-normed (X^2, q) semi-normed by q as follows:

$$3l_{\infty}(\Delta, F, q) = \left\{ (x, y) \in \Omega^3 : \sup_{r, u, v} \left[\left(F_1 \left(\frac{q|\Delta x_{r,uv}|}{\rho} \right) \right) \boxtimes \left(F_2 \left(\frac{q|\Delta y_{r,uv}|}{\rho} \right) \right) \right] < \infty, \text{ for some } \rho > 0 \right\}, \text{ where } \left(F_1 \left(\frac{q|\Delta x_{r,uv}|}{\rho} \right) \right) < \infty \text{ and } \left(F_2 \left(\frac{q|\Delta y_{r,uv}|}{\rho} \right) \right) < \infty.$$

$$3c_0(\Delta, F, q) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r, u, j \rightarrow \infty} \left[\left(F_1 \left(\frac{q|\Delta x_{r,uv}|}{\rho} \right) \right) \boxtimes \left(F_2 \left(\frac{q|\Delta y_{r,uv}|}{\rho} \right) \right) \right] = 0, \text{ for some } \rho > 0 \right\}, \text{ where } \left(F_1 \left(\frac{q|\Delta x_{r,uv}|}{\rho} \right) \right) = 0, \text{ and } \left(F_2 \left(\frac{q|\Delta y_{r,uv}|}{\rho} \right) \right) = 0.$$

And:

$$3c(\Delta, F, q) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r, u, j \rightarrow \infty} \left[\left(F_1 \left(\frac{q|\Delta x_{r,uv} - l_1|}{\rho} \right) \right) \boxtimes \left(F_2 \left(\frac{q|\Delta y_{r,uv} - l_2|}{\rho} \right) \right) \right] = 0, \text{ for some } l_1, l_2 \in \mathbb{C}, \rho > 0 \right\}, \text{ where } \left(F_1 \left(\frac{q|\Delta x_{r,uv} - l_1|}{\rho} \right) \right) = 0 \text{ and } \left(F_2 \left(\frac{q|\Delta y_{r,uv} - l_2|}{\rho} \right) \right) = 0$$

also ,by Asma [1] we can defined difference operator triple sequence spaces $3l_{\infty}, 3c, 3c_0$ on asemi-normed (X^2, q) semi-normed by q , as follows :

$$3l_{\infty}(\boxtimes, \Delta, F, q) = \left\{ (x, y) \in \Omega^3 : \sup_{r, u, v} \left[\left(F_1 \left(\frac{q|\boxtimes_{r,uv} \Delta x_{r,uv}|}{\rho} \right) \right) \boxtimes \left(F_2 \left(\frac{q|\boxtimes_{r,uv} \Delta y_{r,uv}|}{\rho} \right) \right) \right] < \infty, \right.$$

$\left. \text{ for some } \rho > 0 \right\}.$

$$3c(\boxtimes, \Delta, F, q) =$$

$$\left\{ (x, y) \in \Omega^3 : p - \lim_{r, u, j \rightarrow \infty} \left[\left(F_1 \left(\frac{q|\boxtimes_{r,uv} \Delta x_{r,uv} - l_1|}{\rho} \right) \right) \boxtimes \left(F_2 \left(\frac{q|\boxtimes_{r,uv} \Delta y_{r,uv} - l_2|}{\rho} \right) \right) \right] = 0, \text{ for some } l_1, l_2 \in \mathbb{C}, \rho > 0 \right\}$$

$$3c_0 \left(\mathbb{Q}, \Delta, F, q \right) = \left\{ (x, y) \in \Omega^3 : p - \lim_{r, u, j \rightarrow \infty} \left[\left(F_1 \left(\frac{q |\mathbb{Q}_{r, u, j} \Delta x_{r, u, \mathbb{Q}}|}{\rho} \right) \right) \mathbb{Q} \left(F_2 \left(\frac{q |\mathbb{Q}_{r, u, j} \Delta y_{r, u, \mathbb{Q}}|}{\rho} \right) \right) \right] = 0 \text{ for some } p > 0 \right\}, \text{ where } \mathbb{Q} = (\mathbb{Q}_{r, u, j}) \text{ be a triple sequence.}$$

2. Main Results.

Proposition.2.2. Let $F = (F_1, F_2)$ be a double Orlicz-functions satisfy the Δ_2 -conditions, then

$$i) \Omega^3(F_2, \Delta, q) \subset \Omega^3(F_1, \Delta, q) \text{ for } \Omega^3 = 3l_\infty, 3c, 3c_0 \text{ if } F_2(x, y) \leq F_1(x, y)$$

$$\forall (x, y) \in [0, \infty) \times [0, \infty).$$

$$ii) \Omega^3(F_1, \Delta, q) \subset \Omega^3(F \circ F_1, \Delta, q) \text{ for } \Omega^3 = 3l_\infty, 3c, 3c_0.$$

Proof. (i) The proof is obvious.

$$(ii) \text{ Consider } \Omega^3 = 3c. \text{ Let } (x, y) \in 3c(\Delta, F_1, q).$$

Then for some $\rho > 0$,

$$3c(\Delta, F, q) =$$

$$\left\{ (x, y) \in \Omega^3 : p - \lim_{r, u, j \rightarrow \infty} \left[F_1 \left(q \left(\frac{|\Delta x_{r, u, \mathbb{Q}} - l_1|}{\rho} \right) \right) \mathbb{Q} F_2 \left(q \left(\frac{|\Delta y_{r, u, \mathbb{Q}} - l_2|}{\rho} \right) \right) \right] = 0 \text{ for some } \rho > 0 \right\}, \text{ where}$$

$$F_1 \left(q \left(\frac{|\Delta x_{r, u, \mathbb{Q}} - l_1|}{\rho} \right) \right) \rightarrow 0, \text{ as } r, u, j \rightarrow \infty$$

$$F_2 \left(q \left(\frac{|\Delta y_{r, u, \mathbb{Q}} - l_2|}{\rho} \right) \right) \rightarrow 0 \text{ as } r, u, j \rightarrow \infty.$$

By (i) we get $[F(x, y) \leq F_1(x, y)]$.

$$\text{Then } F \left[F_1 \left(q \left(\frac{|\Delta x_{r, u, \mathbb{Q}} - l_1|}{\rho} \right) \right) \right] \rightarrow 0, \text{ as } r, u, j \rightarrow \infty.$$

$$(F \circ F_1) \left(q \left(\frac{|\Delta x_{r, u, \mathbb{Q}} - l_1|}{\rho} \right) \right) \rightarrow 0, \text{ as } r, u, j \rightarrow \infty.$$

Hence $(x, y) \in \Omega^3(F \circ F_1, \Delta, q)$. We can be proved the spaces $3l_\infty, 3c_0$ by a similar way.

This complete the prove.

Theorem2.3. The space $\Omega^3(\Delta_n^v, F, p, q, \varphi)$ are paranormed space , paranormed by

$$g(x, y) = \sum_{r,u,j=1}^{m,l,n} |x_{r,u,j} + y_{r,u,j}| + \inf \left\{ \rho^{\frac{p_{r,u,j}}{H}} : \sup_{r,u,j} \left[F_1 \left(q \left(\frac{|\Delta_n^v x_{r,u,j}|}{\rho} \right) \right) \boxplus F_{2r} \left(q \left(\frac{|\Delta_n^v y_{r,u,j}|}{\rho} \right) \right) \right] \leq 1 \right\},$$

where $H = \max(1, \sup_{r,u,j} p_{r,u,j})$, and $\Omega^3 = 3l_\infty, 3c, 3c_0$.

Proof. We prove this theorem for the space $3l_\infty(\Delta_u^v, F, p, q, \varphi)$ and the spaces $3c_0, 3c$ proved with a similar way.

Clearly $g(-x) = g(x), g(-y) = g(y)$,

let $(x_{r,u,j})$ and $(y_{r,u,j})$ be any two triple sequences belong to any one of the spaces $\Omega^3(\Delta_n^v, F, P, q, \varphi)$, for $\Omega^3 = 3l_\infty, 3c$ and $3c_0$. Then we have $\rho_1, \rho_2 > 0$ such that

$$\sup_{r,u,j} F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho_1} \right) \leq 1$$

And

$$\sup_{r,u,j} F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho_2} \right) \leq 1$$

Let $\rho = \rho_1 + \rho_2$. Then by convexity of F , we have

$$\begin{aligned} \sup_{r,u,j} F \left(\frac{q|\Delta_n^v(x_{r,u,j} + y_{r,u,j})|}{\rho} \right) &\leq \left(\frac{\rho_1}{\rho_1 + \rho_2} \right) \sup_{r,u,j} F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho_1} \right) \\ &+ \left(\frac{\rho_2}{\rho_1 + \rho_2} \right) \sup_{r,u,j} F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho_2} \right) \leq 1. \end{aligned}$$

Hence we have

$$\begin{aligned} g(x + y) &= \sum_{r,u,j}^{m,l,n} |x_{r,u,j} + y_{r,u,j}| + \inf \left\{ \rho^{\frac{p_{r,u,j}}{H}} : \sup_{r,u,j} F \left(\frac{q|\Delta_n^v (x_{r,u,j} + y_{r,u,j})|}{\rho} \right) \leq 1 \right\} \\ &\leq \sum_{r,u,j=1}^{m,l,n} |x_{r,u,j}| + \inf \left\{ \rho_1^{\frac{p_{r,u,j}}{H}} : \sup_{r,u,j} F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho_1} \right) \right\} \\ &+ \sum_{r,u,j=1}^{m,l,n} |y_{r,u,j}| + \inf \left\{ \rho_2^{\frac{p_{r,u,j}}{H}} : \sup_{r,u,j} F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho_2} \right) \leq 1 \right\} \end{aligned}$$

This implies that

$$g(x + y) \leq g(x) + g(y).$$

The continuity of the scalar multiplication follows from the following inequality:

$$\begin{aligned} g(k'x) &= \left\{ \sum_{r,u,j=1}^{m,l,n} |k'x_{r,u,j}| + \inf \left\{ \rho_1^{\frac{p_{r,u,j}}{H}} : \sup_{r,u,j} F_1 \left(\frac{q|\Delta_n^v k'x_{r,u,j}|}{\rho_1} \right) \right\} + \right. \\ &\left. \sum_{r,u,j=1}^{m,l,n} |k'y_{r,u,j}| + \inf \left\{ \rho_2^{\frac{p_{r,u,j}}{H}} : \sup_{r,u,j} F_2 \left(\frac{q|\Delta_n^v k'y_{r,u,j}|}{\rho_2} \right) \right\} \right\} \leq 1 \end{aligned}$$

$$= |k'| \sum_{r,u,j=1}^{m,l,n} |x_{r,u,j}| + \inf\{(T|k'|)^{\frac{p_{r,u,j}}{H}} : \text{Sup}_{r,u,j} F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho_1} \right) + |k'| \sum_{r,u,j=1}^{m,l,n} |y_{r,u,j}| + \inf\{(T|k'|)^{\frac{p_{r,u,j}}{H}} : \text{Sup}_{r,u,j} F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho_1} \right) \} \leq 1.$$

Hence the spaces $\Omega^3(\Delta_n^v, F, p, q, \varphi)$ for $\Omega^3 = 3l_\infty, 3c_0$ and $3c$ are paranormed spaces. ■

Theorem.2.4. Suppose a semi-normed space (X^2, q) which is complete, where $\Omega^3 = 3l_\infty, 3c, 3c_0$ are complete semi-normed spaces semi-normed by

$$\varphi(x, y) = \inf \left\{ \rho > 0 : \text{sup}_{r,u,j} \left[F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho_1} \right) \boxtimes F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho_1} \right) \right] \leq 1 \right\}, \text{ where}$$

$$F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}|}{\rho_1} \right) \leq 1, \text{ and } F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}|}{\rho_1} \right) \leq 1.$$

Proof.

Let $(x_{r,u,j}^i, y_{r,u,j}^i)$ be any triple Cauchy sequence in $3\boxtimes_\infty(\Delta_n^v, F, q)$ where x^i, y^i is a Cauchy sequence, such that $(x_{r,u,j}^i)$ and $(y_{r,u,j}^i)$ be a triple Cauchy sequence in $3\boxtimes_\infty(\Delta_n^v, F_1, q), 3\boxtimes_\infty(\Delta_n^v, F_2, q)$ respectively.

Let fixed numbers $m_1, m_2 > 0$, then for all $\frac{\epsilon}{m_1 m_2} > 0, \exists$ a positive integers N such that

$$(\|x^i - x^t\|_{\Delta_n^v}, \|y^i - y^t\|_{\Delta_n^v}) < \frac{\epsilon}{m_1 m_2}, \text{ for all } i, t \geq N, \text{ also for } m_2 > 0, \text{ choose } F\left(\frac{m_1 m_2}{2}\right) \geq 1$$

Using the definition of seminorm, we have,

$$\left[\text{sup}_{r,u,j} \left[\left(F_1 \left(\frac{q|\Delta_n^v x_{r,u,j}^i - \Delta_n^v x_{r,u,j}^t|}{\varphi(x,y)(x^i - x^t)} \right) \right) \boxtimes \left(F_2 \left(\frac{q|\Delta_n^v y_{r,u,j}^i - \Delta_n^v y_{r,u,j}^t|}{\varphi(x,y)(y^i - y^t)} \right) \right) \right] \right] \leq F\left(\frac{m_1 m_2}{2}\right),$$

for all $i, t \geq N$, where

$$\left[\text{sup}_{r,u,j} \left[q \left(F_1 \left(\frac{|\Delta_n^v x_{r,u,j}^i - \Delta_n^v x_{r,u,j}^t|}{\varphi(x,y)(x^i - x^t)} \right) \right) \right] \right] \leq F\left(\frac{m_1 m_2}{2}\right),$$

$$\left[q \left(F_2 \left(\frac{|\Delta_n^v y_{r,u,j}^i - \Delta_n^v y_{r,u,j}^t|}{\varphi(x,y)(y^i - y^t)} \right) \right) \right] \leq F\left(\frac{m_1 m_2}{2}\right) \cdot \left[\text{sup}_{r,u,j} \right]$$

So,

$$q \left[\left(F_1 \left(\frac{|\Delta_n^v x_{r,u,j}^i - \Delta_n^v x_{r,u,j}^t|}{\varphi(x,y)(x^i - x^t)} \right) \right) \boxtimes \left(F_2 \left(\frac{|\Delta_n^v y_{r,u,j}^i - \Delta_n^v y_{r,u,j}^t|}{\varphi(x,y)(y^i - y^t)} \right) \right) \right] \leq 1, \text{ such that}$$

$$q \left[\left(F_1 \left(\frac{|\Delta_n^v x_{r,u,j}^i - \Delta_n^v x_{r,u,j}^t|}{\varphi(x,y)(x^i - x^t)} \right) \right) \boxtimes \left(F_2 \left(\frac{|\Delta_n^v y_{r,u,j}^i - \Delta_n^v y_{r,u,j}^t|}{\varphi(x,y)(y^i - y^t)} \right) \right) \right] \leq$$

$$q \left[(F_1 \left(\frac{m_1 m_2}{2} \right)) \boxtimes (F_2 \left(\frac{m_1 m_2}{2} \right)) \right] \geq 1.$$

The implies that

$$\left| (\Delta_u^v x_{r,u}^i - \Delta_u^v x_{r,u}^t, \Delta_u^v y_{r,u}^i - \Delta_u^v y_{r,u}^t) \right| \leq \frac{1}{2} m_1 m_2 \times \frac{1}{m_1 m_2} \epsilon = \frac{1}{2} \epsilon, \text{ for all } r, u \text{ and } j \text{ we get}$$

$$\left| \Delta_n^v x_{r,u,j}^i - \Delta_n^v x_{r,u,j}^t, \Delta_n^v y_{r,u,j}^i - \Delta_n^v y_{r,u,j}^t \right| \leq \frac{\epsilon}{2}, \text{ for all } i, t \geq N.$$

Hence $(\Delta^v x_{r,u,j}^i), (\Delta^v y_{r,u,j}^i)$ are triple Cauchy sequence in \mathbb{R} such that $(\Delta_n^v x_{r,u,j}^i, \Delta_n^v y_{r,u,j}^i)$ atriple Cauchy sequence in $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$, such that

$(\Delta^v x_{r,u,j}^i), (\Delta^v y_{r,u,j}^i)$ are triple Cauchy sequence in $3\boxtimes_\infty (\Delta_u^v, F_1, q)$ and

$3\boxtimes_\infty (\Delta_u^v, F_1, q)$. Therefore each $0 < \epsilon < 1$), there exist a positive integer N such that

$$\left| (\Delta^v x_{r,u,j}^i - \Delta^v x_{r,u,j}^t, \Delta^v y_{r,u,j}^i - \Delta^v y_{r,u,j}^t) \right| < \epsilon \text{ for all } i, t \geq N$$

Now, using the continuity of F_1, F_2 for each r, u we get :

$$\sup_{r,u,j \geq N} \left[q \left[(F_1 \left(\frac{\Delta_n^v x_{r,u,j}^i - \lim_{t \rightarrow \infty} \Delta_n^v x_{r,u,j}^t}{\rho} \right)) \boxtimes (F_2 \left(\frac{\Delta_n^v y_{r,u,j}^i - \lim_{t \rightarrow \infty} \Delta_n^v y_{r,u,j}^t}{\rho} \right)) \right] \right] \leq 1.$$

Thus

$$\sup_{r,u,j \geq N} \left[q \left[(F_1 \left(\frac{|\Delta_n^v x_{r,u,j}^i - \Delta_n^v x_{r,u,j}^t|}{\rho} \right)) \boxtimes (F_2 \left(\frac{|\Delta_n^v y_{r,u,j}^i - \Delta_n^v y_{r,u,j}^t|}{\rho} \right)) \right] \right] \leq 1$$

Taking infimum of ρ^s we have

$$\inf \left\{ \rho > 0 : \left[\sup_{r,u,j \geq N} q \left[(F_1 \left(\frac{|\Delta_n^v x_{r,u,j}^i - \Delta_n^v x_{r,u,j}^t|}{\rho} \right)) \boxtimes (F_2 \left(\frac{|\Delta_n^v y_{r,u,j}^i - \Delta_n^v y_{r,u,j}^t|}{\rho} \right)) \right] \right] \leq 1 \right\} \leq \epsilon \text{ for all } i \geq N \text{ and } t \rightarrow \infty$$

Since $(x^i, y^i) \in 3\boxtimes_\infty (\Delta_n^v, F, q)$ and F_1, F_2 be an a double Orlicz functions , then

$F = (F_1, F_2)$ is an a double Orlicz functions for each r, u and by continuous , we get that $(x, y) \in 3\boxtimes_\infty (\Delta_n^v, F, q)$ is linear .Then $3\boxtimes_\infty (\Delta_n^v, F, q)$ is seminorm.

The rest of proof 3c, $3c_0$ is like the previous case $3l_\infty$.

Conflict of Interests.

There are non-conflicts of interest

Reference.

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الخلاصة

الهدف الرئيسي من هذا البحث هو انتاج فضاء شبه معياري مؤثر لاختلاف فضاءات المتتابعات الثلاثية باستخدام دالة اورليسز المضاعفة, وندرس بعض الخصائص المختلفة مثل الكمال , الصلابة , الرتابة, التناظر وغيرها .