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New Type of Neutrosophic Points

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Abstract

Through the presentation of [1] in his research about Neutrosophic set theory as well as through his definition of Neutrosophic points . We have shown that it doesn't make its sets through the union which defined by the researcher . so we need to define Neutrosophic point which achieved principle that the set represents its union points by definition an union which serve this principle as well as integration of Neutrosophic point in the Neutrosophic topological spaces and defined (interior, exterior, boundary limit and closure) points.

Keywords: Neutrosophic topological space

Introduction

The notion of neutrosophic sets was by Smarandache [3].My research consists after studying the theory of Neutrosophic sets, some of the main definitions of this theory and the development of anew point that achieves the principle of union and intersection (the set represe-nts the union of its points) as well as the integration of this point in the Neutrosophic topological spaces to find the (interior , exterior and boundary) points and also to find (limited closure) points and give some examples and proposition that belong to these points .

1. Basic Definition and proposition in Neutrosophic set theory

Definition 1.1[1]

Let Ӿ be a non – empty fixed sample space. A Neutrosophic crisp set denoted by (Ncs), w is an object having the form $\acute{W} = <\acute{w}_1, \acute{w}_2, \acute{w}_3>$ where $\acute{w}_1$, $\acute{w}_2$ and $\acute{w}_3$ are sub set of Ӿ.

Definition 1.2[1]

$\acute{W}$ is called a Neutrosophic crisp set if it satisfying $\acute{w}_1 \cap \acute{w}_2 \cap \acute{w}_3 = \Phi$ and $\acute{w}_1 \cup \acute{w}_2 \cup \acute{w}_3 =$ Ӿ.

Example 1.3

Let Ӿ= {â, ƅ, ç, đ } , $\acute{W}$=<{â,ƅ}, {ç}, {đ}> where $\acute{w}_1$={â,ƅ}, $\acute{w}_2$= {ç} and $\acute{w}_3$={đ} then we have $\acute{w}_1 \cap \acute{w}_2 \cap \acute{w}_3 = \Phi$, { â , ƅ }∩ { ç} ∩ {đ}=Φ and $\acute{w}_1 \cup \acute{w}_2 \cup \acute{w}_3$ = Ӿ , {â, ƅ}∪{ç} ∪ {đ} = Ӿ. ∴$\acute{W}$ is Ncs

Definition 1.4[1]

A Ncs of Φ_N , $Ӿ_N$, in Ӿ may be define as three types :-

a-Φ_N = < Φ,Φ, Ӿ > or Φ_N = <∅ ,Ӿ , Φ> or Φ_N=< Φ, Ӿ ,Ӿ >

b- $Ӿ_N$ = < Ӿ, Φ,Φ > or $Ӿ_N$ = < Ӿ ,Ӿ ,Φ > or $Ӿ_N$ = < Ӿ ,Φ ,Ӿ >

Proposition 1.5[1]

Let {Ẃĵ, ĵ ∈ Ĵ} be an arbitrary family of Neutrosophic crisp sub sets in Ӿ , then:-
Ẃĵmay be defined as two kinds :- 1. ∩

a- ∩Ẃĵ = <∩ $ẃĵ_1$, ∩$ẃĵ_2$, ∩$ẃĵ_3$> or

b- ∩Ẃĵ= <∩$ẃĵ_1$, ∪$ẃĵ_2$, ∪$ẃĵ_3$>

2. ∪Ẃĵ may be defined as two kinds :-

a-∪Ẃĵ = <∪ $ẃĵ_1$, ∩ $ẃĵ_2$, ∩ $ẃĵ_3$>

b-∪Ẃĵ = <∪ $ẃĵ_1$,∪ $ẃĵ_2$, ∩ $ẃĵ_3$>

Example 1.6

Let Ӿ = {â, ƅ, ç}, $ẃ_1$= < {â},{ƅ},{ç}>

$ẃ_2$= < {â},{ç},{ƅ}>

$ẃ_3$= <{ƅ},{â},{ç} >

$ẃ_4$= < {ƅ},{ç},{â}>

$ẃ_5$ = < {ç}, {â},{ƅ}>

$ẃ_6$= <{ç},{ƅ}, {â}>

Then ∩ $ẃĵ_1$ ∩$ẃĵ_2$, ∩$ẃĵ_3$= < Φ,Φ,Ӿ > =$Φ_{Ṇ}$ Or ∩$ẃĵ_1$ ∪$ẃĵ_2$, ∪$ẃĵ_3$ = < Φ , Ӿ ,Ӿ >=$Φ_{Ṇ}$

Example 1.7

Let Ӿ = {â, ƅ, ç, đ}, $ẃ_1$=< {â} , {ƅ} , {ç ,đ} >, $ẃ_2$ = < {â},{ç ,đ},{ƅ} >, $ẃ_3$ = < {ƅ} ,{â} , {ç ,đ} > , $ẃ_4$=< {ƅ} ,{ç ,đ} , { â} >,$ẃ_5$=< {ç ,đ}, { â}, {ƅ} > ,$ẃ_6$=<{ç ,đ},{ƅ},{â}>

Then we have ∪ $ẃĵ_1$∩ $ẃĵ_2$ ∩ $ẃĵ_3$= < Ӿ , Φ,Φ> =$Ӿ_{Ṇ}$

Or ∪ $ẃĵ_1$∪ $ẃĵ_2$ ∩ $ẃĵ_3$ = < Ӿ ,Ӿ ,Φ > = $Ӿ_{Ṇ}$

Definition 1.8[1]

Let Ӿ be anon – empty set , Ẃ= <$ẃ_1$, $ẃ_2$, $ẃ_3$> if Ẃ is Ncs , in Ӿ, then the complement of the set w is $Ẃ^C$ may be defined by three kinds of complement.

a) $Ẃ^C$ = < $ẃ_1^c$, $ẃ_2^c$,$ẃ_3^c$ > or

b) $Ẃ^C$=< $ẃ_3$,$ẃ_2$,$ẃ_1$> or

c) $Ẃ^C$= < $ẃ_3$, $ẃ_2^c$,$ẃ_1$>

Example 1.9

Let Ӿ= { 1,2,3,4,5,6} Ẃ= < {1,2} ,{3,4},{5,6}> find $Ẃ^C$

Solution:- by definition of complement we have .

=<{3,4,5,6}, {1,2,5,6}, {1,2,3,4} > $Ẃ^C$

Or $Ẃ^C$= <{5,6}, {3,4},{1,2} >

Or $Ẃ^C$=<{5,6}, {1,2,5,6}, {1,2}>

Definition 1.10[1]

Let Ӿ be anon – empty set and the Neutrosophic sets Ẃ and Ɗ be in the form Ẃ= <$ẃ_1$, $ẃ_2$, $ẃ_3$> , Ɗ= < $đ_1$,$đ_2$, $đ_3$> we consider two possible definition for subsets (Ẃ ⊆ Ɗ). SO (Ẃ ⊆ Ɗ) may be defined as two kinds :-

1- Ẃ $\underline{C}$ Ɗ ⇔ $ẃ_1$$\underline{C}$ $đ_1$, $ẃ_2$$\underline{C}$ $đ_2$ and $ẃ_3$⊒ $đ_3$

2- Ẃ $\underline{C}$ Ɗ ⇔ $ẃ_1$$\underline{C}$ $đ_1$, $ẃ_2$⊒ $đ_2$ and $ẃ_3$⊒ $đ_3$

Definition 1.11[1]

Let Ӿ be anon – empty set and the Ncs Ẃ and Ɗ be of the form Ẃ= <$ẃ_1$, $ẃ_2$,$ẃ_3$>,Ɗ= < $đ_1$,$đ_2$, $đ_3$> be Ncs, then :-

1- Ẃ∩Ɗ may be defined as two kinds :-

a) Ẃ∩Ɗ = < $ẃ_1$∩$đ_1$, $ẃ_2$∩$đ_2$, $ẃ_3$∪$đ_3$>

b) Ẃ∩Ɗ=< $ẃ_1$∩$đ_1$,$ẃ_2$∪$đ_2$, $ẃ_3$∪$đ_3$>

2-Ẃ∪Ɗ may be defined as two kinds :-

a) Ẃ∪Ɗ = < $ẃ_1$∪$đ_1$, $ẃ_2$∩$đ_2$, $ẃ_3$∪$đ_3$>

b) $Ẃ \cup Ɗ = < ẃ_1 \cup đ_1, ẃ_2 \cap đ_2, ẃ_3 \cap đ_3 >$

Proposition 1.12[1,2]

Let $\{Ẃĵ : ĵ \in Ĵ\}$ be an arbitrary family of Neutrosophic crisp sub sets in Ӿ , then :-

1- $\cap\, Ẃĵ$ may be defined as the following two kinds:-

a) $\cap\, Ẃĵ = < \cap\, ẃĵ_1, \cap\, ẃĵ_2, \cup\, ẃĵ_3 >$

b) $\cap\, Ẃĵ = < \cap\, ẃĵ_1, \cup\, ẃĵ_2, \cup\, ẃĵ_3 >$

2- $\cup\, Ẃĵ$ may be defined as the following two kinds:-

a) $\cup\, Ẃĵ = < \cup\, ẃĵ_1, \cap\, ẃĵ_2, \cap\, ẃĵ_3 >$

b) $\cup\, Ẃĵ = < \cup\, ẃĵ_1, \cap\, ẃĵ_2, \cup\, ẃĵ_3 >$

Example 1.13

Let Ӿ = {â, ƅ, ç, đ}, $ẃ_1 = <$ {â, ƅ}, {ƅ}, {ƅ, ç } >, the all sub sets possible of Ẃ

$ẃ_1 = <$ {â, ƅ}, Φ , {ƅ, ç }>

$ẃ_2 = <${â},{ç}, {ƅ, ç }>

$ẃ_3 = <${â}, {ç},Φ >

$ẃ_4 = <${â} ,Φ ,{ƅ, ç}>

$ẃ_5 = <$ {â,ƅ},Φ, {ç}>

$ẃ_6 = <$ {â,ƅ},{ç}, {ƅ, ç }>

$ẃ_7 = <$ {â,ƅ},{ç} , {ƅ}>

$= < \cup\, ẃĵ_1, \cap\, ẃĵ_2, \cap\, ẃĵ_3 > \; \cup\, Ẃĵ$

= < {â,ƅ}, Φ,Φ>

Or $\cup\, Ẃĵ = < \cup\, ẃĵ_1, \cap\, ẃĵ_2, \cup\, ẃĵ_3 >$

= < {â,ƅ}, Φ, {ƅ,ç}>

And $\cap Ẃĵ = < \cap\, ẃĵ_1, \cap\, ẃĵ_2, \cup\, ẃĵ_3 >$

Ẃĵ =< Φ,Φ, {ƅ,ç} >∩

$= < \cap\, ẃĵ_1, \cup\, ẃĵ_2, \cup\, ẃĵ_3 > = <$ Φ,{ç}, {ƅ,ç} >

From this example , we notice that the union and intersection don't achieve the concept of Neutrosophic set which acts the union of all the sub sets , then will define the union and intersection which achieve the concept of the Neutrosophic sets as follows:-

$Ẃĵ = < \cup\, ẃĵ_1, \cup\, ẃĵ_2, \cup\, ẃĵ_3 > \cup^*$

and $\cap_* ẃj = < \cap\, wĵ_1, \cap\, wĵ_2, \cap\, wĵ_3 >$

Ẃĵ = < {â,ƅ},{ç}, {ƅ,ç}>$\cup^*$

Ẃĵ =< Φ,Φ,Φ>$\cap_*$

Definition 1.14[1,2]

Let $Ẃ = < ẃ_1, ẃ_2, ẃ_3 >$ be a Neutrosophic crisp sets Ӿ , then $Ᵽ = <\{p_1\},\{p_2\},\{p_3\}>$, $p_1 \neq p_2 \neq p_3 \in$ Ӿ is called a neutrosophic crisp point. An Ncs , $Ᵽ = <\{p_1\},\{p_2\},\{p_3\}>$ belong to a Neutrosophic crisp sets $Ẃ = < ẃ_1, ẃ_2, ẃ_3 >$ of Ӿ denoted by $p \in Ẃ$ may be defined by two kinds

a) $\{p_1\} \subseteq ẃ_1$, $\{p_2\} \subseteq ẃ_2$ and $\{p_3\} \subseteq ẃ_3$

b) $\{p_1\} \subseteq ẃ_1$, $\{p_2\} \supseteq w_2$ and $\{p_3\} \subseteq ẃ_3$

Example 1.15

Let Ӿ= {â,ƅ,ç}, let Ẃ=<{â},{ç},{ƅ,ç}> the all Ncp of Ẃ

$p_1 = <$ {â},Φ,Φ> $p_2 = <${â} ,Φ, {ƅ}> $p_3 = <${â},{ç},Φ>

$p_4 = <$ {â},{ç},{ƅ}> $p_5 = <$ {Φ,Φ,ƅ} > $p_6 = <$ Φ,{ç},{ƅ}>

$p_7 = <$ Φ ,{ç},Φ >

$\{ p_1 : p_1 \in ẃ_1\}, \cap \{ p_2 : p_2 \in ẃ_2\}, \cup \{ p_3 : p_3 \in ẃ_3\} > < \cup$

= <{â},∅,{ƅ}>

Or $< \cup \{ p_1 : p_1 \in ẃ_1\}, \cap \{ p_2 : p_2 \in ẃ_2\}, \cap \{ p_3 : p_3 \in ẃ_3\} >$

= < {â},Φ,Φ>

And $<\cap \{ p_1 : p_1 \in ẃ_1\}, \cap \{ p_2 : p_2 \in ẃ_2\}, \cup \{ p_3 : p_3 \in ẃ_3\}>$
$= <\Phi,\Phi,\{ƅ\}>$
Or $<\cap \{ p_1 : p_1 \in ẃ_1\},\cup\{ p_2 : p_2 \in ẃ_2\}, \cup \{ p_3 : p_3 \in ẃ_3\}> = <\Phi, \{ç\}, \{ƅ\}>$

Throughout this example, the union and intersection also don't achieve the concept of the Neutrosophic sets which acts their union points , that's it necessary to define the union and intersection as following depending on anew point which serves his concept. $\cup^* Pĵ = <\cup \{ p_1 : p_1 \in ẃ_1\},\cup \{p_2 : p_2 \in ẃ_2\}, \cup\{p_3 : p_3 \in ẃ_3\}>$
$\cap_* Pĵ = <\cap\{ p_1 : p_1 \in ẃ_1\}, \cap\{p_2 : p_2 \in ẃ_2\}, \cap\{p_3 : p_3 \in ẃ_3\}>$

So , we can defined anew point of Neutrosophic crisp points , satisfy the concept union.

$$Pî = \begin{cases} P = <\{p_1\},\{p_2\},\{p_3\}> \\ \Phi \\ \text{single point and at least one of } Pî \neq \Phi \end{cases}$$

Example 1.16

Let $X = \{1 , 2\}$,let $Ẃ = < \{1\},\{2\} ,\{1,2\}>$ the all points of Neutrosophic crisp $p_1 = < \{ 1\} , \Phi, \Phi>$ $p_2 = < \{ 2\} , \Phi, \Phi>$ $p_3 = < \{ 1,2\} , \Phi, \Phi>$
$p_4 = <\Phi,\{1\} , \Phi>$ $p_5 = < \Phi ,\{2\} , \Phi>$ $p_6 = < \Phi ,\{1,2\} , \Phi>$
$p_7 = <\Phi, \Phi , \{1\} >$ $p_8 = <\Phi, \Phi , \{2\} >$ $p_9 = <\Phi, \Phi , \{1,2\} >$
$Pĵ = <\cup pĵ_1,\cup pĵ_2 ,\cup pĵ_3>$ $\cup^*$
$= < \{1\},\{2\} ,\{1,2\}> = Ẃ = \cup \{ P : P \in Ẃ\}$
$Pĵ = <\cap p_{ĵ1}, \cap p_{ĵ2} ,\cap p_{ĵ3}> = <\Phi, \Phi, \Phi>$ $\cap_*$

2. Basic definition and proposition of neutrosiphic topological space

The topology which the researcher defined it [1] is depended on the concept of the union and intersection which was defined by him , but in our definition it will be depended on the concept of the union and intersection which has been defined by us.

Definition 2.1

A neutrosophic topology denoted by (NŤ*) is a non –empty set X and let $\mathcal{T}^*$ is a family of neutrosophic (sub sets in X) satisfying the following axioms.

ṆŤ$_1^*$ $X_N , \Phi_N \in \mathcal{T}^*$
ṆŤ$_2^*$ $Ĝ_1 \cap Ĝ_2 \in \mathcal{T}^*$ for any $Ĝ_1, Ĝ_2 \in \mathcal{T}^*$
ṆŤ$_3^*$ $\cup Ĝ_î \in \mathcal{T}^* \forall \{Ĝ_î , î \in Ĵ\} \subseteq \mathcal{T}^*$

The pair (X,$\mathcal{T}^*$) is called a Neutrosophic topological space. Denoted by ṆŤ* s) and any Neutrosophic set in$\mathcal{T}^*$ is known as Neutrosophic open set denoted by (Nos) in X. The element of $\mathcal{T}^*$ are called open Neutrosophic sets .A Neutrosophic sets Ƒ is closed if and only it C(Ƒ) is Neutrosophic open .

Definition 2.2

Let (X,$\mathcal{T}^*$) be a Neutrosophic topological space Ă ⊆ X. A point P∈Ă is called a Neutrosophic interior point of Ă if there exists a Neutrosophic open set $Ú \in \mathcal{T}^*$ containing $p_î$ such that $p_î \in Ú \subseteq Ă$.

The set of all Neutrosophic interior points of Ă is called Neutrosophic interior points of Ă and denoted by Ṇ int(Ă)

Ṇ int (Ă) = $\{p_î \in Ă : \exists Ú \in \mathcal{T}^* ; p_î \in Ú \subseteq Ă\}$

Example 2.3

Let Ӿ ={â,ƅ,ç} ,$\mathcal{T}^*$ = <{Ӿ$_{Ņ}$, Φ$_{Ņ}$, < {â}, Φ, Φ >}

Ă=<{â}, {â}, {ç}> find Ņ ɨnt (Ă)

Sol// by define of Ņ ɨnt (Ă)= { p$_{î}$ ∈Ă : ∃Ú ∈ $\mathcal{T}^*$; p$_{î}$ ∈ Ú⊆ Ă }

Ņ ɨnt (Ă)= <{â} ,Φ, Φ > ∴

Proposition 2.4

Let (Ӿ , $\mathcal{T}^*$) be a Neutrosophic topological space , and Ă, Ḅ be two Neutrosophic sets in Ӿ , then the following properties hold

1- Ņ ɨnt (Ă) ⊆ Ă

2- Ă ⊆ Ḅ⟹Ņ ɨnt (Ă) ⊆ Ņ ɨnt (Ḅ)

3- Ă∈ $\mathcal{T}^*$ (i .e Ă is Nos)⟺Ņ ɨnt (Ă) = Ă

4- Ņ ɨnt (Ă) ∩ Ņ ɨnt (Ḅ) = Ņ ɨnt (Ă∩Ḅ)

5- Ņ ɨnt (Ă)∪ Ņ ɨnt (Ḅ) ⊆ Ņ ɨnt (Ă∪Ḅ)

6- Ņ ɨnt (Ă)=∪ { Ú ∈ $\mathcal{T}^*$, Ú⊆ Ă }

Proof 1) by definition of Ņ ɨnt (Ă)

Proof 2) suppose that Ă ⊆ Ḅ to prove Ņ ɨnt (Ă)⊆ Ņ ɨnt (Ḅ) ,let p$_{î}$∈ Ņ ɨnt (Ă) ⟹ ∃ ∪ ∈ $\mathcal{T}^*$, p$_{î}$∈ Ú ⊆ Ă (by def of Ņ ɨnt (Ă) ⟹ ∃Ú ∈ $\mathcal{T}^*$; p$_{î}$∈ Ú ⊆ Ḅ (since Ă ⊆ Ḅ)

p$_{î}$∈ Ņ ɨnt (Ḅ) (by def of Ņ ɨnt (Ḅ)∴ Ņ ɨnt (Ă) ⊆ Ņ Ņ ɨnt(Ḅ) ∴

Proof 3) & 4) & 5) and 6) obvious & clearly .

Definition 2.5

Let (Ӿ ,$\mathcal{T}^*$) be a Neutrosophic topological space and Ă⊆Ӿ. A point p$_{î}$∈C(Ă) is called a Neutrosophic Exterior point of Ă if there exists a Neutrosophic open set Ú ∈ $\mathcal{T}^*$ containing p$_{î}$ such that p$_{î}$∈ Ú ⊆C(Ă) .(C(Ă) is complement of Ă).The set of all Neutrosophic exterior points of Ă and denoted by Ņ ɇxt (Ă) , Ņ ɇxt(Ă)= { p$_{î}$∈C(Ă) ; ∃Ú ∈ $\mathcal{T}^*$∋ p$_{î}$∈ Ú ⊆C(Ă) }.

Example 2.6

Let Ӿ = {â, ƅ, ç} , $\mathcal{T}^*$ = {Ӿ$_{Ņ}$, Φ$_{Ņ}$, < Φ , {C} , Φ >}

And Ă = < {â} , {ƅ} , {ƅ, ç} > then

Sol C(Ă) = < {ƅ , ç} , {â , ç} , {â} > so by definition of Ņ ɇxt (Ă)

we have

Ņ ɇxt (Ă) = < Φ , {C} , Φ >∴

Remark 2.7

From definition of Ņ ɇxt (Ă) we have Ņ ɇxt (Ă) ⊆ C(Ă) or Ņ ɇxt (Ă) ∩Ă = Φ and Ņ ɇxt (Ă)= Ņ ɨnt (C(Ă))

Proposition 2.8

Let (Ӿ , $\mathcal{T}^*$) be a Neutrosophic topological space , and Ă, Ḅ be two Neutrosophic sets in Ӿ , then the following properties hold

1- Ņ ɨnt (Ă) ∩ Ņ ɇxt (Ă)=Φ

2- Ă ⊆ Ḅ⟹ Ņ ɇxt (Ḅ) ⊆ Ņ ɇxt (Ă)

3- Ņ ɇxt (Ă∪Ḅ) = Ņ ɇxt (Ă) ∩ Ņ ɇxt (Ḅ)

4- C(Ă) ∈ $\mathcal{T}^*$ (i .e Ă closed) ⟺ Ņ ɇxt (Ă)= C(Ă)

5- Ņ ɇxt (Ă)∪ Ņ ɇxt (Ḅ) ⊆ Ņ ɇxt (Ă∪Ḅ)

proof 1) by definition of Ņ ɨnt (Ă) ⟹ Ņ ɨnt (Ă) ⊆ Ă and Ņ ɇxt (Ă) ⊆ C(Ă)

Ņ ɨnt (Ă) ∩ Ņ ɇxt (Ă) ⊆ Φ⟹ Ņ ɨnt (Ă) ∩ Ņ ɇxt (Ă) =Φ ⟹

proof 2 & 3& 4 nd 5 obvious

Definition 2. 9

Let (Ӿ,$\mathcal{T}^*$) be a Neutrosophic topological space Ă ⊆ Ӿ. A point pᵢ∈ Ӿ is called a Neutrosophic boundary point of Ă of every Neutrosophic open set in Ӿ containing pᵢ contain at least one point of Ă , and least one point of C(Ă)

The set of all boundary Neutrosophic points of Ă is called the Neutrosophic boundary of Ă and denoted by Ņ ɓɗ (Ă)

Ņ ɓɗ(Ă)= { pᵢ∈ Ӿ :∀Ú ∈ $\mathcal{T}^*$ { p_2 : p_2∈$ẃ_2$}, ᵢ∈ Ú; Ú ∩ Ă ≠ Φ ^ Ú ∩ C(Ă) ≠ Φ

Remark 2.10

Ņ ɓɗ(Ă)= $Ӿ_N$/ Ņ ɨnt (Ă) ∩ Ņ ɇxt (Ă)

Proposition 2.11

Let (Ӿ , $\mathcal{T}^*$) be a Neutrosophic topological space , and Ă, Ḅ be two Neutrosophic sets in Ӿ , then the following properties hold

1- Ņ ɓɗ(Ă) ∩Ņ ɨnt (Ă) = Φ and Ņ ɓɗ(Ă) ∩ Ņ ɇxt (Ă) = Φ

2- Ņ ɓɗ(Ă)= Ņ ɓɗ(C(Ă))

3-Ņ ɓɗ(Ă∪Ḅ) ⊆ Ņ ɓɗ(Ă)∪ Ņ ɓɗ(Ḅ)

4- Ă ∈ $\mathcal{T}^*$⟺N ɓɗ(Ă) ⊆C(Ă) or Ņ ɓɗ(Ă) ∩ Ă=Φ

5- C(Ă) ∈ $\mathcal{T}^*$ ⟺Ņ ɓɗ (Ă) ⊆Ă or Ņ ɓɗ(Ă) ∩ C(Ă) = Φ

6- Ă , C(Ă)∈ $\mathcal{T}^*$⟺ Ņ ɓɗ (Ă) =Φ

Proof 1) Suppose that Ņ ɓɗ (Ă) ∩ Ņ ɨnt(Ă) ≠ Φ

pᵢ∈ Ņ ɓɗ (Ă) ∩ Ņ ɨnt(Ă)⟹pᵢ∈ Ņ ɓɗ(Ă)^ Ᵽ ∈ Ņ ɨnt(Ă) ⟺ ∃Ú ∈ $\mathcal{T}^*$,pᵢ∈⊆Ă(def of ∃ Ņ

(Ă)) Ᵽ ∉ Ņ ɓɗ (Ă)⟹C!! contradiction ɨnt

Ņ ɓɗ(Ă) ∩ Ņ ɨnt(Ă) = Φ ∴

By similar way to prove Ņ ɓɗ (Ă) ∩ Ņ ɇxt (Ă) = Φ and proof 2 & 3 & 4 & 5 and 6 obvious & clearly

Definition 2.12

Let (Ӿ,$\mathcal{T}^*$) be a Neutrosophic topological space Ă ⊆ Ӿ. A point pᵢ∈ Ӿ is called a Neutrosophiclimit point of Ă if every Neutrosophic open set containing pᵢ contains at least one point of Ă different from pᵢ.

The set of all Neutrosophic limit points of Ă is called the Neutrosophic derived set of Ă and denoted by Ņ ℓɨm(Ă),Ņℓɨm(Ă)= { pᵢ∈ Ӿ :∀Ú ∈ $\mathcal{T}^*$pᵢ∈ Ú^ Ú/{pᵢ} ∩ Ă ≠ Φ

Proposition 213

Let (Ӿ , $\mathcal{T}^*$) be a Neutrosophic topological space , and Ă , Ḅ be two Neutrosophic sets in Ӿ , then the following properties hold

1-Ă ⊆ Ḅ⟹Ņ ℓɨm(Ă)⊆ Ņ ℓɨm(Ḅ)

2- Ņ ℓɨm(Ă∪ Ḅ) = Ņ ℓɨm(Ă) ∪ Ņ ℓɨm(Ḅ) (The converse is not true)

3- Ņ ℓɨm (Ă ∩ Ḅ) ⊆ Ņ ℓɨm(Ă) ∩ Ņ ℓɨm(Ḅ) (In general the equality isn't true) 4- C(Ă)∈ $\mathcal{T}^*$⟺ Ņ ℓɨm(Ă) ⊆ Ă or Ă is closed⟺Ņ ℓɨm(Ă) ⊆ Ă

Proof 2 :-

Ă ⊆ Ă∪Ḅ(by def of union)

Ņ ℓɨm(Ă) ⊆ Ņ ℓɨm(Ă ∪ Ḅ) (by part 1) ⟹

and Ḅ ⊆ Ă∪Ḅ (by def of union)

Ņ ℓɨm(Ḅ) ⊆ Ņ ℓɨm(Ă ∪ Ḅ) by part 1 ∴

Ņ ℓɨm(Ă) ∪ Ņ ℓɨm (B) ⊆ Ņ ℓɨm(Ă ∪ Ḅ) …………(1) ⟹

Let pᵢ∈ Ņ ℓɨm(Ă) ∪ Ņ ℓɨm(Ḅ) ⟹ pᵢ∈ Ņ ℓɨm(Ă)^ p ∈ Ņ ℓɨm(Ḅ)

* ; p∈ Ú ^ Ú /{pᵢ} ∩ Ă ≠ Φ (by def of Neutrosophic limit point)∃Ú ∈ $\mathcal{T}$

* ; pᵢ∈ Ṿ ^ Ṿ /{Ᵽᵢ} ∩ Ḅ ≠ Φ∃ Ṿ ∈ $\mathcal{T}$

* ; pᵢ∈ Ú ∩ Ṿ ^(Ú ∩ Ṿ) /{Ᵽᵢ} ∩ (Ă∪Ḅ) ≠ Φ∴ Ú ∩ Ṿ ∈ $\mathcal{T}$

pᵢ∈ Ņ ℓim(Ă ∪ Ḅ) ∴

Ņ ℓim(Ă ∪ Ḅ) ⊆ Ņ ℓim(Ă) ∪ Ņ ℓim(Ḅ)……………(2)

from (1) & (2) we have Ņ ℓim(Ă ∪ Ḅ) ⊆ Ņ ℓim(Ă) ∪ Ņ ℓim(Ḅ)

proof 3 & 4 are clearly

Definition 2.15

Let (Ӿ,$\mathcal{T}^*$) be a Neutrosophic topological space Ă ⊆ Ӿ. the Neutrosophic closure of a set Ă is Ă ∪ Ņ ℓim(Ă) and is denoted by Ņ cℓ (Ă) i.e Ņ cℓ (Ă) i.e Ņ cℓ (Ă) = Ă ∪ Ņ ℓim(Ă)

Remarket 2.16

Ņ cℓ (Ă) = ∩{ Ƒ ⊆ Ӿ ; C(Ƒ)∈ $\mathcal{T}^*$^ Ă ⊆ Ƒ }

Example 2.17

Let Ӿ ={ â,ƅ,ç}$\mathcal{T}^*$ = {Ӿ$_{Ņ}$, Φ$_{Ņ}$, < Φ , {â , ç} , Φ >} , Ă = < {â} , {â , ƅ} , {ç} >

Ƒ= { Φ$_{Ņ}$, Ӿ$_{Ņ}$< Ӿ , {ƅ} , Ӿ >}.So by definition of Ņ cℓ (Ă) ∴ Ņ cℓ (Ă) = {∩ Ƒ : Ƒ is closed , Ă ⊂ Ƒ} we have

Ņ cℓ (Ă) = < Φ, Φ, Φ >

Proposition 2.18

Let (Ӿ , $\mathcal{T}^*$) be a Neutrosophic topological space , and Ă, Ḅ be two Neutrosophic sets in Ӿ , then the following properties hold

1-Ă ⊆ Ņ cℓ (Ă)

2- Ă ⊆ Ḅ ⟹ Ņ cℓ (Ă) ⊆ Ņ cℓ (B) (The converse is not true)

3- Ņ cℓ (Ă∪ Ḅ) = Ņ cℓ (Ă) ∪Ņ cℓ (Ḅ)

4- Ņ cℓ (Ă∩ Ḅ) ⊆ Ņ cℓ (Ă) ∩Ņ cℓ (Ḅ) (the quality is not true true)

5- Ņ cℓ (Ă) = ∩ { Ƒ ⊆ Ӿ ; C(Ƒ)∈ $\mathcal{T}^*$^ Ă ⊆ Ƒ }

6- C(Ă)∈ $\mathcal{T}^*$ (i.e Ă is closed) ⟺ Ņ cℓ (Ă) =Ă

7- Ņ cℓ (cℓ (Ă)) = Ņ cℓ (Ă)

proof (1) Ņ cℓ (Ă) = Ă∪ Ņ ℓim(Ă) (by define of Ņ cℓ (Ă))

Ă ⊆ Ņ cℓ(Ă) ⟹

proof (2) suppose that Ă ⊆ Ḅ to prove Ņ cℓ (Ă) ⊆ Ņ cℓ (Ḅ), since Ă ⊆ Ḅ, Ņ ℓim(Ă) ⊆ Ņ ℓim(Ḅ) (Property of Ņ cℓ (Ă))

Ă ⊆ Ḅ and Ņ ℓim(Ă) ⊆ Ņ ℓim(Ḅ)

Ă ∪Ņ ℓim(Ă) ⊆ Ḅ ∪ Ņ ℓim(Ḅ)

Ņ cℓ (Ă) ⊆ Ņcℓ (Ḅ) (by definition of Ņ cℓ (Ă) proof 3 &4&5&6 and 7 is clearly.

Conflict of Interests.

There are non-conflicts of interest

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الخلاصة

من خلال ما تم عرضه من قبل الباحث احمد سلامة لموضوعة نظرية المجموعة النتروسفكية وكذلك بتعريفه الى النقاط النتروسفكية قد بينا انها لا تكوّن مجموعتها من خلال الاتحاد الذي عرضه الباحث .

لذا تم الحاجة الى تعريف نقطة النتروسفكية تحقق مبدأ ان المجموعة تمثل اتحاد نقاطها بتعريف اتحاد يخدم هذا المبدأ وكذلك ادماج النقطة النتروسفكية في الفضاءات التبولوجيةالنتروسفكية وتعريف النقاط (الداخلية , الخارجية, الحدودية) ونقاط التجمع والانغلاق .

الكلمات الدالة: الفضاء التبولوجي النتروسفكي.