



Article

Diversity Based on Oversampling Technique for PAM-4 in Optical Wireless Communication Systems

Kidsanapong Puntsri^{1,*}, Ekkaphol Khansalee¹, and Puripong Suttisopapan²

¹ Department of Electronics and Telecommunication, Faculty of Engineering, Rajamangala University of Technology Isan, Khon Kaen Campus, Khon Kaen, 40000, Thailand

² Department of Electrical Engineering, Faculty of Engineering, Khon Kaen University, Khon Kaen, 40000, Thailand

*E-mail: kidsanapong.pu@rmuti.ac.th (Corresponding author)

Abstract. This paper utilizes the concept of oversampling to achieve diversity for optical wireless communication (OWC) systems using four levels pulse amplitude modulation format. We propose two contributions to the development of OWC applications. The first one is the implementation of realtime PAM-4 transmitter (Tx) using field-programmable gate array (FPGA). The second one is diversity using oversampling method which can be used to improve the system bit error rate performance. In this case, each symbol is sampled more than two times; and then, averages them all together. The proposed method is fully verified by experimental demonstration for both Tx realtime implementation and the receiver (Rx). The digital PAM format from FPGA is converted to analog format by using a digital-to-analog converter (DAC). With the update rate of 400 Msps, the proposed OWC systems can provide the communication speed over 800 Mbps. Furthermore, only a few FPGA resource usages are consumed. The experimental results are shown that the bit error rate free can be achieved when applying oversampling diversity with four taps. Additionally, the communication distance up to 14 m is successfully transmitted.

Keywords: Diversity, realtime implementation, pulse amplitude modulation, optical wireless communication systems.

ENGINEERING JOURNAL Volume 24 Issue 4

Received 4 February 2020

Accepted 15 May 2020

Published 31 July 2020

Online at <https://engj.org/>

DOI:10.4186/ej.2020.24.4.133

1. Introduction

Over the last decade, optical wireless communications (OWC) has attracted a significant amount of research attention because it has many advantages when compared with the radio frequency (RF) systems, e.g., free license to use and no interference from RF. Especially, OWC has a very large bandwidth, which is up to THz [1]. Therefore, a very high speed of wireless communication systems can be realized, as reported in [1-3]. Generally, intensity modulation and direct detection (IM/DD), which is regarded as a simple modulation scheme for light communication systems [4-5], is conventionally employed to establish OWC systems. It is worth to mention that the communication speed for this type of modulation strongly depends on the bandwidth of the light source [6].

In order to further enhance communication speed in OWC systems, an advance modulation format and multi communication systems, such as orthogonal frequency division multiplexes (OFDM) [7-9] and/or single carrier frequency division multiplexing (SC-FDM) [10], are applied. However, the problem is that the hardware design and implementation would be very complex when compared with the single carrier communication, such as PAM-4. This motivates us to present the IM/DD OWC system with low complexity and cost that can exploit many inherent benefits mentioned above.

IM/DD with PAM modulation format is adopted in this work. The realtime PAM-4 with Gray mapping transmitter implementation using field-programmable gate array (FPGA) is considered. Additionally, a diversity technique, which is a key to improve a communication capacity, is also included. To achieve the diversity, it is conventional to increase the number of Tx and Rx. However, this results in a very expensive cost, especially for low speed and short-range applications.

C. G. Xi and et al. [11] has presented a FPGA processing for PAM-16 in visible light communication (VLC) with light emitting diode (LED) array. The short-range video transmitting is experimentally successful and its data rate is up to 40 Mbps can be achieved. D.J.F. Barros and et al. [12] has conducted an analytical comparison of OFDM and PAM for indoor OWC. The communication using PAM signal outperforms OFDM in term of complexity in some cases. Additionally, the data rate of 300 Mbps is achieved. Y. Chu and et al. [13] has presented a mixture of WiFi and optical wireless communication systems using PAM, where the precoder for PAM mapping is used for channel estimation. Then, the decision PAM level is done by decoding the thresholds.

In recent years, Asanka Nuwanpriya and et al. [14] proposed a single carrier system with frequency-domain equalization (SC-FDE) using PAM modulation format (called PAM-SCFDE) for OWC. The results showed that the signal-to-noise ratio (SNR), required achieving the BER of 10^{-4} , is lower than that of OFDM. However, the

performance in terms of power is excellent only for the data rates below 3 bit/s/Hz. K.Puntsri and et al. [15], had presented an experimental of realtime transmitter PAM-4 for OWC which can provide low data rate transmission. However, the Gray mapping and bit error rate (BER) has been not considered yet. Additionally, S. M. Navidpour and et al. [16] has presented the spatial diversity method for OWC systems. The theoretical result shows that the diversity gains the SNR in this work. The number of Tx is increased whereas the bit error rate decreases. T. Fath and et al. [17] has presented the theoretical comparison of multiple-input multiple-output (MIMO) scheme, where the number of MIMO of 4×4 is considered. The system performance is verified by numerical simulation method. The Repetition Coding (RC), Spatial Multiplexing (SMP) and Spatial Modulation (SM) are included for comparison purpose. The results show that diversity gains and the lower BER can be achieved.

From the literature reviews, the MIMO would be very importance for OWC to gain the SNR up, which the communication systems can be provided higher speed and capacity. In this work, the diversity based on oversampling method is proposed for OWC using PAM signaling. At the Tx, the realtime PAM-4 with Gray mapping transmitter processing using FPGA is designed and implemented. To verify the implemented transmitter, a full system experimental is adopted. A digital-to-analog converter (DAC) with the update rate of 400 Msps is employed (we have only 400 Msps in our Laboratory). Therefore, the communication speed of 800 Mbps can be achieved. A laser diode with the lambda of 405 nm and 50 mW power is used in this work, where only zero and positive voltage are needed to drive a laser diode (LD). Additionally, the invisible light likes infrared can be better; however, it is hardly to alignment. The experimental results showed that communication distance of 14 m can be reached perfectly. Additionally, the FPGA resource usages are only a few. Please note that there is no lens at the received end and this work is aimed to focus only on low speed and low cost applications.

The rest of this paper is organized as follows: Section II presents the background of this work. The method to achieve diversity by using oversampling is described in Section III. We present our experimental results of the proposed OWC system in Section IV. Finally, conclusions are given in Section V.

2. PAM-4 Transmitter Details

In this section, PAM-4 with Gray mapping is discussed. Firstly, the PAM description is shown in section 2.1 and design and realtime implementation using FPGA are detailed in section 2.2 in the following.

2.1. Descriptions of PAM Signal

The amplitude of PAM train is expressed by

$$s(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - KT), \quad (1)$$

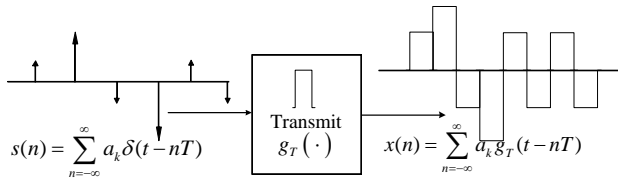


Fig. 1. Conception of PAM with weighted impulse functions.

where a_k is the amplitude with k level. For example, $K \in \{A_1, A_2\}$ is for two levels (PAM-2). A_1 and A_2 are the arbitrary amplitude and $\delta(\cdot)$ is Dirac delta function. The number of bit per symbol (b/s) is deepened on the number of levels of PAM, which is denoted by L . For example, PAM-4 is consisted of 4 levels ($L=4$); therefore, the number of bit per second is 2 bits ($=\log_2(L)$). If L is increased, the number of transmitted bit will be increased. Additionally, to represent the PAM pulse train with weighted impulse functions is given by

$$x(n) = \sum_{n=-\infty}^{\infty} a_k g_T(t - nT). \quad (2)$$

$x(n)$ is the transmitted PAM signal. All of them is expressed in time domain waveform, and, for clearly understand, Eq. (1)-(2) are detailed in practical way as shown in Fig. 1. Let's denoted $g_T(\cdot)$ is a weighted impulse function.

2.2. PAM-4 Transmitter Implementation using FPGA

For the FPGA design, the realtime PAM-4 transmitter implementation is consisted of two main units, which are pseudorandom binary sequence (PRBS) generators and Gray mapping processing. The whole transmitter system is shown in Fig. 2. Additionally, in this work, ML650 FPGA evaluation board from Xilinx is used.

From Fig. 2, four parallel processing are employed. Each unit is consisted of two bits, which are denoted by b_0 and b_1 , respectively. Each bit is generated by PRBS-15 pattern, where the b_1 is delayed by 16 bits when compared with b_0 . Both b_0 and b_1 act as an addresses of

the look-up table (LUT). Next, the address is fed to the Gray mapping, where its mapping is expressed by

$$\begin{aligned} bg_0 &= b_0 \\ bg_1 &= b_0 \text{ xor } b_1 \end{aligned}, \quad (3)$$

where the bg_0 and bg_1 are the Gray mapping output and **xor** is an exclusive or operation. The Gray mapping output are fed to the LUT, where its output bits are set corresponding to the number of DAC input bits resolution. In this work, 12 bits resolution is employed. The DAC output power is divided for 4 levels for PAM-4 or 16 for PAM-8; and, each power level calculated from the number of bit resolution. The DAC power is precalculated in Matlab and stored in the LUT or read only memory (ROM) inside the FPGA. Additionally, DAC works with double data rate (DDR) format; therefore, the clock needed is only a half of the full clock rate. In this case, if the DAC sampling rate of 400 MHz is considered; then, the clock input to the DAC is 200 MHz. As can be seen that, there is four parallel processing and it is computed at the same time with a very low internal clock (50 MHz). This is extremely advantage for time scrolling in FPGA design issue. Before sending it out to the DAC, the four parallel streams are multiplexed in to one high speed stream using serialize primitive inside FPGA (OSERDESE1). From now on, the DAC output is PAM-4 in analog signal, which is ready to transmit it out to the communication channels. In this work, the DAC3162 evolution board from TI is used.

3. Experimental Setup

In this section an experimental setup is detailed and it shown in Fig. 3. At the transmitter (Tx), the implemented PAM-4 transmitter signal using FPGA (ML506 from Xilinx [18]) is tested in realtime. The digital PAM-4 signal format is fed to the DAC with the update rate of 400 Msps. Additionally, the PAM-4 analog signal is consisted of positive and negative components. However, the laser diode (LD) needs only zero and positive sign. It is known as IM/DD system. Therefore, a bias-T is used to offset the negative signal to zero. In this work, the bias-T is self-developed in our laboratory, as detailed in [1 - 2]. Additionally, the laser diode (LD) power of 50 mW with lambda of 405 nm is used, where the communication distance is varied. Additionally, since this is experiment result, there for, all the impairments including the free space optical channel is comprised

At the receiver part (Rx), the avalanche photodetector (APD) from Hamamatsu [19] with the bandwidth of 1 GHz is used. The received electrical signal is saved into oscilloscope for further processing in

Matlab, where the recovery signal processing is followed: firstly, the received signal is sampled and amplified to gain up signal power. Next, to minimize the noise, a digital filter is used. In the digital filter unit, a Matlab designed toolbox is employed, where a low-pass fourth-order Butterworth filter with 800 MHz bandwidth is adopted. In the following, the diversity scheme is employed to minimize the noise power as explained in section 3.1. Then, the PAM level decision is calculated for demapping the levels into bit streaming, where the power level alignment mapping is depicted in Fig. 4.

From Fig. 4, \mathbf{b} is demapping bit vector, denoted by $\tilde{b}g_0$ and $\tilde{b}g_1$. $y_{av}(n)$ is the received signal after

averaging, as detailed in section 3.1. A, B and C are the decision threshold. Then, the Gray demapping is expressed by

$$\begin{aligned} \tilde{b}_0 &= \tilde{b}g_0 \\ \tilde{b}_1 &= \tilde{b}g_0 \text{ xor } \tilde{b}g_1 \end{aligned} \tag{4}$$

where \tilde{b}_0 and \tilde{b}_1 are estimated bit. Finally, bit error is computed at the last process.

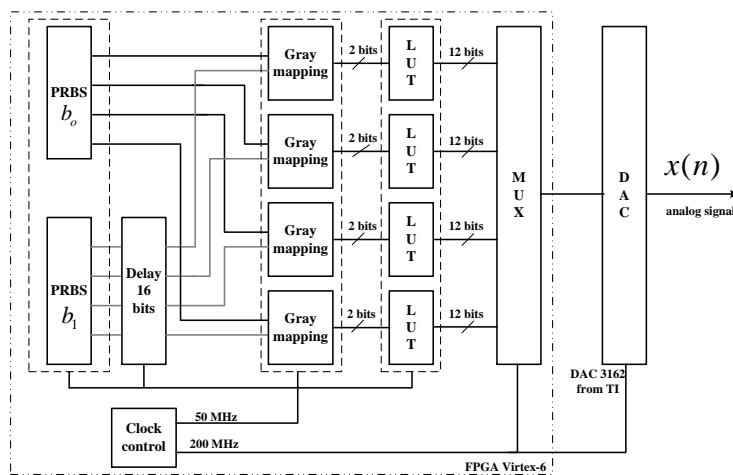


Fig. 2. Conception of PAM with weighted impulse functions.

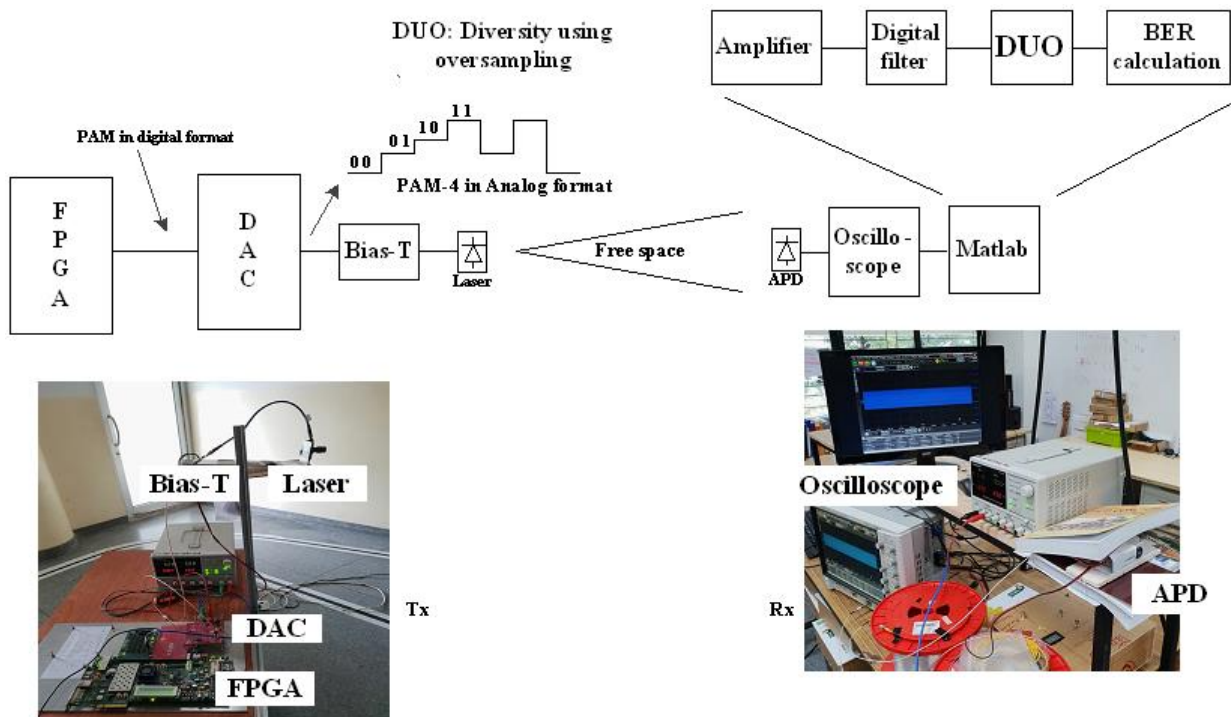


Fig. 3. Experimental setup for PAM signalling with Gray mapping in OWC systems.

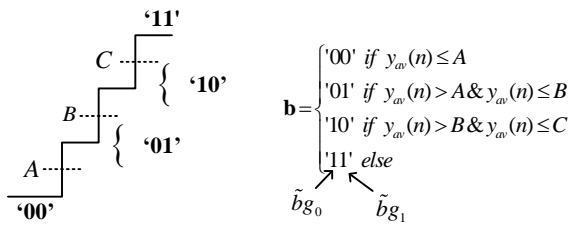


Fig. 4. Diction power levels for bit demapping.

3.1. Diversity Using Oversampling

In this section, the diversity using oversampling method is detailed, as depicted in Fig. 5. Let's defined that $y(n)$ is the received PAM-4 signal, $h(n)$ free space channel and $z(n)$ is Additive white Gaussian noise (AWGN) component with zero mean and variant of σ_z^2 . \otimes is convolution operator. The diversity using oversampling method is detailed in Fig. 5.

From Fig. 5, it is very simple; each PAM-4 symbol is sampled more than two times. Then, every sampling signal calculate average, which is expressed by

$$y_{av}(n) = \frac{\sum_{l=0}^{N-1} y_l(n)}{N}, \quad (5)$$

where $y_{av}(n)$ is an average received sampled signal, $y_l(n)$ is the sampled received signal of $y(n)$ at the l th time and N is the sampling time. As you can be seen from Eq. (5), after averaging the received signal, the noise component would be reduced, as shown in the experimental results. If low speed and short-reach are used, the impact of optical free space channel is very weak.

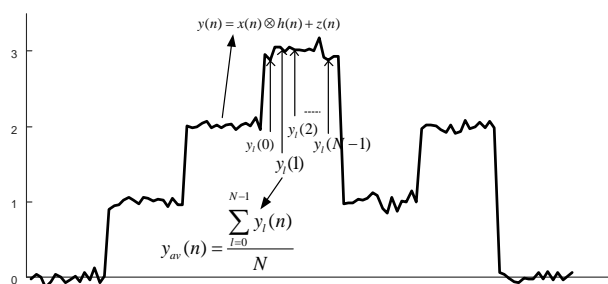


Fig. 5. The diversity scheme using oversampling method.

3.2. Laser Bias Point Characteristic

The bias point is very importance for optical wireless communication. Actually, for both in optical cable and free space should be considered to get the highest modulation index (MI) and extinction ratio. The

characteristic transfer function is given in Fig. 6. As can be seen, to avoid nonlinearity, the optimum bias current, denoted by I_{op} , has to be at the middle point, which provides also the optimum light power, P_{op} . Additionally, the current must be beyond current threshold, I_{th} . Denote that P_1 and P_2 are high and low power. In this case, the power slope is $\Delta_p = P_2 - P_1$ and the current slope is $\Delta_I = I_2 - I_1$, where I_1 and I_2 are high and low current region. Therefore, the laser characteristic is expressed by [20]

$$\eta = \Delta_p / \Delta_I, \quad (6)$$

and the extinction ratio is given by [21]

$$ER_{dB} = 10 \log_{10} (P_2 / P_1). \quad (7)$$

4. Experimental Results

Experimental results are shown in this section, where the BER performance against crucial communication systems such as bias voltage, communication length and diversity gain are considered. Firstly, the optimal bias voltage is investigated. The communication distance of 1 m is placed and the bias voltage (V) of the Bias-T is varied from 4.9V to 5.5V, where PAM-4 with Gray mapping is used. The system performance result is illustrated in Fig. 7.

From Fig. 7, the BER at the bias voltage of 4.9 V to 5.1 V, is non-linear because the bias voltage to the laser is not enough for working with linear region. However, it can be seen that the best bias point is 5.2 V, where it gives the lowest BER and the optical current is 30 mA. Especially, there is no error at the 5.2 V. This experiment implies that the linear region of the laser is 5.2 V and it covers all PAM-4 levels. Therefore, the bias voltage is very importance; and from now on, the bias voltage to the Bias-T is set to 5.2 V for all experiments.

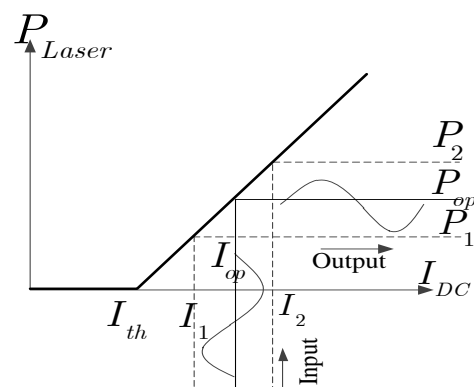


Fig. 6. The diversity scheme using oversampling method.

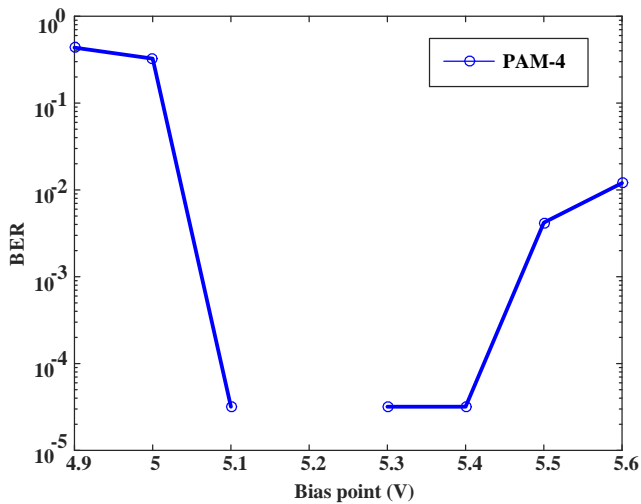


Fig. 7. BER against bias point of the bias-T for PAM-4.

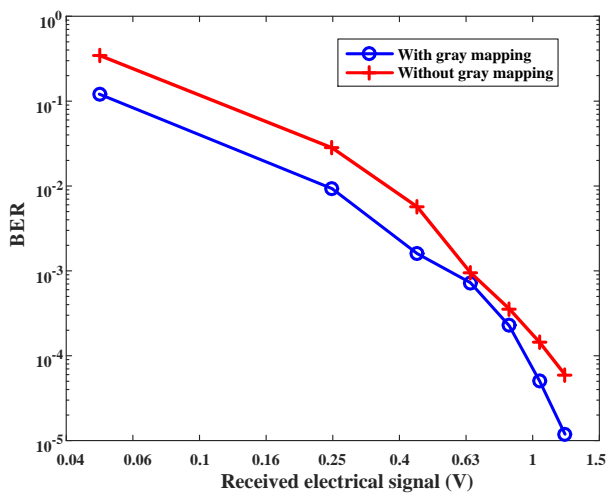


Fig. 8. BER versus received electrical signal in dB for with and without Gray mapping.

Next, the received electrical power signal at the receiver end is examined. The received electrical power is varied from 50 mV to 1,300 mV. In this experiment, the BER performance of PAM-4 with Gray and without Gray mapping are compared, where it is shown in Fig. 8. As can be seen, the BER of PAM-4 with Gray mapping gives lower BER. Additionally, when the received electrical power is increased, the BER is reduced. At the forward error correction (FEC) limit (BER of 2.8×10^{-3}), the received voltage of 400mV is required.

Then, the distance between the transmitter and the receiver is investigated. The distance is varied from 4 to 14 m. The BER result is depicted in Fig. 9, where PAM-4 and PAM-8 are compared. As can be seen, the noise power is increased as the distance is increased. As a result, PAM-8 has higher BER than PAM-4. It may need more noise margin because each level is very close to each other, when compared with PAM-4. PAM-8 is more sensitive to noise. However, PAM-8 gives higher

transmitter speed; but, it would need coding technique to compensate this noise. Therefore, these would make the communication speed is decreased. In other way round, PAM-4 may not need any coding for the distance up to 6 m, as shown Fig. 10.

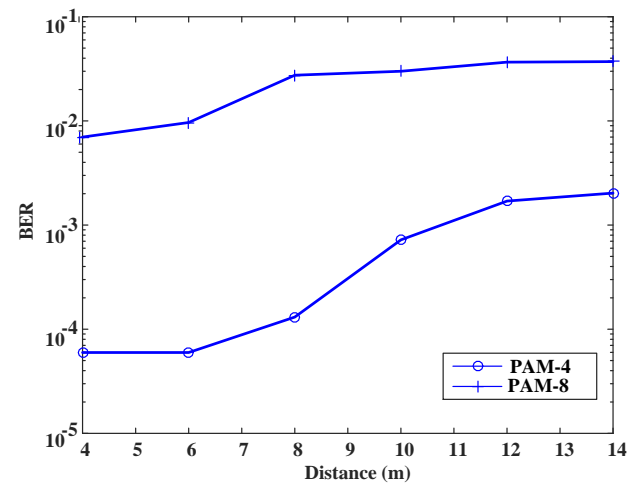


Fig. 9. BER against distance for PAM-4 and PAM-8.

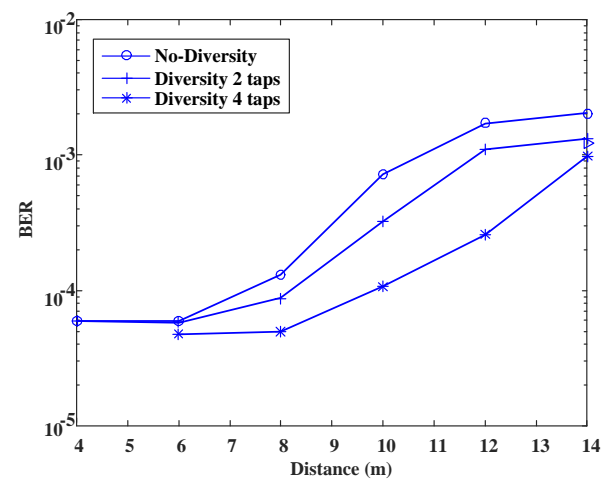


Fig. 10. BER improvements versus distance by using diversity scheme.

In Fig. 10, the diversity using oversampling technique is applied to the received PAM-4 signal. The oversampling points of 2 and 4 taps are considered. As can be seen from Fig. 10, when the numbers of diversities are increased up to 4 taps, the BER is reduced significantly and it is errors free at the communication distance up to 5 m. The number diversity taps can be increased more than 4 taps; but, in the real practical, it is hardly to implement. For example, in this work the signal bandwidth is 400 MHz, and, if the number of oversampling is 8 taps the minimum sampling clock of 3.2 Gbps is needed for the analog-to-digital converter. This leads to very expensive systems.

Finally, to confirm the signal quality and bandwidth, the measurements of fast Fourier transform (FFT) power and eye diagram at the receiver end are shown in Fig. 11.

The received electrical signal of 11.3 V and the distances of 2 m are obtained in this test. As can be seen that the eye diagram is open and clear enough to distinguish the

PAM-4 level and the bandwidth of 400 MHz is met, as shown in Fig. 11A and Fig. 11B, respectively.

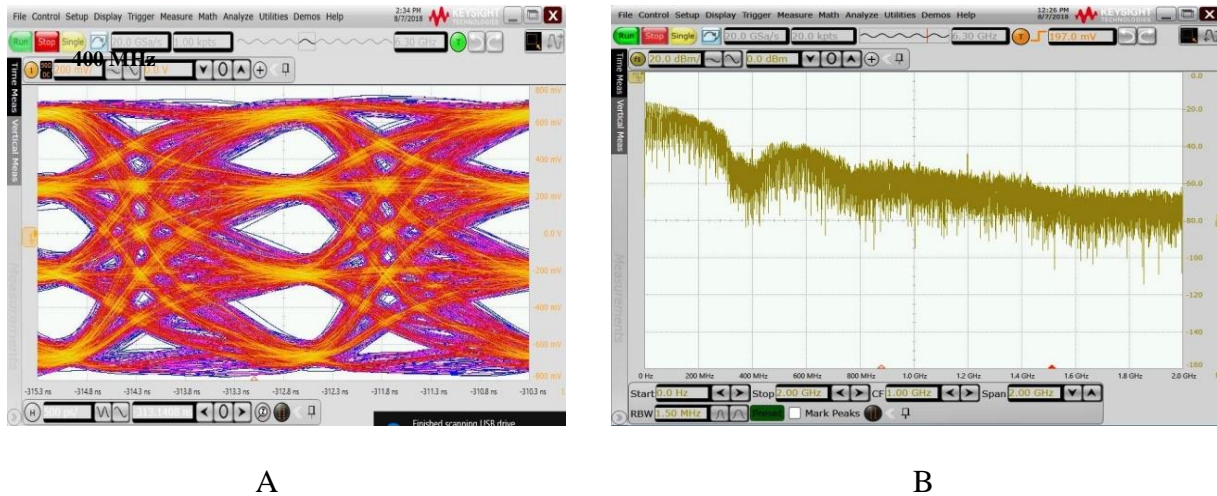


Fig. 11. Quality of PAM signalling at the received end A.) PAM-4 eye diagram B.) Signal bandwidth.

5. Conclusions

This work has been presented a diversity using oversampling method for PAM-4 modulation format in OWC. For the transmitter part, a design and development of realtime PAM-4 signalling transmitter using FPGA is proposed. The DAC update rate of 400 Msps is employed. Therefore, the communication speed of 800 Mbps can be achieved for 14 m. Especially, it was BER free up to the length of 5 m. The quality of PAM-4 signalling is suitable and the bandwidth is met the requirement. Additionally, in this work, only a few FPGA resources are utilized and PAM-4 with Gray mapping transmitter is running in realtime perfectly.

Acknowledgement

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions to improve the quality of the paper. Additionally, thank you very much for all guys supported. This work is partly funding by Rajamagala University of Technology Isan, Khonkhaen Campus for both Master and senior project.

References

- [1] N. Xiang, Z. Zhang, J. Dang and L. Wu, "A novel receiver design for PAM-DMT in optical wireless communication systems," *IEEE Photonics Technology Letters*, vol. 27, no. 18, pp. 1919-1922, Sept. 15, 2015.
- [2] A. Gomez et al., "Design and demonstration of a 400 Gb/s indoor optical wireless communications link," *Journal of Lightwave Technology*, vol. 34, no. 22, pp. 5332-5339, Nov. 15, 2016.
- [3] K. Wang, A. Nirmalathas, C. Lim and E. Skafidas, "High-speed optical wireless communication system for indoor applications," *IEEE Photonics Technology Letters*, vol. 23, no. 8, pp. 519-521, April 15, 2011.
- [4] M. Rahaim and T. D. C. Little, "Interference in IM/DD optical wireless communication networks," *IEEE/OSA Journal of Optical Communications and Networking*, vol. 9, no. 9, pp. D51-D63, Sept. 2017.
- [5] L. Wu, Z. Zhang, J. Dang and H. Liu, "Capacity lower bounds of IM/DD AWGN optical wireless channels based on Fano's inequality," in *2015 International Conference on Wireless Communications & Signal Processing (WCSP)*, Nanjing, 2015, pp. 1-5.
- [6] S. Rajbhandari et al., "High-speed integrated visible light communication system: Device constraints and design considerations," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 9, pp. 1750-1757, Sept. 2015.
- [7] S. D. Dissanayake and J. Armstrong, "Comparison of ACO-OFDM, DCO-OFDM and ADO-OFDM in IM/DD systems," *Journal of Lightwave Technology*, vol. 31, no. 7, pp. 1063-1072, April 1, 2013.
- [8] Z. Jiang, C. Gong, and Z. Xu, "Clipping Noise and Power Allocation for OFDM-Based Optical Wireless Communication Using Photon Detection," *IEEE Wireless Communications Letters*, vol. 8, no. 1, pp. 237-240, Feb. 2019.
- [9] S. M. Rajendrakumar and M. Karruppaswamy, "Analysis of link availability in FSO-OFDM system

- under various climatic conditions,” *Engineering Journal.*, vol. 19, no. 1, pp. 85-95, Jan. 2015.
- [10] K. Puntsri, “Experimental demonstration of high spectral efficiency SC-FDMA with soft clipping for optical wireless communication systems,” *IET Optoelectronics*, vol. 12, no. 2, pp. 80-85, 4 2018.
- [11] C. Xi, A. Mirvakili, and V. J. Koomson, “A visible light communication system demonstration based on 16-level pulse amplitude modulation of an LED array,” in *2012 IEEE Symposium on Photonics and Optoelectronics*, Shanghai, 2012, pp. 1-4.
- [12] D. J. F. Barros, S. K. Wilson and J. M. Kahn, “Comparison of orthogonal frequency-division multiplexing and pulse-amplitude modulation in indoor optical wireless links,” *IEEE Transactions on Communications*, vol. 60, no. 1, pp. 153-163, Jan. 2012.
- [13] Y. Chu, T. Nguyen, and Z. N. Stark, “WiFO: Hybrid WiFi and free-space optical communication system with PAM optimal decoding,” in *2016 25th International Conference on Computer Communication and Networks (ICCCN)*, Waikoloa, HI, 2016, pp. 1-6.
- [14] A. Nuwanpriya, S. Ho, J. A. Zhang, A. J. Grant, and L. Luo, “PAM-SCFDE for optical wireless communications,” *Journal of Lightwave Technology*, vol. 33, no. 14, pp. 2938-2949, July 15, 2015.
- [15] K. Puntsri, E. Khansalee, P. Khophon, and P. Siripakdee, “Experimental of realtime transmitter PAM-4 for optical wireless communication systems,” in *IEEE International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology*, Chiang Mai, Thailand, 2016, pp. 1-4.
- [16] S. M. Navidpour, M. Uysal, and M. Kavehrad, “BER performance of free-space optical transmission with spatial diversity,” *IEEE Transactions on Wireless Communications*, vol. 6, no. 8, pp. 2813-2819, Aug. 2007.
- [17] T. Fath and H. Haas, “Performance comparison of MIMO techniques for optical wireless communications in indoor environments,” *IEEE Transactions on Communications*, vol. 61, no. 2, pp. 733-742, Feb. 2013.
- [18] “Virtex-6 FPGA SelectIO Resources.” https://www.xilinx.com/support/documentation/user_guides/ug361.pdf (accessed 22 May 2018).
- [19] “APD Module C5658.” https://www.hamamatsu.com/resources/pdf/ssd/c5658_kacc1023e.pdf (accessed: 22 May 2018).
- [20] A. Brillant, *Digital and Analog Fiber Optic Communication for CATV and FTTx Applications*. SPIE Press, 2008.
- [21] S. Kumar and M. Jamal Deen, *Fiber Optic Communications: Fundamentals and Applications*. John Wiley & Sons, 2014.





Kidsanapong Puntsri received B.Eng. degree in telecommunication engineering from Mahanakorn University of Technology (MUT), Thailand, in 2002, and M.Eng. degree in telecommunication engineering from King Mongkut's Institute of Technology Ladkrabang (KMUTT), Thailand, in 2004. In 2014, he obtained Dr.-Ing in optical communication from university of paderborn, Germany. At the present, he is an assistant professor at department of electronics and telecommunication engineering, Rajamangala University of Technology Isan, Khon Kaen Campus, Thailand. His main research interests include multi-carrier communication in both optical and wireless systems, and realization of communication systems by field-programmable gate array (FPGA). He has published in IEEE more than 40 contribution papers. He is a senior IEEE member.



Ekkaphol Khansalee received B.Eng. degree in electronics and telecommunication engineering from Rajamangala University of Technology Isan, Khon Kaen Campus (RMUTI KKC), Thailand, in 2008, and M.Sc. degree in communication engineering from King Mongkut's University of Technology North Bangkok (KMUTNB), Thailand, in 2013. At the present, he is a lecturer at department of electronics and telecommunication engineering, RMUTI KKC, Thailand. His main research interests include radio frequency and microwave circuit design in wireless power transfer systems, radio frequency energy harvesting systems and optical wireless communication systems. He has published in IEEE 12 contribution papers.



Puripong Suthisopapan received the B.Eng., M.Eng. and Ph.D. degrees in Electrical Engineering from Khon Kaen University, Thailand in 2007, 2009 and 2012, respectively. Since January 2020, he has been an assistant professor in the Department of Electrical Engineering, Faculty of Engineering, Khon Kaen University, Thailand. His current research interests are error correction codes, signal processing techniques for modern digital communications and quantum error correcting codes.