

Conceptual Design of Low-Boom Low-Drag Supersonic Transports

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This paper introduces a supersonic aircraft design model developed in a multidisciplinary design analysis and optimization (MDAO) environment. Low- to medium- fidelity methods are applied to the conceptual design model. A family of different classes supersonic aircraft are designed with similar general layout, including a single seat supersonic demonstrator, a 10-passenger supersonic business jet and a 50-seat supersonic airliner. These concepts prove the feasibility of low-boom low-drag supersonic transport designs in a multidisciplinary perspective. There are some challenges to balance the volume with packaging and control requirements. The low-boom design for configuration with nacelles are also very challenging.

Nomenclature

a_0	=	ambient sound speed
A	=	ray tube area
A_e	=	equivalent area
A_v	=	Mach plane cross sectional area
C_D	=	total drag coefficient
C_{DF}	=	friction drag coefficient
C_{DLV}	=	lift induced vortex drag coefficient
$C_{D_{wave}}$	=	supersonic wave drag coefficient
C_L	=	lift coefficient
C_F	=	skin friction coefficient
FF	=	form factor
F_{lift}	=	F-function due to lift
F_{volume}	=	F-function due to volume
l	=	overall aircraft length
L/D	=	lift to drag ratio
$L(x, \theta)$	=	lift on a spanwise strip per unit chordwise length
M	=	Mach number
p	=	local overpressure
p_0	=	ambient pressure
δp	=	$p - p_0$
q_∞	=	ambient dynamic pressure
r	=	radial coordinate
SSBJ	=	supersonic business jet
S_{ref}	=	reference area

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S_{wet}	=	wet area
x	=	longitudinal axis location
β	=	$\sqrt{M^2 - 1}$
γ	=	ratio of specific heat
θ	=	angle between the Y-axis and a projection onto the Y-Z plane of a normal to the Mach plane
χ	=	$x - \beta r$, the location on the axis of the equivalent body of the Mach plane translated field point

I. Introduction

THERE has been a renewed interest to design an environmentally friendly, economically viable and technologically feasible civil supersonic transport. It is desirable for the next-generation supersonic transport to have not only low sonic boom level but also high fuel efficiency. There has been a progress in low-boom technology by tailoring the lift and volume distribution [1]. However, the future supersonic civil transport can be viable only if overland supersonic restrictions are relaxed or entirely lifted [2]. NASA is working on a single-seat quiet supersonic demonstrator X-59 [3] to prove the low-boom design technology and support the potential changes in FAA regulations. BOOM Technology is building a demonstrator XB-1 [4] to test the supersonic technologies.

The supersonic business jet (SSBJ) is regarded as the pioneer for the next-generation supersonic transport [5]. Aerion Supersonic [6] has been updating its supersonic natural laminar flow SSBJ concept AS-2 to get a higher cruise aerodynamic efficiency. Spike Aerospace [7] is developing a low-boom SSBJ concept with innovative digital cabin. There have been many studies focusing on the business class supersonic transport concepts [8-11].

The supersonic airliner will be a next-step after the civil supersonic market is reopen. BOOM Technology [12] is developing a 55-75 passenger supersonic aeroplane with business-class fares. JAXA has been conducting research and experiments on next-generation supersonic transports, including 36-50 passenger supersonic aircraft [13].

The research is conducted in a multidisciplinary design analysis optimization environment. In the previous studies, the authors established the multidisciplinary methodologies for supersonic transport [14], evaluated the sonic boom and aerodynamics of current supersonic transports [1, 15], and optimized the Cranfield E-5 supersonic business jet for low-boom and low-drag objectives [16]. This paper is a further study based on the previous research, designing a single-seat supersonic demonstrator, a 10-seat SSBJ concept and a 50-seat supersonic airliner concept with low-boom and low-drag features.

II. Design Methods

The supersonic transport model is developed in a conceptual design environment called GENUS [17]. It is the aim of the GENUS aircraft conceptual design environment is to provide a modular, flexible framework both for designers to use existing, and for researchers to develop new, or use existing methods for aerospace vehicle design. The environment can synthesize and analyse most aerospace vehicle concepts. The modules in GENUS are linked together tightly, which makes the multidisciplinary design analysis and optimization possible. GENUS has proven its capabilities through a solar UAV design [18], a hypersonic transport design [19], a blended-wing-body airliner design [20], a conventional airliner design [21] and a combat UAV design [22].

A. Geometry

The geometry parts are abstracted into lifting surface and body component arrays. The lifting surface array is then specified as a wing, horizontal tail, vertical tail or canard. The body component array is specified as a fuselage, engine pod, external tank or tail boom. In this way, the geometry module can represent most of the configurations.

B. Mission

The mission module specifies the flight requirements for a design. These requirements are derived from the market or customer specifications. They are assumed to be the appropriate initial requirements for environmentally friendly and economically viable supersonic transport.

C. Mass

The mass breakdown module provides a general framework for estimating the mass of various components of an aircraft. Another important function of this module is to decouple the mass components from the packaging module that uses them. At this stage, only empirical methods are used for SSBJ design. These empirical methods include Howe's mass prediction method [23], Raymer's mass prediction method [24], Torenbeek's method [8], Cranfield in-house mass prediction method, etc.

D. Aerodynamics

The main aerodynamic analysis tool is PANAIR [25]. PANAIR can predict inviscid subsonic and supersonic flows of arbitrary configurations by solving a linear partial differential equation numerically. For aerodynamic analysis, PANAIR can provide lift coefficients and induced drag coefficients. The drag components for supersonic flight is denoted by Eq. (1).

$$C_D = C_{D_F} + C_{D_{wave}} + C_{D_{LV}} \quad (1)$$

where C_D is the total drag coefficient, C_{D_F} is the friction drag coefficient, $C_{D_{wave}}$ is the supersonic wave drag coefficient, $C_{D_{LV}}$ is the lift induced vortex drag coefficient.

The form factor method [26] is modified to calculate the zero-lift skin friction and form drag. The result comes from the contribution of each component, as shown in Eq. (2).

$$C_{D_F} = \sum_{j=1}^N \frac{FF_j C_{F_j} S_{wet_j}}{S_{ref}} \quad (2)$$

where N is the number of components used to model the configuration, FF_j is the form factor of component j , C_{F_j} is the skin friction coefficient of component j , S_{wet_j} is the wet area of component j , S_{ref} is the reference area of the total aircraft.

The supersonic area rule [27] is applied to calculate wave drag due to volume, as indicated in Eq. (3) and Eq. (4). For accurate wave drag calculation, the Mach plane cross sectional area intersecting with the geometry is required.

$$C_{D_{wave}}(\theta) = -\frac{1}{2\pi} \int_0^l \int_0^l A''(x_1) A''(x_2) \ln|x_1 - x_2| dx_1 dx_2 \quad (3)$$

$$C_{D_{wave}} = \frac{1}{2\pi} \int_0^{2\pi} C_{D_{wave}}(\theta) d\theta \quad (4)$$

where θ is the angle between the Y-axis and a projection onto the Y-Z plane of a normal to the Mach plane, l is the overall aircraft length, $A(x)$ is the ray tube area at longitudinal coordinate x .

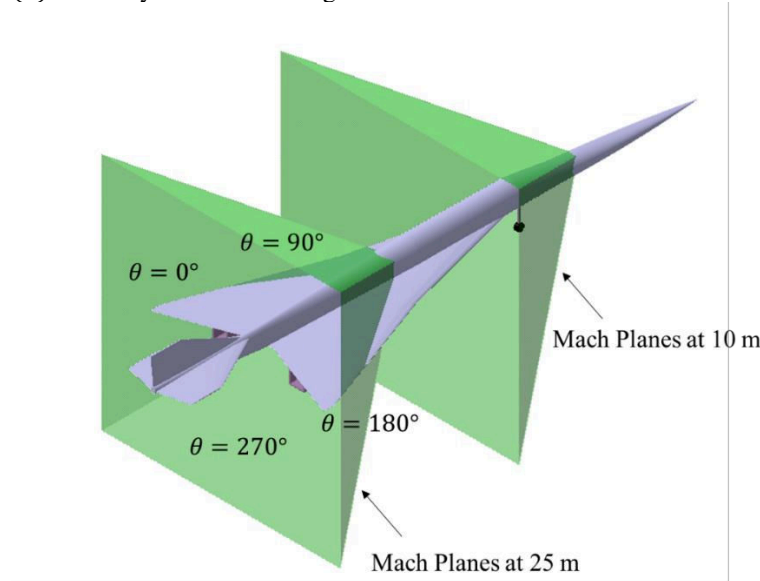


Fig. 1 Illustration of wave drag computing procedure

E. Propulsion

The propulsion module calculates the engine performance at any flight condition. The engine available thrust and corresponding specific fuel consumption (SFC) are stored in two matrices. Users can extract the engine performance at given flight condition by interpolating the Thrust-Mach-Altitude-Throttle matrix and the SFC-Mach-Altitude-Throttle matrix. The supersonic propulsion module uses the NASA EngineSim model [28] to build rubber engine

system. EngineSim calculates the engine performance based on engine parameters, including the geometry, material property, flight condition, throttle setting and so on.

F. Packaging and C.G.

Packaging is a novel and essential module in the GENUS environment. The function of the packaging and CG module is, firstly, to position inner components to detect and avoid interference with other components and the external surface, secondly, to calculate the CG positions for various conditions (with and without fuel or payload). The CG positions are calculated by positioning the mass components from the mass module and they are then used for stability evaluation in this module.

G. Performance

The performance module is modified to be flexible for different segment combinations. Each segment consists of 11 control points by default. The Mach number, altitude, thrust request, aerodynamic coefficients, throttle, C.G. position, etc. at each point are calculated in a physical based approach. The time, distance, fuel consumption, flight path angle, etc. between every two points are calculated for field performance analysis.

H. Stability and Control

This module is designed to evaluate the stability characteristics and trim abilities of the aircraft at various flight conditions. Digital DATCOM [29] is an appropriate tool for stability and control analysis in aircraft conceptual design. The longitudinal and lateral-directional stability characteristics are provided by Digital DATCOM. High-lift devices and control characteristics can be calculated. Trim options are available for subsonic speeds.

I. Sonic Boom Prediction

In this research, there are two steps to predict the sonic boom intensity. The first step is to get the near-field pressure distribution. The second step is to propagate the near-field pressure to the ground through considering the nonlinear characteristics of the atmosphere.

The Whitham theory [30] is used for the near-field pressure calculation. The equivalent area due to volume and equivalent area due to lift are required for the near-field pressure calculation. The equation for the total effective area calculation is indicated in Eq. (5).

$$A_e(x, \theta) = A_v(x, \theta) + \frac{\beta}{2q_\infty} \int_0^x L(x, \theta) dx \quad (5)$$

where $A_e(x, \theta)$ is the equivalent area at coordinate x and angle θ , $A_v(x, \theta)$ is the Mach plane cross sectional area at coordinate x and angle θ , $\beta = \sqrt{M^2 - 1}$, q_∞ is the ambient dynamic pressure, $L(x, \theta)$ is the lift on a spanwise strip per unit chordwise length.

The F-function derives from the equivalent area, as shown in Eq. (6).

$$F(x) = \frac{1}{2\pi} \int_0^x \frac{A_e''(\bar{x}, \theta)}{\sqrt{x - \bar{x}}} d\bar{x} \quad (6)$$

In this research, we decompose the F-function to F-function due to volume and F-function due to lift, as indicated by Eq. (7). This helps to study their individual effects on sonic boom intensity.

$$F(x, \theta) = F_{volume}(x, \theta) + F_{lift}(x, \theta) = \frac{1}{2\pi} \int_0^x \frac{A_v''(\bar{x}, \theta)}{\sqrt{x - \bar{x}}} d\bar{x} + \frac{\beta}{4\pi q_\infty} \int_0^x \frac{L'(\bar{x}, \theta)}{\sqrt{x - \bar{x}}} d\bar{x} \quad (7)$$

where F_{volume} is the F-function due to volume, F_{lift} is the F-function due to lift.

The near-field pressure is then calculated based on the Whitham theory, as shown in Eq. (8).

$$\delta p(x) = p_0 \frac{\gamma M^2 F(x)}{(2\beta r)^{1/2}} \quad (8)$$

where $\delta p = p - p_0$, p_0 is the ambient pressure, γ is the ratio of specific heat, M is the flight Mach number, r is the radius in polar coordinate.

The sonic boom propagates method is the waveform parameter method [31]. This method is based on geometrical acoustics and calculates the sonic boom signature directly with distance steps along a ray, which is more suitable for automatic computation. It is reprogrammed in JAVA and implemented as a special module method. A weakness of the method is that the code expects one shock formation or coalescence at a time. For a complex signature with large numbers of points specified, there is a big chance of failure. This becomes a problem when using CFD data as F-function inputs.

III. Conceptual Design Concepts

This part introduces the conceptual design concepts of a supersonic demonstrator, a supersonic business jet and a supersonic airliner. The results are arranged in a comparison pattern so that the readers can get intuitions among different concepts.

A. Design Mission

The design mission specifies the flight requirements for designs. These requirements are derived from the market or customer specifications. The mission requirements, shown in Table 1, for supersonic transport have been analysed by the authors in a review paper [5], considering the environmental impacts, technological challenges, and market analysis.

Table 1 Mission requirements for the designs

Operational Specification	Supersonic Demonstrator	Supersonic Business Jet	Supersonic Airliner
Design Range	1,500 nm	4,000 nm	4,500 nm
Design Cruise Speed	1.4 Mach	1.8 Mach	1.8 Mach
Maximum Cruise Altitude	55,000 ft	55,000 ft	55,000 ft
Take-off Field Length	1,500 m	1,800 m	2,000 m
Landing Field Length	1,000 m	1,200 m	1,500 m
Environmental Specification			
Sonic Boom intensity	<0.5 psf	<1.0 psf	<1.5 psf
Dimensions			
Wing Span	8.483 m	14.605 m	18.811 m
Wing Gross Area	53 m ²	154 m ²	251 m ²
Leading-edge sweep (inboard)	75.5°	75.5°	75.5°
Leading-edge sweep (outboard)	70°	70°	70°
Fuselage Overall Length	28.600 m	45.670 m	61.100 m
Fuselage maximum Width	1.030 m	1.980 m	2.200 m
Capacities			
Maximum Take-off Mass	9,291 kg	45,000 kg	78,000 kg
Maximum Landing Mass	7,433 kg	36,000 kg	62,400 kg
Maximum Zero-Fuel Mass	6,235 kg	19,990 kg	38,572 kg
Operating Empty Mass	5,963 kg	17,990 kg	32,322 kg
Design Payload	272 kg	1,250 kg	5,750 kg
Crew + Passenger	1+0	2+10	4+50
Propulsion			
Power Plant	1 Mixed Flow Low-Bypass Turbofan	2 Mixed Flow Low-Bypass Turbofan	2 Mixed Flow Low-Bypass Turbofan
Sea Level Static T/O Thrust	55 kN	110 kN	225 kN

B. General Geometry

The authors have analysed several different wing and fuselage combinations in a previous study [1] and find that a low-boom low-drag design should have uniform longitudinal volume and lift distributions. The fuselage is expected to be more slender and a highly-swept wing is expected to flat the lift distribution. Horizontal tail is preferred rather than the canard because horizontal tail can help to reduce sonic boom signature in the aft part. The design of the supersonic demonstrator (Fig. 2) is based on the NASA X-59 concept. The SSBJ (Fig. 3) and supersonic airliner (Fig. 4) share the same wing shape with the demonstrator. The fuselage is optimized to not only meet the requirements of passenger comfortability and packaging requirement, but also for low-boom and low-drag objectives.

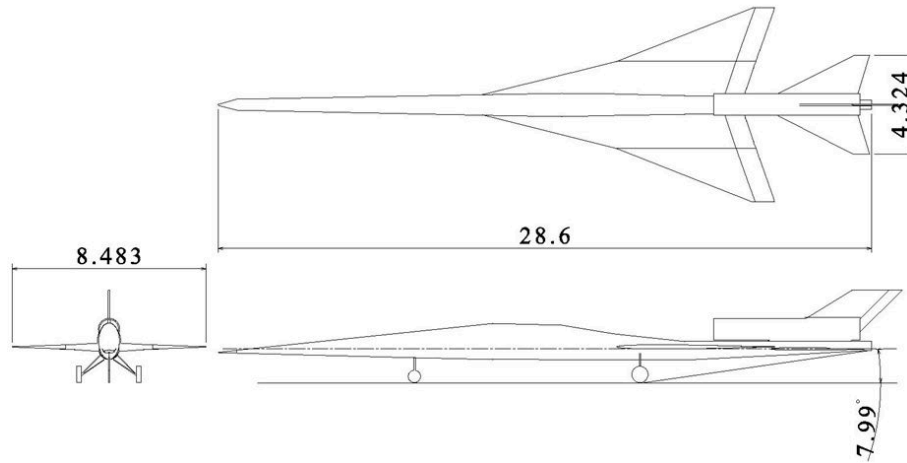


Fig. 2 General geometry of the supersonic demonstrator design

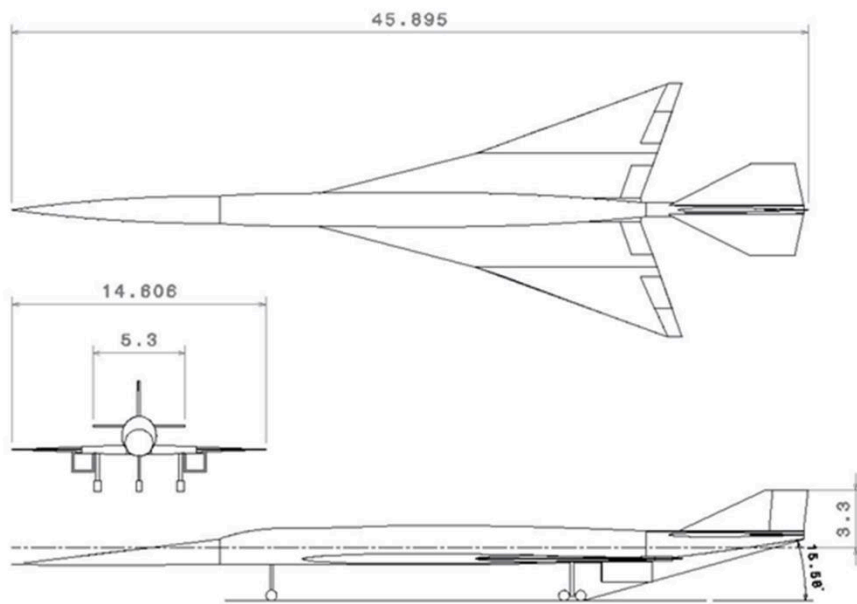


Fig. 3 General geometry of the SSBJ design

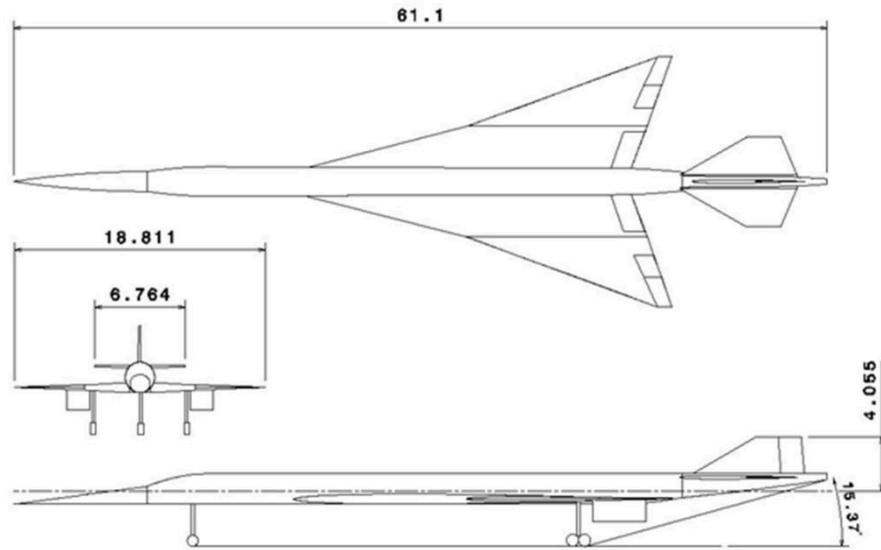


Fig. 4 General geometry of the supersonic airliner design

C. Packaging

Unlike the subsonic aircraft, the supersonic aircraft have very thin wings. The wing tank volume is usually not enough for mission fuel. Fuselage fuel tanks are required. Packaging is an important aspect of supersonic aircraft fuselage volume allocation. On the one hand, the supersonic wave drag reduction tends to an area-ruled fuselage. On the other hand, the passenger cabin and the fuselage fuel tanks require large blocks of volume. One of the most important function of the packaging is to check the available wing tank volume and arrange the fuselage fuel tanks to avoid interface with other inner components. The trim fuel tanks are also designed to control the centre of gravity directly. As can be found from Fig. 5, the packaging challenge for the supersonic demonstrator is relatively weak without a passenger cabin. On the other hand, it is very challenging for the supersonic airliner to achieve an area-ruled fuselage, as shown in Fig. 7.

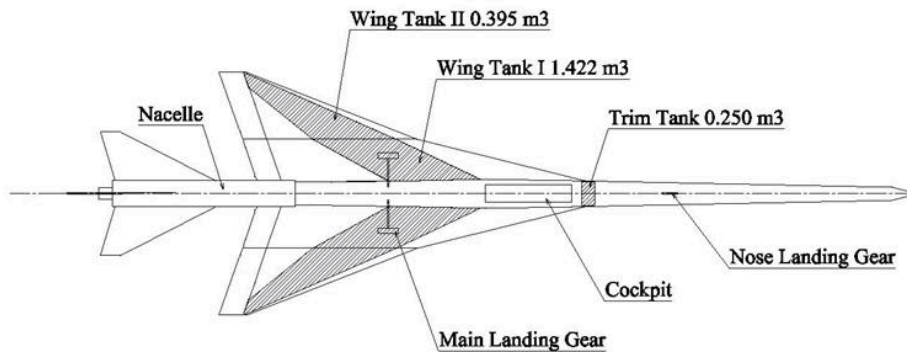


Fig. 5 Packaging of the supersonic demonstrator design

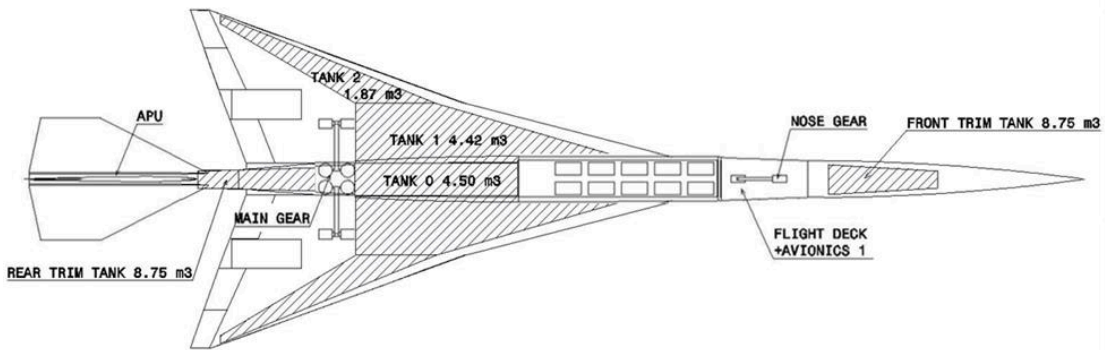


Fig. 6 Packaging of the SSBJ design

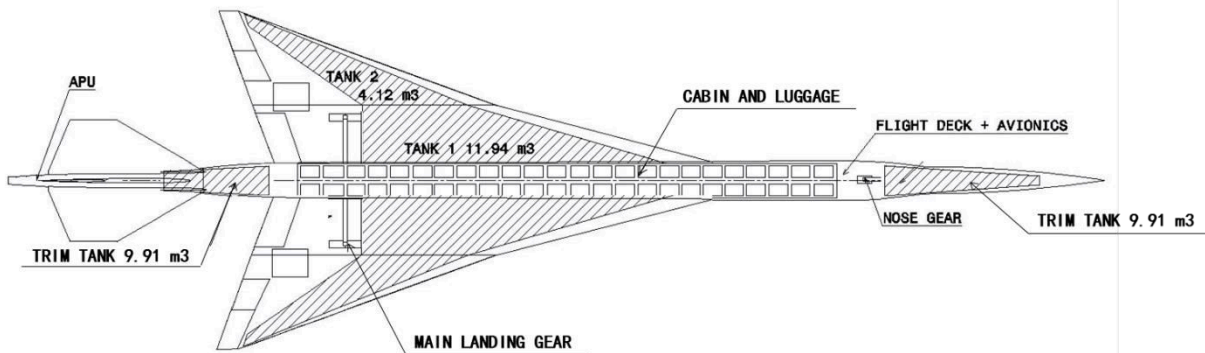


Fig. 7 Packaging of the supersonic airliner design

D. Aerodynamics

The aerodynamic coefficients are important for the performance estimation in the conceptual phase. For the supersonic aircraft, the low-drag optimizations mainly focus on decreasing the supersonic wave drag through geometry variation. Therefore, the supersonic cruise efficiencies (demonstrator $L/D = 12.17$, SSBJ $L/D = 10.77$, airliner $L/D = 11.89$) of these concepts (with nacelles) are higher than the last-generation supersonic transport (e.g. Concorde $L/D = 7$).

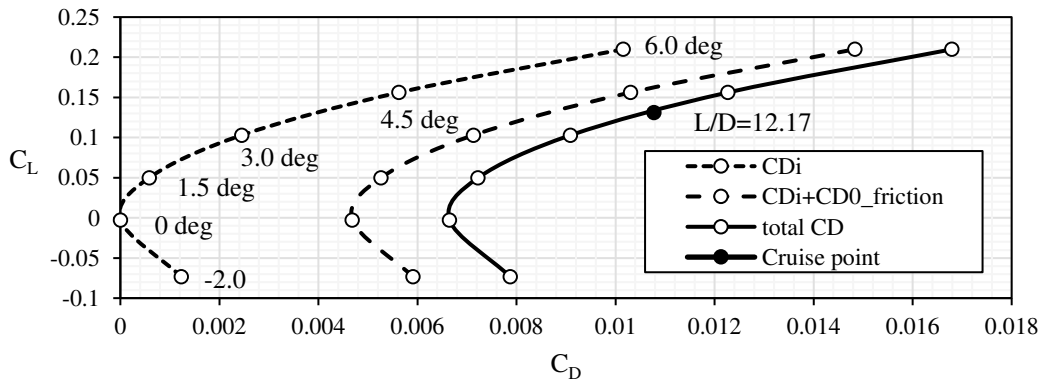


Fig. 8 Supersonic demonstrator drag components and drag polar

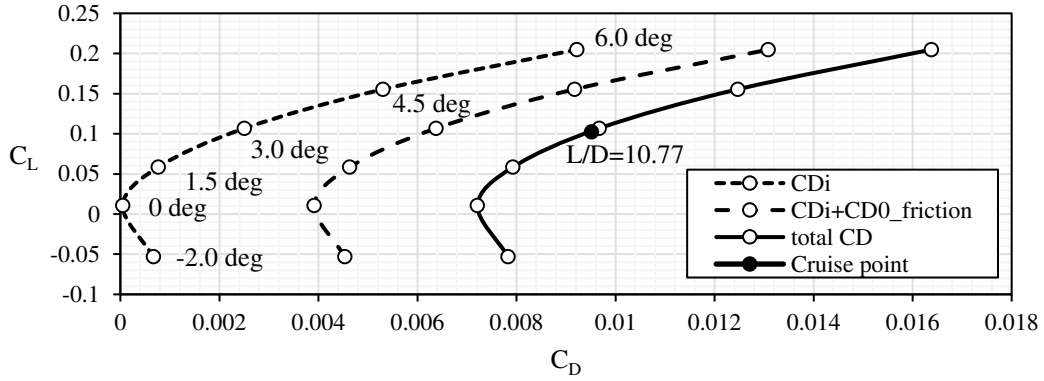


Fig. 9 SSBJ drag components and drag polar

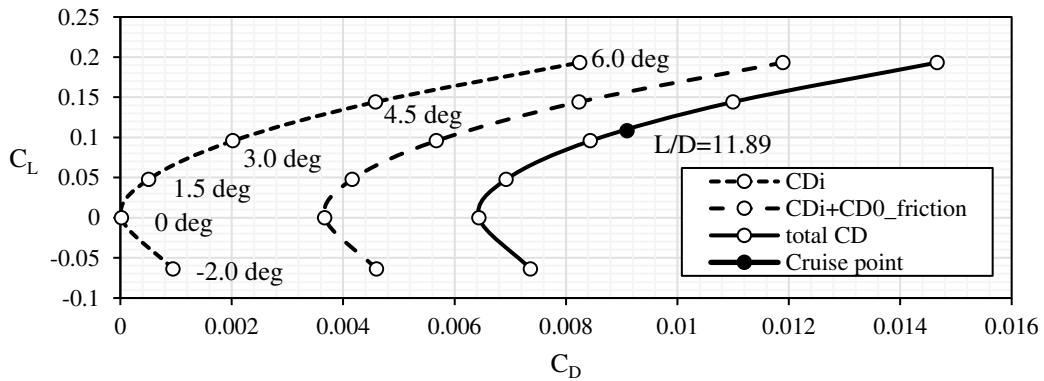


Fig. 10 Supersonic airliner drag components and drag polar

E. Sonic Boom Intensity

The idea of low-boom optimization is to tailor the lift and volume distribution to achieve a low near-field pressure, thus a low ground signature. As can be found in Fig. 11 and Fig. 12, the near-field signatures are optimized to have no single highest peak, especially the overpressure. The sonic boom intensity for the three concepts (without nacelles) are 0.47 psf, 0.90 psf and 1.08 psf, respectively.

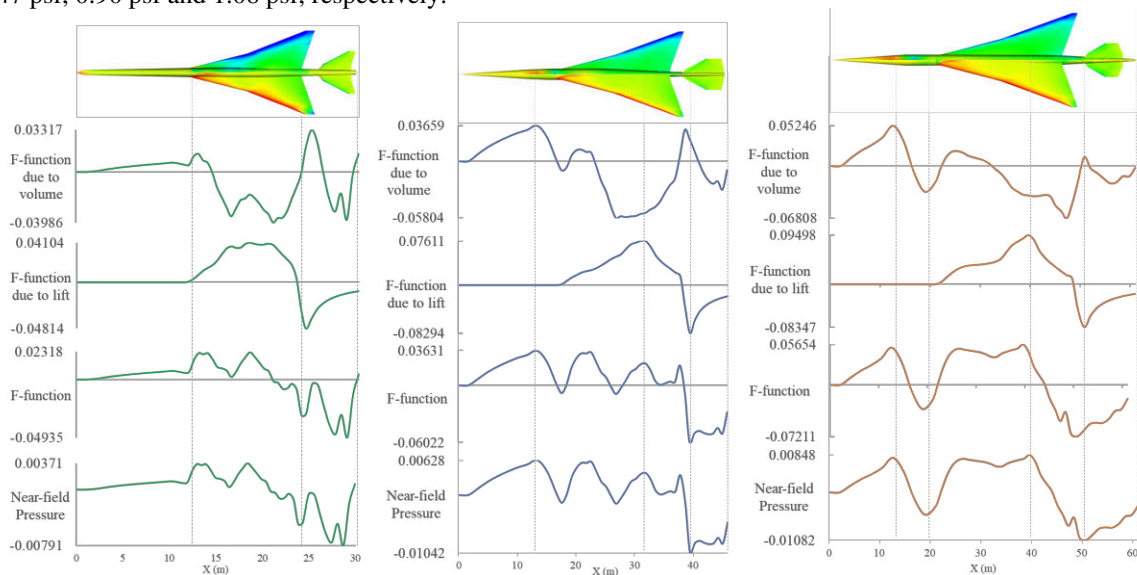


Fig. 11 Sonic boom near-field pressure generation

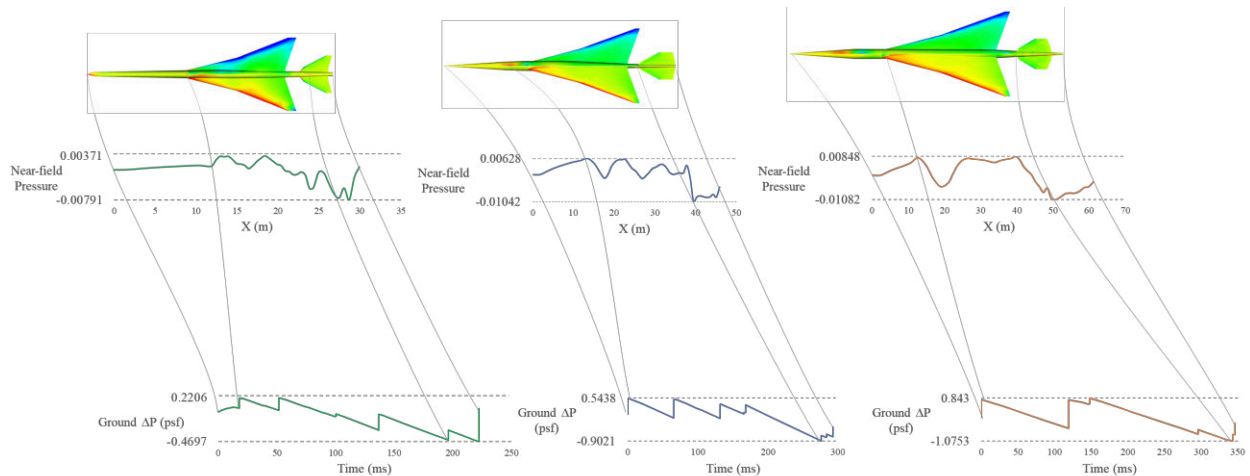


Fig. 12 Sonic boom propagation

IV. Conclusion

Through the study, we demonstrate a supersonic transport model in a multidisciplinary design analysis and optimization environment. With this model, the design space of the low-boom low-drag SSBJ and supersonic airliner are explored.

As a result of the small wing and fuselage volume, the packaging design for supersonic aircraft is much more challenging than subsonic aircraft. The low-boom and low-drag constraints add difficulty to the design.

A single-seat supersonic demonstrator is designed in a multidisciplinary view and optimized for low sonic boom and low supersonic drag. The sonic boom intensity of this design is 0.47 psf. The supersonic lift to drag ratio is 12.17. A 10-passenger SSBJ concept is designed and optimized following the same procedure. The sonic boom intensity of this design is optimized to be 0.90 psf. The supersonic cruise lift to drag ratio is 10.77. For the 50-passenger supersonic airliner, the sonic boom intensity is 1.08 psf and the supersonic cruise lift to drag ratio is 11.89.

However, the low-boom optimization for supersonic aircraft with nacelles is still very challenging. The relatively bulky volume of the nacelles has a huge impact on the volume distribution, thus the sonic boom intensity. More efforts are required to study the low-boom design with nacelles or to study supersonic configurations with distributed propulsion system.

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