Investigating Tail-Risk Dependence in the Cryptocurrency Markets: A LASSO Quantile Regression Approach

Linh Hoang Nguyen* Thanaset Chevapatrakul† Kai Yao‡

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Abstract

We construct the complete network of tail risk spillovers among major cryptocurrencies using the Least Absolute Shrinkage and Selection Operator (LASSO) quantile regression. We capture important features of the network, including major risk-driving and major risk-receiving currencies, and the evolution of the tail dependence among the currencies over time. Importantly, we reveal a striking finding that the right tail dependence among the cryptocurrencies is significantly stronger than the left tail counterpart. This unique characteristic may have contributed to the rise in popularity of cryptocurrencies over the last few years. Our portfolio analysis reveals that diversification in cryptocurrency investment can be accomplished simply by employing the naïve equal-weighted scheme even when transaction costs are taken into account.

JEL classification : C20; C51; C53; G11; G12; G17

Keywords : Tail risk; spillovers; cryptocurrency; network

*Leicester Castle Business School, The Gateway, Leicester, LE1 9BH. Email: linh.nguyen@dmu.ac.uk.

†Corresponding author at Nottingham University Business School, Jubilee Campus, University of Nottingham, Nottingham, United Kingdom, NG8 1BB. Email: Thanaset.Chevapatrakul@nottingham.ac.uk. Tel: +44 (0) 115 8232490.

‡School of Economics and China Center for Behavioral Economics and Finance, Southwestern University of Finance and Economics, 555 Liutai Avenue, Chengdu, 611130, China. Email: yaokai@swufe.edu.cn

1 Introduction

On Monday 5th January 2015, Bitstamp – the second largest Bitcoin exchange at the time – suspended its operation following a security breach by a group of hackers who stole up to $5.2 million worth of Bitcoin. The incident caused the BTC/USD exchange rate to plunge by approximately 21% on 14th January 2015. The impact of the sell-off rippled through the entire cryptocurrency markets, causing prices of other popular cryptocurrencies to crash. Data from `www.coinmarketcap.com` show that more than ten cryptocurrencies saw their prices decline by more than 20% on the day: Dash fell by 20%, Litecoin by 25%, and Monero by 28%, to name a few.

Since the sharp decline in the cryptocurrency prices in 2015, there have been several more similar episodes where prices moved sharply in either direction.[1] These "tail events" as described in the example above are – by definition – rare and high-impact events. They have a small probability of manifesting.[2] However, anecdotal and empirical evidence suggest that tail events in the cryptocurrency markets occur quite frequently. Trading data point to unusually frequent price jumps and crashes which could partly be explained by market participants' lack of understanding about fundamental factors that have potential to move prices. Traders therefore tend to overreact to news, leading to dramatic price action.

Factors which have been documented to cause price jumps in the cryptocurrency markets are illiquidity, order flow imbalance, and the dominance of aggressive traders. Study by Scaillet et al. (2017), using tick data on the BTC/USD exchange rate, extracted from the Mt. Gox database leak, identifies a total of 124 jump days out of 888 sample days between June 2011 and November 2013 – an average of about one jump per week. The finding also reveals that positive jumps occurred more frequently than negative jumps with about 1.3 positive jumps observed for every negative jump. Papers by Osterrieder & Lorenz (2017), Phillip et al. (2018) and Chaim & Laurini (2018) report that the distributions of returns on cryptocurrencies exhibit greater volatility, a larger degree of non-normality, and much heavier tails than those of traditional fiat currencies. Chaim & Laurini (2018) warn that cryptocurrencies are susceptible to sudden and violent price swings and cite formative events, such as hacks and unsuccessful fork attempts, as main drivers behind negative price crashes frequently observed in the cryptocurrency markets.

Despite strong evidence of frequent occurrences of discontinuous price jumps in the cryptocurrency markets, little research has been conducted to investigate the contagious nature of tail events among cryptocurrencies and the mechanism by which tail risk permeates the

[1] For example, the BTC/USD exchange rate rallied from $2,300 to $2,930 per Bitcoin on 20th July 2017 – an increase of about 27% in one day. On the same day, Ethereum rose by 14% while Ripple went up by 12%. Analysts cited improving outlook for Bitcoin's technology which would see an upgrade to the cryptocurrency's infrastructure to accommodate more transactions as the main cause of the sharp rally.

[2] Research in the area of asset pricing has shown that several puzzling phenomena in financial markets can be explained by incorporating probability of tail events in the analysis. For example, by taking into account the effect of low-probability market crashes, Rietz (1988) demonstrates that the extreme event risk helps solve the equity premium puzzle discussed in Mehra & Prescott (1985). Furthermore, Barro (2006) shows that high equity risk premium, low risk-free rate, and volatile stock returns can be explained when data are calibrated with disaster probabilities, associated with sharp contractions during World War I, the Great Depression, and World World II.

cryptocurrency markets. This paper fills in the gap. Specifically, we examine the characteristics of tail risk connectedness in the cryptocurrency markets by constructing a network of tail risk spillovers among the most popular cryptocurrencies and identifying the most important shock-driving and shock-sensitive currencies in the network. We employ the Least Absolute Shrinkage and Selection Operator (LASSO) quantile regression technique to analyse how tail risk is transmitted, received, and evolves over time. Our econometric strategy allows models of tail risk for each of the cryptocurrencies in the sample to be identified in a data-driven way where only relevant tail events with nontrivial impacts are included in the estimation of parameters, thereby significantly reducing the number of parameters to be estimated and greatly simplifying the interpretation of results.[3] Related to our paper is research by Borri (2019) who employs the Conditional Value-at-Risk (CoVaR), first proposed by Adrian & Brunnermeier (2016), to estimate the conditional tail risk in the cryptocurrency markets, using data on Bitcoin, Ether, Ripple, and Litecoin. The results in Borri (2019) indicate that cryptocurrencies are highly exposed to tail risk within the cryptocurrency markets but are disconnected from other global assets.

As a preview of our main results, we detect significant tail risk spillovers among cryptocurrencies. We find that tail connectedness is stronger at the less extreme tail thresholds – supporting the findings reported in Gkillas et al. (2018). Thus, there is a tendency for idiosyncratic jumps to occur at the more extreme tail thresholds. Importantly, we uncover overwhelming evidence showing that the degree of tail risk spillovers at the right tail is more pronounced than that at the left tail, pointing to a tendency for euphoria to spread during market upswings. Our analysis also reveals the importance of Bitcoin as a major shock-driving cryptocurrency during periods of positive market sentiments and Ethereum as a main tail risk driver during downturns. Results from the rolling window analysis indicate that cryptocurrency markets have become more segregated over time with the strength of the right tail connectedness continuing to outstrip that of the left tail connectedness.

Evidence of the idiosyncratic nature of extremely large jumps and the dominance of co-jumps at the right tail strongly hints at the diversification benefit in cryptocurrency investment. To examine how diversification is achieved, we compare the performance of two cryptocurrency portfolios: (i) the naïve, equal-weighted portfolio and (ii) the mean-conditional Value-at-Risk (mean-CVaR) optimised portfolio whose objective is to minimise the portfolio's expected loss conditional on the loss exceeding the VaR.[4] For the mean-CVaR optimisation, we adopt a framework, developed by Rockafellar & Uryasev (2000, 2002), which simultaneously estimates VaR and optimises CVaR to obtain the optimum allocation of cryptocurrencies .

Our portfolio analysis results show that, for almost all of the holding periods under examina-

[3]The Least Absolute Shrinkage and Selection Operator (LASSO) quantile regression technique is pioneered by Belloni & Chernozhukov (2011) and subsequently employed by Hautsch et al. (2015) to study the network of tail risk spillovers among the US financial institutions.

[4]The Conditional Value-at-Risk (CVaR) is also known as Mean Excess Loss, Mean Shortfall, Expected Shortfall or Tail VaR. The CVaR measures the expected loss of a portfolio conditional on the loss exceeding the portfolio's VaR at a pre-specified probability level.

tion, a hypothetical equal-weighted buy-and-hold portfolio that is formed every month earns higher average returns than a corresponding buy-and-hold mean-CVaR portfolio. We further examine the performance of hypothetical active portfolios with initial investment of $1 which are periodically rebalanced using up-to-date information. Mimicking the real-world application, we also incorporate transaction costs in the portfolio formation process. Four different cost scenarios are assumed: 0, 10, 25, and 50 basis points (bps) per trade. Performance of the hypothetical portfolios that are constantly rebalanced is assessed by their total returns, average returns, volatility, CVaR, Sharpe ratios, and the modified Sharpe ratios, calculated as the excess returns per one unit of CVaR. Our results indicate that the same diversification benefit delivered by the mean-CVaR optimisation can be straightforwardly achieved by a naïve, equal-weighting strategy. Under an assumption of no transaction fee, the mean-CVaR portfolios, which are specifically designed to actively minimise the expected shortfalls, outperform the equal-weighted portfolios by only a slim margin. When transaction costs are incorporated into the mean-CVaR optimisation, however, the equal-weighted portfolios surpass the mean-CVaR portfolio in all of the performance criteria.

Our paper contributes to research on cryptocurrencies in a number of ways. It confirms not only the presence of tail dependence among cryptocurrencies but also the dominance of the right tail connectedness over the left tail dependence. This unique characteristic of cryptocurrencies is diametrical of the typical characteristics of traditional asset classes. In addition to their widely reported high risk and returns, the discovery that cryptocurrencies possess a great degree of right tail dependence should attract the attention of investors. Further, we provide strong evidence demonstrating how benefits of diversification can be achieved through a naïve, simple investment strategy of equal weighting. Our findings also corroborate the arguments in Corbet et al. (2018) and Borri (2019) which suggest that cryptocurrencies are largely disconnected from mainstream asset classes, making them excellent risk diversifiers in portfolios of conventional assets. Our paper also complements growing research on return and volatility spillovers as well as literature relating to speculative bubbles and co-explosivity in the cryptocurrency markets.[5]

This paper is organised as follows. We discuss our data in Section 2. The method of LASSO quantile regression is explained in Section 3. In Sections 3.1–3.3, we report and discuss results of the full sample analysis, the rolling window analysis, and the predictive connection of the tail risk connectedness network, respectively. We dedicate Section 4 to the portfolio analysis. Finally, Section 5 concludes the paper.

[5]See Yi et al. (2018), Koutmos (2018), and Ji et al. (2019), among others, for studies on return and volatility spillovers in the cryptocurrency markets. The majority of the papers in this area of research employ the forecast-error variance decomposition techniques, popularised by Diebold & Yilmaz (2009, 2012, 2014), to quantify the degrees of volatility spillovers. Papers in the areas of co-explosivity and bubbles in the cryptocurrency markets include Cheah & Fry (2015), Gkillas & Katsiampa (2018), Gkillas et al. (2018), and Bouri et al. (2018), among others.

2 Data

We download time series of cryptocurrency prices from `www.coinmarketcap.com`.[6] To ensure that cryptocurrencies in our sample are well established cryptocurrencies and can be reasonably easily accessed by regular investors, we include in our sample only the cryptocurrencies that have at least three years worth of daily price data and a minimum market capitalisation of 50 million USD as of 31st December 2019. We exclude all the stable coins – even if they satisfy the above criteria – from our sample, since their prices are pegged to the values of fiat currencies (e.g., USD or EUR).[7] After the filtering process, our sample comprises 21 cryptocurrencies with daily return observations available from November 2016.

Table 1 gives an overview of the 21 cryptocurrencies in our sample. The five most popular cryptocurrencies are Bitcoin, Ethereum, XRP (Ripple), Litecoin and Stellar, all of which have been extensively analysed in a number of studies.[8] Ranked first in the list, Bitcoin is the most popular cryptocurrency with market capitalisation of around 130 billion US dollars – almost 10 times that of the second most popular cryptocurrency, Ethereum. The market capitalisation of the cryptocurrency at the bottom of the list, MonaCoin, is approximately 50 million US dollars – around 0.03% of that of Bitcoin. Figure 1 shows the time series plots of prices and volumes (in millions of US dollars) of the five most popular cryptocurrencies during November 2016 and December 2019. It is clear from the plots that all five of them have enjoyed the same level of popularity, moving together in tandem in both price and volume. It is worth noting how prices of the other four currencies peaked at around the same time when Bitcoin hit the all-time high of just over $20,000 on 17th December 2017.

While Bitcoin is the most popular cryptocurrency, its daily average return of 0.3% is slightly below that of the daily average of all the cryptocurrencies in the sample of around 0.5%. Interestingly, both Verge and Bytecoin have relatively large expected daily returns of 1.5% and 1.1% with large return standard deviations of approximately 16% and 19%, respectively, making them the riskiest cryptocurrencies in the sample. Zcash, however, has the largest left tail risk with the lowest daily return ever recorded of -72% while ByteCoin emerges as the cryptocurrency with the highest degree of right tail risk with the highest daily return of almost 400% realised sometime during 2016–2019.

3 The Least Absolute Shrinkage and Selection Operator (LASSO) Quantile Regression

The use of quantile regression to explain the behaviour of asset returns at different quantiles of the return distribution is ideal for examining tail risk and is one of the main tools commonly

[6]The website provides up-to-the-minute updates for all market data and all the price series are volume-weighted averages of prices from different exchanges, where every minute, the various exchanges are queried for their most recent market data. As of 23rd March 2020, `www.coinmarketcap.com` aggregates price data from a total of 322 exchanges. A full list of exchanges can be found on `https://coinmarketcap.com/rankings/exchanges/`.

[7]Noteable stable coins are Tether, USD Coin, Stasis Euro, among others.

[8]See Gkillas & Katsiampa (2018), Yi et al. (2018), and Ji et al. (2019), among others.

employed by researchers to analyse the tail risk of asset returns (Adrian & Brunnermeier 2016, Adams et al. 2014, Högholm et al. 2011). In fact, the Value-at-Risk (VaR) of an asset is by definition a quantile of the asset's return distribution and is widely used as a measure of tail risk.

Employing the quantile regression technique, this study examines how the tail risk of a cryptocurrency, as measured by its VaR, is affected by tail events in the cryptocurrency markets. Specifically, for each cryptocurrency in the sample, we model its VaR at the various tail thresholds as a function of the loss exceedance, computed as returns lower than a pre-determined tail threshold, of the other cryptocurrencies in the network. Our model has the following functional form:

$$\text{VaR}^{i}_{q,t} = \alpha^{i} + \boldsymbol{\theta}^{i\intercal}\boldsymbol{E}^{-i}_{t} + \omega^{i}X^{i}_{t-1} \tag{1}$$

where $\text{VaR}^{i}_{q,t}$ is the VaR (also called the conditional quantile function) of the return on cryptocurrency i at the qth quantile at time t, X^{i}_{t-1} is the return on cryptocurrency i at time $t-1$, and $\boldsymbol{E}^{-i}_{t}$ is a vector whose elements are the loss exceedance of all the other cryptocurrencies in the network except cryptocurrency i at time t, where, following Hautsch et al. (2015), the loss exceedance of cryptocurrency j, denoted by E^{j}_{t}, is defined as follows:

$$E^{j}_{t} = \begin{cases} 0, X^{j}_{t} \geq \text{the unconditional } q\text{th sample quantile of } X^{j} \\ X^{j}_{t}, \text{otherwise.} \end{cases} \tag{2}$$

Thus, the jth element in the coefficient vector $\boldsymbol{\theta}^{i}$ in Eq. (1), denoted by θ^{i}_{j}, shows the extent of tail risk spillover from cryptocurrency j to cryptocurrency i when the return of cryptocurrency j falls below the qth quantile of the return distribution. The higher value of θ^{i}_{j} implies that when cryptocurrency j experiences a more severe shock (e.g., a large negative return), the VaR of cryptocurrency i increases in absolute value by a larger amount. Since we define the VaR as the quantile of returns rather than the quantile of loss value, therefore a lower value of VaR implies a larger amount of tail risk and a higher value of θ^{i}_{j} implies a stronger tail risk spillover.

Even though the quantile regression technique is well suited for analysing the tail risk of asset returns, the approach is not consistent in models with large numbers of regressors, small numbers of observations, and more importantly when only some of the regressors in the model have nonzero impacts on the dependent variable (Belloni & Chernozhukov 2011). These shortcomings hinder the applicability of the traditional quantile regression technique to analyse the tail risk dependence in a large network of assets where each of the assets can potentially influence and be influenced by the other assets in the network. Belloni & Chernozhukov (2011) address these drawbacks by introducing the Least Absolute Shrinkage and Selection Operator (LASSO) method to the conventional quantile regression technique. By allowing "data to speak for itself", the LASSO approach selects only relevant regressors that have nonzero impacts on the dependent variable in the final conditional quantile model in the data-driven way. By removing covariates that have very little explanatory power from the model, the number of parameters to be estimated in the network is reduced significantly.

In implementing the LASSO quantile regression technique to obtain only the relevant drivers of the VaR of a cryptocurrency, we follow the approach outlined in Hautsch et al. (2015). We first standardise all the variables to eliminate the impact of scale on the coefficient values[9] and define

$$\boldsymbol{\xi}^{i\intercal}\boldsymbol{W}_t^i = \alpha^i + \boldsymbol{\theta}^{i\intercal}\boldsymbol{E}_t^{-i} + \omega^i X_{t-1}^i \tag{3}$$

where vectors $\boldsymbol{W}_t^i$ and $\boldsymbol{\xi}^i$ contain all the standardised regressors for the VaR of cryptocurrency i and their corresponding coefficients, respectively. The parameter vector $\boldsymbol{\xi}^i$ is estimated by minimising the following term in the corresponding ℓ_1-penalised quantile regression model:

$$\frac{1}{T}\sum_{t=1}^{T}\left[q - I\left(X_t^i \leq \boldsymbol{\xi}^{i\intercal}\boldsymbol{W}_t^i\right)\right]\left(X_t^i - \boldsymbol{\xi}^{i\intercal}\boldsymbol{W}_t^i\right) + \lambda^i\frac{\sqrt{(q(1-q))}}{T}\sum_{k=1}^{K}\left|\xi_k^i\right| \tag{4}$$

where $I(\cdot)$ is the indicator function which takes the value of 1 when $X_t^i \leq \boldsymbol{\xi}^{i\intercal}\boldsymbol{W}_t^i$ and 0 otherwise, T is the number of observations in the sample, K is the number of regressors in $\boldsymbol{W}^i$, and ξ_k^i is the kth element in the coefficient vector $\boldsymbol{\xi}^i$. Following Hautsch et al. (2015), we set the predetermined cut-off limit below which the coefficients in $\boldsymbol{\xi}^i$ along with their corresponding regressors are removed from the model to 0.0001.

The cryptocurrency-specific penalty parameter λ^i in Eq. (4) governs the rate at which irrelevant regressors are removed from the final model for cryptocurrency i. A higher penalty value depresses the coefficient values to be closer to zero and therefore more regressors are likely to be eliminated from the model.[10] For each cryptocurrency, we determine the optimum value for λ^i in a data driven way. Specifically, we obtain λ^i for a currency i to maximises the backtesting performance of its estimated VaR in the following steps:

Step 1. For each c in the ν-equidistant grid $C = \{c_1 < \cdots < c_h = c_1 + (h-1)\nu < \cdots < c_L\}$, we determine the penalty parameter $\lambda^i(c)$ using four sub-steps.[11]

Step 1a. Draw T i.i.d. observations from the uniform distribution $U[0,1]$ and denote them as $u_1, u_2, \ldots, u_T$. These observations are independent from the timing of the observations in the dataset. Calculate the following variable:

$$\Lambda^i = T \times \max_{1\leq k\leq K}\frac{1}{T}\left|\sum_{t=1}^{T}\frac{W_{t,k}^i\left(q - I\left(u_t \leq q\right)\right)}{\sqrt{q\left(1-q\right)}}\right| \tag{5}$$

Step 1b. Repeat Step 1a for 500 times to obtain an empirical distribution of Λ^i (conditional on $\boldsymbol{W}^i$). Given a confidence level $1-\alpha$, the penalty parameter is calculated as

$$\lambda^i(c) = c \times Q\left(\Lambda^i, 1-\alpha\right) \tag{6}$$

[9]When the scale of a regressor is significantly larger than that of the dependent variable, the magnitude of the estimated coefficient is small by construction regardless of the explanatory power of the regressor.

[10]When $\lambda^i = 0$, we revert to the traditional quantile regression model without any deselection of variables.

[11]We set $c_1 = 0.5$, c_L=20, and $\nu = 0.5$.

where $Q\left(\Lambda^i, 1-\alpha\right)$ is the $1-\alpha$ quantile of the empirical distribution of Λ^i. We follow Belloni & Chernozhukov (2011) recommendation to use $\alpha = 0.1$ in this calculation.

Step 1c. Estimate the ℓ_1-penalized quantile regression according to Eq. (4) and remove regressors in $\boldsymbol{W}^i$ whose coefficient's absolute value is smaller than 0.0001. Estimate the post-LASSO quantile regression (to be defined below) with the remaining regressors to obtain the post-LASSO estimated coefficients. The fitted value of the regression is the estimated VaR of the currency i over time.

Step 1d. Backtest the estimated VaR with Hautsch et al. (2015) log likelihood ratio test: estimate the logistic regression model for the VaR exceedance $\mathrm{VE}_t = I\left(X_t^i < \widehat{\mathrm{VaR}}_{q,t}\right)$:

$$\mathrm{VE}_t = \beta_0 + \left(\mathrm{VE}_{t-1}, \mathrm{VE}_{t-2}, \mathrm{VE}_{t-3}, \widehat{\mathrm{VaR}}_{q,t-1}\right)\boldsymbol{\beta} + \varepsilon_t = \beta_0 + \boldsymbol{V}_t^\intercal \boldsymbol{\beta} + \varepsilon_t \tag{7}$$

The log likelihood ratio test statistic for the null hypothesis that the VaR exceedance is i.i.d. Bernoulli distributed with success probability q is

$$\mathrm{LR} = -2\left(\ln \mathcal{L}_r - \ln \mathcal{L}_u\right) \overset{a}{\sim} \chi_5^2 \tag{8}$$

where

$$\ln \mathcal{L}_u = \sum \left[\mathrm{VE}_t \ln F_{\log}\left(\beta_0 + \boldsymbol{V}_t^\intercal \boldsymbol{\beta}\right) + \left(1 - \mathrm{VE}_t\right) \ln\left(1 - F_{\log}\left(\beta_0 + \boldsymbol{V}_t^\intercal \boldsymbol{\beta}\right)\right)\right] \tag{9}$$

$$\ln \mathcal{L}_r = \sum \mathrm{VE}_t \ln(q) + \left(T - \sum \mathrm{VE}_t\right) \ln(1-q) \tag{10}$$

and $F_{\log}\left(\beta_0 + \boldsymbol{V}_t^\intercal \boldsymbol{\beta}\right)$ is the fitted value of the logistic regression. Record the p-value of the test $p(c)$ corresponding to the current c value.

Step 2. Repeat Step 1 for all c in the C grid and select the c that produces the highest $p(c)$ to be the optimum value of c. The corresponding value of λ^i is the optimum penalty parameter for the LASSO quantile regression.

We also examine different approaches of selecting the values of λ. In Appendix B, we test the robustness of our results using either cross-validated λ for each currency or the same λ for all currencies.

After regressors with negligible impacts are deselected from the model, the remaining regressors are employed in the conventional quantile regression model – now referred to as the post-LASSO quantile regression – to explain the VaR of the cryptocurrency at the specified tail thresholds. The estimated coefficient of currency j in the post-LASSO quantile regression is regarded as the spillover coefficient of that currency in $\boldsymbol{\theta}^i$ in Eq. (1). The coefficients in $\boldsymbol{\theta}^i$ of the deselected currencies take the value of 0. The estimated spillover coefficients for every cryptocurrency in the network obtained from the estimation of Eq. (1) are collected to form the tail risk connectedness matrix, denoted by $\boldsymbol{A} = \{A_{ij}\}$. The entry $\{A_{ij}\}$ of row i and column j in

$\boldsymbol{A}$ takes the value of θ_j^i. Using the information in the connectedness matrix, we calculate the tail risk in-degree and the tail risk out-degree of cryptocurrency i as the number of cryptocurrencies which transmit tail risk to i and the number of cryptocurrencies which receive i's tail risk, respectively. The difference between i's tail risk out-degree and i's tail risk in-degree is its tail risk net-degree. Finally, the total degree of tail risk connectedness in the entire network is computed as the number of nonzero entries in matrix $\boldsymbol{A}$.

3.1 The Tail Risk Connectedness Network: Full Sample Analysis

Table 2 summarises the estimated total degrees of tail risk connectedness, calculated using the full sample of 21 cryptocurrencies, at the 1%, 5%, 10% and the 20% VaR thresholds. It is worth noting that while the thresholds at the left tails are the significance levels of the VaR, denoted by q, in Eq. (1), those at the right tails equal 100% minus the corresponding VaR significance levels. For example, the 1% threshold at the right tail is in fact the 99% VaR.

Several interesting features of the tail risk network can be highlighted. First, consistent with Borri (2019), cryptocurrencies appear to be highly exposed to tail risk within the cryptocurrency markets. At the 10% tail threshold, the VaR at the left tail of a cryptocurrency is found to be affected by the tail events originating from an average of $140/21 \approx 7$ other cryptocurrencies in the network whereas its right-tail VaR is also affected by at least $174/21 \approx 8$ other currencies on average. Second, tail risk connectedness tends to be more popular at the less extreme tail thresholds although this relation is not strictly monotonic. At the right tails, the number of connections among the cryptocurrencies at the 20% tail threshold is more than double the corresponding number at the 1% threshold while at the left tails, the same ratio is almost triple. These findings suggest that drastic tail events – both positive and negative – are more likely caused by the cryptocurrencies' idiosyncratic shocks than from tail events originating from the other cryptocurrencies in the system. Third, judging by the number of connections at both the right and the left tails at all the tail thresholds under examination, the degree of connectedness at the right tails is much more pronounced than that at the left tails. While we observe, at the 10% VaR, 24% more connections at the right tails than those at the left tails, the figure is as high as 114% at the 5% VaR threshold. These findings highlight the attractiveness of investment in cryptocurrencies: investors benefit not only the segmentation of idiosyncratic tail risk in the cryptocurrency markets during declines but also market-wide rallies during upswings. These characteristics may have contributed to the impressive emergence of cryptocurrencies as an alternative asset class over the last few years.

Table 3 shows additional results of the analysis of the degree of tail risk connectedness network. The table lists the top five cryptocurrencies with the largest values of in-degree, out-degree, and net-degree at the various tail thresholds. Cryptocurrencies with large out-degrees or large in-degrees are refereed to as tail risk drivers and tail risk receivers, respectively, while cryptocurrencies with both large out-degrees and in-degrees are referred to as tail risk connectors. Tail risk drivers, receivers, and connectors all play important roles in the cryptocurrency network as they significantly contribute to the propagation of tail events across the system. In other words, they account for the majority of the aggregate tail risk of the entire cryptocurrency markets.

According to the results presented in Panel A of Table 3, Siacoin and NEM appear to be the most prominent risk receivers as they consistently rank among the top five receivers for a number of tail thresholds. Interestingly, Lisk is found to possess a desirable property as a right tail risk receiver since it ranks among the top right tail risk receivers while never ranks among the top left tail risk receivers. Meanwhile, as shown in Panel B of Table 3, the consistent tail risk drivers in the network are Ethereum and Litecoin. In fact, Ethereum appears in the list of top tail risk drivers at all the tail thresholds. The role of Bitcoin is also noticeable as it appears in the list of top right tail risk drivers but not in the list of top left tail risk drivers. Across all the right tail thresholds, Bitcoin is the biggest driver of right tail shocks with the average out-degree of 13. Panel C lists the top five net tail risk drivers in the system as captured by their net-degrees. We can see that Litecoin, Ethereum, NEM, Decred are among the most common net tail risk drivers across the different tail levels. Ethereum is the most popular left tail risk drivers, appearing in 3 out of 4 lists, at the 1%, 10%, and 20%. Bitcoin is, again, the most popular right tail risk drivers, where it features in most of the lists of right tail drivers. Across the lists of top drivers and receivers, Litecoin and Ethereum Classic tend to appear frequently in multiple tail thresholds, suggesting that they are major tail connectors in the system, playing key roles in both generating and receiving tail risk. Full results of the degree analysis for all the cryptocurrencies in the network at all the tail thresholds are available in Tables A1–A3 in Appendix A.

To illustrate graphically the extent to which tail risk in the cryptocurrency markets is intertwined, Figure 2 depicts the tail risk connectedness of the entire network at the 5% and the 95% tail thresholds. Each node in the figures represents a cryptocurrency, whose out-degree is signified by the node size. The directional tail risk connection from one cryptocurrency to another is characterised by an arrow pointing from the risk-driving cryptocurrency to the risk-receiving cryptocurrency. The magnitude of the spillover coefficient for each cryptocurrency pair is portrayed by the width of the arrow. According to Figure 2, it is evident that the tail risk network at the 95% VaR is noticeably denser than the network at the 5% VaR, indicating much stronger connectedness and a larger number of relevant connections at the 95% VaR than at the 5% VaR. Notably, the roles which the most popular cryptocurrencies play in the tail risk networks are clearly visible in the figure: while Bitcoin and Litecoin are the cryptocurrencies with the highest value of out-degree at the 95% tail threshold, Ethereum and Etherium Classic appears to be the most prominent risk driver at the 5% threshold. To put it differently, Bitcoin and Litecoin play a major role in the booming market while Ethereum and Etherium Classic take the driving seat during turbulent periods

We present the distributions of the degree measures using histograms of the in-degrees, the out-degrees, and the net-degrees at the various VaR thresholds in Figures 3 and 4. We observe from the histograms in Figure 3 that the majority of the cryptocurrencies in the sample have very low left-tail in-degrees and out-degrees as depicted by the high frequencies of the first few bins in most of the histograms. This finding points to the idiosyncratic nature of the tail events in the cryptocurrency markets – which we have previously discussed. Furthermore, the net-degree distributions reveal the popularity of small and moderate net tail risk receivers. In all of the net-degree histograms, except for that at the 1% VaR threshold, there are more negative net-degree currencies than positive net-degree currencies. Since the sum of all net-degrees in the system must equal zero by definition, this finding suggests that, even though there are fewer risk drivers in the system, they are likely to be major drivers with large, positive net-degrees.

Compared to the histograms for the left-tail networks, the histograms for the right-tail networks shown in Figure 4 paint a different picture. First, the total number of connections for the right-tail networks are higher than those of the corresponding left-tail counterparts, indicating a larger degree of tail risk dependence at the right tails of the return distributions than at the left tails. Second, as is visible from the shape of the histograms, we observe a higher proportion of cryptocurrencies with large right-tail in-degrees and out-degrees. Especially in the case of the 95% VaR, the number of cryptocurrencies with moderate and high degrees of spillover is much larger than the number of currencies with low levels of spillover. These results show that the network of cryptocurrencies contains a greater number of major right tail risk drivers and risk receivers than the left tail counterparts. Third, the distributions of the net-degrees of the right tail risk spillovers are, in general, less skewed than those of the left tail spillovers. Our calculation shows that the average skewness of the right-tail spillover net-degree distributions is 0.047 whereas the average skewness of the left-tail spillover net-degree distributions is 0.302. We can also report a larger number of cryptocurrencies with positive right-tail net-degrees than those with negative right-tail net-degrees after we exclude currencies with zero net-degrees, suggesting that right-tail spillovers are more common among cryptocurrencies than left-tail spillovers. These features confirm the dominance of the right-tail connectedness over the left-tail connectedness.

To further examine if our findings hold in a larger network of cryptocurrencies, we expand our sample to include currencies with at least two years of price data, taking the total number of cryptocurrencies in the network up to 44. Our extended sample thus starts from November 2017. We present the summary statistics of the currencies in the extended sample in Table 4 and the LASSO quantile regression results for the tail risk network of the 44 cryptocurrencies in Table 5. Similar to our main results for the 21 cryptocurrencies previously reported, we continue to observe that the number of significant left tail risk spillover coefficients increases monotonically from the 1% VaR to the 20% VaR levels, implying that there is a tendency for the tail events at the extreme tail thresholds to be idiosyncratic tail shocks. More importantly, tail risk spillover at the right tails continues to dominate spillover at the left tails at all the VaR significance levels. Taken together, the results for the extended sample of 44 cryptocurrencies confirm the robustness of our previous findings.

3.2 The Tail Risk Connectedness Network: Rolling Window Analysis

In this section, we employ a rolling window analysis to investigate whether the tail dependence behaviour in the cryptocurrency markets is time varying. Specifically, using daily returns of cryptocurrencies that came into existence and investible within each calendar quarter between 2017 and 2019, we estimate the LASSO quantile regression model for the currencies and construct the connectedness matrices for the different tail thresholds.[12] Thus, as shown in the second column of Table 6, our sample size grows from 21 cryptocurrencies during the first quarter under investigation (2017Q1) to 69 currencies in the last quarter ending December 2019 (2019Q4). It is worth noting that since there are no observations with loss exceeding the 1% VaR thresholds in each of the quarters within the sample period, we do not estimate the connectedness matrix for the 1% VaR thresholds. Columns 3–11 of Table 6 show the number of connections in the tail risk networks at the 5%, 10%, and 20% VaR thresholds from the first quarter of 2017 to the last quarter of 2019. For each tail threshold, we report the number of connections at both the left tails and the right tails along with their differences.

In general, we observe a downward trend in the total of connection level in the cryptocurrency networks. During the first quarter of 2017, an average of 34.7 connections is observed in the network across all the tail thresholds at both sides of the distributions. The figure accounts for 8.3% of the total number of possible connections among the 21 currencies.[13] The corresponding value in the fourth quarter of 2019 is 241 connections, accounting for 5.1% of the total number of possible connections among the 69 currencies. These findings suggest that the cryptocurrency markets have become more segregated over time. Interestingly, similar to the results for the full sample analysis, the right tail dependence continues to dominate the left tail connectedness. In the majority of cases, the total numbers of connections at the right tails are higher (in some cases, more than double) than those at the left tails at all the tail thresholds. As this period encompasses the bear market which started just before 2018, the findings that spillovers at the right tails dominate those at the left tails during this period suggest that the dominance of the right tail spillovers in the cryptocurrency markets is not a result of bubbles.

We further examine if the dominance of the right-tail connectedness over the left-tail connectedness over time are driven by positive skewness of the return distributions coupled with a "peso problem" effect. We first compute the average skewness statistics of all the cryptocurrencies in each quarter between 2017 and 2019 and present them in the last column of Table 6. It can be seen that the average skewness of the return distributions of all the cryptocurrencies follows a downward path from 2017Q1 to 2019Q4 – though the trend is not strictly monotonic. This suggests that occurrences of extreme positive returns have become less frequent over time. We then calculate the correlation coefficients between the average skewness and the differences between the right-tail and the left-tail dependence at the 5%, 10%, and 20% VaR thresholds. The correlation coefficients are found to be -0.12, -0.35, and -0.45 for the 5%, 10%, and 20% levels,

[12] This approach allows us to examine the robustness of our results to the variation in the network composition as more currencies are added to the sample when they became available in the quarter under examination.

[13] The total number of possible connections in a network of 21 currencies is $21 \times 20 = 420$.

respectively.[14] These results indicate that right-tail spillovers in the cryptocurrency markets continue to dominate left-tail spillovers even as the return distributions have become less skewed over the sample period, providing strong evidence against the hypothesis that spillovers at the right tails are a consequence of positively skewed return distributions coupled with a potential peso problem. However, since the time series of the cryptocurrencies in our sample are relatively short, we cannot completely rule out a possibility that our results are caused by a peso problem and therefore they need to be interpreted with some caution.[15]

3.3 The Tail Risk Connectedness Network: The Predictive Connection between Cryptocurrencies

Thus far, our main results point to the contemporaneous tail risk linkages among the cryptocurrencies in the network. They also indicate that the tail fatness of the conditional distribution of a cryptocurrency is affected by concurrent tail events originated from other cryptocurrencies. Although the issue of endogeneity due to simultaneity of the model in Eq. (1) can be safely ignored (Hautsch et al. 2015), we further test the robustness of our findings by using the lagged tail events of other cryptocurrencies as the explanatory variables in Eq. (1) thereby completely obviating simultaneity.[16] Specifically, we estimate the following predictive LASSO quantile regression model for each cryptocurrency:

$$\mathrm{VaR}^i_{q,t} = \alpha^i + \boldsymbol{\theta}^{i\intercal}\boldsymbol{E}^{-i}_{t-1} + \omega^i X^i_{t-1} \tag{11}$$

where the only difference between Eq. (1) and Eq. (11) is that we use lagged loss exceedances, denoted by $\boldsymbol{E}^{-i}_{t-1}$, instead of the contemporaneous values as the independent variables. This setting allows us to examine if tail events of a cryptocurrency can be predicted using lagged loss exceedances of the other cryptocurrencies in the network.

Results for the predictive tail connectedness at the different tail thresholds are reported in Table 7. Similar to the results for the contemporaneous tail connectedness reported in Tables 2 and 5, the total number of connections at the right tails are found to be consistently higher than those at the left tails. The differences between the number of tail connections at the right tails and that at the left tails are even more prominent than the differences calculated under the contemporaneous setting. Across all the VaR thresholds, the total number of connections at the right tails approximately triples (doubles) that at the left tails in the 21 (44) currency network. Therefore, our central finding that tail risk spillovers among the cryptocurrencies at the right tails dominate that at the left tails remains robust under the predictive set-up. In comparing the results in Table 7 with those in Tables 2 and 5, it is evident that the predictive tail risk

[14] We also obtain qualitatively similar results when using the ratios of the number of connections at the right tails and the left tails instead of their differences to compute the correlation coefficients.

[15] We thank the referee for the suggestion regarding this investigation.

[16] The reason why Eq. (1) does not suffer from simultaneity is that although a highly negative return on cryptocurrency j causes the VaR of cryptocurrency i to rise, it does not necessarily imply a higher loss exceedance of cryptocurrency i because the relationship between a specific quantile and the conditional distribution of exceedances, given a fixed threshold, is not known.

networks are less connected than the contemporaneous tail risk networks since, in most cases, the numbers of total connections of the predictive networks are less than half of those of the contemporaneous networks. This is not surprising: as cryptocurrencies are traded round-the-clock, a tail event affecting a cryptocurrency most likely impacts other cryptocurrencies in the markets instantaneously.

We reach similar conclusions when applying rolling window analysis to the predictive quantile regression model in Eq. (11). Judging by the results in Table 8, tail risk dependence at the right tails is still much more popular than that at the left tails in almost all the connectedness matrices for each of the quarters and at all the VaR significance levels under examination. In fact, only two instances, observed in 2018Q4 and 2019Q4, point to the dominance of the left tail risk dependence over the right tail risk dependence at the 5% VaR threshold. Lastly, consistent with the results for the full sample investigation, we document a general reduction in the number of connections in the connectedness matrices under the predictive framework as compared to the contemporaneous setting.

4 Portfolio Analysis: Equal-Weighting versus Mean-Conditional Value-at-Risk Optimisation (Mean-CVaR)

In this section, we compare the risk and return performance of a cryptocurrency portfolio, constructed using the the naïve, equal weighting scheme to that of a portfolio, formed based on a more advanced mean-Conditional Value-at-Risk (mean-CVaR) optimisation algorithm. While the weights of currencies in the equal-weighted portfolio all equal $1/n$ where n is the number of currencies, the weights of the constituent cryptocurrencies in the mean-CVaR portfolio are calculated such that the trade off between reward and risk, as measured by the expected returns and the CVaR, is optimised. The optimisation problem assumes extreme downside risk as the relevant risk measure which is of much more concern to some investors than the traditional dispersion risk as captured by the variance in the canonical mean-variance optimisation framework.

The mean-CVaR problem can take a number of alternative representations, including minimising portfolio risk with a minimum portfolio expected return constraint, maximising portfolio expected return with a maximum portfolio risk constraint, and maximising a utility function defined as the portfolio expected return minus a penalty for risk. These three alternative problems are equivalent in the sense that they generate the same efficient frontier when varying the expected return constraint, the risk constraint and the risk aversion coefficient, respectively (see Krokhmal et al. 2002, Brandtner 2013). A widely used framework for the mean-CVaR optimisation problem is developed by Rockafellar & Uryasev (2000, 2002) where the optimisation is transformed into a linear programming problem and therefore can be used to obtain optimised weights for a large number of assets in the portfolio. Rockafellar & Uryasev (2000, 2002) show that the loss value of CVaR of a portfolio, denoted here by $CVaR_v$, can be obtained through minimising a function with respect to the loss value of VaR, denoted as VaR_v, as follows:

$$F_\alpha\left(\boldsymbol{w},\, VaR_v\right) = VaR_v + \frac{1}{\alpha}\int_{X \in R^N} \left[-\boldsymbol{w}^\intercal \boldsymbol{X} - VaR_v\right]^+ p\left(\boldsymbol{X}\right) d\boldsymbol{X} \tag{12}$$

where α is the significance level of the VaR and CVaR; $\boldsymbol{w} = (w_1, w_2, \ldots, w_N)^\intercal$ and $\boldsymbol{X} = (X_1, X_2, \ldots, X_N)^\intercal$ are the weights and returns of the N assets in the portfolio.[17] The returns are random variables distributed according to some multivariate distribution with probability density function $p\left(\bullet\right)$ and $[t]^+$, where $[t]^+ = \max\{t, 0\}$, is a function that takes the value of t when $t > 0$ and 0 otherwise.

Rockafellar & Uryasev (2000, 2002) further show that minimising $CVaR_v$ with respect to $\boldsymbol{w}$ is equivalent to minimising $F_\alpha\left(\boldsymbol{w},\, VaR_v\right)$ over all $\left(\boldsymbol{w},\, VaR_v\right)$:

$$\min_{\boldsymbol{w} \in \boldsymbol{W}} CVaR_v\left(\boldsymbol{w}\right) = \min_{\left(\boldsymbol{w},\, VaR_v\right) \in \boldsymbol{W} \times R} F_\alpha\left(\boldsymbol{w},\, VaR_v\right) \tag{13}$$

where $F_\alpha\left(\boldsymbol{w},\, VaR_v\right)$ is convex with respect to $\left(\boldsymbol{w},\, VaR_v\right)$ and $CVaR_v\left(\boldsymbol{w}\right)$ is convex with respect to $\boldsymbol{w}$. Given that $\boldsymbol{W}$ is also a convex set, which is generally true, minimising $CVaR_v$ is a convex programming problem. As a result, the full mean-CVaR optimisation problem can be formed by combining the risk minimisation objective and the minimum expected return constraint as follows:

$$\min_{\left(\boldsymbol{w},\, VaR_v\right) \in \boldsymbol{W} \times R} \left(VaR_v + \frac{1}{\alpha}\int_{X \in R^N} \left[-\boldsymbol{w}^\intercal \boldsymbol{X} - VaR_v\right]^+ p\left(\boldsymbol{X}\right) d\boldsymbol{X}\right) \tag{14}$$

subject to

$$\sum_{i=1}^{N} w_i = 1,$$
$$\boldsymbol{w}^\intercal \boldsymbol{\mu} \geq \mu_0,$$
$$0 \leq w_i \leq 1, \forall i = 1, 2, \ldots, N.$$

Moreover, Rockafellar & Uryasev (2000, 2002) recommend transforming the above problem into a linear programming problem via using asset return scenarios in order to eliminate the probability density component and introduce the new variables $\{z_j\}$ to replace the values of the $[\bullet]^+$ function in these return scenarios:

$$\min_{\left(\boldsymbol{w},\, VaR_v, \boldsymbol{z}\right) \in \boldsymbol{W} \times R \times \boldsymbol{R}^{\boldsymbol{J}+}} \left(VaR_v + \frac{1}{\alpha}\sum_{j=1}^{J} \pi_j z_j\right) \tag{15}$$

[17] $CVaR_v$ and VaR_v are essentially equal to -1 times $CVaR$ and VaR, respectively.

subject to

$$z_j \geq -\boldsymbol{w}^\intercal \boldsymbol{X_j} - VaR_v, \forall j = 1, 2, \ldots, J,$$
$$z_j \geq 0, \forall j = 1, 2, \ldots, J,$$
$$\sum_{i=1}^{N} w_i = 1,$$
$$\boldsymbol{w}^\intercal \boldsymbol{\mu} \geq \mu_0,$$
$$0 \leq w_i \leq 1, \forall i = 1, 2, \ldots, N$$

where $\boldsymbol{X}_j$ is a vector of the constituent asset returns in scenario j; J is the number of all investigated scenarios which could be obtained from historical observations of asset returns; π_j is the probability that scenario j occurs; the components of $\boldsymbol{z} = (z_1, z_2, \ldots, z_J)^\intercal$ are the variables that replace the value of the unsmooth $[\bullet]^+$ function in each return scenario. We set the minimum return constraint to the average risk-free rate interest rate over the sample period, which is available in Kenneth French's online database.[18] We obtain similar results using other minimum return constraints such as zero and the average return of the cryptocurrency market over the sample period. It should be noted that we do not allow short-selling in this optimisation by restricting the value of any weight to be bound between 0 and 1. This is to reflect the fact that cryptocurrencies are an emerging risky alternative investment and short selling is restricted.

We first examine the difference between returns of the equal-weighted portfolio and the mean-CVaR portfolio. In this exercise, the portfolio construction starts from 1st May 2017. To construct the mean-CVaR portfolio, we compute the mean-CVaR optimal weights for each of the cryptocurrencies in the portfolio at the beginning of each month using the currencies' realised daily returns over the last 6 months as input in the optimisation. We include only currencies whose historical return data are available for the full six-month period prior to the portfolio formation. Moreover, we allow the number of cryptocurrencies in the portfolio to increase over time when new currencies became investible. We keep the optimisation problem parsimonious and close the industry practice by calculating the expected returns of the cryptocurrencies on the construction date as the simple historical averages. For the naïve, equal-weighted portfolio, each currency in the portfolio is assigned a weight of $1/n$ where n is the number of currencies which become investible on the portfolio construction date. In order to examine the portfolio performance for different holding periods, portfolio returns are recorded for investment horizons of one to twelve months. As a result, we obtain a series of post-formation holding period returns for both the mean-CVaR portfolio and the equal-weighted portfolio.

Table 9 reports the average holding period returns for the naïve and the mean-CVaR portfolios at the different investment horizons, ranging from one month to twelve months. We also report the differences in the average returns as well as the corresponding t-statistics calculated using Newey & West (1987) standard errors to account for overlapping holding period returns. For

[18]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

brevity, we only report the results for the 5% CVaR.[19] It can be seen in Table 9 that the naïve portfolio generates higher expected returns than the mean-CVaR portfolio at all the investment horizons except for the one-month holding period. Although the return differences are not statistically significant at the conventional significance levels, we are able to conclude that the equal-weighted portfolio performs no worse than the mean-CVaR portfolio.

To further investigate the relative performance of the two portfolio weighting schemes, we construct a hypothetical equal-weighted portfolio and a hypothetical mean-CVaR portfolio with $1 of intial investment each at the beginning of May 2017. Both portfolios are periodically rebalanced based on their weighting strategy. While the naïve portfolio is rebalanced to maintain the equal weight of $1/n$ for each currency at the beginning of each rebalancing period, the mean-CVaR optimal weights are estimated using historical returns realised during the previous six months prior to the portfolio formation regardless of the rebalancing frequency. This approach differs from the above approach in that both portfolios are held for a period of one, three, or six months, depending on the rebalancing period under examination, before they are rebalanced on the last day of the holding period where allocation of cryptocurrencies in the portfolios are obtained based on the portfolios' composition on the rebalancing date. This approach not only closely mimic real-world investing but also allows us to incorporate transaction costs into the rebalancing process and backtest both weighting strategies.[20]

We modify the optimisation problem to factor in transaction costs based on the technique discussed in Adcock & Meade (1994) where the cost is a linear function of the absolute weight changes.[21] Since it is in investors' interest to minimise transaction costs, it is straightforward to include the transaction cost term into the objective function. The modified mean-CVaR problem is therefore specified as follows:

$$\min_{(\boldsymbol{w}, VaR_v, \boldsymbol{z}) \in \boldsymbol{W} \times R \times \boldsymbol{R^{J}}+} \left(VaR_v + \frac{1}{\alpha} \sum_{j=1}^{J} \pi_j z_j \right) + \sum_{i=1}^{N} P_0 c_i \left| w_i - w_{0i} \right| \tag{16}$$

subject to

$$z_j \geq -\boldsymbol{w}^\intercal \boldsymbol{X_j} - VaR_v, \forall j = 1, 2, \ldots, J,$$
$$z_j \geq 0, \forall j = 1, 2, \ldots, J,$$
$$\sum_{i=1}^{N} w_i = 1,$$
$$\boldsymbol{w}^\intercal \boldsymbol{\mu} \geq \mu_0,$$
$$0 \leq w_i \leq 1, \forall i = 1, 2, \ldots, N$$

[19] Results for the other VaR levels are qualitatively similar and available upon request.

[20] In practice, an optimised portfolio may produce poor performance after trading costs are factored in if it frequently needs rebalancing and therefore incurs large trading fees.

[21] Kolm et al. (2014) stress the necessity of incorporating transaction costs directly into the optimisation problem to obtain the optimum asset allocation. The optimised weights in a frictionless condition is guaranteed to be suboptimum in the world with transaction costs.

where c_i is the transaction cost (in decimals) incurred when trading cryptocurrency i, P_0 is the portfolio value before rebalancing, and w_{0i} is the weight of cryptocurrency i in the portfolio just before rebalancing. Thus, w_{0i} is the weight after the holding period, and w_i is the target weight after rebalancing (i.e., at the beginning of the following holding period). Given that w_{0i} is a constant and is known before the optimisation, $|w_i - w_{0i}|$ is therefore convex with respect to w_i. Since $P_0 > 0$ and $c_i > 0$, it follows that the transaction cost term in the objective function is a convex function with respect to w_i. Therefore, the modified mean-CVaR is still a convex programming problem. As the average transaction cost for Bitcoin is reported to be 27 bps Borri & Shakhnov (2018), we investigate the portfolio performance using transaction cost values ranging from 0 bp (frictionless market) to a more conservative figure of 50 bps. Specifically, $c_i \in \{0, 0.001, 0.0025, 0.005\}$.

To compare the performance of the equal-weighed portfolio to that of the mean-CVaR portfolio, we report portfolio total returns, average returns, volatility, CVaR, and two (annualised) Sharpe ratios calculated using the volatility and the CVaR measures. To calculate the two Sharpe ratios, we use the average of the daily yields on the one-month Treasury bills during the whole sample period from November 2016 to December 2019.[22]

We report the performance of both portfolios for the rebalancing frequency of 1-, 3-, and 6-month in Table 10. Similar to the previous investigation, we report only the results for the 5% tail threshold for the sake of brevity.[23] First, the most striking finding is the stellar performance of both cryptocurrency portfolios during 2017–2019. Assuming one-month holding period and frictionless trading (i.e., no transaction cost), the equal-weighted portfolio and the optimised mean-CVaR portfolio generate total returns of approximately 4,900% and 6,100%, respectively! Both portfolios generate slightly lower returns when they are rebalanced less frequently. Without trading costs, the risk-adjusted performance of the naïve, equal-weighted portfolio, as measured by the traditional and the CVaR-adjusted Sharpe ratios, is slightly worse than that of the mean-CVaR portfolio. The difference is, however, modest.

When trading costs are taken into account, even at a charge of only 10 bps per trade, the equal-weighted portfolio easily outperforms the mean-CVaR portfolio. While the mean-CVaR portfolio generates over 3,600% of total return between May 2017 and December 2019, the naïve portfolio earns around 4,900% in the same period – a difference of 1,300%. The underperformance of the mean-CVaR portfolio undoubtedly stems from frequent trading as a result of significant changes in the target allocation of currencies at the beginning of each rebalancing period, causing the portfolio performance to be heavily eroded by trading charges. When the transaction cost is assumed to be 50 bps, the total returns of the naïve scheme are more than 9 times those of the mean-CVaR. Interestingly, the impact of transaction costs on the portfolio performance of the mean-CVaR portfolio appears to be flat as the costs become higher. This can be explained by the fact that as trading in order to rebalance the portfolio becomes more costly, the "optimal" strategy is to trade less frequently. All the conventional and the CVaR Sharpe ratios for the

[22] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
[23] Results for the other tail thresholds are qualitatively similar and are available from the authors upon request.

naïve portfolio are much larger than those of the mean-CVaR portfolio. The only performance measure which ranks the mean-CVaR portfolio higher than the equal-weighted portfolio is the tail risk measure – CVaR. This should not come as a surprise as the mean-CVaR portfolio is constructed with the aim to minimise CVaR. However, when both portfolios are rebalanced monthly, the equal-weighted portfolio of cryptocurrencies still succeeds in outperforming the mean-CVaR portfolio as judged by this measure.

Figure 5 compares the value of the equal-weighted portfolio to the value of the mean-CVaR portfolio over time between May 2017 and December 2019. Figure 5a shows the portfolio values assuming no transaction cost while Figure 5b shows the portfolio values assuming a charge of 25 bps per trade. It is evident that without transaction costs, the performance of the equal-weighted portfolio is almost identical to that of the mean-CVaR portfolio. When transaction cost is accounted for, the equal-weighted portfolio outperforms the mean-CVaR portfolio for most of the sample periods – even during the collapse of the cryptocurrency markets in 2018.[24]

Our portfolio analysis results suggest an important implication for cryptocurrency investors. While the mean-CVaR optimisation strategy specifically aims to minimise tail risk, the equal weighting of the cryptocurrencies in the equal-weighted portfolio is a naïve approach to diversification. The finding that the simple, equal-weighting strategy succeeds in achieving similar or better risk and return profile as the mean-CVaR optimisation which actively seeks to minimise tail risk indicates that diversification can be straightforwardly accomplished by means of equally weighting cryptocurrencies in the portfolio. This finding undoubtedly should be welcome by regular, retail investors, given their disadvantages in the information processing and complex portfolio construction capability.

5 Conclusions

In this article, we examine the nature of tail risk spillovers among the most actively traded cryptocurrencies. We model the VaR of each cryptocurrency in the sample at various thresholds as a function of loss exceedances of the other cryptocurrencies in the network. The model is estimated using the LASSO quantile regression technique which permits the deselection of explanatory variables with negligible impacts from the final model in a data-driven way, thereby significantly reducing the number of parameters to be estimated and greatly easing the interpretation of the results. Our findings show that Bitcoin and Litecoin is the major risk-driving cryptocurrencies during the bull markets while Ethereum and Ethereum Classic appear to drive down the entire markets during bearish periods. Results from the dynamic rolling window analysis point to increasing segmentation of tail risk dependence in the cryptocurrency markets over time.

Our results show that rare event risk becomes sparser at the more extreme tail thresholds, pointing to the idiosyncratic nature of extreme tail risk among cryptocurrencies. Interestingly,

[24] In 2018, the naïve portfolio value falls by 83% while the mean-CVaR portfolio value drops by 90% for the year.

contrary to the phenomena typically observed in traditional assets, such as stocks and bonds, we find that tail risk dependence at the right tails of the return distributions is more prominent than that at the left tails, suggesting that portfolios of cryptocurrencies are less prone to collective downside risk and should enjoy strong performance during positive market sentiments. Our portfolio analysis further suggests that diversification in cryptocurrency investment can be effectively and easily achieved by means of equal weighting of cryptocurrencies in the portfolio.

Our research has important implications for both cryptocurrency traders as well as policymakers. We reveal different layers of the unique attractiveness of this asset class to both institutional and retail investors. Even though markets for cryptocurrencies account for a small proportion of the entire financial markets, it is a matter of time before they become more mainstream as financial institutions start to offer products linked to cryptocurrencies (e.g., traditional passive or active mutual funds or exchange-traded funds (ETFs) linked to the cryptocurrency markets). Policymakers should therefore be aware of the potential risk, originating from the cryptocurrency markets, which could destabilise the financial markets and affect the health of the economy as a whole.

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Table 1

Descriptive Statistics
Full Sample: 21 Cryptocurrencies

This table reports the descriptive statistics for the 21 cryptocurrencies in the sample. The market capitalisation (MC) is shown in million of dollars. The table presents the means, the standard deviations (SD), the minima (Min), the 5th percentile (5% VaR), the 10th percentile (10% VaR), the medians (Median), the 90th percentile (90% VaR), the 95th percentile (95% VaR), and the maxima (Max) for each cryptocurrency. The sample period is from November 2016 to December 2019.

Names	MC(in millions $)	Mean	SD	Min	5% VaR	10% VaR	Median	90% VaR	95% VaR	Max	Skewness	Kurtosis
Bitcoin	130446.00	0.003	0.042	-0.187	-0.065	-0.112	0.002	0.048	0.069	0.252	0.309	4.077
Ethereum	14139.77	0.004	0.058	-0.271	-0.081	-0.147	0.000	0.065	0.103	0.337	0.796	4.850
XRP	8359.62	0.006	0.093	-0.460	-0.082	-0.146	-0.003	0.063	0.109	1.794	7.868	130.060
Litecoin	2635.70	0.004	0.065	-0.326	-0.082	-0.133	-0.001	0.065	0.106	0.666	2.245	16.585
Stellar	906.96	0.007	0.095	-0.307	-0.104	-0.175	-0.004	0.082	0.127	1.061	4.154	36.268
Monero	774.58	0.004	0.064	-0.254	-0.097	-0.142	0.000	0.073	0.111	0.538	1.061	7.265
Neo	612.87	0.007	0.095	-0.369	-0.105	-0.178	-0.002	0.089	0.149	1.228	3.814	37.544
Ethereum Classic	523.06	0.004	0.067	-0.353	-0.089	-0.163	-0.001	0.069	0.111	0.580	1.088	9.057
Dash	380.82	0.003	0.062	-0.216	-0.085	-0.158	-0.001	0.067	0.098	0.549	1.569	10.444
NEM	289.06	0.005	0.089	-0.303	-0.097	-0.152	-0.001	0.079	0.115	1.706	7.162	121.803
Dogecoin	248.88	0.004	0.070	-0.389	-0.089	-0.163	0.000	0.067	0.115	0.612	1.993	15.239
Zcash	230.20	-0.002	0.091	-0.719	-0.104	-0.183	-0.005	0.071	0.110	1.826	7.109	147.342
Decred	181.23	0.006	0.080	-0.290	-0.103	-0.150	-0.002	0.092	0.142	0.554	1.643	7.140
Waves	101.72	0.003	0.070	-0.248	-0.100	-0.166	0.000	0.079	0.120	0.466	0.951	4.795
Augur	98.35	0.003	0.073	-0.268	-0.098	-0.162	0.000	0.081	0.115	0.922	2.333	24.500
Lisk	67.62	0.004	0.074	-0.336	-0.094	-0.161	-0.003	0.084	0.127	0.601	1.254	7.260
DigiByte	66.95	0.007	0.112	-0.303	-0.113	-0.189	-0.003	0.101	0.149	2.208	7.724	135.329
Verge	55.73	0.015	0.159	-0.500	-0.161	-0.310	0.000	0.121	0.250	1.647	3.519	26.119
Siacoin	54.37	0.005	0.088	-0.356	-0.108	-0.188	-0.002	0.083	0.132	0.794	2.094	12.907
Bytecoin	52.66	0.011	0.192	-0.467	-0.135	-0.208	-0.001	0.106	0.157	3.942	13.091	240.959
MonaCoin	50.17	0.007	0.106	-0.287	-0.101	-0.162	-0.005	0.076	0.133	1.345	5.214	46.189

Table 2

The Contemporaneous Total Tail Risk Connectedness of the Cryptocurrency Network Full Sample Static Analysis: 21 Cryptocurrencies

This table presents the estimated degrees of tail risk connectedness in the network of 21 cryptocurrencies, computed as the total number of relevant spillover coefficients in the connectedness matrix $\boldsymbol{A}$ at the 1%, the 5%, the 10%, and the 20% tail thresholds. The thresholds at the left tails are the significance levels of the VaR in Eq. (1) while those at the right tails equals 100% minus the corresponding VaR significance levels. This means that the 1%, the 5%, the 10%, and the 20% right tail threshold are the 99%, the 95%, the 90%, and the 80% VaR thresholds, respectively.

	Right Tail	Left Tail	Difference
1% VaR	73	39	34
5% VaR	182	85	97
10% VaR	174	140	34
20% VaR	162	106	56

Table 3

The In-Degrees, the Out-Degrees, and the Net-Degrees of the Top 5 Cryptocurrencies at the Different VaR Thresholds Full Sample Static Analysis: 21 Cryptocurrencies

This table presents the top 5 cryptocurrencies in terms of their in-degrees, out-degrees, and net-degrees at the various tail thresholds. Panel A lists the top 5 tail risk receivers in the network, captured by their in-degrees. Panel B lists the top 5 tail risk drivers in the network, captured by their out-degrees. Panel C lists the top 5 net tail risk drivers in the network, captured by their net-degrees.

Cryptocurrency	1% Left Tail	Cryptocurrency	5% Left Tail	Cryptocurrency	10% Left Tail	Cryptocurrency	20% Left Tail	Cryptocurrency	20% Right Tail	Cryptocurrency	10% Right Tail	Cryptocurrency	5% Right Tail	Cryptocurrency	1% Right Tail
Panel A: In-Degree															
Dogecoin	8	Siacoin	12	Waves	13	Siacoin	12	NEM	16	NEM	16	Bitcoin	13	Ethereum Classic	9
Ethereum Classic	7	Dash	11	Siacoin	12	Zcash	11	MonaCoin	14	Dogecoin	13	Dogecoin	13	DigiByte	8
Monero	7	Bytecoin	8	Bytecoin	10	Litecoin	10	Stellar	13	Ethereum	13	Lisk	13	Lisk	8
Augur	6	NEM	7	Litecoin	9	Monero	10	Bytecoin	11	MonaCoin	12	NEM	13	Verge	8
Neo	4	Litecoin	7	NEM	9	Ethereum Classic	9	Siacoin	11	Lisk	11	Siacoin	13	Augur	7
Panel B: Out-Degree															
NEM	7	Ethereum Classic	9	Ethereum	16	NEM	12	Ethereum	18	Lisk	17	Litecoin	16	Lisk	8
DigiByte	3	Decred	8	Ethereum Classic	12	Ethereum	11	Bitcoin	15	Bitcoin	15	Bitcoin	15	Bitcoin	7
Ethereum	3	Augur	7	Litecoin	12	Siacoin	8	Lisk	15	Litecoin	14	Dash	13	Waves	6
Ethereum Classic	3	Ethereum	7	NEM	12	Dash	7	Waves	14	Waves	14	Ethereum	12	Ethereum	5
Litecoin	3	Waves	7	Monero	8	Monero	7	Litecoin	13	Ethereum	13	Lisk	12	Ethereum Classic	5
Panel C: Net-Degree															
NEM	7	Ethereum Classic	9	Ethereum	11	Ethereum	11	Bitcoin	15	Bitcoin	13	Augur	11	NEM	5
DigiByte	3	Decred	8	Decred	7	NEM	11	Ethereum	13	Decred	7	Decred	9	Waves	5
Ethereum	3	Waves	7	Ethereum Classic	4	Waves	5	Litecoin	12	Ethereum Classic	6	DigiByte	5	Dash	4
Litecoin	3	Augur	4	Litecoin	3	Lisk	4	Lisk	11	Lisk	6	Litecoin	4	Bitcoin	3
Stellar	3	Neo	4	NEM	3	Neo	4	Dash	8	Dash	4	Zcash	3	Dogecoin	3

Table 4

Descriptive Statistics
Extended Sample: 44 Cryptocurrencies

This table reports the descriptive statistics for the 44 cryptocurrencies in the sample. The market capitalisation (MC) is shown in million of dollars. The table presents the means, the standard deviations (SD), the minima (Min), the 5th percentile (5% VaR), the 10th percentile (10% VaR), the medians (Median), the 90th percentile (90% VaR), the 95th percentile (95% VaR), and the maxima (Max) for each cryptocurrency. The sample period is from November 2017 to December 2019.

Names	MC(in millions $)	Mean	SD	Min	5% VaR	10% VaR	Median	90% VaR	95% VaR	Max	Skewness	Kurtosis
Bitcoin	130446.00	0.001	0.042	-0.169	-0.066	-0.110	0.001	0.043	0.072	0.252	0.416	4.020
Ethereum	14139.77	0.000	0.051	-0.187	-0.081	-0.150	-0.001	0.056	0.085	0.265	0.075	2.642
XRP	8359.62	0.002	0.072	-0.298	-0.080	-0.144	-0.003	0.060	0.095	0.835	3.837	35.286
BitcoinCash	3723.64	0.000	0.070	-0.336	-0.105	-0.168	-0.003	0.065	0.106	0.512	1.221	8.831
Litecoin	2635.70	0.001	0.060	-0.191	-0.083	-0.130	-0.003	0.056	0.093	0.476	1.885	11.839
EOS	2443.24	0.003	0.072	-0.260	-0.099	-0.177	0.000	0.075	0.120	0.415	1.125	5.883
BinanceCoin	2135.15	0.005	0.065	-0.290	-0.081	-0.137	0.000	0.065	0.102	0.620	2.206	17.824
Tezos	938.73	0.002	0.077	-0.358	-0.103	-0.211	0.000	0.087	0.127	0.766	1.310	14.215
Stellar	906.96	0.002	0.068	-0.264	-0.093	-0.145	-0.004	0.069	0.107	0.587	1.891	11.953
TRON	887.25	0.007	0.104	-0.301	-0.108	-0.177	-0.002	0.084	0.125	1.196	5.328	52.643
Cardano	851.58	0.004	0.093	-0.251	-0.098	-0.148	-0.003	0.070	0.100	1.367	6.436	77.782
Monero	774.58	0.000	0.057	-0.228	-0.099	-0.148	-0.001	0.063	0.097	0.282	0.188	2.523
Chainlink	619.31	0.006	0.081	-0.272	-0.103	-0.180	-0.004	0.094	0.132	0.623	1.526	8.239
Neo	612.87	0.000	0.064	-0.233	-0.097	-0.150	-0.002	0.067	0.109	0.367	0.626	3.520
Ethereum Classic	523.06	0.000	0.059	-0.297	-0.095	-0.165	0.000	0.057	0.096	0.320	0.065	4.471
IOTA	445.10	0.000	0.068	-0.314	-0.098	-0.164	-0.002	0.067	0.102	0.468	1.148	8.238
Maker	430.96	0.002	0.063	-0.293	-0.089	-0.140	-0.001	0.068	0.099	0.582	1.531	12.884
Dash	380.82	-0.002	0.054	-0.195	-0.087	-0.153	-0.003	0.053	0.088	0.333	0.711	5.437
NEM	289.06	0.001	0.089	-0.303	-0.102	-0.168	-0.001	0.059	0.099	1.706	9.460	173.948
BasicAttentionToken	257.28	0.003	0.069	-0.298	-0.103	-0.171	0.000	0.084	0.118	0.316	0.294	2.203
Dogecoin	248.88	0.002	0.063	-0.308	-0.083	-0.151	-0.002	0.060	0.101	0.577	2.026	15.942
Zcash	230.20	-0.002	0.057	-0.210	-0.092	-0.146	-0.005	0.063	0.094	0.298	0.398	2.692
Decred	181.23	0.000	0.059	-0.225	-0.095	-0.144	-0.002	0.062	0.102	0.333	0.509	3.305
Qtum	155.02	0.000	0.077	-0.347	-0.102	-0.176	-0.003	0.065	0.099	0.751	2.374	20.005
Ox	109.91	0.002	0.071	-0.289	-0.103	-0.155	-0.004	0.078	0.122	0.404	0.848	3.572
Waves	101.72	0.000	0.063	-0.224	-0.093	-0.165	-0.002	0.067	0.093	0.466	1.032	7.516
Augur	98.35	0.001	0.073	-0.268	-0.100	-0.163	-0.002	0.070	0.113	0.922	3.052	34.436
BitcoinGold	92.77	-0.004	0.063	-0.290	-0.096	-0.167	-0.004	0.056	0.087	0.596	1.566	16.679
Nano	85.86	0.006	0.093	-0.306	-0.118	-0.182	-0.001	0.097	0.156	0.784	1.990	11.319
OmiseGO	85.01	-0.001	0.063	-0.248	-0.102	-0.161	-0.002	0.070	0.095	0.264	0.190	2.307
KuCoinShares	74.89	0.004	0.086	-0.245	-0.104	-0.176	0.000	0.080	0.133	0.962	3.606	32.702
Horizen	74.39	0.000	0.062	-0.230	-0.098	-0.156	-0.002	0.075	0.107	0.250	0.172	1.664
Lisk	67.62	-0.002	0.063	-0.244	-0.093	-0.153	-0.004	0.065	0.098	0.298	0.704	3.871
DigiByte	66.95	0.002	0.076	-0.286	-0.109	-0.168	-0.002	0.087	0.119	0.682	1.728	13.217
Bytom	65.82	0.002	0.081	-0.369	-0.104	-0.186	-0.002	0.075	0.122	0.938	2.664	27.646
MCO	63.57	0.002	0.076	-0.285	-0.101	-0.176	0.000	0.074	0.114	0.910	2.855	31.816
EnjinCoin	63.00	0.006	0.103	-0.333	-0.116	-0.171	-0.002	0.082	0.124	1.156	4.534	41.940
BitcoinDiamond	59.79	0.002	0.169	-0.692	-0.143	-0.323	-0.004	0.067	0.137	3.215	10.081	177.642
Komodo	58.59	0.000	0.067	-0.279	-0.103	-0.158	-0.001	0.077	0.110	0.437	0.586	3.784
ICON	58.19	0.000	0.083	-0.321	-0.124	-0.200	-0.002	0.081	0.123	0.592	1.298	8.248
Verge	55.73	0.004	0.101	-0.365	-0.118	-0.201	-0.003	0.079	0.121	1.158	3.806	34.086
Siacoin	54.37	0.001	0.079	-0.356	-0.108	-0.174	-0.002	0.067	0.114	0.794	2.322	18.813
Bytecoin	52.66	0.005	0.152	-0.314	-0.123	-0.191	-0.003	0.094	0.125	3.452	15.644	344.135
MonaCoin	50.17	0.001	0.094	-0.287	-0.093	-0.161	-0.007	0.052	0.093	1.046	5.302	45.958

Table 5

The Contemporaneous Total Tail Risk Connectedness of the Cryptocurrency Network Extended Sample Analysis: 44 Cryptocurrencies

This table presents the estimated degrees of tail risk connectedness in the network of 44 cryptocurrencies, computed as the total number of relevant spillover coefficients in the connectedness matrix $\boldsymbol{A}$ at the 1%, the 5%, the 10%, and the 20% tail thresholds. The thresholds at the left tails are the significance levels of the VaR in Eq. (1) while those at the right tails equals 100% minus the corresponding VaR significance levels. This means that the 1%, the 5%, the 10%, and the 20% right tail threshold are the 99%, the 95%, the 90%, and the 80% VaR thresholds, respectively.

	Right Tail	Left Tail	Difference
1% VaR	293	186	107
5% VaR	450	247	203
10% VaR	439	338	101
20% VaR	392	352	40

Table 6

The Contemporaneous Total Tail Risk Connectedness of the Cryptocurrency Network Rolling Window Analysis

This table presents the total number of connections in the tail risk network for each of the tail thresholds during every quarter from the first quarter of 2017 to the last quarter of 2019 estimated using Eq. (1). For each tail threshold, we report the total number of connections at both the left tails and the right tails along with their differences.

Time	# of Currencies	5% VaR			10% VaR			20% VaR			Average Skewness
		Right Tail	Left Tail	Difference	Right Tail	Left Tail	Difference	Right Tail	Left Tail	Difference	
2017Q1	21	38	25	13	52	32	20	38	23	15	1.79
2017Q2	23	36	60	-24	43	55	-12	61	50	11	2.21
2017Q3	29	101	59	42	72	50	22	75	46	29	0.91
2017Q4	38	151	83	68	92	52	40	110	89	21	1.92
2018Q1	44	123	86	37	163	47	116	135	97	38	0.95
2018Q2	52	206	84	122	257	97	160	268	130	138	0.64
2018Q3	55	225	213	12	182	142	40	243	130	113	0.68
2018Q4	58	272	195	77	205	123	82	193	195	-2	0.33
2019Q1	60	162	95	67	217	156	61	237	185	52	0.74
2019Q2	63	168	200	-32	214	166	48	192	125	67	1.11
2019Q3	67	262	293	-31	139	131	8	220	146	74	−0.06
2019Q4	69	234	351	-117	237	140	97	273	211	62	1.00

Table 7

The Predictive Total Tail Risk Connectedness of the Cryptocurrency Network Full Sample Static Analysis

This table presents the estimated degrees of tail risk connectedness in the network of 21 and 44 cryptocurrencies respectively, computed as the total number of relevant spillover coefficients in the connectedness matrix $\boldsymbol{A}$ at the 1%, the 5%, the 10%, and the 20% tail thresholds. The thresholds at the left tails are the significance levels of the VaR in Eq. (11) while those at the right tails equals 100% minus the corresponding VaR significance levels. This means that the 1% right tail threshold is, in fact, the 99% VaR

	21 Cryptocurrencies			44 Cryptocurrencies		
	Right Tail	Left Tail	Difference	Right Tail	Left Tail	Difference
1% VaR	30	11	19	147	75	72
5% VaR	100	32	68	217	86	131
10% VaR	86	25	61	152	73	79
20% VaR	53	15	38	96	45	51

Table 8

The Predictive Total Tail Risk Connectedness of the Cryptocurrency Network
Rolling Window Analysis

This table presents the total number of connections in the tail risk network for each of the tail thresholds during every quarter from the first quarter of 2017 to the last quarter of 2019 estimated using Eq. (11). For each tail threshold, we report the total number of connections at both the left tails and the right tails along with their differences.

Time	# of Currencies	5% VaR			10% VaR			20% VaR			Average Skewness
		Right Tail	Left Tail	Difference	Right Tail	Left Tail	Difference	Right Tail	Left Tail	Difference	
2017Q1	21	10	5	5	23	19	4	17	7	10	1.79
2017Q2	23	31	20	11	53	35	18	39	22	17	2.21
2017Q3	29	30	21	9	31	24	7	22	14	8	0.91
2017Q4	38	84	51	33	60	42	18	58	27	31	1.92
2018Q1	44	48	27	21	46	17	29	19	17	2	0.95
2018Q2	52	66	30	36	73	26	47	45	34	11	0.64
2018Q3	55	100	82	18	92	39	53	39	31	8	0.68
2018Q4	58	80	104	-24	84	52	32	64	50	14	0.33
2019Q1	60	69	22	47	78	38	40	56	39	17	0.74
2019Q2	63	102	102	0	122	84	38	68	53	15	1.11
2019Q3	67	153	70	83	106	29	77	109	36	73	−0.06
2019Q4	69	131	173	-42	117	80	37	106	84	22	1.00

Table 9

Buy-and-Hold Portfolio Performance at Different Holding Periods

This table reports the average holding period returns for the naïve, equal-weighted portfolio and the mean-CVaR portfolio, optimised at the 5% VaR threshold, at the different investment horizons, ranging from one month to twelve months. The table also reports the differences in the average returns as well as the corresponding t-statistics calculated using Newey & West (1987) standard errors to account for overlapping holding period returns.

Holding period	Equal-Weighted	Mean-CVaR (5% VaR)	Difference	t-statistics
1 month	38.293	38.519	−0.226	−0.109
2 months	85.737	66.891	18.846	1.368
3 months	139.629	129.704	9.924	1.287
4 months	182.245	133.459	48.785	1.389
5 months	180.824	147.527	33.297	1.227
6 months	178.688	161.935	16.753	0.719
7 months	236.711	186.618	50.093	0.941
8 months	789.454	294.879	494.576	1.058
9 months	450.558	262.750	187.808	1.084
10 months	367.009	231.507	135.503	1.096
11 months	270.473	148.155	122.318	1.166
12 months	406.685	184.025	222.660	1.169

Table 10

Portfolio Performance when Portfolios are Rebalance Periodically

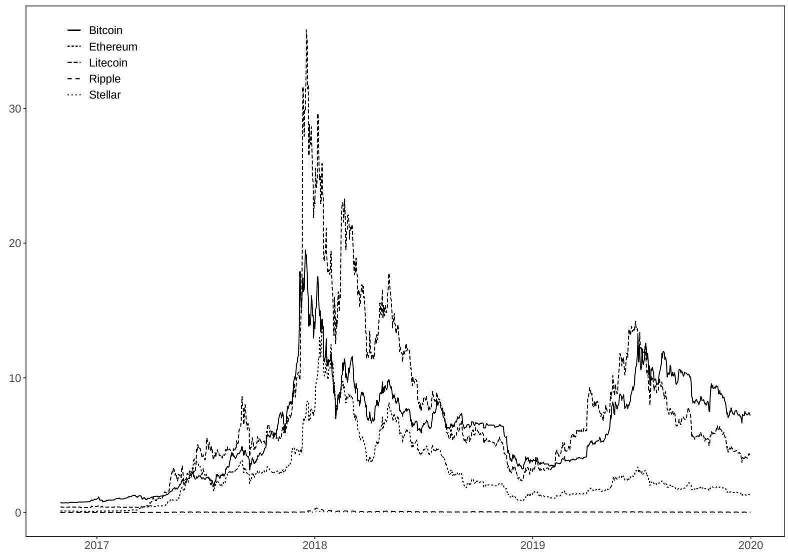
This table reports the performance of a hypothetical equal-weighted portfolio and a hypothetical mean-CVaR portfolio, optimised at the 5% VaR threshold, with $1 of intial investment each at the beginning of May 2017. Both portfolios are periodically rebalanced based on their weighting strategy. While the naïve portfolio is rebalanced to maintain the equal weight of $1/n$ for each currency at the beginning of each rebalancing period, the mean-CVaR optimal weights are estimated using historical returns realised during the previous six months prior to the portfolio formation regardless of the rebalancing frequency. For each of the balancing frequency of 1, 3, and 6 months, we report the total returns, the average returns, portfolio volatility, CVaR, and the two (annualised) Sharpe ratios calculated based on the volatility and the CVaR measures.

	1 Month		3 Months		6 Months	
	Equal-Weighted	Mean-CVaR	Equal-Weighted	Mean-CVaR	Equal-Weighted	Mean-CVaR
			Transaction cost = 0 bps			
Total return (%)	4,959.086	6,121.068	4,882.809	4,500.913	3,521.085	3,735.224
Average return (%)	0.576	0.586	0.596	0.564	0.575	0.566
Volatility (%)	5.916	5.726	6.284	5.890	6.465	6.235
CVaR (%)	-12.588	-12.212	-13.414	-12.491	-14.011	-13.357
Sharpe ratio	1.840	1.935	1.793	1.809	1.682	1.715
CVaR Sharpe ratio	0.865	0.907	0.840	0.853	0.776	0.801
			Transaction cost = 10 bps			
Total return (%)	4,913.298	3,610.819	4,853.566	274.817	3,505.563	269.728
Average return (%)	0.575	0.708	0.595	0.337	0.575	0.333
Volatility (%)	5.915	8.442	6.285	6.445	6.465	6.378
CVaR (%)	-12.588	-15.815	-13.416	-13.275	-14.011	-13.023
Sharpe ratio	1.837	1.588	1.791	0.982	1.681	0.980
CVaR Sharpe ratio	0.863	0.848	0.839	0.477	0.776	0.480
			Transaction cost = 25 bps			
Total return (%)	4,845.409	488.607	4,810.031	306.066	3,482.408	307.332
Average return (%)	0.573	0.395	0.594	0.348	0.574	0.350
Volatility (%)	5.915	6.602	6.285	6.481	6.465	6.496
CVaR (%)	-12.589	-13.097	-13.420	-13.052	-14.011	-13.098
Sharpe ratio	1.832	1.125	1.788	1.010	1.679	1.011
CVaR Sharpe ratio	0.861	0.567	0.838	0.501	0.775	0.501
			Transaction cost = 50 bps			
Total return (%)	4,734.344	486.966	4,738.339	307.611	3,444.161	304.891
Average return (%)	0.571	0.396	0.593	0.349	0.573	0.349
Volatility (%)	5.915	6.628	6.286	6.483	6.466	6.499
CVaR (%)	-12.591	-13.118	-13.425	-13.056	-14.011	-13.099
Sharpe ratio	1.825	1.124	1.784	1.011	1.675	1.009
CVaR Sharpe ratio	0.857	0.568	0.835	0.502	0.773	0.501

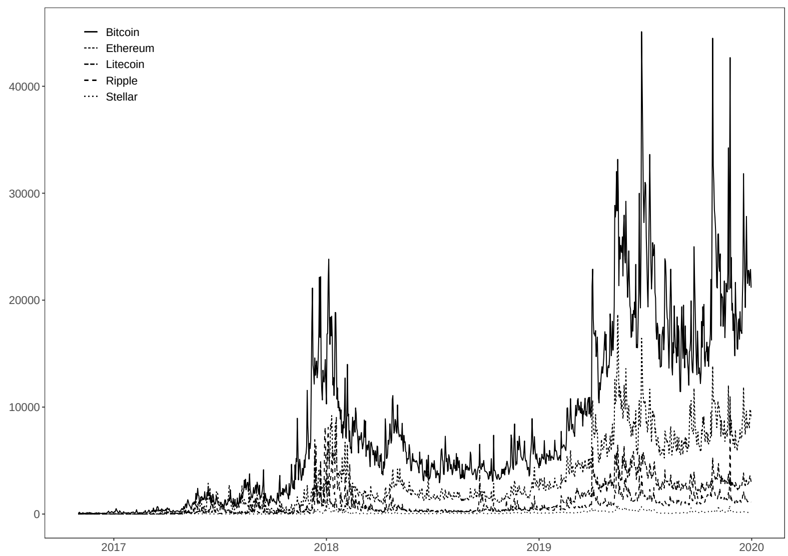
Figure 1

Time Series of Prices And Trading Volumes

This figure shows the time series of prices (in US dollars) and trading volumes (in millions of US dollars) for Bitcoin, Ethereum, Litecoin, Ripple, and Stella during November 2016 to December 2019. The prices are scaled as follows: the price of Bitcoin is divided by 1,000, the price of Ethereum is divided by 100, the price of Ripple is multiplied by 10,the price of Litecoin is divided by 10, and the price of Stella is divided by 10.



(a) Price

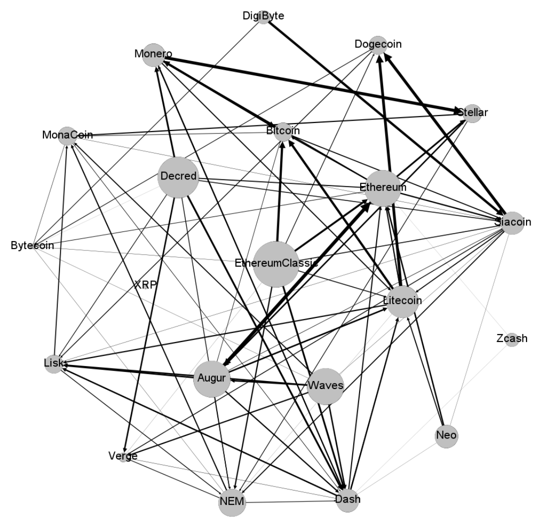


(b) Volume

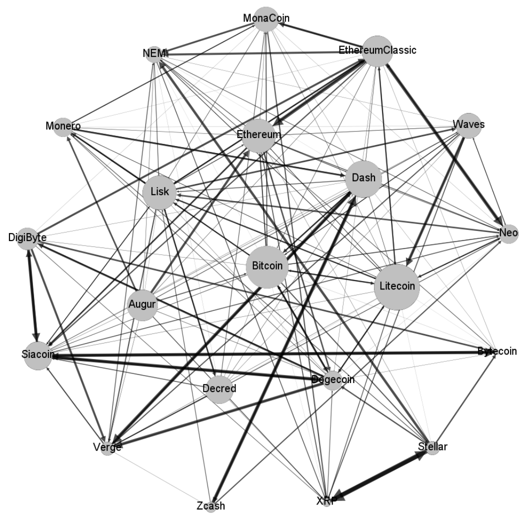
Figure 2

The Tail Connectedness Network at the 5% and 95% VaR Thresholds

This figure shows the tail risk connectedness network of 21 cryptocurrencies at the 5% VaR and the 95% VaR thresholds. Each node in the figure depicts a cryptocurrency, whose out-degree is signified by the size of the node. The directional connection from one currency to another is illustrated by an arrow going from the source cryptocurrency to the target cryptocurrency. The width of the arrow is proportional to the magnitude of the spillover coefficient for the pair of the cryptocurrencies in the connectedness matrix.



(a) 5 VaR%

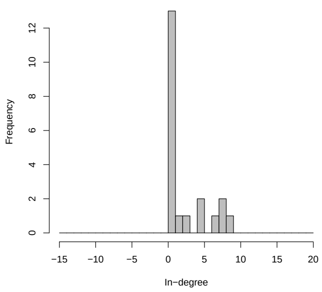


(b) 95 VaR%

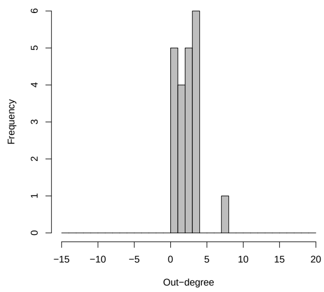
Figure 3

The Distributions of the Left Tail Spillovers

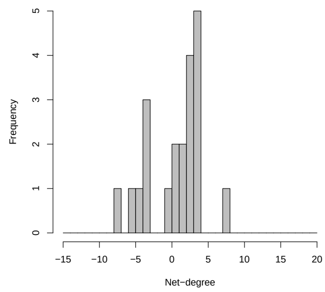
The histograms below show the distributions of the in-degrees, the out-degrees, and the net-degrees of all the 21 cryptocurrencies in the network at the left tails at the 1%, the 5%, the 10%, and the 20% tail thresholds.



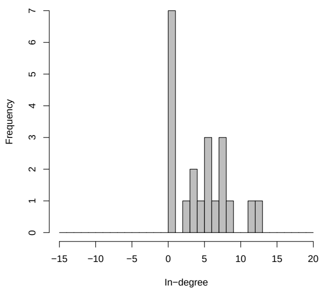
(a) 1% VaR in-degree



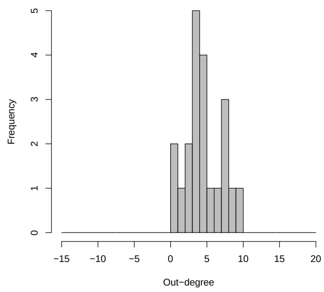
(b) 1% VaR out-degree



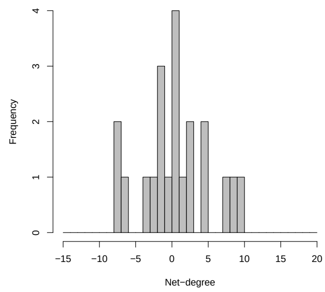
(c) 1% VaR net-degree



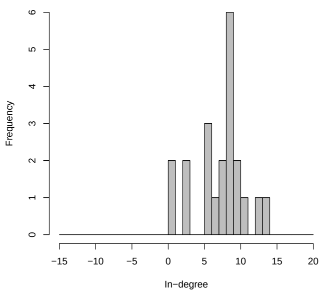
(d) 5% VaR in-degree



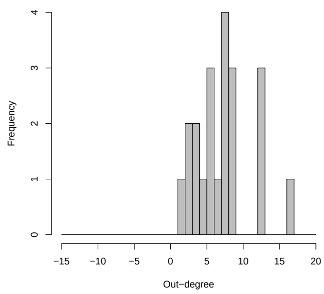
(e) 5% VaR out-degree



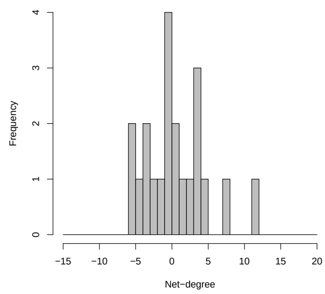
(f) 5% VaR net-degree



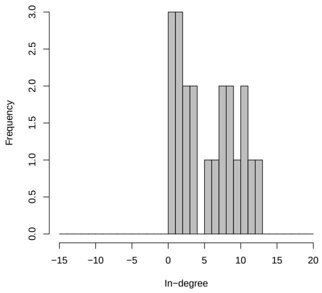
(g) 10% VaR in-degree



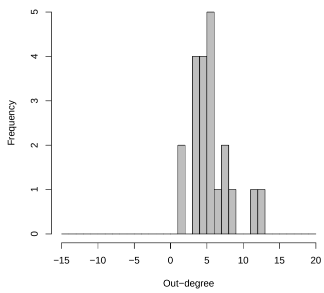
(h) 10% VaR out-degree



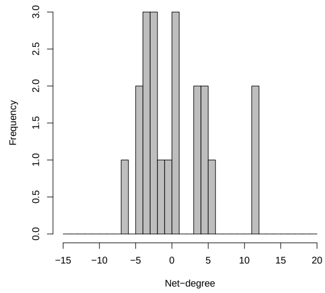
(i) 10% VaR net-degree



(j) 20% VaR in-degree



(k) 20% VaR out-degree

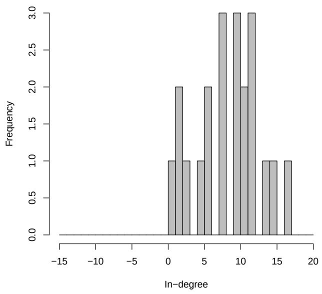


(l) 20% VaR net-degree

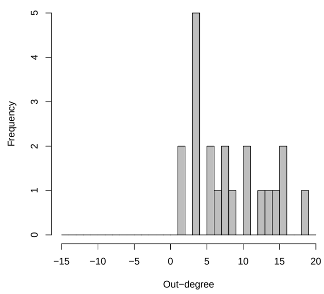
Figure 4

The Distributions of the Right Tail Spillovers

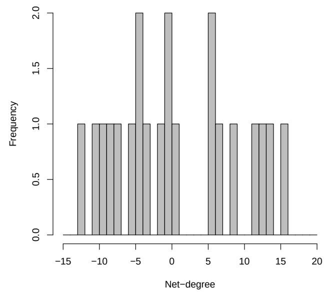
The histograms below show the distributions of the in-degrees, the out-degrees, and the net-degrees of all the 21 cryptocurrencies in the network at the right tails at the 1%, the 5%, the 10%, and the 20% tail thresholds.



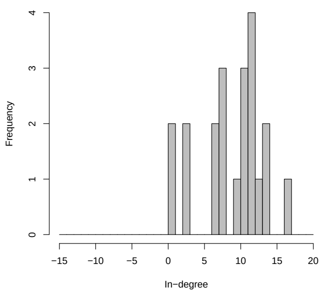
(a) 80% VaR in-degree



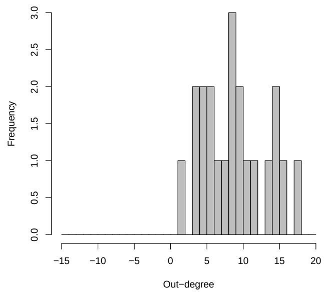
(b) 80% VaR out-degree



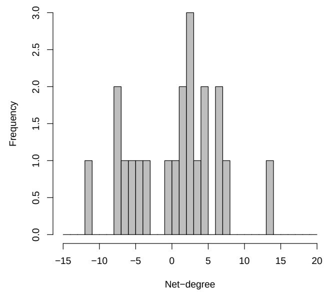
(c) 80% VaR net-degree



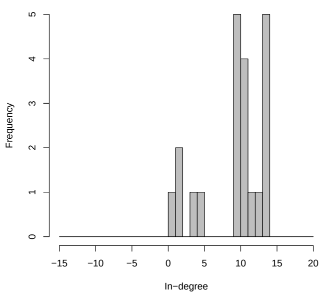
(d) 90% VaR in-degree



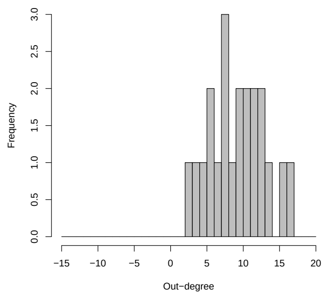
(e) 90% VaR out-degree



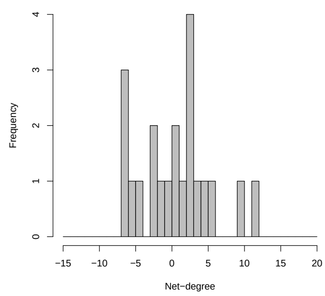
(f) 90% VaR net-degree



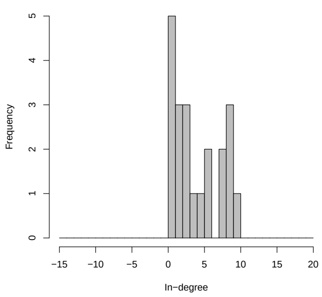
(g) 95% VaR in-degree



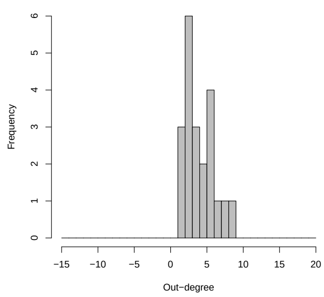
(h) 95% VaR out-degree



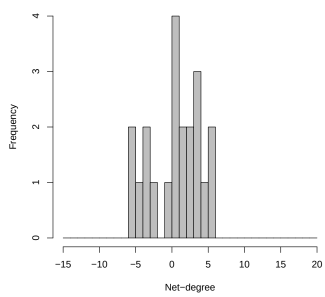
(i) 95% VaR net-degree



(j) 99% VaR in-degree



(k) 99% VaR out-degree

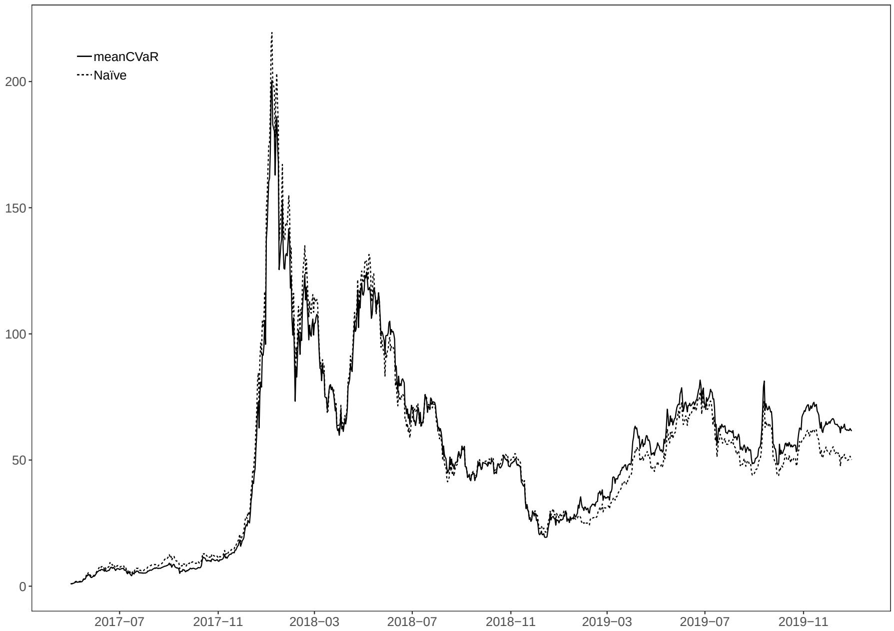


(l) 99% VaR net-degree

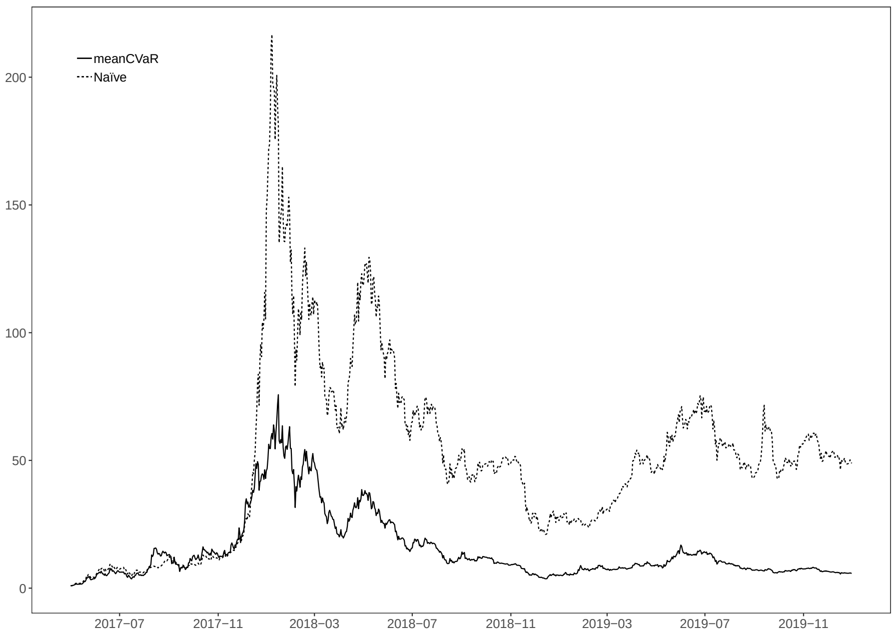
Figure 5

Portfolio Performance Over Time

The figure compares the value of the equal-weighted portfolio to the value of the mean-CVaR portfolio over time between May 2017 and December 2019. Panel A shows the portfolio values assuming no transaction cost while Panel B shows the portfolio values assuming a charge of 25 bps per trade.



(a) Transaction Cost = 0 bps



(b) Transaction Cost = 25 bps

A In-Degrees, Out-Degrees, and Net-Degrees at the Different VaR Thresholds

Table A1

In-Degrees at the Different VaR Thresholds

This table shows the details of in-degrees of all the 21 cryptocurrencies in the system at the various tail thresholds. The thresholds at the left tails are the significance levels of the VaR in Eq. (1) while those at the right tails equal 100% minus the corresponding VaR significance levels. This means that the 1% right tail threshold is, in fact, the 99% VaR.

No˙	Cryptocurrency	Left Tail				Right Tail			
		1%	5%	10%	20%	20%	10%	5%	1%
1	Augur	6	3	7	2	7	6	0	7
2	Bitcoin	2	5	7	7	0	2	13	4
3	Bytecoin	0	8	10	8	11	0	9	2
4	Dash	0	11	8	7	2	7	11	0
5	Decred	0	0	0	8	7	0	1	1
6	DigiByte	0	0	6	1	9	10	3	8
7	Dogecoin	8	3	5	5	10	13	13	0
8	Ethereum	0	7	5	0	5	13	10	3
9	Ethereum Classic	7	0	8	9	1	2	9	9
10	Lisk	0	6	5	0	4	11	13	8
11	Litecoin	0	7	9	10	1	10	12	5
12	MonaCoin	0	5	2	3	14	12	9	2
13	Monero	7	4	8	10	7	9	9	1
14	NEM	0	7	9	1	16	16	13	0
15	Neo	4	0	8	1	10	10	10	0
16	Siacoin	0	12	12	12	11	7	13	5
17	Stellar	0	2	8	3	13	7	4	2
18	Verge	4	5	2	2	9	11	10	8
19	Waves	0	0	13	0	9	11	9	1
20	XRP	0	0	8	6	11	11	10	7
21	Zcash	1	0	0	11	5	6	1	0

Table A2

Out-Degrees at the Different VaR Thresholds

This table shows the details of out-degrees of all the 21 cryptocurrencies in the system at the various tail thresholds. The thresholds at the left tails are the significance levels of the VaR in Eq. (1) while those at the right tails equal 100% minus the corresponding VaR significance levels. This means that the 1% right tail threshold is, in fact, the 99% VaR.

No˙	Cryptocurrency	Left Tail				Right Tail			
		1%	5%	10%	20%	20%	10%	5%	1%
1	Augur	1	7	5	5	6	8	11	1
2	Bitcoin	2	3	4	4	15	15	15	7
3	Bytecoin	0	0	6	3	1	1	2	2
4	Dash	2	4	2	7	10	11	13	4
5	Decred	2	8	7	3	7	7	10	2
6	DigiByte	3	2	5	4	5	6	8	3
7	Dogecoin	0	3	7	5	5	8	7	3
8	Ethereum	3	7	16	11	18	13	12	5
9	Ethereum Classic	3	9	12	5	7	8	11	5
10	Lisk	1	3	5	4	15	17	12	8
11	Litecoin	3	6	12	6	13	14	16	2
12	MonaCoin	2	3	3	1	3	5	9	1
13	Monero	1	4	8	7	12	10	7	4
14	NEM	7	5	12	12	3	4	6	5
15	Neo	0	4	7	5	1	4	7	2
16	Siacoin	1	4	8	8	10	9	10	5
17	Stellar	3	3	7	3	8	9	5	2
18	Verge	0	1	1	1	3	3	5	2
19	Waves	3	7	8	5	14	14	9	6
20	XRP	2	0	2	3	3	3	3	3
21	Zcash	0	2	3	4	3	5	4	1

Table A3

Net-Degrees at the Different VaR Thresholds

This table shows the details of net-degrees of all the 21 cryptocurrencies in the system at the various tail thresholds. The thresholds at the left tails are the significance levels of the VaR in Eq. (1) while those at the right tails equal 100% minus the corresponding VaR significance levels. This means that the 1% right tail threshold is, in fact, the 99% VaR.

No˙	Cryptocurrency	Left Tail				Right Tail			
		1%	5%	10%	20%	20%	10%	5%	1%
1	Augur	-5	4	-2	3	-1	2	11	-6
2	Bitcoin	0	-2	-3	-3	15	13	2	3
3	Bytecoin	0	-8	-4	-5	-10	1	-7	0
4	Dash	2	-7	-6	0	8	4	2	4
5	Decred	2	8	7	-5	0	7	9	1
6	DigiByte	3	2	-1	3	-4	-4	5	-5
7	Dogecoin	-8	0	2	0	-5	-5	-6	3
8	Ethereum	3	0	11	11	13	0	2	2
9	Ethereum Classic	-4	9	4	-4	6	6	2	-4
10	Lisk	1	-3	0	4	11	6	-1	0
11	Litecoin	3	-1	3	-4	12	4	4	-3
12	MonaCoin	2	-2	1	-2	-11	-7	0	-1
13	Monero	-6	0	0	-3	5	1	-2	3
14	NEM	7	-2	3	11	-13	-12	-7	5
15	Neo	-4	4	-1	4	-9	-6	-3	2
16	Siacoin	1	-8	-4	-4	-1	2	-3	0
17	Stellar	3	1	-1	0	-5	2	1	0
18	Verge	-4	-4	-1	-1	-6	-8	-5	-6
19	Waves	3	7	-5	5	5	3	0	5
20	XRP	2	0	-6	-3	-8	-8	-7	-4
21	Zcash	-1	2	3	-7	-2	-1	3	1

B Robustness Check: the Impact of λ

In this appendix, we examine the robustness of our main results when the value of the penalty parameter λ^i in Eq. (4) is determined using alternative methods. In the first robustness check, we "cross-validate" the value of λ^i by replacing λ^i, the penalty parameter of currency i, with the average of the optimised λ values calculated for all other currencies in the network. This approach allows us to investigate if using sub-optimum values of λ values in the LASSO quantile regression models has any significant impacts on our main findings – at least, qualitatively.

As can be seen in Table B1 which summarises the tail connectedness of the network of 21 cryptocurrencies, the robustness results remain qualitatively similar to our main results: we observe smaller degrees of connectedness at more extreme tail thresholds and the dominance of the right tail connectedness over the left tail counterpart. The numbers of right tail connections are noticeably higher than the numbers of the left tail connections at all the VaR levels – except at the 1% VaR where the number of connections at the left tail is only marginally higher than that at the right tail.

In our second robustness exercise, we examine the sensitivity of our main results when the same λ values are used for all the currencies in the sample in the estimation of the LASSO quantile regression models. To identify a set of λ values which can be best used across all the currencies, we do the following.

Step 1. We compute all the optimum values of λ for all the 21 cryptocurrencies in the sample at the 1%, 5%, 10%, and 20% VaR significance levels for both the left and the right tails. This step gives us a total of $21 \times 8 = 168$ values of λ.

Step 2. Using each of the 168 values of λ calculated in Step 1 as the common λ value, we estimate the LASSO quantile regression models for all the 21 currencies at the 1%, 5%, 10%, and 20% VaR significance levels. We therefore estimate a total of $21 \times 8 \times 168 = 28{,}224$ models with all the possible combinations of cryptocurrencies, the VaR significance levels, and the values of λ.

Step 3. We backtest all the 28,224 estimated LASSO quantile regression models using the log likelihood ratio test discussed in Hautsch et al. (2015).

Figure B1 plots, for each value of λ, the number of estimations that the log likelihood ratio test of Hautsch et al. (2015) fails to reject the null hypothesis that the VaR exceedance is identically and independently distributed following the Bernoulli distribution with the success probability of q (i.e., they are models with good VaR estimations). According to the plot in Figure B1, the reasonable λ values to be used in the LASSO quantile regression for all the currencies are found to lie within a range of 17 and 20 where the numbers of cases that the log likelihood ratio test fails to reject the null hypothesis peak. We therefore report results for the networks of 21 cryptocurrencies estimated using the λ values of 17, 18, 19, and 20 in Table B2. It can be seen that the robustness results and the main results, reported in Table 2, are qualitatively identical. In all four cases, we observe stronger tail risk connectedness at the less extreme tail threshold levels. The right tail connectedness is also found to dominate the left tail counterpart in all four networks.[25]

[25] We also obtain qualitatively similar results when applying the above robustness checks to the extended network of 44 currencies, the rolling window analysis, and the predictive LASSO quantile regression analysis. These results are available upon request.

Table B1

Cross Validation of the Robustness of the Penalty Parameter λ

This table shows the LASSO quantile regression results when the value of λ^i, the penalty parameter for currency i, is replaced with the average of the optimised λ values computed for all other currencies in the network.

	Right tail	Left tail	Difference
1% VaR	70	78	-8
5% VaR	180	104	76
10% VaR	166	155	11
20% VaR	157	129	28

Table B2

The Contemporaneous Total Tail Risk Connectedness of the Cryptocurrency Network for Different Values of λ
Full Sample Static Analysis: 21 Cryptocurrencies

This table reports the degree of tail risk connectedness results for the networks of 21 cryptocurrencies estimated using the λ values of 17, 18, 19, and 20.

	$\lambda = 17$			$\lambda = 18$			$\lambda = 19$			$\lambda = 20$		
	Right Tail	Left Tail	Difference	Right Tail	Left Tail	Difference	Right Tail	Left Tail	Difference	Right Tail	Left Tail	Difference
1% VaR	125	104	21	120	99	21	112	100	12	111	101	10
5% VaR	243	197	46	239	194	45	237	193	44	234	190	44
10% VaR	254	204	50	250	209	41	249	206	43	249	208	41
20% VaR	265	218	47	264	216	48	261	217	44	255	216	39

Figure B1

The Performance of the LASSO Quantile Regression Model at the Different Values of λ

The figure plots, for each value of λ, the number of estimations that the log likelihood ratio test of Hautsch et al. (2015) fails to reject the null hypothesis that the VaR exceedance is identically and independently distributed following the Bernoulli distribution with the success probability of q (i.e., they are models with good VaR estimations).