Article:

Dai, Y., Susmel, L. orcid.org/0000-0001-7753-9176 and Qin, F. (2020) Sharp V-notches in viscoplastic solids: strain energy rate density rule and fracture toughness. Fatigue and Fracture of Engineering Materials and Structures. ISSN 1460-2695

https://doi.org/10.1111/ffe.13319

Sharp V-notches in viscoplastic solids: strain energy rate density rule and fracture toughness

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Abstract

Different from Neuber's rule or Glinka's energy method which are always adopted to characterize the notch tip field under elastoplastic condition, in this paper the strain energy rate density (SERD) rule is used for viscoplastic materials. In particular, based on the definition of generalized notch stress intensity factor (G-NSIF) for sharp V-notch in viscoplastic solids, the concept of SERD for sharp V-notch in viscoplastic solids is presented. Subsequently, by taking as a starting point the SERD, the averaged strain energy density (SED) for sharp V-notch in viscoplastic solids is derived with integration of time. The fracture toughness relation between sharp V-notch specimens and crack specimen in viscoplastic materials is given based on the transformation of SERD. A numerical approach is presented to compute the SERD and SED based on finite element method. Some crucial comments on the G-NSIF have been discussed. Some typical solutions for SERD and SED for sharp V-notched specimens are investigated.

Keywords: Strain energy rate density; Fracture toughness; Sharp V-notch; Viscoplastic; Generalized notch stress intensity factor.

1 Introduction

Due to the stress concentration and high stress gradient around the notch tip, cracking easily occurs around the notch tip region for materials and structures under viscoplastic conditions which becomes a crucial issue under high temperature. Fracture assessments of notched component in linear-elastic materials and plastic solids have drawn a lot of attentions in available literatures. Some theories have been developed to evaluate and predict the fracture behavior of notched components in linearly elastic or plastic materials [1-6]. However, as far as the authors' knowledge, theories to characterize the fracture strength of sharp V-notch in viscoplastic solids still need to be developed thoroughly.

The most common method to evaluate the stress field of sharp V-notch in viscoplastic materials or creeping materials is based on Neuber's rule [7]. Note that the solutions with Neuber's rule is considered to be an upper bound solution to predict the stress and strain of a notch. Different from Neuber's rule, Molski and Glinka [8] presented an energy-based method which is always adopted to be an alternative method compared with that of Neuber's method. Since then, some studies have been performed based on those two methods which have been discussed by Moftakhar et al. [9], by Hyde et al. [10] and by Nuñez and Glinka [11].

Lately, the strain energy density (SED) method or averaged SED presented by Lazzarin, Berto and coworkers [12-14] have been extended to the stress and strain evaluation for sharp and blunted notches in creeping solids by Gallo and coworkers [15-17] with the concept of plastic zone correction. The concept of SED has been successfully applied to many conditions for the notch problem which has been originally developed for linearly elastic or elastoplastic solids [18, 19]. Note that this method is directly introduced to non-localized creep stresses ahead of notches from the concept of averaged SED, which is generally valid for elastic or plastic notch stress and strain analyses. It needs a sequence of numerical computations based on analysis of strain and stress increment with creep time under viscoplastic condition.

Different from those methods based on Neuber's rule or energy-based method, some researchers are trying to solve the stress and strain characteristics of notch tip in viscoplastic solids through asymptotic analysis. For example, the asymptotic solution given by Bassani [20] for the notch tip was solved under mode III loading with hyperbolic-sine-law creeping law. The enclosed contour integral method was proposed by Zhu et al. [21] for sharp V-notch in power-law creeping materials. Recently, Dai et al. [22] reported an asymptotic method to evaluate the tip field of sharp V-notch in power-law creep solids, i.e. viscoplastic materials, and a notch stress intensity factor was recently discussed by Dai et al. [23] which was presented to characterize the stress intensity of a notch tip obeying power-law creep.

In this paper, the concept of strain energy rate density (SERD) is presented to characterize the sharp V-notch tip field in viscoplastic solids. With the extension of the concentration factor of SED defined for viscoplastic solids based on the concept of SED developed by Lazzarin and coworkers [12-14], the framework of SERD is presented with the theoretical analysis and numerical computation. A relation of fracture toughness between sharp V-notch and crack in viscoplastic solids is also given based on the concept of SERD. With the extension of the SERD, the averaged strain energy density (SED) for sharp V-notch in viscoplastic materials can be obtained directly, where this method is rather different from traditional methods given by Neuber's rule [7] or energy based method by Glinka [8]. The applications of Neuber's rule or energy-based method by Glinka to creep notch problem need a series of calculations for strain and stress increments or plastic zone correction. However, the method presented in this paper does not need increment calculations of strain or stress calculations which can be computed based on the SERD.

The aim of this investigation is to present the concept of SERD and fracture toughness correction for sharp V-notch in viscoplastic solids or creeping solids. To this end, the structure of this paper is organized as following. The basic equations and generalized notch stress intensity factor for sharp V-notch under viscoplastic condition are given in Section 2 and Section 3, respectively. The definition and discussion on the SERD are presented in Section 4. The fracture toughness correction is given in Section

5. A numerical scheme to compute SERD of sharp V-notch in viscoplastic solids are studied in Section 6. The results and discussion are given in Section 7. The conclusions are drawn in the last section.

2 Basic equations of the viscoplastic material

A typical viscoplastic material with the following form is adopted. This kind of constitutive equation is also usually named as power-law creep equation.

$$\dot{\varepsilon}_{ij} = \frac{1+v}{E}\dot{S}_{ij} + \frac{1-2v}{E}\dot{\sigma}_{kk}\delta_{ij} + \frac{3}{2}\dot{\varepsilon}_0\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\frac{S_{ij}}{\sigma_0} \tag{1}$$

where the quantities with dot form represent the differentiative of time. E, $\dot{S}_{ij}$, n, v, $\dot{\sigma}_{ij}$ and $\dot{\varepsilon}_{ij}$ are the Young's modulus, deviatoric stress, creep exponent, Poisson's ratio, stress tensor and strain tensor, respectively. σ_e is the von Mises equivalent stress which is calculated as following.

$$\sigma_e = \sqrt{\frac{3}{2}S_{ij}S_{ij}} \tag{2}$$

In fact, coefficients of Eq. (1) are dependent on the specific material as well as various temperatures. For example, the material constants for Inconel 800H at 649°C adopted from Chao et al. [24] have been listed in Table 1. The reference stress σ_0, creep coefficient $\dot{\varepsilon}_0/\sigma_0^n$, creep exponent n, Young's modulus E, references strain rate $\dot{\varepsilon}_0$ and Poisson's ratio v are 416.8 MPa, $1.352\times10^{-16}\mathrm{MPa}^{-n}h^{-1}$, 5, 153606 MPa, $1.70065\times10^{-3}h^{-1}$ and 0.33, respectively.

Table 1 Material properties for a viscoplastic material used in this paper

Creep exponent	Young's modulus	Reference stress	Poisson's ratio	Creep coefficient	Reference strain rate
n=5	153606 MPa	416.8 MPa	0.33	$1.352\times10^{-16}\,\mathrm{MPa}^{-5}h^{-1}$	$1.70065\times10^{-3}\,h^{-1}$

3 Generalized notch stress intensity factor and tip field for sharp V-notch

For typical sharp V-notch specimens such as single edge V-notched (SEVN) specimen and single edge V-notch bending specimen (SEVB) shown in Fig. 1, the stress field of the apex of the notch is expressed as following according to the recent investigations reported by Dai et al. [22, 23].

$$\dot{\sigma}_{ij}(r,\theta) = \sigma_0 \left(\frac{K(t)}{\sigma_0 \dot{\varepsilon}_0 r} \right)^{-s} \dot{\tilde{\sigma}}_{ij}(n,\alpha,\theta) \tag{3}$$

in which $K(t)$, s and $\tilde{\sigma}_{ij}(n,\alpha,\theta)$ are the amplitude factor, eigenvalue and dimensionless stress distribution function, respectively. Those eigenvalues and dimensionless stress distribution function have been solved with asymptotic analysis, and some typical solutions have been listed in Dai et al. [22, 23] which is not repeated here. It should be emphasized that $\dot{\tilde{\sigma}}_{ij}(n,\alpha,\theta)$ is only dependent on the asymptotic solutions when the viscoplastic behaviors are under large extent. Herein, $K(t)$ defined for sharp V-notch is analogy to $C(t)$-integral [25] defined for crack front in viscoplastic solids, and $K(t)$ is identical to $C(t)/I_n$ when notch angle is zero. I_n is an integral constant which depends on the stress state of the creep crack front. If the extensive viscoplastic behavior occurs for a sharp V-notch, the singular exponent, s, can be approximated by the following formula.

$$s = \frac{2(1-\lambda)}{n+1} \tag{4}$$

where λ and n are the singular eigenvalues for sharp V-notch in linearly elastic solids [25] and viscoplastic exponent defined in Eq. (1), respectively. Although this relation is easily found in tip field of elastoplastic solids [13, 26, 27], however, the relation is first time to be confirmed here for viscoplastic materials and the detail comparison is presented in Fig. 2.

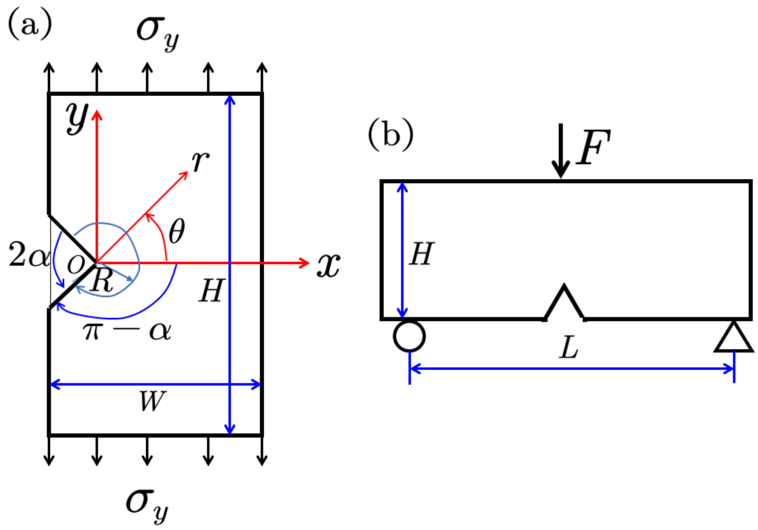

Fig. 1 Configuration of sharp V-notch specimens: (a) single edge V-notched (SEVN) specimen and (b) single edge V-notch bending (SEVB) specimen

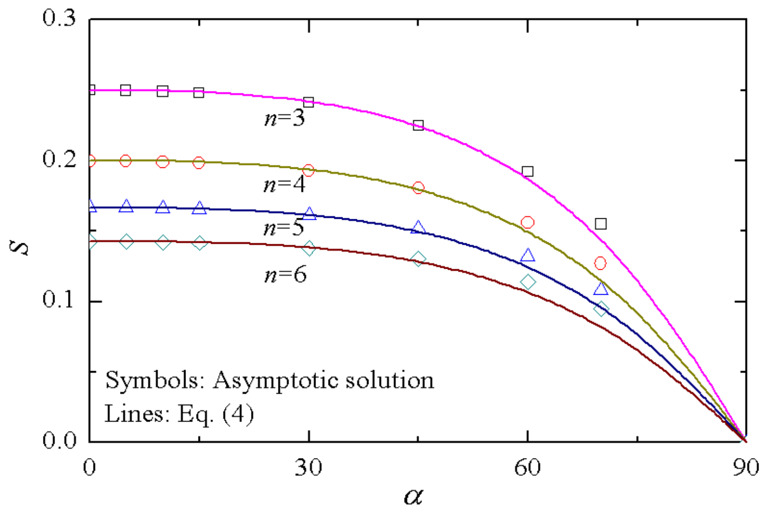

Fig. 2 Comparisons of eigenvalue s with asymptotic solution and Eq. (4) for viscoplastic V-notch

Herein, the generalized notch stress intensity factor for a sharp V-notch is defined as below based on Eq. (3).

$$\dot{\sigma}_{ij}(r,\theta) = K_{\mathrm{N}}(t) r^{-s} \dot{\tilde{\sigma}}_{ij}(n,\alpha,\theta) \tag{5}$$

$$\dot{\sigma}_{e}(r,\theta) = K_{\mathrm{N}}(t) r^{-s} \dot{\tilde{\sigma}}_{e}(n,\alpha,\theta) \tag{6}$$

in which $K_{\mathrm{N}}(t)$ and $\tilde{\sigma}_e(n,\alpha,\theta)$ are the generalized notch stress intensity factor (G-NSIF) and dimensionless equivalent stress, respectively. The condition discussed in current paper is defined for viscoplastic materials under mode I loading. It is noted that $K_{\mathrm{N}}(t)$ should be dependent on the extent of the behavior of viscoplasticity ahead of notch tip. The relation between $K_{\mathrm{N}}(t)$ and $K(t)$ is expressed as following.

$$K_N(t)=\sigma_0\left(\frac{K(t)}{\sigma_0\dot{\varepsilon}_0}\right)^{-s} \tag{7}$$

For viscoplasticity bearing long time, the generalized notch stress intensity factor $K_{\mathrm{N}}(t)$ becomes $K_{*\mathrm{N}}$, i.e. constant. In order to denote the variations of time between $K_{\mathrm{N}}(t)$ and $K_{*\mathrm{N}}$, a critical time t_{CV} is defined as below [25]:

$$t_{CV}=t_{\mathrm{red}}=\frac{(1-v)K_{\mathrm{I}}^2}{(n+1)EC^*} \tag{8}$$

where t_{red} and C^* are the redistribution time and C^*-integral of a crack front with the same geometry size as that of notch front. Based on the definition of Eq. (8), $K_{\mathrm{N}}(t)$ becomes $K_{*\mathrm{N}}$ when the time exceeds t_{red}. A following relation is proposed to characterize the ratio between $K_{\mathrm{N}}(t)$ and $K_{*\mathrm{N}}$.

$$\frac{K_N(t)}{K_{*\mathrm{N}}}=\frac{(1+t/t_{CV})^{n+1}}{(1+t/t_{CV})^{n+1}-1} \tag{9}$$

This formulation of Eq. (9) is similar to the relation that of $C(t)/C^*$ for creep crack [28, 29]. Different from Eq. (9), a new form to describe the relation between $K_{\mathrm{N}}(t)$ and $K_{*\mathrm{N}}$ is given as:

$$\frac{K_N(t)}{K_{*\mathrm{N}}}=(1+t_{\mathrm{red}}/t)^{\frac{2}{n+1}} \tag{10}$$

4 Strain energy rate density theory for sharp V-notch in viscoplastic solids

4.1 Definition of strain energy rate density

The concept of strain energy rate density (SERD) is defined as below:

$$\dot{W}=\int^{\dot{\varepsilon}_{ij}}\sigma_{ij}\mathrm{d}\dot{\varepsilon}_{ij} \tag{11}$$

where σ_{ij} and $\dot{\varepsilon}_{ij}$ are the stress tensor and strain tensor with rate form, respectively. The explicit form for definition of the SERD for viscoplastic solids is given as below by neglecting component of elasticity.

$$\dot{W}_{VP}=\frac{n}{n+1}\frac{\dot{\varepsilon}_0\dot{\sigma}_e^{n+1}(\theta)}{\sigma_0^n} \tag{12}$$

Substituting Eq. (6) into Eq. (12), the concept of SERD is extended to sharp V-notch tip field in viscoplastic solids as following:

$$\dot{W}_{VP}=\frac{n}{n+1}\frac{\dot{\varepsilon}_0}{\sigma_0^n}K_{\mathrm{N}}^{n+1}(t)r^{-s(n+1)}\dot{\tilde{\sigma}}_e^{n+1}(\theta) \tag{13}$$

Based on the studies given by Lazzarin and Zambardi [13], the elastic strain energy density (SED) is presented as

$$W_E=\frac{1}{2E}K_V^2\cdot r^{2(\lambda-1)}\cdot f_e(\theta) \tag{14}$$

where K_V, λ and $f_e(\theta)$ are notch stress intensity factor (NSIF), eigenvalue of sharp V-notch in linearly elastic solids and equivalent stress function, respectively. Herein, K_V and $f_e(\theta)$ for sharp V-notch in linearly elastic materials are defined as below [30, 31]:

$$\begin{cases} K_V=\sqrt{2\pi}\lim\limits_{r\to 0}\sigma_{\theta\theta}(\theta=0^\circ)r^{(1-\lambda)} \\ f_e(\theta)=\hat{\sigma}_{\theta\theta}^2+\hat{\sigma}_{rr}^2+\hat{\sigma}_{zz}^2-2v(\hat{\sigma}_{\theta\theta}\hat{\sigma}_{rr}+\hat{\sigma}_{\theta\theta}\hat{\sigma}_{zz}+\hat{\sigma}_{zz}\hat{\sigma}_{rr})+2(1+v)\hat{\sigma}_{r\theta}^2 \end{cases} \tag{15}$$

where $\hat{\sigma}_{ij}$ is the eigenvalue of distribution function for sharp V-notch in linearly elastic solids. Herein, the quantities listed in Eq. (15) can be obtained analytically with the theoretical analysis for V-notch tip of linearly elastic materials.

The SED for those locations of the nominal region in the sharp V-notch plate are expressed as [13]:

$$W_E^{Nom}=\frac{\sigma_{nom}^2}{2E} \tag{16}$$

where σ_{nom} is the nominal stress located far away from the notch tip region. The SERD for the nominal locations of the notched plate in the viscoplastic solids is given as following:

$$\dot{W}_{VP}^{Nom} = \frac{\sigma_{nom}^2}{2E} + \frac{n}{n+1}\frac{\dot{\varepsilon}_0 \sigma_{nom}^{n+1}}{\sigma_0^n} \tag{17}$$

in which σ_{nom} is the nominal stress for a sharp V-notch in viscoplastic solids and it varies with time of stress relaxation if the applied loading is a constant at the outer boundary.

With the presented form in Eq. (12) and Eq. (17), the strain energy density (SED) at nominal region and apex region for a sharp V-notch in viscoplastic solids can be directly obtained by integration within the time range t based on SERD, which can be rewritten as the following forms.

$$W_{VP}^{Nom} = \frac{\sigma_{nom}^2}{2E} + \int_0^t \frac{n}{n+1}\frac{\dot{\varepsilon}_0 \dot{\sigma}_{nom}^{n+1}}{\sigma_0^n} \mathrm{d}t \tag{18}$$

$$W_{VP} = \int_0^t \frac{n}{n+1}\frac{\dot{\varepsilon}_0}{\sigma_0^n} K_{\mathrm{N}}^{n+1}(t) r^{-s(n+1)} \dot{\tilde{\sigma}}_e^{n+1}(\theta) \mathrm{d}t \tag{19}$$

where W_{VP}^{Nom} and $W_{VP}(r,\theta)$ are the SED at nominal region and apex region, respectively. The quantities in Eqs. (18) and (19) represent the same mechanical meaning as those defined previously. Eq. (18) can be expressed with an explicit form which is presented as following formula if σ_{nom} is kept as a constant.

$$W_{VP}^{Nom} = \frac{\sigma_{nom}^2}{2E} + \frac{n}{n+1}\frac{\dot{\varepsilon}_0 \dot{\sigma}_{nom}^{n+1}}{\sigma_0^n} t \tag{20}$$

However, Eq. (19) can be only solved with implicit form as $K_N(t)$ and $\dot{\tilde{\sigma}}_e(\theta)$ vary with time. The computation of Eq. (19) can be achieved with interpolation scheme which will discussed in Section 6. With the presented Eq. (20), it is found amazingly that Eq. (20) possesses the similar expression form as the method given by Molski and Glinka [8] which can be seen from Hyde et al. [10] where the elastic stress concentration factor, K_t, is given for creeping condition as below [8, 10]:

$$K_t^2 = \frac{\dfrac{\sigma_{\max}^2}{2E} + \dfrac{n}{n+1} A \sigma_{\max}^{n+1} t}{\dfrac{\sigma_{\mathrm{nom}}^2}{2E} + \dfrac{n}{n+1} A \sigma_{\mathrm{nom}}^{n+1} t} \tag{21}$$

in which A, K_t and $\sigma_{\max}$ are creep coefficient, the stress concentration factor and maximum stress ahead of a notch, respectively. It represents that the form of Molski and Glinka [8] can be obtained with the presented concept of SERD, directly.

4.2 Averaged SED for viscoplastic condition based on SERD rule

Based on Eq. (19) and SED theory studied by Lazzarin and Zambardi [13], the concept of the averaged SED within a R radius range ahead of notch tip for linearly elastic material is given as below:

$$\overline{W}_E = \frac{1}{2E} K_V^2 \frac{\overline{I}_E}{(\pi - \alpha) R^2} \tag{22}$$

where $\overline{I}_E$ is an integral based on the selection of the integration path R and α is the half notch angle of the notched plate. The path R is shown in Fig. 1. Note that Eq. (22) represents the SED in a controlled volume. The explicit integral of $\overline{I}_E$ is given as below:

$$\begin{aligned} \overline{I}_E &= \int_0^R \int_{-(\pi-\alpha)}^{\pi-\alpha} r^{2(\lambda-1)} \cdot f_e(\theta) r \, \mathrm{d}r \, \mathrm{d}\theta \\ &= \frac{R^{2\lambda}}{2\lambda} \int_{-(\pi-\alpha)}^{\pi-\alpha} f_e(\theta) \, \mathrm{d}\theta \end{aligned} \tag{23}$$

Based on the concept of SERD defined in Eq. (13), the averaged SERD in viscoplastic material, denoted as $\dot{\overline{W}}_{VP}$, should be defined within a radius R ahead of notch which is defined as following.

$$\dot{\overline{W}}_{VP} = \frac{n}{n+1} \frac{\dot{\varepsilon}_0}{\sigma_0^n} K_{\mathrm{N}}^{n+1}(t) \frac{\dot{\overline{I}}_{VP}}{(\pi - \alpha) R^2} \tag{24}$$

Note that the terms $K_{\mathrm{N}}(t)$ and $\dot{\tilde{\sigma}}_e(r,\theta)$ are dependent on the time of the occurrence of viscoplastic behaviors. According to the studies reported by Dai et al. [23], $K_{\mathrm{N}}(t)$ and $\dot{\tilde{\sigma}}_e(\theta)$ become unchanged only if the extensive viscoplastic behaviors for sharp V-notch are obtained. For small scale viscoplastic behaviors, Eq. (19) should vary with time. The integral $\dot{\bar{I}}_{VP}$ is written as below:

$$\dot{\bar{I}}_{VP} = \frac{R^{2-s(n+1)}}{2-s(n+1)} \int_{-(\pi-\alpha)}^{\pi-\alpha} \dot{\tilde{\sigma}}_e^{n+1}(\theta)\,\mathrm{d}\theta \tag{25}$$

where $\dot{\tilde{\sigma}}_e(\theta)$, s and α are the dimensionless distribution function of equivalent stress, eigenvalue and half notch angle for sharp V-notch defined in Section 3, respectively.

With the presented Eq. (25), the averaged SED in a controlled volume within a radius R with thickness of unit is obtained by integrating Eq. (24), which can be presented as following form.

$$\bar{W}_{VP} = \frac{n}{n+1}\frac{\dot{\varepsilon}_0}{\sigma_0^n}\int_0^t K_{\mathrm{N}}^{n+1}(t)\frac{\dot{\bar{I}}_{VP}}{(\pi-\alpha)R^2}\mathrm{d}t \tag{26}$$

In order to characterize the stress concentration extent of the sharp V-notch, the concentration factor is defined based on averaged SED concept under linearly elastic condition which is expressed as below [13]:

$$K_E = \frac{W_E}{W_E^{Nom}} = \frac{K_V^2 \bar{I}_E}{(\pi-\alpha)\sigma_{nom}^2 R^2} \tag{27}$$

The SED concentration factor under viscoplastic condition is presented as following.

$$K_{VP} = \frac{\bar{W}_{VP}}{W_{VP}^{Nom}} = \frac{\frac{n}{n+1}\frac{\dot{\varepsilon}_0}{\sigma_0^n}\int_0^t K_{\mathrm{N}}^{n+1}(t)\frac{\dot{\bar{I}}_{VP}}{(\pi-\alpha)R^2}\mathrm{d}t}{\frac{\sigma_{nom}^2}{2E} + \frac{n}{n+1}\frac{\dot{\varepsilon}_0\dot{\sigma}_{nom}^{n+1}}{\sigma_0^n}t} \tag{28}$$

where the quantities in Eq. (28) present the same definition with Eqs. (7)-(26).

Based on the analogy between elastoplastic material and viscoplastic solids, e.g. power-law plasticity and power-law creep, the SED concentration factor should be the same regardless of the material types for the sharp V-notch with presence of the same geometry. Hence, it is assumed that the following relation is satisfied.

$$K_E = K_{VP} = K_t^2 \tag{29}$$

i.e.

$$\frac{K_V^2 \overline{I}_E}{(\pi-\alpha)\sigma_{\text{nom}}^2 R^2} = \frac{\frac{n}{n+1}\frac{\dot{\varepsilon}_0}{\sigma_0^n}\int_0^t K_{\text{N}}^{n+1}(t)\frac{\dot{\overline{I}}_{VP}}{(\pi-\alpha)R^2}\text{d}t}{\frac{\sigma_{\text{nom}}^2}{2E}+\frac{n}{n+1}\frac{\dot{\varepsilon}_0\sigma_{\text{nom}}^{n+1}}{\sigma_0^n}t} \tag{30}$$

In fact, Eq. (30) cannot be solved analytically except for steady state. Eq. (30) is rewritten as below by simplifying Eq. (30).

$$\int_0^t K_{\text{N}}^{n+1}(t)\,\dot{\overline{I}}_{VP}\text{d}t = K_V^2\,\overline{I}_E\left[\frac{n+1}{n}\frac{\sigma_0^n}{2E\dot{\varepsilon}_0}+\sigma_{\text{nom}}^{n-1}t\right] \tag{31}$$

If the steady state is approached for a sharp V-notch, then the G-NSIF, $K_{\text{N}}(t)$, will become a constant, $K_{*\text{N}}$, which will not change with accumulation of time. Thereafter, the notch stress intensity factor for sharp V-notch in viscoplastic materials is obtained with the following form for those cases with extensive viscoplasticity:

$$K_{*\text{N}} = \left\{\frac{K_V^2\,\overline{I}_E}{\dot{\overline{I}}_{VP}}\left[\frac{n+1}{n}\frac{\sigma_0^n}{2E\dot{\varepsilon}_0 t}+\sigma_{\text{nom}}^{n-1}\right]\right\}^{\frac{1}{n+1}} \tag{32}$$

With the foundation of asymptotic solutions reported by Dai et al. [23], integral $\dot{\overline{I}}_{VP}$ is defined with the following form if it is under extensive viscoplasticity:

$$\dot{\overline{I}}_{VP} = \frac{R^{2-s(n+1)}}{2-s(n+1)}I_P \tag{33}$$

where I_P has been listed by Lazzarin and Zambardi [13]. The ratio of $\overline{I}_E$ and $\dot{\overline{I}}_{VP}$ is presented as below:

$$\frac{\overline{I}_E}{\dot{\overline{I}}_{VP}} = \frac{2-s(n+1)}{2\lambda}\frac{R^{2(\lambda-1)+s(n+1)}\int_{-(\pi-\alpha)}^{\pi-\alpha} f_e(\theta)\text{d}\theta}{\int_{-(\pi-\alpha)}^{\pi-\alpha}\dot{\tilde{\sigma}}_e^{n+1}(\theta)\text{d}\theta} \tag{34}$$

It should be pointed out that there is the R defined in Eq. (34). Based on the relation given in Eq. (4), the R related term vanishes in Eq. (34). Therefore, Eq. (34) is rearranged as

$$\frac{\overline{I}_E}{\dot{\overline{I}}_{VP}} = \frac{\displaystyle\int_{-(\pi-\alpha)}^{\pi-\alpha} f_e(\theta)\,\mathrm{d}\theta}{\displaystyle\int_{-(\pi-\alpha)}^{\pi-\alpha} \dot{\tilde{\sigma}}_e^{n+1}(\theta)\,\mathrm{d}\theta} \tag{35}$$

It indicates that the ratio of $\overline{I}_E$ and $\dot{\overline{I}}_{VP}$ is determined by the eigenvalues of sharp V-notch in linearly elastic and elastoplastic solids if it is under extensive viscoplasticity. The reason is that $f_e(\theta)$ is only determined by notch angle and its explicit form under plane strain condition is found in Dunn et al. [31]. The term $\dot{\tilde{\sigma}}_e(\theta)$ varies with notch angle, exponent of the viscoplastic materials and occurrence time of the viscoplasticity. It surely changes with the time and becomes steady when the extensive behaviors of the viscoplasticity form [23]. There are some points which should be emphasized as following regarding to SERD:

Point 1: SERD is the rate form of energy stored in a unit volume, which is significantly dependent on the occurrence time of the viscoplasticity. This verifies the influence of rate effect on sharp V-notch tip field in viscoplastic solids. With the integration of SERD, SED for viscoplastic solids is obtained directly.

Point 2: With the concept of SERD, the generalized notch stress intensity factor (G-NSIF) for viscoplastic condition is presented. An explicit form is built with the concept of SERD for G-NSIF under extensive viscoplasticity.

5 Extension of fracture toughness to sharp V-notch based on SERD rule

Based on the concept of averaged SERD, it supposes that the material fails when the averaged SED of the notched component approaches to a critical value $\dot{\bar{W}}_{C}$ which is presented as following.

$$\dot{\bar{W}}_{VP}(t) \leqslant \dot{\bar{W}}_{C}(t,\ T) \tag{36}$$

where $\dot{\bar{W}}_{C}(t,\ T)$ is a parameter which depends on the time of the occurrence for viscous plasticity as well as temperature. For material bearing viscously plastic deformation, critical value $\dot{\bar{W}}_{C}(t,\ T)$ should vary with the viscous time t if the temperature is fixed. Based on Eq. (23), the critical G-NSIF $K_N^C(t)$ for viscoplastic material is expressed as below when the critical $\dot{\bar{W}}_{C}$ is obtained.

$$K_N^C(t) = \left[\frac{n+1}{n}\frac{\sigma_0^n}{\dot{\varepsilon}_0}\frac{(\pi-\alpha)R^2}{\dot{\bar{I}}_{VP}}\dot{\bar{W}}_C(t)\right]^{1/(n+1)} \tag{37}$$

As a special case, the aforementioned Eq. (37) is expressed as below when the condition degenerates to be a crack.

$$K_N^C(t) = K_{mat}^C = \left[\frac{n+1}{n}\frac{\sigma_0^n}{\dot{\varepsilon}_0}\frac{(\pi-\alpha)R^2}{\dot{\bar{I}}_{VP}^C}\dot{\bar{W}}_C(t,\alpha=0^\circ)\right]^{1/(n+1)} \tag{38}$$

$\dot{\bar{I}}_{VP}^C$ is the integral defined in Eq. (25) for the crack in viscoplastic material, and $\dot{\bar{W}}_C(t,\alpha=0^\circ)$ is the critical SERD for crack case in viscoplastic materials. Note that K_{mat}^C for a creep crack is also defined as below according to Budden and Ainsworth [32] as well as Davies and coworkers [33].

$$K_{\mathrm{mat}}^C = \left[K_{\mathrm{I}}^2 + \frac{n}{n+1}\frac{EP\Delta_c}{B_n(L-a)}\eta\right]^{1/2} \tag{39}$$

where a, n, B_n, E, K_I, P, L, η and Δ_c are the crack depth, creep exponent, specimen thickness, Young's modulus, stress intensity factor, applied load, specimen width, correction factor and loading line displacement, respectively. Eq. (39) can be obtained through experimental test according to Davies et al. [32].

With equality of Eq. (38) and Eq. (39), the R of the controlled volume is obtained as below:

$$R = \left[\frac{n}{n+1} \frac{\dot{\varepsilon}_0}{\sigma_0^n} \frac{\dot{\bar{I}}_{VP}^C (K_{mat}^C)^{n+1}}{(\pi - \alpha)\dot{\bar{W}}_C(t, \alpha = 0^\circ)} \right]^{1/2} \tag{40}$$

Substituting Eq. (40) into Eq. (37), the following formula is obtained for the critical G-NSIF $K_N^C(t)$, or the fracture toughness for a sharp V-notch specimen.

$$K_N^C(t) = \beta K_{mat}^C \tag{41}$$

where the coefficient β is presented as following:

$$\beta = \frac{\dot{\bar{I}}_{VP}^C}{\dot{\bar{I}}_{VP}} \frac{\dot{\bar{W}}_C(t)}{\dot{\bar{W}}_C(t, \alpha = 0^\circ)} \tag{42}$$

in which $\dot{\bar{I}}_{VP}$, $\dot{\bar{I}}_{VP}^C$, $\dot{\bar{W}}_C(t)$ and $\dot{\bar{W}}_C(t, \alpha = 0^\circ)$ are the integral parameter defined in Eq. (25) and SERD for notch and crack, respectively. It indicates that the fracture toughness of sharp V-notch can be estimated via the evaluation of creep toughness with the same specimen size based on the concept of SERD.

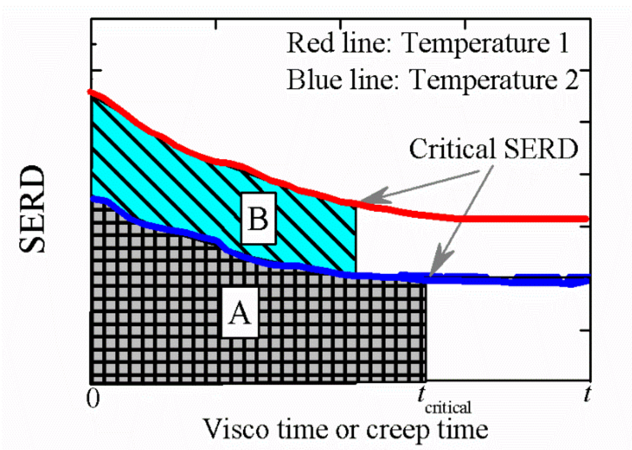

Fig. 3 Concept of critical SERD

Fig. 3 is presented to illustrate the concept of SERD more clearly. For a notch or crack, SERD should vary with the time and become unchanged when the extensive viscoplasticity approaches. The area of enclosed region with curve of SERD and coordinates is the SED at a fixed time *t*. For a specific material, it fails when the critical SED is obtained. The critical SERD can be estimated when the integral region is approaching to the critical SED where the critical time t_{critical} is obtained. If there is no viscous time, the SED is the traditional SED.

6 Numerical solutions to determine strain energy rate density and generalized notch stress intensity factor

Although the concepts of G-NSIF and SERD are proposed, the method to compute these parameters are needed. Herein, a finite-element based method is presented to extract the G-NSIF and SERD.

6.1 Determination of generalized notch stress intensity factor

The strain rate component for sharp V-notch with viscoplasticity is written as below:

$$\frac{\dot{u}_i}{\dot{\varepsilon}_0 L} = \left(\frac{K(t)}{\dot{\varepsilon}_0 \sigma_0}\right)^{ns} r^{ns+1} \dot{\tilde{u}}_i \tag{43}$$

in which $\dot{\tilde{u}}_i$ is the dimensionless displacement function and it is expressed as following form.

$$\begin{cases} \dot{\tilde{u}}_r = \dfrac{\dot{\tilde{\varepsilon}}_{rr}}{ns+1} \\ \dot{\tilde{u}}_\theta = \dfrac{2\dot{\tilde{\varepsilon}}_{r\theta} - \dot{\tilde{u}}_{r,\theta}}{ns} \\ \dot{\tilde{\varepsilon}}_{ij} = \dfrac{3}{2}\left(\dot{\tilde{\sigma}}_e\right)^{n-1} \dot{\tilde{S}}_{ij} \end{cases} \tag{44}$$

Herein, an average contour extraction method is presented as following. Firstly, the surrounded paths are extracted around the apex of notch tip field shown in Fig. 1 and Fig. 4. According to normalized equivalent stress, the G-NSIF can be represented as [23]:

$$K_{\mathrm{N}}(t)=\frac{\dot{\sigma}_e^{FE}(r,\theta)}{\hat{r}^{-s}\dot{\tilde{\sigma}}_e(\theta)} \tag{45}$$

in which $\hat{r}$ is the distance between notch tip and the extraction contour. With a number of G-NSIF determined by extraction contours, the G-NSIF can be calculated by averaging a serial of extraction values, i.e.

$$K_{\mathrm{N}}(t)=\frac{\sum_{i=1}^{m}K_{\mathrm{N}i}(t)}{m} \tag{46}$$

where m is the number of extracted contours. Thus, the dimensionless distribution functions of von Mises equivalent stress are calculated as following.

$$\dot{\tilde{\sigma}}_e(\theta)=\frac{\dot{\sigma}_e^{FE}(r,\theta)}{K_{\mathrm{N}}(t)\hat{r}^{-s}} \tag{47}$$

The method presented above has been verified by Dai et al. [22, 23] that a good accuracy can be obtained even under a coarse FE mesh. The maximum value of $\dot{\tilde{\sigma}}_e(\theta)$ is normalized to be identical to 1 under viscoplastic condition.

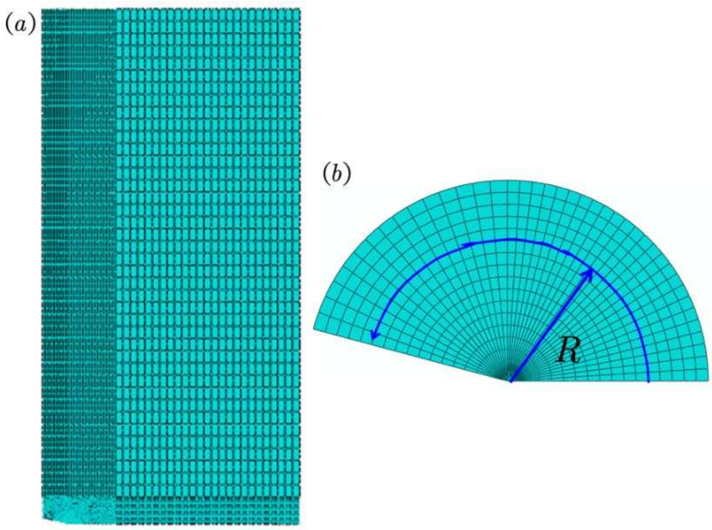

Fig. 4 Typical FE meshes for a sharp V-notch (a) symmetric model and (b) local mesh of notch tip

6.2 Determination of SERD

The most important stage to determine the SERD is to determine the integral given in Eq. (25). A simple method with Newton-Cotes integral is presented to compute the integral below:

$$\dot{\bar{I}}_{VP} = \frac{R^{2-s(n+1)}}{2-s(n+1)} \sum_{i=1}^{m} \dot{\bar{\sigma}}_e^{n+1}(\theta_i) \Delta\theta_i \tag{48}$$

where $\dot{\bar{\sigma}}_e(\theta_i)$ is the value of the dimensionless equivalent von Mises stress at θ_i with the integral increment $\Delta\theta_i$ at the fixed time t. Herein, $\dot{\bar{\sigma}}_e(\theta_i)$ can be determined with Eq. (47) with a serial number of discrete points based on finite element solutions.

6.3 Numerical procedures and verification

In order to present the solutions proposed in this paper. The entire finite element numbers of the sharp V-notch plate are between 3757 and 5598 where the typical finite element mesh of the entire V-notch plate is shown in Fig. 3. The element type is eight node plain strain element with reduced integration (CPE8R). Two typical V-notched specimens are adopted, e.g. single edge V-notch specimen (SEVN) and single edge V-notch bending (SEVB) specimen where the specimen configurations have been given in Fig. 1. The specimen width (*W*) and height (*H*) of SEVN specimens are kept as $50\,\text{mm}$ and $200\,\text{mm}$, and the height (*H*) and length (*L*) of SEVB specimens are adopted as $25\,\text{mm}$ and $200\,\text{mm}$, respectively. Two kinds of ratio between notch depth and height of the specimen are adopted in the computations, i.e. a/W = 0.1 and a/W = 0.5, which represent the shallow notched and deep notched conditions, respectively. The materials listed in Table 1 are adopted in the computations of this paper. In the analysis, the symmetric model of SEVN and SEVB specimens are adopted in the calculations. The applied loading for SEVN and SEVB specimens are kept as 50 MPa and 50 N, respectively.

The solutions given in Fig. 5 are presented to state the mesh quality effect on the acquisition of $\dot{\tilde{\sigma}}_e(\theta)$ with different mesh qualities, where the typical finite element meshes are given in Fig. 3. It can be found that the values of $\dot{\tilde{\sigma}}_e(\theta)$ shown in Fig. 5 agree quite reasonably with the extracted values under various viscos time. Compared with the asymptotic solutions solved by Dai et al. [23], the extracted solutions with finite element analysis here agree quite closely with the asymptotic solutions especially in the crack front region. The differences in region $90^\circ \leqslant \theta \leqslant 165^\circ$ between asymptotic solution and numerical solutions are because of the neglecting of elastic strain for asymptotic solution.

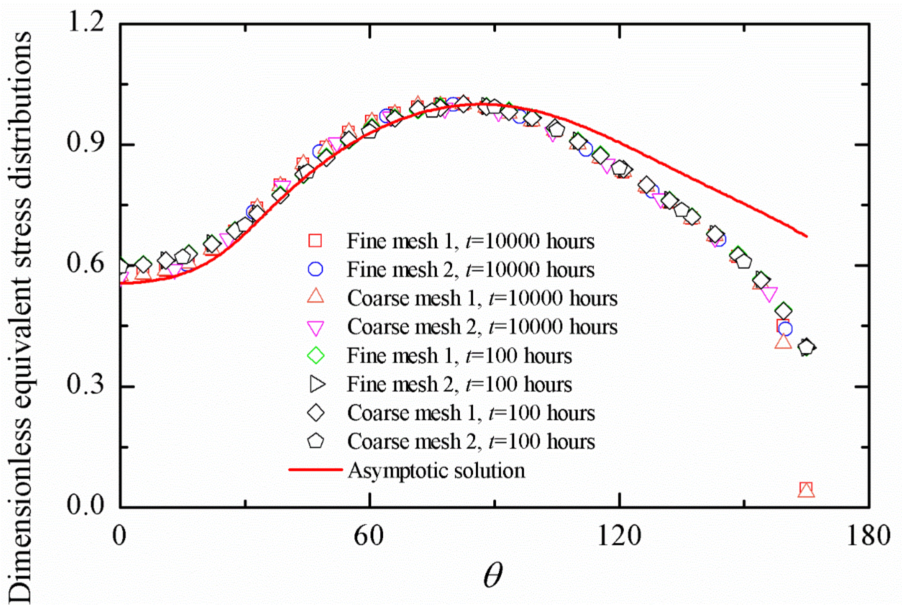

Fig. 5 Verifications of mesh quality on the dimensionless equivalent stress distribution functions extracted from numerical solutions for $2\alpha = 30^\circ$

Table 2 Different integrals of dimensionless equivalent stress with Eq. (48)

	Fine mesh 1	Fine mesh 2	Coarse mesh 1	Coarse mesh 2	t (h)
$\sum_{i=1}^{m} \dot{\tilde{\sigma}}_e^{n+1}(\theta_i)\Delta\theta_i$	1.1739	1.16605	1.1682	1.1262	100
$\sum_{i=1}^{m} \dot{\tilde{\sigma}}_e^{n+1}(\theta_i)\Delta\theta_i$	1.1593	1.1564	1.1515	1.1137	10000

With the presented Eq. (48), the integrals of dimensionless equivalent stress with various finite element meshes shown in Fig. 6 and viscos time are given in Table 2. It is found that the integral solutions agree quite closely with each other. The average values for $\sum_{i=1}^{m} \dot{\tilde{\sigma}}_e^{n+1}(\theta_i)\Delta\theta_i$ at 100 h and 10000 h are 1.1586 and 1.1452, respectively. The maximum relative errors between the computed results and the average values given in Table 2 are 2.795% and 2.753%, respectively. It indicates that the integration method to obtain integral given in Eq. (48) is reasonable.

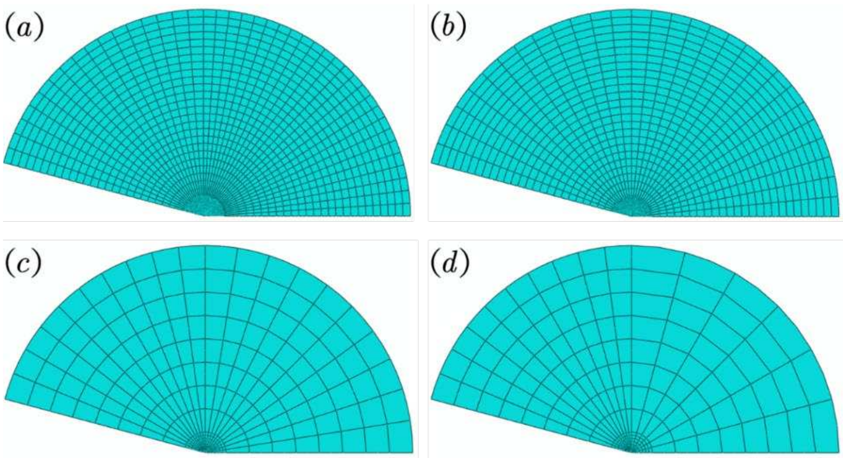

Fig. 6 Different mesh qualities of sharp V-notch tip region (a) Fine mesh 1; (b) Fine mesh 2; (c) Coarse mesh 1; (d) Coarse mesh 2

7 Results and discussion

7.1 Generalized notch stress intensity factor (G-NSIF)

Due to the role of G-NSIF in SERD, the G-NSIF for several typical specimens will be discussed. Based on the method given in Section 6.1, the variations of G-NISF with time for SEVN specimens and SEVB specimens are presented in Fig. 7. It can be seen that the values of G-NSIF for sharp V-notch specimens decreases with the increase of time. If the time is long enough, the value becomes nearly unchangeable. The value levels of G-NSIF for sharp V-notch specimens are higher than those of crack conditions. The variation tendencies of G-NSIF for sharp V-notch specimens are the same to those of the cracked specimens.

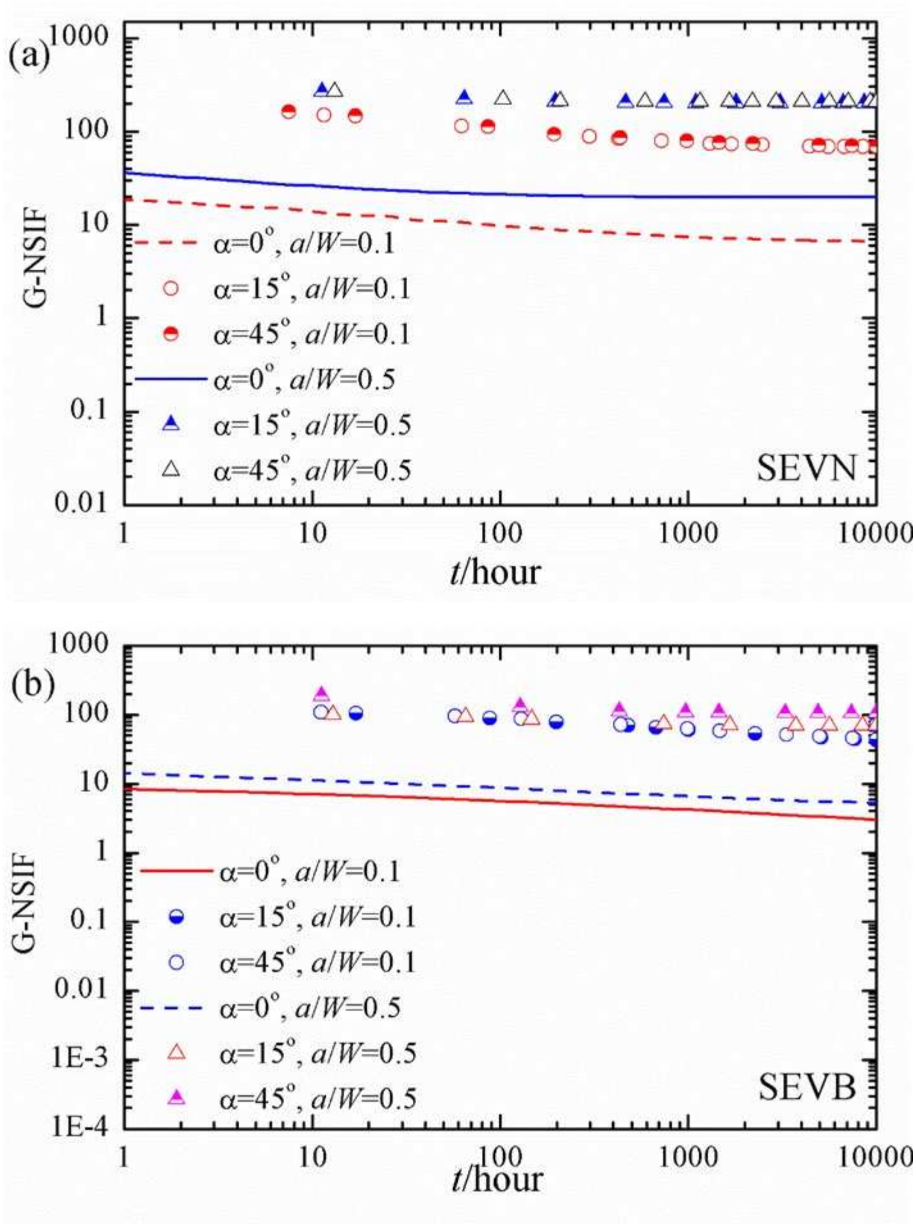

Fig. 7 Variations of G-NSIF with creep time for (a) SEVN and (b) SEVB

Table 3 G-NSIF of sharp V-notch in linearly elastic solids

	$\alpha=0^\circ$ $(\mathrm{MPa}\cdot\mathrm{mm}^{0.5})$	$\alpha=15^\circ$ $(\mathrm{MPa}\cdot\mathrm{mm}^{0.49855})$	$\alpha=45^\circ$ $(\mathrm{MPa}\cdot\mathrm{mm}^{0.45552})$
SEVN, *a*/*W*=0.1	235.70	236.16	254.02
SEVN, *a*/*W*=0.5	1251.80	1243.90	1271.65
SEVB, *a*/*W*=0.1	68.19	135.40	148.82
SEVB, *a*/*W*=0.5	218.11	427.14	442.52

It should be noted that the G-NSIF under viscoplastic condition should be identical to G-NSIF in linearly elastic solids for a sharp V-notch when the time is 0. Note that all the values of G-NSIF at time 0 have been listed in Table 3. It can be found that the values of G-NSIF for notched specimens are always higher than the SIF of cracked ones ($\alpha = 0^{\circ}$) when the time is 0. Herein, the geometries of the cracked specimens are the same to the SEVN and SEVB specimens. SIF of cracked SEVN and SEVB are directly extracted from ABAQUS with contour integration.

For SEVN specimens, G-NSIF is very close to that of crack condition. However, the G-NSIF of SEVB specimens are much higher than those of cracked ones. With the increase of time for viscoplasticity, the G-NSIF decreases with the accumulation of time, and the steady values for G-NSIF can be approached with the increase of time. The G-NSIF of different conditions at 10000 h are given in Fig. 8. It can be seen that the values of G-NSIF under steady state are much less than those of G-NSIF at 0 h regardless of SEVN and SEVB specimens.

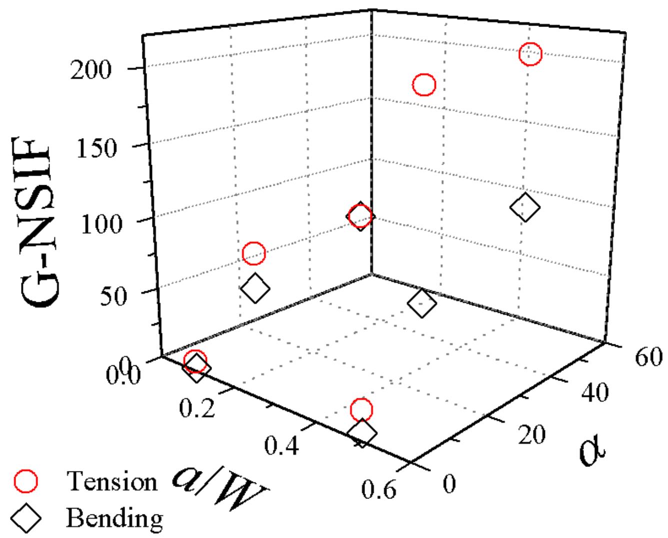

Fig. 8 Variations of G-NSIF with various notch depths and notch angles at 10000 hours

Based on the concept of SERD, a prediction formula has been given for estimation of K_{*N} with proposed Eq. (32). The comparisons of the values of K_{*N} computed with numerical solutions and Eq. (32) have been given in Fig. 9. It can be seen that the predicted solutions of K_{*N} for SEVB specimens with $\alpha = 0^\circ$, $\alpha = 15^\circ$ and $\alpha = 45^\circ$ agree quite closely with the numerical computations. The same tendency can be found for SEVN specimens. However, the accuracy of prediction solutions deviates from the numerical solutions slightly. It indicates that the presented Eq. (32) based on SERD concept can be used to estimate the G-NSIF under extensive occurrence of vsicoplasticity, i.e. K_{*N} is approached.

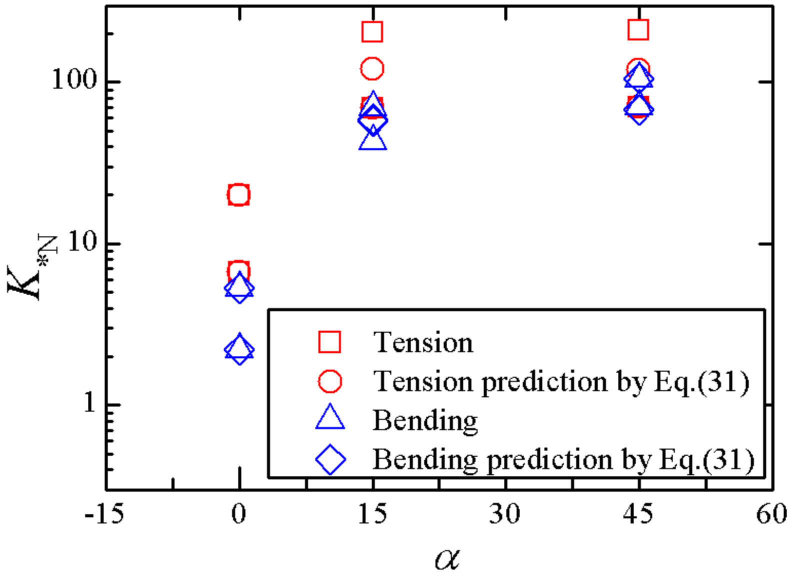

Fig. 9 Comparisons of the K_{*N} between numerical solutions and proposed Eq. (31)

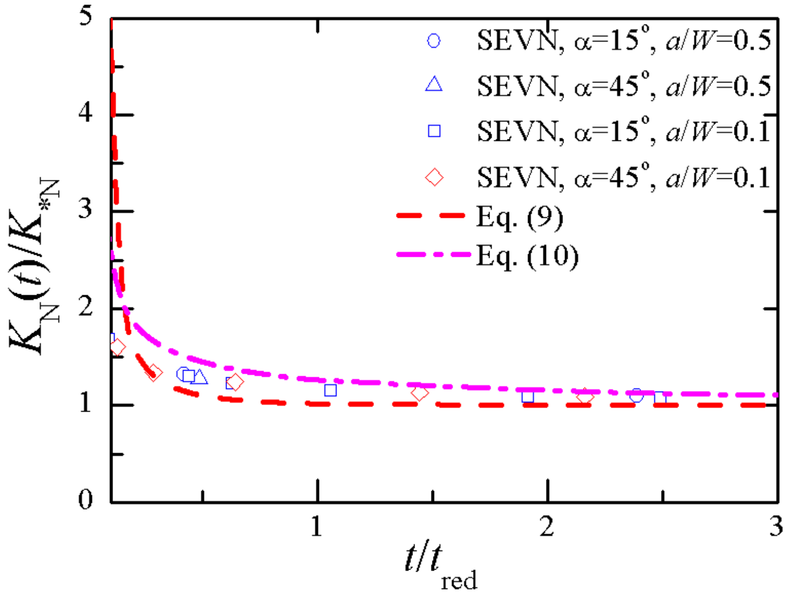

Fig. 10 Variations of G-NSIF with various notch depths and notch angles at 10000 hours

According to the characteristics of the stress field of sharp V-notch in viscoplastic solids, the variations of the ratio between $K_N(t)/K_{*N}$ for SEVN specimens are presented in Fig. 10. Note that the redistribution time, t_{red}, can be estimated through the computation of retribution time for the crack front with the same specimen geometry [23]. The formula (9) is much higher than the numerical solutions especially for the very short time region. Eq. (9) is originally to evaluate the $C(t)/C^*$ relation. It can be seen that the presented Eq. (10) coincide with the numerical results quite closely. Hence, Eq. (10) can be adopted to simulate the relation of $K_N(t)/K_{*N}$.

7.2 Strain energy rate density and strain energy density

The variations of SERD for SEVN and SEVB are presented in Fig. 11. It can be seen clearly that SERD decreases with the increase of time, and it becomes steady state when the time is long enough. Similar tendencies are found for various specimens with notches and cracks. It is confirmed that the SERD for notched specimen is higher than that of cracked one with the same geometry size compared with the notched specimen. It is also found that the rule of notch (or crack) depth on the SERD is more significant than that of the notch angle.

Table 4 SERD of sharp V-notch in viscoplastic solids under steady state

	$\alpha = 0°$ (MPa · h^{-1})	$\alpha = 15°$ (MPa · h^{-1})	$\alpha = 45°$ (MPa · h^{-1})
SEVN, a/W=0.1	7.93×10^{-12}	8.97×10^{-6}	1.15×10^{-5}
SEVN, a/W=0.5	5.63×10^{-9}	2.04×10^{-2}	6.33×10^{-3}
SEVB, a/W=0.1	1.04×10^{-14}	5.73×10^{-7}	7.23×10^{-7}
SEVB, a/W=0.5	2.03×10^{-12}	9.50×10^{-6}	1.31×10^{-4}

The steady values for SERD of crack conditions for single edge crack specimens with $a/W = 0.1$ and $a/W = 0.5$ under tension and single bending cracked specimens with $a/W=0.1$ and $a/W=0.5$ under bending have been listed in Table 4, respectively. The SERD values under steady state for sharp notched specimens under tension loading and bending loading with extensive viscoplasticity are also given in Table 4.

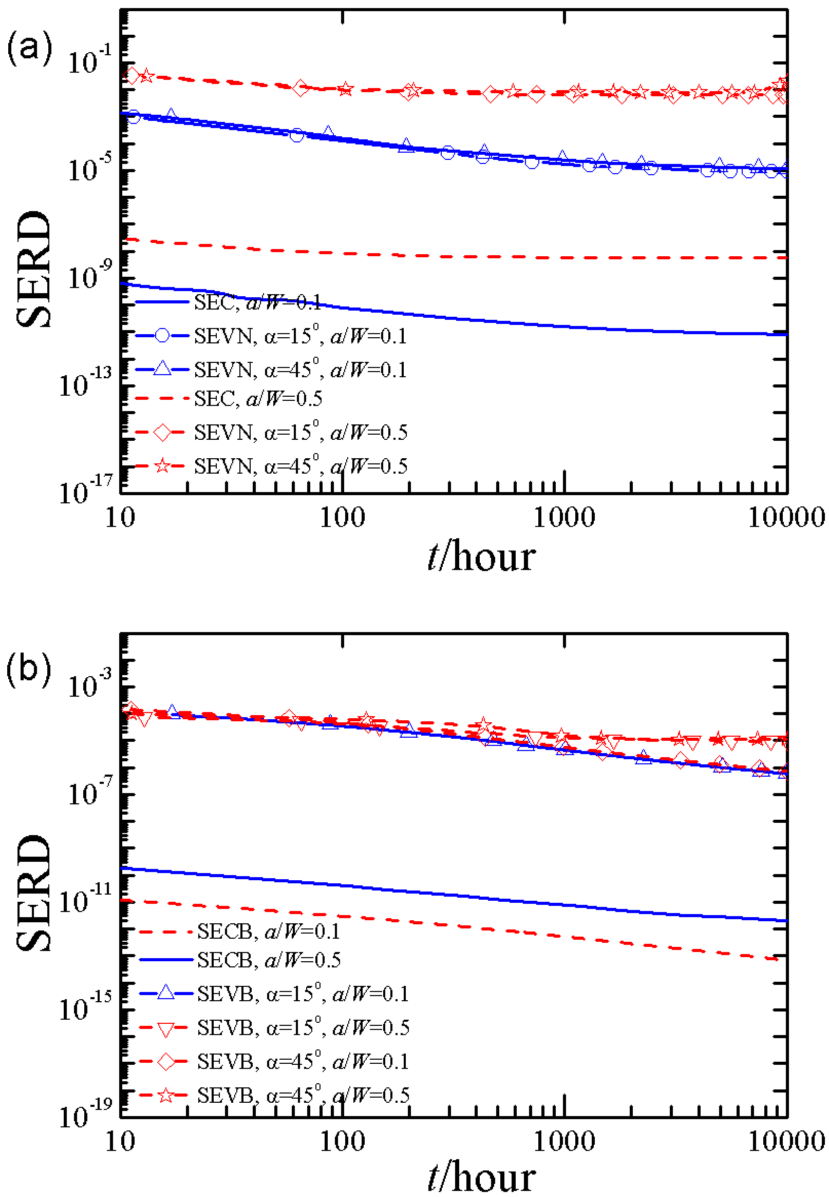

Fig. 11 Variations of SERD with time for (a) SEVN and (b) SEVB

With the concept of SERD, the averaged SED for a sharp V-notch under viscoplastic condition is obtained through integration of SERD within a specific time by Eq. (26). The specific values of the integrated SED have been listed in Table 5, where the averaged SED for sharp V-notch and crack specimens at time 0 h and 10000 h are presented, respectively. For conditions with time 0 h, the SED is obtained under linearly elastic condition, i.e. viscoplasticity is not considered. For condition with time 10000 h, the averaged SED is obtained under viscoplastic solids. It is seen clearly that the SED under viscoplastic condition increases with the improvement of notch angle regardless of tension loading and bending loading. It should be emphasized that the averaged SED under viscoplastic condition does not take the elastic SED into account. For a viscoplastic solids considering linearly elastic behavior, the entire SED for a sharp V-notch should be superposed with the averaged SED under linearly elastic condition and averaged SED under viscoplastic solids. For example, the total averaged SED for case $\alpha = 15°$ under tension loading at time 10000 h is 6.9387 while the averaged SED under linearly elastic solids and viscoplastic solids are 1.41889 and 5.5198, respectively.

Table 5 Variations of averaged SED at various occurrence of viscous time (unit of time: *h*)

	$\alpha = 0°$		$\alpha = 15°$		$\alpha = 45°$	
	t=0	*t*=10000	*t*=0	*t*=10000	*t*=0	*t*=10000
Tension, *a*/*W*=0.1	0.04864	1.61E-07	0.04834	2.08E-01	0.05160	2.73E-01
Tension, *a*/*W*=0.5	1.3719	5.98E-05	1.41889	5.5198	1.3676	7.2834
Bending, *a*/*W*=0.1	0.004071	3.55E-09	0.01862	3.56E-02	0.02108	4.90E-02
Bending, *a*/*W*=0.5	0.04165	5.80E-08	0.1926	1.25E-01	0.1735	1.52E-01

7.3 Discussion on fracture toughness for sharp V-notch under viscoplastic condition

Based on Eq. (38), the fracture toughness of a sharp V-notch can be estimated through the evaluation of fracture toughness of a crack specimen with the same sizes as that of notched one by transformation of SERD. The creep toughness defined in Eq. (38) provides an experimental way to obtain the fracture toughness of a cracked body within viscoplastic solids. For some typical steel at high temperature, the fracture toughness can be obtained via experimental test [33, 34]. The typical creep toughness of the 316H steel has been tested by Davies and coworkers [33, 34], which has been listed in Fig. 12. Creep toughness of a creeping solids is dependent on the creep time as it generally decreases with the increase of creep time due to accumulation of creep damage.

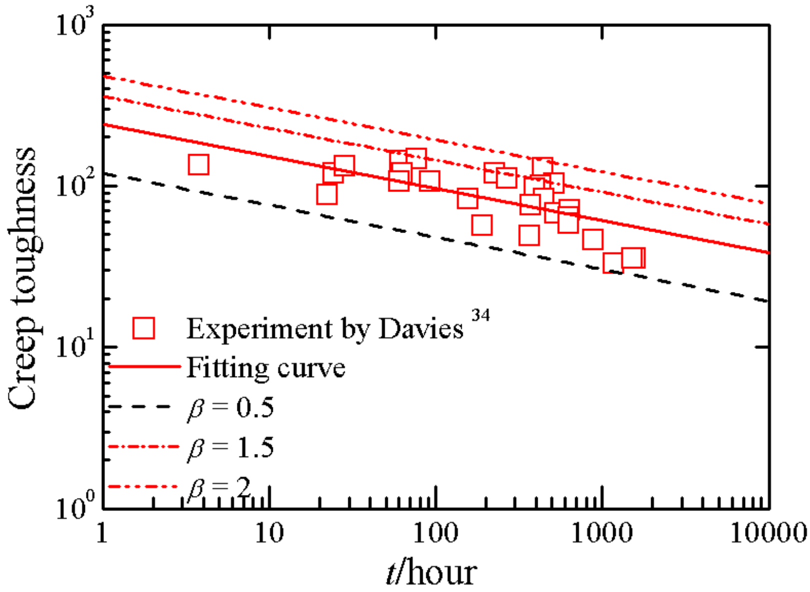

Fig. 12 Variations of creep toughness with different β

For the transformation of creep toughness between sharp V-notch and creep crack, the most important factor is to calculate the ratio of $\dot{\bar{I}}_{VP}^{C}/\dot{\bar{I}}_{VP}$. For the ratio of $\dot{\bar{I}}_{VP}^{C}/\dot{\bar{I}}_{VP}$, it can be seen that the difference between $\dot{\bar{I}}_{VP}^{C}$ and $\dot{\bar{I}}_{VP}$ is not that significant. The ratio of $\dot{\bar{I}}_{VP}^{C}$ and $\dot{\bar{I}}_{VP}$ can be treated to be close to 1. Hence, the transformation of fracture toughness for sharp V-notch is mainly dependent on the ratio of SERD for notch and the crack with the same geometry, i.e. coefficient β. The variations of creep toughness with time with different values of β have been presented in Fig 12. It can be seen that the influence of coefficient β on the creep toughness is remarkable. Herein, the creep toughness with symbols were tested by Davies [34] for creep crack with 316H stainless steel. If the notch effect is considered, the creep toughness varies differently. With the correlated form given in Section 5, the creep toughness of sharp V-notch can be evaluated by the averaged SED. According to the computations, $\dot{\bar{W}}_C(t)$ is always higher than $\dot{\bar{W}}_C(t, \alpha = 0°)$, which indicates that the fracture toughness of a sharp V-notch will be greater than that of a cracked one.

8 Conclusions

With the definition of strain energy rate density (SERD), the averaged strain energy density (SED) is derived for sharp V-notch in viscoplastic solids. The concept of SERD is a more direct form to characterize the notch tip field in viscoplastic solids or creeping materials. This concept can be easily extended to mixed mode conditions although the mode I loading condition is only given in this paper. The concept of SERD is also very hopeful to be extended to notch fatigue evaluations for notch problem in viscoplastic solids. With theoretical analysis and numerical computations, the conclusions are drawn as following.

1) The concept of SERD is proposed for sharp V-notch in viscoplastic solids. By extending the concept of energy concentration factor, an energy concentration factor is proposed based on SERD for viscoplastic solids. The averaged SED is found to be achieved with an integration form of SERD for viscoplastic solids.
2) A numerical method is also presented to calculate the SERD based on the finite element method, which is found with good accuracy even under a relative rough finite element mesh. SERD and SED vary with the occurrence of time under viscoplastic condition. SERD is found to be equivalent to the method presented by Hyde et al. [10] for viscoplastic solids.
3) An explicit relation to descript SERD and the G-NSIF is also given, and an analytical relation between $K_N(t)$ and K_{*N} is presented. The singularity exponent for sharp V-notch in viscoplastic materials is confirmed.
4) The fracture toughness for sharp V-notch specimen is presented based on the concept of SERD under viscoplastic condition. With this form, the fracture toughness of sharp V-notch specimens can be estimated easily. The presented relation reveals that the fracture toughness of sharp V-notch should be higher than that of cracked specimen under the same specimen geometry.

Acknowledgement

The authors acknowledge the supports from the National Natural Science Foundation of China (11902009), the Beijing Natural Science Foundation (2204074) and the Scientific Research Common Program of Beijing Municipal Commission of Education (KM202010005034).

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