R&D Investment under Financial Constraints and Competition

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Abstract

I develop a dynamic model to examine how financial constraints and competition affect firms' research and development (R&D) strategies. Contrary to conventional wisdom, I show that financially constrained firms can optimally invest more intensively in R&D projects than unconstrained firms. Financial constraints introduce a risk that a firm may run out of money before its ongoing R&D project bears fruit, which forces the firm to abandon the project. For firms that rely on risky cash flows to keep their R&D projects alive, early success can be relatively important. When the discovery process can be expedited by heavier investment ("accelerable" R&D), a financially constrained firm may find it optimal to "over"-invest in order to raise the probability of project survival. Moreover, when firms with different financial constraints compete in an R&D race, their strategic interactions lead to an unconstrained firm having a hump-shaped response of investment rate against its constrained competitor. As a result, a constrained firm can preempt its unconstrained competitor in market equilibrium. The model also generates new testable implications regarding how project characteristics and cash flow risks impact R&D decisions.

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1. Introduction

Research and development ("R&D") investment is an important corporate decision. Unlike traditional corporate investment, R&D projects often take years, if not decades, before bearing the fruits of innovation. The maturity uncertainty associated with the long investment horizon and the innate technical risk make R&D decisions more complex, even without any real world friction acting as the "Sword of Damocles". This challenge may explain why the following question remains open: How do financial constraints and competition affect firms' R&D decisions?

My goal in this paper is to provide a theoretical framework for analyzing firms' R&D decisions in the presence of financing frictions and innovation competition. I focus my analysis on the corporate decision of R&D investment per period, which will be referred to as "investment intensity" and "investment rate" interchangeably from hereinafter. In practice, investment intensity often affects the speed as well as the economic significance of discoveries, both of which are essential for innovation. The extent to which an R&D project can be accelerated and/or scaled up by more intensive investment are important characteristics of the innovation technology, but have been largely ignored in the economics and finance literature. The dynamic and strategic nature of the investment intensity decision causes non-trivial tradeoff for an innovative firm. These considerations lead to the specific research questions of this paper: how does the impact of financial constraints on a firm's R&D investment rate depend on the characteristics of innovation technologies? How do firms with different financing constraints compete in R&D projects, and how do their strategic interactions vary with the R&D project characteristics?

To answer these questions, I build a dynamic model of R&D investment. An all-equity firm with stochastic cash flows faces a now-or-never R&D investment opportunity. It decides whether or not to start an R&D project, and chooses an investment intensity which is invariant over time to maximize its value. The time of discovery is random and follows an exponential distribution. Both the rate parameter of the discovery time and the expected magnitude of the breakthrough depends positively on the chosen investment intensity. The firm is financially constrained, and has to rely on internal cash flows to pay for R&D investment each period. Once the firm fails to pay, it has to terminate the project with no scrap value. Meanwhile, if a competitor reaches a discovery first, the firm's project becomes obsolete. A key departure from existing models is that

I distinguish two typical aspects of R&D project characteristics: *accelerability* versus *scalability*. An *accelerable project* can be expedited by more intensive investment. Consider a pharmaceutical company searching the best chemical compound for a drug. Hiring more technicians will likely help find the most suitable compound sooner. A *scalable project*'s expected payoff can be raised by more intensive investment. One example is an R&D project by an automobile firm which aims at improving a car model that is expected to release at a certain time. Higher investment rate will probably result in cars with more-attractive functions and generating higher profits. I assume that the firm can assess the accelerability and scalability of the R&D project when making decisions.

The first key result is that financial constraints can induce more aggressive R&D investment if the innovation project is accelerable. Comparing with an unconstrained ("UC") firm, a financially constrained ("FC") firm may invest more intensively in R&D in the absence of competition. This optimal "over"-investment strategy results from an FC firm's motive to increase the likelihood of project survival. Intuitively, financial constraints impose a risk that a firm may run out of money when its valuable project is still in progress. If this happens, the firm has to terminate the project and forgo any potential future cash flows associated with a discovery. When determining investment intensity, an FC firm weighs the cost of investing, the risk of having to abandon the project due to funding shortages, and the benefit of an earlier and/or better discovery. A speedier discovery can be relatively important for a constrained firm with cash flow risks. Therefore, if the R&D project is accelerable, an FC firm may find it optimal to invest more heavily in order to expedite the discovery, although the higher burning rate of internal cash flows makes the financing constraints bind earlier.

The aforementioned rationale of over-investment from financial constraints does not require the alleviation of the constraints upon a successful innovation (e.g., Aghion et al., 1999; Povel and Raith, 2001; Almeida et al., 2011), or the distortion from debt financing on inefficient liquidation of assets (e.g., Eisdorfer and Hsu, 2011; Lyandres and Zhdanov, 2010), or the agency problem between shareholders and managers for constraints to play a disciplinary role (e.g., Almeida et al., 2014). It also differs from the effect of a larger discount rate due to an exogenous obsolescence risk (e.g., Merton, 1976; McDonald and Siegel, 1986). The most closely related work perhaps is by Boyle and Guthrie (2003), who show that a threat of future cash shortfall introduces the risk of not being able to fund a growth option later, thus reduces the value of waiting and may lead to an earlier exercise of the growth option. My result on investment intensities complements their finding of the investment timing decision. Instead of focusing on the option value of delaying investment, I emphasize the innovation technology characteristics which can arguably be more relevant to R&D type of investment.

The second key result concerns strategic choices of investment intensities in an R&D race among firms with heterogeneous financing capabilities. I find that, as an equilibrium outcome, an FC firm can preempt its UC rival when the two compete on accelerable innovation. My model reveals novel patterns of a firm's choice of investment intensity as a response to its rival (of different constraints): it is monotonically increasing for the FC contestant, but it follows a hump-shape for the unconstrained contestant. Together, they suggest that the counter-intuitive preemption happens as a UC firm chooses not to escalate the speed contest in R&D, but instead invests less intensively and waits for the FC firm to drop out of the race, so it can achieve discovery with a relatively low cost. This result is related to the weaker status of the FC competitor: a UC firm can achieve innovation after an FC firm drops out of the race, but an FC firm's only possibility of achieving an innovation is by winning the race. This new channel of preemption may help explain "the standard folklore that smaller firms are more aggressive about entering new markets or launching new products than bigger, safer, and less financially constrained firms" (Boyle and Guthrie, 2003) in a competitive setting.

The preemption result comes from my pioneering approach of modeling endogenous R&D competition with financing consideration. In practice, it is common that only a handful of firms compete in an innovation race, which makes their strategic interactions crucial in the understanding of R&D decisions. Thus, the simplified way of modeling competition as an exogenous obsolescence risk (e.g., Hackbarth et al., 2014; Gu, 2016) has its limitations in examining R&D strategies. I join a few recent studies which model innovation competition endogenously (e.g., Bena et al., 2016; Ma et al., 2018; Malamud and Zucchi, 2019), and take a step further to allow for heterogeneity in firms' financing capabilities. This approach combined with a closer examination of technology characteristics of R&D projects also helps pinpoint exceptions to the status quote of the literature that competition motivates innovation. If an R&D project is hardly accelerable, then for a UC firm, having a rival in R&D is equivalent to adding an exogenous obsolescence risk to its project, which changes the marginal cost and benefit of investment intensity equally. In this case, competition has no effect on the investment rate of the UC firm. On the contrary, there is a positive effect of competition on R&D investment for an FC firm, regardless of whether or not the project is accelerable.

My model on R&D investment provides a unified framework of examining “termination risk”. The risk of having to terminate an otherwise valuable project acts like the Sword of Damocles, which is a significant concern for any firms that conduct R&D. Although it is related to the maturity uncertainty and technical risk in recent R&D models (e.g., Berk et al., 2004; Malamud and Zucchi, 2019), a careful inspection of termination risk coming from market competition, or financing frictions in combination with various internal cash flow risks in the same model is useful. For example, contrasting to the negative impact of a termination risk from cash flow volatility (diffusion risk) on investment, a jump risk on cash flows raises investment intensity for an FC firm. Intuitively, catastrophic events on internal cash flows causes a project termination regardless of how much liquidity the firm had. They alleviate the negative impact of heavier investment on earlier constraints binding, and give an FC firm extra incentives to invest, similar to a larger discount rate effect from exogenous competition (e.g., Hackbarth et al., 2014). However, my model shows the severity of termination risk from competition, and therefore its effect, depends on the characteristics of all competing firms’ cash flows, financing capacity, as well as their competing projects’ characteristics. As a tradeoff of model tractability, my model is silent on optimal liquidity management which is relevant for R&D firms.

My model on R&D investment contributes to at least three strands of literature. Firstly, by providing a new understanding of investment level decisions, this paper contributes to the corporate investment literature and complements endogenous timing models (e.g., McDonald and Siegel, 1986; Boyle and Guthrie, 2003; Bolton et al., 2011; Hugonnier et al., 2015) in which “the investment level is not a choice variable”(Gu, 2016). Secondly, by introducing strategic interactions among R&D competitors with different financing constraints, this paper can enlighten more in-depth studies in the booming literature on the interaction of finance and industrial organization (e.g., Lambrecht, 2001; Phillips and Zhdanov, 2013; Hackbarth et al., 2014; Malamud and Zucchi, 2019). Thirdly, by recognizing the consequence of investment rate decisions on both the timing and scale of innovation for the first time in the literature, it provides an analytical tool to the growing literature on innovation and entrepreneurship (e.g., Krishnan and Wang, 2018;

Balmaceda, 2018).

The theoretical framework in this paper generates new empirical implications regarding R&D strategies. For example, it is more likely to observe higher R&D investment rates by financially constrained firms when (1) their cash flow volatility is low; (2) there is a looming challenge on their existing business or from an external funding source; and (3) they operate in an industry where innovation technologies are quite accelerable and the focus of competition is speed as opposed to quality. A more subtle implication is that we may expect unconstrained firms to cut R&D rate more heavily than the constrained ones when the product market becomes more volatile. Regarding the decision to initiate a project, an unconstrained firm is less likely to join an R&D race when its constrained rivals have a high asset growth rate and/or a low cash flow volatility.

This paper has broader applications in the finance and economics research. It demonstrates the relevance of cross-industry studies in examining the real effects of financial market frictions on corporate innovation. A potentially fruitful way of separating industries is by asking whether the new technologies in development are more likely to be accelerable or scalable. Challenging as it may sound, this can add a new line of research to the growing empirical literature studying the role of finance in the innovation process (e.g., Hellmann and Puri, 2000; Lerner et al., 2011; Manso, 2011; Tian and Wang, 2014; Nanda and Rhodes-Kropf, 2017; Malamud and Zucchi, 2019). The model may also bring new insights for R&D-driven mergers and acquisitions (e.g., Bena and Li, 2013; Phillips and Zhdanov, 2013). For instance, it shows the possibility of linking characteristics of R&D opportunity to the valuation gaps between acquirers and targets when explaining the waves of acquisition for innovation. In addition, we may also explain time-varying composition of young and private firms making successful innovation in the economy by examining how the characteristics of innovation change over time.

The paper proceeds as follows. Section 2 presents a baseline model examining the effects of financial constraints on firms' R&D investment strategies. Section 3 presents the full model with an endogenous R&D race. Section 4 discusses the robustness of the baseline model, and Section 5 concludes the paper. The Appendix contains proofs and additional graphs.

2. A Baseline Model

Consider an all-equity firm that cannot get external financing and faces a potential downfall of the internal stochastic cash flows from assets in place ("AIP"). This financially constrained ("FC") firm is run by a risk-neutral agent who maximizes the firm value when making decisions. Upon the arrival of a non-deferrable one-time innovation opportunity, the agent decides whether or not to start a project, and if she does, she chooses an R&D investment intensity throughout the duration of the project and decides when to abandon the project, if ever. We compare this firm's investment decision with one that can issue new equity at no extra cost, that is, financially unconstrained ("UC"). Whether the firm is financially constrained or not is exogenous to the model, and financial constraints are defined based on the cost of accessing financial markets (as in, e.g., Kaplan and Zingales, 1997 and Bolton et al., 2011). We do not interpret the constraints as collateral constraints (as in, e.g., Li, 2011 and Rampini and Viswanathan, 2013), thus the liquidation of AIP does not prohibit a UC firm from getting external financing for its innovation project.

The AIP Cash Flows At any time t, the firm's assets in place generate a cash flow X_t (≥ 0) which follows a combined geometric Brownian motion/jump process (see Chapter 5.B in Dixit and Pindyck, 1994):

$$dX_t = \mu X_t dt + \sigma X_t dZ_t - X_t dq_1, \tag{1}$$

where $Z = \{Z_t; 0 < t < \infty\}$ is a standard Brownian motion, and dq_1 is the increment of a Poisson process with mean arrival rate λ_j. The diffusion process associated with dZ_t represents uncertainty from the firm's daily operations, and σ^2 is a measure for the conventional cash flow risk. Meanwhile, the process $-X_t dq_1$ captures an extreme downward jump risk on the cash flows, and can represent a negative shock on the AIP which wipes out future cash flows and is expected in $\frac{1}{\lambda_j}$ years. Due to the nature of the shock, I use "jump risk" and "catastrophe risk" interchangeably hereafter. [1] Note that when the catastrophe hits, an FC firm can no longer fund an ongoing R&D project and has to terminate it. Consistent with the notion of a firm being unconstrained in the model, a UC firm is capable of raising funds through external financial markets and continuing

[1]Here are a few examples of a catastrophe. An car manufacturer is found to have fatal defects in its models, or a smartphone company is blown by a recall crisis, or a pharmaceutical firm loses its dominant status in a market when its patent expires and its competitor successfully manufactured a generic drug. The diffusion and the jump processes are independent from each other.

the project, although it has no AIP left. We can also think of Eq.(1) as stochastic cash flows from a financier, so the model can apply not only to an established firm that has an new R&D investment opportunity, but also to a start-up that uses venture capital funding to develop its first big idea.

The R&D Project Cash Flows The fixed cost of the R&D project is normalized to zero, and the R&D cash flows do not affect the AIP cash flows, [2] but the latter may impact the former via the optimal choice of R&D intensity. If the firm starts the project upon its arrival and decides on an investment intensity R, we assume that this flow cost is constant and continuous before discovery (similar to Malamud and Zucchi, 2019), unless the project is terminated.[3] The project generates a random one-time payoff $\tilde{u}$ at an uncertain discovery time τ_d, which is modelled as the first jump time of a Poisson process with parameter λ_d. The smooth investment assumption is based on the fact that firms usually cannot cut variable expenses of R&D projects without seriously compromising project outcomes (see Brown and Petersen, 2011 for empirical evidence), and the characterizations of an innovation project are close to that of typical R&D models in finance (such as Schwartz and Moon, 2000 and Berk et al., 2004), but with two key distinctions. Firstly, the investment intensity is chosen endogenously. Secondly, both the project payoff and the project discovery time depend on the investment intensity R. In particular, the one-time payoff of an R&D project $\tilde{u}$ and the discovery rate λ_d follow

$$\tilde{u} = \tilde{A} \times f(R), \quad \lambda_d = \eta \times I(R). \tag{2}$$

The expected payoff is $u = E(\tilde{u}) = A \times f(R)$. The scaling factor $A > 0$ and the acceleration factor $\eta > 0$ are constants, and the functions $f(R)$ and $I(R)$ are weakly increasing and concave. We assume the R&D investment has decreasing returns to scale, that is, $\frac{\partial(u\lambda_d)^2}{\partial^2 R} < 0$, to avoid infinite optimal investment size. To separate the effect of investment intensity R on the project payoff and on the project discovery time, we define two kinds of R&D projects accordingly:

Definitions. A scalable R&D project is a project whose final payoff can be scaled up by more intensive investment. An accelerable R&D project is a project whose discovery can be expedited

[2] The main results will not change qualitatively if there is a moderate cannibalization cost (e.g., Hackbarth et al., 2014), that is, a negative effect on the AIP cash flows from a successful innovation.

[3] One can also interpret R as the searching intensity in the search models (e.g., Mortensen, 1986). More-intense searching leads to speedier discovery and/or better search outcome.

by more intensive investment.

Going beyond the extant research on R&D investment, we capture both the scalability (of the final outcome) and the accelerability (of the discovery speed) of the input-output relationship for innovation investment. This novelty allows us to separate the scalable-only projects ($I'(R) = 0$, $f'(R) > 0$) and accelerable-only projects ($f'(R) = 0$, $I'(R) > 0$), and disentangle the effects of financial constraints on R&D investment through different aspects of project characteristics for the first time in the literature. Note that the two kinds of projects are not mutually exclusive. Later in the numerical analysis and comparative statics, I use the simple forms of $f(R) = R^{\beta}$ and $I(R) = R^{\gamma}$ with $\beta, \gamma \in (0, 1)$ and $\beta + \gamma < 1$, so that the production technology has decreasing returns to scale. We can interpret β as *project scalability*, which measures the degree to which the project is scalable, and interpret γ as *project accelerability*, which measures the degree to which the project is accelerable.[4]

If the project is discontinued before maturity, we assume that the scrap value is zero. The assumption seems extreme, but the huge uncertainty during innovative project development and the exclusiveness of accumulated knowledge make it very difficult to evaluate the resale value of underdeveloped intangible assets. The results are qualitatively the same if the scrap value is positive. Furthermore, the R&D project has no effect on the firm's financing ability, before or after the discovery (as in Boyle and Guthrie, 2003). This is to exclude the investment incentive from a motive to relax financing constraints (Almeida et al., 2011).

The random cash flows from AIP in Eq.(1) can be used to pay for the R&D investment expense each period, and any residual is paid out as dividends. Following earlier literature on corporate investment (such as Hennessy et al., 2007), we do not consider liquidity management (as in e.g., Bolton et al., 2013; Hugonnier et al., 2015) to keep the model parsimonious but still insightful.[5]

2.1 The Firm's Problem

The agent maximizes the firm value by choosing an R&D strategy $\{\mathbb{1}_{\text{invest}},\ R,\ \underline{X}\}$, where $\underline{X}$ is a threshold on the AIP cash flows below which the firm abandons the project. [6] By backward

[4] More generally, this model framework can study the tradeoff between a better-but-slower discovery versus a worse-but-speedier one (i.e., $\beta\gamma < 0$).

[5] Admittedly, cash holding policy and liquidity management can be relevant for R&D strategies (see e.g., Brown and Petersen, 2011; Schroth and Szalay, 2010).

[6] We can show that the agent optimally abandons an ongoing project if its AIP cash flows falls below a certain level.

induction, we solve the abandonment threshold $\underline{X}(R)$ for any investment intensity R, then picks the optimal R, and check whether or not the project value is positive to determine the project initiation decision.

FC firm After an FC firm starts the R&D project, the agent maximizes the firm value as follows:

$$\sup_{R,\underline{X}\geq R} E\Big[\int_0^{\tau_j} \mathrm{e}^{-rt}X_t\mathrm{dt} + \int_0^{\tau_d\wedge\tau_c\wedge\tau_j} \mathrm{e}^{-rt}(-R)\mathrm{dt} + \mathrm{e}^{-r\tau_d}\tilde{u}\mathbb{1}_{\{\tau_d<\tau_c\wedge\tau_j\}}\Big]. \tag{3}$$

The first term in Expression (3) represents the firm value from AIP cash flows, which ends at the random jump time τ_j. The second term is the present value of the future R&D cost. The firm stops paying for the investment at the earliest time of (1) R&D discovery, (2) project abandonment, either voluntary or forced by the financial constraints that require $X_t \geq R$, and (3) liquidation of the firm induced by the AIP catastrophe. This is denoted as $\tau_d \wedge \tau_c \wedge \tau_j$, where $\tau_c \equiv \inf\{t : X_t \leq \underline{X}|\underline{X} \geq R\}$ is the first time that the AIP cash flows hit the abandonment threshold. The last term is the present value of the project payoff, which is only realized if the project reaches discovery before it is terminated, i.e., $\tau_d < \tau_c \wedge \tau_j$.

UC firm For a UC firm, the agent maximizes the firm value by solving

$$\sup_{R,\underline{X}\geq 0} E\Big[\int_0^{\tau_j} \mathrm{e}^{-rt}X_t\mathrm{dt} + \int_0^{\tau_d\wedge\tau_c} \mathrm{e}^{-rt}(-R)\mathrm{dt} + \mathrm{e}^{-r\tau_d}\tilde{u}\mathbb{1}_{\{\tau_d<\tau_c\}}\Big]. \tag{4}$$

Exp.(4) differs from Exp.(3) in two ways: (1) the earlier time of project abandonment and AIP catastrophe is replaced with the project abandonment (i.e., $\tau_c \wedge \tau_j \to \tau_c$), because a UC firm can raise external funds after the AIP cash flows jump to zero whilst an FC firm cannot. For the same reason, (2) there is no lower bound for the project abandonment threshold, which we set as zero for the geometric Brownian motion, i.e., $\underline{X} \geq 0$. We can simplify the analysis for both types of firms by focusing on the strategies which maximize the R&D project value, given the value of AIP cash flows is independent of R&D decisions in the model. Denote the project value as $V(X)$,

$$V_{UC}(X) = \sup_{R,\underline{X}\geq 0} E\Big[\int_0^{\tau_d\wedge\tau_c} \mathrm{e}^{-rt}(-R)\mathrm{dt} + \mathrm{e}^{-r\tau_d}\tilde{u}\mathbb{1}_{\tau_d<\tau_c}\Big]. \tag{5}$$

$$V_{FC}(X) = \sup_{R,\underline{X}\geq R} E\Big[\int_0^{\tau_d\wedge\tau_c\wedge\tau_j} \mathrm{e}^{-rt}(-R)\mathrm{dt} + \mathrm{e}^{-r\tau_d}\tilde{u}\mathbb{1}_{\tau_d<\tau_c\wedge\tau_j}\Big]. \tag{6}$$

2.2 Optimal R&D Strategies - Initiation, Investment intensity and Abandonment

Intuitively, the firm does not abandon an ongoing project voluntarily. Because the project discovery is random and memoryless, given any chosen investment intensity, the project value is higher if it lasts longer[7]. It indicates that the firm optimally sets the lowest possible abandonment threshold (i.e., $\underline{X}_{FC} = R$ and $\underline{X}_{UC} = 0$) and invests in an ongoing project for as long as it can. The formal proofs of this intuition and the following result on project value are provided in the appendix.

Proposition 1. *The firm does not voluntarily abandon an ongoing R&D project, and the R&D project values are*

$$V_{UC}(X) = \sup_R \frac{u(R)\lambda_d(R) - R}{\lambda_d(R) + r}, \tag{7}$$

$$V_{FC}(X) = \sup_R \frac{u(R)\lambda_d(R) - R}{\lambda_d(R) + \lambda_j + r}(1 - (\frac{X}{R})^{\alpha_1}), \tag{8}$$

for a UC firm and an FC firm respectively, where $\alpha_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2\left(\lambda_d(R)+\lambda_j+r\right)}{\sigma^2}} < 0.$

The UC firm's project value in Eq.(7) follows a formula of the present value of a perpetuity, with per-period payments of $u\lambda_d - R$ and a discount rate of $\lambda_d + r$. The discovery rate λ_d increases the discount rate because the project cash flows end once the discovery occurs. The firm can tap the financial market at no extra cost, so the project value for a UC firm does not depend on AIP cash flows including the jump risk. The project value of an FC firm in Eq.(8) is a product of two terms. The first term is similar to Eq.(7), except that the discount rate is further raised by the Poisson intensity λ_j which reflects the jump risk of AIP cash flows (as in Dixit and Pindyck, 1994). The second term can be interpreted as the FC firm's probability of project discovery before the firm runs out of money. Besides in the discount rate, λ_j also appears in the second term through α_1. Consistent with Merton (1976) and McDonald and Siegel (1986), the jump risk causes a sudden ruin of the project and acts like an obsolescence risk which effectively increases the interest rate.

We use the simplex search method by Lagarias et al. (1998) to obtain the optimal investment intensities numerically. Table 1 shows the model parameters used in this exercise. The baseline firm's AIP cash flows expect to decline 20% annually with a 30% annual volatility, and a catas-

[7]The model may appear static due to the no-active-abandonment result, but the nature of the tradeoff that the firm faces is dynamic.

trophe on the AIP occurs with a 10% probability per year. The parameter values reflect the high uncertainty typically associated with innovating firms' internal cash flows or future funding, and are close to a few previous studies (e.g., Morellec and Schürhoff, 2011; Hackbarth et al., 2014) except the growth rate of AIP μ. We check a wide range of parameter values of μ to ensure our study is relevant for industries at large, instead of only the sharply declining industries. The baseline values for the R&D project parameters (i.e. η, γ, A, β) are chosen so that the optimal investment decisions fall within a reasonable range. [8]

[Insert Table 1 here.]

At the baseline example, a constrained company invests much more intensively than an unconstrained one, by a non-trivial of 103% (13.8 vs. 6.8). The following "over"-investment result is based on the model solutions from a wide range of parameter values.

Result 1. *A financially constrained firm can optimally invest more intensively in an R&D project than if it is not constrained ("the first best" level).*

The main intuition is that by investing more intensively, a constrained firm may be able to increase its chance of retaining the project value if it is accelerable. Although the financial constraints are exogenous, the expected time that the FC firm runs out of internal cash flows is determined by the firm's choice of the investment intensity: a higher cash burning rate on the R&D investment leads to an earlier instance of hitting the firm's constraints and triggering the project abandonment. If the project is accelerable and depending on the characteristics of the project and the firm's cash flows, then by investing more heavily each period, the firm may be able to push the discovery even sooner than the already earlier constraint hitting time. When speeding up the project gives the constrained firm the highest expected value, the firm optimally invests more aggressively comparing with the first best.

This finding may seem similar to what has been recognized in the endogenous investment timing models, which suggests that the risk of obsolescence or preemption reduces the value of waiting to invest, thus triggers an earlier exercise of a growth option. More closely related, Boyle

[8] At the baseline optimum, an FC (UC) monopoly spends around 14% (7%) of its instantaneous AIP cash flows on R&D, and the project value is around 13% (18%) of what its AIP are worth, at the time of project initiation. The discovery takes about four (five) years for the FC (UC) monopoly. The FC firm is expected to run out of money in about ten years, which is slightly sooner than an anticipated catastrophe.

and Guthrie (2003) find that the threat of future funding shortfalls encourages acceleration of investment in some states, where the benefits of delay are outweighed by the risk of losing the ability to finance the project. Instead of investment timing, I focus on the investment level decision, which is arguably as practical and can be more relevant for innovative firms. When determining the R&D investment intensity, an FC firm weighs the cost to run the project, the risk of having to involuntarily terminate the project after its launch, and the benefit of a better and/or an earlier R&D outcome. New to the literature, the effect of financial constraints on R&D investment rate goes beyond the effect of a larger interest rate, and it is also distinctive from maximizing the project survival.

We use Figure 1 to visualize the trade-offs faced by an FC firm in this model. The parameter values are set as in Table 1. The solid blue lines in Figure 1 plot a sample path of the AIP cash flows, and the dashed lines depict R&D expenses. When the former crosses the latter from above, involuntary abandonment happens. In the left panel, the firm invests at the optimal level as if it were not constrained. Consequently, involuntary project abandonment occurs in Year 5, and it is earlier than the expected R&D discovery. In the right panel, the FC firm invests at its optimal level, which moves the project abandonment forward to Year 4. However, the more aggressive R&D investment speeds up the discovery and pushes the expected maturity earlier than abandonment. Therefore, it is possible that the FC firm is able to finish the R&D project before it runs out of money in the second but not the first case.

[Insert Figure 1 here.]

Next we examine two potential determinants in an FC firm's tradeoff: the payoff characteristics of the R&D project and the AIP cash flow risks, separately.

Accelerability Versus Scalability More intense investment leads to earlier burn-out of internal capital. If a project is not accelerable (that is, if $\frac{\partial \lambda_d(R)}{\partial R} = 0$), then the negative marginal effect of a more intense R&D investment on the firm value always forces a constrained firm to reduce its R&D investment from the first best level. The following result (proved in the appendix) establishes project accelerability as a necessary condition for over-investment by an FC firm.

Result 2. *If an R&D project is scalable but not accelerable, an FC firm always invests at a lower level than a UC firm– that is,* $R^*_{FC} < R^*_{UC}$.

Cash Flows Risks Both the diffusion risk and jump risk in AIP can cause project termination for an FC firm, but the mechanisms differ. Lowering investment intensity delays project termination caused by diffusion risk, however, it has no effect on the jump-risk-induced termination. On the contrary, a catastrophe risk in AIP effectively reduces the cost of financial constraints by lowering the concern of hitting the constraints early when the investment level is high, so it increases optimal investment scale. More formally, I prove the following result in the appendix.

Result 3. *If an R&D project is only scalable, then a downward jump risk on AIP cash flows motivates an FC firm to invest more intensively, – that is,* $\frac{\partial R^*_{FC}}{\partial \lambda_j} \geq 0$.

I conjecture that the motivational effect of a jump risk is stronger if the project can be accelerated modestly, which implies a positive effect of the jump risk λ_j on the endogenous discovery rate λ_d. Project accelerability provides an additional incentive for an FC firm to invest, and the same additional incentive for a UC firm is not large enough to eliminate the over-investment. At the baseline, if the jump on AIP cash flows is expected in two years instead of ten, then an FC firm invests two times more in a scalable-only project than without the jump. When we turn on the project accelerability, the FC firm invests three times more than without the jump. However, I find evidence from numerical exercises that instead of being a necessary condition for over-investment, the jump risk only enhances it. Without project accelerability, having a jump risk does not change the fact that the marginal effect of investment intensity on project survival is negative, so the marginal benefit of R on project payoff has to be greater than 1 which implies underinvestment. This argument also shows that the over-investment induced by financial constraints is not just driven by a discount effect from obsolescence. To conclude from Result 1 - 3:

Proposition 2. *Financial constraints can make a firm invest more intensively in R&D. Project accelerability is necessary for over-investment induced by financial constraints. A downward cash flow jump risk always increases a constrained firm's R&D investment, but it only exacerbates the over-investment incentive instead of being a necessary condition.*

Going back to the R&D project initiation decision, the firm optimally starts a non-deferrable innovative project if its value is positive. We can calculate such project value from Proposition 1 with the optimal R. If the project requires some initial investment cost κ, the firm optimally carries out the project if its value exceeds the fixed cost. Ceteris paribus, a UC firm is more likely to initiate investment than an FC firm because the UC firm's project value is always higher.

2.3 Implications On R&D Investment Intensity

I study comparative statics regarding how investment intensity is affected by the characteristics of innovation technology and the firm's cash flows. I show in Figure 2 the investment intensity with respect to the project scaling factor (A), scalability (β), acceleration factor (η), and accelerability (γ) around the baseline. The solid blue lines correspond to an unconstrained firm's R&D investment, and the dashed red lines correspond to a constrained firm. All these parameters have positive effects on the investment, regardless of the financial constraints. Panel (a) suggests that an FC firm's investment is more sensitive to the project scaling factor A when A is low, compared with a UC firm. When A improves from a low level (e.g., from 50 to 100), an FC firm's investment increases (e.g., from 3 to 14) more than a UC firm's (e.g., from 2 to 6). However, as A becomes larger, an FC firm's investment becomes less sensitive. It is probably because sustaining project development at a high level is more and more difficult for an FC firm. Overall, the over-investment is more apparent when A is at a medium level.

[Insert Figure 2 here.]

Panel (b) uncovers the effect of changes in the project scalability β. When the project is not scalable ($\beta = 0$), $R^*_{UC} = 2$ and $R^*_{FC} = 12.5$. As the project becomes more scalable (e.g., β rises to 0.08), R^*_{UC} increases much more (e.g., to $R^*_{UC} = 24$) than R^*_{FC} (e.g., $R^*_{UC} = 28$). In general, a UC firm is more responsive to β than an FC firm because it can always take the full advantage of project scalability without the concern on its financial ability to support R&D.

In comparison, Panels (c) and (d) plot the effects of the accelerability related aspect of R&D projects. Panel (c) shows that the acceleration factor η influences investment in a similar way as A, and Panel (d) shows a contrasting impact of project accelerability γ, different from scalability β in Panel (b). Unlike how firms respond to β, an FC firm is more responsive to γ than a UC firm, especially as γ gets closer to 1. When γ improves (e.g., from 0.4 to 0.8), R^*_{FC} increases more (e.g., from 2.4 to 27) than R^*_{UC} (e.g., from 2 to 15). This is consistent with the discussion proceeding Proposition 2 regarding the different effects of the scalable and accelerable aspects of a project. Accelerability may help the FC firm avoid involuntary abandonment, and thus has a larger effect on the FC firm's R&D intensity than a UC firm's.

I also examine the impact of a firm's AIP cash flows characteristics on its innovation strategy. In Figure 3, I plot optimal investment intensities by changing one aspect of AIP cash flows whilst keeping the other parameters at the baseline values. Clearly, all four panels confirm that a UC firm's investment is invariant with changes in AIP cash flows. Panel (a) shows a positive response of an FC firm's investment with respect to AIP cash flows level X at the project arrival. A larger profit from existing business enables an FC firm to spend more on R&D. As X increases, the FC firm's investment converges to what would have been the optimal investment for a UC firm if it were not subject to the jump risk. Panel (b) shows that as the cash flows decline faster, the FC firm reduces its investment. Numerical solutions with a large set of parameter values confirm that, although it is helpful to develop our intuition, the deterioration of AIP cash flows is not a necessary condition for over-investment induced by financial constraints.

[Insert Figure 3 here.]

Panels (c) and (d) plot the effects of two sources of uncertainty regarding firm liquidity: a jump risk and a volatility risk. The effects are opposite on a constrained firm's R&D investment. As the intensity of a catastrophe risk increases (e.g., $\lambda_j \uparrow\uparrow$ from 0.1 to 0.5) in Panel (d), an FC firm chooses a much larger R&D investment (e.g. increases from 14 to 28). On the contrary, when the cash flow volatility σ increases (e.g., doubles from 0.2 to 0.4) in Panel (c), an FC firm reduces its investment intensity (e.g., from about 14 to 13). A larger jump risk reduces the marginal cost of investment, thus motivates investment. A larger volatility risk makes the endogenous constraint hitting happens sooner, and leads to a more conservative innovation strategy. In contrast, using an investment timing model, Boyle and Guthrie (2003) find that greater uncertainty about firm liquidity increases current investment, because it raises the risk of future funding shortfalls, thereby lowering the value of waiting.

Figure 2 and Figure 3 together reveal that financial constraints are more likely to induce R&D investment when it is relatively easy to accelerate project development but difficult to scale up the discovery payoff ($\gamma \uparrow,\ \beta \downarrow$), and when the firm has better AIP cash flow prospects ($X \uparrow$, $\mu \uparrow$, $\sigma \downarrow$) but faces more imminent catastrophe risk ($\lambda_j \uparrow$).

2.4 Value Implications

The frictions in financial markets force a firm to deviate from the first-best decisions. Thus, ceteris paribus, project values of an FC firm are always lower than those of a UC firm. The baseline model allows us to gauge the difference of project values due to financial frictions, and how it depends on cash flows characteristics and R&D investment characteristics.

Figure 4 shows how project value changes with AIP cash flows. It highlights the relevance of a firm's AIP cash flows in the valuation of an R&D project when the firm is constrained. The project value of a UC firm is invariant of the AIP cash flows. For an FC firm, the upper panels imply that its R&D project value increases with the level and growth rate of AIP cash flows, and the lower panels reveal negative effects of cash flow volatility and the downward jump risk on its project value. A higher cash flow volatility usually indicates a higher value of a growth option, and this makes the pattern in Panel (c) seem counter-intuitive. Unlike the investment timing model, the investment scale decision is endogenous in this model, and a higher volatility leads to a lower optimal investment intensity. The pattern in Panel (c) thus reflects the direct effect of a higher volatility and an indirect effect through a lower level of investment. Panel (d) illustrates the same directional impact of a negative cash-flow shock as the cash flow volatility in Panel (c), only that the project value is concave in σ but convex in λ_j.

[Insert Figure 4 here.]

Figure 5 plots project values with respect to changes in R&D project characteristics. We can see that project values increase with the project's scaling factor, scalability, acceleration factor, and accelerability ($A \uparrow$ in Panel (a), $\beta \uparrow$ in Panel (b), $\eta \uparrow$ in Panel (c), and $\gamma \uparrow$ in Panel (d)) regardless of financial frictions. All four parameters can be interpreted as measures of project quality, which have positive effects on project values.

[Insert Figure 5 here.]

The comparative statics on project values may shed light on the patterns of R&D - driven mergers and acquisitions. Recent studies such as Phillips and Zhdanov (2013) and Bena and Li (2013) documented that large and more financially capable firms acquire smaller and young innovative firms, especially when the credit market freezes. My findings add to this literature by showing how such acquisitions may depend on the smaller and young innovative firms' cash flows, as well as their project characteristics. One can think of a non-deferrable R&D project being available only to an FC firm but not to a UC firm. Figure 4 and numerous unreported results based on large sets of model parameters suggest that the valuations from the two types of firms differ the most when the FC firm's cash flows are low ($X_{FC} \downarrow$), decline at a faster rate ($\mu_{FC} \downarrow$), being more volatile ($\sigma_{FC} \uparrow$), and face a higher catastrophe risk ($\lambda_j \uparrow$). Therefore, more R&D - motivated acquisitions are expected in those situations due to the larger gains to acquisitions. Meanwhile, we need to interpret such testable implications cautiously, because the model excludes an explicit motivation of being acquired or to acquire innovation.

3. The Model With An R&D Race

Firms often compete with others on innovative products or technologies in terms of speed, and the number of innovation competitors is usually not small. Like financial constraints, the competition also imposes a termination risk to firms: one firm's successful innovation makes its competitors' R&D projects obsolete in a winner-takes-all market. Unlike financial constraints, the termination risk from competition is usually a result of strategic interactions in market equilibrium. That's why I do not regard competition as an exogenous risk as previous studies (e.g.,Eisdorfer and Hsu, 2011; Hackbarth et al., 2014; Gu, 2016). But instead, I endogenize the obsolescence risk from competition by examining firms' equilibrium R&D strategies in a duopoly setting. Built on my baseline model, now two firms conduct R&D investment and compete in innovation as in a patent race (e.g., Loury, 1979; Weeds, 2002). The firms decide on their R&D investment intensities simultaneously with complete information.

3.1 Competition Among UC Firms

As a benchmark case, both competitors are financially unconstrained, and we call it "homogeneous duopoly".[9] Proposition 3 presents the project values with its proof in the Appendix.

Proposition 3. *When two financially unconstrained firms compete against each other in an R&D*

[9] A duopoly with two constrained competitors is an interesting case to study. However, adding this complex two-dimension free boundary problem is not necessary for the main insights of the paper. I therefore omit this case for potential future studies.

project, the equilibrium project values are

$$V_1 = \frac{u_1\lambda_{d,1} - R_1}{r + \lambda_{d,1} + \lambda_{d,2}} \tag{9}$$

$$V_2 = \frac{u_2\lambda_{d,2} - R_2}{r + \lambda_{d,1} + \lambda_{d,2}} \tag{10}$$

where (R_1, R_2) *is a pair of equilibrium R&D investment levels of the two unconstrained firms,* u_i *is the expected one-time project payoff for Firm* i*, and* $\lambda_{d,i}$ *is Firm i's discovery rate for* $i \in \{1, 2\}$.

Project values are independent of a firm's own and its rival's cash flows because both firms are unconstrained. The project value of a UC duopoly is similar to the baseline model (Eq.(7) in Proposition 1), except that the competitor's success rate ($\lambda_{d,-i}$) enters the discount rate. If the project is not accelerable (i.e., λ_d does not depend on R), then competing against another UC firm is equivalent to introducing an exogenous termination risk. In this case, competition reduces the marginal benefit and marginal cost of investment equally, and the firm's optimal R&D strategy remains the same as the baseline.

Result 4. *Competition among unconstrained firms motivates higher levels of R&D investment only if the innovative project is accelerable. The motivational effect of competition in an accelerable project is stronger if the project has a higher expected payoff, or if the project is more accelerable.*

The widely accepted notion that competition enhances innovation (e.g.,Weeds, 2002) holds with conditions related to project characteristics. When the maturity of an innovation project is fixed ex ante, competition between two UC firms does not make them more aggressive even if the project is scalable. However, if the project is accelerable, then the marginal benefit of speeding up the discovery and winning the competition exceeds the marginal cost of investment. This makes a duopoly firm invest at a higher rate than at the baseline. The positive effect from competition is stronger if the firm is able to shorten the project maturity more easily ($\gamma \uparrow$ or $\eta \uparrow$).

3.2 Competition Between An FC Firm And A UC Firm

Firms competing in innovation usually vary in their financing abilities. It can appear in the form of a small firm competing against a large one, or a young firm competing against a mature one, and so on. I abstract from the various scenarios and focus on analyzing market equilibria in which an FC company competes with a UC company in their R&D projects. We call the market "heterogeneous duopoly". Proposition 4 presents the project values in such cases.

Proposition 4. *In a heterogeneous duopoly, the R&D project values for the FC firm ("Firm 1") and the UC firm ("Firm 2") are*

$$V_1(X_1) = \frac{u_1\lambda_{d,1} - R_1}{r + \lambda_{d,2} + \lambda_{d,1} + \lambda_j}(1 - (\frac{X_1}{R_1})^\alpha) \tag{11}$$

$$V_2(X_1) = \frac{u_2\lambda_{d,2} - R_2 + \lambda_j V_2^m(R_2)}{r + \lambda_{d,2} + \lambda_{d,1} + \lambda_j}(1 - (\frac{X_1}{R_1})^\alpha) + V_2^m(R_2)(\frac{X_1}{R_1})^\alpha \tag{12}$$

respectively. (R_1, R_2) *is a pair of the equilibrium investment levels,* $\lambda_{d,i}$ *and* u_i *are the equilibrium discovery rate and expected project payoff of Firm* i*,* $i \in \{1, 2\}$*, and* λ_j *is the jump density of the FC firm's AIP cash flows.* $\alpha = \frac{1}{2} - \frac{\mu_1}{\sigma_1^2} - \sqrt{(\frac{1}{2} - \frac{\mu_1}{\sigma_1^2})^2 + \frac{2(r+\lambda_{d,2}+\lambda_{d,1}+\lambda_{j,1})}{\sigma_1^2}}$. $V_2^m(R_2)$ *is the UC firm's project value if it invests at* R_2 *as a baseline firm in Proposition 1 —that is,* $V_2^m = \frac{u_2\lambda_{d,2}-R_2}{r+\lambda_{d,2}}$.

The only state variable for both competitors is the constrained firm's AIP cash flows X_1. The UC firm's cash flows do not affect its own investment decision, and thus should have no impact on its FC rival's strategy. There are two possible market structures at the time of a project discovery. If both firms were still developing the project right before the discovery, then it was a duopoly. If the FC firm was already forced to abandon the project due to shortages of funds, then it was a monopoly by the UC firm. Correspondingly, the project value in Eq.(12) is a weighted average of the two scenarios for the UC firm, with weights being the probabilities. Likewise, Eq.(11) is a weighted average of FC firm's project values under the two scenarios, with the project value being zero in the second case.

It is clear from an inspection of Eq.(12) and Eq.(11) that a heterogeneous competition does not incentivize a UC firm to change its investment strategy from the baseline if the discovery rate is fixed. This observation strengthens the result from Result 4, and can be stated as follows:

Result 5. *Regardless of the rival firm's financing ability, an innovation competition only changes a UC firm's R&D investment if the project under competition is accelerable.*

3.3 Market Equilibrium

To find the equilibrium investment strategies, I examine firms' best responses to their competitor's R&D intensity. New to the literature, numerical solutions on a wide range of parameters for the model reveal two interesting patterns of R&D investment as a result of strategic interactions:[10] 1) a UC firm's best response is non-monotonic in a heterogeneous duopoly, and 2) regardless of being financially constrained or not, a firm always reacts to a UC competitor's more aggressive investment by investing more in R&D.

Figure 6 illustrates the two firms' best responses regarding their R&D investment intensities. Panel (a) graphs a heterogeneous duopoly, and Panel (b) graphs a homogeneous duopoly. The blue solid line in Panel (a) shows a hump-shaped investment intensity of a UC firm, as responses to its constrained competitor. The graph demonstrates a force that reduces the UC firm's optimal investment when its FC competitor invests beyond a certain level. As the FC firm invests more, the UC firm perceives a higher likelihood of project abandonment by the FC firm as a result of the faster burning rate of the cash flows. Consequently, competing head-to-head against the FC firm becomes less appealing. Instead, the UC firm invests at a level closer to its baseline level to maximize the project value.

[Insert Figure 6 here.]

On the other hand, a firm always becomes more aggressive in accelerable projects when facing an UC competitor, which can be seen from the monotonicity of the green circled lines in both panels. The intuition is similar to the effect of a catastrophe risk on an FC firm in Result 3. A firm has to forgo any potential profit associated with the project once its UC rival, who never abandons the project, makes a discovery. Therefore, the firm is motivated to expedite its discovery in order to lower the probability of losing the contest.

Regarding market equilibrium, examining where best responses intersect in Figure 6 help with the intuition. The symmetry of best response functions in Panel (b) leads to identical equilibrium investments in a homogeneous duopoly, at which point, the slopes of the best-response functions are positive. However, evident from Panel (a), best responses in a heterogeneous duopoly can intersect at a point where the slope of the UC firm's best response is almost flat. The UC firm has a motive to profit from the project when the FC competitor drops out, which pushes down its best-response curve and makes it less sensitive to its FC rival's investment. It could appear that a UC firm is sitting on the sideline of an innovation competition against an FC firm.

[10]Numerical solutions suggest that the fixed point for firms' best-response correspondences is unique when model parameters are within reasonable ranges.

In Table 2, I list the equilibrium R&D investment intensities and project values under different market structures at the parametrization in Table 1. Both an FC firm and a UC firm increase their investment in a duopoly, compared with their baseline levels. They also both experience a large drop in the project value as a result of competition. In addition, competing against a UC firm is more fierce than it is against an FC firm, and it leads to even more aggressive investment strategy. Column (2) shows that for an FC firm, competition doubles the investment (from 13.8 to 29.4) and cuts down one-third of the project value (from 37.6 to 21.8). Column (3) shows a threefold increase in investment (from 6.8 in the baseline to 24.4 in heterogeneous duopoly) and a half-cut of the project value (from 52.6 to 25.3) for a UC firm.

[Insert Table 2 here.]

3.4 Investment Decision And Project Value

To understand market equilibria more broadly, I conduct comparative statics analysis on equilibrium investments and project values. Figure 7 depicts equilibrium investments in heterogeneous duopoly from some of the analyses. Panel (a) graphs the equilibrium investment as a function of an FC duopoly's cash flow volatility while keeping all other parameters at the baseline. A market equilibrium is represented by a pair of investment intensities sharing the same *x-axis* value, with the UC firm on the solid line and the FC firm on the dotted line. As the FC firm's cash flow volatility increases ($\sigma_{FC} \uparrow$), both firms reduce their R&D investments. However, the UC firm reduces its investment more than the constrained competitor. It is because the probability of the UC firm ending up being the monopoly in the market is likely to be higher in such cases. The UC firm's R&D investment correspondingly moves toward its baseline level.

[Insert Figure 7 here.]

Panel (b) and Panel (c) in Figure 7 graph market equilibria as functions of project scaling factor A of the UC firm and FC firm respectively. Both firms react to the UC firm's project quality measured by A positively in Panel (b). Nevertheless, the UC firm stops increasing its R&D investment when the project of the FC competitor improves beyond a certain level in Panel (c). This pattern highlights the different investment incentives for the UC and FC firms regarding the rival's project payoff characteristics in a duopoly competition.

Moreover, firms invest more heavily in R&D in equilibrium if either their own project quality or their rival's project quality is better ($A \uparrow, \eta \uparrow, \beta \uparrow, \gamma \uparrow$). They also both invest more if the FC competitor's cash flows start at a higher level, have lower diffusion and/or jump risks, and deteriorate at a slower rate ($X \uparrow$,$\sigma \downarrow$, $\lambda_j \downarrow$,$\mu \uparrow$). In terms of the investment sensitivity to its own project characteristics, a UC firm always reacts more positively than an FC firm. However, an FC firm's investment is more sensitive to its rival's project characteristics compared with a UC firm. Figure A.1 and Figure A.2 present these comparative statics around the baseline.

Regarding project values for the two firms in a heterogeneous duopoly, they always move in the opposite directions whenever one's project characteristics or AIP cash flows change. This confirms the intuition that if one firm's project quality improves, its own project value increases while its opponent's project value decreases in general. In addition, the UC firm's AIP has no effect on both firms' strategies and project values in equilibrium. However, the FC firm's project worths more and the UC firm's project worths less if the FC firm's AIP cash flows have higher growth and lower volatility.

The model has the potential to extend the baseline implications on R&D-driven mergers and acquisitions. Table 2 shows a large increase in the UC firm's project value, if the project is carried out by the UC firm alone as opposed to be by both firms ($V = 25.3 \rightarrow 52.6$). Similarly, there is a value increase to the FC firm if there is no R&D competition. More importantly, the project value for a UC monopoly exceeds the sum of project values in the heterogeneous duopoly (52.6 vs. $25.3 + 20.4 = 45.7$). This gap could indicate a profitable acquisition by the UC firm, and its magnitude depends on innovation project characteristics and firm cash flow characteristics.

3.5 Preemption

In the context of my model, one firm preempts another in R&D competition when it achieves the discovery first. [11] With preemption defined this way, I find that an FC firm may preempt a UC firm in a head-to-head R&D competition. For example, Row (3) in Table 2 shows $R_{FC} > R_{UC}$ (29.4 vs. 24.4), which indicates an earlier expected discovery by the constrained firm. This is not a mere extension of the over-investment result in the baseline model. Competition motivates a UC firm's R&D investment more than an FC firm because a UC firm has no funding restriction. However, this

[11] Alternatively, a preemption in a heterogeneous duopoly can be understood as a firm investing more aggressively than it would in a homogeneous duopoly or in a monopoly in this setup. See Fudenberg et al. (1983) for a dynamic setting of preemption, in which the relative position during the race matters for firms' decisions.

competition driven investment incentive does not fully eliminate the over-investment from financial constraints. When it is very likely for the constrained rival to blow up the cash flows before its project discovery, the unconstrained firm pulls back its R&D activity from the homogeneous duopoly level, and behaves more like in baseline. Meanwhile, the only way for an FC duopoly to retain project value from competition is to be aggressive. Such strategic considerations lead to preemption by a constrained firm.

Result 6. *In an equilibrium with heterogeneous duopoly on R&D, an FC firm can preempt a UC firm by investing more intensively.*

The novelty of the preemption result is also reflected by the fact that the conditions for preemption is different than those for over investment in the baseline model. From unreported large numbers of numerical solutions, a constrained firm is more likely to preempt its unconstrained competitor when its cash flow risk is high, deteriorates faster, and starts at a higher level ($\sigma \uparrow$, $\lambda_j \uparrow$, $\mu \downarrow$, $X \uparrow$). Recall that in the baseline model, an FC firm is more likely to over-investment comparing with a UC firm when its AIP cash flow diffusion risk is low and it declines more slowly ($\sigma \downarrow$, $\mu \uparrow$). The opposite conditions for preemption in heterogeneous competition are a result of the strategic interactions in the duopoly equilibrium.

In the model, we exclude new entries in the innovation competition, which can be justified by the high barrier of entry due to the specific knowledge or human capital needed to develop R&D projects. If instead, firms can enter the innovative market by paying a fixed cost, and there is a large pool of potential contestants, then the profits should be competed away. Because an unconstrained firm always values an R&D project more than an otherwise identical but constrained firm, we expect R&D investments to only come from unconstrained firms in a fully competitive market. The fact that a more constrained firm makes R&D investment in a competitive environment with endogenous entry is likely to suggest that its project is superior in some aspects or it has a low fixed cost to start. Moreover, the results on over-investment and preemption can imply a socially negative or positive effect of financial constraints on innovation, depending on whether one takes the view that there is too much innovation (Biais, Rochet, and Woolley, 2015) or too little innovation (Hall and Lerner, 2010) than socially optimal.

4. Model Robustness

As a robustness check for the baseline model, I examine an FC firm's R&D strategy if it can access external financing at a cost. Consider a firm which can finance its cash flow gap $(R - X_t)$ at some cost $g((R - X_t)^+)$ when its instantaneous AIP cash flow X_t falls short of R&D investment cost R. We call it a costly external financing firm, or a "CEF firm". The cost function $g(\cdot)$ captures a cash bribery to the existing equity holders when it issues new equity or a flotation cost at issuance, and it is increasing and convex.[12] In the numerical exercise $g((R - X_t)^+) = \delta((R - X_t)^+)^2$. The scalar δ (> 0) indicates the extent to which the firm is constrained. Assume the CEF firm follows a threshold strategy and keeps investing in the R&D project until $X_t \leq \underline{X}_{CEF}$,[13] and there is no catastrophic risk—that is, $\lambda_j \to \infty$. The R&D project value for a CEF firm is

$$V_{CEF}(X) \quad = \quad \sup_{\tau_c, R} E\Big\{ \int_0^{\tau_c \wedge \tau_d} [-R - g(X_t)] e^{-rt} dt + 1_{\{\tau_d < \tau_c\}} u e^{-r\tau_d} \Big\}, \tag{13}$$

where abandonment time is defined similarly as before $\tau_c = \inf\{t : X_t \leq \underline{X}_{CEF}\}$. Using either dynamic programming or the contingent claim approach, we can get the Hamilton-Jacobi-Bellman (HJB) equation of the project value before discovery or abandonment:

$$(r + \lambda_d)V = \mu X V_X + \frac{1}{2}\sigma^2 X^2 V_{XX} + \lambda_d u - R - g(X). \tag{14}$$

The Appendix shows how to solve this ordinary differential equation (ODE) with boundary conditions. Figure 8 illustrates two examples of abandonment thresholds $\underline{X}_{CEF}$. The green 45-degree line plots the abandonment threshold for an FC firm ($\delta = \infty$), which shows that $\underline{X}_{CEF} = R$ and it is consistent with Proposition 1. One can view the horizontal axis $\underline{X}_{CEF} = 0$ as the case for a UC firm. The blue line corresponds to the abandonment threshold when the financing cost parameter $\delta = 2$. The red line corresponds to the abandonment threshold when the financing cost is $\delta = 0.5$. The flat parts of the colored lines indicate that a firm does not carry out the project, because it has negative NPV at these investment levels.

[12] The same assumptions are used in papers on financial constraints such as Kaplan and Zingales (1997) and Hennessy and Whited (2007).

[13] Another possible threshold strategy is based on the state of accumulative investment, as it is used in Berk et al. (2004). I eliminate this alternative threshold strategy because the project success intensity is exogenous, and the Poisson process implies a constant success possibility in any instance.

[Insert Figure 8 here.]

Notice that a CEF firm's optimal financing threshold is lower than a constrained firm and higher than an unconstrained firm—that is, $\underline{X}^*_{CEF}(R) \in (0, R)$. This means that a CEF firm makes use of the external financing partially. In addition, the more costly it is to obtain external financing, the higher the abandonment threshold is, and the earlier the expected endogenous project abandonment. We can then use the optimal abandonment threshold and solve for R_{CEF} numerically, and verify that it lies between R_{UC} and R_{FC}.

5. Final Remarks

Using a parsimonious model, I examine the impacts of financial constraints and competition on corporate innovation with a focus on firms' R&D investment intensity decision. While financial constraints often reduce firms' incentive to invest in R&D, I show that they can also raise R&D investment if the project is accelerable. Preemption by constrained firms can occur in market equilibrium as a result of strategic interactions among firms with different financing constraints. The model yields novel testable implications on values and R&D investment rate based on the characteristics of a firm's cash flows and its R&D investment opportunity, and those of its competitor's.

By incorporating both project accelerability (widely considered in the patent race literature) and project scalability (often assumed in the investment literature), my paper shows that the characteristics of innovation projects can significantly impact a firm's R&D decisions. This study calls for more careful empirical investigations on R&D investment through the consideration of innovation technology, and can help understand corporate decisions which not only affect the levels but also the timing of cash flows.

There are a few directions for related future research. One regards endogenous choice of innovation technology by firms with different financing frictions, especially if firms can trade off between innovation speed and scale. A second one regards the impact of having sequential investment opportunities on the R&D investment decision. Another significant omission in this paper is cash holding. I leave the study of the optimal liquidity management policy for R&D firms to future research.

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Figure 1 – A Constrained Firm's Tradeoff in the Baseline Model

This figure illustrates the trade-off of a constrained firm at the baseline model, using parameter values in Table 1. The solid blue line plots a sample path of the AIP cash flows, and the dashed lines depict R&D expenses. The left panel uses an unconstrained firm's optimal investment intensity and the right panel uses a constrained firm's optimal investment intensity. When the AIP cash flows cross the R&D investment from above, involuntary abandonment happens (represented by a red dot). Expected R&D discovery is marked as a green dot.

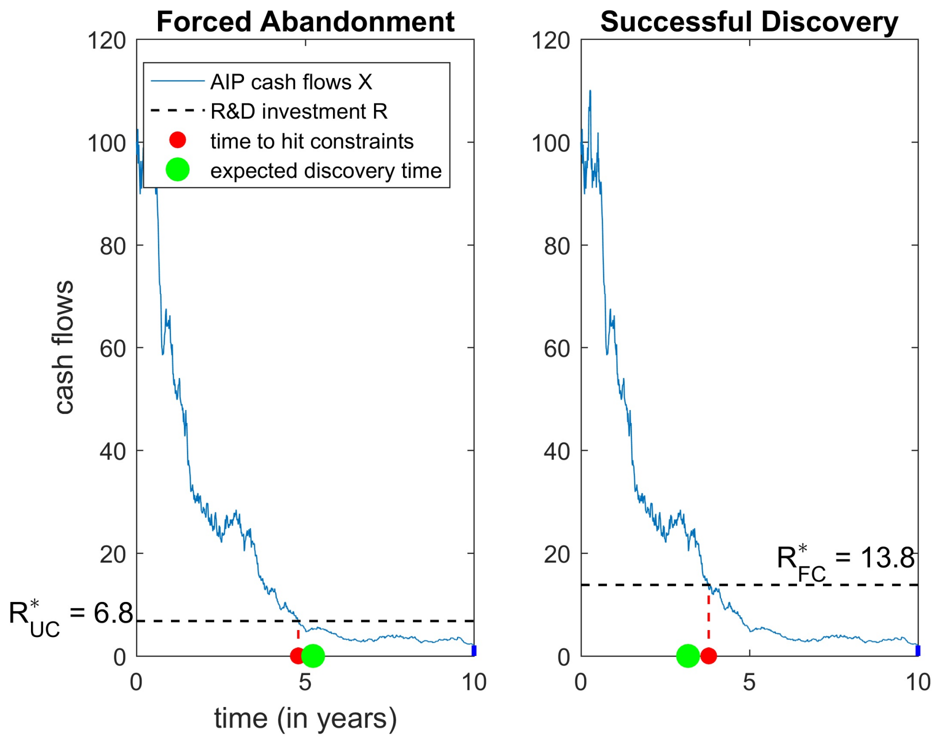
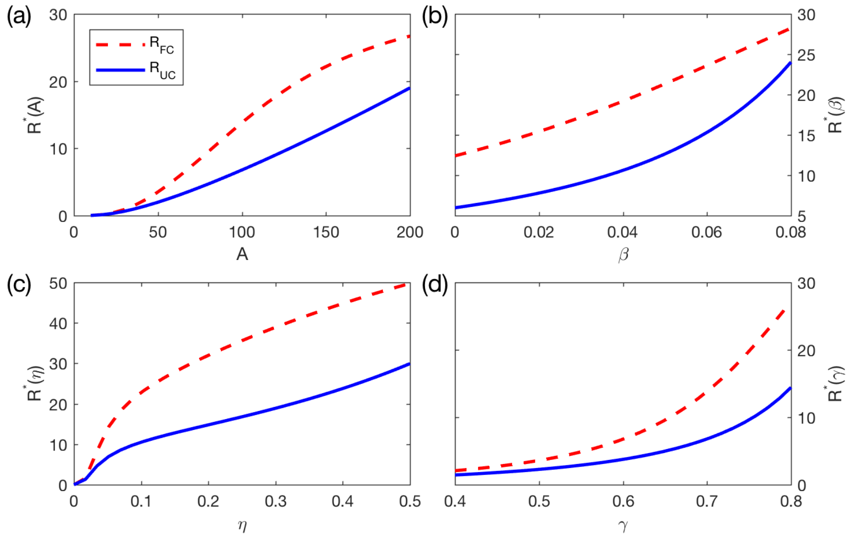
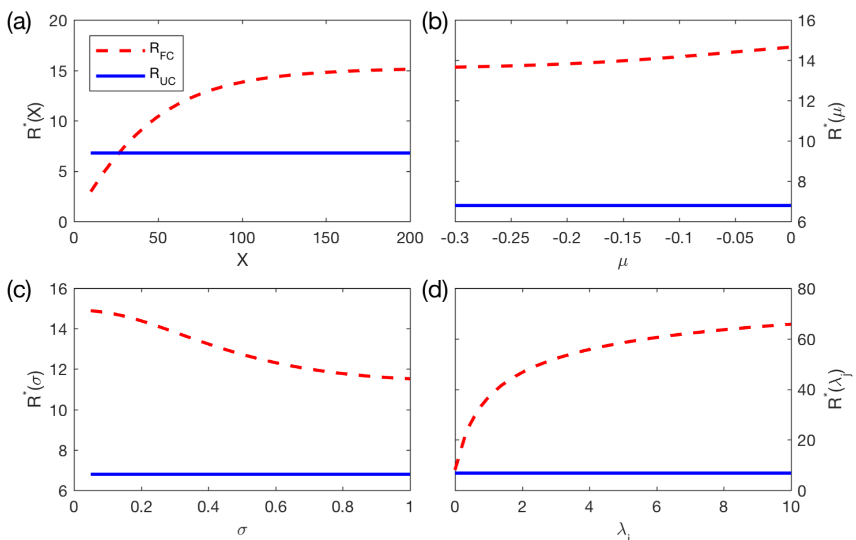


Figure 2 – R&D Investment Intensities with respect to Project Parameters



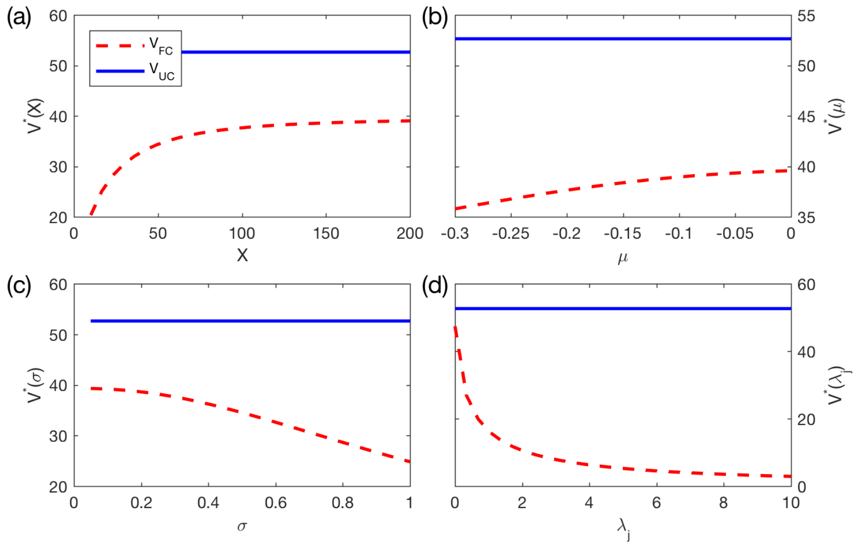
Baseline: A=100; β=0.01; η=0.05; γ=0.7; X=100; μ=-0.2; σ=0.3; r=0.05; λ_j=0.1
E(payoff) = $A R^{\beta}$, $\lambda_d = \eta R^{\gamma}$; R^*_{FC}=13.82; R^*_{UC}=6.79

Figure 3 – R&D Investment Intensities with respect to Cash Flow Parameters



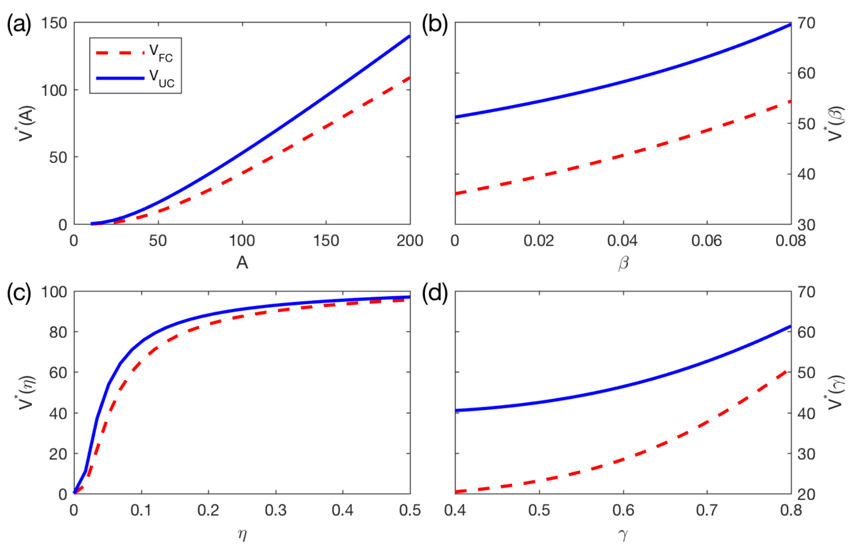
Baseline: A=100; β=0.01; η=0.05; γ=0.7; X=100; μ=-0.2; σ=0.3; r=0.05; λ_j=0.1
E(payoff) = $A R^{\beta}$, $\lambda_d = \eta R^{\gamma}$; R^*_{FC}=13.82; R^*_{UC}=6.79; $\tau_{d,fc}$=3.2; τ_c=8.08

Figure 4 – R&D Project Values with respect to Cash Flow Parameters



Baseline: A=100; β=0.01; η=0.05; γ=0.7; X=100; μ=-0.2; σ=0.3; r=0.05; λ_j=0.1
E(payoff) = A R^{β}, $\lambda_d = \eta R^{\gamma}$; V^*_{FC}=37.64; V^*_{UC}=52.63; $\tau_{d,fc}$=3.2; τ_c=8.08

Figure 5 – R&D Project Values with respect to Project Parameters



Baseline: A=100; β=0.01; η=0.05; γ=0.7; X=100; μ=-0.2; σ=0.3; r=0.05; λ_j=0.1
E(payoff) = A R^{β}, $\lambda_d = \eta R^{\gamma}$; V^*_{FC}=37.64; V^*_{UC}=52.63

Figure 6 – Best Responses in Heterogeneous vs. Homogeneous Competition

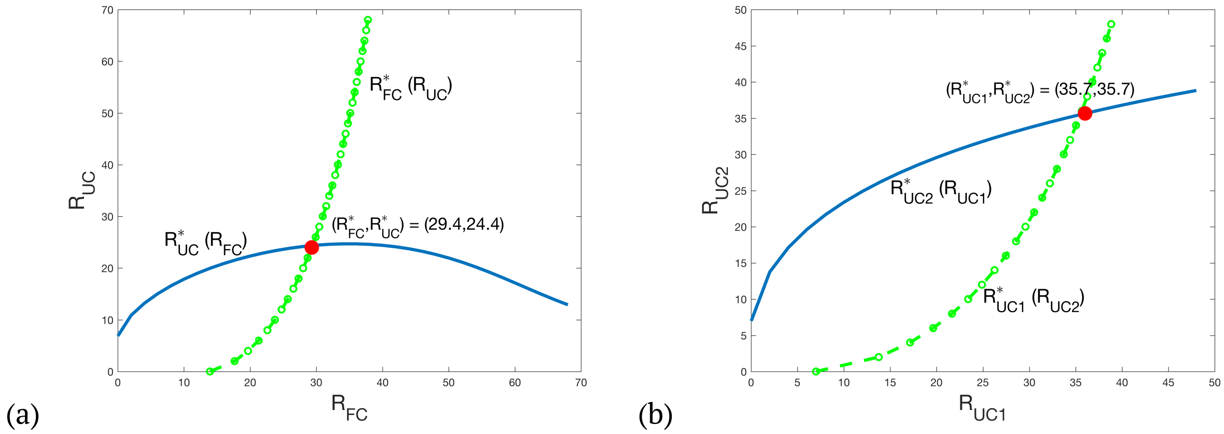


Figure 7 – Market Equilibria in Heterogeneous Duopolies

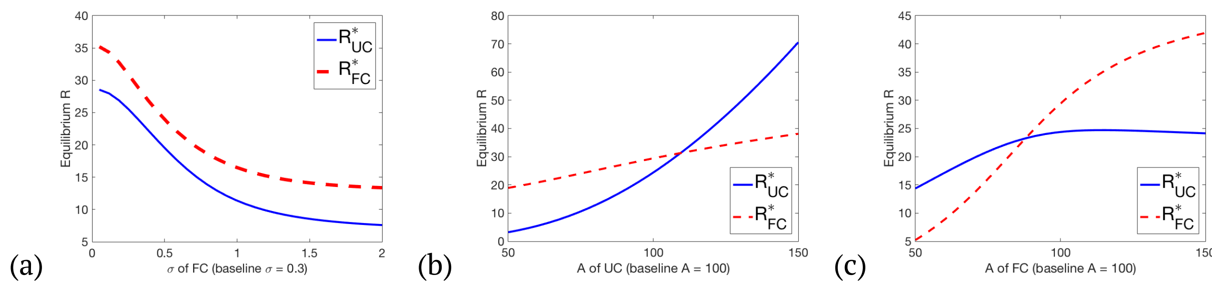
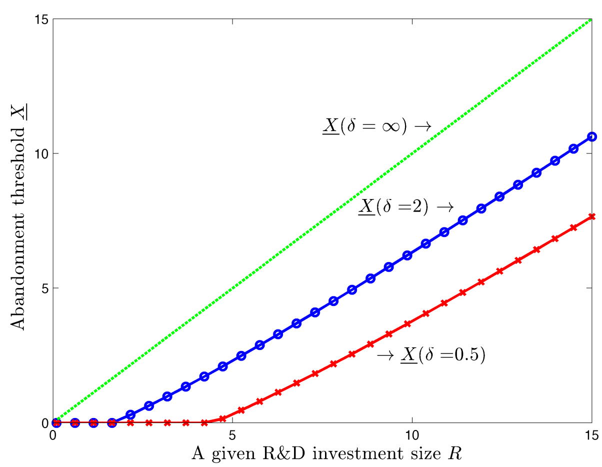


Figure 8 – Optimal Abandonment Threshold



Baseline: A=100; η=0.05; β=0.01; γ=0.7; μ =-0.2; σ=0.3; X=100; r=0.05

External financing cost = δ * max((R-X),0)^2

Table 1 – Baseline Parameter Values for Numerical Solutions

Parameter	**Value**
Discount rate	$r = 0.05$
Discovery rate ($\lambda_d = \eta R^\gamma$)	$\eta = 0.05$, $\gamma = 0.7$
Expected project payoff ($u = AR^\beta$)	$A = 100$, $\beta = 0.01$
AIP cash flows at the project's arrival	$X = 100$
Decline rate of AIP cash flows	$\mu = -0.2$
Volatility of AIP cash flows	$\sigma = 0.3$
Catastrophe risk of AIP cash flows	$\lambda_j = 0.1$

Table 2 – Investment Intensities and Project Values

	The constrained firm	The unconstrained firm
Baseline	$R = 13.8$, $V = 37.6$	$R = 6.8$, $V = 52.6$
Heterogeneous duopoly	$R = 29.4$, $V = 21.8$	$R = 24.4$, $V = 25.3$
Homogeneous duopoly	–	$R = 35.5$, $V = 21.7$

Appendix A. Proofs and derivations

Proof of Proposition 1

Proof. Recall that the state variable for the project value is the cash flow from assets in place which follows a mixed Poisson-Wiener process of the form $\frac{dX}{X} = \mu dt + \sigma dZ + dq_j$, where dq_j takes the value of -1 with probability λdt and 0 with probability $1 - \lambda dt$. By Itô's lemma (Chapter 3.6 of Dixit and Pindyck, 1994), the expected change in the project value given any choice of R is

$$E\mathscr{D}V = \Big\{\frac{\partial V}{\partial X}\mu X + \frac{1}{2}\frac{\partial^2 V}{\partial X^2}\sigma^2 X^2 + \lambda_j[V(0) - V]\Big\}dt$$

For the optimal strategy, the required rate of return for investing in the project should equal to the expected rate of capital gain minus the flow payment to the project and plus the expected payoff of the project at discovery whilst taking the discovery probability into consideration. Therefore, we can write the Hamilton-Bellman-Jacobi (HJB) equation on the project value as

$$rV = \frac{E\mathscr{D}V}{dt} - R + \lambda_d(u - V). \tag{15}$$

If the firm is unconstrained, then the value of the project does not depend on the internal cash flow. Thus $V_X = 0, V_{XX} = 0$, and $V(0) = V$. Then from Eq.(15), we can get the project value as $V_{UC} = \sup_R \frac{\lambda_d(R)u(R) - R}{\lambda_d(R) + r}$. Note that we can also get the same expression by formulating the unconstrained firm's problem directly as $\sup_R E(e^{-r\tau_d}\tilde{u} - \int_0^{\tau_d} Re^{-rt}dt)$.

If the firm is constrained, then the value of the project when there is no internal cash flow should be zero. With $V(0) = 0$, Eq.(15) becomes

$$(r + \lambda_d + \lambda_j)V = \frac{\partial V}{\partial X}\mu X + \frac{1}{2}\frac{\partial^2 V}{\partial X^2}\sigma^2 X^2 + \lambda_d u - R. \tag{16}$$

The solution to the ordinary differential equation follows the form of $V(X) = A_1 X^{\alpha_1} + A_2 X^{\alpha_2} + \frac{u\lambda_d - R}{r + \lambda_d + \lambda_j}$, where α_1, α_2 are the solutions of the quadratic function $\frac{1}{2}\sigma^2\alpha(\alpha - 1) + \mu\alpha - (r + \lambda_d + \lambda_j) = 0$, i.e., $\alpha_1, \alpha_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(\lambda_d + r + \lambda_j)}{\sigma^2}}$. Suppose $\alpha_1 < 0$ and $\alpha_2 > 0$. The project value is subject to the two following boundary conditions:

$$\lim_{X \to \infty} V(X) = \frac{u\lambda_d - R}{r + \lambda_d + \lambda_j} \tag{17}$$

$$V(X = \underline{X}) = 0. \tag{18}$$

The first no-bubble condition gives us A_2=0, and the second value matching condition at the abandonment threshold gives us the value of A_1, which is a function of $\underline{X}$. Together,

$$V_{FC}(X) = \sup_{\{(R, \underline{X}_{FC}); \underline{X}_{FC} \geq R\}} \frac{u\lambda_d - R}{\lambda_d + r + \lambda_j}\left(1 - \left(\frac{X}{\underline{X}_{FC}}\right)^{\alpha_1}\right). \tag{19}$$

A usual smooth pasting condition at the abandonment is not used to solve the problem because voluntary abandonment is not optimal. It is obvious that the lower the abandonment thresholds are, the higher the project values will be, therefore $\underline{X}^*_{FC} = R$.

□

Proof of Result 2

Proof. Non-accelerability implies that the discovery rate λ_d does not depend on R. The first-order condition for the FC firm is

$$\frac{\partial(1-(\frac{X}{R})^{\alpha_1})}{\partial R} \times \frac{u(R)\lambda_d - R}{\lambda_d + r + \lambda_j} + \frac{\partial \frac{u(R)\lambda_d - R}{\lambda_d + r + \lambda_j}}{\partial R} \times (1-(\frac{X}{R})^{\alpha_1}) = 0. \tag{20}$$

The first of the two terms is negative, because $u(R)\lambda_d > R$ (for a firm to start the project), and the derivative of $1-(\frac{X}{R})^{\alpha_1}$ with regard to R is always negative. We know $1-(\frac{X}{R})^{\alpha_1} > 0$, so it has to be $u'(R)\lambda_d - 1 > 0$ at R^*_{FC} for Eq.(20) to hold. It is clear from a UC firm's problem that $u'(R)\lambda_d - 1 = 0$ at R^*_{UC}. From the concavity of f, $R^*_{FC} < R^*_{UC}$. The second-order derivative shows that a sufficient condition for the project value to be concave everywhere is $\alpha_1 < -1$:

$$\begin{aligned}\frac{\partial^2 V_{FC}}{\partial R^2} &= \underbrace{\frac{u''(R)\lambda_d}{\lambda_d + r + \lambda_j}}_{<0} \underbrace{(1-(\frac{X}{R})^{\alpha_1})}_{>0} + \underbrace{\frac{u'(R)\lambda_d - 1}{\lambda_d + r + \lambda_j}}_{>0} \times \underbrace{2\alpha_1 X^{\alpha_1} R^{-1-\alpha_1}}_{<0} \\ &\quad + \underbrace{\frac{u(R)\lambda_d - R}{\lambda_d + r + \lambda_j}}_{<0} \alpha_1(-1-\alpha_1) \underbrace{X^{\alpha_1} R^{-2-\alpha_1}}_{>0}.\end{aligned}$$

□

Proof of Result 3

Proof. Denote $h(R^*, \lambda_j) = \left.\frac{\partial V_{FC}}{\partial R}\right|_{R=R^*} = 0$ as the first order condition for the FC firm. When λ_d does not depend on R, we can write

$$h(R^*, \lambda_j) = (u'\lambda_d - 1)((1-(\frac{X}{R^*})^{\alpha_1}) + (u\lambda_d - R^*)\alpha_1 X^{\alpha_1} R^{*-\alpha_1-1}$$

To prove $\frac{dR^*}{d\lambda_j} \geq 0$, we apply the Implicit Function Theorem $\frac{dR^*}{d\lambda_j} = -\frac{\frac{\partial h(R^*,\lambda_j)}{\partial \lambda_j}}{\frac{\partial h(R^*,\lambda_j)}{\partial R^*}}$. The denominator $\frac{\partial h(R^*,\lambda_j)}{\partial R^*} = u''\lambda_d((1-(\frac{X}{R^*})^{\alpha_1}) + 2(u'\lambda_d - 1)\alpha_1 X^{\alpha_1} R^{*-\alpha_1-1} + (u\lambda_d - R^*)\alpha_1(-\alpha_1 - 1)X^{\alpha_1} R^{*-\alpha_1-2} < 0$ with a sufficient condition $\alpha_1 < -1$. The numerator

$$\begin{aligned}\frac{\partial h(R^*, \lambda_j)}{\partial \lambda_j} &= \frac{\partial h}{\partial \alpha_1} \times \frac{\partial \alpha_1}{\partial \lambda_j} = \frac{\partial \alpha_1}{\partial \lambda_j} \times (\frac{X}{R^*})^{\alpha_1}\Big[-(u'\lambda_d - 1)\ln(\frac{X}{R^*}) + \frac{(u\lambda_d - R^*)}{R^*}\Big(1 + \alpha_1 \ln(\frac{X}{R^*})\Big)\Big] \\ &= \frac{\partial \alpha_1}{\partial \lambda_j} \frac{(\frac{X}{R^*})^{\alpha_1}}{1-(\frac{X}{R^*})^{\alpha_1}}\Big(\frac{u\lambda_d}{R^*} - 1\Big)\Big[\alpha_1(\frac{X}{R^*})^{\alpha_1}\ln(\frac{X}{R^*}) + \Big((1-(\frac{X}{R^*})^{\alpha_1}\Big)\Big(1 + \alpha_1 \ln(\frac{X}{R^*})\Big)\Big]\end{aligned}$$

$$= \underbrace{\frac{\partial \alpha}{\partial \lambda_j}}_{-} \times \underbrace{\frac{(\frac{X}{R^*})^{\alpha_1}}{(1-(\frac{X}{R^*})^{\alpha_1}}(\frac{u\lambda_d}{R^*}-1)}_{+} \Big[1-(\frac{X}{R^*})^{\alpha}+\alpha\ln(\frac{X}{R^*})\Big]. \tag{21}$$

$h(R^*, \lambda_j) = 0$ is used to get Eq.(21) from the equation before that. The last term in Eq. (21) is a decreasing function of $\frac{X}{R^*}$ on its domain $\frac{X}{R^*} \in [1, \infty]$, and it is zero when $\frac{X}{R^*} \to 1$ whilst it is negative when $\frac{X}{R^*} \to \infty$. Therefore, the last term is non-positive on its domain, and Eq.(21) is non-negative. Consequently, $\frac{dR^*}{d\lambda_j} \geq 0$. □

Proof of Proposition 3

Proof. Let's call the two UC firms Firm 1 and Firm 2. The two firms' problems are the same, so we just prove the project value for Firm 1. The subscript i denote the firms, and we use τ to denote discovery time without confusion here.

$$\begin{aligned}
\sup_{R_1} E\Big(\mathbb{1}_{\{\tau_1<\tau_2\}}e^{-r\tau_1}\tilde{u}_1 - \int_0^{\tau_1\wedge\tau_2} R_1 e^{-rt}dt\Big) &= \sup_{R_1} E\int_0^{\tau_2} \lambda_1\tilde{u}_1 e^{-(r+\lambda_1)\tau_1}d\tau_1 - \int_0^{\tau_2} R_1 e^{-(r+\lambda_1)t}dt \\
&= \sup_{R_1} E_{\tau_2}\int_0^{\infty}(\lambda_1 u_1 - R_1)\mathbb{1}_{\{t<\tau_2\}}e^{-(r+\lambda_1)t}dt \\
&= \sup_{R_1} \frac{u_1\lambda_1 - R_1}{\lambda_1+\lambda_2+r}
\end{aligned}$$

Alternatively, we can also use the Bellman equation which describes the condition that the required rate of return for the investment equals the expected rate of capital gain minus the flow payment, plus the expected probability weighted payoff at project discovery and subtract the expected probability weighted loss at rival's discovery:

$$rV_1 = E\mathscr{D}V_1 - R_1 + \lambda_1(u_1 - V_1) + \lambda_2(0 - V_1).$$

Because the cost of external financing and internal financing is the same for a UC firm, its project value does not depend on its own cash flow or its rival's cash flow. Also the project value is not time dependent. Therefore $E\mathscr{D}V_1 = 0$, and we can get the project value.

□

Proof of Result 4

Proof. To show the effect of competition on a UC firm, say Firm 1, we compare $\hat{R}_1 = \text{argmax}_R \frac{u_1\lambda_1 - R_1}{r+\lambda_1+\lambda_2}$ with competition and $\bar{R}_1 = \text{argmax}_R \frac{u_1\lambda_1 - R_1}{r+\lambda_1}$ without competition. Because the rate of the rival's success λ_2 acts as an added discount factor, comparing the two investment scales is equivalent to derive the sign of $\frac{\partial \bar{R}_1}{\partial r}$. If it is positive, then competition on R&D increases the UC firm's investment.

If the project is accelerable, that is, $\lambda_1 = \lambda_1(R_1)$, and for simplicity on the derivation, let's focus on the projects that are not scalable, i.e., $\frac{\partial u_1}{\partial R_1} = 0$. The first order condition for $\bar{R}_1$ is that $h(\bar{R}_1) = \lambda_1'(u_1 r + \bar{R}_1) - (r + \lambda_1) = 0$, which implies that $\lambda_1' u_1 - 1 = \frac{u_1\lambda_1 - R_1}{u_1 r + R}$. The last expression is positive given the project was started. Because $\frac{\partial h}{\partial r} = \lambda_1' u_1 - 1 > 0$ and $\frac{\partial h}{\partial \bar{R}_1} = \lambda_1''(u_1 r + \bar{R}_1) < 0$, we apply the Implicit Function Theorem and get $\frac{\partial \bar{R}_1}{\partial r} = -\frac{\frac{\partial h}{\partial r}}{\frac{\partial h}{\partial \bar{R}_1}} > 0$. Back to the effect of competition, this implies that $\hat{R}_1 > \bar{R}_1$ for any $\lambda_2 > 0$, i.e., competition increases a UC firm's R&D investment scale if the project is accelerable.

However, if the project is not accelerable, that is, λ_1 does not depend on R_1, then the first order condition indicates that both $\hat{R}_1$ and $\bar{R}_1$ satisfy $u_1'(R)\lambda_1 = 1$. Since $u_1'(R)$ is monotonic by assumption, $\hat{R}_1 = \bar{R}_1$, that is, R&D competition does not affect a UC firm's R&D investment scale.

□

Proof of Proposition 4

Proof. Denote the FC firm as Firm 1 and the UC firm as Firm 2. Replace λ_d by λ_d to simplify the subscripts. The project value during innovation race for Firm 1 satisfies the following HJB equation at the optimum:

$$rV_1 dt = E\mathscr{D}V_1 - R_1 dt + \lambda_1(u_1 - V_1)dt + \lambda_2(0 - V_1)dt, \tag{22}$$

where $E\mathscr{D}V_1(X_1) = \left\{\frac{\partial V_1}{\partial X_1}\mu_1 X_1 + \frac{1}{2}\frac{\partial^2 V_1}{\partial X_1^2}\sigma_1^2 X_1^2 + \lambda_j[V_1(0) - V_1]\right\}dt$ and $V_1(0) = 0$. The solution of the corresponding ODE on V_1 follows the form of $V_1(X_1) = c_1 X_1^{a_1} + c_2 X_1^{a_2} + \frac{\lambda_1 u_1 - R_1}{r + \lambda_1 + \lambda_2 + \lambda_j}$, where a_1, a_2 are the roots of the quadratic function $\frac{1}{2}\sigma_1^2 \alpha(\alpha - 1) + \mu_1 \alpha - (r + \lambda_1 + \lambda_2 + \lambda_j) = 0$. The project value is subject to the two boundary conditions:

$$\lim_{X_1 \to \infty} V_1(X_1) = \frac{\lambda_1 u_1 - R_1}{r + \lambda_2 + \lambda_1 + \lambda_j} \tag{23}$$

$$V_1(X_1 \to \underline{X}_1) = 0. \tag{24}$$

The first condition states that when the FC firm's AIP cash flows become very high, its project value almost converges to that of an unconstrained firm, except that the discount rate includes the jump rate of AIP cash flow because such a jump stops the project cash flow for the FC firm but not for a UC firm. The second is a value matching condition. We then obtain Eq.(11) in the proposition, provided that the optimal abandonment threshold $\underline{X}_1$ equals its minimal level R_1.

For the UC firm in the innovation race, its project value satisfies the HJB equation:

$$rV_2 dt = E\mathscr{D}V_2 - R_2 dt + \lambda_2(u_2 - V_2)dt + \lambda_1(0 - V_2)dt + \lambda_j(V_2^m - V_2)dt, \tag{25}$$

$$rV_2 dt = E\mathscr{D}V_2 - R_2 dt + \lambda_2(u_2 - V_2)dt + \lambda_1(0 - V_2)dt, \tag{26}$$

where $E\mathscr{D}V_2(X_1) = \left\{\frac{\partial V_2}{\partial X_1}\mu_1 X_1 + \frac{1}{2}\frac{\partial^2 V_2}{\partial X_1^2}\sigma_1^2 X_1^2 + \lambda_j[V_2(X_1 = 0) - V_2(X_1)]\right\}dt$ and $V_2(X_1 = 0)$ is the UC firm's monopolistic project value given it invests at R_2. For simplicity, we denote it as V_2^m. The ODE can be solved with the two boundary conditions:

$$\lim_{X_1 \to \infty} V_2(X_1) = \frac{u_2 \lambda_2 - R_2 + \lambda_j V_2^m}{r + \lambda_2 + \lambda_1 + \lambda_j} \tag{27}$$

$$V_2(X_1 = \underline{X}_1) = V_2^m. \tag{28}$$

Eq.(27) states that is the condition that the UC firm's project value converges to what is similar to an unconstrained duopoly in a homogeneous duopoly, if the FC duopoly has a high level of cash flows from its AIP. There are two variations as a result of the FC firm's AIP cash flows jump process: we have $\lambda_j V_2^m$ in the annuity term, and λ_j in the discount rate. This is because such a jump terminates the FC firm's project, and effectively changes the duopoly to a UC monopoly. Eq.(28) is a value matching condition that the UC firm recovers its monopolistic project value when the FC rival abandons the project. Solving V_2 yields Equation (12) in the proposition, provided that $\underline{X}_1 = R_1$. □

The case with with costly external financing in Section 4

The HJB equation Equation (14) is a linear second-order ODE. Similar to Liu and Loewenstein (2002), we follow Boyce and DiPrima (2000) to get its solution.[14] If the functions p,q and g are continuous on an open interval I, and if the functions y_1 and y_2 are linearly independent solutions of the homogeneous equation $y" + p(t)y' + q(t)y = 0$ corresponding to the non-homogeneous equation $y" + p(t)y' + q(t)y = g(t)$, then a particular solution of the non-homogeneous equation is

$$Y(t) = -y_1(t)\int_{t_1}^{t}\frac{y_2(s)g(s)}{W(y_1,y_2)(s)}ds + y_2(t)\int_{t_2}^{t}\frac{y_1(s)g(s)}{W(y_1,y_2)(s)}dt$$

where the Wronskian $W = y_1 y_2^{'} - y_1^{'} y_2$. The general solution for Equation (14) is

$$V(X) = c_1 X^{\alpha_1} + c_2 X^{\alpha_2} + V_p(X)$$

where $\alpha_1, \alpha_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(\lambda_d + r)}{\sigma^2}}$ with $\alpha_1 < 0$ and $\alpha_2 > 0$. A particular solution for Equation (14) is

$$V_p(X) = -X^{\alpha_1}\int_{t_1^*}^{X}\frac{2(R+g(t)-u\lambda_d)}{(\alpha_2-\alpha_1)t^{\alpha_1+1}\sigma^2}dt + X^{\alpha_2}\int_{t_2^*}^{X}\frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_2+1}\sigma^2}dt. \tag{29}$$

Let's set both of the lower bounds at the convenience level $\underline{X}_{CEF}$, i.e. $t_1^* = t_2^* = \underline{X}_{CEF}$.

Rewrite $V(X)$ by plugging the particular solution of Equation (29), substituting t_1^* and t_2^* in the general solution, and sorting the terms:

$$V(X) = X^{\alpha_1}(c_1 - \int_{\underline{X}_{CEF}}^{X}\frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_1+1}\sigma^2}dt) + X^{\alpha_2}(c_2 + \int_{\underline{X}_{CEF}}^{X}\frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_2+1}\sigma^2}dt) \tag{30}$$

The boundary conditions for the HJB equation are

$$\lim_{X\to\infty} V(X) = \frac{u\lambda_d - R}{\lambda_d + r} \tag{31}$$

$$V(\underline{X}_{CEF}) = 0 \text{ (value matching)} \tag{32}$$

$$\frac{dV(X)}{dX}\Big|_{X=\underline{X}_{CEF}} = 0 \text{ (smooth pasting).} \tag{33}$$

[14] Theorem 3.7.1. in the 12th edition of the book.

The first boundary condition gives us that the coefficient associated with term X^{α_2} is zero as $X \to \infty$, given $\lim_{X\to\infty} \int_{\underline{X}_{CEF}}^{X} \frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_2+1}\sigma^2}\mathrm{dt}$ has a finite limit. If g is a polynomial as assumed in the extended model, consisting the highest degree of h, then a sufficient condition for having a finite limit is $\alpha_2 > h$. Thus,

$$c_2 = -\int_{\underline{X}_{CEF}}^{\infty} \frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_2+1}\sigma^2}\mathrm{dt}. \tag{34}$$

We need to verify that c_2 satisfies the first boundary condition. By plugging the expression of c_2 into $V(X)$ in Equation (30) and taking its limit, we can get

$$\begin{aligned}\lim_{X\to\infty} V(X) &= \lim_{X\to\infty}\{c_1 X^{\alpha_1} - X^{\alpha_2}\int_{\underline{X}_{CEF}}^{\infty} \frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_2+1}\sigma^2}\mathrm{dt} \\ &- X^{\alpha_1}\int_{\underline{X}_{CEF}}^{X} \frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_1+1}\sigma^2}\mathrm{dt} + X^{\alpha_2}\int_{\underline{X}_{CEF}}^{X} \frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_2+1}\sigma^2}\mathrm{dt}\} \\ &= \lim_{X\to\infty}\{-\underbrace{X^{\alpha_1}}_{\to 0}\underbrace{\int_{\underline{X}_{CEF}}^{X} \frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_1+1}\sigma^2}\mathrm{dt}}_{\to\infty \text{ if } \alpha_1+1<0} - \underbrace{X^{\alpha_2}}_{\to\infty}\underbrace{\int_{X}^{\infty} \frac{2(R+g(t)-\lambda_d u)}{(\alpha_2-\alpha_1)t^{\alpha_2+1}\sigma^2}\mathrm{dt}}_{\to 0}\}.\end{aligned} \tag{35}$$

By L'Hopital's Rule and provided that $\alpha_1\alpha_2 = -\frac{2(r+\lambda_d)}{\sigma^2}$, the first boundary condition is verified:

$$\begin{aligned}\lim_{X\to\infty} V(X) &= \lim_{X\to\infty}\{-\frac{\frac{2(R+g(X)-\lambda_d u)}{(\alpha_2-\alpha_1)X^{\alpha_1+1}\sigma^2}}{-\alpha_1 X^{-\alpha_1-1}} - \frac{-\frac{2(R+g(X)-\lambda_d u)}{(\alpha_2-\alpha_1)X^{\alpha_2+1}\sigma^2}}{-\alpha_2 X^{-\alpha_2-1}}\} \\ &= \frac{2(R-\lambda_d u)}{(\alpha_2-\alpha_1)\alpha_1\sigma^2} - \frac{2(R-\lambda_d u)}{(\alpha_2-\alpha_1)\alpha_2\sigma^2} \\ &= \frac{u\lambda_d - R}{\lambda_d + r}\end{aligned}$$

The second boundary condition gives the solution for c_1:

$$V(\underline{X}_{CEF}) = c_1\underline{X}_{CEF}^{\alpha_1} + c_2\underline{X}_{CEF}^{\alpha_2} = 0 \Rightarrow c_1 = -c_2\underline{X}_{CEF}^{\alpha_2-\alpha_1}$$

And the third condition leads to the condition:

$$\begin{aligned}0 &= -c_2\underline{X}_{CEF}^{\alpha_2-\alpha_1}\alpha_1\underline{X}_{CEF}^{\alpha_1-1} + c_2\alpha_2\underline{X}_{CEF}^{\alpha_2-1} - \underline{X}^{\alpha_1}\frac{2(R+g(\underline{X})-\lambda_d u)}{(\alpha_2-\alpha_1)\underline{X}^{\alpha_1+1}\sigma^2} \\ &+ \underline{X}_{CEF}^{\alpha_2}\frac{2(R+g(\underline{X}_{CEF})-\lambda_d u)}{(\alpha_2-\alpha_1)\underline{X}_{CEF}^{\alpha_2+1}\sigma^2} \\ &\Rightarrow c_2(-\alpha_1+\alpha_2)\underline{X}_{CEF}^{\alpha_2-1} = 0\end{aligned} \tag{36}$$

Notice that this is not an optimality condition. To verify $\underline{X}_{CEF}$ is the optimal strategy, we need to check the second-order condition: $\frac{\partial^2 V(X, \underline{X}_{CEF}; R)}{\partial \underline{X}_{CEF}^2}|_{\underline{X}^*_{CEF}} < 0$.

Equation (36) suggests two possible solutions: one is $\underline{X}_{CEF} = 0$ and the other is $c_2 = 0$. The first solution is not sensible given we haven't imposed any restriction on the cost function g. The second solution is more sensible; together with Equation (36), we can get

$$-\int_{\underline{X}_{CEF}}^{\infty} \frac{2(R + g(t) - \lambda_d u)}{(\alpha_2 - \alpha_1)t^{\alpha_2+1}\sigma^2}\mathrm{dt} = 0 \Rightarrow \int_{\underline{X}_{CEF}}^{\infty} \frac{(R + g(t) - \lambda_d u)}{t^{\alpha_2+1}}\mathrm{dt} = 0. \tag{37}$$

This indicates that the threshold should be a function of the investment scale, i.e. $\underline{X}_{CEF}(R)$, and a zero-NPV kind of stopping rule. Because both c_1 and c_2 equal zero, the project value before success and abandonment (from Equation (30)) is

$$\begin{aligned} V(X; \underline{X}_{CEF}(R), R) &= X^{\alpha_2}\int_{\underline{X}_{CEF}}^{X} \frac{2(R + g(t) - \lambda_d u)}{(\alpha_2 - \alpha_1)t^{\alpha_2+1}\sigma^2}\mathrm{dt} - X^{\alpha_1}\int_{\underline{X}_{CEF}}^{X} \frac{2(R + g(t) - \lambda_d u)}{(\alpha_2 - \alpha_1)t^{\alpha_1+1}\sigma^2}\mathrm{dt} \\ &= \frac{2}{(\alpha_2 - \alpha_1)\sigma^2}[\int_{\underline{X}^*_{CEF}}^{X} (R + g(t) - \lambda_d u)(\frac{X^{\alpha_2}}{t^{\alpha_2+1}} - \frac{X^{\alpha_1}}{t^{\alpha_1+1}})\mathrm{dt}]. \end{aligned} \tag{38}$$

With $g((R - X_t)^+) = \delta((R - X_t)^+)^2$, Equation (37) can be further written as

$$\int_{\underline{X}_{CEF}}^{\infty} \frac{(R + g(t) - \lambda_d u)}{t^{\alpha_2+1}}\mathrm{dt} = 0 \tag{39}$$

$$\Rightarrow \int_{\underline{X}_{CEF}}^{\infty} \frac{R - \lambda_d u}{t^{\alpha_2+1}}\mathrm{dt} + \int_{\underline{X}_{CEF}}^{R} \frac{\delta(R - t)^2}{t^{\alpha_2+1}}\mathrm{dt} = 0 \tag{40}$$

$$\Rightarrow [R - \lambda_d u]\frac{t^{-\alpha_2}}{-\alpha_2}\Big|_{\underline{X}_{CEF}}^{\infty} + \frac{R^2\delta t^{-\alpha_2}}{-\alpha_2}\Big|_{\underline{X}_{CEF}}^{R} - \frac{2R\delta t^{-\alpha_2+1}}{-\alpha_2+1}\Big|_{\underline{X}_{CEF}}^{R} + \frac{\delta t^{-\alpha_2+2}}{-\alpha_2+2}\Big|_{\underline{X}_{CEF}}^{R} = 0$$

$$\Rightarrow [R - \lambda_d u]\frac{\underline{X}_{CEF}^{-\alpha_2}}{\alpha_2} - \frac{\delta R^2}{\alpha_2}[R^{-\alpha_2} - \underline{X}_{CEF}^{-\alpha_2}] - \frac{2R\delta}{1 - \alpha_2}[R^{-\alpha_2+1} - \underline{X}_{CEF}^{-\alpha_2+1}]$$

$$+ \frac{\delta}{2 - \alpha_2}[R^{-\alpha_2+2} - \underline{X}_{CEF}^{-\alpha_2+2}] = 0. \tag{41}$$

From Equation (39), $R + g(\underline{X}_{CEF}) - \lambda_d u > 0$ because $g(X)$ is decreasing in X and is bounded from above. This shows that with costly financing being possible, firms are willing to endure a negative expected cash flow at times. After we solve for $\underline{X}$ from Equation (41) as a function of R, we can find the optimal investment R by maximizing Equation (38).

Appendix B. Additional Figures

Figure A.1—A.4 show comparative statics of equilibrium investment scales and project values in a heterogeneous duopoly, with baseline parameters listed in Table 1. $R_{UC,opponent}$ denotes a UC duopoly's investment level in equilibrium as a function of its opponent's parameter. $R_{FC,self}$ denotes an FC duopoly's investment level in equilibrium as a function of a parameter of itself. Other notations are similar.

Figure A.1 – R_{UC} and R_{FC} on AIP Parameters in a Heterogeneous Duopoly

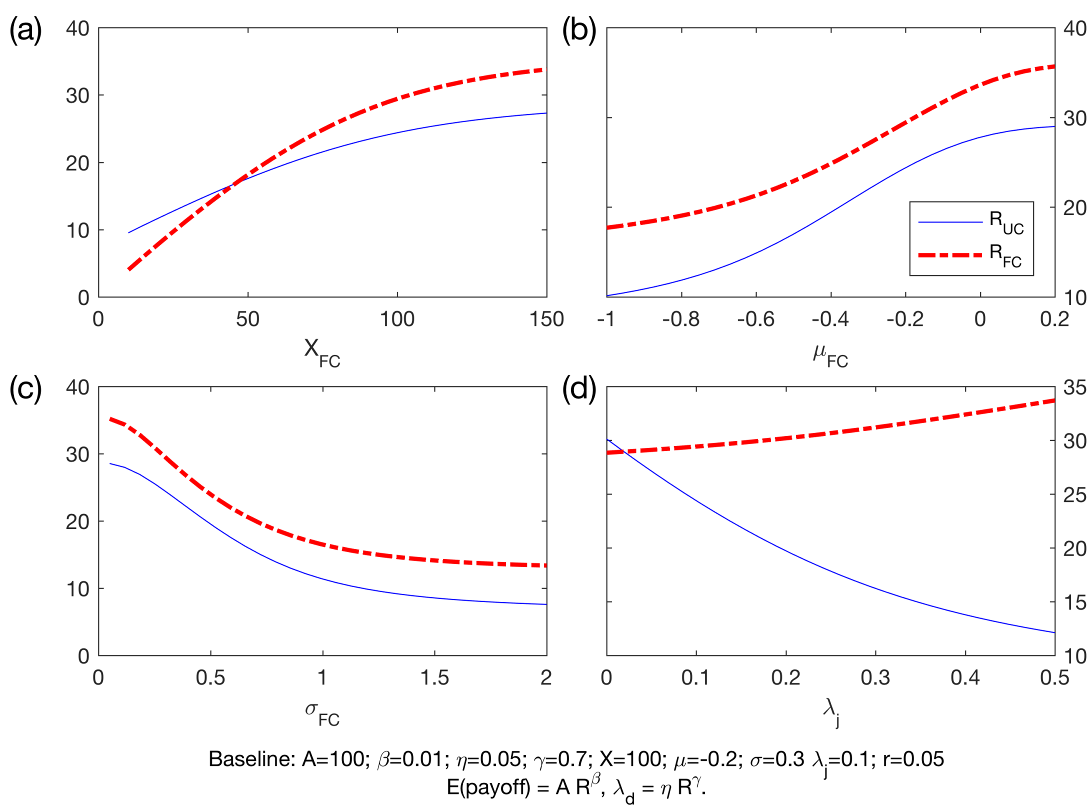


Figure A.2 – R_{UC} and R_{FC} on Project Parameters in a Heterogeneous Duopoly

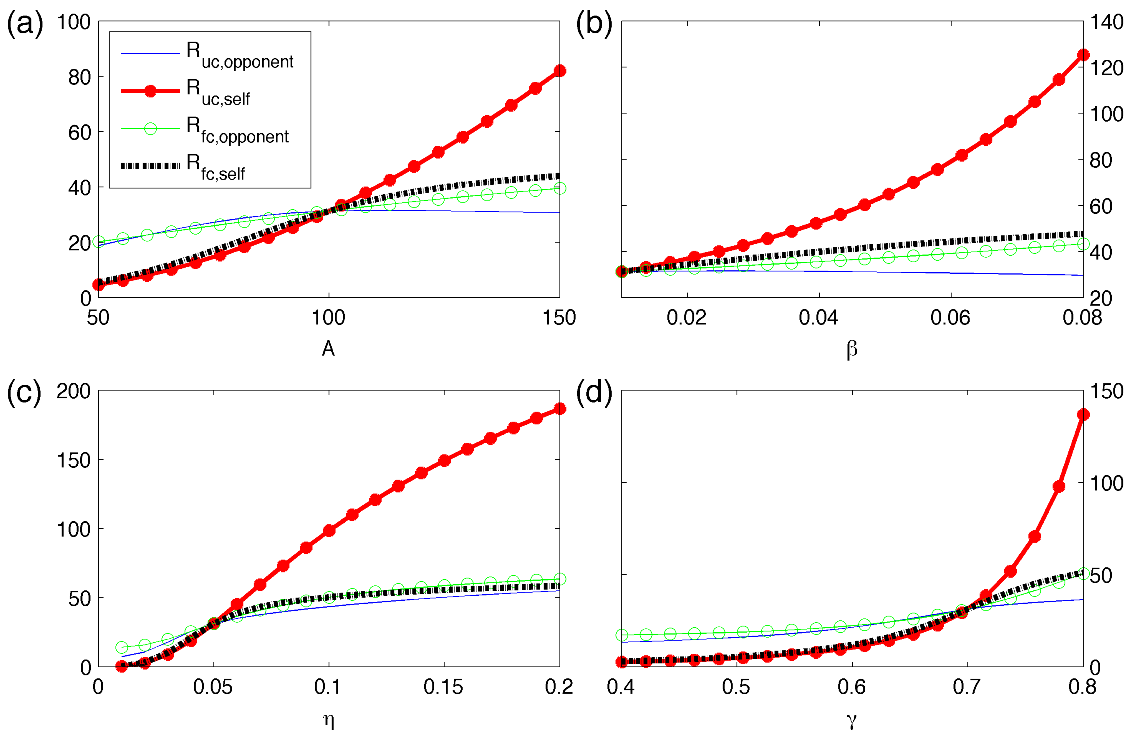
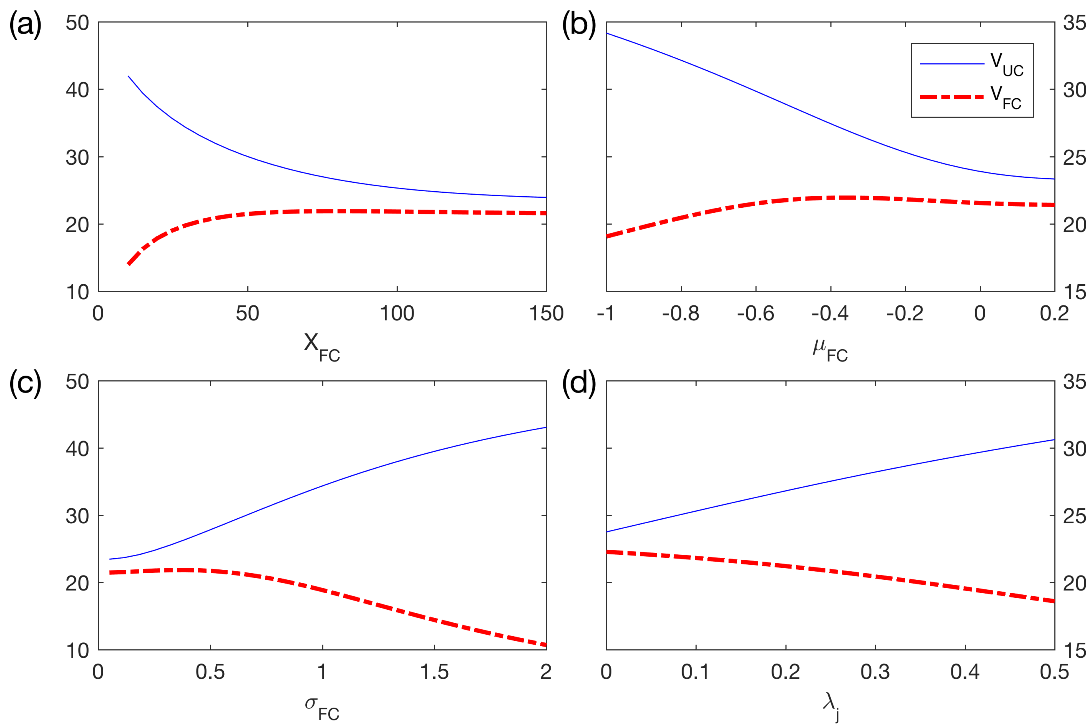


Figure A.3 – V_{UC} and V_{FC} on AIP parameters in a Heterogeneous Duopoly



Baseline: A=100; β=0.01; η=0.05; γ=0.7; X=100; μ=-0.2; σ=0.3 λ_j=0.1; r=0.05
E(payoff) = A R^{β}, $\lambda_d = \eta R^{\gamma}$.

Figure A.4 – V_{UC} and V_{FC} on Project Parameters in a Heterogeneous Duopoly