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Citation: Feng, Huijuan, Kang, Rongjie and Chen, Yan (2016) Workspace Analysis of a Reconfigurable Mechanism Generated from the Network of Bennett Linkages. In: Advances in Reconfigurable Mechanisms and Robots II. Mechanisms and Machine Science (36). Springer, Cham, pp. 153-163. ISBN 9783319233260, 9783319233277

Published by: Springer

URL: https://doi.org/10.1007/978-3-319-23327-7_14 <https://doi.org/10.1007/978-3-319-23327-7_14>

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Workspace Analysis of a Reconfigurable Mechanism Generated from the Network of Bennett Linkages

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Abstract In this paper, a workspace triangle is introduced to evaluate the workspace of a reconfigurable mechanism generated from the network of Bennett linkages. Three evaluation indexes of workspace including motion path of the joint, surface swept by the link and helical tube enveloped by the workspace triangle have been discussed. The workspace comparison between the reconfigurable mechanism and the sum of five resultant 5*R*/6*R* linkages including generalized Goldberg 5*R* linkage, generalized variant of the *L*-shape Goldberg 6*R* linkage, Waldron's hybrid 6*R* linkage, isomerized generalized *L*-shape Goldberg 6*R* linkage and generalized Wohlhart's double-Goldberg 6*R* linkage is accomplished by using the evaluation indexes and mapping the workspace to the joint space defined by a vector whose components are joint variables, which is much easier to evaluate.

Keywords Reconfiguration, workspace triangle, workspace, joint space

1 Introduction

As there is an ever-growing need for reducing production time and saving energy to meet the changing market, reconfigurable mechanisms that can reconfigure to meet various requirements in production present a new way of developing mechanisms. In the past few years, the possibility to design such advanced mechanisms using overconstrained spatial linkages as elements has been revealed [1, 2]. Recently, the reconfigurable mechanism generated from a network of four Bennett linkages which was firstly discussed by Goldberg [3] has been proposed [4]. Reconfiguration among five types of overconstrained 5*R* and 6*R* linkages, including the generalized Goldberg 5*R* linkage, generalized variant of the *L*-shape Goldberg 6*R* linkage, Waldron's hybrid 6*R* linkage, isomerized case of the generalized *L*-

shape Goldberg 6*R* linkage and generalized Wohlhart's double-Goldberg 6*R* linkage, can be fulfilled with modification of topology of this Bennett network by rigidifying certain joints. In evaluating the performance of this reconfigurable mechanism, concern should be given to the workspace factor.

IFToMM terminology defines workspace as totality of points that can be reached by the reference point of a robot arm. Accordingly, workspace of a serial manipulator is the volume capable of being reached by the end-effector which includes reachable workspace and dextrous workspace [5]. Complete workspace of a parallel manipulator can be considered as a set of all poses of the platform under all constraints, which is embedded in a six-dimensional space [6, 7]. As it is generally difficult to represent the complete workspace, some subsets such as reachable position workspace, reachable orientation workspace, constant-position workspace and constant-orientation workspace are determined to represent the workspace in a three-dimensional space. Research about the workspace of closed loop manipulators has been conducted with similar methodology as parallel manipulators by treating the link to be analyzed as the platform [8, 9]. Furthermore, the workspace helical tube formed by the workspace triangle representing the palm configuration was defined to represent the workspace of the metamorphic hand [10].

Recent study shows that during the reconfiguration of the mechanism generated from the network of Bennett linkages, particular attention should be paid to the operation when fixing and releasing the joints since an incorrect sequence would lead to the failure of reconfiguration process. This phenomenon is closely related to the workspace of the reconfigurable mechanism. In order to explain the question, the evaluation of the workspace of the reconfigurable mechanism and the workspace comparison between the reconfigurable mechanism and the five aforementioned resultant 5*R*/6*R* linkages are focused in this paper.

The layout is arranged as follows. In section 2, the reconfigurable mechanism generated from the network of four Bennett linkages is briefly reviewed. Section 3 deals with the evaluation indexes of workspace by introducing the workspace triangle. Section 4 illustrates the workspace comparison between the reconfigurable mechanism and the sum of the resultant 5*R*/6*R* linkages. Conclusions are enclosed in section 5.

2 Generation of the Reconfigurable Mechanism from the Bennett Network

The Bennett linkage is a spatial overconstrained 4*R* linkage with zero offsets, in which alternative links have identical lengths and twists while the lengths are proportional to the sine values of the corresponding twists, as shown in Fig. 1. It is the only spatial overconstrained 4*R* linkage with joint axes neither concurrent nor parallel. Its geometry conditions are

$$\begin{aligned}&a_{12} = a_{34} = a,\ \ a_{23} = a_{41} = b,\\&\alpha_{12} = \alpha_{34} = \alpha,\ \ \alpha_{23} = \alpha_{41} = \beta,\\&\sin\alpha/a = \sin\beta/b,\\&R_i = 0\ (i = 1, 2, 3, 4).\end{aligned} \tag{1}$$

where a_{ij} is the length, α_{ij} is the twist, R_i is the offset in accordance with Denavit and Hartenberg's parameters [11].

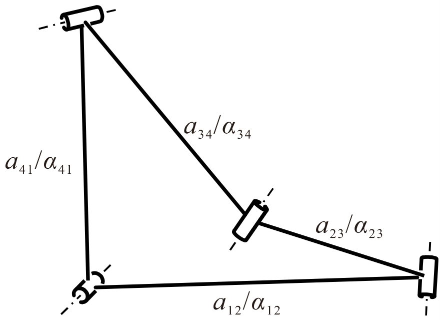

Fig. 1 Geometry of a Bennett linkage

A spatial network of four Bennett linkages A, B, C and D can be constructed as shown in Fig. 2, which are made of links a/α, b/β, c/γ and d/δ with the same Bennett ratio [12],

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{\sin\delta}{d}. \tag{2}$$

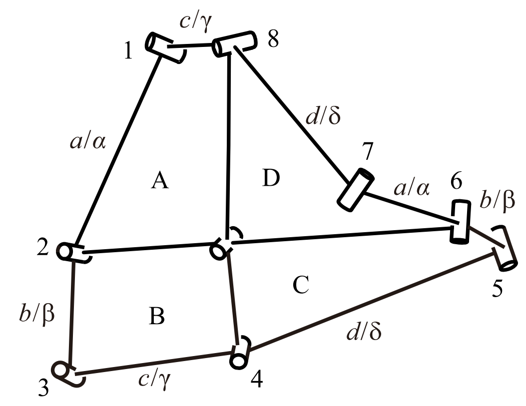

Fig. 2 The network of four Bennett linkages A, B, C and D

As a Bennett linkage has one degree of freedom (DoF), three independent inputs are needed to determine the configuration of the network. Therefore, the network of four Bennett linkages in Fig. 2 has three DoFs.

In order to achieve a single-loop mechanism with one DoF, two joints on the peripheral loop are selected and fixed to reduce the number of links and then the four links inside the network are removed. By means of different choice of the combination of two fixed joints, five Bennett-based linkages can be obtained [4], including generalized Goldberg 5*R* linkage, generalized variant of the *L*-shape Goldberg 6*R* linkage, Waldron's hybrid 6*R* linkage, isomerized generalized *L*-shape Goldberg 6*R* linkage and generalized Wohlhart's double-Goldberg 6*R* linkage.

Specifically, when the fixed joints are chosen as *n/n+1* or *odd/odd+2* with the sequence as shown in Fig. 2, the resultant linkage is essentially a generalized Goldberg 5*R* linkage; when the fixed joints are chosen as *n/n+3*, the resultant linkage can be considered as a generalized variant of the *L*-shape Goldberg 6*R* linkage; when the fixed joints are chosen as *odd/odd+4*, the resultant linkage belongs to Waldron's hybrid 6*R* linkage; when the fixed joints are chosen as *even/even+2*, the resultant linkage is an isomerized case of the generalized *L*-shape Goldberg 6*R* linkage; when the fixed joints are chosen as *even/even+4*, the resultant linkage is in fact a generalized Wohlhart's double-Goldberg 6*R* linkage. As the construct process of the five linkages is common, reconfiguration between different resultant linkages can be accomplished by changing the fixed joints.

3 Workspace Evaluation of the Reconfigurable Mechanism

The reconfigurable mechanism generated from the Bennett network is a spatial single-loop 8*R* linkage with three DoFs [4]. Compared with the general spatial 8*R* linkage with two DoFs, the DoF of this linkage increases as a result of redundant constraints. Similar as parallel mechanisms, its workspace can be evaluated geometrically with the indexes below.

A workspace triangle [10] is defined by selecting any three joints of the spatial 8*R* linkage as the vertexes. As shown in Fig. 3, joints 4, 6, 7 are chosen as an example to represent workspace triangle of the reconfigurable mechanism. Here three indexes can be applied to evaluate the workspace. Firstly, totality of the points that can be reached by the reference point on the regarded platform indicates the position workspace of the reconfigurable mechanism. Supposing that link 67 is regarded as the platform, the movement curve of any point on link 67 can be used to evaluate the workspace of this linkage, where the chosen point is regarded as the reference point in the definition of workspace and then the dimension of workspace can be reduced to three regardless of the orientation. Secondly, surface swept by link 67 demonstrates complete workspace of the reconfigurable mechanism. Differently, orientation transformation is taken into account through direction of link 67, while the previous one only gives the position workspace. Thirdly, the helical tube enveloped by triangle 467 illustrates the most information about the workspace of the reconfigurable mechanism. In addition to the complete workspace, it reveals the distance change between joint 4 and link 67, which provides information about compactness of the mechanism.

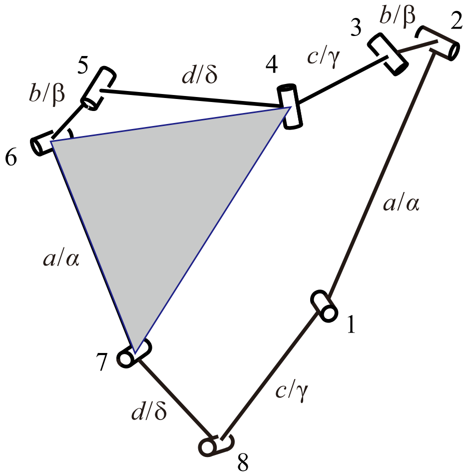

Fig. 3 A workspace triangle of the reconfigurable mechanism

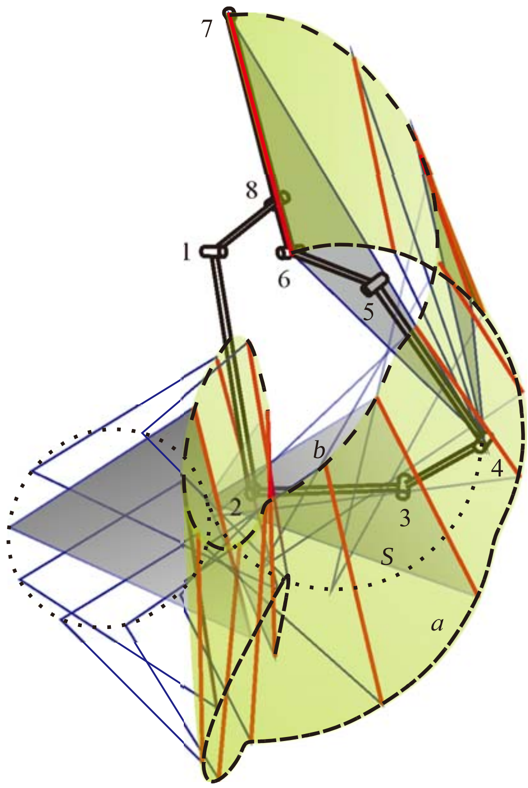

Fig. 4 The workspace evaluation of an exemplified reconfiguration process of the reconfigurable mechanism

Taking a reconfiguration process as an example, the workspace evaluation of the reconfigurable mechanism is demonstrated as shown in Fig. 4, where the curves a and b are the movement paths of joints 6 and 7, the full thick line represents the pose of link 67 and the triangle 467 in thin solid line represent the workspace triangle. The reconfiguration starts from the generalized Goldberg 5*R* linkage with joints 1 and 2 fixed as illustrated in Fig. 4. When it moves to a desired configuration, joint 4 is fixed and then joint 2 is released, resulting in a generalized variant of the *L*-shape Goldberg 6*R* linkage. Similarly, this linkage is reconfigured into Waldron's hybrid 6*R* linkage, isomerized generalized *L*-shape Goldberg 6*R* linkage and generalized Wohlhart's double-Goldberg 6*R* linkage subsequently. During the process, the workspace is evaluated with the curves a and b, the surface S and the helical tube enveloped by the pose variation of triangle 467.

4 Comparison between the Workspace of the Reconfigurable Mechanism and Resultant Linkages

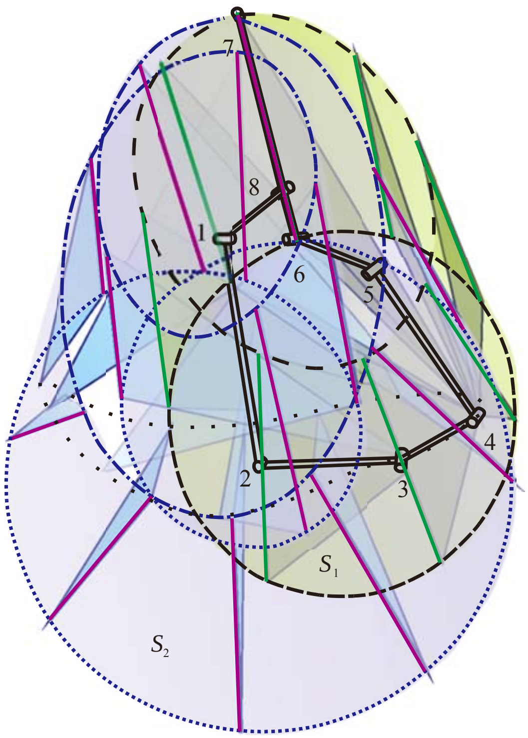

Fig. 5 The workspace comparison between the reconfigurable mechanism and the generalized Goldberg 5*R* linkage

Applying the evaluation indexes, the workspace comparison between the reconfigurable mechanism and the resultant 5*R*/6*R* linkages can be accomplished. As shown in Fig. 5, the workspace of the generalized Goldberg 5*R* linkage with given values of θ_1 and θ_2, the joint angles in accordance with Denavit and Hartenberg's parameters, is measured as surface S_1. As the generalized Goldberg 5*R* linkage is generated from the reconfigurable mechanism, its workspace is contained in the workspace of the reconfigurable mechanism. Besides, the workspace of the reconfigurable mechanism also includes surface S_2 obtained by releasing the fixed joints 1 and 2, which cannot be achieved by the generalized Goldberg 5*R* linkage. This applies to other generalized Goldberg 5*R* linkages with different parameters and other resultant linkages as well. Therefore, the workspace of the reconfigurable mechanism is larger than the sum of the five resultant 5*R*/6*R* linkages.

Besides, the comparison also can be carried out by mapping the workspace to the joint space. As the 8*R* linkage formed by the peripheral loop of Bennett network has three DoFs, its configuration would be well-determined once the three input joint angles were given. It means that the joint space defined by the joint variables in three-dimension space has a one-to-one mapping relationship with the configuration of the mechanism.

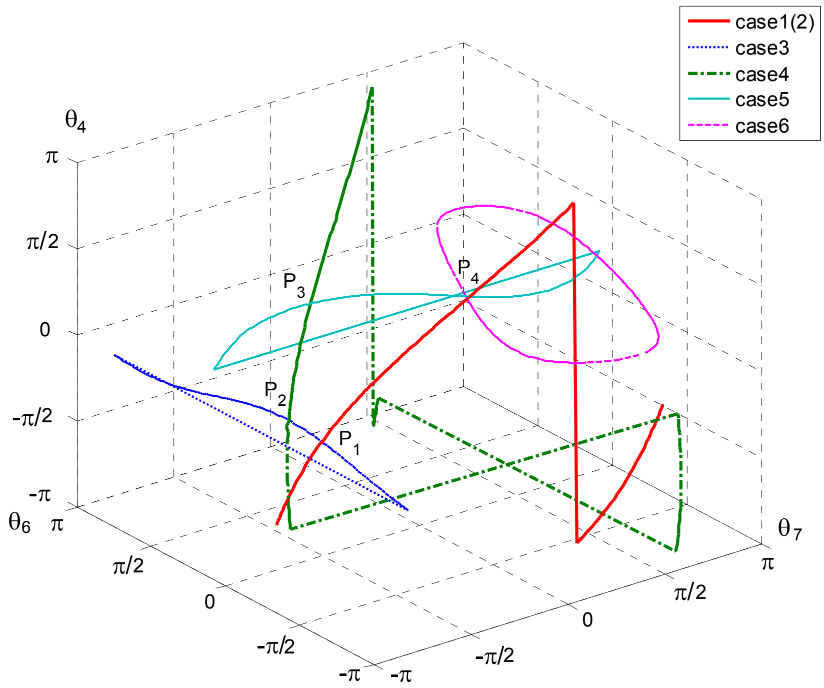

Fig. 6 Kinematic paths of the resultant linkages during a reconfiguration process, where case 1(2) is a generalized Goldberg 5*R* linkage, case 3 is a generalized variant of the *L*-shape Goldberg 6*R* linkage, case 4 is a Waldron's hybrid 6*R* linkage, case 5 is an isomerized generalized *L*-shape Goldberg 6*R* linkage, case 6 is a generalized Wohlhart's double-Goldberg 6*R* linkage

Choosing θ_4, θ_6 and θ_7 as the three free variables which can be equal to any value among $-\pi$ to π, the joint space of the reconfigurable mechanism can be measured as a cube with length 2π as shown in Fig. 6, where a point in the cube corresponds to a configuration of the mechanism. In terms of the resultant 5*R*/6*R* linkages, the joint space corresponds to the point on the kinematic path. The kinematic paths of an example of the reconfiguration process is illustrated in Fig. 6 with the parameters formulated as

$$\begin{aligned} &a = 1.0000,\ \alpha = -125.00\pi/180; \\ &b = 0.6104,\ \beta = -150.00\pi/180; \\ &c = 0.4175,\ \gamma = -20.00\pi/180; \\ &d = 0.7847,\ \delta = -40.00\pi/180. \end{aligned} \tag{3}$$

As shown in Fig. 6, the reconfiguration process is the same as the previous section and the coordinates of each reconfiguration points are

$$\begin{aligned} &\mathrm{P}_1(-\frac{123.25\pi}{180}, \frac{44.50\pi}{180}, -\frac{30.00\pi}{180}),\ \mathrm{P}_2(-\frac{118.66\pi}{180}, \frac{2.92\pi}{180}, -\frac{30.00\pi}{180}), \\ &\mathrm{P}_3(-\frac{50.77\pi}{180}, \frac{65.41\pi}{180}, \frac{40.00\pi}{180}),\ \mathrm{P}_4(\frac{48.10\pi}{180}, \frac{6.01\pi}{180}, \frac{40.00\pi}{180}). \end{aligned} \tag{4}$$

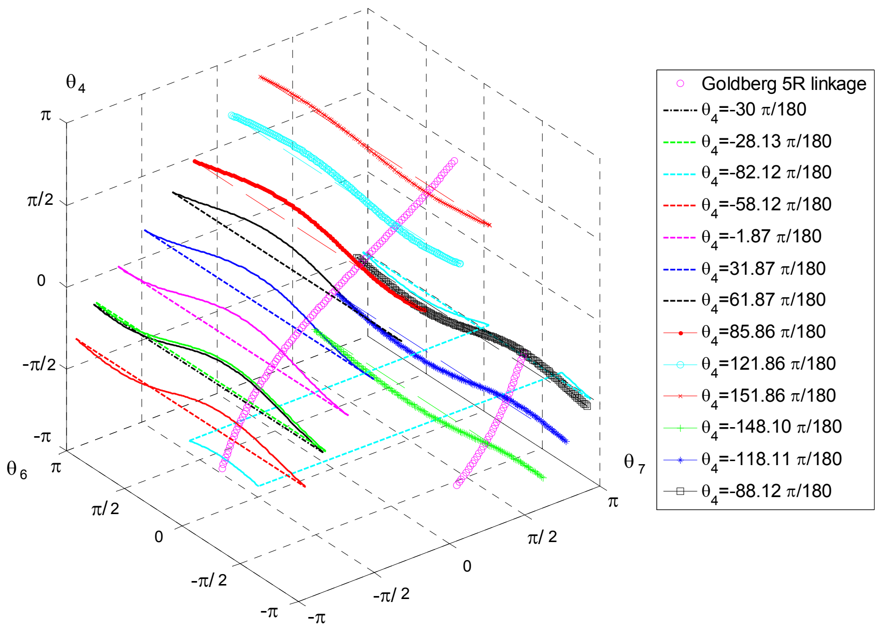

Fig. 7 Kinematic paths of the generalized Goldberg 5*R* linkage and the generalized variant of the *L*-shape Goldberg 6*R* linkage with different discrete values of angle θ_4

It should be pointed out that the reconfiguration point could be any point on the corresponding kinematic path and the resultant kinematic paths are different. Taking reconfiguration between the generalized Goldberg 5*R* linkage and the generalized variant of the *L*-shape Goldberg 6*R* linkage as an example, several kinematic paths of the resultant generalized variant of the *L*-shape Goldberg 6*R* linkages can be plotted as shown in Fig. 7 by fixing θ_4 at some discrete values. When every possible value of θ_4 is adopted, a surface constituted by all the resultant kinematic paths is obtained as C1 as shown in Fig. 8. The surface is acquired under the condition that θ_1 is constant. Different values of θ_1 generate different surfaces such as C1 and C2 and so on. Considering the variation of θ_1, series of curved surfaces will be derived, which represent the joint space of the generalized variant of the *L*-shape Goldberg 6*R* linkage.

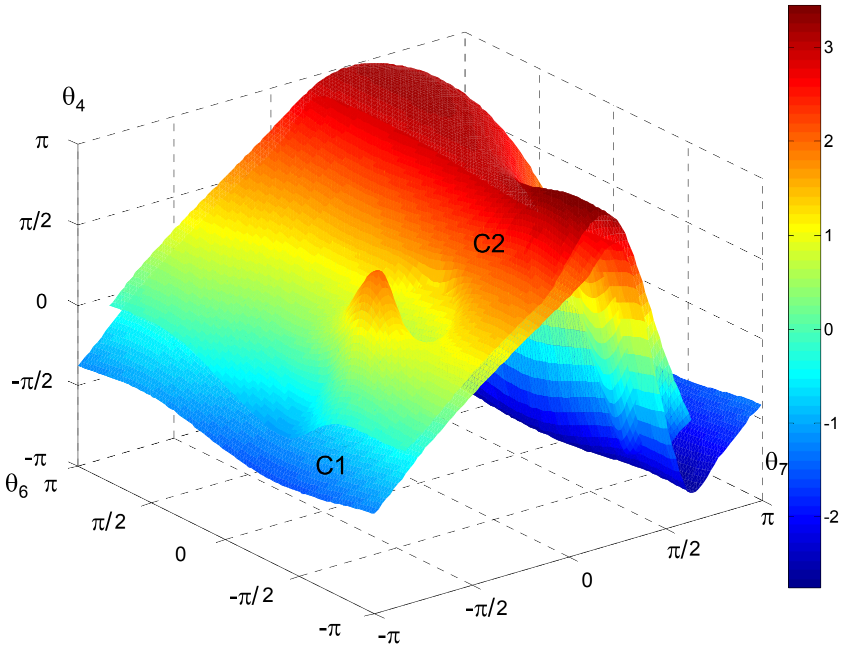

Fig. 8 Kinematic surfaces of the generalized variant of the *L*-shape Goldberg 6*R* linkage with different discrete values of angle θ_1

Similarly, four more series of curved surfaces could be figured out for the generalized Goldberg 5*R* linkage, Waldron's hybrid 6*R* linkage, isomerized case of the generalized *L*-shape Goldberg 6*R* linkage and generalized Wohlhart's double-Goldberg 6*R* linkage. As the sum of joint space of resultant 5*R*/6*R* linkages are several series of curved surfaces, which is less than the whole cube possessed by the reconfigurable mechanism, the workspace of the reconfigurable mechanism is larger than the sum of five resultant 5*R*/6*R* linkages.

5 Conclusions

The workspace of a reconfigurable mechanism generated from the network of Bennett linkages can be evaluated by three indexes including motion path of the joint, surface swept by the link and helical tube enveloped by the workspace triangle. By applying the evaluation indexes, the workspace comparison between the reconfigurable mechanism and the sum of five resultant $5R/6R$ linkages can be accomplished. As there is a one-to-one mapping relationship between the joint space defined by the joint variables and the configuration of the mechanism, the workspace can be mapped to the joint space. The comparison result reveals that workspace of the reconfigurable mechanism is larger than the sum of five resultant $5R/6R$ linkages, which explained the necessity of fixing the joints first before releasing the previously fixed joint during the reconfiguration.

Acknowledgments The authors would like to acknowledge the support of the National Natural Science Foundation of China under grant codes of 51275334 and 51290293 and of Tianjin Municipal Science and Technology Department Program under grant codes of 13JCZDJC26400 and 14JCYBJC19300. The first author would like to acknowledge the support of Tianjin University for providing the University Graduate Scholarship.

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