INFLUENCE OF CALCULATION PARAMETERS ON NONLINEAR DYNAMICS MEASURES

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Nonlinear analysis methods based on a dynamical systems approach have become more prevalent in recent biomechanics studies. The aim of this study is to identify the range of time delay and embedding dimension values estimated for gait data, and how sensitive Lyapunov exponent and correlation dimension are to the range of these values. A participant walked at 3 km/h and ran at 9 km/h on a treadmill for 2 minutes. Lyaponuv exponent and correlation dimension were calculated based on a combination of the optimal and mean average time delay (TD) and embedding dimension (ED) for both angle and marker data. In the majority of literature, only one ED and TD is considered for LyE estimation, based on an average across data or values suggested in previous papers. However, the results of our study show that every angle or position, has an optimum ED and TD, and the use of these values affects the nonlinear dynamics values in non-trivial ways.

KEYWORDS: Nonlinear Dynamics, Lyapunov,Correlation Dimension.

INTRODUCTION: Nonlinear dynamics and dynamical systems methodologies have been developed for analysing, understanding and capturing complexity in neurobiological systems. In the case of human movement analysis, and in particular gait analysis, nonlinear dynamics methods are applied to identify attractor states including fixed point, limit cycle and chaotic attractors which exist in this cyclic action (Strogatz 2018). The characteristics of these attractor dynamics are then quantified and interpreted to increase our understanding of the coordination and complexity of the systems evolution in time (Strogatz 2018). Key examples in the literature include studies by Dingwell and colleagues, who have estimated differences in average maximum finite-time Lyapunov exponents between the walking gait of diabetic neuropathic populations and healthy populations (Dingwell and Cusumano 2000). Findings show there was a decreased joints range of motion and stability of neuropathic populations compared to healthy populations. While these methods seem useful for characterising healthy from impaired movement patterns, there are, as yet, no set rules for the calculation of parameters that underpin these nonlinear dynamics methods in the context of gait analysis.

Importantly, phase space reconstruction permits the identification of attractor states. For studying systems with deterministic properties, it is vital to reconstruct phase space with optimum time delay (TD) and embedding dimension (ED) (Kantz and Schreiber 2004). However, there are no mathematical *rules* for selecting the ‘correct’ values for TD and ED to reconstruct the precise phase space; rather, some recommendations have been suggested for their estimation (Huffaker, Bittelli et al. 2017). For the majority of literature published in this area, authors make reference to the algorithms for estimating ED and TD and Theiler window, but rarely report the actual values when reconstructing and subsequently quantifying characteristics of the attractor state.

Since ED and TD are essential for the reconstruction and subsequent quantifying characteristics of the attractor state, but can only be ‘estimated’, it is imperative to further understand the influence of these values on outcome measures. For example, Lyapunov exponent (LyE) and correlation dimension (CD) are outcome measures that have an essential role in studying these systems. In addition, we want to engage in a discussion on how we should address the estimation of these variables in our field.

METHODS: Ethical approval was obtained from the University of Exeter. Informed consent was acquired from the participant prior to testing. A male participant (age 28 years, mass 75.2 kg, height 1.90 m, regular runner 45 mins x 4 times per week) walked at 3 km/h and ran at 9 km/h on a motorized treadmill for 2 minutes. An automated 3D motion capture system (CODA, Charnwood Dynamics Ltd, UK) was used for collecting kinematic data during all gait trials at 100 Hz. Three CX1 CODA scanners provided a field of view exceeding 2.5 m around the treadmill. CODA was aligned such that the x axis was the direction of travel of the participant. Active markers on the lateral aspect of the estimated centre of rotation of the hip, posterior superior iliac spine (PSIS), femoral condyle (knee), lateral malleolus (ankle), and calcaneus (heel) and the first distal phalanx of the foot (toe). Positional data of markers (x direction) and hip, knee and ankle joint angles were analysed using R (http://www.r-project.org).

The first step in using nonlinear methods is to concisely reconstruct the phase space by calculating some nonlinear dynamics variable such as the Theiler window, TD, and number of ED (Huffaker, Bittelli et al. 2017). In the present study, the estimation of the TD, Theiler window and ED respectively were performed using the average mutual information (AMI), autocorrelation function (ACF), and false nearest neighbour (FNN) analysis, respectively (Huffaker, Bittelli et al. 2017). After determining nonlinear dynamics variables, the estimation of the largest Lyapunov exponent (LyE) as a nonlinear dynamics measure of stability can be conducted (Wolf et al., 1985). The calculation of the LyE was performed for all-time series using a computer code to estimate the maximum characteristic Lyapunov exponent (MCLE) implemented in 'tseriesChaos' based on the Rosenstein algorithm.

CD was calculated for all-time series using the tseriesChaos (Huffaker, Bittelli et al. 2017).The CD is a nonlinear dynamics measure of how the data points of a dynamical system are organized within a state space (Grassberger and Procaccia 1983).

For position and angle of ankle, knee, and hip are shown in Table 1, LyE and CD were calculated based on combinations of the mean and estimated TD and ED values; with TD and ED estimated, with TD estimated and mean ED, with mean TD and estimated ED. The range and mean of Theiler windows was 58 to 225 for walking data and 29 to 113 for running data. The mean Theiler window of 111 and 65 were used in all calculations for walking and running, respectively.

RESULTS: Figure 1. shows the difference values between the LyE for angles (hip, knee, ankle) and markers (hip, knee ankle in the x direction) calculated with three variations of ED and CD: 1) based on using the mean TD and mean ED; 2) based on using mean TD and the estimated optimum ED; 3) based on using estimated TD and mean ED. Table 1. shows the estimated TD and ED and resulting LyE and CD in these conditions. The range of TD and ED calculated in walking conditions were, from 15 to 40 and with mean value of 30, and from 7 to 9 with mean value of 8, respectively. Also, the range of TD and ED in running conditions were from 15 to 40 and with mean value of 30 and from 6 to 14 with mean value of 10, respectively. We had to change other values such as Theiler window and minimum number of data points to make the ED to converge.

Generally, using mean TD and estimated ED condition, LyE reduced in the majority of cases. Mostly, a decrease in LyE in both mean TD and mean ED conditions was seen, however, mostly during walking such as hip angle, ankle position, knee angle, knee position, and ankle position there is increase in LyE.

Figure 2. demonstrates the difference values between the CD values with the same combinations of mean and estimated optimum TD and ED as above. As it is shown, there is mostly decrease in CD in 1) and 3) conditions. In addition, it can be seen a slight increase in CD in mean TD and mean ED and mean TD with estimated ED conditions for knee ankle during walking.

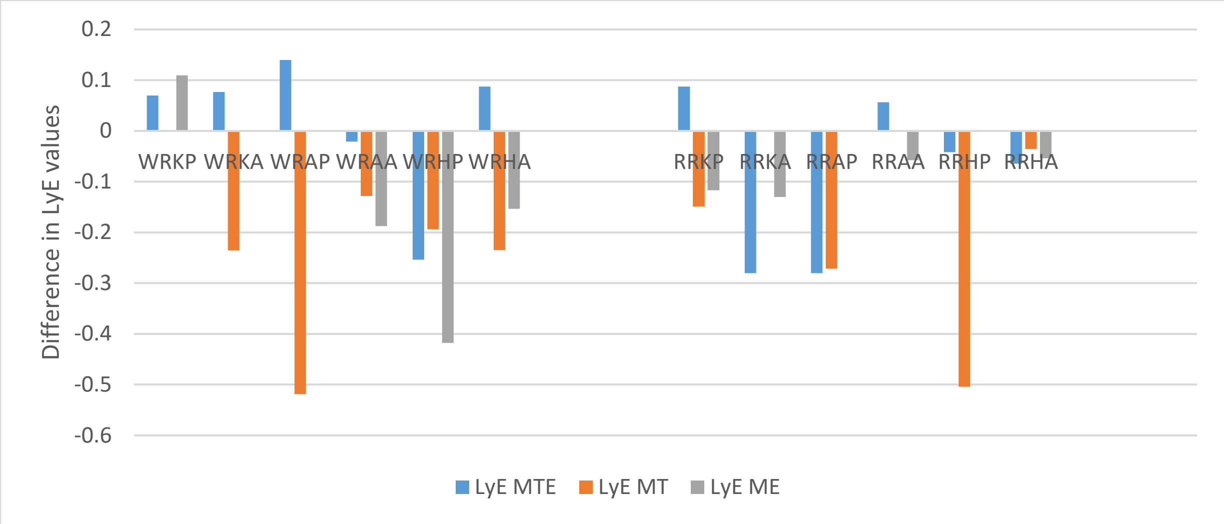

Figure 1: Difference value of LyE for right hip angle in running (RRHA), right hip position in running (RRHP), right ankle angle in running (RRAA), right ankle position in running (RRAP), right knee angle in running (RRKA), right knee position in running (RRKP), right knee hip angle in walking (WRHA), right hip position in walking (WRHP), right ankle angle in walking (WRAA), right ankle position in walking (WRAP), right knee angle in walking (WRKA), right knee position in walking (WRKP)

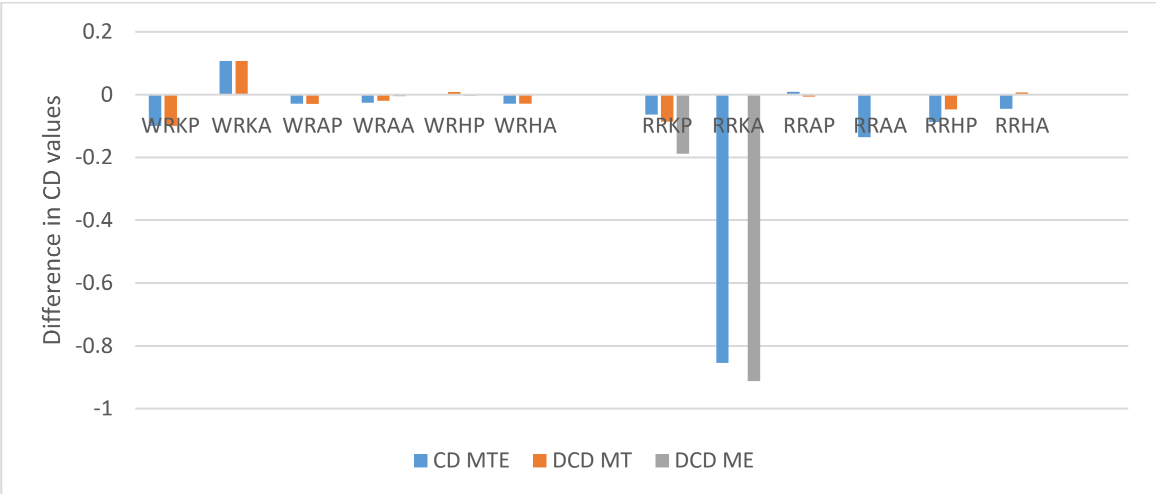

Figure 2: Difference value of CD for right hip angle in running (RRHA), right hip position in running (RRHP), right ankle angle in running (RRAA), right ankle position in running (RRAP), right knee angle in running (RRKA), right knee position in running (RRKP), right knee hip angle in walking (WRHA), right hip position in walking (WRHP), right ankle angle in walking (WRAA), right ankle position in walking (WRAP), right knee angle in walking (WRKA), right knee position in walking (WRKP)

DISCUSSION: The aim of this study was to further understand the influence of TD and ED values on the outcome measures LyE and CD. In addition, we want to generate discussion about the calculation of these variables in our field.

Changes in LyE and CD were not-trivial based on changes in TD and ED. For positional data (ankle, knee, hip marker in the X direction) at walking speed, optimum TD value was consistently 15. When using the mean TD value of 30, LyE values which is the stability measure, were always higher and less stable than with the optimum TD of 15, which could mean that we are overestimating the instability of gait data by using higher mean values of TD. The CD with mean TD and optimum ED increased or decreased compared to the value using the optimum. This may be result of the variables in CD calculation, such as Theiler window and minimum number of data points that we had to revise to make the ED to converge. Estimated LyE and CD values with mean ED and optimum TD, were mostly decreased or stayed unchanged, however there was an increase in LyE value in knee position which is because of the increase in ED compare to its optimum value.

Table 1: LyE and CD values when calculated with three conditions as: Mean time delay (TD) and mean embedding dimension (ED) (MTE), Mean time delay and optimum embedding dimension (MT), mean embedding dimension and optimum time delay (ME). LyE = Lyaponuv exponent, CD = correlation dimension, LyE MTE = estimated LyE in MTE condition, DLyE MTE = Difference in estimated LyE in MTE condition, repeated for CD.

Condition	TD	ED	LyE	CD	LyE MTE	CD MTE	DLyE MTE	DCD MTE	LyE MT	CD MT	DLyE MT	DCD MT	LyE ME	CD ME	DLyE ME	DCD ME
WRKP	15	7	0.2406	1.92	0.1708	2.02	0.0698	-0.1	0.2422	2.02	-0.0016	-0.1	0.1309	1.92	0.1097	0
WRKA	67	8	0.2036	1.95	0.1271	1.847	0.0765	0.107	0.4388	1.847	-0.2352	0.107	0.2036	1.954	0	0
WRAP	15	8	0.1824	1.97	0.04292	2.001	0.13948	-0.029	0.7011	2.002	-0.5187	-0.03	0.1824	1.972	0	0
WRAA	27	9	0.1549	1.94	0.1759	1.961	-0.021	-0.026	0.2833	1.955	-0.1284	-0.02	0.3423	1.94	-0.1874	-0.005
WRHP	15	9	0.08945	2.04	0.3429	2.035	-0.25345	0.003	0.2833	2.03	-0.19385	0.008	0.5073	2.042	-0.41785	-0.004
WRHA	40	7	0.1297	1.96	0.04216	1.984	0.08754	-0.029	0.3645	1.984	-0.2348	-0.029	0.2831	1.955	-0.1534	0
Mean Average	30	8														
RRKP	11	6	0.1297	1.94	0.04216	1.998	0.08754	-0.063	0.2785	2.021	-0.1488	-0.086	0.2466	2.123	-0.1169	-0.188
RRKA	10	4	0.1297	1.01	0.4099	1.865	-0.2802	-0.854	0.1297	1.011	0	0	0.2598	1.923	-0.1301	-0.912
RRAP	8	7	0.1297	2.02	0.4099	2.008	-0.2802	0.009	0.4012	2.023	-0.2715	-0.006	0.1297	2.017	0	0
RRAA	10	9	0.2279	1.9	0.1715	2.032	0.0564	-0.136	0.2279	1.896	0	0	0.2854	1.895	-0.0575	0.001
RRHP	6	7	0.1297	1.96	0.1715	2.048	-0.0418	-0.088	0.6336	2.007	-0.5039	-0.047	0.1297	1.96	0	0
RRHA	14	8	0.1479	1.97	0.2123	2.018	-0.0644	-0.045	0.1837	1.966	-0.0358	0.007	0.2018	1.975	-0.0539	-0.002
Mean Average	10	7														

For angle data in walking, estimated TD was much higher (range from 27-67) than for position data (15), while the CD values were similar (7-9 dimensions for both position and angle data). These values are higher than those reported in previous literature. TD was more stable across angle and position when looking at running data (range between 6-14). CD had a higher range in running; between 4 and 9. It is important to open discussion about what we expect and can rationalise these numbers to be in our field.

For position data in running, estimated LyE with mean TD and mean CD always decreased, however mean TD for knee position is bigger than the estimated one. For angle data in running, estimated LyE and CD in all three conditions decreased, however estimated CD with mean ED and estimated TD increased. Again, there seems to be an interesting relationship between TD and ED and resulting nonlinear dynamics measures.

As it is shown, in the mean TD and optimum ED condition, although some of the ED and TD were increased and some of them decreased, LyE always decreased. In the majority of literature only one ED and TD is considered for LyE estimation, based on an average across data or values suggested in previous papers. However, every joints and their angle or position, as we estimated in this study, has an optimum ED and TD, and the use of these values affects the nonlinear dynamics values in non-trivial ways. We look forward to discussing this work at the conference and hope to move this area forward with other researchers in our field.

CONCLUSION: Nonlinear dynamics methods such as LyE and CD, are being used increasing to show relevant changes in biomechanical variables. However, we need to carefully consider the estimation of parameters that are part of the calculation of these variables in order to make sure our results are valid across studies. Specifically, for calculation of stability measures such as LyE and CD, it is needed the estimation of calculation parameters such as, Theiler window, minimum number of data points to make the ED converge, number of neighbours considered, number of points in series taken into account, iterations along which the neighbours of each point are followed, radius of the ball inside which nearest neighbours. Our field would benefit from a discussion as to how to go about estimating and reporting these calculation parameters in a way to best answer our research questions, and ensure that results are clinically relevant.

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