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Fraction models used by primary school teachers

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Abstract. Fractions and anything related to fractions are among the mathematics subjects that are challenging for primary school students to comprehend probably due to the fact that fractions have multiple meanings in different circumstances. Primary school teachers could overcome this challenge using various fraction models to facilitate students' understanding the concept of fraction. With this in mind, the current study aimed to reveal the fraction models used by these teachers when teaching fractions. Accordingly, those used by 14 teachers were analyzed to see whether the meaning of fraction differs depending on the models used. This qualitative study adopted the single case research model. The data were collected from primary school teachers through a semi-structured interview form and descriptively analyzed. The study has concluded that the cluster model, the area model, and the length model are the most preferred models by the teachers and that their choice of models in teaching fractions differs across the sub-constructs of fractions.

Keywords: Primary school teacher, mathematics education, fractions, sub-construct, fraction models

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INTRODUCTION

Teachers are expected to notice the commonly observed incomplete and / or mis-developed concepts related to the subject area and field education in the students, and to be able to provide satisfactory answers to students' questions provided that they are within their area of expertise. Shulman (1986) advocates that this is closely related to pedagogic field knowledge, a synthesis of field knowledge and pedagogic knowledge that allows distinguishing between field specialists and educators. The researcher goes on to state that it 'includes an understanding of what makes the learning of specific topics easy or difficult' (p. 9) underlying the significance of the 'knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners' (p. 10). In this sense, teachers are required to anticipate such mistakes and to take precautions to prevent them. In other words, they are expected to plan and implement their teaching activities to present these concepts with their different meanings and dimensions.

Fraction is among the fundamental concepts of mathematic and related to other mathematics subjects. Despite allocating a great amount of time during the learning and teaching process to the formation and development, fractions and other subjects related to fractions are among the leading mathematical subjects that are difficult for the students to comprehend (Bayazit, Aksoy & Kırnap, 2011; Charalambous & Pantazi, 2005; Doğan & Işık-Tertemiz, 2019; Doğan & Işık-Tertemiz, 2018). The difficulties faced during the teaching and learning of the fraction concept are accounted for such reasons as the difference between fractions and the whole numbers, topic initially learned by the initial construction of fractions in the form of part-whole relation and quotient, and possibility of presenting them through different symbols and expressions (Olkun & Toluk, 2003; Pantziara & Philippou, 2012; Van de Walle, Karp & Bay-Williams, 2014).

In general, studies on the concept of fractions have concluded that a/b expression can be interpreted in five different ways: (i) part-whole, (ii) measure, (iii) ratio, (iv) quotient (division), and (v) operator (Behr et al., 1993; Pantziara & Philippou, 2012). These sub-constructs of fraction have been confirmed by Charalambos and Pitta-Pantazi (2007) by using structural equation modelling techniques.

Part–Whole Meaning: This is the most frequently used meaning of fractions and conceptually the easiest to understand. Charalambous and Pitta-Pantazi (2005) stated that the part-whole meaning of fraction is necessary but not sufficient for learning the concept of fraction.

Measure Meaning: This is based on the principle of determining a fixed unit of measure, repeating the action and dividing the unit into smaller parts. For instance, to 1/5-unit fraction is defined to present 3/5 fraction teach 3/5 fraction is the length model is commonly recommended for the measurement meaning of fractions as the units on the number line are easily repeated and divided into equal parts (Martinie, 2007).

Ratio Meaning: Generally defined as the numeric presentation of proportional relationship of the same or different quantities (Van de Walle, Karp & Bay-Williams, 2014), the ratio meaning of fraction is exemplified with the relationship between the numbers of apples and oranges in a fruit basket.

Quotient Meaning: It expresses the result of a division such as a/b. It can also be considered as the value elicited from the division of "a" by "b" (Kieren, 1993). The result acquired in the quotient sub-construct represents a numeric value rather than equally divided parts, and therefore the fractional dimension should not necessarily be taken into consideration.

Operator Meaning: Fractions can be considered as the result of two multiplicative functions or two separate functions that are actually related while operator sub-construct could support developing a conceptual understanding of the multiplicative operations on these fractions (Charalambos & Pitta-Pantazi, 2005). Operator sub-construct is also defined as a function where rational numbers are applied to a number, object or cluster (Behr et al., 1993).

It is advocated that students can learn a subject more efficiently through multiple presentation, different approaches, descriptions and proper reasoning (Pantziara & Philippou, 2012). In this vein, use of various models, visualization and charts and creating cases where different meanings of fractions can be compared enable students to discover the meanings of fraction. In order to achieve this, teachers need to have in-depth pedagogical content knowledge and a high-level awareness of using the visuals in concern.

Fractions are one of the most important subjects for students to comprehend story problems and to succeed in mathematics during primary schooling and onwards, respectively. Operations and calculations related to fractions are getting harder in other content areas of fractions, especially in using algebra. Therefore, effective teaching of fractions requires primary school teachers, in particular, to be aware of the importance of fractions and to help students understand fractions in all meanings.

As visualization, embodiment and modelling of the fractions have a positive effect on students' learning process, teachers are supposed to use fraction models in their teaching. There is a growing body of research that highlights the significance of using models in teaching fractions (Cramer & Henry, 2002; Olkun & Toluk 2003; Van de Walle et al, 2014). Teachers can help primary school students internalize the concept of fraction by using different models. It could be claimed that the students are confused on fractions when the fraction models are superficially taught whereas using different models provides different teaching and learning opportunities for them. Using different fraction models can provide better learning opportunities for students. Namely, the area model allows students to visualize parts of a whole and the length model indicates that other fractions can always be found between any two fractions while the cluster model can show multiplicity ratios with more ease (Van de Walle et al, 2014). Furthermore, some of these models are likely to appeal to certain students more than others since they all come to the learning environment with different levels of readiness and prior knowledge.

Models Used in Fraction Teaching

The most frequently used models when presenting fraction expressions are the area model (territorial model), the cluster model and the length model.

The Area (Territory) Model: In this model, fraction number is embodied as a certain part of an area. It might be the most appropriate model particularly when introducing the part-whole sub-construct. It should be carefully implemented considering certain criteria such as using a figure that can be divided into equal parts or avoiding the use of circle to model fractions with an odd-number denominator.

The Cluster Model: In this model, fraction corresponds to a certain number of objects found within a cluster. While the objects inside the cluster form the whole of the cluster, a group of objects that comprise the sub-cluster of the main cluster form the fraction. Successful use of this model largely depends on students' ability to group the cluster objects. Students mostly fail to distinguish between fractions since they tend to focus on the size of the objects rather than their number (NCTM, 2000). Hence, they are supposed to have well-developed division skills to prevent from that.

The Length Model: In this model, the given line, length or the number line is divided into parts. Subsequently, each fraction is used as a number and placed onto the correct point on the number line. Fraction number is taken as a tangible and real number. The most recent studies on fractions report that the length model facilitates students' comprehension of fractions as numbers and development of other fraction concepts (Van de Walle et al, 2014). It also allows students to gain knowledge about fractions by positioning them on the number line and considering the size of the fraction based on its distance, particularly the distance to nought, half and one.

Conceptual internalization of fractions might take longer particularly in primary schooling. It is only possible for students to understand a fraction in different circumstances, in other words, to grasp its different meanings, by gaining experience in different problem situations. The concept in concern can be well-established through the embodiment of different meanings of fractions for the students (Olkun & Toluk, 2003). The existing literature indicates that each fraction model is more suitable to express at least one of the sub-constructs. Cramer, Wyberg and Leavitt (2008) claim that the area model is more effective in presenting the part-whole sub-construct while Van de Walle et al. (2014) advocates that the cluster model is more suited to present the fractional parts of sub-clusters within a cluster of objects (e.g. the operator sub-construct). On the other hand, the length model is reported to be a higher-level model that needs to be considered for presenting the numerical values of fractions (e.g. the measure, ratio and quotient sub-constructs) (Bright, Behr, Post and Wachsmuth, 1988; Clarke et al, 2008). Moving from these viewpoints, primary school teachers are expected to be aware of which fraction model(s) should be used for effective teaching of a particular sub-construct.

Students tend to suffer from misconception on fractions and to make mistakes on size, comparison and sequencing of fractions. Previous research has concluded that using inter-relations of fractions, fraction models and mental images for fractions can facilitate students' effective understanding of fractions with all dimensions (Pantziara & Philippou, 2012). Embodiment and visualisation of fractions by using different fraction models provide a better comprehension of the subject for the students. Mathematical visualization is mostly realized through models. Olkun and Toptaş (2007) define the concept of model as "actual objects, drawings or symbols used for presenting mathematical concepts or relations". Primary school teachers need to have sufficient amount of knowledge about these models to use while teaching fractions.

As the subject of fractions occupies an important place in primary mathematics curriculum and forms the basis of several other subjects such as measuring, area and ratio, the current research was motivated to reveal the models used by primary school teachers when teaching fractions and, more specifically, to reveal the particular models used for fraction sub-constructs.

METHODS

Research Model

The present research was designed as a case study with the aim of revealing the fraction models used by primary school teachers to teach fractions and seeing whether the preferred models differ depending on the meaning of the fraction. Defined as the in-depth portrayal and analysis of a limited system (Merriam, 2013), a case study "explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in depth data collection involving multiple sources of information… and reports a case description and case themes" (Creswell, 2013, p. 97). A qualitative case study, on the other hand, adopts a single case method, a means of obtaining specific and detailed information from one person's experience to gain a greater awareness of an issue or concern (Doughty-Horn et al., 2016). It allows educators and teachers to conduct research to identify the causes of problems encountered in students, curricula and teaching (Leymun, Odabaşı & Yurdakul, 2017).

Participants

The study was conducted with voluntary participation of 14 primary school teachers chosen through the purposive sampling method, which is useful in explaining facts and events in many cases (Yıldırım & Şimşek, 2013). The participants in concern were chosen among the teachers who had taught from 1st to 4th graders at least once and who were still teaching at state schools in Turkey at the time of data collection (Female: 9; Male:5). 13 of the teachers held BA degree while only one of them held MA degree. Their professional experience in teaching ranged from 8 to 30 years, with an approximate mean experience of 19 years. For the sake of confidentiality, they were coded as T1, T2, T3, ….., T14.

Data Collection Tool

The research data were collected from the afore-mentioned teachers through a semi-structured interview form comprised of questions about the sub-constructs of fraction and the preferred fraction models. In line with the research objective, they were kindly requested to provide verbal and written responses to the interview questions and to illustrate the fraction teaching models they prefer based on the given sub-construct. It is significant to note that one of the six open-ended questions on the form was about the general meaning of fraction while the remaining five were about its sub-construct.

A preliminary meeting was held with the participant teachers to brief them about the content of the study and interview. They were also informed that their sincere responses would help realizing the research objective. The interviews were held on the predefined dates on an individual basis. The teachers were kindly requested to write their responses on the semi-structured form and to provide responses to the further questions –if any. Lastly, the audio records and interview documents were digitally transcribed for data analysis.

Data Analysis

The transcribed data were descriptively analysed by the researchers. This study used descriptive and systematic analysis, which is a version of the descriptive analysis techniques. This method involves transcription of the dialogues and categorization and evaluation of the participants' responses more systematically (Yıldırım & Şimşek, 2013). The first stage of data analysis required coding data based on the predefined screening and selection criteria and identifying various themes accordingly. Subsequently, the data were re-arranged, grouped into themes and digitalized. The inter-rater reliability in coding was found high (0,82). Codes were finalized after discussion in the cases of disagreement. Inductive analysis method was used while interpreting data-based findings (Patton, 2014). Finally, codes were tabulated for the sake of clarification and the outlined with quotes drawn from the participants' responses to the interview questions.

FINDINGS

The data obtained from 14 primary school teachers' responses to the semi-structured interview questions were descriptively analyzed to see the fraction models used by the teachers in relation to fraction sub-constructs (e.g. part/ whole, ratio, measure, quotient, length and cluster). The elicited findings are presented in the following section.

Primary School Teachers' Knowledge in Fraction Sub-constructs

The participants were asked the meaning of fractions such as 2/3 to reveal their knowledge in the above-mentioned sub-constructs. The analysis results showed that a total of 43 codes were developed based on the teachers' responses to the interview questions. The results in concern are illustrated in Table 1.

Table 1. *Teachers' responses on fraction sub-constructs*

Theme	Codes	Participants
Sub-constructs	Part-whole (f:14)	T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, T13, T14
	Operator (f:6)	T2, T5, T7, T9, T13, T14
	Measure (f:6)	T5, T7, T9, T12, T13, T14
	Ratio (f:8)	T2, T4, T5, T7, T9, T12, T13, T14
	Quotient (f:9)	T1, T4, T5, T7, T8, T9, T12, T13, T14

As indicated in Table 1, the participant teachers expressed the meaning of the fraction 2/3 using the part-whole sub-construct (f=14), followed by the quotient sub-construct (f=9), the ratio sub-construct (f=8) and the operator and measure sub-constructs (f=6).

The answers provided by primary school teachers regarding their general knowledge in the part-whole and operator sub-constructs are respectively given below.

T1; T6, T7; T9; T10; T13; T14: "It means that a whole is divided into three parts and two of these parts are taken." (Part-whole sub-construct)

T2: "Again, if I were to divide 30 by 3 and then multiply it by 2, it would be the operator sub-construct. One of the sub-constructs mentioned here is the operator sub-construct. But I do not know a lot about this sub-construct."

Only 6 of the interviewed teachers informed that they recognized such a fraction can also have the operator sub-construct. Their responses indicated that they interpreted it as a quantity or the certain parts of a given whole. The following are intended to depict their responses with a focus on the measure and ratio sub-constructs, respectively.

T13: "It is like two thirds of the depot is full. It is also used for measuring fluids."

T9: "It is something like the ratio of female teachers to the male teachers in the school."

Some of the participating primary school teachers who stated that a fraction such as 2/3 could mean a ratio between 2 and 3 have expressed this through examples.

As in the case of the operator sub-construct, 6 teachers stated that 2/3 fraction can have the measure sub-construct. Among them, 2 teachers regarded it as a general means of measurement while other teachers associated it with specific units of measurement such as length and liquid. The following statement was extracted from a teacher who expressed the given fraction with its quotient sub-construct:

T4: "It reminds me of sharing 2 apples among 3 people, by dividing them."

It is noteworthy that some of the teachers stated that it could also mean dividing 2 by 3. In fact, this could be attributed to the teachers' misunderstanding of the fraction in concern or his/ her inadequate conceptual knowledge in fractions. T1, on the other hand, attempted to clarify the meaning of the fraction by dividing a watermelon into 3 equal parts while T4 illustrated it by dividing 2 apples among 3 people. As suggested in responses, the teachers have certain amount of knowledge about each sub-construct and they are able to exemplify it.

Models Used by Primary School Teachers based on Fraction Sub-constructs

In this section, the models used by primary school teachers for each sub- constructs of the fraction are explained.

Fraction models used in teaching the part-whole sub-construct

With the purpose of revealing the fraction models used for the part-whole sub-construct, the participants posed the questions "How do you teach the part-whole sub-construct of a fraction? What fraction models do you use?" Table 2 provides the analysis results.

Table 2. *Fraction models used in teaching part-whole sub-construct*

Theme	Fraction Models	Presentation Types	Participants
Part-Whole sub-construct	Cluster (f:8)	Drawing a figure	T2, T4, T5, T6, T12, T13, T14
		Using objects	T3
	Area (f:13)	Using geometrical shapes	T1, T2, T5, T6, T8, T9, T10, T11, T13, T14
		Presenting on objects	T4, T7, T12
	Length (f:10)	Drawing a number line	T1, T2, T3, T4, T5, T6, T9, T13, T14
		Using a strip	T7
	Using realia (f:7)	Presenting with real objects	T1, T3, T4, T5, T8, T9, T14

As seen in Table 2, the teachers' responses related to the part-whole sub-construct has yielded four models from 14 sources: (i) the cluster model, (ii) the area model, (iii) the length model and (iv) use realia. The results revealed that the teachers employed various presentation types while using different models in teaching fractions: cluster model (e.g. drawing a model & using objects); the area model (e.g. geometrical figures and using objects); the length model (e.g. drawing a number line, using a strip); and using realia (e.g. using a material). All in all, it was observed that the teachers mostly use the area model while teaching the part-whole sub-construct of the fraction.

The length model revealed the second most frequently preferred model by the teachers, followed by the part-whole sub-construct and the cluster model. Those who use the cluster model stated that they mostly draw clusters on the board or construct student clusters in the classroom. The ones who prefer the area model informed that they teach fractions by drawing various geometric shapes, rectangle in particular, and/ or using the classroom-objects. T7, who is presenting the part-whole sub-construct on a number line, stated that she makes a strip out of an object, hence creating her own length model to present the subject while some of her colleagues prefer to use such objects as apple, bread and watermelon to teach the part-whole construct. Figure 1, Figure 2, Figure 3 and Figure 4 illustrate presentation of the part-whole construct by using the cluster model, the area model, the length model and using realia, respectively.

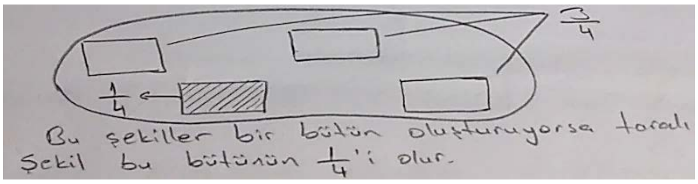

FIGURE 1. *Presenting part-whole sub-construct through the cluster model (T4)*

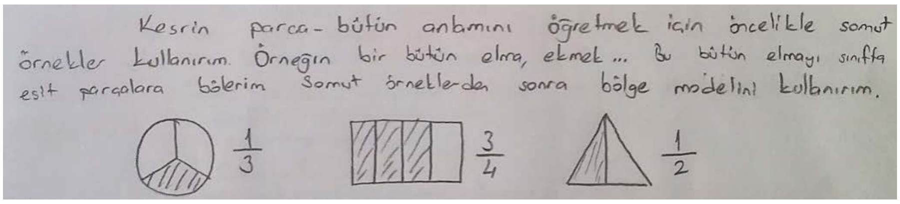

FIGURE 2. *Presenting part-whole sub-construct by using the area model (T14)*

FIGURE 3. *Presenting part –whole sub-construct by using the length model T3)*

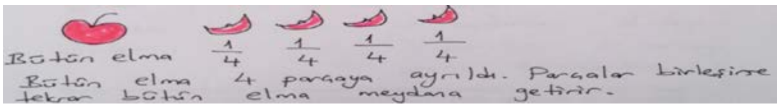

FIGURE 4. *Presenting part-whole sub-construct by using an object (T3)*

The findings reported in this sub-section showed that 13 out of 14 participant teachers mostly prefer the area model when presenting the part-whole sub-construct while 10 teachers use the length model in teaching fractions. Lastly, 8 and 7 teachers reported that they use the cluster model and objects use to present fractions respectively. That the area model was used in teaching the part-whole sub-construct with the highest frequency might be attributed to its functionality and easiness and the extensive use of the area model in teaching the part-whole sub-construct in the textbooks.

Fraction models used in teaching the operator sub-construct

In order to reveal the teachers' choice of fraction models to teach the operator sub-construct, they were asked the following questions: "What sub-construct does 2/5 of 30 correspond to? sub-construct? How would you explain, exemplify and illustrate this sub-construct? Their responses were analyzed and the related results are given in Table 3.

Table 3. *Fraction models used in teaching the operator sub-construct*

Theme	Fraction Models	Presentation Types	Participants
Operator sub-construct	Cluster (f:14)	Drawing figures	T1, T2, T3, T4, T5, T6, T9, T10, T12, T13
		Using objects	T3, T7, T8, T14
	Area (f:6)	Using geometric shapes	T2, T4, T8, T9, T12, T13
	Length (f:7)	Drawing an number line	T1, T2, T4, T5, T9, T12, T14
	Using operations (f:6)	Presenting with operations	T1, T2, T3, T6, T6, T12

As demonstrated in Table 3, four models were obtained from 14 sources regarding the use of operator sub-construct in teaching fractions: (i) the cluster model, (ii) the area model, (iii) the length model and (iv) using operations. The cluster model proved the mostly used model to teach the operator sub-construct (f=14), followed by the length model (f=7), the area model (f=6) and using operations (f=6). The findings also indicated that the teachers who preferred to use the cluster model by drawing clusters outnumbered those who used real objects to make clusters. Below are the illustrations taken from the teachers' responses on the interview forms.

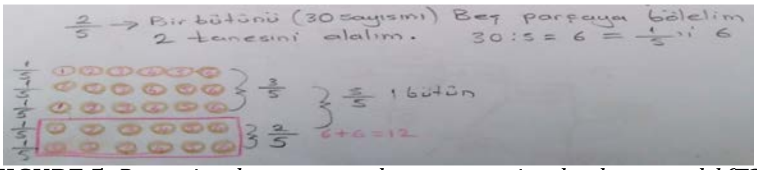

FIGURE 5. *Presenting the operator sub-construct using the cluster model (T3)*

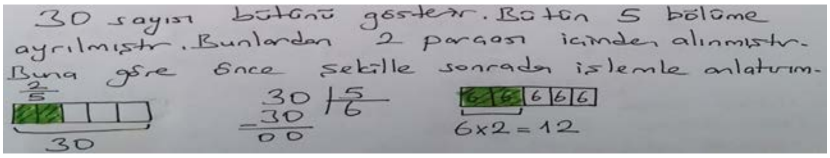

FIGURE 6. *Presenting operator sub-construct using the area model (T8)*

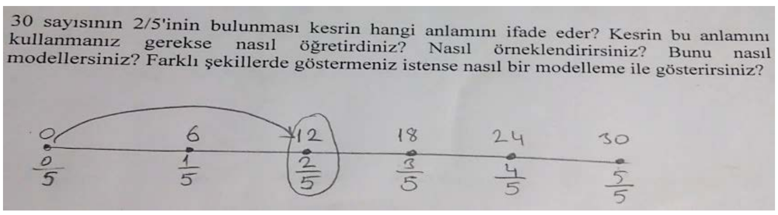

FIGURE 7. *Presenting operator sub-construct using the length model (T9)*

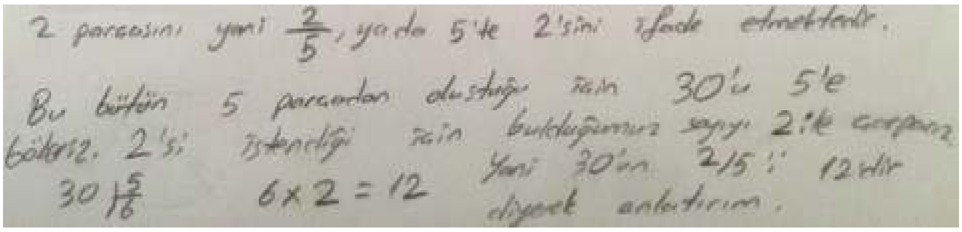

FIGURE 8. *Presenting operator sub-construct through operations (T6)*

As displayed in Table 3 and the figures, using the cluster model involved drawing a cluster shape and making a cluster with objects while the area model required using geometric shapes. The use of the length model comprised of drawing a number line while using operations involved doing a mathematical operation. As the operator sub-construct of fractions requires processing a certain amount of a quantity and doing mathematical operations (multiplication-division or division-multiplication), primary school teachers might have presented this sub-construct only through operations.

Fraction models used in teaching the measure sub-construct

The teachers were asked how they would explain, exemplify and illustrate the measure sub-construct in teaching fractions to reveal the fraction models they prefer to use when teaching the sub-construct in concern. The analysis results of their responses are shown in Table 4.

Table 4. *Fraction models used in teaching the measure sub-construct*

Theme	Fraction Models	Presentation Types	Participants
Measure sub-construct	Area (f:8)	Using geometric figures	T1, T2, T3, T5, T7, T12, T13, T14
	Length (f:9)	Drawing a number line	T1, T2, T3, T4, T5, T7, T9, T11
		Using a strip	T14
	Using realia (f:6)	Presenting with objects	T1, T4, T7, T8, T10, T12

Table 4 shows that three models were drawn from 14 sources related to the measure sub-construct: (i) the area model, (ii) the length model and (iii) using realia. It was observed that the teachers mostly used the length model to teach the measure sub-construct (f=9), followed by the area model (f=8) and using realia (f=6). It is a noteworthy finding of the study that none of the participant teachers preferred the cluster model to teach the measure sub-construct. This might stem from the fact that the measure sub-construct is mostly used to teach measuring such phenomena as liquids and length. Therefore, the teachers might have overwhelmingly used the length and area models rather than the cluster model in their teaching. The following are the illustrations taken from the interview forms.

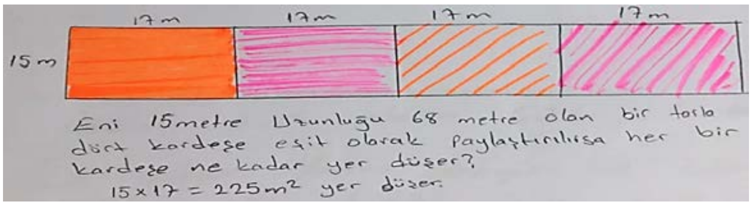

FIGURE 9. *Presenting the measure sub-construct using the area model (T13)*

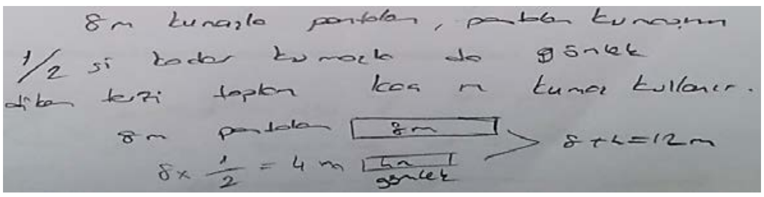

FIGURE 10. *Presenting the measure sub-construct by using the length model (T1)*

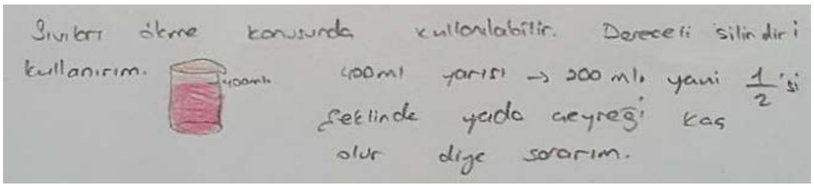

FIGURE 11. P*resenting measure sub-construct by using realia (T10)*

As depicted in Table 4 and the figures, the teachers who utilized the length model to teach the measure sub-construct either drew a number line or used a strip. Those who used the area model drew geometric shapes while those who preferred using realia employed real objects to teach the measure sub-construct in their classroom. More specifically, 6 of the teachers reportedly used real objects such as measuring cups and beakers as measurement tools when teaching the measure sub-construct. These objects were exclusively employed to teach the measure sub-construct regardless of the cluster, area or length model models.

Fraction models used in teaching the ratio sub-construct

The participants were requested to express how they would teach, exemplify and illustrate the ratio sub-construct. Table 5 displays the models and presentation types they preferred to teach the ratio sub-construct.

Table 5. *Fraction models used in teaching the ratio sub-construct*

Theme	Fraction Models	Presentation Types	Participants
Ratio sub-construct	Cluster (f:9)	Drawing clusters	T2, T3, T8, T9, T12, T13
		Clustering objects	T1, T5, T8, T14
	Alan (f:10)	Drawing geometric shapes	T1, T2, T3, T4, T5, T7, T10, T11, T12, T13
	Length (f:6)	Drawing a number line	T1, T2, T3, T5, T9, T14

As provided in Table 5, three models were elicited from 14 sources concerning teaching the ratio sub-construct: (i) the cluster model, (ii) the area model and (iii) the length model. It was revealed that the area model proved the most frequently used model (f=10), followed by the cluster model (f=9) and the length model (f=6). Use of the three models are exemplified in Figure 12, Figure 13 and Figure 14, respectively.

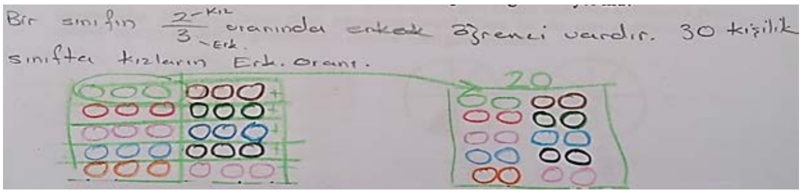

FIGURE 12. *Presenting the ratio sub-construct by using the cluster model (T5)*

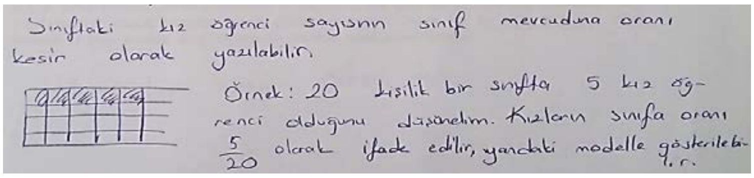

FIGURE 13. *Presenting ratio sub-construct using the area model (T3)*

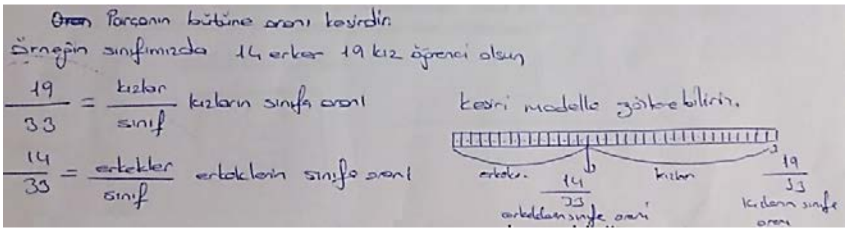

FIGURE 14. *Presenting ratio sub-construct using the length model (T5)*

The teachers who preferred the cluster model either drew shapes onto the board or used real objects to create clusters. Those who employed the area model tended to draw geometric shapes; while those who used the length model drew a number line.

Fraction models used in teaching the quotient sub-construct

The teachers were asked the following question in order to reveal the fraction models they use to teach the quotient sub-construct: "If a student asked whether the fraction 2/3 means dividing 2 by 3, how would you respond? How would you explain it? How would show it?". Analysis results of their responses are provided in Table 6.

Table 6. *Fraction models used in teaching the quotient sub-construct*

Theme	Fraction Models	Presentation Types	Participants
Quotient sub-construct	Cluster (f:3)	Drawing a cluster with figures	T8, T9, T14
	Area (f:5)	Drawing geometric figures (square, rectangle etc.)	T1, T2, T8, T9, T14
	Length (f:4)	Drawing a number line	T2, T4, T9, T14
	Use of realia (f:4)	Using real objects	T4, T6, T9, T10,
	Division (f:5)	Presenting through division	T5, T6, T11, T12, T13

As demonstrated in Table 6, five models were drawn from 14 sources based on the participants' responses to the question on quotient sub-construct: (i) the cluster model, (ii) the area model, (iii) the length model, (iv) use of realia and (vi) division. It was observed that the area model and division were most frequented models employed by the teachers (f=5), followed by the length model (f=4), using realia (f=4) and the cluster model (f=3), respectively. Even though the number of models used to teach the quotient sub-construct is relatively high, the presentation types used by the teachers were quite limited. Figure 15 displays the presentation of quotient sub-construct through the cluster model.

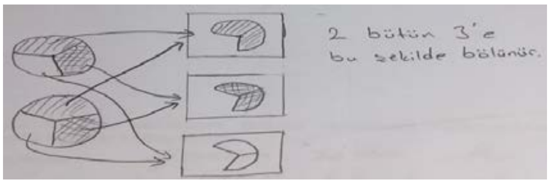

FIGURE 15. *Presenting quotient sub-construct using the cluster model (T9)*

As seen in in Figure 15, T9 presented the quotient sub-construct by grouping areas in the form of clusters. The teachers who used this model were observed to make clusters with figures while those who preferred the area model drew geometric figures such as squares, rectangles and etc. The teachers who employed the length model drew a number line whereas those who preferred to use realia utilized real object in their teaching. Finally, the teachers who adopted division introduced the quotient sub-construct simply by performing a division.

Nonetheless, some of the teachers failed to present this sub-construct using the area model, as illustrated in Figure 16.

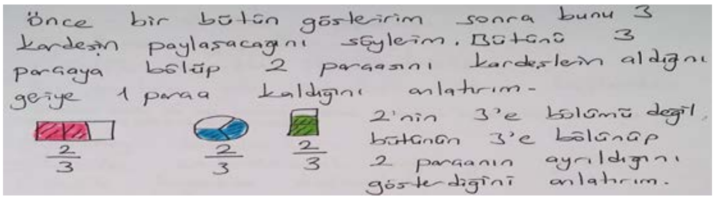

FIGURE 16. *Presenting quotient sub-construct using the area model (T1)*

In this example, T1 was observed to illustrate division a whole by 3 when she was asked to teach 2/3 fraction using the quotient sub-construct although she was expected to divide 2 wholes by 3. Likewise, T14 committed a similar mistake by presenting the quotient sub-construct using the length model on a stick, as shown in Figure 17.

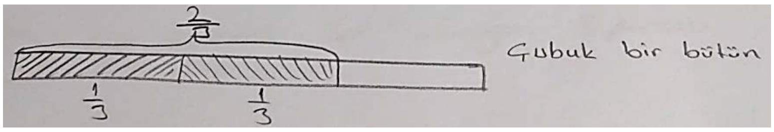

FIGURE 17. *Presenting quotient sub-construct using the length model (T14)*

As seen in Figure 17, T14 attempted to teach 1/3 fraction rather than 2/3 fraction using the length model. It has also been observed that the participant teachers used some real objects such as food and cutlery with the ultimate aim of teaching the quotient sub-construct by increasing students' attention through visualization, as depicted in Figure 18.

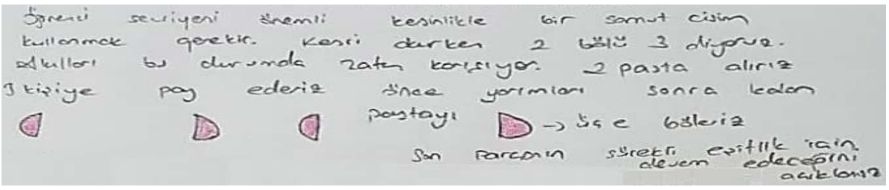

FIGURE 18. *Presenting quotient sub-construct by using an object (T10)*

Finally, the teachers utilized 'division function', which is peculiar to the quotient sub-construct, to teach the fraction in concern. As a fraction simultaneously refers to a numerical value and result of a division, it was employed so frequently as the area model by the teachers, as in Figure 19.

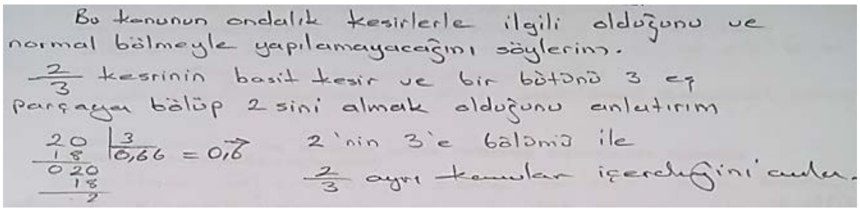

FIGURE 19. *Presenting quotient sub-construct by performing division (T6)*

As seen in Figure 19, T6 attempted to introduce the quotient sub-construct through division rather than using the above-mentioned models extensively reported in the existing literature.

In conclusion, this particular research was conducted to reveal the fraction models used by primary school teachers in teaching the five sub-constructs of fractions (e.g. part-whole, operator, measure, ratio and quotient). The overall findings are summarized in Table 7.

Table 7. *Fraction models used by primary school teachers in teaching sub-constructs*

Fraction models/meanings	Cluster Model	Area Model	Number Line	Use of Realia	Use of Operations	Total
Part-whole	8	13	10	7	-	38
Operator	14	6	7	-	6	33
Measure	-	8	9	6	-	23
Ratio	9	10	6	-	-	25
Quotient	3	5	4	4	5	21
Total	34	42	36	17	11	140

A general review of the models used by the teachers to teach sub-constructs indicated that the part-whole sub-construct revealed the mostly employed sub-construct introduced through all models except using operations (f=38), followed by the operator sub-construct introduced through four models (f=33), the ratio sub-construct presented through three models (f=25), the measure sub-construct using three models (f=23) and the quotient sub-construct via all models (f=21). The following section offers discussion of the findings presented so far, concluding remarks, practical implications developed in the light of the present findings and the existing literature and a couple of suggestions for further research.

DISCUSSION and CONCLUSIONS

This study exclusively scrutinized the Turkish primary school teachers' use of models they use in teaching fractions based on its sub-meanings through a semi-structured interview form developed by the researchers. Findings of the study have revealed that the area model, the cluster model and the length model are the most commonly used models in teaching fractions. This particular finding is in congruence with the existing literature (Çelik & Çiltaş, 2015; Kadhi, 2005; Toptaş, Han & Akın, 2017). The finding in concern conforms to Gökkurt, Soylu and Demir (2015) who previously reported that most of the teachers start teaching fractions with appropriate activities and recommended the use of the cluster, length and area models when presenting the concept of fraction. Furthermore, it approves Pesen (2008) who suggested the use of the area, cluster and length models, respectively in teaching fractions. Lastly, it is in partial agreement with Clarke, Roche and Mitchell (2011) who informed that students are able to present fractions with various models and partially competent in visualising fractions using the length model.

The findings of the study also show that the participants use all models, particularly the area model, to visualize the part-whole sub-construct and the models that are similar to those preferred by mathematics teachers reported in Çelik and Çiltaş (2015). The former finding largely overlaps with Cramer, Wyberg and Leavitt (2008) who favoured the use of the area model in teaching part-whole meaning of fractions in comparison to the other models. It also approves Behr et al. (1993) who reported that the area and cluster models were the most frequently used models to teach the part-whole meaning of fractions while the length model was mostly preferred to teach their measure meaning.

The current findings have yielded that the area model was the second most frequented model used by the teachers following the area model. This finding is quite in line with Bright, Behr, Post and Wachsmuth (1988) stated that the length model requires a higher level of cognition. There are several studies suggesting that the length model needs to be attached more attention when teaching fractions (Clarke et al, 2008). This might be attributed to the fact that it significantly differs from other models as it functions in daily life through its measure meaning and allows indicating the relative greatness of the numerical meaning of fraction.

One of the striking findings of the study was that the participant teachers used realia and operations rather than the fraction models informed by the literature to teach certain sub-constructs of the fractions. Hence, it is hoped to contribute to the existing literature via these findings.

Considering the findings informed by the relevant literature, the teachers are incompetent in teaching fractions as they do not pay adequate attention to conceptual knowledge (Işık & Kar, 2012). Therefore, different meanings of fractions must be emphasized and different models must be used when teaching fractions in order to ensure that students fully comprehend the fractions. This necessitates the teachers to take different meanings of fraction and fraction models into consideration when planning the relevant teaching process. Accordingly, primary school teachers are recommended to get in-service training programmes on the concept of fraction and to put what they learn from these trainings into effect in their teaching (Toptaş, Han & Akın, 2017).

It is highly important that primary school teachers possess instructional field knowledge and modelling ability to teach the concept of fractions. Considering its reflection on students, they must have in-depth knowledge in fractions, sub-meanings of fractions and fraction model. Constituting the core focus of the present study, fractions have a prominent place in primary and secondary mathematics. In consequence, it is highly significant for primary school teachers to have in-depth professional and content knowledge for ensuring the quality of primary mathematics education. Hence, pre-service primary school teachers are recommended to go through a training process that enables them to have adequate knowledge in sub-meanings of fractions and fraction models. Finally, it might be suggested that fractions should be taught with their sub-meanings in conceptual, operational and relational contexts.

The current research was confined to the investigation of fraction models used by a limited number of primary school teachers working at state schools in Turkey to teach sub-constructs. So, further studies could be conducted with larger sampling in order to reveal what fraction models are more appropriate for the effective use of the sub-constructs outlined in this research.

Note: This paper was produced from the doctoral dissertation entitled "Primary school teachers' knowledge toward fractional meanings and models used in fraction teaching", submitted to Graduate School of Educational Sciences, Gazi University, Ankara, TURKEY.

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