Behaviour of Reinforced Concrete Beams with Circular Transverse Openings under Static Loads

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ABSTRACT

Transverse opening in a reinforced concrete beam allows the crossing of mechanical and electrical services through the beam. However, it affects the strength of a beam. Understanding its structural behaviour is crucial to ensure a safe design of the beam. For that, an experimental study was carried out on reinforced concrete beams with circular transverse openings. The four-point load test was conducted to study the effects of the size and the position of the opening on the beam performance under the shear and flexural loads. In addition, three reinforcing methods for the opening were tested. The beams were evaluated in terms of the load-displacement responses, mechanical properties, deflections, and failure modes. The opening with the diameter not exceeding 0.25 times beam height affected about 20% of beam strength (without reinforcements at the opening). The diagonal bar reinforcing method effectively restored the beam strength for the opening size not exceeding 1/3 of beam height. The equation model proposed conservatively predicted the ultimate capacity of the beam with a transverse opening.

Keywords: Reinforced Concrete Beam, Circular Transverse Opening, Flexural and Shear Load Paper type: Research paper

INTRODUCTION

Transverse opening provides a passage for the mechanical and electrical services through a reinforced concrete (RC) beam when the ceiling space is insufficient. It reduces the required length of the services and the headroom of the building. With that, fewer loads are to be transferred to the foundations, and this reduces the building cost [1]. The cost savings could be substantial when it is repeatedly applied over a multi-story building or a large scale project [2].

The opening in RC beams have been extensively studied through experiments, numerical modelling and analytical derivation [3-6], (a) in various types of beam, such as simply supported beams [7-14], deep beams [15-27], continuous beams [28, 29], cantilever beams [2, 30], T-beams [31-36], curved beams [37-39] and etc, and (b) subjected to various types of load, such as the flexural and shear loads [18, 40-44], torsional load [45, 46], impact load [31], and seismic load [2, 30].

The opening affected the structural performance of a beam. The beams had lower strength and stiffness, endured excessive deflections, and demonstrated premature cracking and failure [2, 8, 31, 36, 47-49]. The opening caused a discontinuity of the cross-sectional configuration and led to the concentration of stress surrounding it [36]. This caused the beams to fail in poor ductility immediately after the diagonal shear cracks propagated through the opening [50]. The opening was more critically placed at the shear zone than the flexural zone of the beam [41].

The opening is considered small when the beam-type behaviour is maintained, which means the beam theory is still applicable [2, 51]. For a marginal strength reduction of the beam (not exceeding 10%), the opening size should not exceed 0.25 or 0.2 times the beam's depth [17, 52,]. In rectangular shape, the opening height should not exceed 0.2 times the beam depth and the opening length should be limited to 0.05 times the beam length [53].

The beam strength was more severely affected by large opening size [50]. When the size increased from 0.3 to 0.5 times the beam height, a 51% average strength reduction was reported [50]. The beam reduced 75% capacity as the opening size reached 0.6 times the beam depth [54].

The opening can be in various shapes [51, 55]. Circular openings are commonly used for the passage of service pipes, while rectangular openings are for air-conditioning ducts [51]. Beams with circular openings offer 8% higher strength than square openings [7, 10, 41]. The orthogonal corners attracted stress concentration, and this affected the beam strength [41, 46].

The opening is preferably placed at the upper corners of a deep beam, which is far from the arching action and the flexure region [16]. For the shallow beams, openings should avoid the regions with maximum displacement and shear [48].

Despite the numerous studies, the understanding of the beams with openings is still fragmented. For some reason, discrepancies of the findings by different researchers are observed. Some recommended the opening size not to exceed 0.2 to 0.25 times the beam depth [17, 52], while the others suggested 0.44 to 0.48 [7, 10]. On top of

that, the current design codes, such as ACI-318 [56], provide no explicit guide to design a beam with openings [2]. Engineers who are unconfident to design such beams tend to avoid it or provide very conservative designs.

This paper presents an experimental study to investigate the behaviour of RC beams with a circular opening. The purposes are (a) to investigate the effects of the size and position of the opening on beam's performance, and (b) to determine a suitable reinforcing method for the opening.

METHOD

Specimen details

Eleven RC beams were tested under the four-point load test (Fig. 1). This included (a) 2 control beams without opening, (b) 6 beams with openings, and (c) 3 beams with reinforced openings (Table I). The beam size was 150 mm (width) x 300 mm (height) x 1650 mm (length). The clear span between supports was 1500 mm.



Fig. 1: Test setup

	Tra	nsverse opening	Point load	Shear	
Specimen ^a	Diameter,	Distance from support,	Distance from support,	reinforcement	Remarks
	d_o (mm)	x (mm)	<i>a</i> (mm)	remoreement	
C1/S	-	-	500	R8-250	Control, shear
C2/F			600	D8 150	Control,
CZ/Γ	-	-	000	K0-130	flexural
S1/100	100	300	500	R8-250	Shear
S2/75	75	300	500	R8-250	Shear
S3/50	50	300	500	R8-250	Shear
F1/100	100	750	600	R8-150	Flexural
F2/75	75	750	600	R8-150	Flexural
F3/50	50	750	600	R8-150	Flexural
	100	200	500	D8 250	Shear,
KI/DK	100	500	500	K0-230	reinforced
D2/GI	100	300	500	D8 250	Shear,
K2/01	100	500	500	K0-230	reinforced
P3DS	100	300	500	D8 250	Shear,
K3D5	100	300	500	K0-230	reinforced

TABLE 1. SPECIMEN DETAILS

^a C - control, S - shear, F - flexural, R - reinforcement to opening

All specimens were reinforced with 2T12 and 2T10 high yield steel bars ($f_{y,b} = 460 \text{ N/mm}^2$) as the bottom and top reinforcements, respectively. The shear links were R8-150 and R8-250 mild steel bars ($f_{y,sl} = 250 \text{ N/mm}^2$) for flexural and shear load tests, respectively. The concrete cover was 25 mm.

The beam specimens were horizontally cast in plywood moulds. Ready-mixed concrete of grade 25 (maximum aggregate size = 20 mm, designed slump = 60 mm to 180 mm) was used. The specimens were cured for 7 days at an atmospheric temperature of $30 \pm 5^{\circ}$ C and tested after 28 days of casting.

Polyvinyl Chloride (PVC) pipes (diameters, $d_o = 50 \text{ mm}$, 75 mm and 100 mm) were used to create the opening. It was placed at the midheight of the beam (150 mm from soffit) and distance, x, from the support (Table 1 and Fig. 1). The opening was placed at the midspan and the support for the shear and flexural load tests, respectively. To encourage shear failure for the shear load test, the point loads were placed closer to the support (a = 500 mm) and fewer shear links (R8-250) were provided. Three types of opening reinforcements were provided, namely the diagonal reinforcement, Galvanised Iron (G. I.) pipe and diagonal square reinforcement (Fig. 2). The diagonal reinforcement bars were placed at 25 mm offset distance from the opening and the G.I. pipe replaced the PVC pipe. The effectiveness of these reinforcement methods was evaluated.



Fig. 2: Reinforcing method for opening

Test setup:

A static load was applied on the beam by using a hydraulic cylinder (Fig. 3). The steel I-beam distributed the load into 2 point loads acting on the beam. The load cell placed between the hydraulic cylinder and the steel I-beam was used to measure the load. Three linear variable differential transducers (LVDT) were placed below the beam at the midspan and the point loads to measure the beam deflection. All measuring devices were connected to a data logger for data acquisition. The specifications of the instruments used are outlined in Table 2.

TABLE 2. INSTRUMENT SPECIFICATIONS								
Instruments	Brand, Model	Description	Data					
			accuracy					
Hydraulic Cylinder	Enerpac, RR-10018	Push +933 kN, Pull -435 kN	-					
Hydraulic Pump	Enerpac P464	Manual hand pump	-					
Displacement	TML, CDP-100	100 mm	0.01 mm					
transducer								
Load Cell	TML, CLJ-300KNB	Capacity 300 kN	0.01 kN					
Data Logger	TML, TDS-530	30 Channels	-					



Fig. 3: Experimental setup in the laboratory

Test procedure:

Before the test started, all readings were set to zero. The beam was preloaded to 10% of the estimated beam capacity for 5 minutes to consolidate the test setup. The load was then released for another 5 minutes to observe the reading recovered to zero for the validity check of the measuring devices. This process was repeated twice.

Then, the readings were reinitialised to zero before the test started. The load increased at an interval of 7 kN load or 0.5 mm deflection (whichever reached first) until the specimen failed. Readings were taken after holding the load for at least 1 minute. The load-displacement response of the beam and the propagation of cracks were monitored throughout the test.

TEST RESULTS

Material properties:

The test results were considered acceptable based on the following justifications: (Tables 3 and 4)

- a. The concrete strength, *f_{c,u}*, ranged from 24.7 N/mm² to 26.9 N/mm², which was close to the designed strength of 25 N/mm².
- b. The bar strength was constantly higher than the specified yielding strength of 460 N/mm² and 250 N/mm² for the main bars and shear links, respectively.
- c. The concrete density was quite consistent. It ranged between 2300 kg/m³ and 2400 kg/m³.

TABLE 3. TEST KESULTS OF CONCRETE CUBES KEPKESEN HING DIFFEKENT TEST SPECT
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Testing	Specimen	Compressive	Density
davi	a	strength of cube,	$ ho_c$
uay		$f_{c,u}$ (N/mm ²)	(kg/m^3)
28	C1/S	25.1	2320
29	C2/F	25.9	2329
30	S1/100	24.7	2320
31	S2/75	24.9	2367
35	S3/50	25.4	2344
36	F1/100	25.7	2329
37	F2/75	25.0	2347
38	F3/50	26.9	2335
42	R1/DR	26.7	2367
43	R2/GI	26.2	2320
44	R3DS	25.8	2373

^a One 150 mm concrete cube was tested on the day that the beam specimen was tested to represent its concrete strength.

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Bar type	Т	ensile strengt	h,	Average strength,
	f_s,u	or $f_{sl,u}(N/mn$	n ²)	$f_{s,u,avg}$ or $f_{sl,u,avg}$ (N/mm ²)
	Sample 1	Sample 2	Sample 3	
High yield steel bar	532	551	547	543
Mild steel bar	295	281	278	285

TABLE 4. EXPERIMENTAL RESULT OF TENSILE STRENGTH

Test results:

Table 5 presents the test results. The observed results comprised (a) the loads when the shear and flexural cracks were first detected, and (b) the load when the cracks first reached the opening. These results were obtained from the markings made to demonstrate the propagation of cracks during the test (Fig. 4).

TABLE 5. TEST RESULTS

	Observed results			Measured results				Computed Results				
	First	First	Creat			Disp	lacement ((mm)	Firs	t crack	Ultimate	e state ^c
Specimen	shear crack , P _{ic,s} (kN)	flexural crack, P _{ic,f} (kN)	reached opening, P _{ic,o} (kN)	Failure mode ^a	Ult. load, P _u (kN)	At point load 1, $\delta_{l,u}$	At mid- span, $\delta_{2,u}$	At point load 2, $\delta_{3,u}$	Load, P _{ic} (kN) ^b	Moment, M _{ic} (kNm)	Shear load, V _u (kN)	Moment, M _u (kNm)
C1/S	96	47	-	F	163.1	8.53	10.2	9.17	47	11.8	81.6	40.8
C2/F	34	34	-	F	156.8	9.62	10.42	10.09	34	10.2	78.4	47.0
S1/100	44	39	46	S	108.0	5.20	5.76	5.56	39	9.8	54.0	27.0
S2/75	69	38	70	S	126.7	6.35	6.99	6.79	38	9.5	63.4	31.7
S3/50	85	30	126	S	135.8	8.71	9.34	9.17	30	7.5	67.9	34.0
F1/100	40	33	40	F/S	102.3	8.36	8.61	8.15	33	9.9	51.2	30.7
F2/75	50	39	125	F/S	127.2	8.12	8.30	8.01	39	11.7	63.6	38.2
F3/50	69	30	44	F/S	134.3	9.52	10.07	9.42	30	9.0	67.2	40.3
R1/DR	52	40	89	F	141.1	17.86	17.65	14.39	40	10.0	70.6	35.3
R2/GI	53	43	71	S	101.6	7.39	7.89	7.90	43	10.8	50.8	25.4
R3/DS	54	32	81	S	119.0	8.76	9.59	9.17	32	8.0	59.5	29.8

*Note: ${}^{a}F$ – flexural failure, S – shear failure, F/S – flexural and shear failure

^bFirst crack load, $P_{ic} = \min(P_{ic,s}, P_{ic,f}, P_{ic,0})$

 $^{\circ}$ The results were computed from the readings of the load cell. It ignored the weight of the beam, the load cell (4.9 kg), the rockers (2 x 8.3 kg) and the steel I-beam beam (20.7 kg) sitting on top of the specimen.

The measured results were the ultimate load and the beam displacement, which were obtained directly from the load cell and the LVDTs. The computed results were the calculated bending moment and shear load based on the load applied to the specimen (refer to (1) and (2)).



Fig. 4: Crack pattern of test specimens

From Table 5:

- a. The first crack was always the flexural crack. The load, $P_{ic,f}$, was always lower than $P_{ic,s}$ and $P_{ic,o}$.
- b. As the opening size decreased from 100 mm to 50 mm, (Table 6)
 - i. The first shear crack delayed ($P_{ic,s}$ increased).
 - ii. The shear crack reaching the opening delayed ($P_{ic,o}$ increased). No delay was noticed under the flexural load.
 - iii. The ultimate capacity of the beam, P_u , increased.
 - iv. The ultimate midspan displacement, $\delta_{2,u}$, increased. It developed at a faster rate when the opening was placed at the support.
- c. The first flexural crack occurred at about the same time (about 9.6 kNm moment load) regardless of the size, location, and reinforcement of the opening.

TABLE 6. RESPONSE OF BEAM WHEN THE OPENING SIZE REDUCED FROM 100 MM TO 50 MM (WITHOUT REINFORCE	MENT AT THE OPENING)

Paspansa	Location	of the opening
Response	Near the support	At midspan
The occurrence of the first shear crack	Delayed from 44 kN to 85 kN load, <i>P_{ic,s}</i> (93% load increment)	Delayed from 40 kN to 69 kN load, $P_{ic,s}$ (73% load increment).
The occurrence of the first flexural crack	About the same time. Occurred around 8.9 kNm average moment.	About the same time. Occurred around 10.2 kNm average moment.
Crack first reaching the opening	Delayed from 46 kN to 126 kN load, $P_{ic,o}$ (274% load increment).	No specific trend. Depended on the propagation path of the crack.
The ultimate capacity of the beam	Increased by 26% (from 108.0 kN to 135.8 kN ultimate load, P_u)	Increased by 31% (from 102.3 kNm to 134.3 kNm moment)
Deflection of beam	Increased by 60% (from 5.76 mm to 9.34 mm, $\delta_{2,u}$)	Increased slightly 17% (from 8.61 mm to 10.07 mm, $\delta_{2,u}$)

The reinforcements at the opening led to: (Table7)

a. An increase of the load to initiate the shear crack, $P_{ic.s}$, by 20%.

b. A substantial increase in the load for the crack to reach the opening (54% to 93% increment of $P_{ic,o}$).

The diagonal bar reinforcing method (specimen R1/DR) had effectively strengthened the beam with opening. It controlled the propagation of cracks surrounding the opening and increased the beam strength by 30%. The G.I. reinforcing method was not effective. It did not increase the beam strength.

TABLE 7. COMPARISON OF DIFFERENT REINFORCEMENT METHOD WITH RESPECT TO THE BEAM WITH UNREINFORCED OPENINGS

Response		Reinforcement methods	
	Diagonal bar reinforcement	G.I. pipe reinforcement	Diagonal square reinforcement
Load to initiate the shear crack, $P_{ic,s}$		Similar for all methods	
	About 20%	% higher (44 kN to 53 kN av	erage $P_{ic,s}$)
Load to initiate the flexural crack, $P_{ic,f}$		No specific trend.	
Load for the shear crack to first reach the opening, $P_{ic,o}$	Increased by 93% (from 46 kN to 89 kN)	Increased by 54% (from 46 kN to 71 kN)	Increased by 76% (from 46 kN to 81 kN)
Ultimate load capacity, P_u	Increased by 30% (from 108.0 kN to 141.1 kN)	No notable effect	Increased by 10% (from 108.0 kN to 119.0 kN)

Load-Displacement Response:

The load-displacement (*P*- δ) responses of the specimens are presented in Fig. 5. In general, the beam with opening behaved alike the solid beam, except it had lower stiffness, yield strength, ultimate strength, and ductility.

Fig. 5: Load-displacement response

The beam typically initiated with a high degree of stiffness, as represented by the gradient of the P- δ curve. While the beam was in the elastic stage, small deflection developed proportionally to the applied load.

As the first crack developed, the stiffness decreased slightly. The flexural crack developed at the beam midspan from its soffit as the deformability limit of the concrete was exceeded. As the load increased, (a) the cracks extended and widened, (b) more cracks were observed, and (c) the cracking regions expanded sideways until a diagonal shear crack developed.

The progressive development of the cracks gradually degraded the beam stiffness. This led to a faster rate of beam deflection. Then, the beam yielded due to excessive cracking of concrete or excessive deformation of the tension bars. As the tension bars lost bonding with concrete, the deflection accelerated and the stiffness dropped drastically. The beam continued to deteriorate until the critical damage occurred. By then, the beam strength peaked and the ultimate state was reached.

Without opening reinforcement, most beams failed almost immediately after the yield point. These beams were generally less ductile than a solid beam. The diagonal reinforcement bars in specimen R1/DR prolonged the post-yielding stage of the beam for better ductility. The G. I. pipe and the diagonal square reinforcing methods (R2/GI and R3/DS) showed no notable effects on the ductility of the beam.

The beam properties were then computed from the *P*- δ curves, as demonstrated in Fig. 6 and given in Table VIII. The ultimate capacity, *P_u*, was the highest point of the curve. The corresponding displacement was the total displacement, δ_u .

The yield point was determined based on the method used by Park (1988) [57] and Noushini et. al (2014) [58]. Two horizontal lines were drawn at P_u and $0.75P_u$. A straight line connecting the Origin and the interception of $0.75P_u$ line and the *P*- δ curve was extended and intercepted with line P_u . The line gradient represented the secant stiffness, *E*, and the point on the *P*- δ curve below the interception was taken as the yield point (P_v , δ_v).

The beam properties were then re-computed into several ratios to indicate beam performance (Table VIII). These ratios reflected (a) the occurrence of the first cracks with respect to the yielding and ultimate strengths (i.e. P_{ic}/P_y , P_{ic}/P_u , $P_{ic,o}/P_y$ and $P_{ci,o}/P_u$), (b) the yield strength relative to the ultimate strength (i.e. P_y/P_u), and (c) the ductility of the specimens (i.e. δ_u/δ_y).

Fig. 6: Beam properties of a typical load-displacement response

	Secant	Yield	Yield			Performat	nce ratios		
Specimen	stiffness, <i>E</i> (kN/mm)	strength, P_y (kN)	Displacement, δ_y (mm)	P_{ic}/P_y	P_{ic}/P_u	$P_{ic,o}/P_y$	$P_{ci,o}/P_u$	P_y/P_u	δ_u/δ_y
C1/S	25.8	152.2	6.32	0.31	0.29	-	-	0.93	1.61
C2/F	23.3	140.0	6.73	0.24	0.22	-	-	0.89	1.55
S1/100	21.4	99.1	5.05	0.39	0.36	0.46	0.43	0.92	1.14
S2/75	20.7	120.7	6.12	0.31	0.30	0.58	0.55	0.95	1.14
S3/50	20.8	120.7	6.53	0.25	0.22	1.04	0.93	0.89	1.43
F1/100	14.8	93.8	6.91	0.35	0.32	0.43	0.39	0.92	1.25
F2/75	19.6	113.7	6.49	0.34	0.31	1.10	0.98	0.89	1.28
F3/50	18.7	122.9	7.18	0.24	0.22	0.36	0.33	0.92	1.40
R1/DR	23.2	123.1	6.08	0.32	0.28	0.72	0.63	0.87	2.90
R2/GI	16.5	94.0	6.16	0.46	0.42	0.76	0.70	0.93	1.28
R3/DS	18.5	105.8	6.43	0.30	0.27	0.77	0.68	0.89	1.49

TABLE 8. PROPERTIES AND PERFORMANCE RATIOS OF BEAM SPECIMENS

Table 8 shows that:

- a. The opening affected the beam stiffness more significantly when it is placed at the midspan, where the moment was predominant.
- b. The occurrence of the first crack was not notably affected by the opening. It occurred around 0.24 to 0.39 times P_y or 0.22 to 0.36 times P_u .
- c. Depending on the pathway of the first crack, it reached the opening at various load levels $(0.36-1.10P_y \text{ or } 0.33-0.98P_u)$. If the opening was not in the way of the crack, $P_{ic,o}$ would be higher, as demonstrated by specimens F2/75 and S3/50.
- d. The opening reinforcements controlled the propagation of the cracks, and thus, considerably increased the $P_{ic,o}/P_y$ and $P_{ic,o}/P_u$ ratios of the specimens.
- e. The yield strength, P_y , was about 89% to 95% of the ultimate capacity, P_u . Should the ultimate capacity of the beams with an opening are analytically predicted, the design strength could be conservatively taken as 0.85 times the ultimate capacity.
- f. The ductility ratio of the beam, δ_{u}/δ_{y} , was generally low. It barely exceeded 1.43. It increased to 2.9 as the diagonal reinforcement bars were provided.

From these observations, the following principles could be extracted:

- a. For the beam to remain uncracked, the load should not exceed $1/4P_y$ or $1/5P_u$.
- b. For a higher beam capacity, the opening should avoid the beam midspan and the support, where the maximum moment and shear occurs. It should also avoid the potential pathway of the first crack and the critical shear crack. Thus, the opening could be placed somewhere between the midspan and the shear crack (Fig. 7), subjected to verification in future studies.
- c. Diagonal bars can be adopted to reinforce the beam with opening. The bars should be arranged perpendicularly to the diagonal cracks.
- d. The opening size should not exceed 0.25 times the beam's height. The relevant beams (S2/75 and S3/50) are more likely to escape the pathway of the first crack.

Failure Modes:

From Fig. 4, the critical causes of failure were identified based on the severity of the cracks, as defined from the concentration of crack lines and the crack width. Specimens S1/100, S2/75 and S3/50 implied the shear failure. There were shear cracks propagated diagonally from the support, passing through the opening, and to the point load (Fig. 4(c), (d) and (e)). The crack width was larger than the flexural cracks at the midspan.

Specimens F1/100, F2/75 and F3/50 demonstrated a combined shear and flexural failure (Fig. 4(f), (g) and (h)). The flexural cracks occurred at the midspan, propagated upward, and passed through the opening and to the top surface of the beam. Meanwhile, shear cracks developed diagonally from the support to the point load. Their crack widths were similar.

In the presence of the opening, the shear cracks were generally more severe (Fig. 4(c), (d) and (e)). It created void in the beam, reduced the effective cross-sectional area, led to the concentration of stress surrounding it, and thus, the beam failed earlier.

With the diagonal reinforcements (specimen R1/DR), the severity of the shear cracks reduced (Fig 4(i)). The reinforcements had effectively controlled the diagonal cracks and subsequently strengthened the beam. The G. I. pipe reinforcing method (specimen R2/GI) failed to control the shear cracks (Fig. 4(j)). The high compressive strength of the opening did not strengthen the beam at all. The diagonal square reinforcement (specimen R3/DS) slightly delayed the beam failure by disrupting the propagation of the first shear crack. However, the next diagonal crack, which developed at an offset distance from the first crack, bypassed the reinforcement and resulted in the beam failure (Fig. 4(k)). For that, the reinforcement should cover a wider beam region.

Deflection:

The deflection profiles of the beams are shown in Fig. 8. For the symmetrical load setup, the largest deflection always occurred at the midspan.

Fig. 8: Deflection profile

Fig. 8: Deflection profile (cont)

In general, the deflection profile remained symmetrical prior to the occurrence of the first shear crack ($\delta_l \approx \delta_3$). Then, the differences between the two displacements increased gradually when a diagonal crack propagated at one side of the beam. The beam side with the opening normally had a larger shear crack, therefore, a larger deflection occurred. When the opening was placed at the midspan, the shear cracks at both sides of the beam were similar.

Effects of opening to RC beam:

The beams with opening are compared with the control specimens C1/S and C2/F in Table 9. The parametric responses are demonstrated in Fig. 9.

	Specimen	Stiffness E_i/E_c	Yield strength $P_{y,i}/P_{y,c}$	Ultimate strength $P_{\mu}/P_{\mu c}$	Yielding displacement $\delta_{y,y}/\delta_{y,c}$	Ultimate displacement $\delta_{\mu\nu}/\delta_{\mu c}$	Ductility Δ_i/Δ_c
	C1/S	1.00	1.00	1.00	1.00	1.00	1.00
Effects of transverse opening near	S1/100	0.83	0.65	0.66	0.80	0.56	0.71
the support	S2/75	0.80	0.79	0.78	0.97	0.69	0.71
	S3/50	0.81	0.79	0.83	1.03	0.92	0.89
	C2/F	1.00	1.00	1.00	1.00	1.00	1.00
Effects of transverse opening at	F1/100	0.64	0.67	0.65	1.03	0.83	0.81
midspan	F2/75	0.84	0.81	0.81	0.96	0.80	0.83
	F3/50	0.80	0.88	0.86	1.07	0.97	0.9
	S1/100	1.00	1.00	1.00	1.00	1.00	1.00
Effects of reinforcement for	R1/DR	1.08	1.24	1.31	1.20	3.06	2.54
transverse opening under shear load	R2/GI	0.77	0.95	0.94	1.22	1.37	1.12
	R3/DS	0.86	1.07	1.10	1.27	1.66	1.31

TABLE 9. PERFORMANCE OF BEAM RELATIVE TO CONTROL SPECIMENS

It is observed that:

- a. The opening affected the stiffness, yield strength, ultimate strength, and the ductility of the beam, regardless of the position whether at the midspan or the support. Such detrimental effects were more pronounced as the opening size increased.
- b. The increasing opening size at the support affected the beam ductility more significantly than the midspan.
- c. The stiffness was significantly affected by a large opening at the midspan. The opening size of 1/3h affected 36% of stiffness, as demonstrated by specimen F1/100.
- d. The reduction of beam strength was about proportion to the opening size. The beam lost approximately 1/3 of its strength when the opening size reached 1/3 of beam height. It lost 1/6 of the strength as the opening size was 1/6h.

e. To maintain at least 80% beam performance, the opening size should not exceed 0.25*h*.

Fig. 9: Effects of opening size on beam properties

The comparison of the reinforcing methods shows that (Table 9):

- a. The diagonal bar reinforcing method (R1/DR) was the most effective of all. It increased stiffness, yield strength, ultimate strength and ductility by 8%, 24%, 31%, and 154% respectively.
- b. The G.I. pipe reinforcing method (R2/GI) did not strengthen the beam in any way.
- c. The diagonal square reinforcing method (R3/DS) was less effective than the diagonal bar reinforcing method. It only increased by 10% of the ultimate strength compared with a 31% increase by the diagonal bar reinforcing method.

Predicting the ultimate capacity:

For predicting the ultimate capacity of the beam with opening, the equation model proposed by Mansur and Tan (1999) [51] was tested. The model was modified from ACI-318 [56] and was popularly referred by the researchers [3, 5, 35, 43, 47, 49, 54, 59], with or without modifications.

The nominal shear strength of the beam is the sum of the shear strength given by the concrete, V_c , the shear reinforcement, V_{sl} , and the diagonal reinforcement, V_{sd} (3).

$$V_n = V_c + V_{sl} + V_{sd} \tag{3}$$

where
$$V_c = \frac{1}{\epsilon} \sqrt{f_c} b(d - d_o)$$
 (4)

$$V_{sl} = \frac{A_{sl}f_{y,sl}}{c} (d_v - d_o) \tag{5}$$

$$V_{sd} = A_d f_{vd} \sin\alpha \tag{6}$$

The nominal flexural strength of the beam is derived from the stress block diagram in Fig. 10, as given in (7) [51]. b

Fig. 10: Stress diagram by Mansur and Tan (1999)

$$M_n = A_s f_{y,b} \left(d - \frac{a_{sb}}{2} \right) \tag{7}$$

where
$$a_{sb} = \frac{A_s f_{y,b}}{0.85 f_c b}$$
 (8)

The nominal shear and flexural strengths are converted into functions of the equivalent load acting on the beam based on the experimental setup of this study, as expressed in (9) and (10).

$$P_{u,v} = 2V_n \tag{9}$$

$$P_{u,M} = \frac{2M_n}{a} \tag{10}$$

The ultimate strength of the beam is theoretically governed by the smaller value of the nominal shear or flexural strength (11).

$$P_{u,pre} = \min\{P_{u,V}, P_{u,M}\} \tag{11}$$

To determine the reliability of the equation model, the data from the experiments were substituted into the equations. For predicting the ultimate strength of beam, the ultimate stresses of concrete and steel bars, $f_{c,u}$, $f_{s,u,avg}$ and $f_{sl,u,avg}$, were used to substitute f_c , $f_{y,b}$, $f_{y,d}$ in (4) to (8), as demonstrated in Table 10.

The predicted load capacities of the beams, $P_{u,pre}$, were then compared with the load capacities of the experiment, $P_{u,exp}$ in Table XI. The equation model was found not reliable in predicting the loading capacity of the beam;

a. Only 1 out of 11 specimens had the reliability ratios, R_r , within the acceptable range of 0.9 to 1.1.

b. Only 55% of the predict failure mode was as per the failure mode observed in the test.

	Computing data				Shear strength			Beam Capacity	
Specimen	Concrete	Opening	Spacing of	Area of opening	Concrete	Shear	Diagonal	Shear	Moment
	strength, f_c	diameter,	shear links,	reinforcement,	V (kN)	reinforcement,	reinforcement,	capacity,	capacity,
	$(N/mm^2)^a$	$d_o (\mathrm{mm})$	s (mm)	$A_d (\mathrm{mm}^2)$	V_c (KIN)	V_{sl} (kN)	V_{sd} (kN)	V_n (kN)	M_n (kNm)
Eq.					(4)	(5)	(6)	(3)	(7)
C1/S	20.1	0	250	0	29.3	25.7	0	55	29.1
C2/F	21.0	0	150	0	29.9	42.8	0	72.7	29.2
S1/100	19.7	100	250	0	17.9	14.2	0	32.1	29
S2/75	19.9	75	250	0	20.7	17	0	37.7	29.1
S3/50	20.4	50	250	0	23.8	19.9	0	43.7	29.1
F1/100	20.8	100	150	0	18.4	23.6	0	42	29.2
F2/75	20.0	75	150	0	20.8	28.4	0	49.2	29.1
F3/50	22.0	50	150	0	24.7	33.2	0	57.9	29.3
R1/DR	21.8	100	250	226	18.8	14.2	86.8	119.8	29.3
R2/GI	21.3	100	250	0	18.6	14.2	0	32.8	29.3
R3DS	20.9	100	250	226	18.4	14.2	86.8	119.4	29.2

*Note: ^a f_c = compressive strength of cylinder, which was computed through a correlation between the strength of concrete cube and cylinder based on the standard concrete grade in Eurocode 2, C20/25, C25/30, C30/37.

^b Computing data: b = 150 mm, d = 261 mm, $A_{sl} = 101$ mm², $f_{y,sl} = f_{sl,u,avg} = 285$ N/mm², $d_v = 223$ mm, $f_{y,b} = f_{y,d} = f_{s,u,avg} = 543$ N/mm², $\alpha = 45^{\circ} = 0.785$ rad., $A_s = 226$ mm²

TABLE 11. RELIABILITY OF THE PREDICTED LOAD CAPACITY AND FAILURE MODE OF BEAM WITH RESPECT TO EXPERIMENTAL RESULTS

	Load capacity						Failure mode		
Specimen	Due to shear, $P_{u,v}$ (kN)	Due to moment, $P_{u,M}$ (kN)	Prediction, $P_{u,pre}$ (kN)	Experiment, P _{u,exp} (kN)	Reliability ratio, $R_r = P_{u,pre}/P_{u,exp}$	Remarks ^a	Prediction ^b	Experiment	Remarks [°]
C1/S	110.0	116.4	110	163.1	0.67	NA	S	F	NA
C2/F	145.4	97.4	97.4	156.8	0.62	NA	F	F	А
S1/100	64.2	116.1	64.2	108.0	0.59	NA	S	S	А
S2/75	75.4	116.2	75.4	126.7	0.60	NA	S	S	А
S3/50	87.4	116.5	87.4	135.8	0.64	NA	S	S	А
F1/100	84.0	97.3	84.0	102.3	0.82	NA	S	F/S	NA
F2/75	98.4	96.9	96.9	127.2	0.76	NA	F	F/S	NA
F3/50	115.8	97.8	97.8	134.3	0.73	NA	F	F/S	NA
R1/DR	239.6	117.3	117.3	141.1	0.83	NA	F	F	А
R2/GI	65.6	117.0	65.6	101.6	0.65	NA	S	S	А
R3DS	238.8	116.8	116.8	119.0	0.98	А	F	S	NA
Reliability d						1/11			6/11

Note: ^aA - reliable prediction ($0.9 \le R_n \le 1.1$), NA – unreliable prediction ($R_n < 0.9$ or $R_n > 1.1$).

^bF – Flexural failure (when $P_{u,M} < P_{u,V}$), S – Shear failure (when $P_{u,V} < P_{u,M}$)

^cA - predicted failure mode = experimental failure mode, NA - predicted failure mode \neq experimental failure mode ^dNumber of applicable prediction out of the total specimen tested.

From Table 11:

- a. The predicted results were consistently lower than the experimental results. The reliability ratios, R_r , were always less than 1.0. Thus, the equation model gave conservative estimations of the strength of the beams with opening.
- b. The equation model is less accurate in predicting the responses of (i) the solid beams (specimens C1/S and C2/F), and (ii) the beams that failed in shear (specimens S1/100, S2/75, S3/50, and R2/GI).
- c. The model predicted the beam to be governed by the shear strength when (i) the opening was placed at the support (specimens S1/100, S2/75, and S3/50), (ii) large-diameter opening ($\geq 1/3h$) be placed at the midspan without proper reinforcements (specimens F1/100 and R2/GI).

CONCLUSION

This study was carried out to investigate the effects of opening on beam performance. Based on the experimental results, the following conclusions can be drawn:

- a. The opening affected the beam performance in terms of stiffness, yield strength, ultimate strength, and ductility, particularly for large the opening size.
- b. For a marginal strength reduction ($\leq 20\%$), the opening size should not exceed 0.25 times the beam height.
- c. The diagonal bar reinforcing method was effective in reinforcing the beams with the opening of 1/3 beam height.

The equation model proposed by Mansur and Tan (1999) [51] predicted the ultimate capacity of the beam with opening conservatively. The predicted values were generally lower than the experimental results.

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SYMBOLS

а	Distance between point load and support, mm						
a_{sb}	Depth of equivalent rectangular stress block, mm						
b	Width of beam, mm						
d	Effective beam depth, mm						
d_o	Diameter of transverse opening, mm						
d_v	Centre-to-centre distance between the top and bottom steel bar, mm						
f_c	Compressive strength of concrete, N/mm ²						
f _{c,u}	Ultimate compressive stress of concrete cube, N/mm ²						
$f_{s,u}$	Ultimate tensile stress of top and bottom steel bar, N/mm ²						
f _{sl,u}	Ultimate tensile stress of shear link, N/mm ²						
fs,u,avg	Average value of ultimate tensile stress of top and bottom steel bar, N/mm ²						
fsl,u,avg	Average value of ultimate tensile stress of shear link, N/mm ²						
$f_{y,b}$	Specified yield strength of bottom steel bar, N/mm ²						
$f_{y,d}$	Specified yield strength of diagonal steel, N/mm ²						
$f_{y,sl}$	Specified yield strength of shear links, N/mm ²						
ĥ	Height of beam, mm						
l	Clear span of beam between support, mm						
S	Spacing of shear links, mm						
to	Thickness of opening pipe, mm						
x	Distance between transverse opening and support, mm						
A_d	Area of diagonal reinforcement, mm						
A_s	Area of bottom steel bar, mm ²						
A_{sl}	Area of shear link, mm						
Ε	Stiffness of beam, kN/mm						
E_c	Stiffness of control specimen, kN/m						
E_i	Stiffness of the respective test specimens, kN/m						
M	Moment, kNm						
M_{ic}	Moment when the first flexural crack occurred, kNm						
M_n	Nominal flexural strength, kNm						
M_u	Moment at ultimate state, kNm						
Р	Point load generated by hydraulic cylinder acting on beam, kN						
P_{ic}	Applied load when the first crack initiated, kN						
$P_{ic,f}$	Applied load when flexural crack initiated, kN						
$P_{ic,o}$	Applied load when crack reached the transverse opening, kN						
D							

 $P_{ic,s}$ Applied load when shear crack initiated, kN

- P_y Applied load when the beam yielded, kN
- $P_{y,c}$ Yielding load of control specimen, kN
- $P_{y,i}$ Yielding load of respective specimen, kN
- P_u Ultimate load generated by hydraulic cylinder acting on beam, kN
- $P_{u,c}$ Ultimate load of control specimen, kN
- $P_{u,exp}$ Load capacity of beam obtained from experiment, kN
- $P_{u,pre}$ Predicted load capacity of beam based on equation model, kN
- $P_{u,i}$ Ultimate load of respective test specimen, kN
- R_r Reliability ratio
- V Shear load, kN
- V_c Shear strength of beam due to the concrete, kN
- V_{ic} Shear load when the first shear crack occurred, kN
- *V_{sd}* Shear strength of beam due to diagonal reinforcement, kN
- V_{sl} Shear strength of beam due to the shear reinforcement, kN
- V_u Shear load at ultimate state, kN
- V_n Nominal shear strength, kN
- α Angle of inclination of diagonal bars, rad.
- δ_u Vertical displacement of beam at midspan at ultimate state, mm
- $\delta_{u,c}$ Ultimate displacement of control specimen, mm
- $\delta_{u,i}$ Ultimate displacement of respective specimen, mm
- δ_y Vertical displacement of beam at midspan at yielding state, mm
- $\delta_{y,c}$ Displacement at yield point of control specimen, mm
- $\delta_{y,i}$ Displacement at yield point of respective specimen, mm
- δ_l Vertical displacement of beam at midspan, mm
- δ_2 Vertical displacement of beam at below the point load no. 1, mm
- δ_3 Vertical displacement of beam at below the point load no. 2, mm
- $\delta_{l,u}$ Vertical displacement of beam at midspan at ultimate state, mm
- $\delta_{2,u}$ Vertical displacement of beam at below the point load no. 1 at ultimate state, mm
- $\delta_{3,u}$ Vertical displacement of beam at below the point load no. 2 at ultimate state, mm
- ρ_c Density of concrete, kg/m³
- Δ_c Ductility of control specimen
- Δ_i Ductility of respective specimen

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