THE EVALUATION OF AN INSTRUCTIONAL FRAMEWORK USING THE VAN HIELE LEVELS FOR LEARNING AND TEACHING GEOMETRY: A STUDY IN FIVE RURAL SENIOR SECONDARY SCHOOLS

by

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ABSTRACT

The Constitution (1996) of the Republic of South Africa forms the basis for social transformation in our new society. The Constitution (Act 108 of 1996) and the amendments that followed provided a basis for curriculum transformation and development in South Africa. The Department of Education introduced the present curriculum known as the National Curriculum Statement, which is modern and internationally benchmarked, in grade 10 in 2006. It required the learners to do seven subjects in grades 10 to 12 of which mathematics or mathematical literacy was prescribed as compulsory subjects. To attain social transformation, the South African Government attached a great deal of importance to the learning and teaching of mathematics and sciences in the South African schools. This study was undertaken in an effort to improve the understanding of geometry and, consequently, the performance and achievement of senior secondary school learners in geometry.

The study was inspired by the van Hiele theory. The study made use of the different levels of the van Hiele theory for the development of an instructional framework for geometry in senior secondary schools. The research was conducted in a previously disadvantaged area in South Africa. Given the setting of this study and the wider application of it, the use of 'hands-on' and practical approach to use manipulatives and worksheets to improve the geometric understanding was tried and tested in this study. The assumption was that such experiences would make the learning of geometry more relevant and enjoyable for learners from limited financial and underprivileged circumstances.

A quasi-experimental design was chosen. A total of 359 learners from five purposively selected schools in Mthatha district in the Eastern Cape Province participated in this study. Qualitative data through interviews were gathered. The data were analysed using IBM SPSS Version 19 and Microsoft Excel.

Findings indicated that there was a notable improvement in the performance of learners who were taught by the application of the van Hiele theory. The results revealed that most of the learners were not ready for the application of deductive principles of geometry in terms of formal proof in senior secondary school geometry. Based on the results, some recommendations are made to enhance the teaching and learning of geometry in senior secondary schools.

DECLARATION

I, Jogymol Kalariparampil Alex, Student Number 209189355 solemnly declare that this thesis entitled "The Evaluation of an Instructional Framework Using the Van Hiele Levels for Learning and Teaching Geometry: A Study in Five Rural Senior Secondary Schools" is the result of my own research, except where otherwise acknowledged, and that this thesis has not been submitted for a higher degree to any other institution.

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I dedicate this work to my loving husband, Anil, who has been a pillar of strength throughout.

ACRONYMS

CDASSG	Cognitive Development and Achievement in Secondary School Geometry
DBE	Department of Basic Education
DHET	Department of Higher Education and Training
DoE	Department of Education
FET	Further Education and Training
GET	General Education and Training
IMAGES	Improving Measurement And Geometry in Elementary Schools
LO	Learning Outcome
MALATI	Mathematics Learning And Teaching Initiative
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
RNCS	Revised National Curriculum Statement
SAARMSTE	Southern African Association for Research in Mathematics, Science and Technology Education
VHGT	Van Hiele Geometry Test

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CHAPTER 1
OVERVIEW OF THE STUDY

1.1. Introduction

The Constitution (1996) of the Republic of South Africa forms the basis for social transformation in our new society. According to the South African Department of Education (DoE), in an ever changing society, it is vital that all learners passing through the Further Education and Training (FET) band acquire a functioning knowledge of the Mathematics that empowers them to make sense of the society (DoE, 2003a). "Competence in Mathematics contributes to the personal, social, scientific and economic development" (DoE, 2003a, p.9). To attain social transformation, the South African Government attached a great deal of importance to the teaching and learning of mathematics and sciences in the South African schools.

According to Sherard (1981, p.23), "Knowledge of mathematics is being called the *critical filter*, which permits entry into a wide variety of many different careers". National Council of Teachers of Mathematics (NCTM) "Standards 2000" documents states that "In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive future. A lack of mathematical competence keeps those doors closed…" (NCTM, 2000, p.50). Mathematics being a gateway subject to tertiary study can be described as a critical mass in secondary education, and adequate learning facilitation in this subject is of pivotal importance in any country (van der Walt & Maree, 2007). Mathematics ensures access to an extended study of the mathematical sciences and a variety of career paths (DoE, 2003a). "It is an essential element in the curriculum of any learner who intends to pursue a career in the physical, mathematical, computer, life, earth, space and environmental sciences or in technology" (DoE, 2003a, p.11).

Geometry is an essential part of the mathematics curriculum. Geometry is connected to every strand in the mathematics curriculum and to a multitude of situations in real life (Yegambaram & Naidoo, 2010). Geometry focuses on the development and application of spatial concepts through which children learn to represent and make sense of the world (Thompson, 2003). Spatial understandings are necessary for interpreting, understanding and appreciating our inherently geometric world (NCTM, 1989). Geometry is the mathematics of space and mathematicians search for mathematical interpretations of space (Bishop, 1983). School geometry is the study of those spatial objects, relationships and transformations that have been mathematised, and the axiomatic mathematical systems that have been constructed to represent them (Clements & Battista, 1992). Geometry provides a vehicle for developing mathematical reasoning abilities about visual concepts (Burger, 1985). It also serves as a vehicle for making connections among various mathematical subjects or between mathematics and other subjects (Senk & Hirschhorn, 1990). Geometry is one of the best opportunities that exist to learn how to mathematise reality (Freudenthal, 1973). Not only do children learn many useful geometrical concepts and skills, but this field provides a foundation for many other topics in higher mathematics (DoE, 2003a).

Geometric skills are important in architecture and design, in engineering, and in various aspects of construction work. The knowledge of geometry remains a pre-requisite for study in such fields as "physics, astronomy, art, mechanical drawing, chemistry (for atomic and molecular structure), biology (for cell structure), and geology (for crystalline structure)" (Sherard, 1981, p.20). The fields of study mentioned above play a major role in the development of any given country.

Snyders (1995) observed that geometry is regarded as a problematic branch of mathematics around the world. Malloy (1999) also stated that historically, understanding geometric concepts and developing and reproducing proofs had been problematic for many teachers and students and both these groups considered geometry to be the most dreaded topic in high school mathematics. In a study conducted in underperforming schools in Tshwane North, Mji and Makgato (2006, p.261) reported on factors affecting the poor performance in mathematics and one of the responses from participants was "…we spend most of the time learning algebra which is easy but what about geometry which is difficult? That is why we do little geometry…". More recently, Atebe and Schafer (2009) have stated that the teaching and learning of geometry is one of the most disappointing experiences in many schools across nations.

Research has documented that most high school learners are not ready for formal proofs in the senior secondary schools and has stressed the need for more informal geometry instruction in junior secondary schools (Hoffer, 1981; Senk, 1985; Shaughnessy & Burger, 1985; Clements & Battista, 1992; De Villiers, 1996; Siyepu, 2005). According to Murray (1997), the geometry studied in the primary school has traditionally been a preparation for the formal geometry in senior schools. The geometry at primary school was therefore regarded as "triangles, circles and squares" (Pegg & Davey, 1998).

How children develop their understanding of geometry and their spatial sense has been an area of research over the past 60 years. One of the models that were proposed in the 1950's was the theoretical perspective put forward by two Dutch mathematicians, Pierre van Hiele and his wife Dina van Hiele-Geldof. The most prominent feature of the model is a five level hierarchy of ways of understanding spatial ideas. Each of these five levels (levels 1 – 5) describes the thinking processes used in geometric contexts (van de Walle, 2001).

A number of causes have been identified and reported by researchers concerning geometry learning (Usiskin, 1982; Burger & Shaughnessy, 1986; van Hiele, 1986; Fuys, Geddes & Tischler, 1988; Clements & Battista, 1992; King, 2003; Atebe, 2008). The major factors identified are curricular, textual and instructional factors.

This study addressed at least some of the challenges amongst those cited above and was based on van Hiele levels for geometry. The purposes of this study were to find out the van Hiele level of geometric thinking of the learners in the sample and to develop an instructional framework in relation to the van Hiele levels to increase the level of geometrical thinking (see van Hiele theory, Chapter 2). These purposes were, to a large extent, inspired by the following factors: (a) the poor performance of geometry in our schools; (b) van Hiele's recent book, 'Structure and Insight- A Theory of Mathematics Education' (1986); (c) interpretation of the van Hiele theory by Fuys, et al., (1988); and (d) research by Atebe (2008) in the Eastern Cape.

This study intended to achieve three major objectives, which are to: (a) determine the van Hiele levels of geometric thinking of the selected grade 10 learners in the participating schools; (b) develop an instructional framework in line with the van Hiele levels to introduce geometry in senior secondary schools and (c) assess the effectiveness of the instructional framework. The methodology adopted a quasi-experimental design with experimental and control groups. Qualitative data was also gathered. The whole research process progressed in six phases within an ethical framework, the details of which are given later in section 1.9.

1.2. Historical context

"Geometry is a complex interconnected network of concepts, ways of reasoning, and representation systems that is used to conceptualise and analyse physical and imagined special environment".

(Battista, 2007, p.843)

Geometry comes from two Greek words, "geo" meaning "earth" and "metria" for "to measure". So the meaning of geometry is earth measuring. It is thus the study of space and systematisation of the way we view the world around us (Yee, 2006). Bishop (1983) states that geometry is the study of spatial relationships that can be found in the three-dimensional space we live in and on any two-dimensional surface in this three-dimensional space. Geometry is one of the oldest branches of mathematics embraced by several ancient cultures such as Indian, Babylonian, Egyptian and Chinese, as well as Greeks (Jones, 2002). In these cultures geometry was based on the relationships between lengths, areas and volumes of physical objects. In those times, one and two-dimensional geometric patterns were used by people to adorn their dwellings, clothes and implements (De Villiers, 1996). Barkley and Cruz (2001) suggested that one practice shared by all people is that of using geometry to decorate everyday objects and Native American beadwork exhibits a high degree of sophistication when it is examined in light of specific symmetrical patterns. Mathematicians and geometers found geometry a worthwhile branch of mathematics to study, which culminated in the compilation of Euclid's *Elements* as a systematisation of the geometric knowledge in 300 B.C. (Jones, 2002). Euclid, in his book, 'The Elements' developed a formal and somewhat rigid approach to the study of geometry that relied almost exclusively on logico-deductive reasoning (Atebe, 2008). De Villiers (1987) again observed that the influence of Euclid's *Elements* became particularly strong when parts of it were being used in the 14th century as prescribed books in European universities and from the 18th century in European schools. Since most of the countries in Africa were colonised by European countries, the geometry education that was introduced by the colonisers had roots in Euclid's *Elements* as well (Mateya, 2008).

In the 19th century, geometry went through a period of growth that was near "*cataclysmic*" in proportion (Jones, 2002, p.15). It resulted in the content of geometry and its internal diversity increasing almost beyond recognition. De Villiers (1996) pointed out that many exciting results like theorems of Morley, Miquel etc in Euclidean geometry were discovered in the 19th and 20th centuries and during this time non Euclidean geometries of Lobachevsky-Bolyai and Riemann were also developed. In the 20th century the axiomatic development of the projective geometry had also taken place. Differential geometry, hyperbolic geometry, fractal geometry, elliptic geometry, algebraic geometry, plane geometry, taxicab geometry, and co-ordinate geometry are some of the different types of geometries that are developed so far. Copeland (1974), states that even though there are many geometries, those most closely related to children's experiences are topology, Euclidean geometry, projective geometry and metric geometry or measurement. Even though Euclidean geometry was the only geometry that was taught in the school curriculum in many countries for many decades (De Villiers, 1997; French

2004), the present South African mathematics curriculum also offers analytical geometry and transformation geometry along with Euclidean geometry as the core of the geometry curriculum.

1.3. Objectives of geometry in the mathematics curriculum

According to van de Walle (2001, p.309), the reasons for the inclusion of geometry in the school curriculum are:

- **Geometry can provide a more complete appreciation of the world.** Virtually everything that humans create and our natural universe have elements of geometric form.
- **Spatial reasoning is an important form of problem solving and geometric explorations can develop problem solving skills**. Problem solving is one of the major reasons for studying mathematics.
- **Geometry plays a key role in the study of other areas of mathematics.** Ratio and proportion are directly related to the geometric concept of similarity.
- **Geometry is used daily by many people.** Professionals like engineers, architects, artists, scientists, land developers are a few that use geometry regularly.
- **Geometry is enjoyable.** If geometry increases learners' liking for mathematics more in general, that makes the effort worthwhile.

Jones (2002) supports this view and suggests that geometry helps the students to develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. In the literature, spatial sense, spatial perception, spatial insight, spatial visualisation and spatial orientation have been used for reference to spatial skills in geometric thinking (Bennie, 1998a). In South Africa, one of the aims of teaching mathematics is to develop an understanding of spatial concepts and relationships (DoE, 2003b). According to Bennie (1998a), the ability to perceive spatial relationships is important for everyday activities like reading maps and playing sports, technical and scientific occupations and the study of mathematics itself as in studying about surface area and volume.

French (2004) emphasises that students' general mathematical competencies have been closely linked to their geometric understanding. This implies that geometric knowledge is important for the students to perform well in mathematics in general.

Van de Walle (2001, p.309) has defined spatial sense as "… an intuition about shapes and the relationships among shapes". Individuals with spatial sense have a feel for the geometric aspects of their surroundings and the shapes formed by objects in the environment. Spatial sense is enhanced by an understanding of shapes, what they look like, and even what they are named. According to Smith (1998), it would be difficult to exist in this world without spatial sense as we would not be able to communicate about position, relationships between objects, giving and receiving directions and size of shapes. Children's earliest mathematical experience is spatial in nature as they physically explore the space around them by moving within it and discovering their relation to it (Nickson, 2000). The concepts of symmetry, congruence, and similarity contribute to understanding our geometric world (van de Walle, 2001).

It is for the above mentioned reasons that South African learners should study geometry as part of their experience with mathematics in order for them to have a wide range of options in choosing appropriate occupations. Despite geometry being an important branch of mathematics, there are many challenges in teaching and learning it.

Learners experience a lot of difficulties in understanding terminology in plane geometry, identifying and classifying shapes, properties of shapes and proof writing (Usiskin, 1982, Fuys, et al., 1988; Clements & Battista, 1992; Siyepu, 2005; Atebe, 2008).

Research over the past 60 years discussed how children develop their understanding of geometry and their spatial sense. A variety of models to describe children's spatial sense and thinking has been proposed and researched and they include Piaget and Inhelder's topological primacy thesis, van Hiele's levels of geometric thinking and cognitive science models (Clements & Battista, 1992). These three general models had the greatest impact on school mathematics curricula.

Piaget and Inhelder's (1967) and Piaget, Inhelder and Szeminska's (1960) research explored how children represent space and suggested two major themes in their topological primacy thesis. The first theme states that children's representations of space are constructed through progressive organisation of their motor and internalised actions resulting in operational systems. The second theme states that children's progressive organisation of geometric ideas follows a definite order and this order is more logical than historical and in that topological relations such as connectedness, closure and continuity are constructed first. Children later construct projective relationships and Euclidean relationships such as angularity, parallelism and distance (Clements & Battista, 1992).

The second major perspective, the van Hiele model forms the basis for this study. It is discussed in detail in the second chapter of this thesis.

The third major perspective that has been applied to understanding students' learning of geometry is that of cognitive science. These precise models of geometric knowledge and processes attempt to integrate research and theoretical work from psychology, philosophy, linguistics and artificial intelligence (Clements & Battista, 1992).

Spatial ability plays a very important part in the development of geometric concepts and their representations (Nickson, 2000). Hershkowitz, Parzysz and Dormolen (1996) argue that meaningful interaction with real shapes in our space has three main goals: to discover similarities and differences among objects, to analyse components of form, and finally recognise shapes in different representations. They refer to Senechal (1990) who suggested a strand of topics related to shapes, through all school years, guided by three main tools: (a) identification and classification of shapes, (b) analysis of forms and representations and (c) visualisation of shapes. According to Freudenthal (in NCTM, 1989, p.48), "Geometry is grasping space … that space in which the child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe, and move better in it". According to van Niekerk (1995), geometric figures are hidden in spatial objects, thus spatial skills activities provide excellent opportunities for dealing with two and three dimensional figures. The Mathematics Learning and Teaching Initiative Thinkshop (MALATI, 1997) refers to the NCTM Draft "Standards 2000" document which suggests that mathematics instruction programmes should pay attention to geometry and spatial sense so that, learners, among other things, use visualisation and spatial reasoning to solve problems both within and outside of mathematics.

1.4. Senior secondary school geometry in the South African context

Until recently (2010), the South African Education was politically and administratively controlled at the national level by a single department of education known as Department of Education (DoE). Currently, there are two departments of education each under a separate Minister: one deals with basic education known as Department of Basic Education (DBE) and the other one deals with higher education known as Department of Higher Education and Training (DHET). The DBE handles the education from grade 0 to grade 12.

Since the inception of the new democratic government in South Africa in 1994, the ministry of education has embarked on a number of educational policy reformations. Central to these transformations was the need for equal educational opportunities to all South Africans. This process resulted in the implementation of an interim core syllabus in 1995, a document that was succeeded by Curriculum 2005's implementation in 1998 (King, 2003). The Curriculum 2005 was based on the ideal of lifelong learning for all the South Africans, regardless of colour, race or sex. This came out as a result of the South African government's wish to provide quality education which would ensure that the learners gain the skills, knowledge and values that allow them to contribute to their own success as well as to the success of their family, community and the nation as a whole (Horn, 2009). This was a progressive model of education based on the principles of Outcomes Based Education (OBE), which was a learner-centered, educator-driven system (Jansen & Taylor, 2003). Teaching practices, adopted through Curriculum 2005 required that the learners participate in classroom activities, become more involved in the learning process, and take responsibility for their own learning (Aldridge & Fraser, 2004). The introduction of continuous formative assessments rather than once-off examination came into effect. This was a direct response to the apartheid curriculum, which was teacher centered, authority-driven, content and examination based. Teaching and learning had to focus on the attainment of learning outcomes at the end of a particular period of instruction. Owing to the practical impossibility of directly training all educators, a core of trainers at higher level was done and they were meant to take it down to the various levels of the education system (Jansen & Taylor, 2003). This curriculum was heavily criticised due to its high inaccessibility, lack of resources in underprivileged schools and incompetence of teachers. This ultimately contributed to the partial failure of Curriculum 2005 to achieve its intended outcomes. This was followed by the curriculum review in 2000, which resulted in the release of a document called "Draft National Curriculum Statement (NCS) by the education minister in 2001 and a Revised National Curriculum Statement (RNCS) in 2002 (King, 2003). This curriculum came into effect in Further Education Band (FET) in 2006 and the first cohort of matric learners wrote their National Senior Certificate Examination in 2008.

According to King (2003), dissatisfaction with the secondary school geometry curriculum and the poor performance of learners in geometry in South Africa has been a topic of concern over the past four decades. During 1997, the Geometry working Group of a South African Non Governmental Organisation called Mathematics Learning And Teaching Initiative (MALATI) tried to re-conceptualise the teaching and learning of geometry (Bennie, 1998b). For that re-conceptualisation to happen and to propose changes to the curriculum, the group felt that a means to understand the geometric thinking of learners would be needed (King, 2003). The group found that the van Hiele model of geometric thinking could be used as a framework to understand the geometric thinking of learners. The idea of re-conceptualising the approach to geometry teaching and learning was placed in the foreground of the introduction of Curriculum 2005 in 1998 by the South African National Ministry (King, 2003).

Breen (1997) also corroborated the situation in South Africa where in primary schools the geometry instruction was insufficient in terms of providing learners with the necessary skills to function at the level of axiomatic thinking in senior secondary schools. De Villiers (1997) suggested that a revision of the primary school geometry curriculum along the van Hiele levels would ensure success in the senior secondary school.

De Villiers (2010) states that we used to have a geometry curriculum heavily loaded in the senior secondary school with formal geometry, and relatively little content done informally in the primary school. Although tessellations are recently introduced in the primary school, many teachers and textbook authors do not appear to understand its relevance in relation to the van Hiele theory (De Villiers, 2010). The present South African mathematics curriculum offers analytical geometry and transformation geometry along with Euclidean geometry as the core of the geometry curriculum.

The National Curriculum Statement (NCS) emphasises learning outcomes (DoE, 2003a). Each subject has its own learning outcomes and each learning outcome has its own assessment standards. A learning outcome describes the knowledge, skills and values the learner should acquire in a phase and assessment standards are criteria that define the knowledge, attitude, values and skills that a learner should know and be able to demonstrate at a specific grade. There are five learning outcomes in mathematics, namely, Learning Outcome (LO) 1, LO 2, LO 3, LO 4 and LO 5. One of the mathematical learning outcomes (which is LO 3) is the mastery in space and shape (DoE, 2003a). Within the NCS, geometry is part of the attainment target currently entitled as `space, shape and measurement'. An understanding of measurement, proportional reasoning, algebra and integers among others is necessary to develop an understanding of space and shape (Kotze, 2007).

The mathematics learning outcomes of the National Curriculum Statement (NCS) Grades R-9 and the NCS Grades 10-12 are linked as follows:

Table 1.1: Mathematics learning outcomes

Learning outcome	Grade R-9	Grade 10-12
LO 1	Number and Number relationships	Number and Number relationships
LO 2	Patterns, Functions and Algebra	Functions and Algebra
LO 3	Shape and Space	Shape, Space and Measurement
LO 4	Data Handling and Probability	Data Handling and Probability
LO 5	Measurement	

Feza and Webb (2005) indicate that the current South African National Curriculum Statement (NCS) at the intermediate phase reflects levels 1, 2 and 3 in the van Hiele hierarchy of thinking levels (see Chapter 2, Section 2.2.1) as learners are required to describe and represent the characteristics and relationships between two-dimensional and three-dimensional objects in a variety of orientations and positions.

In the learning programme for grade 10 to grade 12, to achieve learning outcome 3, learners are expected to (1) work with a wide range of patterns and transformations and solve related problems and (2) describe, represent and analyse shape and space in two and three-dimensions with justifications using geometry and trigonometry. A further description of learning outcome 3 states that "the treatment of formal Euclidean geometry is staged through the grades so as to assist the gradual development of proof skills and an understanding of logical axiomatic systems" (DoE, 2003a, p.54). The above criteria for achieving learning outcome 3 is closely linked to levels 1, 2, and 3 and to a certain extent of level 4 (means no formal proof is required for examination purposes) of van Hiele levels of geometric thinking. The above prescribed learning outcome 3 is part of the compulsory paper 2 in the National Senior Certificate examination in mathematics. In addition to this, learners in grade 12 can opt for an additional optional paper (paper 3), which examines optional assessment standards in LO 1, LO 3, and LO 4. The optional assessment standards in LO 3, contribute 40% of the examination mark which comprises of learners learning different aspects of Euclidean geometry including proving theorems (formal proof) in similarity, proportionality and circle geometry they learn in grades 11 and 12. A detailed version of the learning outcomes with assessment standards is shown in Appendix B. It is to be noted that this is closely linked to the content standards and grade level expectations explained for grades 9 -12 geometry curriculum in the NCTM (2000) document entitled 'Principles and Standards for School Mathematics'.

In the work schedule for grade 10 mathematics under the content of triangles, quadrilaterals and other polygons, the learning outcomes and assessment standards specify that learners, through investigations, produce conjectures and generalisations related to triangles, quadrilaterals and other polygons and attempt to validate, justify, explain or prove them, using any logical method (Euclidean, coordinate and/or transformation)(DoE, 2008).

A further revision of the curriculum is underway and it will come into effect in the FET band in 2012. This is called Curriculum and Assessment Policy Statement (CAPS), which replaces the old Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines (DBE, 2011). This cannot be considered as old wine in new sheath, as major changes are happening in the mathematics curriculum. There will be no more optional paper in mathematics and all will be combined in paper 1 and 2. This means that Euclidean geometry with its formal proof is brought back into the compulsory curriculum of paper 2. Under Euclidean geometry in grade 10 learners are expected to (a) investigate and form conjectures about the properties of special triangles, quadrilaterals and other polygons. They need to try to validate or prove conjectures using any logical method (Euclidean, coordinate or transformation geometry from Grade 9), disapprove false conjectures by producing counter examples and (b) investigate alternative definitions of various polygons (including the isosceles, equilateral and right-angled triangles, the kite, parallelogram, rectangle, rhombus and square). In grade 11, learners are expected to (a) revise grade 9 and 10 work on the necessary and sufficient conditions for polygons to be similar and (b) prove (i) proportional intercept theorem (midpoint theorem as a special case) (ii) similar triangles theorem and its converse, and corollaries. In grade 12, learners are expected to (a) investigate and prove theorems of the geometry of circles as a mini-axiomatic system and (b) solve circle geometry problems, providing reasons for statements when required (DBE, 2011). This means that even though the content and assessment standards of grade 10 remain almost the same, in grades 11 and 12 major changes are expected due to the merging of optional assessment standards into the compulsory papers.

This is mentioned here to show that we are heading towards a curriculum change that requires all learners to perform fully at a higher level (level 4 – Deduction: with formal proof) of the van Hiele levels in major aspects of Euclidean geometry in all grades. This makes this study more relevant as this curriculum change is a cause for deeper concern on the future of secondary school geometry in South Africa.

1.5. Problem statement

Studies conducted in different parts of the world (Hoffer, 1981; Usiskin, 1982; Senk, 1985; Shaughnessy & Burger, 1985; Fuys, et al., 1988; Clements & Battista, 1992; King, 2003; Atebe, 2008) highlighted the following inferences:

- Most of the learners are not ready for the formal deductive study of school geometry.
- Learners' poor performance in geometry holds account for geometry classroom teaching and learning.
- van Hiele (1986) believes that students' difficulty with school mathematics generally and geometry in particular is caused largely by teachers' failure to

deliver instruction that is appropriate to the learners' geometric level of thinking.

- Educators' familiarity with the instructional cycle of the van Hiele levels will contribute to the effectiveness of their effort to assist the learners in making progress with their learning.

- Regardless of pockets of excellence, innovation and productive energy, poor results create the impression that many of our mathematics and science classrooms are characterised by an inertia that is not conducive to effective teaching and learning in these fields (Schafer, 2009).

In view of the foregoing, the problem is that learning and instructional strategies based on research in South Africa is minimal and additional research aimed at improving the learning and instructional strategies need to be developed.

In many western countries, the van Hiele theory has become the most influential factor in their geometry curriculum (van de Walle, 2004), but only a few studies have utilised this instructional model in the South African context.

1.6. Research objectives

This study intended to achieve three major objectives:

1. Determine the van Hiele levels of geometric thinking of the grade 10 learners in the selected schools.
2. Develop an instructional framework in line with the van Hiele levels for introducing geometry in senior secondary schools and the implementation of it in the participating schools.
3. Assess the effectiveness of the instructional framework.

1.7. Research questions

1. What are the van Hiele levels of geometric thinking of the learners participating in the study?
2. Can a researcher-designed instructional framework in line with the van Hiele levels improve the level of geometric thinking of the participating learners?

These research questions will be dealt with as Focus one and Focus two in Chapters 4, 6 and 7.

1.8. Rationale for the study

Under the section on the background and problem statement, several challenges on geometry education in general and in special reference to the South African scenario were explored which led to the problem statement. Solutions to such challenges need to be sought and the main rationale for this study is based on such challenges. Furthermore, there was a personal reason and motivation too. As an educator in senior secondary school mathematics for 19 years and as a senior certificate examination marker for mathematics paper 2, I have repeatedly encountered low levels of achievement in mathematics and geometry in particular by the learners. I was also a part-time facilitator for teacher training courses offered by three universities at different times since 2006. The mathematical proficiency level of those in-service teachers was also disappointing. This made me to believe that the teachers are inadequately prepared for themselves which in turn has an impact on their instructional practice.

Mji and Makgato (2006) state that it has been reported that outdated teaching practices and lack of basic content knowledge have resulted in poor teaching standards. Also, the process of teaching and learning at the secondary school level (traditional method), which gives less opportunity to learn at the students' own pace is a potential factor hindering students' achievement in mathematics and the prevailing learning activities in schools consisting of mainly listening, watching and imitating the teacher are not supportive of efficient learning in mathematics (Akinsola & Ifamuyiwa, 2008).

According to Shulman (1987), teaching necessarily begins with a teacher's understanding of what is to be learned and how is it to be taught. He places a special emphasis on pedagogical content knowledge as one of the main categories of the knowledge base of teachers as it identifies the distinctive bodies of knowledge for teaching. According to Rossouw and Smith (1997), the rich base developed from research on van Hiele levels and how students learn geometry is an important source of understanding teachers' pedagogical content knowledge of geometry teaching. The teachers' pedagogical content knowledge of geometry teaching is a distinctive knowledge that teachers need to have in order to transform the geometry content to make it interesting and comprehensible to the learners (Rossouw & Smith, 1997).

It is imperative that efforts are made by practitioners engaged in learning and teaching to find constructive solutions to face the challenges alluded to thus far. In line with the above concerns, I was inspired to undertake a research on the van Hiele theory and its teaching implications.

In senior secondary schools, learners are expected to work with (1) a wide range of patterns and transformations and solve related problems and (2) describe, represent and analyse shape and space in two and three dimensions using geometry and trigonometry to achieve the learning outcome of space and shape. This curriculum for geometry consists of a mixture of unrelated concepts with no systematic progression to higher levels of thinking that are required for sophisticated concept development and substantive geometric problem solving. The aim of this study is to address the deficiencies in the current senior secondary school geometry instruction by developing a research based instructional framework based on the theoretical framework of van Hiele.

A further revision of the curriculum (CAPS) as mentioned earlier under section 1.4 will come into effect in the FET band in 2012. Euclidean geometry with its formal proof is brought back into the compulsory curriculum of paper 2. This means that even though the content and assessment standards of grade 10 remain almost the same, in grades 11 and 12 major changes are expected due to the merging of optional assessment standards into the compulsory papers. This curriculum change requires all the learners to perform fully at a higher level (level 4 – Deduction: with formal proof) of the van Hiele levels in major aspects of Euclidean geometry in all grades. This makes the study more relevant as this curriculum change is a cause for deeper concern on the future of secondary school geometry.

Again, in our present curriculum, the learning outcomes and the assessment standards for geometry curriculum are closely linked to the NCTM's standards for geometry and the teaching principle of NCTM (NCTM, 2000, p.12) which states that "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well". In this context, being an educator in a senior secondary school, I trust that, using the van Hiele theory to understand what learners know and supporting them to learn what is needed by providing instructional tasks will definitely contribute to an effective mathematics teaching.

1.9. Methodology and research design

This study made use of a quasi-experimental design. For this, a control group was employed to compare with the experimental group, but the participants were not randomly selected and assigned to the groups (Creswell, 1994). According to McMillan and Schumacher (2006), non-equivalent groups pretest-posttest designs are very prevalent and useful in education as it is often impossible to randomly assign subjects. For this research, the experimental group included learners who were instructed with the van Hiele instructional framework, while the control group comprised learners who were instructed with traditional method. A detailed description of it is given in Chapter 3.

The research was done in six phases:

Phase 1

- The present geometrical thinking level was established through a base line test (pretest) on van Hiele levels among 359 grade 10 learners from five purposively selected schools from the senior secondary schools in Mthatha in the Eastern Cape Province of the Republic of South Africa.

Phase 2

- Development of the instructional framework based on van Hiele levels.

Phase 3

- Workshop for the educators of the schools participated in the base line test.

Phase 4

- Implementation of the instructional framework for the experimental group in all the five schools.

Phase 5

- Evaluation of the instructional framework through a posttest that was administered on the participating learners. Interviews with educators to collect their opinion on the instructional framework.

Phase 6

- Interviews with selected learners from the five schools on their levels of thinking to enrich the study by giving it a qualitative element.

1.10. Significance of the study

This study is significant and novel in many ways.

As far as I have been able to ascertain, this study is the first attempt to use van Hiele levels to develop an instructional framework to teach geometry in senior secondary schools in South Africa, more particularly in the Eastern Cape.

Studies conducted in different parts of the world (Hoffer, 1981; Usiskin, 1982; Senk, 1985; Shaughnessy & Burger, 1985; Fuys, et al., 1988; Clements & Battista, 1992; King, 2003; Atebe, 2008) have identified the causes and sources of learning and teaching difficulties in geometry. Most of these studies highlighted the factors as curricular, textual and instructional (Fuys, et al., 1988; Clements & Battista, 1992; King, 2003; Siyepu, 2005; Atebe, 2008).

According to Clements and Battista (1992), the curriculum with regard to what topics are treated and how they are treated has far reaching implications for students' performance in geometry. One of the major reasons for learners' poor performance in senior secondary school is their limited exposure to geometry due to the lack of rich and well sequenced geometry curriculum in the primary school level (Clements & Battista, 1992; De Villiers, 1997; Siyepu, 2005). Cox (1985) comments that geometric content is often neglected in the elementary, middle and junior school levels and it is overshadowed by an emphasis on computational skills. This results in many students entering tenth grade having a very weak grasp of geometric concepts.

The above said curricular content is experienced by learners and teachers through the textbooks that they use in their classrooms and for homework (Atebe, 2008). Suydam (1985), states that teachers generally depend very heavily on the textbook and follow it very closely for content and sequencing while Atebe (2008) warns that if learners are to have learning experiences consonant with the expectations of the geometry curriculum, textbooks should be chosen very carefully so that its contents reflect the curriculum objectives. Van Hiele (1986, p.45) mentions the method of "telescoped teaching" where in the textbooks the subject matter is repeated many times and each time it is dealt with from the very beginning, can assist to attain higher level, which in turn can make teaching much better. According to the principles of NCS, learners are meant to achieve learning outcomes based on the knowledge, skills and values that are specific for that outcome. Educators are meant to choose from textbooks that are available to them to look for content that are relevant to achieve the learning outcomes. Since no particular textbook is prescribed by the Department of Education, it becomes difficult for the educators to rely on different textbooks for their teaching and is tempted to use the one that is available to them and to the learners. Due to the financial constraints in most of the underprivileged areas learners are even attending schools without the necessary textbooks. This in turn, may not help the learners to achieve the prescribed learning outcomes.

According to Atebe (2008), the classroom is one of the most important educational focal point where curricular intentions are transformed into potential learning experiences and the amount of learning that takes place in the classroom depends mostly on teachers' own knowledge of the subject matter to be learned. "What the teacher knows is one of the most important variables that impact on what is done in the classroom" (van der Sandt & Nieuwoudt, 2003, p. 199). The more a teacher knows about and the way students learn, the more effective that individual will be in nurturing mathematical understanding (Swafford, Jones & Thornton, 1997). According to Swafford, Jones and Thornton (1997), very little is known about teachers' knowledge of student cognition in geometry or the impact of that knowledge on instruction.

The curriculum reforms and changes in South Africa within the past 10 years have a major contribution to the factors that are mentioned above.Changes in the curriculum, buying of textbooks with the changes, and training of in-service teachers are affecting the morality and the limited financial circumstances of most of the underprivileged schools.

The present study is significant as it tries to address the above said problems of curricular, textual and instructional factors by looking at the learners' cognition in geometry by identifying their levels of understanding in geometry first and use that knowledge in developing an instructional framework to enhance the geometry instruction.

This study has made a significant contribution towards closing the perceived gap in the above said issues by providing activities and worksheets which can be used as an integral part of the instruction. The sequential and hierarchical order of concepts will presumably close the gap in the insufficient preparation of learners from the junior schools as they enter the senior secondary schools. The development of the instructional framework and the subsequent training given to the educators will hopefully empower them for the effectiveness of their teaching.

The findings from the study can be utilised by mathematics educators and curriculum developers for their attempt to improve the instructional strategies in our schools.

A few other aspects of significance are further discussed under section 7.6 in Chapter 7.

1.11. Limitations of the study

The study had the following limitations:

- The relatively small sample that was involved in the study limits the generalisability and the wider application of the findings from this study.

- The assessment instrument that was chosen might not have provided the learners with opportunities to show what they are capable of. It did not cater for the varied cultural and mathematical background of the learners. The van Hiele Geometry Test evaluated only the understanding of some major concepts. The findings of the study cannot be applied to all geometry topics.

- Even though the learners were exposed to a variety of geometrical concepts in the instructional framework, enormous time was taken to guide the learners through the activities. It is possible that the learners might have failed to give their “best” in all the activities and that might have affected their performance in one way or other.

- The activities in the instructional framework used only manipulatives and worksheets. It may be viewed as a limitation for this study. For example, De Villiers (1994, 2004) suggested that dynamic software has the potential to bring out better responses from learners. But given the settings of the study, it was impossible to have access to computers to all the learners involved in the study. The majority of the South African learners are in the previously disadvantaged areas and the study was focusing on these learners. Therefore, the study developed activities that could be implemented without expensive or sophisticated materials. But it is hoped that the instructional framework that is developed from the study may work in similar contexts.

- The present study limits its scope to the first four levels namely, visualisation, analysis, informal deduction and deduction (labelled as levels 1 – 4), with the possibility of the inclusion of level 0 (pre-recognition). The learners in senior secondary school are not expected to establish theorems in different systems (level 5). The van Hiele levels are explained in section 2.2.1 of Chapter 2.

A few other limitations as emerged as the study was completed are also discussed under section 7.7 in Chapter 7.

1.12. Ethical considerations

Ethical issues play an important role in any research investigations in the social sciences where human subjects are used as suggested by (Cohen & Manion, 1994; McMillan, 2000). Subjects have the right to privacy, or nonparticipation, confidentiality, expect experimental responsibility and to remain anonymous (Tuckman, 1994). So for this research study, a formal approval from the Department of Education was obtained to conduct research in five senior secondary schools in Mthatha. Permission from school principals was also sought and obtained. A research information sheet was given to the parents. A voluntary informed consent, with the freedom of the learner to withdraw from the study at any stage was obtained from the learners and learners' parents of those under 18 years. Anonymity of both the schools as well as the research participants was assured as suggested by Cohen, Manion and Morrison (2007).

1.13. Reliability and validity

External validity refers to the generalisability or representativeness from the findings of a study (Tuckman, 1994; Struwig & stead, 2001). For this study, generalisability was a limitation as cited in section 1.10. Drew, Hardman and Hosp (2008) state that it is important for the researcher to ensure the validity of the instruments used in the study. Schumacher and McMillan (1993) state that validity depends on the purpose, population and situational factors in which measurement take place. The validity and reliability are explained in Chapter 3 under each instrument used in the study.

1.14. Definition of terms

Van Hiele theory based instruction

It is the geometry instruction in which the researcher designed teaching materials based on the educational theory based on the van Hiele theory, more particularly, its levels of geometric thinking. The van Hiele levels of geometric thinking concerns the learners' levels of understanding in geometric concepts and the instructional programme concerns the classroom teaching to improve the levels of understanding of those geometric concepts.

Traditional method of instruction

It is the geometry instruction in which the researcher did not implement the characteristics of the van Hiele theory in the presentation of geometry.

Instructional framework

A framework can be defined as a hypothetical description of a complex entity of process or the underlying structure or a structure supporting or containing something. Instruction refers to those curriculum-related, professionally-informed decisions that teachers purposefully enact to enhance learning opportunities for students. Effective instruction is interactive and designed to accommodate student learning needs and styles through a variety of teaching practices. An instructional framework can be regarded as a series of processes (or practice) of maximising the effectiveness, efficiency and appeal of instruction and other learning experiences (source: http://mag.ofi.hu/instructional-approaches retrieved from internet on 5/6/2011).

Evaluation

It means the systematic study of a particular programme or set of events over a period of time in order to assess the effectiveness (Hitchcock & Hughes, 1995).

1.15. Thesis overview

To clarify the scope and to give an idea of the study, a brief description is given outlining the content of this thesis.

Chapter 1 has provided an introduction to the study. It explained the historical context and the objectives of teaching geometry in the mathematics curriculum. It also provided an explanation of senior secondary school geometry in the South African context. The problem statement, research goals, research questions, rationale for the study and the methodology have been explained in limited details. The significance of the study in the South African context has also been explained.

Chapter 2 reviews the literature in the field of geometry teaching and learning. It focuses mainly on the van Hiele theory, the different aspects of it and its relevance in geometry instruction. The research done in the field of geometry and van Hiele theory are explained in detail.

Chapter 3 elaborates on the research methodology employed in this study and it also discusses the instructional framework developed for the study. The study is conducted in six phases in which the first phase concentrates on the first research question and the rest concerns the second research question.

Chapter 4 presents the data analysis and results of the quantitative data in detail. The results for the pretest and posttest of the experimental group as well as the control group are presented in detail. It also details the performance of the learners in school-wise and gender-wise basis. An overall idea of the levels of geometric thinking of the participating learners is also given. It also provides information on the effectiveness of the van Hiele-based instruction.

Chapter 5 presents the data analysis and results of the qualitative data gathered from the interviews of the learners and the educators. The analysis of the triangle activities and quadrilateral activities of three learners are presented in detail. The analysis of the interviews with the five educators from the five different schools is also presented in this chapter.

Chapter 6 presents the discussions on the quantitative and qualitative data based on the analysis presented in Chapters 4 and 5.

Chapter 7 presents the conclusions and recommendations based on the analysis presented in Chapters 4 and 5 and the discussions presented in Chapter 6.

CHAPTER 2

LITERATURE REVIEW ON THE THEORY UNDERPINNING THE STUDY

" A literature review, if conducted carefully and presented well, will add much to an understanding of the research problem and help place the results of a study in a historical perspective" .

(McMillan & Schumacher, 2006, p.75)

2.1. Introduction

The first chapter provided an explanation on the historical context and the objectives of teaching geometry in the mathematics curriculum and also explained senior secondary school geometry in the South African context. Therefore, this chapter only deals with the literature concerning the theory of teaching of geometry. The van Hiele theory appeals as an ideal theory for use as a theoretical framework, as well as a frame of reference, to link geometry to educational principles (King, 2003). It is acclaimed as one of the best frameworks for studying teaching and learning processes in geometry (Atebe, 2008). Various aspects of the van Hiele theory, which serve as the stem of this study are explained in this chapter. The theory proposes that geometrical thinking develops in a series of five levels which are distinguished by the characteristic of the thinking process. In this study, the levels of geometric thinking and its implication on geometry instruction are utilised. The literature is arranged with the main headings as follows:

2.2. The van Hiele theory

2.3. Some characteristics, features and properties of the van Hiele levels

2.4. The van Hiele phases

2.5. The importance of language in the van Hiele theory

2.6. Empirical research on the van Hiele theory

2.7. Comparison of van Hiele with other theories

2.8. Teaching implications and instructional ideas of the van Hiele theory

2.9. A critique of the van Hiele theory

2.2. The van Hiele theory

"Theories are useful if they are used – and contested, attacked and modified. By this criterion, van Hiele's theory is a useful theory".

(Clements, 2004, p.60)

Due to their experience in classroom teaching in the Netherlands in the 1950s, the husband and wife van Hiele team (Pierre van Hiele and Dina van Hiele – Geldof) put forward a theoretical perspective for the teaching and learning of geometry, which is universally referred to as the van Hiele theory (Pegg & Davey, 1998). This model of thought levels provides useful empirically-based descriptions of what are likely to be relatively stable, qualitatively different, states or levels of understanding in learners (Ding & Jones, 2007).

Even the time when he was a student, van Hiele helped his classmates with their difficulties at school and when they seemed to lack all understanding, he was curious about the causes. He had the following to say:

> I had great difficulties with geometry, though I got good grades, because I did not understand what axioms and definitions were good for. When after some time I began to understand, there were new difficulties: my understanding turned out to be misguided…..(van Hiele, 1986, p.1).

As experienced teachers in Montessori secondary schools, the van Hieles were concerned about the difficulties their students encountered with secondary school geometry (Fuys, et.al., 1988). Concerned with these difficulties, they began to think that the content they were teaching was too advanced for many of their students to understand fully (Malloy, 2002). They believed that secondary school geometry involved thinking at a relatively "higher level" and the students had not had sufficient experiences in thinking at prerequisite "lower levels" (Fuys, et al., 1988, p.4). They investigated the prerequisite reasoning abilities needed to successfully engage a logical-deductive system of thought. In 1957, the van Hieles completed companion dissertations at the University of Utrecht on levels of thinking and the role of insight in learning geometry (Fuys, et al., 1988). They realised that the learning of facts could not be the only purpose of teaching mathematics, but more importantly, the development of insight ought to be the main purpose (King, 2003). Dina van Hiele – Geldof's work was on a didactic experiment aimed at raising a student's thought level, while Pierre van Hiele formulated a structure of thought levels and principles designed to help students gain insight into geometry (Fuys, et al., 1988). They described these levels of thought as five levels of thought that characterised the thinking of children as they become more sophisticated in their understanding of geometric relationships (Malloy, 2002). This is the most prominent feature of the theory. The levels describe "how one thinks about, rather than how much knowledge one has" (van de Walle, 2001, p.309).

The van Hiele theory was primarily directed at improving teaching as well as the geometric understanding of learners by organising instruction in such a way that it would take learners' thinking ability into account whilst the new work is being introduced. The model clarifies many of the shortcomings in traditional instruction and offers ways to improve it by focussing on getting students to the appropriate level to be successful in high school geometry (Pittalis, Mousoulides & Christou, 2009). Pegg (1997) suggests that the levels have proved a useful tool in identifying the problems in students' understanding of certain geometrical concepts, evaluating the structure or development of geometric content in secondary school textbooks and guiding the development of syllabi.

The van Hiele theory has been extensively investigated with different research groups such as learners from different grades, pre-service teachers and in-service teachers in various parts of the world since the early 1980's (Hoffer, 1981; Smith, 1987; Clements & Battista, 1992). This theory is particularly relevant in South Africa, where mathematics remains a problematic learning area, as Fuys, et al., (1988) suggest that "its emphasis on developing successively higher thought levels appears to signal direction and potential for improving the teaching of mathematics"(p.191).

The van Hiele theory comprises of two main components: levels of geometric thinking and their characteristics and phases of learning (Crowley, 1987). Each of these will be discussed in the following sections in this chapter.

2.2.1. The van Hiele levels

"*Tracing the levels of thinking that play a part in geometry is not a simple affair, for the levels are situated not in the subject matter but in the thinking of man".*

(van Hiele, 1986, p.41)

Even though there had been attempts as early as 1920's to improve the manner in which subject matter was structured and developed in order to make geometry instruction more accessible, and the existence of stages in the development of geometry (Pegg, 1995), the theory of thought levels, with significant pedagogical implications in geometry thinking proposed by the van Hieles in the 1950's is the one that attracted considerable interest among researchers (Usiskin, 1982; Hoffer, 1983; Burger & Shaughnessy, 1986; Senk, 1989; Genz, 2006; Atebe, 2008). The theory proposes that an individual passes through five separate thought levels on the way to a complete mastery of the subject matter. Van Hiele believes that the levels are not situated in the subject matter, but in the thinking of man (van Hiele, 1986). The formulation of the levels was founded by the van Hieles in response to an analysis of their own teaching and was initially aimed at developing insight in their students as an effort at improving instructional practice (King, 2003).

According to the van Hieles, there are 5 levels in children's geometric understanding.

Table 2.1: Van Hiele levels of geometric thinking

Levels	Known as…….	Description: Learner will be able to ………
Level 1	Visualization	recognise and name figures
Level 2	Analysis	describe the attributes of shapes
Level 3	Ordering	classify and generalise by attributes
Level 4	Deduction	develop proofs using axioms and definitions
Level 5	Rigor	work in various geometrical systems

The levels are further described as follows:

Level 1: Recognition (or Visualisation)

Students recognise a figure by its appearance (or shape/form). It is the appearance of the shape that defines it for the student. A square is a square, "because it looks like a square". And a child recognises a rectangle by its form and a rectangle seems different to him than a square (van Hiele, 1999, p.311). Or, "It is a rectangle because it looks like a door" (van der Sandt & Nieuwoudt, 2005, p. 109). Since the appearance is dominant at this level, appearances can overpower properties of a shape. As an example, for a student operating at this level, a square that has been rotated so that all sides are at 45^0 angles to the vertical may not appear to be a square (van de Walle, 2001, p.309). Students can identify, name, compare and operate on geometric shapes such as triangles, squares, rectangles, angles, intersecting or parallel lines according to their appearance (Fuys, et al., 1988). They do not explicitly identify the properties of these shapes (De Villiers, 1996). The students reason about basic geometric concepts such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components (Burger & Shaughnessy, 1986). Properties of a figure play no explicit role in its identification (Pegg & Davey, 1998). Students at this level will sort and classify shapes based on their appearances – may say "I put these together because they all look sort of alike" (van de Walle, 2001, p.309).

Van Hiele (1999, p.311) points out that

> … all rational thinking has its roots in non verbal thinking and many decisions are made with only that kind of thought. We observe some things without having any words for them. The 'visual level' starts with non verbal thinking and the figures are judged by their appearance. We say "It is a square, I know that it is one because "I see it is" and children might say, "It is a rectangle because it looks like a box" …

Level 2: Analysis (or descriptive level)

Students at this level are able to consider all shapes within a class rather than a single shape. By focusing on a class of shapes, students are able to think about "what makes a rectangle a rectangle" (van de Walle, 2001, p.309). Students at this level identify a figure by its properties, which are seen as independent of one another (Pegg & Davey, 1998). Students analyse the attributes of shapes and the relationship among the attributes of shapes and discover properties and rules through observation (Malloy, 2002). Students reason about geometric concepts by means of an informal analysis of component parts and attributes (Burger & Shaughnessy, 1986). The irrelevant features fade into the background. Students begin to appreciate that a collection of shapes goes together because of properties (van de Walle, 2001). Students can recognise and name properties of geometric figures, but they do not yet understand the difference between these properties and between different figures (van Hiele, 1986). This means that students at this level may be able to list all the properties of squares, rectangles and parallelograms but will not be able to see that these are sub-classes of one another that all squares are rectangles and all rectangles are parallelograms (van de Walle, 2001). They will be able to analyse figures in terms of their components and relationships among components and discover properties or rules of a class of shapes empirically by folding, measuring, using grids or diagrams (Fuys, et al., 1988). At this level, students begin to identify properties of shapes and learn to use appropriate vocabulary related to properties, but do not make connections between different shapes and their properties (Teppo, 1991). The properties are seen as separate entities that cannot be combined together to describe a particular figure. As an example, Pegg (1995, p.90) notes, "an isosceles triangle can have two equal sides, two equal angles and an axis of symmetry, but no property implies another". The students have not yet mastered which properties are necessary and which are sufficient to describe a geometric shape (Mason, 1998). Class inclusion is not yet understood.

Van Hiele (1999, p.311) states that

> …at this level, figures are the bearers of their properties. A figure is no longer judged because "it looks like one" but rather because it has certain properties. At this level language is important for describing shapes. At this level, however the properties are not yet logically ordered, so a triangle with equal sides is not necessarily one with equal angles…

Level 3: Informal deduction (or order)

Students at this level discover and formulate generalisations about previously learned properties and rules and develop informal arguments to show those generalisations to be true (Malloy, 2002). They no longer see properties of figures as independent. They recognise that a property proceeds or follows from other properties. They also understand relationship between different figures (Pegg & Davey, 1998). "If all four angles are right angles, the shape must be a rectangle. If it is a square, all angles are right angles. If it is a square, it must be a rectangle" (van de Walle, 2001, p.310). The students logically order the properties of concepts, form abstract definitions, and can distinguish between the necessary and sufficiency of a set of properties in determining a concept (Burger & Shaughnessy, 1986). Class inclusions are understood at this level (van Hiele, 1999). With greater ability to engage in "if-then" reasoning, shapes can be classified using only minimum characteristics. Observations go beyond properties themselves and begin to focus on logical arguments about the properties. They may be able to follow and appreciate an informal deductive argument about shapes and their properties (van de Walle, 2001). In other words, they logically interrelate previously discovered properties or rules by giving or following informal arguments (Fuys, et al., 1988). But the role and importance of formal deduction, however, is not yet understood (Mason, 1998).

According to van Hiele (1999, p.311)

> …at the informal deduction level, properties are logically ordered. They are deduced from one another; one property precedes or follows from another property. Students use properties that they already know to formulate definitions and use them to justify relationships. However, at this level, the intrinsic meaning of deduction, that is, the role of axioms, definitions, theorems, and their converses, is not understood...

Level 4: Deduction

Students at this level prove theorems deductively and understand the structure of the geometric system (Fuys, et al., 1988; Malloy, 2002). The students reason formally within the context of a mathematical system, complete with undefined terms, axioms an underlying logical system, definitions, and theorems (Burger & Shaughnessy, 1986). They use the concept of necessary and sufficient conditions and can develop proofs rather than learning by rote. They can devise definitions (Pegg & Davey, 1998). They are able to make conjectures and prove them (De Villiers, 2003). They beginto appreciate the need for a system of logic that rests on a minimum set of assumptions and from which other truths can be derived. They will be able to work with abstract statements about geometric properties and make conclusions based more on logic than intuition. "They can clearly observe that for a rectangle, the diagonals bisect each other just like a student operating at level 3, but there is an appreciation of the need to prove this using a series of deductive arguments" (van de Walle, 2001, p.310).

Level 5: Rigor

This is the highest level in the van Hiele hierarchy (Teppo, 1991). Students at this level can establish theorems in different systems of postulates and compare and analyse deductive systems (Fuys, et al., 1988; Malloy, 2002). The students can compare systems based on different axioms and can study various geometries in the absence of concrete models (Burger & Shaughnessy, 1986). There is an appreciation of the distinctions and relationships between different axiomatic systems. According to Hoffer (1981), students at this level understand the importance of precision in dealing with foundations and interrelationships between structures. Non-Euclidean geometries can be studied and different systems can be compared (Mayberry, 1983). This is the most advanced level and is generally the level of a college mathematics major who is studying geometry as a branch of mathematical science (van de Walle, 2001).

Two different numbering systems are commonly used in the literature to describe the van Hiele levels: level 0 to level 4 (Yee, 2006) and level 1 to level 5 (Senk, 1989; Pegg & Davey, 1998; Malloy, 2002). The van Hieles originally made use of the level 0 to level 4 system claiming that all students are at least at level 0 (Senk, 1989). Their subjects were secondary school students and since others have tried this model to elementary school students, inclusion of a level "below 0" became necessary (Pusey, 2003). However, the more recent writings of van Hiele (1986) use level 1 to level 5 numbering system. In this study, all references and all results from the research using the 0 – 4 numbering system have been changed to the numbering system as level 1 to level 5.

Meanwhile, as a result of learners not achieving even the basic level (level 1), such as 'not yet at level 1' or 'weak level 1', researchers have suggested the introduction of another level, called level 0 (pre-recognition level). Halat (2007) points out that even though the existence of level 0 is the subject of some controversy (e.g., Usiskin, 1982; Burger & Shaughnessy, 1986), van Hiele (1986) himself does not talk and acknowledge the existence of such a "non-level". Instead, he asserts that all students enter at ground level, that is, at level 1, with the ability to identify common geometric figures by sight. But Usiskin's (1982) research project has shown that level 0 exists. Usiskin (1982, p.99) found that out of the 2361 participants of his study 222 participants were at level 0. This is also talked about by Clements and Battista (1992). Clements and Battista (1992, p.429) point out that "the bulk of the evidence from the van Hiele-based research along with research from the Piagetian perspective, indicated the existence of thinking more primitive than, and probably prerequisite to, van Hiele's level 1". They named this level 0 as "pre-recognition". They defined it as "children initially perceive geometric shapes, but may attend to only a subset of a shape's visual characteristics and they are unable to identify many common shapes" (p.429). Therefore level 0 can be explained as follows using the descriptions of Clements and Battista (1992).

Level 0: Pre-recognition

At this level, children perceive geometric shapes, but perhaps due to the deficiency in perceptual activity, may attend to only a subset of a shape's visual characteristics. They are unable to identify many common shapes due the lack of ability to form requisite visual images. They may differentiate between a circle and a square, but not between a square and a triangle. At this level, the objects about which students reason are specific visual or tactile stimuli and the product of this reasoning is a group of figures recognised visually as 'the same shape'.

In this study, the existence of level 0 was taken into consideration in the assignment of the van Hiele levels to the learners who participated in the study.

Even though the levels are grade-invariant, in an ideal world, students from pre-kindergarten through high school are meant to think and reason about geometry in a similar progression (Malloy, 2002). According to Malloy, students in pre-kindergarten to grade 2 should focus on the visualisation level, grade 2 to grade 5 on the analysis level, grade 5 to grade 8 on the informal deduction level and high school students are supposed to be on the deduction level. But this is not usually the case.

Even though these descriptions are content specific, van Hiele's levels are actually stages of cognitive development (Pegg & Davey, 1998). "The levels are situated not in the subject matter but in the thinking of man" (van Hiele, 1986, p.41) which suggests that the levels are in fact stages of cognitive development. Progression from one level to the next is not the result of maturation or natural development. It is the quality and nature of the experience in the teaching and learning program that influences a genuine advancement from a lower to a higher level.

According to Pegg and Davey (1998), the van Hiele theory is aimed at improving teaching by organising instruction to take into account learner's thinking which is described by the hierarchical series of levels and if the student's level of thinking is addressed in the teaching process, students will have the ownership of the encountered material and the development of insight will also be enhanced. For the van Hieles, the main purpose of the instruction is the development of such insight (van Hiele, 1986). The theory also offers a model of teaching that teachers can apply in order to promote their learners' levels of understanding of geometry (Atebe, 2008).

The present study limits its scope to the first four levels namely, visualisation, analysis, informal deduction and deduction (labelled as level 1-4), with the possibility of the inclusion of level 0 (pre-recognition). The learners in senior secondary school are not expected to establish theorems in different axiomatic systems (Level 5). Furthermore, the non-testability of van Hiele level 5 is the reason often cited for its non-inclusion in many studies (see Usiskin, 1982; Atebe, 2008).

Van Hiele himself refrains from describing levels higher than the fourth (in the old numbering system, which is now level 5). According to him those higher levels are much more difficult to discern than levels 2, 3 and 4 and if our students do not understand us at these levels, they are not going to understand us on the fifth or higher levels. Van Hiele (1986, p.47) appears to question the existence of level 5:

> ...I am unhappy if, on the grounds of my levels of thinking, investigations are made to establish the existence of fifth and higher levels…I would much prefer that a beginning be made on the improvement of education with the aid of the levels of thinking…

2.3. Some characteristics, features and properties of the van Hiele levels

According to van Hiele, the levels have five distinctive properties (van Hiele, 1959/1986, 1999). Some others (Usiskin, 1982; Mayberry, 1983; Burger & Shaughnessy, 1986; Senk, 1989; Pegg, 1995) also have affirmed it through their research. The characteristics are:

- **Intrinsic and extrinsic properties:** At each level, there appears in an extrinsic way which was intrinsic at the preceding level. At the base level (level 1), figures were in fact also determined by their properties, but someone thinking at this level is not aware of these properties (van Hiele, 1986).

- **Hierarchic arrangement:** The ways of thinking of the levels have a hierarchic arrangement. "Thinking at the second level is not possible without that of the first level; thinking at the third level is not possible without thinking at the second level" (van Hiele, 1986, p.51). The levels are hierarchical in that a student cannot operate with understanding on one level without having been through the previous levels. Mayberry (1983), Senk (1989) and Pegg (1995) confirm that a student who has not attained level n may not understand thinking of level n +1 or higher. Therefore, for students to function adequately at one of the advanced levels in the van Hiele hierarchy, they must have mastered large portions of the lower levels (Hoffer, 1981). Senk (1989) states that two persons reasoning at different levels may not understand each other. This can be taken as one of the reasons for a learner not being able to understand the teacher.

- **Discontinuity:** According to van Hiele (1986), the most distinctive property of the levels of thinking is their discontinuity, the lack of coherence between their networks of relations. He observed that at certain points in instruction, the learning process has stopped and later on it would continue itself as it was. A student having reached a given level remains at the level for a time, as if maturing (Pegg, 1995). The teacher does not succeed in explaining the subject. Clements and Battista (1992) support this by explaining that there are jumps in the learning curve which reveal the presence of discrete, qualitatively different levels of thinking. Further, Pegg (1995) cautions that forcing a student to perform at a higher level will not succeed until the maturation process have occurred.

- **Linguistic character:** Each level has its own linguistic symbol and its own system of relations connecting these signs. A relation which is "correct" at one level can reveal itself to be incorrect at another level (van Hiele, 1986). Two people operating at two different levels speak a very different language (van Hiele, 1986). Each level has its own language (De Villiers & Njisane, 1987; Fuys, et al., 1988; Burger & Shaughnessy, 1986; Senk, 1989). Two people who reason at two different levels cannot understand each other (Atebe, 2008). This is what often happens between teacher and student (van Hiele, 1986). Neither of them can manage the thought process of the other and their dialogue can only proceed if the teacher attempts to form for himself an idea of the student's thinking and to match to it. Van Hiele observed that the teacher seems to speak a language which cannot be understood by pupils who have not reached the new level. The pupils might accept the explanations of the teacher, but it might not sink into their minds (van Hiele, 1986). If a teacher is making a presentation at his/her own level while asking learners to respond to his/her questions, is actually making a monologue, as the teacher considers all the answers which do not belong to his system of relations as stupid or misplaced (van Hiele, 1986). To establish a dialogue the teacher must start at the level of the learners.

- **Advancement:** The transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program. "The transition is not possible without the learning of a new language" (van Hiele, 1986, p.50). The maturation which leads to the higher level happens in a special way. Adequate and effective learning experiences are required at lower levels in order to learn how to think and reason at the higher levels.

In addition to the above characteristics, the researchers like Usiskin (1982) and van de Walle (2004) also have identified a few other features and characteristics of the levels from their studies. A few are mentioned in the following sections.

For example, van de Walle (2004, p.348) has affirmed the following additional feature as suggested by van Hiele (1986):

> "The levels are not age dependent in the sense of the developmental stages of Piaget. A third grader or a high school student could be at level 1. In fact, some students and adults remain forever at level 1 and a significant number of adults never reach level 3. But age is certainly related to the amount and type of geometric experiences that we have".

Van de Walle (2001) again suggested that each of the five levels describes the thinking processes used in geometric contexts. As an individual progresses from one level to the next, the objects of his/her geometric thinking change as illustrated in Table 2.2.

Table 2.2: Objects of thought and products of thought

Level	Objects of thought are….	Products of thought are …..
Level 1	shapes and what they "look like"	classes or groupings of shapes that seem to be "alike"
Level 2	classes or groupings of shapes that seem to be "alike" rather than individual shapes	the properties of shapes
Level 3	the properties of shapes	the relationships among properties of geometric objects
Level 4	the relationships among properties of geometric objects	deductive axiomatic systems for geometry
Level 5	deductive axiomatic systems for geometry	comparisons and contrasts among different axiomatic systems of geometry

(Source: van de Walle, 2001, pp. 309-310)

The products of thought at one level become the object of thought at the next level. This means that the objects (ideas) must be created at one level so that relationships among these objects can become the focus of the next level.

Usiskin (1982, p.4) suggests that "it is inherent in the van Hiele theory that, in understanding geometry, a person must go through the levels in order". This is called the fixed sequence property of the levels. These properties are summarised as follows:

- **Fixed sequence** – a student cannot operate, with understanding, at van Hiele level *n+1* without having passed through level *n.*

- **Adjacency** – at each level of thought what was intrinsic in the preceding level becomes extrinsic in the current level.

- **Distinction** – each level has its own linguistic symbols and its own network of relationships connecting those symbols.

- **Separation** – two persons who reason at different levels cannot understand each other.

According to Usiskin (1982, P.12), the theory is considered to be one of the best for the teaching and learning of geometry because it posses three appealing characteristics such as "elegance, comprehensiveness and wide applicability". These characteristics as described by Usiskin (1982, p.12) are summarised as follows:

- **Elegance** means that the theory involves a rather simple structure described by reasonably concise statements, each with broad effect. For example, the same principles apply for movement from level 1 to 2 as from 2 to 3 and so on, displaying an elegance of form. The simplicity of structure is evident when one notes that the figures of level 1 are the building blocks for properties at level 2 which in turn are ordered at level 3, the ordering is a prerequisite for level 4 and so on.

- **Comprehensiveness** means that the theory covers the whole of learning of geometry. It seeks to explain why students have trouble in learning and also what could be done to remove these stumbling blocks. According to Usiskin, van Hiele asserts that the theory applies to all of mathematical understanding and gives examples involving the learning of functions and other non-geometric notions.

- **Wide applicability** means that the theory is seen as widely and easily applicable. For example, the theory is widely applied in geometry curricula in countries as diverse as the Netherlands, the Soviet Union and the United States of America. And it can be applied in South Africa also.

Again, Burger and Shaughnessy (1986), suggest that the levels are not as discrete as suggested by the descriptions, rather it appears that students can be in transition between levels and that they will oscillate between levels during the transition period. It is sometimes possible that students can achieve higher levels by learning rules or definitions by rote or by applying routine algorithms that they do not understand (Pegg, 1995). Pegg however emphasises that rote learning or applying routine algorithms without understanding does not represent the achievement of a particular level. Therefore, any information or knowledge acquired without understanding cannot be regarded as the attainment of a certain level of thinking. Pegg (1995) again clarifies that, except perhaps when one deals with gifted or exceptional students, to move a student from one level to the next requires direct instruction, exploration and reflection by the student. This means that to succeed in moving a student from one level to the next, adequate time should be allowed for this growth to occur. Mayberry (1983), Burger and Shaughnessy (1986) and Mason (1998) indicate that there is also evidence that students can be on different van Hiele levels for different concepts. That is, a student's level of thinking might vary across topics and according to how recently a topic was studied. But Mason (1998) suggests that once a student's thought has been raised to a certain level in one concept, it becomes easier for the student to think at that level for other concepts. Clements (2004) states that, as postulated by the van Hieles, progress from one level to the next depend little on biological maturation or development; instead, it proceeds under instructional experience. According to the theory, an important characteristic of mathematical reasoning is that growth in age does not necessarily imply growth in a student's level of reasoning. Here, instruction, assisted by the learning phases, as proposed by the van Hieles, plays a major role (Jaime & Gutierrez, 1995). A brief discussion on the implications of the levels and the learning phases on the instruction of geometry follow in the next section.

2.4. The van Hiele phases

According to the van Hiele theory (1986), knowledge is strengthened and added to within the learning phases between each level and each instructional learning stage builds upon and adds to the thinking of the level before it (Genz, 2006). Van Hiele recommended five phases for guiding learners from one level to the higher one in a given topic in geometry classroom instruction (van Hiele, 1986; Fuys, et al., 1988). These phases are phases that a learner should go through within each level in order to move from one level to the next. A learner's progress from one level to the next is the result of purposeful instruction organised into five phases (Hoffer, 1983; Mayberry, 1983; van Hiele, 1986). These five phases are of "sequenced activities that emphasise exploration, discussion, and integration" (Teppo, 1991, p.212). Therefore, the instruction at each learning phase fully and clearly describes that which was implied at the previous phase. The learning phases are useful in designing learning and instructional activities (Mateya, 2008). These phases are described as follows:

Phase 1: Information/Inquiry

The students become acquainted with the context domain (van Hiele, 1986; Clements & Battista, 1992).This is achieved by the teacher engaging the learners in conversations about the topic of study (King, 2003). As a result, this process causes the learners to discover a certain structure (Fuys, et al., 1988). During this phase, questions are asked and observations are made from both sides about the topic of the study. The teacher acquaints with the background and the prior knowledge of the learners. This helps the teacher to learn how learners interpret the language and provides the necessary and appropriate vocabulary while setting the scene for further study (Clements & Battista, 1992).

Phase 2: Directed orientation

In this phase, students become acquainted with the objects from which geometric ideas are abstracted. They perform tasks involving different relations of the network that is to be formed. These tasks are carefully sequenced for exploration to point students in the intended direction of the study (King, 2003). The students begin to realise what direction their learning is taking. This helps the students to become familiar with "the principal connection of the network of relations to be formed" (van Hiele, 1986, p.177). In short, this implies that the students are becoming familiar with the structures of the topic such as the figures, vocabulary, symbols, definitions, properties and relations. The teacher's role is to direct students' activity by guiding them in appropriate explorations (Clements & Battista, 1992).This activity helps the students to explore the field of investigation using the material, for example, by folding, measuring, and looking for symmetry (Mason, 1998).Therefore teachers should choose materials and tasks in which the targeted concepts and procedures are salient.

Phase 3: Explicitation/Explanation

This phase is often referred to as the phase of "explanation" which seems to suggest that the teacher is conveying information (King, 2003). In this phase, the students have gained insights in working with the structures of the topic. Students become explicitly aware of their geometric conceptualisations, describe these conceptualisations in their own language and learn some of the traditional mathematical language for the subject matter (Clements & Battista, 1992; Mason, 1998). This is the time during which the student is provided with opportunity to express his/her own opinion about the tasks as well as the new relations observed whilst materials are used. The students make their observation known through verbalising about them and begin to use more accurate and appropriate vocabulary under the teacher's guidance (King, 2003).

Phase 4: Free orientation

In this phase, students solve problems in which the solution requires the synthesis and utilisation of those concepts and relations previously elaborated (Clements & Battista, 1992). Therefore, students prepare themselves for multi-step tasks in addition to the one-step tasks they were familiar with. The students are now involved in multi-step tasks which can be completed in different ways (King, 2003). Van Hiele (1986) points out that it can be said that this is the further development of the second phase in which the student, for example, learns to find his or her way in a network of relations with the help of the connections he or she has at his or her disposal. The tasks are organised in such a way that the students are encouraged to see the connection and observe relations more explicitly (King, 2003). Fuys, et al. (1988) support the above statement by stating that the field of investigation or network of relations is still largely unknown at this stage, but the student is given more complex tasks to find his or her way round this field. A student might know about the properties for a new shape, for example, a kite. The teacher's role is to select appropriate materials and geometric problems – with multiple solution paths, to give instructions to permit various performances and to encourage students to reflect and elaborate on these problems and their solutions, and to introduce terms, concepts and relevant problem-solving processes as needed.

Phase 5: Integration

According to van Hiele (1986), the teaching process comes to an end with this final phase indicating that the students have reached a new level of thought, and have increased their thought level in the new subject matter. They now review the methods at their disposal and form an overview of the concept learned (King, 2003). This means that the student summarises all that he or she has learnt about the subject, reflects on his or her actions and thus obtains an overview of the whole network or field that has been explored, for example, summarises and synthesises the properties of a figure (Fuys, et al., 1988). In this phase, the language and conceptualisations of mathematics are used to describe the network (Clements & Battista, 1992). At this stage the students are expected to have unified and internalised the object and the relations studied (Hoffer, 1983). Hoffer (1983) also elaborates that the teacher provides summaries of some of the main points of the subject that are already known by the students to help this process. The teacher assists the process by providing global surveys of what the students have learned (Hoffer, 1983; Crowley, 1987). In other words, this phase represents the stage where the teaching-learning process is evaluated.

As the process in which the old domain of thinking has been replaced by the new, the students have technically attained a new level of thought and are ready to begin the phases of learning at the next level (King, 2003).

The role of the teacher at different phases is different. From planning tasks, directing children's attention to geometric qualities of shapes, introducing terminology and engaging children in using these terms through discussions and encouraging explanations and problem-solving approaches that make use of children's descriptive thinking about shapes (van Hiele, 1999). More precisely, in phase 1, the information phase, the teacher introduces new material and interviews the students, on both one –to-one basis as well as in groups, regarding their personal conceptions and understanding of the new ideas. In the second phase, the direct orientation phase, the teacher may assign short activities which will provide more clarity on issues at hand. In the explicitation phase, the teacher moderates a discussion among the students and assists them to reach consensus. In the last phase, the free orientation phase, the teacher once again clarifies the activities and helps students to find solutions and discover new results. Here, the teacher leads the discussion concluding in summarising the results (King, 2003).

According to Atebe (2008), the phases are invariant with respect to any two adjacent van Hiele levels and this offers teachers a chance to identify clear starting and ending points in their efforts to raise students' thought at any given level to the next higher level during instruction in geometry. In applying learning phases to an instructional unit, the teacher may consciously remain in a particular phase for several lessons, and may cycle through phases several times according to the needs of the students before completing the unit (King, 2003).

2.5. The importance of language in the van Hiele theory

"Language is very important to thinking. Without language, thinking is impossible. Without language, there is no development of sciences".

(van Hiele, 1986, p.9)

Van Hiele (1986) states that when he began teaching, there were parts of subject matter, that he could explain and explain, but students could not understand it even after trying. In the years that followed, he changed his explanation many times, but the difficulties remained. He felt that he was speaking a different language. The solution to this concern was the different levels of thinking. Again, he speaks of an unavoidable situation in class, where we find that a group of learners having started homogeneously do not pass the next level of thinking at the same time that half of the class might speak a language which the other half may not understand (van Hiele, 1986). According to van Hiele, each level has its own language. Two people operating at two different levels speak a very different language (van Hiele, 1986). A teacher beginning to teach geometry should address himself to the learners in a language they understand and by doing that the teacher inspires their confidence and the learners will understand him (van Hiele, 1986). In stressing the importance of language, van Hiele notes that many failures in teaching geometry result from a language barrier-the teacher using the language of a higher level than is understood by students (Fuys, et al., 1988). The findings from a few researchers on implications on language are already discussed under section 2.3 on the linguistic characteristics of the van Hiele theory.

A few research on the van Hiele theory is discussed in the following section.

2.6. Empirical research on the van Hiele theory

The van Hiele theory of thinking which was developed and structured by Pierre van Hiele and Dina van Hiele-Geldof in the period from 1957 to 1986 focuses on the teaching and learning of geometry (Mateya, 2008). Challenges in their own classrooms inspired these Dutch educators to do companion doctoral dissertations. Dina van Hiele-Geldof's research (1959) as cited in Fuys, et al. (1988) was an inquiry into the didactical possibilities of geometry instruction. The inquiry led to the assumption that students interpret structures and objects according to their level of understanding. Dina van Hiele-Geldof's research provided specific examples of students' behaviours at the levels in response to many specific instructional tasks. It strove to explain from a teaching perspective how to help children make progress with the levels and described five teaching phases within each level (Pusey, 2003).

A considerable amount of research projects were focused on testing the validity of the theoretical underpinning of this theory with all its properties (King, 2003). Besides a significant amount of research studies into students' understanding of geometric proofs, the van Hiele theory stands out as one of the best recognised frameworks for the teaching and learning of geometry (Dindyal, 2007). As a result, this model is often considered as the foundation for curricula implemented in mathematics classrooms in many countries. As an example, an article written by van Hiele in 1959, "the Child's Thought and Geometry" attracted the attention of Soviet psychologists and educators who have been concerned about student difficulty with geometry and had long been studying how children learn (Fuys, et al., 1988). Since the mid 1980's there has been a growing interest in the area of teaching and learning geometry (Mayberry, 1983).

With the said interest, a number of research studies were and still are conducted in different parts of the world. A few of them are shown below:

2.6.1. Some milestones in the research on the van Hiele theory

Table 2.3: Some milestones in the research on the van Hiele theory

Researcher	Year	Research Area/Findings
Van Hiele	1959	Levels of thinking
Dina van Hiele-Geldof	1959	Phases of teaching
Usiskin	1982	Tested the validity of the levels in school children
Mayberry	1983	Testing levels of pre-service elementary school teachers
Burger Shaughnessy	1986	Focused on the characteristics of the van Hiele levels of reasoning
Fuys, et al.	1986	Effect of instruction on the levels- developed instructional modules
Gutierrez, Jaime & Fortuny	1991	Assessed the geometric ability of students as a function of van Hiele levels
Clements, Battista & Sarama	2001	The effects of logo on children's conceptualisation of angle and polygons.

Since the proposal of the van Hiele theory, studies have focused on various components of this learning model at different grade levels. According to Pusey (2003), there are three different lines of work, namely, research that focused on the testing of the van Hiele theory, research that focused to find appropriate ways to assess the levels and implications of these assessments, research that examined the validity of the van Hiele theory in terms of curricula and research that focused on the effects of interventions on van Hiele model with students and teachers. Of these, a few are chosen and discussed below due to the fact that their target population were high school students, pre-service and in-service teachers and were testing the effects of van Hiele-based instruction. King (2003) suggests that some of them have been recognised and acclaimed as ground-breaking projects in the field (For example, Usiskin, 1982; Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988; Clements & Battista, 1992).

2.6.2. Research that focused on the testing of the van Hiele theory

One of the first major studies done with the van Hiele model was led by Usiskin (1982) at the University of Chicago which was on the van Hiele levels and achievement in secondary school geometry known as the CDASSG (Cognitive Development and Achievement in Secondary School Geometry) project. It used the van Hiele theory to explain why many students have trouble in learning geometry. Usiskin developed a multiple choice test to measure learners' van Hiele level of reasoning. This test has been widely used by others. Usiskin also introduced the '3 of 5 correct' criterion to assign a student to a van Hiele level with a weighted sum. The test that was conducted on about 2700 grade 10 learners from 13 different high schools at the beginning and the end of the school year revealed that the learners were not ready for high school geometry. It was reported that 9%, were operating on level 0, 46% were level 1, 28% on level 2, 12% on level 3 and 4% were on level 4. Crowley (1990) questioned the validity and reliability of the van Hiele results from Usiskin's study. According to Usiskin and Senk (1990), their purpose of the test was to determine whether the van Hiele theory was accurate in describing the thinking level and success in geometry. Usiskin comments that the van Hiele model's elegance, comprehensiveness and wide applicability would attract mathematics educators. Usiskin (1982) also points out that the theory not only provides an explanation of why pupils have problems, but also suggests a remedy for these problems.

Senk (1989) also examined the relationship between the achievements in writing geometry proof and the van Hiele levels. She revisited Usiskin's (1982) Cognitive Development and Achievement in Secondary School Geometry (CDASSG) and reached the conclusion that there was a positive relationship between high school students' achievement in writing geometry proofs and van Hiele levels of geometric thought.

2.6.3. Research that focused on finding appropriate ways to assess the levels and implications of these assessments

Burger and Shaughnessy (1986) examined three specific questions related to the van Hiele theory of learning in geometry. Firstly, they wanted to know whether these levels were useful for classifying students' thinking in geometry. Secondly, they were looking for specific indicators in students' reasoning that might be aligned with each of the levels. Finally they were interested in designing an interview procedure that could reveal predominant levels of reasoning on specific geometry tasks, instead of a written test. Their subjects varied in age from primary school to college level. Their findings did support much of van Hiele's description and characteristic of the levels. However, they cautioned that sometimes students could be in transition from one level to the next and in that instance, reliable classification could be problematic. They also could not detect that the levels were discrete structures. The levels appeared to be more dynamic rather than static and more of a continuous nature. They were able to assign certain behaviours to each level. According to them, the van Hiele levels were useful in describing students' reasoning process for polygons. They were successful in designing an interview protocol and they developed a series of indicators for each level for assessing students' geometric understanding in task based interviews. These level indicators as suggested by Burger and Shaughnessy (1986, pp. 43-45) are shown below.

Level 1

1. Use of imprecise properties (qualities) to compare drawings and to identify, Characterise and sort shapes.
2. References to visual prototypes to characterize shapes.
3. Inclusion of irrelevant attributes when identifying and describing shapes, such as orientation of the figure on the page.
4. Inability to conceive of an infinite variety of shapes.
5. Inconsistent sorting; that is, sorting by properties not shared by the sorted shapes.

6. Inability to use properties as necessary conditions to determine a shape; for example, guessing the shape in the mystery shape task after far too few clues, as if the clues triggered a visual image.

Level 2

1. Comparing shapes explicitly by means of properties of their components.
2. Prohibiting class inclusions among several general types of shapes, such as quadrilaterals.
3. Sorting by single attributes, such as properties of sides, while neglecting angles, symmetry and so forth.
4. Application of a litany of necessary properties instead of determining sufficient properties when identifying shapes, explaining identifications, and deciding on a mystery shape.
5. Descriptions of types of shapes by explicit use of their properties, rather than by type names, even if known.
6. Explicit rejection of textbook definitions of shapes in favour of personal characterisations.
7. Treating geometry as physics when testing the validity of a proposition; for example relying on a variety of drawings and making observations about them.
8. Explicit lack of understanding of mathematical proof.

Level 3

1. Formation of complete definitions of types of shapes.
2. Ability to modify definitions and immediately accept and use definitions of new concepts.
3. Explicit references to definitions.
4. Ability to accept equivalent forms of definitions.
5. Acceptance of logical partial ordering among types of shapes, including attributes.
6. Ability to sort shapes according to a variety of mathematically precise attributes.
7. Explicit use of "if, then" statements.

8. Ability to form correct informal deductive arguments, implicitly using such logical forms as the chain rule (if p implies q and q implies r, then p implies r) and the detachment.
9. Confusion between the roles of axiom and theorem.

Level 4

1. Clarifications of ambiguous questions and rephrasing of problem tasks into precise language.
2. Frequent conjecturing and attempts to verify conjectures deductively.
3. Reliance on proof as the final authority in deciding the truth of a mathematical proposition.
4. Understanding the roles of the components in a mathematical discourse, such as axioms, definitions, theorems and proof.
5. Implicit acceptance of the postulates of Euclidean geometry.

Burger and Shaughnessy's research also supported Mayberry (1983) who conducted a study of 19 pre-service elementary school teachers. She developed a set of 62 items that she used in clinical interviews with her pre-service teachers. She examined the ability of a student to reason at a specific level in geometric concepts like squares, triangles, circles, parallel lines, congruency and similarity. Her findings were that, 52% of the response patterns of the students who had taken high school geometry were below level 2. She concluded that the van Hiele levels of thinking could be assigned to students for each of the topics tested and that the levels were hierarchical in nature. This means that a student could not attain a higher van Hiele level of thinking before first mastering the lower levels; this was also supported by Burger and Shaughnessy.

Gutiérrez, Jaime and Fortuny (1991) studied 9 eighth graders and 41 pre-service elementary teachers. The major goal of their study was to find an alternative way of assigning the van Hiele levels of thinking to students who were in transition between van Hiele levels. They theorised that the van Hiele levels of thinking were not discrete and that the transition between levels needed more in-depth attention. They presented an alternative method to evaluate and identify those students who are in transition. Their method of evaluation tried to measure the acquisition of levels quantitatively and qualitatively. Their method of evaluation allowed the possibility that a student could develop two successive levels of reasoning at the same time, and that the acquisition of the lower level was more complete than the acquisition of the upper level.

As a Master's thesis completed at the University of Maine, Knight's (2006) study with pre-service elementary and secondary mathematics teachers found that their reasoning stages were below level 3 and level 4, respectively. These results are consistent with the findings of Gutierrez, et al. (1991) and Mayberry (1983). None of these pre-service elementary and secondary mathematics teachers demonstrated a level 4 reasoning stage in geometry. This is surprising because the van Hiele levels of pre-service elementary and secondary mathematics teachers were lower than the expected levels for students completing middle school and high school, respectively (Hoffer, 1983; Crowley, 1987; NCTM, 2000).

2.6.4. Research that examined the validity of the van Hiele theory in terms of curricula

Genz's (2006) study was on determining high school geometry students' geometric understanding using van Hiele levels and to find out whether there was a difference between standards-based curriculum (which is based on van Hiele theory) students and non-standards based curriculum students in Utah. She used the tasks developed by Burger and Shaughnessy (1986) and she concluded that there was no significant difference between the two samples. But Billstein and Williamson (2003) and Chapbell (2003) as cited by Halat (2007), agree that standards based curricula have positive impact on students' performance and motivation in mathematics.

Halat (2007) reported on a study on acquisition of the levels of students engaged in instruction using a reform based curriculum (which was designed on the van Hiele theory) against students using a traditional curriculum. His study also revealed that there was no significant difference detected in the acquisition of levels between the two groups.

2.6.5. Research that focused on the effects of interventions on the van Hiele model with students and teachers

Fuys, et al., (1988) were involved in a three year study to determine whether the van Hiele model could be used to describe how students learn geometry. They conducted their research to achieve four main objectives. Firstly, they developed and documented a working model of the van Hiele levels after translating the van Hieles' doctoral thesis and some selected writings. This working model and the level descriptors were examined by van Hiele himself and other two van Hiele researchers, Alan Hoffer and William Burger. They developed and validated three modules based on the model and designed it for use as a research tool in clinical interviews. The modules included instructional activities along with key assessment tasks that were correlated with specific level descriptors. Later, clinical interviews were conducted with 16 sixth grades, 16 ninth graders, eight pre-service and five in-service teachers. They also analysed the geometric curriculum of American textbook series in line with the van Hiele model. The results of the study showed that most of the grade six was at level 1 and some were at level 2. This could be due to a lack of exposure to geometry in school. In grade nines, two out of 16 were at level 1, seven were between levels 1 and 2 and the rest were at level 2. Following the study, they suggested some modifications of the original descriptors and provided guidelines to avoid misinterpretation of their model (Bennie, 1998b).

The study by Erdogan and Durmus (2009) also tested the effect of van Hiele-based instruction on pre-service elementary teachers taken from eight classes in Turkey and found that the intervention had a significant positive effect.

2.6.6. South African researchers in the field of geometry

It might be of interest to know the research that has been conducted in the South African context. A few are shown below:

Table 2.4: Some South African researchers in the field of geometry

Researcher	Year	Research Area/findings
De Villiers	1997	Dynamic geometry/ Geometer sketchpad
Bennie	1998	Testing grade 9 learners' geometrical understanding with Fuys et al.'s interpretation of the van Hiele theory
Mc Auliffe	1999	Impact of a geometry course on pre-service teachers' understanding of geometry
King	2003	Development of an instructional model for primary school geometry
Schafer	2004	Worldview theory and the conceptualisation of space in mathematics education
Feza & Webb	2005	Van Hiele levels and grade 7 learners' understanding of geometry
Siyepu	2005	Using the van Hiele theory to explore the problems encountered by grade 11 learners in circle geometry
Atebe	2008	Levels of geometric thinking of senior secondary school learners comparative study of Nigerian and South African schools

Table 2.4 is an over simplification of the enormous work that has been done in this field of research. A few recent research are discussed below:

De Villiers M. D. (1986 - 2010)

Michael de Villiers is a prominent South African researcher in dynamic geometry and has been involved in it as early as 1986. He has pointed out that just knowing the definition of a concept using direct teaching of geometry definitions does not at all guarantee understanding of the concept and that research conducted with geometer sketchpad do indicate some improvement and positive gains in students' understanding of the nature of definitions as well as their ability to define geometric concepts such as quadrilaterals themselves (De Villiers, 2010). De Villiers suggests that the geometer sketchpad or Cabri are incredible computer programmes for exploring geometry (De Villiers, 2010). With geometer sketch pad, explorations of the properties of triangles, quadrilaterals, circles and other configurations are very easy. It allows the learners to dynamically transform their figures while preserving the geometric relationship of their constructions. Generalisations are easily possible and it is useful for learning proofs. It encourages the process of discovery where learners first visualise and analyse a problem and make conjectures before attempting a logical explanation of why their observations are true. De Villiers has presented and published many research papers on van Hiele theory and its implications in teaching geometry, geometer sketchpad and South African Senior Secondary School geometry (De Villiers, 1986, 1987, 1994, 1996, 1997, 1998, 2003, 2004, 2010).

Bennie K. (1998)

Bennie was part of the Geometry Working Group of a South African Non-Governmental Organisation called Mathematics Learning and Teaching Initiative (MALATI) during 1997 which tried to re-conceptualise the teaching and learning of geometry (Bennie, 1998b). The group found that van Hiele model of geometric thinking could be used as a framework to understand the geometric thinking of learners. The idea of re-conceptualising the approach to geometry teaching and learning was placed in the foreground of the introduction of Curriculum 2005 in 1998 by the South African National Ministry (King, 2003). The group has identified the importance of spatial skills in the study of geometry and developed a Spatial Skills Package for use in grades 4 to 7. Responses from the workshops conducted were used to explore the MALATI approach to spatial skills (Bennie, 1998a). These were used for the implementation of Curriculum 2005. Bennie has presented many research papers on spatial abilities and spatial sense (1999) based on van Hiele theory in Junior Secondary School learners.

King L.C.C. (2003)

As a Doctorate study completed at Curtin University of Technology, this study used the van Hiele theory to develop an instructional model to enhance students' understanding of primary school geometry. The main themes of the study were to establish whether the performance of primary school learners could be improved by a structured classroom intervention programme and whether they were able to talk about their geometric conceptual knowledge sensibly. The study was conducted in a medium-sized, co-educational urban primary school in Port Elizabeth in the Eastern Cape Province. It used a quasi-experimental research design and the results indicated that there had been a significant improvement in the performance of the experimental group. The study also tested whether primary school mathematics teachers were prepared to adopt and implement the structured package of geometry activities in their classrooms. Results obtained indicated that the participating teachers were positively inclined towards using such a package of activities with their students.

Feza N. & Webb P. (2005)

As a published article in Pythagoras, this study investigated whether a sample of grade 7 learners in previously disadvantaged primary schools met both the assessment criteria for geometry as stated by the Revised National Curriculum Statement and the implied van Hiele levels. They conducted open and flexible semi-structured interviews with 30 learners from six different previously disadvantaged schools situated in the ex-Ciskei and Transkei areas. The data generated suggested that none of the 30 learners who participated in the study had attained these requirements and that language competency in general was a barrier to the attainment of higher levels of understanding amongst this group of second language learners. The study suggests that teachers should use the van

Hiele levels and RNCS Assessment Standards to establish their learners' understanding and learners' cultural background and their specific use of words in the vernacular context need to be taken into consideration when developing learning programmes.

Siyepu S.W. (2005)

As a master's research originated at Rhodes University, this study used the van Hiele theory to explore the problems encountered by grade 11 learners in circle geometry. The study was conducted in a rural school in Butterworth in the Eastern Cape Province. His study revealed that many of the grade 11 learners were not prepared for the study of geometry concepts and formal proofs. The study further pointed out that the South African high school geometry curriculum was presented at a higher van Hiele level than what the learners were operating at. He also noted that the structure of the South African geometry syllabus consists of a somewhat disorganised mixture of concepts. It is not sequential and it sequences concepts in a seemingly unrelated manner.

Mateya M. (2008)

This master's thesis was a case study which used the van Hiele theory to analyse geometrical conceptualisation in grade 12 students in the Namibian Context. The study was conducted under Rhodes University in Grahamstown, in the Eastern Cape Province. The study which was conducted in two Namibian Schools with 50 learners, also indicated that the majority of the learners who participated in the study had a weak conceptual understanding of geometric concepts. The study highlighted that the Namibian grade 12 geometry syllabus should be aligned with the van Hiele levels of geometric thinking.

Atebe H.U. (2008)

This PhD study completed at Rhodes University was a case study of Nigerian and South African learners. The aim of the study was to explore and explicate the van Hiele levels of geometric thinking of a selected group of grades 10, 11, and 12 learners in Nigerian and South African senior secondary schools. It also aimed at providing a rich and in-depth description of the geometry instructional practices that might have contributed to the levels of geometric conceptualisation exhibited by the learners. The sample of the study comprised a total of 144 learners and six mathematics teachers from Nigeria and South Africa. The South African subsample was selected from a rural school in Grahamstown in the Eastern Cape Province. Various paper and pen tests were administered on the learners. Learners were assigned to different van Hiele levels based on the criteria used by Usiskin (1982). The results from the study revealed that most of the learners were not ready for formal deductive study of school geometry and the learners' conceptual understanding of geometry was low and most of them were at the pre-recognition level (level 0 as suggested by Clements and Battista (1992)) or van Hiele level 1 and that the South African subsample outperformed their peers in the Nigerian subsample. The findings also provided support for the hierarchical property of the van Hiele levels.

All the research studies above are relevant to my study.

2.7. Comparison of the van Hiele theory with other theories

Many pre-service teachers and practising teachers complain that learning theories are abstract and cannot be applied to real life situations. On the contrary, many of these theories attempt to explain how children develop intellectually, emotionally and socially and that theories can inspire teachers to develop different teaching techniques.

In order to fully appreciate the van Hiele theory, it is necessary to consider other learning theories that are used in research. How children develop their understanding of geometry and their spatial sense has been an area of research over the past 50 years with three general models having the greatest impact on school mathematics curricula. A variety of models to describe children's spatial sense and thinking have been proposed and researched and they include Piaget and Inhelder's topological primacy thesis, van Hiele's levels of geometric thinking and cognitive science models (Clements & Battista, 1992). Jean Piaget's developmental theory is explained in the following section as it talks about hierarchical stages of development which closely links with the topological primacy thesis.

2.7.1. Jean Piaget's developmental theory

Jean Piaget (1896-1980) was the genetic epistemologist originally trained in the areas of biology and philosophy who described the developmental nature of children's thinking in a variety of domains such as space and geometry. He believed that what distinguishes human beings from other animals is our ability to do "abstract symbolic reasoning." His theories about development in the domains were structured using stages of cognitive development that were characteristically connected with certain ages (Pusey, 2003). These stages of development as cited by Huitt and Hummel (2003) are:

- Sensorimotor stage (Infancy) - In this period (which has six stages), intelligence is demonstrated through motor activity without the use of symbols. Knowledge of the world is limited because it is based on physical interactions /experiences. Children acquire object stability at about seven months of age (memory). Physical development allows the child to begin developing new intellectual abilities. Some symbolic or language abilities are developed at the end of this stage.
- Pre-operational stage (early childhood through preschool) - In this period (which has two sub-stages), intelligence is demonstrated through the use of symbols, language use matures, and memory and imagination are developed,

but thinking is done in a non-logical, non-reversible manner. Egocentric thinking predominates at this stage.

- Concrete operational stage (childhood to early adolescence) - In this stage (characterised by seven types of conservation: number, length, liquid, mass, weight, area and volume), intelligence is demonstrated through logical and systematic manipulation of symbols related to concrete objects. Operational thinking develops and egocentric thought diminishes.
- Formal operational stage (adolescence and adulthood) - In this stage, intelligence is demonstrated through the logical use of symbols related to abstract concepts. Early in the period there is a return to egocentric thought. Only 35% of high school graduates in industrialised countries obtain formal operations; many people do not think formally during adulthood.

According to Piaget these stages of cognitive growth are based on the development of a person's mental structures and are not linked with or necessarily influenced by instruction (Lehrer, Jenkins & Osana, 1998). Piaget believed that biological development drives the movement from one cognitive stage to the next. Yoong (2006) states that later research show that these year ranges are too optimistic, that a large percentage of secondary school pupils (12 – 18 years) are still in the concrete operational stage and only a small portion of them reach the formal operational stage.

2.7.2. Piaget and Inhelder's topological primacy thesis

Piaget and Inhelder's (1967) and Piaget, Inhelder, Szeminska's (1960) research explored how children represent space and suggest two major themes in their topological primacy thesis. The first theme states that children's representations of space are constructed through progressive organisation of their motor and internalised actions resulting in operational systems. The second theme states that children's representation of space is not a perceptual 'reading off' of their spatial environment, but is constructed from prior active manipulation of that environment (Clements & Battista, 1992, p.426). The progressive organisation of geometric idea follows a definite order and this order is more logical than historical in that initially topological relations are constructed (Clements & Battista, 1992, p.422). "…Children later construct projective relationships and Euclidean relationships such as angularity, parallelism and distance" (Clements & Battista, 1992, p.421). Clements and Battista comment that researchers have tended not to discuss this theme.

Clements and Battista (1992) cite Piaget and Inhelder (1967) and point out that they claim that inaccurate drawings reflect the inadequacy of mental tools for spatial representation, but Clements and Battista object to it by saying that inaccuracies in drawing might be attributable to motor difficulties.

Another major precept of Piaget's theory is connected with constructivism, whereby a child's representation of space is developed through his own activity and interaction within the environment (Pusey, 2003). Piaget's views are often compared with those of Vygotsky (1896-1934), who looked more to social interaction as the primary source of cognition and behaviour. The writings of Piaget (e.g., 1972, 1990) and Vygotsky (e.g. Vygotsky, 1986; Vygotsky & Vygotsky, 1980), along with the work of Dewey (e.g., Dewey, 1997a, 1997b), Bruner (e.g., 1966, 1974) and Neisser (1967) form the basis of the constructivist theory of learning and instruction (Huitt & Hummel, 2003).

Clements and Battista (1992) iterate that according to Piaget and Inhelder, the development of more sophisticated spatial concepts involves increasingly systematic and coordinated action and that during the first stages of development children are basically passive in their explorations. For Piaget, discovery learning and supporting the developing interests of the child are two primary instructional techniques. It is recommended that parents and teachers challenge the child's abilities, but not to present material or information that is too far beyond the child's level. It is also recommended that teachers use a wide variety of concrete experiences like the use of manipulatives, working in groups etc to help the child learn (Huitt & Hummel, 2003).

Piaget's contribution to the field of education was giving description to children's thinking (Pusey, 2003). According to Clements and Battista (1992), there has not been enough evidence to suggest that Piaget's theory has really been effective in teaching geometry. According to Piaget, learners' development in thinking is of a psychological nature that just happens over time due to maturity and it is not affected or improved by instruction. Planned instructional techniques or activities cannot reverse the already dictated growth of a child (Pusey, 2003). This has been questioned by many such as Clements and Battista (1992) and Pegg and Davey (1998). Another criticism of Piaget and Inhelder is that their use of terms such as topological, separation, proximity, Euclidean as well as the application of these and the related concepts to the design of their studies are not mathematically accurate (Darke, 1982; Martin, 1976 and Kapadia, 1974, as cited in Clements & Battista, 1992). Darke (1982) points out that a replacement of the 'topological primacy thesis' by a 'weird shape primacy thesis' is required as the confusion of terminology originated from the 'shape form'. By 'topological form' Piaget seemed to mean a shape that is strangely irregular and those shapes that have straight sides he called them 'Euclidean'. The above confusion is also strongly criticised by Martin (1976). Another criticism by Darke (1982) by quoting Page (1959) and Fisher (1965) is that even though the theory states that the child's conception of space should reflect the sensory motor activity he uses to reproduce it or to examine it by hand, but it has been noticed by these researchers that very young children 'did not' actively search for features of the shape but simply 'held' it still. The review of the above named researchers and subsequent researchers, Darke (1982, p.140) concluded that

- The category of 'topological figure' is not at all well defined.
- There has been a very loose use of the words topology and topological and so on.
- The experiments conducted by the researchers were often complicated by non-conceptual factors such as examination strategies, verbal labels etc and the children's actions were not as coherent as the theory would suggest.
- The theory fitted very well within genetic epistemology which in turn relies upon structuralism.

2.7.3. Cognitive science

Apart from van Hiele and Piaget's theories, the third major perspective that has been applied to understanding students' learning of geometry is that of cognitive science. Cognitive science is an interdisciplinary field concerned with the mechanics of cognition, or how the mind and brain work to acquire and manipulate knowledge. These precise models of geometric knowledge and processes attempt to integrate research and theoretical work from psychology, philosophy, linguistics and artificial intelligence (Clements & Battista, 1992).

There are a variety of cognitive science models including Anderson's Model of Cognition, Greeno's Model of Geometry Problem Solving and the Parallel Distributed Processing Networks Model. The Anderson's Model of Cognition postulates two types of knowledge, namely declarative knowledge ('knowing that') and procedural knowledge ('knowing how'). According to this model, all knowledge initially comes in declarative form and should be interpreted by general procedures and procedural learning happens only in executing a skill. According to Clements and Battista (1992), the learning of the Anderson Model involves (1) acquisition of declarative knowledge, (2) application of declarative knowledge to new situations by means of search and analogy (3) compilation of domain specific productions and (4) strengthening of declarative and procedural knowledge. Greeno's Model of Geometry Problem Solving is similar to Anderson's Model but is based on computer simulations and the third model explains the holistic template representations of the lower levels in the van Hiele hierarchy.

These cognitive science models bring a precision to models of geometric thinking which are not always present in the theories of van Hiele and Piaget. Battista and Clements (1995) point out that even though these models provide insights and useful metaphors, they have some limitations such as (1) they do not address students' development of qualitatively different levels of thinking and representations, belief systems, motivation and meaningful interpretation of subject matter (2) they de-emphasise the roles of sensory motor activity, intuition and (3) culture in mathematical thinking.

Clements and Battista (1992, p.437) comment that "Piaget's schemes, van Hiele's network of relations, and cognitive science's more explicit declarative networks definitely possess commonalities in their views of knowledge structure" and it is possible that a synthesis of these would yield a richer, more veridical model and that such a model will have the developmental aspects of Piaget and van Hiele perspectives and the explication of the cognitive science.

2.7.4. Comparison of the van Hiele theory to Piaget's theory

Another area of research was to compare and contrast these two theories. One of the most obvious differences is that van Hiele describes levels of thinking and the other describes stages of development (Pusey, 2003). According to Glasersfeld and Kelly (1982) as cited in Pusey (2003), a stage designates a stretch of time and it is characterised by a qualitative change that distinguishes it from adjacent periods and represents one step in a progression. On the other hand, they claim that level is not defined in terms of time and it implies a specific degree or height of some measurable characteristic or performance. According to van Hiele this is true as his levels are not age dependent, that a particular level is possible at any age and can be changed at any time.

The relationship between these two theories is interesting because they both included a study about learning geometry and both propose some form of hierarchical structure. Piaget described how the development of a student's ability to prove ideas formally occurs without considering the curricula and van Hiele analysed progress within the curricula (Battista & Clements, 1995). Piaget's theory describes how thinking in general progresses from being non reflective and unsystematic, to empirical and finally to logical-deductive and van Hiele deals specifically with geometric thought as it develops through several levels of sophistication under the influence of a school curriculum (Clements & Battista, 1992). Both theories suggest that students can understand and explicitly work with axiomatic systems only after they have reached the highest levels in both hierarchies (Battista & Clements, 1995). Denis (1987) (as cited by Clements & Battista, 1992) states that both the theories appear to be connected, because for high school students, the van Hiele levels appear to be hierarchical across concrete and formal operational stages of Piaget.

Clements (2004), states that the van Hiele theory builds on the constructivism and geometric studies of Piaget, but it heads in new directions. Van Hiele himself states that in some chapters of his work 'Structure and Insight' he is critical of certain aspects of Piaget (1986) and asserts that some critics of his earlier articles have recognised that his opinion for Piaget is essentially positive. There are even some reasons that his levels originated with the theories of Piaget. Van Hiele acknowledges that Piaget's 'Structuralism' (1968) gave rise to his writing about structure (1986) and the important parts of the roots of the theory can be found in the theories of Piaget, but most of his ideas of structure he has developed in his book 'Structure and Insight' (1986) are borrowed from Gestalt theory. The setting up of the theory of levels of thinking of Van Hiele is from the levels by Piaget (1986). Although the work of Piaget formed one of their bases, the van Hieles did not accept all that Piaget stated (Orton, 2004). Even though there are so many differences, van Hiele acknowledges that Piaget first introduced levels and by disputing Piaget, he also learned from him.

Van Hiele (1999, p.310) on the misunderstanding of teaching arithmetic in schools, supports Piaget affectionately on his view that "giving no education is better than giving it at the wrong time" as to conclude on providing teaching that is appropriate to the learner's level of thinking.

Van Hiele (1986, p.5) emphasises the differences as follows:

The psychology of Piaget was that of development and not for learning. So how to stimulate the children to go from one level to the next should rather concern him, not Piaget.

- Piaget distinguished only two levels. In geometry, it is necessary for more. Some of Piaget's results would have been more intelligible, if he had distinguished more levels.

- Piaget did not see the very important role of language in moving from one level to the next.

- According to Piaget, human spirit develops in the direction of certain theoretical concepts.

- Piaget did not see structures of a higher level as the result of study of the lower level. In Piaget's theory, the higher structure is primary; children are born with it, and only have to become aware of it. For the van Hieles, structure means everything and the learning of structures is a super goal.

In the van Hiele theory, a structure is a given thing obeying certain laws and if it is a strong structure, it will usually be possible to superpose a mathematical structure onto it. But in Piaget's theory, the mathematical structure always defines the whole structure. Piaget believed that children were born with higher structure and needed only to be aware of it while van Hiele believed that the rules of the lower level became the structure of the higher level.

Van Hiele also believed that the transition from one level to the higher one is not a natural process and that it takes place under the influence of instruction (Orton, 2004), but for Piaget, it is a natural process. Van Hiele saw the role of a teacher in the construction of knowledge, but Piaget did not. According to Piaget, the role of a teacher in a classroom is to provide a rich environment for the spontaneous exploration of the child. According to Pusey (2003), Piaget would say that a child's growth is already dictated and not reversible through planned instructional activities. Both van Hiele and Piaget strongly disagree with the belief that good teachers merely explain clearly to children to teach them (Clements & Battista, 1992). Piaget stresses the role of disequilibrium and resolution of conflicts, while van Hiele implores teachers to recognise students' difficulties.

Van Hiele believed in the importance of language in moving from one level to the next, but Piaget did not see it (van Hiele, 1986). Van Hiele points out that when it was occasionally mentioned to Piaget that the children did not understand his question, he said it could be read from their actions. According to van Hiele, although the actions might be adequate, one cannot read from them the level at which they can think (1986, p.5). Pace (1991) comments that Piaget clearly tied language to the figurative aspect of knowledge and thus took a definite position against any such necessity of language for thought, while for van Hiele, "without language, thinking is impossible".

Both of them were constructivists. Battista and Clements (1995) point out that both Piaget's and van Hiele's theories suggest that students must pass through lower levels of geometric thought before they can attain higher levels and that this passage takes a considerable amount of time. The van Hiele theory, further suggests that instruction should help students "gradually progress through lower levels of geometric thought before they begin with a proof oriented study of geometry" (Battista & Clements, 1995, p.4). This is because students cannot bypass levels and achieve understanding. It further follows that prematurely dealing with formal proof can only lead students to attempt memorisation and to become confused about the purpose of proof (Battista & Clements, 1995).

Clements and Battista (1992) point out that both of them promote students' ownership in building understanding as well as the non-verbal development of knowledge that is organised into complex systems. Therefore this type of learning makes students to not only learn facts, names, or rules, but a network of relationships that link geometric concepts and processes and are eventually organised into schemata. This emphasises the importance of students passing through levels of thinking. Battista and Clements (1995) point out that both theories suggest that students can understand and explicitly work with axiomatic systems only after they have reached the highest levels in both hierarchies. This implies that the explicit study of axiomatic systems is unlikely to be productive for the vast majority of students in high school geometry.

Piaget believed that biological development drives the movement from one cognitive stage to the next. Yoong (2006) states that there is a good number of secondary school students who are still at concrete operational level instead of being at the formal operational level. This situation is similar to the findings of the studies about the van Hiele theory, which indicate that most of the secondary school students are operating at either the pre-recognition level or van Hiele level 1 (Usiskin, 1982; Mayberry, 1983; Burger & Shaughnessy, 1986; Senk, 1989; Teppo, 1991; Mason, 1998).

Clements, Swaminathan, Hannibal and Sarama (1999) point out that van Hielian research was grounded in educational concerns but did not deal with young children. In the original theory and in most of the subsequent research, the focus has been on students in middle school and beyond. But Piagetian studies have not been grounded in educational concerns and it appears that there are present at an early age, certain Euclidean notions such as duplicating and recognising Euclidean features and even preschool children should be able to work with such geometric ideas.

2.8. Teaching implications and instructional ideas of the van Hiele theory

"Empowering students with methods by which they can establish for themselves mathematical truth, and thus, helping students develop intellectual autonomy is a critical goal of geometry instruction and indeed of all mathematics instruction".

(Clements & Battista, 1992, p.457)

As suggested by Clements and Battista (1992), research that describes the development of geometric concepts and thinking in various instructional environments is required. Based on more than five decades of research, it is evident that the van Hiele theory is a well structured and well known theory which has its own reasoning stages and instructional phases in geometry.

Among the variables that affect student learning, researchers have suggested that the educator has the greatest impact on students' motivation and mathematics learning (Halat, 2008). According to the van Hieles, progress from one level to the next depends little on biological maturation or development, but it proceeds under the influence of a teaching and learning process. The teacher plays a special role in facilitating this progress, especially in providing guidance about expectations (Fuys, et al., 1988).

According to van de Walle (2001), the van Hiele theory provides the thoughtful teacher with a framework within which to conduct geometric activities. Even though the theory does not specify the content or the curriculum, it can be applied to most activities. The activities can be designed beginning with the assumption of a particular level and then be raised or lowered by means of the types of questioning and guidance provided by the teacher. The junior secondary school geometry activities should be informal. Those activities should be exploratory and hands-on in order to provide learners with the opportunity to investigate, to build and take apart, to create and make drawing, and to make observations about shapes in the world around them (van de Walle, 2001).

Teppo (1991) explains that systematic geometry instruction in the middle grades is necessary to prevent students from entering high school at low levels of geometric concept development. Systematic geometry instruction would engage students in sequential learning activities during the middle grades that would help students enter high school geometry at a level at which they can comprehend the material, and be prepared to learn deductive geometric proof (Genz, 2006).

Van Hiele suggests that learners must pass through lower levels of geometric thought before they can attain higher levels and instruction should help students through this gradual progress which takes considerable amount of time (Battista & Clements, 1995). The theory furthermore suggests that students can understand and explicitly work with axiomatic systems only after they have reached the highest level and implies that the explicit study of axiomatic systems is unlikely to be productive for the vast majority of students in high school geometry.

Malloy (2002) states that in implementing instruction based on the van Hiele framework, teachers need to recognise and understand the van Hiele levels of their students, they need to help their students' progress through these levels in preparation for the axiomatic, deductive reasoning that is required in high school geometry. Senk (1989) states that much of the students' achievement in proof writing is directly controlled by the teacher and the curriculum.

Burger and Shaughnessy (1986) also claim that the quality of instruction is one of the greatest influences on the students' acquisition of knowledge in mathematics classes. The students' progress from one reasoning level to the next also depends on the quality of instruction more than other factors, such as students' age, environment, and parental and peer support (Crowley, 1987; Fuys, et al., 1988).

Malloy (2002) suggests that in the middle grades, the five instructional phases of the van Hiele theory can be applied to teaching geometry. First, students gather information by working with examples and non-examples of concepts. Then they are provided with appropriate information and complete tasks that develop relationships. Then they become aware of the relationships and explain them using appropriate geometric language. Fourth, they complete additional, more complex tasks to build their understanding of the relationships. Finally, they summarise what they have learned and reflect on it. The five phases occur within each level as students move from one level to the next.

The theory also highlights the necessity of teaching at the learner's level. According to van Hiele, the two major factors that determine a learner's thinking level are ability and prior geometry experience. A learner's response to questions about a topic will provide assessment information about what a learner currently knows about that specific topic. If the learner has little experience with the topic being assessed, it may not give an accurate assessment. Therefore the assessment must focus on progress that a learner might make within a level, or possibly to a higher level as a result of instruction (Fuys, et al., 1988). Usiskin (1982) has found that many students fail to grasp key concepts in and leave their classes without learning basic terminology.

Clements and Battista (1992) propose the following for educational goals for the levels of thinking:

- Opportunities for the construction of ideas should be offered early.

- Level 2 should be attained by the end of the primary grades.

- Instruction should carefully draw distinctions between common usage and mathematical usage.
- Students should manipulate concrete geometric shapes and materials so that they can 'work out geometric shapes on their own. It is imperative that teachers should not rely solely on the text.

Again, Battista and Clements (1995) suggest the following for a secondary school geometry curriculum:

- It should be appropriate for the various thought levels and the students should be guided to learn about significant and interesting concepts.

- It should allow students to use visual justification and empirical thinking to attain higher levels of thought.

- It should encourage students to refine their thinking so that they discover and begin to use some of the critical components of formal proof.

De Villiers (2010) developed an analogy to the van Hiele model of geometry instruction for different branches of mathematics by different researchers. For example, for Boolean algebra, Calculus, Trigonometry and Abstract algebra it is developed as:

Table 2.5: Analogy to the van Hiele model of geometry instruction

Area / Level	Boolean Algebra	Calculus/ language about functions	Trigonometry	Abstract Algebra
Level 1	Interpretation and representation of switching circuits	Level of everyday language	Visualisation	Perceptual
Level 2	Analysis of switching properties	Level of arithmetic	Analysis	Conceptual
Level 3	Logical implication: Deduction	Level of Algebra and Geometry	Primitive definition	Abstract
Level 4		Level of Calculus	Circle definition	
Level 5		Level of Analysis	Spherical Trigonometry	

2.9. A critique of the van Hiele theory

"Being critical of a theory is only meaningful if one agrees with the greater part of it".

(van Hiele, 1986, p viii)

As seen in the previous sections, the van Hiele theory has precipitated a large body of work using, evaluating and modifying the theory itself. It has deepened and expanded research in the learning and teaching of geometry. It has served as a theoretical backbone in a wide range of related topics. It is a model of connections among theory, research, practice of teaching and students' thinking and learning.

Because of its wider applicability in any imaginable field of teaching and learning, it has gathered some responses from researchers around the world.

2.9.1. Criticism on the levels of thinking

One of the areas of the van Hiele theory that has been widely researched is the levels of thinking. Depending upon the topics and the subjects of research many research came up with different inferences. The following questions are the most frequently contested against the original theory as suggested by Clements and Battista (1992).

1. Do the van Hiele levels accurately describe students' geometric thinking and are the levels discrete? – Researchers have validated that the van Hiele levels accurately describe students' geometric thinking and describe geometric development through both interviews and tests (Usiskin, 1982; Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988; King 2003; Halat, 2006; Genz, 2006; Atebe, 2008).

According to van Hiele (1986, p.49), "The most distinctive property of the levels of thinking is their discontinuity, the lack of coherence between their networks of relations." Wirszup (1976) and Hoffer (1983) ascertain that the levels are discrete and there is a discontinuity between the levels. This claim was contested by Burger and Shaughnessy (1986) who argued that the levels are not necessarily discrete. This was because Burger and Shaughnessy's study failed to detect the discontinuity and found instead that the levels appear dynamic rather than being static and of a more continuous nature than their discrete descriptions would lead one to believe. Burger and Shaughnessy (1986) further explain that their study has found that students may move back and forth between the levels quite a few times while they are in transition from one level to the next. This means that students can be in transition between these levels and that they will oscillate during the transition period (Pusey, 2003). But Usiskin (1982) and Fuys et al. (1988) reported that students in transition are difficult to classify reliably. Clements and Battista (1992) report that there was instability and oscillation between the levels and continuity rather than jumps in learning was frequently observed by researchers (like Fuys, et al., 1988). This will pose a difficulty in assigning a level to students who do not seem to fit a particular level or are in transition. A recent South African study (Atebe, 2008) had assumed discreteness to be an attribute of the van Hiele levels.

2. Are there five different levels? – Fuys, et al. (1988) support the characterisation of the model in terms of three levels as visual (previously, 1) analytical (previously 2) and theoretical (previously 3-5). According to Clements and Battista (1992), van Hiele agrees with this interpretation but they caution that the three level model may not be sufficiently refined to characterise thinking. The existence of level 5 is also contested as Usiskin (1982, p.47) expresses that "level 5 is of questionable testability".

3. Do students reason at the same levels across topics? – Mayberry's (1983) study on pre-service elementary teachers revealed that her participants were on different levels for different concepts. Wu and Ma (2006) also reported that the study on elementary students from grade 1 to grade 6 revealed that students were on different levels for different concepts. Burger and Shaughnessy (1986) also reported that students were on different levels on different tasks. Some even oscillated from one level to another on the same task. Therefore, it can be characterised that the levels can be dynamic rather than static and of a more continuous nature than their discrete descriptions (Clements & Battista, 1992). Fuys, et al. (1988) also found that a significant number of participants in their study made some progress towards level 2 with familiar shapes such as squares and rectangles, but encountered difficulties with unfamiliar figures. This made them conclude that progress was marked by frequent instability and oscillation between levels. The study carried out by Mason (1998) also highlighted that some mathematically talented students appeared to skip levels.

Clements and Battista (1992) further stated that researchers hypothesised that as students develop, the degree of the global nature of the levels is not constant, but it increases with level. That is, as children develop, they grasp increasingly large localities of mathematical content and thus understand larger areas of mathematics.

4. **Do the levels form a hierarchy?** – Wirszup (1976) reported that development through the hierarchy appears to proceed under the influence of a teaching learning process. Clements and Battista (1992) stated that the levels seem to be hierarchical and Mayberry's (1983) research affirmed it.

De Villiers (1994, p.17) challenged the van Hiele's claim that class inclusion can only be at level 3, where he explains that "dynamic geometry contexts can facilitate the grasping of class inclusion even as early as level 1". For example, De Villiers (1987), states that students can see a square as a special rectangle at level 1 by simply dragging the rectangle until it becomes a square. According to De Villiers (2010), a serious shortcoming of the van Hiele theory is that there is no explicit distinction between different possible functions of proof such as explanation, discovery and verification and with geometer sketchpad learners can be engaged in proof at levels lower than level 3. Moreover, it seems that a prolonged delay at levels 1 and 2 before introducing proof actually makes the introduction of proof as a meaningful activity later even more difficult (De Villiers, 2010).

5. Does level 0 exist? – Research has reported that learners do not achieve level 1 and they were classified such as 'not yet at level 1' or 'weak level 1', researchers have suggested the introduction of another level, more basic than van Hiele's visual level, called level 0 (pre-recognition level). Wu and Ma (2006) reported that the study on elementary students from grade 1 to grade 6 revealed that some students did not reach visual level of basic figures. Halat (2007) pointed out that even though the existence of level 0 was the subject of some controversy (e.g., Usiskin, 1982; Burger and Shaughnessy, 1986), van Hiele (1986) himself did not talk and acknowledge the existence of such a "non-level". Instead, he asserted that all students enter at ground level, that is, at level 1, with the ability to identify common geometric figures by sight. But Usiskin's (1982) research project had shown that level 0 existed and that the learners who began at level 0 remained at level 0 at the end of the year. Senk (1989) reported that students who entered a geometry course at level 1 performed significantly better at writing proofs than those who entered at level 0. Clements and Battista (1992, p.429) pointed out that "the bulk of the evidence from the van Hiele-based research along with research from the Piagetian perspective, indicated the existence of thinking more primitive than, and probably prerequisite to, van Hiele's level 1". They named this level 0 as "pre-recognition". They defined it as "children initially perceive geometric shapes, but may attend to only a subset of a shape's visual characteristics and they are unable to identify many common shapes" (p.429). Clements and Battista (1992, p.429) state that "the issue is not resolved".

6. Is there an existence of linguistic property? – Clements and Battista (1992) state that the existence of the unique linguistic structures at each level has been supported (for example, Mayberry 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988).

7. What are the other characteristics that are to be considered? – According to the theory, students progress through levels of thought in geometry (van Hiele, 1959/1986). Thinking develops from a level driven by visual patterns through increasingly sophisticated levels of description, analysis, abstraction and proof (Clements & Battista, 1992). According to Clements, Battista and Sarama (2001), it may not be viable to conceptualise a purely visual level, followed and replaced by a purely verbal descriptive level and so on. This is a perspective commonly taken in some discussions of the van Hiele theory. They further suggest that different types of reasoning can coexist in an individual and develop simultaneously but at different rates and along different paths, where each path leads to slightly different combinations of multiple types of knowledge. This view implies a different conceptualisation of levels of geometric thinking and of students' development of these levels (Clements, et al., 2001)

Research by Senk (1989) points out that a proof oriented geometry course requires thinking at least at level 3 in the van Hiele hierarchy. Her research proved that at van Hiele level 4 students master proof, with level 3 being a transitional level.

The students' thinking may not be characterised 'at' a single level. Gutierrez et al. (1991), attempted to take into account students' capacity to use each van Hiele level rather than assign a single level.

Usiskin (1982) has found out that assigning a level to a student is problematic as the level depends upon the criteria used (see section 3.7.1.3 in Chapter 3). This means that a student's level may change even when the questions are not changed. This seems to be a problem because 'if the theory is assumed, a student should have only one level' (Usiskin, 1982).

Van Hiele claims that "a new level cannot be achieved by teaching but through a suitable choice of exercises" (van Hiele, 1986, p.39). But it seems that van Hiele is contradicting himself, as later in the same book it was mentioned that "a teacher beginning the teaching of geometry at a level that the learners are operating will inspire their confidence and the learners will try to understand the teacher" (van Hiele, 1986, p.45). Senk (1989) states that much of the students' achievement in proof writing is directly controlled by the teacher and the curriculum. Burger and Shaughnessy (1986) also claim that the quality of instruction is one of the greatest influences on the students' acquisition of knowledge in mathematics classes. The students' progress from one reasoning level to the next also depends on the quality of instruction more than other factors, such as students' age, environment, and parental and peer support (Crowley, 1987; Fuys, et al., 1988).

2.9.2. Criticism on the phases of instruction

Very little research has been conducted in this area of the van Hiele theory. However, the research conducted raised some concerns and are noted by Clements and Battista (1992) as:

1. How are the phases related to the levels of teaching? – Supposedly, the teacher should lead the students through all the five phases to reach each new level. However, certain phases appear to require of students certain types of thinking that are bound to a given level.

2. Should the teacher attempt to proceed linearly through the phases or approach them as recursive within each level and should the teacher introduce many concepts and guide the students through the levels on each of them in parallel or work through the levels with a single concept? According to Ding and Jones (2007), it is not obvious whether it is necessary for a teacher to go through each and every single phase.

3. The final phases seem to enhance transfer and does the transfer be aided through the provision of systematic spaced reviews? Ding and Jones (2007) comments that the existing van Hiele-based research has yet to address systematically any of the issues concerning the nature and specification of the teaching phases.

Malloy (2002, p.3) suggests that "in the middle grades, the five instructional phases can be applied to teaching geometry". Clements and Battista (1992), state that one study indicated that 20 days of phase-based instruction significantly raised high school students' van Hiele level of thought more from level 1 to level 2 than for any other level , but it did not result in greater achievement in standard content or proof writing. Atebe (2008, p.66) also raises the same concern over the number of lessons to be observed, "as according to Dina van Hiele-Geldof, as many as 50 lessons are needed to move learners from level 2 to level 3".

Van Hiele, (1986, p.53) talks about five "stages" of a learning process as information, guided orientation, explicitation, free orientation and integration. But in the same book, on page 96, he further talks about five "phases" of learning process with the same terms. This creates some confusion as many researchers use these terms under the "phases of instruction". Ding and Jones (2007) also support the idea of Clements and Battista (1992, p 434) that due to the lack of research, many issues remain unclear, including how the phases of teaching relate to the subject matter and the students' prior attainment, whether a different emphasis on particular phases depends on what is being taught and so on. Additional research are needed for the above unresolved questions and concerns regarding the phases so that it can be modified.

Thus the theory has proved that it is useful because it was used, contested, attacked and modified. As Clements stated in 2004, by this criterion, I also acknowledge that van Hiele's theory is a useful theory.

The following section gives a brief description of the present study.

2.10. The present study

The National Council of Teachers of Mathematics (NCTM), in its Curriculum and Evaluation Standards for School Mathematics (1989, p.48), states that "spatial understandings are necessary for interpreting, understanding and appreciating our inherently geometric world". Research has documented that learners are failing to learn basic geometric concepts and geometric problem solving skills (Usiskin, 1982; Mayberry 1983; Burger, 1985; Fuys et al. 1986; Renne, 2004)

In addition to this, "many teachers teach only a portion of the geometry curriculum that is available to them" (Clements, et al., 2001, p. 2). Many believe that our senior secondary school learners are underprepared for their geometry curriculum. For example, Usiskin (1982, p.29) mentioned about U.S. curriculum as "there is no geometry curriculum at the elementary school level. As a result, students enter high school not knowing enough geometry to succeed. There is a geometry curriculum at the secondary level, but only about half of the students encounter it, and only about a third of these students understand it".

As stated in Chapter 1, the South African education system also has the same challenges. In senior secondary schools learners are expected to work with (1) a wide range of patterns and transformations and solve related problems and (2) describe, represent and analyse shape and space in two and three dimensions using geometry and trigonometry to achieve the learning outcome of space and shape. This curriculum for geometry consists of a mixture of unrelated concepts with no systematic progression to higher levels of thinking that are required for sophisticated concept development and substantive geometric problem solving.

The aim of this study is to address the deficiencies in the current senior secondary school geometry instruction by developing a research based instructional framework based on the theoretical framework of van Hiele. Many researchers have used this theory to develop research based curriculum, for example, using graphics based computer programming language, Logo (Clements, et al., 2001) and geometer's sketch pad (De Villiers, 1997). A brief description of both instructional curricula is given below.

2.10.1. Logo Geometry Project

The goal of the Logo Geometry Project, based in the U.S., was to create, implement, and assess a research based curriculum that used logo turtle graphics to develop elementary students' geometric competencies (Clements, et al., 2001; Fraser, 2004). The study used the theoretical frameworks of Piaget and van Hiele. It was spanned over four years to finish, where the background research was conducted, a new curriculum was developed, logo activities were constructed and teacher materials were developed during the first year and in the second year the logo curriculum was field tested. During the third year evaluations were conducted and in the fourth year the materials were revised and published (Clements, et al., 2001). The study found that the curriculum based on logo has a positive effect on student learning (Fraser, 2004). Clements, et al. (2001,) confirm that more recent reviews conducted after their research project was completed generally had been positive. Shields (2002) reviewed the research project and commented that in spite of all these strengths, it is limited only to elementary classroom application. Karakirik and Durmus (2005) comment that proper logo environments may help students make the transition from the visual to the descriptive level of thought in the van Hiele hierarchy.

2.10.2. Geometer's sketchpad

The geometer sketchpad or Cabri are incredible computer programmes for exploring geometry (De Villiers, 1998). It is a dynamic geometric construction kit which takes full advantage of the mouse interface of the Macintosh computer (Olive, 1991). With geometer's sketchpad explorations of the properties of triangles, quadrilaterals, circles and other configurations are very easy. According to De Villiers, it allows the learners to dynamically transform their figures while preserving the geometric relationship of their constructions. Generalisations are easily possible and it is useful for learning proofs. It encourages the process of discovery where learners first visualise and analyse a problem and make conjectures before attempting a logical explanation of why their observations are true (De Villiers, 1998). Even though geometer sketchpad promotes learning of definition of concepts and geometric concepts through construction and measurement, one of the things that De Villiers (2010) has pointed out that certain kinds of construction activities with dynamic geometry software or by pencil and paper are inappropriate at van Hiele level 1 as the learners at level 1 do not yet know the properties of different figures. In dynamic geometry software, at level 1, it would be appropriate to provide children with ready-made sketches of quadrilaterals which they can then easily manipulate and first investigate visually and then they should start using the measure features of the software to analyse the properties to enable them to reach level 2. Olive (1991) also points out that even though it is possible to develop dazzling, animated demonstrations with the sketchpad, the learning power will come from the learners' own attempts at making constructions and investigating the dynamic relationships in ways that they find meaningful.

2.10.3. The present study and the instructional framework

Under the subsections 2.10.1 and 2.10.2 in this chapter the overview of different instructional curricula were given.

It is important that, in order to fully understand the place of this study, a brief overview of the South African schools is needed and it is given below.

Feza and Webb (2005) cite earlier studies (like Davies, 1986; Samuel, 1990 and Hartshorne, 1992) to point out that, in South Africa in 1948, a system of 'Bantu Education' was introduced for black people based in the homelands and they were taught with a different and inferior curricula, usually with no maths or science. But the adoption of the Constitution of the Republic of South Africa (Act 108 of 1996) and the amendments that followed provided a basis for curriculum transformation and development in South Africa (DoE, 2003a). "In the post-apartheid society, social transformation in education is aimed at ensuring that the educational imbalances of the past are redressed and that equal educational opportunities are provided for all sections of our population" (DoE, 2003a, p.2). The Department of Education introduced the present curriculum, known as the National Curriculum Statement, which is modern and internationally benchmarked, into grade 10 in 2006. It requires the learners to do seven subjects in grades 10 to 12 of which mathematics or mathematical literacy is a compulsory subject. This is to ensure that all learners are prepared for life and world in an increasingly technological, numerical and data driven world (Pandor, 2006). Since, mathematics or mathematical literacy was not a compulsory subject in the previously disadvantaged (formerly black and coloured) South African schools and having many of its teachers are the products of Bantu Education, the impact of the Bantu system can be seen even today (Feza & Webb, 2005). Mji and Makgato (2006) state that even the schools that offer mathematics and science do not have facilities and equipments to promote effective teaching and learning. South Africa faces the challenge of providing quality mathematics education for its multi-cultural society of 43 million people (Howie, 2003). Lack of appropriate learner support materials, general poor quality of teachers and teaching are some of the factors that have contributed to the lacking in the necessary informal mathematical knowledge of disadvantaged learners from the impoverished learning environments (Maree, Aldous, Hattingh, Swanepoel & van der Linde, 2006). These have resulted in apparent lack of exposure to mathematics in these under resourced schools.

To highlight the effect of this, the matric pass rate of mathematics for the past three years is shown below:

Table 2.6: The percentage of mathematics passes in the National level and in Eastern Cape

Mathematics passes	Percentage achieved		
	2008	2009	2010
National level	30,5%	30,1%	30,9%
Eastern Cape	22,2%	21,3%	21,3%

The first cohort of the NCS Curriculum which was started in Grade 10 in 2006 wrote their National Senior Certificate (NSC) Exam in 2008. In the national level, mathematics was the worst performed subject in 2008 and in 2010 and the second last in 2009. Eastern Cape is the worst performing province in all three years and is below the national percentage (Department of Basic Education, 2010). According to Mji and Makgato (2006), the Third International Mathematics and Science Study (TIMSS) in 1995 in which South Africa participated with 41 other countries, South African learners came last with a mean score of 351 against the international bench mark of 513 and less than 2% of these learners reached or exceeded the international mean score and a later study in 2003 by TIMSS – R indicated no improvement by South African mathematics and science learners. The TIMSS-R 1999 study also revealed that the South African learners struggled to deal with word problems and experienced great problems with fractions and sums in which geometry had to be used to calculate area (van der Walt & Maree, 2007). Of the South African participants, learners from Eastern Cape Province ranked 7th out of the 9 provinces (Atebe & Schafer, 2008). This is indeed a cause for concern.

The present study was conducted in a previously disadvantaged area of the Eastern Cape Province. The study was undertaken in an effort to improve the geometric understanding and, consequently, the performance and achievement of senior secondary school learners in geometry in particular and mathematics in general. Even though the logo and the geometer sketchpad are found to be effective in improving the geometric understanding of learners by many researchers (Battista & Clements, 1995; De Villiers 1998/2010; Clements, et al., 2001), it may not be possible to have access to computers to all our learners in our South African Schools, where the majority of its learners and schools are in the previously disadvantaged areas. The schools that have been selected for the study represent these schools. So, given the setting of this study and the wider application of it, the use of 'hands-on' and practical approach to use manipulatives and worksheets to improve the geometric understanding was tried and tested in this study. The sequential and hierarchical order of concepts will presumably close the gap in the insufficient preparation of learners from the junior schools as they enter the senior secondary schools. The activities that are designed can be implemented without expensive or sophisticated materials. The process of gradually moving from the concrete and active to abstract and more passive learning under the guidance of the educators will make the learning of geometry more relevant and enjoyable for our learners within the limited financial and underprivileged circumstances. A deep sense of concern on the wellbeing of these underprivileged learners was put in at the heart of this study. An elaborate description of the framework is explained in the next chapter.

2.11. Chapter summary

In this chapter, the theory underpinning the study was discussed in detail. All aspects of the theory such as, the characteristics, features and properties of the van Hiele levels, the van Hiele phases and its implications in geometry teaching, and the importance of language in the van Hiele theory were discussed in the first half of the chapter. Then the empirical research on the van Hiele theory was also discussed. In the later part of the chapter, the comparison of the van Hiele theory with other theories like Piaget's developmental theory, teaching implications of van Hiele levels, instructional ideas of the van Hiele theory and a critique of the van Hiele theory and the present study were discussed.

In the next chapter, the methodology and the instructional framework are discussed in detail.

CHAPTER 3
THE METHODOLOGY

3.1. Introduction

This chapter explains and describes the research processes involved in this study. A brief overview of the study and the research methodology are explained in this chapter. McMillan and Schumacher (2006) state that the methodology explains the general plan of the research, how the research is set up, what happens to the subjects and what methods of data collection are used. The first part of the chapter gives a detailed description of the instructional framework and the second part of the chapter gives the description of the methodology in detail. The reliability and validity of each instrument is discussed under each section of the particular instrument. The whole research was a complex process spanning three years of intensive study from January 2009 to January 2012.

This study was undertaken primarily to develop an instructional framework in line with the van Hiele levels in order to improve the van Hiele levels of learners. Many studies conducted in different parts of the world (Hoffer, 1981; Usiskin, 1982; Senk, 1985; Shaughnessy & Burger, 1985; Fuys, et al., 1988; Clements & Battista, 1992) highlighted that learners' poor performance in geometry holds account for geometry classroom teaching and learning. Atebe (2008) in South Africa also stressed the need for effective classroom teaching. Van Hiele (1986) believes that students' difficulty with school mathematics generally and geometry in particular is caused largely by teachers' failure to deliver instruction that is appropriate to the learners' geometric level of thinking. To improve geometry teaching significantly, teachers need tasks that help them better understand the nature of their students' geometric reasoning and they need to know what research says about such reasoning (Battista, 1999). In many western countries, the van Hiele theory has become the most influential factor in their geometry curriculum (Fuys, et al., 1988; van de Walle, 2004), but only a few studies have utilised this instructional model in the South African context. In South Africa, research aimed at improving the learning and instructional strategies need to be developed and this present study was aimed at looking into the possibilities of improving the geometry education by introducing van Hiele-based instruction after determining the level of geometric thinking.

The other aspects of South African schools and its geometry curriculum deficiencies (see section 2.10.3 in Chapter 2) are considered and served as the rationale behind the development of the instructional framework.

3.2. The instructional framework

A framework can be defined as a hypothetical description of a complex entity of process or the underlying structure or a structure supporting or containing something. Instruction refers to those curriculum-related, professionally-informed decisions that teachers purposefully enact to enhance learning opportunities for students. Effective instruction is interactive and designed to accommodate student learning needs and styles through a variety of teaching practices.

An effective instruction is guided by general pedagogical approaches and specific instructional practices. An effective instruction:

- is eclectic.
- is tied directly to the success of the learning experience.
- is empowered professional practice in action.
- integrates the components of the Core Curriculum.
- is generative and dynamic.
- recognises there is an art as well as a science to teaching.
- acknowledges a comprehensive understanding of the instructional cycle.
- finds best expression when educators collaborate to develop, implement, and refine their professional practices.

Therefore, an instructional framework can be regarded as a series of processes (or practice) of maximising the effectiveness, efficiency and appeal of instruction and other learning experiences. The process consists broadly of determining the current state and needs of the learner, defining the end goals of instruction, and creating some "intervention" to assist the transition. (sources: http://mag.ofi.hu/instructional-approaches & http://en.wikipedia.org/wiki/Instructional_Design) retrieved from internet on 5/6/2011)

For the present study, an instructional framework was developed based on the van Hiele levels and the implementation of it in the classroom teaching is mentioned as the van Hiele levels-based instruction.

3.2.1. The ideas utilised for the development of the instructional framework

"Children whose geometric thinking you nurture carefully will be better able to successfully study the kind of mathematics that Euclid created".

(van Hiele, 1999, p.316)

This section discusses the ideas utilised for the instructional framework which was developed in order to raise the van Hiele levels of school children. The van Hiele's model is utilised for identifying the levels of learners' thinking, designing the instruction for their particular levels and helping them to advance to the next levels. Malloy (2002) states that in implementing instruction based on the van Hiele framework, teachers have two tasks. Firstly, the teachers need to recognise and understand the van Hiele levels of their students, and secondly, they need to help their students' progress through these levels in preparation for the axiomatic, deductive reasoning that is required in high school geometry. Again, in our present curriculum, the learning outcomes and the assessment standards for geometry curriculum are closely linked to the NCTM's standards for geometry and the teaching principle of NCTM. It states that "effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000, p.12). This forms the basis for the development of the instructional framework tried and tested in the current study.

Van Hiele (1999) points out that school geometry has been following the axiomatically fashioned Euclidean geometry and this requires the learners to think on a formal deductive level. This is not usually the case and they lack the prerequisite understandings about geometry and this lack creates a gap between their level of thinking and that required for the geometry that they are expected to learn.

One of the characteristics of the van Hiele levels is that geometric experience is the greatest single factor that influences the advancement through the levels. Activities that permit children to explore, talk about and interact with content at the next level, while increasing their experiences at their current level, have the best chance of advancing the level of thoughts for those children and the van Hiele theory does not tell us what content to teach, but it does provide the thoughtful teacher with a framework in which to conduct geometric activities (van de Walle, 2004).

Van Hiele believes that development of learners' level of thinking is more dependent on instruction than on age or biological maturation and those types of instructional experience can foster, or hamper development (van Hiele, 1999). Instruction intended to foster development should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and concluding in summary activities that help learners assimilate what they have learned into what they already know (van Hiele, 1999). Rich and stimulating instruction in geometry can be provided through playful activities with mosaics and tangram puzzles (van Hiele, 1999). Children should be given ample opportunity for free play and for sharing their creations. Such play gives educators a chance to observe how children use the pieces and to assess informally how they think and talk about shapes. In solving puzzles, children work visually with angles that fit and sides that match. Children who use triangle grid to record solutions to puzzle become aware of equal angles in the grid and also of parallel lines. Activities using paper folding, drawing and pattern blocks can enrich children's store of visual structures. They also develop knowledge of shapes and their properties (van Hiele, 1999).

According to van Hiele (1999), to promote the transition from one level to the next level should follow a five phase sequence of activities.

- Instruction should begin with an **inquiry** phase in which materials lead children to explore and discover certain structures.
- In the second phase, **direct orientation**, tasks are presented in such a way that the characteristic structures appear gradually to the children.
- In the third phase, **explicitation**, the teacher introduces terminology and encourages children to use it in their conversation and written work about geometry.
- In the fourth phase, **free orientation**, the teacher presents tasks that can be completed in different ways and enables children to become more proficient with what they already know.
- In the fifth phase, **integration**, children are given opportunities to put together what they have learned.

As mentioned earlier in section 2.4 in Chapter 2, throughout these phases, the teacher has various roles like planning tasks, directing children's attention to geometric qualities of shapes, introducing terminology and engaging children in discussions using these terms and encouraging explanations and problem solving approaches that make use of children's descriptive thinking about shapes. Cycling through these five phases with materials like the mosaic puzzle enables children to build a rich background in visual and descriptive thinking that involves various shapes and their properties (van Hiele, 1999). Groth (2005) suggests that the use of the five phase framework can result in positive learning outcomes for students and teachers alike.

3.2.2. Characteristics of the instructional framework

According to van Hiele (1986), a suitable choice of exercises can create a situation for the learner favourable to the attainment of the higher level of thinking. A teacher beginning the teaching of geometry at a level that the learners are operating will inspire their confidence and the learners will try to understand the teacher (van Hiele, 1986). Therefore, the instructional framework developed by the present study provides activities starting at the visual level with an introductory game. According to van de Walle (2004), activities on grid papers are second best alternative to real physical objects and this helps the learners to do spatial explorations easily. The framework also has activities where the learners have to do the identification of different geometric shapes and figures from a collection of triangles and to recognise similarity between shapes in a grid paper. The other activities in the framework included a lot of sorting, identifying, and describing a variety of shapes. The framework also took note of the suggestion that spatial sense is enhanced by an understanding of shapes, what they look like, and even what they are named and the concepts of symmetry, congruence, and similarity will contribute to understanding our geometric world as suggested by van de Walle (2001).

3.2.3. Content of the instructional framework

According to van Hiele, the level at which the teaching should begin depends on the level of thinking of the student (van Hiele, 1999). In order to design appropriate teaching materials, a pretest was administered to the participants, analysis of whose result indicated that the majority of them were at van Hiele level 0 (see Chapter 4, section 4.2.4). Due to the fact that most of the learners in the study were at level 0, the instructional framework was designed in such a way that it has activities starting from lower levels to higher levels of thinking in a successive manner. The choice of the activities, to a large extent, was informed by the characteristics of the type of thinking level that occur in the levels 1, level 2, level 3 and to a certain extent at level 4 of the van Hiele levels. In this study, level 4 activities included only stating definitions and making simple inferences and deductions. Much of the literature available do not give specific activities for level 4 as they claim that high school learners do not reach levels 4 and 5 (e.g., IMAGES, 2009; Meng, 2009). In the present study also, it was noted that none of the learners were at level 4 and only 1% was at level 3 (see Chapter 4, section 4.2.4). Even in Dina van Hiele's (1959) project and Fuys, et al.'s (1988) project, students' thinking levels at 4 and 5 were not directly observed (Fuys, et al., (1988).

The instructional activities explained in the following section were greatly influenced by Fuys, et al.'s instructional modules described in the book entitled "The van Hiele Model of Thinking in Geometry among Adolescents" (1988). This particular instructional model was chosen because it had been constructed in consultation with van Hiele and other mathematics educators and researchers working in the field. For the present framework, the construction of the items from different sources like textbooks, worksheets and past research materials and the adaptation of these items after piloting were done in consultation with my supervisor, a researcher (Atebe) on van Hiele theory in the field of geometry, the educators in my school and the educators from the participating schools. However, modifications still had to be incorporated and the actual framework implemented and the reasons for such modifications are cited in Chapter 4 under focus two.

3.2.4. Activities that comprised the instructional framework

Two main categories of activities were designed for the framework namely, informal introductory activities and hands-on approach activities.

Informal introductory activities

The activities were designed to reflect the general structure of the van Hiele levels. The instructional framework opened with an introductory game as an icebreaker activity followed by identifying shapes in pictures and tangram puzzles. These informal activities were presented to provide a non-threatening context for beginning the topic. Sufficient varieties of examples of shapes were given so that irrelevant features become unimportant. Ample opportunities were given to draw, build, make, put together and take apart shapes. These activities provided the learners with the opportunity to develop an understanding of geometric properties and begin to use them naturally.

Hands-on Approach

Activities were two types: dynamic (moving objects, cutting and folding and constructing) and static as in textbooks with verbal information. Cut-out shapes were given for the geometric items sorting activity where the focus was more on the properties of figures rather than on simple identification. As the geometric concepts were learned, connection between shapes and the relationships between shapes were introduced through lots of sorting. Classes of figures were analysed to determine new properties. Circles were constructed with compasses and guided the learners to discuss the terminology associated with circles. Symmetry was introduced through folding shapes. Conjecturing in plane geometry was an investigative approach to explore the understanding of the properties of figures by constructing figures. This encouraged the making and testing of hypothesis or conjectures. The learners examined the necessary and sufficient conditions for shapes and concepts.

In a nutshell, the following are the suggestions for the activities for the first three of the van Hiele levels as suggested by Fuys, et al. (1988), van de Walle (2004), and IMAGES (2009) and for the activities for level 4 as suggested by Fuys, et al. (1988). These suggestions were incorporated into the activities developed by the instructional framework (see section 3.2.6).

Level 1 activities

Activities at this level should include a lot of sorting, identifying, and describing a variety of shapes. Learners should be given lots of physical models that can be manipulated by them. Learners should be provided with opportunities to build, make, put together and take apart shapes. Learners should be seeing different sizes and orientations of the same shape as to distinguish characteristics of a shape and the features that are not relevant.

Level 2 activities

Educators must use problem solving contexts in which properties of shapes are important components. Define, measure, observe and change properties with the use of models. Educators need to classify figures based on properties of shapes as well as by names of shapes.

Level 3 activities

The focus should be on defining the properties, still using the models. It is advisable to make property list and discuss which properties are necessary and which are sufficient conditions for a specific shape or concept.

Level 4 activities

The activities at this level should focus on exploring learners' abilities to state definitions, make simple inferences and deductions and formulate conjectures.

3.2.5. Different activities in the framework

The activities in the framework are summarised as follows:

1. Introductory game
2. Shapes in pictures
3. Tangram puzzles
4. Geometric items sorting activity
5. The family of quadrilaterals
6. Discovering with folding: Paper Shapes
7. Drawing & Construction: Circle
8. Conjecturing in plane geometry

Activity 1: Introductory game

This introductory game is to create a relaxed atmosphere for the educator and the learners to communicate and to informally assess the learners' mathematical language.

For this game, pairs of shapes are presented to the learners. For each pair, the educator says something that is the same about them and the learners say something that is different. The roles are reversed for the next pair. This is repeated for seven pairs.

In this activity, educators are to watch the characteristics of learners such as their ability to initiate ideas or copy ideas. Learners should be given as much opportunity as possible to introduce their own geometric vocabulary spontaneously. No formal vocabulary is supposed to be introduced by the educators.

Activity 2: Shapes in pictures

This activity is to assess learners' familiarity with some basic geometric concepts such as concepts of shapes: triangle, square, rectangle, trapezium and circle. Learners were asked to look at the picture of a building, to find the geometrical shapes, with which they are familiar, in the picture and name them. They were asked to make rough sketches, of the shapes they have recognised on squared paper.

Activity 3: Hidden geometric shapes

This activity is to help the learners with the identification of different geometric shapes and figures from a collection of triangles and for recognising similarity between shapes identified in grid. They were asked to draw any interesting shapes, but focusing on geometric shapes they can "see" hidden in the triangular grid paper. They were asked to use the different colour crayons given to them and then share their discoveries with a partner and, thereafter, with their group. They become aware of equal angles in the grid and also of parallel lines. Trapeziums, which have one pair of parallel sides, can be introduced through this activity.

Activity 4: Tangram puzzles

This activity is to informally assess the understanding of area and to give some experiences in decomposing shapes into other shapes in order to compare areas and provide some information about the learners' visual abilities. The learners have to cut out the seven pieces of the tangram and make the shapes given on the worksheet.

The development of this activity is based on van Hiele's (1999) writing on "Developing Geometric Thinking through Activities That Begin with Play". Van Hiele states that rich and stimulating instruction in geometry can be provided through playful activities like tangram puzzles. This is a playful exploration that deals with certain shapes and their properties, symmetry, parallelism and area. "In solving puzzles like these, children work visually with angles that fit and sides that match" (van Hiele, 1999, p.313). From these activities, the learners can gain more specific understanding of the properties of shapes. They will be able to visualise how the angles will fit together and how to flip and turn the shapes to fit various shapes (http://nrich.maths.org/2487).

Activity 5: **Geometric items sorting activity**

Geometric manipulatives are used in this hands-on activity. The manipulatives are in the form of cut-outs of triangles and quadrilaterals. They are numbered from 1 to 30. There are 10 triangle cards and 20 quadrilaterals. The activity makes the learners to carry out a number of operations such as identifying, naming, classifying and defining of geometric figures. There are five interrelated tasks. These structured questions help to decode the learners' understanding and thoughts of the geometric concepts.

Task 1: Identifying and naming shapes.

Task 2: Sorting of shapes.

Task 3: Sorting by class inclusion of shapes.
Task 4: Defining shapes.
Task 5: Class inclusion of shapes.

This activity has been used in many earlier studies by researchers to understand children's thinking about geometric concepts (Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys & Liebov, 1997; Renne, 2004; Feza & Webb, 2005; Atebe, 2008). It helps the learners to verbalise their thoughts. This activity was adopted from Atebe (2008) with permission.

Activity 6: The family of quadrilaterals

This activity provides learners with familiarising of the properties of shapes, connection between shapes and connection between the properties of shapes. Learners are asked to observe the 2 possible 'routes' as they specialise from a quadrilateral with no particular properties to a square.

Activity 7: Group activity: Discovering with folding: Paper shapes

This activity helps the learners to explore shapes forming from other shapes and learn symmetry through folding.

Activity 8: Drawing & construction: Circle

Learners are asked to draw circles of different sizes with the compass. They are asked to discover the terminology such as chord and diameter by constructing them.

Activity 9: Conjecturing in plane geometry

This activity makes use of an investigative approach to explore learners' understanding of the properties of simple geometric shapes like triangles, squares, rectangles and rhombus. Learners are asked to investigate and discover the properties of these shapes through geometrical construction. This activity was developed in the form of a worksheet with semi-structured open-ended questions which makes the learners to answer the questions in their own words.

This activity has 6 investigations:

Investigation 1: to formulate a conjecture that the sum of the interior angles of a triangle is 180^0.

Investigation 2: to formulate a conjecture that the base angles of an isosceles triangle are equal.

Investigation 3: to formulate a conjecture that all the angles of an equilateral triangle are equal.

Investigation 4: to formulate a conjecture that a parallelogram that has equal diagonals is a rectangle.

Investigation 5: to formulate a conjecture that a parallelogram which had equal diagonals that bisect each other at right angles is a square.

Investigation 6: to formulate a conjecture that a parallelogram which has unequal diagonals that bisect each other at right angles is a rhombus.

In investigations 4, 5 and 6 the learners were asked to list as many properties of these shapes and to formulate a definition of each of the shapes.

According to Senk (1989), making and verifying conjectures is a valuable skill in mathematics and more especially in geometry. Proof is expected to play a much more prominent role throughout the entire school mathematics curriculum and to be a part of the mathematics education for students (Knuth, 2002). Nikoloudakis (2009) suggests that students should be able to write simple and partial proofs before they are taught how to write formal proofs. Good tasks are required for assessing the extent to which students use meaningful justification and understand proof (Galindo, 1998). Therefore the activity aimed at exploring learners' abilities on stating definitions, making simple inferences and deductions and formulating conjectures. These investigations have the scope for motivating learners towards meaningful justifications of their ideas in a way the traditional axiomatic approaches to proof never tried. This activity was also introduced in order to see whether the learners are capable of thinking in a level associated with level 3 and 4 of the van Hiele levels. The measuring instrument comprised of questions involving typical problems on a grade 10 geometry level. This activity was adopted with permission from Atebe (2008).

3.2.6. Validity of the activities in the instructional framework

The activities for the instructional framework were carefully selected to fall within the above categories according to the guidelines suggested by Usiskin (1982), van Hiele (1986) and Crowley (1987) and mainly by Fuys, et al. (1988). The programme complied with the national curriculum statement. It was learner-centered, educator-guided, hands-on activities which guided the learners from the basic level to the higher levels. After the development of the framework, it was taken to my supervisor and to a researcher in the field of geometry for suggestions and it was modified. After piloting it on a class of learners, further modifications were done. During the workshops for the educators from the participating schools, the activities were discussed and modified (see section 3.6.3). This also ensured content validity of the activities as it was coming from experienced educators. The activities are discussed in the following section.

3.2.7. Administering of the instructional framework

This is discussed in detail under methodology in Phases 3, 4, 5 and 6.

3.2.8. Responses in the activities of the learners in the experimental group

The educators in the five schools were asked to collect some of the booklets after it was administered so as to get an idea on the learners' participation and responses. A sample response is portrayed in Appendix D along with the instructional framework as a representation of learners in the experimental group who were instructed with the educator guided activities in the instructional framework.

The methodology and the research design of the study are discussed in the next section.

3.3. The method

This study was aimed at looking into the possibilities of improving the geometry education by introducing van Hiele-based instruction after determining the level of geometric thinking of the participants. For that, it was first needed to identify the present van Hiele levels of geometric thinking of the learners. A group of grade 10 learners in some selected schools in Mthatha District in the Eastern Cape province of South Africa was selected for this purpose. For this, a pretest was conducted and after considering the levels of the learners, the instructional framework was developed and tested. Thus, there were two aspects of this study. The first aspect was concerned about the learners' geometric thinking levels and the second aspect dealt with geometry classroom instruction based on the van Hiele theory.

3.3.1. Orientation

This study is oriented in the interpretive paradigm, which is characterised by a concern for the individual as suggested by Cohen, Manion and Morrison (2007). The interpretive researchers begin with the individuals and set out to understand their interpretations of the world in which they interact. The researchers work directly with experience and understanding to build the theory on them. The interpretive paradigm is characterised by its own ontology, epistemology and methodology (Terre Blanche & Kelly, as cited by Atebe, 2008). Ontology refers to the nature of the reality to be studied and what can be studied about it; epistemology refers to the nature of the relationship between the researcher and what can be studied and 'methodology refers to how the researcher goes about practically studying whatever he or she believes can be known' (Terre Blanche & Durrheim, as cited by Atebe, 2008).

The ontological underpinning of this study consists of knowing the van Hiele levels of geometric thinking of grade 10 learners to develop an instructional framework to teach geometry in senior secondary schools. The interaction between the learners and their educators to know their geometrical understanding (epistemology) was done through test and interviews with educators and learners (methodology).

3.3.2. Research design

According to McMillan and Schumacher (2006, p.117), the term research design refers to "the plan for selecting subjects, research sites, and data collection procedures to answer the research questions". The design shows which individuals will be studied and when, where and under what circumstances the individuals will be studied. In the following session, the research sites and participants are discussed followed by the plan and phases of the study.

3.3.3. Research sites and sample selection

In this section the description of the research sites and the selection of the research participants are discussed.

Research sites

This research was conducted in Mthatha District in the Eastern Cape Province of South Africa. Mthatha Educational District has a total of 60 senior secondary schools. Only five senior secondary schools were purposively selected from a group of nearby rural schools for the study. According to McMillan and Schumacher (2006), in purposive or purposeful sampling, the researcher selects particular elements from the population that will be informative about the topic of interest. Geographical accessibility and proximity, functionality and co-educational schools were also some of the factors that influenced the choice of these schools. These schools represent the schools accessed by the majority of our South African learners. These schools also represent the diverse culture of the South African nation. They accommodate learners with different languages, social backgrounds and learning environments.

For the data collection, I visited the five schools in September 2009 to look for grade 10 educators who were willing to participate in my study. The mathematics educators from these five schools meet once a term for cluster moderation as these schools belong to the same cluster. Cluster is a group of schools from neighbouring areas grouped by the Department of Education, for the smooth running of the administration and moderation of continuous assessment tasks for different learning areas. Since I was known in the district as a cluster leader and examiner for district question papers, they agreed to participate in the study with the consent of their school Principals. Moreover, they were very interested in learning the new framework that was mentioned in the letter of introduction. After that, my supervisor and I wrote letters to the Department of education to get their consent to do research in these five schools. The permission was granted and I took it to the Principals for their further approval. In January 2011, I visited the schools again to meet the educators. Through discussions with each school's mathematics teachers, two grade 10 classes from each school were identified and selected. The only restriction on each learner's possible participation in the study was the learner's willingness to participate and the willingness of their parents or guardians for the learners to participate. Letters of introduction to parents / guardians, information sheets and consent forms were submitted and a time line for disseminating and gathering information sheets and consent forms and for conducting the test were negotiated. It was agreed that the names of the participating learners, teachers as well as the names of the participating schools would be kept confidential and that the names that might appear in the research report would be all pseudonyms as suggested by Cohen, Manion and Morrison (2007).

A short description of the five schools is given below.

School A

This school is a co-educational school under the Department of Education in Eastern Cape which draws learners from low to medium socio-economic background. It is a rural school where its learners come from the nearby area and some travel by taxi from faraway places. Most of the parents of the learners are low income parents or unemployed parents. It caters only for FET band and the enrolment of learners in the year 2011 was 640 of which 266 were in grade 10. It accepts learners who pass from the surrounding junior schools. The school offers mainly mathematics and mathematical literacy is offered in only one class of every grade. There are 26 teachers to teach the different subjects offered in the curriculum. It has an administration block as part of the main building. It has two blocks of classrooms and sport fields. The school has electricity, water and sanitation facilities. The classrooms are equipped with chalkboards, Tables and chairs. It is a school with good discipline and is known for its academic excellence by getting 92% in the matric examinations of 2010. English is done in the second language level, except for one class in each grade where, the learners offer English as a 1st language subject. Even though most of its learners are Black South Africans, the school accommodates a minority of learners who speak different languages, drawn from different cultures, social background and learning environments.

School B

The school is an independent co-educational school under the Department of Education which draws learners from low to medium socio-economic background. It is a rural school, where its learners come from the nearby area, outskirts of the town and some travel by taxi from faraway places. It caters for learners from grade 7 to grade 12. It had an intake of 1391 learners for the year 2011 out of which 387 were grade 10 learners. Even though it caters for the GET band, the main intake is done in grade 10, where it accepts learners from other junior schools also. There are 46 teachers to teach the different subjects offered at the school. The school has a computer lab, a school hall, administration block and 5 blocks of classrooms. The classrooms are equipped with chalkboards, cupboards, Tables and chairs. The school has electricity, water and sanitation facilities. It is a school with good discipline and is known for its academic excellence by getting 97% pass in the matric examinations of 2010. The school offers mainly mathematics and mathematical literacy is offered in only one class of every grade in the FET band. English is done in the second language level, except for one class in each grade where, the learners offer English as a 1st language subject. All of its learners are Black South Africans.

School C

The school is an independent co-educational school under the Department of Education which draws learners from low to medium socio-economic background. It is a rural school, where its learners come from the nearby area, outskirts of the town and some travel by taxi from faraway places. It caters for learners from grade 10 to grade 12. It had an intake of 350 learners for the year 2011 out of which 74 were grade 10 learners. It accepts learners from nearby junior schools. There are 18 teachers to teach the different subjects offered at the school. The school is operating in hired premises with some temporary wooden and prefabricated structures as classrooms. The classrooms are equipped with chalkboards, Tables and chairs. The school has electricity, water and sanitation facilities. The school offers mainly mathematics and mathematical literacy is offered in only one class of every grade in the FET band. English is offered in the second language level. It is a school with good discipline and is known for its academic excellence by getting 83% in the matric examinations of 2010. All of its learners are Black South Africans.

School D

The school is a multi-racial, co-educational school under the Department of Education in Eastern Cape which draws learners from medium to high socio-economic background. It is a semi-rural school where most of its learners are from the nearby area, outskirts of the town and some travel by taxi from faraway places. It is a school with good discipline and is known for its academic excellence by getting 96% pass in the matric examinations of 2010. It has a school hall, administration block, science laboratories, computer laboratory, Library and 3 blocks of classrooms. The school has electricity, water and sanitation facilities. It caters for learners from grade 8 to grade 12. The grade 10 enrolment is 120 out of the total enrolment of 595. Each grade has 3 divisions according to their subject choices in the FET band and learners in the GET band are put into classes according to their choice of second language between isiXhosa and Afrikaans. There are 28 teachers to teach the different subjects offered at the school. All the learners in the FET band do Mathematics as one of the subjects. Mathematical literacy is not offered in the school. Even though most of its learners are isiXhosa speaking Black South Africans, all of its learners do English in first language level. It has a major feeder school with the same facilities from grade 0 to grade 7. About 70% of its learners are Black South Africans and it accommodates learners who speak different languages from different countries, drawn from different cultures, social background and learning environments. In the year 2011, it had learners from 11 different nationalities.

School E

The school is a co-educational school under the Department of Education in Eastern Cape. The school draws learners from low to medium socio economic background. It is a rural school where its learners come from the nearby area and some travel by taxi from faraway places. Most of the parents of the learners are low income parents or unemployed parents. It caters for GET and FET bands and the enrolment of learners in the year 2011 was 811 of which 191 were in grade 10. It caters for learners from grade R to grade 12. Even though it caters for the GET band, the main intake is done in grade 10, where it accepts learners from other junior schools also. The school offers mainly mathematics and mathematical literacy is offered in two classes of every grade. There are 33 teachers to teach the different subjects offered in the curriculum. The school has a school hall, an administration block and has five blocks of classrooms. The school has electricity, water and sanitation facilities. The classrooms are equipped with chalkboards, cupboards, Tables and chairs. It is a school with good discipline and is known for its academic excellence by getting 80% in the matric examinations of 2010. English is done in the second language level. Even though most of its learners are Black South Africans, it accommodates a minority of learners who speak different languages, drawn from different cultures, social background and learning environments.

3.3.4. Mathematics June examination result of the learners in the sample

To get an overall idea of the performance of the learners in the sample, Mathematics marks for the June examination of 2011 of the two grade 10 classes from the participating schools were analysed. The June examination marks is shown in Appendix I.

The following Table shows the average percentage of the June 2011 examination mark out of 100 of all the schools. June examination for grade 10 is internally set and internally moderated. This means that the schools write their own internally set question papers.

Table 3.1: Average percentage of the June 2011 Examination Mathematics mark of participating grade 10 learners in all the schools

School	Number of learners	Participants' average percentage mark
School A	78	26,17%
School B	107	23,04%
School C	57	9,40%
School D	65	23,12%
School E	52	14,54%

3.3.5. Participants and the reasons for selecting the participants for the study

From the above schools that were selected purposively, the participants were selected using convenience sampling. They were selected on the basis of being accessible. McMillan and Schumacher (2006, p.125) explain that "in educational studies, particularly experimental and quasi-experimental investigations, probability samples are not required, rather non probability sampling (convenience sampling, purposeful sampling, and quota sampling) is used, where, the researcher uses subjects who happen to be accessible". So, for conducting the experimental study two classes were selected from each school using convenience sampling, of which one was selected as control group and the other as experimental group. The two groups were selected to test the effectiveness of the instructional framework. The experimental group was given an instruction based on the van Hiele theory and the control group was given the traditional method of geometry instruction where the researcher did not use the characteristics of the van Hiele theory.

The study utilised grade 10 learners as the sample due to two reasons: Most of the literature available discusses the van Hiele Theory in terms of polygons and these are included only in the grade 10 syllabus. The second reason is that grade 10 is the entering level in a senior secondary school and that if their levels of understanding in geometric concepts are investigated and a correct instructional method according to their levels is implemented, it would be beneficial for their success in grade 12 examinations.

3.4. Design of the study

A quasi-experimental design was implemented to check the effectiveness of the instructional framework. In the quasi-experimental design, a control group was employed to compare with the experimental group, but the participants were not randomly selected and assigned to the groups (Creswell, 1994). According to McMillan and Schumacher (2006), non-equivalent groups pretest-posttest control and comparison group designs are very prevalent and useful in education as it is often impossible to randomly assign subjects. For this research, the experimental group included learners who were instructed with the van Hiele instructional framework, while the control group comprised learners who were instructed with conventional method. There were two intact groups of grade 10 class from five schools. This quasi-experimental research method was chosen due to the fact that it provides the best approach to investigate the effectiveness of a particular instruction (McMillan & Schumacher, 2006). In this study, pretest and posttest were given to the participants before and after the instruction as an independent variable. The effects of the instruction on learners' attainment in geometry were investigated in this study. Therefore, the quasi-experimental approach made it possible to evaluate the effectiveness of the instructional framework using the van Hiele theory with the results of the geometry test.

"Convenience" sampling procedure as defined by McMillan and Schumacher (2006) was followed where two classes of learners from five different schools were selected because of availability.

This study made use of both quantitative and qualitative methods. For the quantitative research, quasi-experimental design was used. For the qualitative research, interviews were conducted for a total of 30 learners drawn from the five schools. The selection process of the 30 learners is given in section 3.6.6. The study used mixed methods, which combined the quantitative and qualitative methods to ensure the validity and reliability of the research output. How these methods were used to elicit the geometric thinking levels of the learners is given in section 3.6.1.

The quasi-experimental design is as follows:

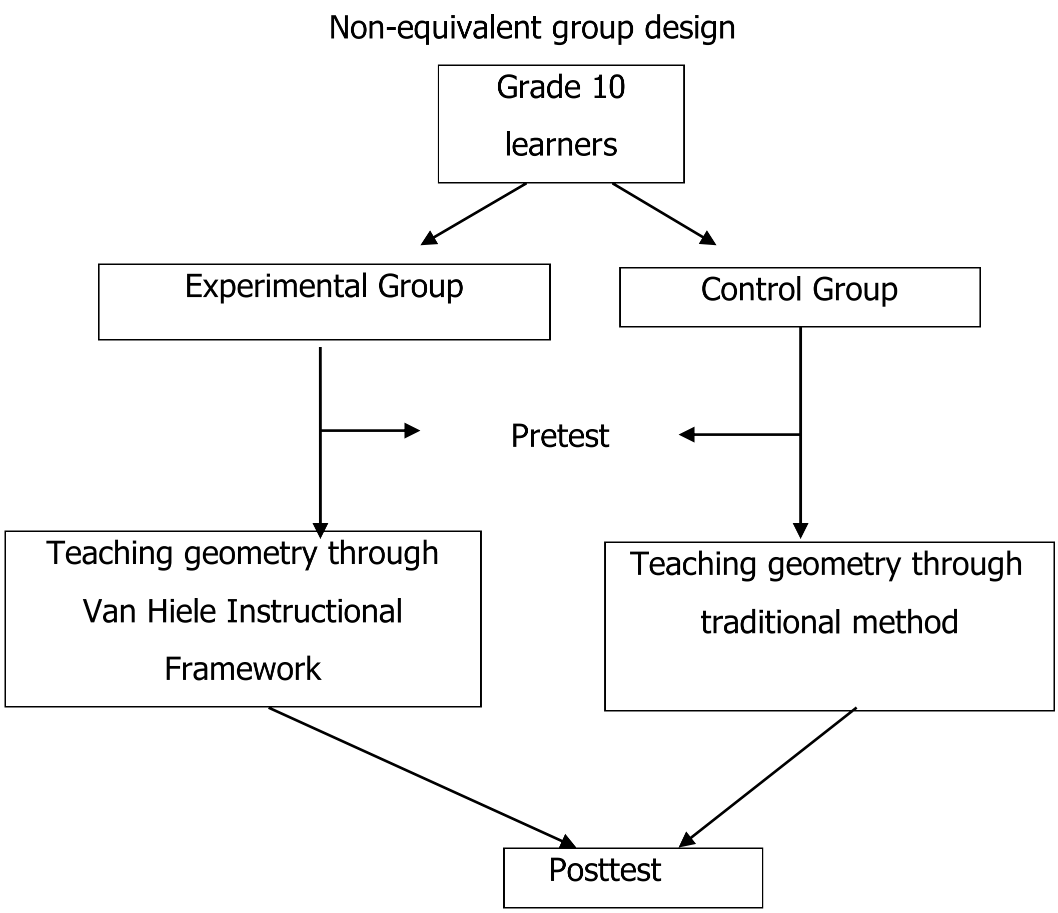


Figure 3.1: Quasi-experimental design

The design can further be shown as suggested by McMillan and Schumacher (2006, p. 274) as:

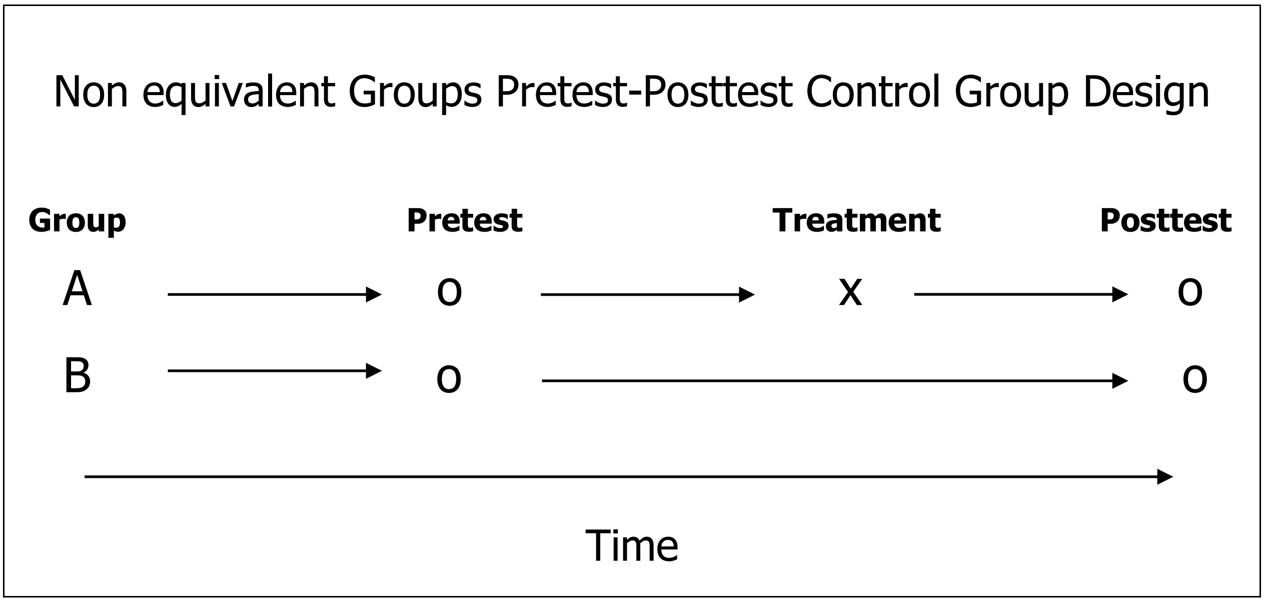


Figure 3.2: Non-equivalent groups pretest-posttest control group design

Where X stands for geometry lessons taught with the van Hiele-based instructional framework applied to the experimental group.

3.5. Validity and reliability of the study

In quantitative research, reliability stands for dependability, consistency and reliability over time, over instruments and over groups of respondents (Cohen, Manion & Morrison, 2007). Joppe (2000) as cited by Golafshani (2003, pp. 598-599) defines reliability as "the extent to which the results are consistent over time and an accurate representation of the total population under study is referred to as reliable and if the results of the study can be reproduced under a similar methodology, then the research instrument is considered to be reliable" and validity "determines whether the research truly measures what it is supposed to measure".

In qualitative research, reliability is also viewed as being synonymous with consistency and validity refers to as trustworthiness or credibility (Struwig & Stead, 2001). When conducting a research, it is important to report the extent to which the instruments employed in the study have reliable and valid scores and whether the research design is valid (Struwig & Stead, 2001).

In the present study, the validity and reliability of the different data collection methods are discussed in their particular sections.

The general validity and reliability strategies used in this study (Cohen, Manion & Morrison, 2007; Struwig & Stead, 2001) are discussed as follows:

- **Triangulation:** Triangulation can be defined as the use of two or more methods of data collection. Methodological triangulation was applied in this study as it used both quantitative and qualitative data. Data triangulation was also applied as multiple sources of data and data collection methods were used in this research to confirm the findings. The reliability of the obtained data was enhanced and validated by a process of triangulation of the data from multiple sources which included the data from the van Hiele Geometry Test and the interviews conducted with 30 learners. This helped the study to establish the validity of the findings through cross-referencing. The mathematics marks of the June examination were also made available and verified against performance of the schools and are presented in appendix I. This method of triangulation enabled to address the question of internal validity and the interpretation of the data from more than one perspective. Lincoln and Guba (1985) suggest that the triangulation is intended as a check on the data, while the member checking can be used as a check on the member's construction of data.

- **Member checks:** The results of the pretest, the posttest and interviews were taken back to the learners and teachers after each analysis to see whether they agreed with them.

- **Peer review:** Each stage of the study was discussed with the supervisor, a researcher in the field of geometry, colleagues and the five teachers who participated in the study. Their valuable suggestions were incorporated into the study.

- **Adequate collection of data:** It was ensured that adequate data were collected to validate the findings of the study. Data were collected from five different research sites to validate the findings.

3.6. Data collection procedures

This section describes the processes followed and the instruments used to collect the data. I structured my interaction with the schools into six phases as these phases define the data collection procedures.

3.6.1. Phase 1

Data collection to determine the van Hiele levels of geometric thinking

It concerns determining the van Hiele levels of geometric thinking of the participating learners. As agreed upon by the educators, I visited the schools in June 2011 and delivered the question papers and answer sheets to the teachers. The pretest on the van Hiele Geometry Test was written during mathematics lessons. Two intact classes of grade 10 from each of the five schools participated in the pretest.

The use of quantitative and qualitative methods to elicit the geometric thinking levels of learners

To devise a methodology that best suits the process of eliciting the learners' thinking level was a major concern for me when I started. After consulting a lot of research studies in this field (Hoffer, 1981; Usiskin, 1982; Senk, 1985; Shaughnessy & Burger, 1985; Fuys et al., 1988; Clements & Battista, 1992; King, 2003; Atebe, 2008), it was decided that a quantitative study involving paper and pen test best suits to elicit the level of geometric thinking. Most of the van Hiele writings (van Hiele, 1986; 1999) and subsequent research in the 1980's (Usiskin, 1982; Fuys, et al., 1988) have used pen and paper test to determine the van Hiele levels of geometric thinking. Jaime and

Gutierrez (1994) comment that we can hardly meet any researcher on the van Hiele model who has not needed to assess the van Hiele levels of the students, and this implies the use of a test which can be written or oral. Crowley (1990) raised concerns about the possibility of measuring reasoning by the kind of tests like Usiskin's (1982). But this test has its main advantage that it can be administered to many individuals and it is easy and quick to assess the level of reasoning of learners (Jaime & Gutierrez, 1995). The Burger and Shaughnessy's (1986) test has to be administered by an interview, but it is time consuming, which makes it unsuitable for assessing many learners. But it has a great advantage of obtaining the results which have deeper knowledge of reasoning of learners and can be more reliable (Jaime & Gutierrez, 1995).

Considering both views above, this study first used pen and paper test to investigate and interpret the van Hiele levels of geometric thinking of participating learners. The test that was used in this study was adopted with permission from a similar study done in the Grahamstown area (South Africa) to determine the van Hiele levels of geometric thinking of senior secondary school learners (Atebe, 2008), which was adapted from the CDASSG Project (Usiskin, 1982).

Van Hiele himself acknowledges the fact that "tracing the levels of thinking that play a part in geometry learning is not a simple affair, as the levels of thinking are not situated in the subject matter but in the thinking of man" (van Hiele, 1986, p.41). Here, van Hiele seems to be suggesting that, to elicit the geometric thinking of a learner is a complex process where a simple administration of a paper and pen test may not give a full detail of the thinking levels of a learner (Atebe, 2008).

Therefore, to further enrich the study, a qualitative domain in the form of one-on-one interview with some selected learners was also conducted. Walker (1993) suggests that in many projects the most significant findings have emerged from points at which different methods have complemented each other. According to Creswell and Garrett (2008, p.322), "when researchers bring together both quantitative and qualitative research, the strengths of both approaches are combined, it can be assumed, to a better understanding of research problems than either approach alone".

3.6.2. Phase 2

Development of the instructional framework

This was discussed under the section 3.2.

3.6.3. Phase 3

Workshop for the educators of the schools who participated in the pretest

There were five educators from the participating schools. All of them were trained graduates in mathematics. Their teaching experience ranged from two years to 20 years with an average of 8 years of teaching. All of them are presently teaching in senior secondary schools with grade 10 as their main teaching grade.

Procedure

Each educator spent approximately five to six hours over the 3 sessions with me. At the outset, the educators were given an orientation on the work of van Hiele, the van Hiele levels of thought including level descriptors and an overview of the research project. They were given guidelines and theoretical outline of the van Hiele theory during the orientation. They were quite interested to know the levels of the geometric thinking of their learners as they were also dissatisfied with the performance of learners in mathematics and particularly in geometry. As agreed upon by the Principals of the chosen schools, a date in June was chosen for the pretest as mentioned earlier.

Table 3.2: Workshop time schedule for educators

Date	Workshop session
11 June 2011	Orientation
20 June 2011-24 June 2011	Administering the pretest in 5 schools
15 July 2011	Workshop 1
23 July 2011	Workshop 2

The workshop for the educators were conducted towards the end of the July holidays and on a Saturday as it was felt that it was the best time to get to know the programme as to minimise the disturbance of conducting it during the school days. During the workshops, the activities were discussed and modified. This also ensured the content validity of the activities as it was coming from experienced educators. It was agreed that the activities like tangram puzzles can be done as homework as it would give more time to the learners to experiment it at home in a more relaxed environment. From 25 July to 23 August they implemented the instructional framework in their schools in their mathematics lessons. A detailed description of the implementation program is discussed below.

3.6.4. Phase 4

Implementation of the instructional framework for the experimental group

For the implementation of the instructional framework, as discussed earlier, two grade 10 classes were taken from each school. One grade 10 class was designated as the experimental group with the other grade 10 class used as a control group. Both these groups wrote their pretest during the mathematics lesson to cause minimal disruption to the school program. The aim of the pretest was discussed in section 3.3.

Time schedule for the implementation of the instructional framework and the administration of the posttest

Table 3.3: Time schedule for the implementation of the instructional framework and the administration of the posttest

Date	Classroom session: Experimental group	Classroom session: Control group
25 July 2011 - 22August 2011	Experimental group was taught during normal mathematics lessons with the instructional framework	Control group was taught during normal mathematics lessons using conventional method
29 August 2011	Posttest	Posttest

The third term was chosen due to the fact that the work schedule of grade 10 caters for geometry teaching only in the third term. The experimental group was taught with the instructional framework and the control group was taught with the conventional method as shown in the above time schedule. Each class got 5 hours of lessons per week as per the normal time Table, which gave the educators 20 hours of normal school hours. Evaluation of the instructional framework was done through a posttest that was administered on the experimental group and control group on the week of the 29th of August 2011.

3.6.5. Phase 5

Evaluation of the instructional framework through a posttest that was administered on the participating learners

On 22 August, the instructional framework officially came to an end. It was agreed that the learners would write their posttest in the following week. The data was analysed the same way as the pretest and the data analysis is shown in Chapter 4. The initial analysis using Microsoft Excel showed that the experimental groups' mean scores were higher than the control groups' mean scores and that the instructional framework had a significant effect as suggested by McMillan and Schumacher (2006). The tests were marked according to the same criteria that are mentioned in the pretest.

The interviews with educators

The five teachers from the five schools were instrumental in the implementation of the framework. After the whole program, interviews with educators were also conducted to collect their opinions, attitudes and suggestions regarding the implementation and effectiveness of the van Hiele framework. Practicability, usefulness, suitability, time allocated for each activity and participation from learners were some of the questions that were asked to the educators. The interview schedule is shown in Appendix E.

3.6.6. Phase 6

The interviews with learners

As mentioned earlier, a learner's level of thinking was determined mainly by his/her responses in the van Hiele Geometry Test. Analysis of the responses on the written test proved more difficult than expected and some interesting features emerged as a result. In cases where there appeared to be certain trends in a learner's understanding, it was decided to conduct a structured interview with the learners in order to explore these features in greater detail and to obtain clarification on his/her understanding in geometric shapes. Van Dalen (1979) suggests that through respondents' incidental comments, facial and bodily expressions, and tone of voice, an interviewer can acquire information that would not be conveyed in written replies. So as to confirm the levels, I interviewed 30 learners individually on one-on-one interviews. The 30 learners were taken from the five different schools participated in the study (6 learners each from a school - 3 from the experimental group and 3 from the control group). I described the interview process and its purpose to the cooperating educators and then asked the educators to select some of their learners according to their performance in the van Hiele Geometry Test as one from each level. The selection was therefore purposive. The number of learners in each gender was specified as to have a gender ratio of 2:1 of girls and boys. The interview tasks were first piloted in the researcher's own school prior to the commencement of the interviews.

The Interview procedure

The interview tasks were administered to each learner by the researcher in an audio taped one-on-one interview. The learners were told that they were going to be interviewed on some questions about geometric shapes mainly triangles and quadrilaterals. Pencils, papers, erasers and mathematics instrument boxes were made available. The learners were encouraged to use any of these instruments at any time during the interview. The interviewer presented the tasks to each learner in the same order according to the script. On completion of each task the interviewer was free to follow up on any response. The data for the interview consisted of the audio files, the learners' drawings and the interviewer's notes.

The interviews took place in the learners' classrooms after school under the guidance of the mathematics educators. Only the learner and the interviewer were present at each interview. Each interview took about 40 to 60 minutes to complete. The interviews took place from 24th October 2011 to 10th November 2011, which was two weeks after the reopening of the fourth term.

The interview followed a script, written by Burger and Shaughnessy (1986), designed to prevent any influence of the interviewer from skewing the results of the interview and to give the interviewer control over the way the questions are asked. The script was followed as closely as possible, which made the interviews as similar as possible among the 30 learners. The structured, conversational interview format enabled me to gather similar information from each learner while at the same time to explore how each learner has come to his/her current understanding. Crucial to this interview format is the use of open ended questions or statements in between the tasks such as, "Why did you choose …?", "How will you explain …?", "What do you think …?", "Tell me about …". Mitchelmore and White (2000) in particular, discuss how open ended questions encouraged subjects to describe and discuss their thinking.

Thus the data from the learners' interviews were collected from three sources:

- audio recording of each interview that provided further data about the learners' conversation;
- learners' pencil and paper recordings from various tasks, particularly written or drawn responses to interview tasks;
- field notes for each interview that contained annotations about surprising or unexpected responses and indicators about learner confidence.

A detailed description of the instruments used in the study, namely, the van Hiele Geometry Test and interview schedule for learners are given in the following sections.

3.7. Instruments

This section describes the instruments used and the processes followed in collecting data. To address research questions 1, a pretest was conducted to determine the current level of geometric thinking in the five schools. The van Hiele test with 20 items on level 1-level 4 were used for this purpose.

3.7.1. Van Hiele Geometry Test

According to McMillan and Schumacher (2006, p.189), the term "test" means that a standard set of questions presented to each subject that requires completion of cognitive tasks. The responses or answers are summarised to obtain a numerical value that represents a characteristic of the subject. McMillan and Schumacher (2006) explain that the advantage of using standardised tests is that they are prepared by experts and careful attention will be paid to the nature of the norms, reliability and validity and are intended to use in a wide variety of settings.

The van Hiele model was developed in the 1950s by Pierre van Hiele and Dina van Hiele-Geldof. In the 1970s it became popular in the United States (Hoffer, 1983; Wirszup, 1976). Following that, Usiskin (1982) developed the van Hiele Geometry Test which is known as Cognitive Development and Achievement in Secondary School Geometry (CDASSG) to test the theory and since then both the test and theory gets refined and thousands of people get tested with it. For this study, the test which was constructed by the staff of the CDASSG project, and adapted by Atebe (2008), was adopted with permission.

The research instrument, the van Hiele Geometry Test, which was used to determine the van Hiele level of geometric thinking, was done on the following topics:

1. Basic geometric concepts like identification, classification and properties of triangles and quadrilaterals.
2. Angle measurement, angle sums of lines, triangles and quadrilaterals.

The van Hiele Geometry Test that was used as a multiple choice test comprised 4 subtests. Each subtest consists of 5 items based on one van Hiele level. There were 20 items in the test, with item numbers 1-5, 6-10, 11-15, and 16-20 for testing learner's attainment of van Hiele levels 1, 2, 3, and 4 respectively.

The following is a sample question from van Hiele level 1 subtest:

Question1. Which of these are triangles?

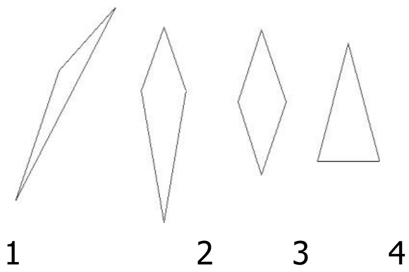


Figure 3.3: Sample item from van Hiele level 1 subtest

A. All are triangles

B. 4 only

C. 1 and 2 only

D. 3 only

E. 1 and 4 only

The following is a sample question from van Hiele level 2 subtest:

Question 10: RSTU is a square. Which of these properties is **not true** in all squares?

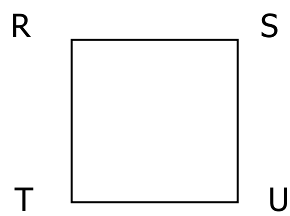


Figure 3.4: Sample item from van Hiele level 2 subtest

A. RS and SU have the same measure.
B. The diagonals bisect the angles.
C. RT and SU have the same measure.
D. RT and Su are lines of symmetry.
E. The diagonals intersect at right angles.

The following is a sample question from van Hiele level 3 subtest:

Question 12: Which is **true**?

A. All properties of rectangles are properties of all parallelograms.

B. All properties of squares are properties of all rectangles.

C. All properties of squares are properties of all parallelograms.

D. All properties of rectangles are properties of all squares.

E. None of (A) – (D) is true

The following is a sample question from van Hiele level 4 subtest:

Question 17: Examine these statements.

i). Two lines perpendicular to the same line are parallel.

ii). A line perpendicular to one of two parallel lines is perpendicular to the other.

iii). If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines S and P are perpendicular and lines T and P are perpendicular.

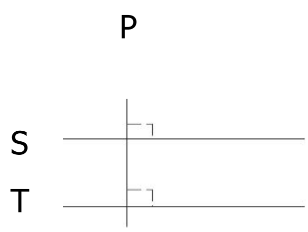


Figure 3.5: Sample item from van Hiele level 4 subtest

Which of the above statements could be the reason that line S is parallel to line T?

A. (i) only
B. (ii) only
C. (iii) only
D. Either (ii) or (iii)
E. Either (i) or (ii)

3.7.1.1. Rationale for the van Hiele Geometry Test

The rationale for the van Hiele Geometry Test was based on the assumption that the learners' understanding of geometry can be described by their attainment in the van Hiele levels of geometric thinking. According to van Hiele (1986), the studies that investigate the geometric thinking levels should be content specific. Therefore the reason for adopting the adapted test was that the above geometrical aspects form the basis for the content of geometry (space and shape) in the senior secondary schools. The original CDASSG test of Usiskin (1982) was developed to assess the level of thinking across different concepts and it was presented at different van Hiele levels- levels 1, 2, 3, 4 and 5. The adapted version of Atebe (2008), used test items for level 1-4, as many researchers (Burger & Shaughnessy, 1986; van Hiele, 1986; Senk, 1989) suggest that the highest van Hiele level attainable by a student in senior secondary school is level 4. Van Hiele himself painfully argues that testing beyond the fourth level is not appropriate and said, "Some people are now testing students to see if they have attained the fifth or higher levels. I think this is only a theoretical value... so I am unhappy if, on the ground of my levels of thinking, investigations are made to establish the existence of fifth and higher levels" (van Hiele, 1986, p.45). The present study also used test items on levels 1 – 4.

3.7.1.2. Collection of Data/administration of the van Hiele Geometry Test

The VHGT was administered on the learners by their mathematics educators in June 2011. The test was written during school hours in their classes. The educators were well aware of the instructions as I explained beforehand to them and they reported that the learners also followed the instructions carefully while they answered each item. They were each given a question paper and a multiple choice answer sheet where they crossed the answers of their choice. It is shown in Appendix C.

3.7.1.3. Marking of answer sheets

The tests were scored by a computer assistant from the school where the researcher is working, using Microsoft Excel and were later verified by the researcher and a grade 10 mathematics educator. Each correct response was given 1 mark each. Hence each learner's total score was between 1 and 20 in the VHGT. The percentage score was calculated and an analysis of correct responses was also done using Microsoft excel.
The grading of the VGHT was done again using a second method which was based on the '3 of 5 correct" success criterion as suggested by Usiskin (1982, p.22).
By this criterion, if a learner answers correctly at least 3 out of 5 items in any of the 4 subtests within the VHGT, the learner was considered to have mastered that level. According to this grading system the learners' scores were weighted as follows:
1 point for meeting criterion on item 1-5 (level 1)
2 points for meeting criterion on item 6-10 (level 2)
4 point for meeting criterion on item 11 - 15 (level 3)
8 point for meeting criterion on item 16-20 (level 4)
This makes the maximum score for any learner to be 1+2+4+8 = 15 points. This weighted sum helps to determine upon which van Hiele levels the criterion has been met from the weighted sum alone. For example, a score of 7 indicates that the learner met the criterion at level1, 2 and 3 (i.e., 1+2+4 =7). This grading system helped to assign the learners into various van Hiele levels based on their responses. The weighted sum and the corresponding levels are as shown below.

Table 3.4: Van Hiele levels and weighted sums

Weighted sum	Corresponding levels
0	0
1	1
3	2
7	3
15	4

Again, a weighted sum of 0 indicates that a learner has not achieved any levels, as the learner did not get at least 3 out of any subtests of the van Hiele Geometry Test. This learner will be rather operating at a lower level known as level 0 or pre-recognition level (see section 4.3.4, Chapter 4).

A weighted sum of 2 indicates that the learner achieved at least 3 out of 5 only at level 2. But, because of skipping level 1, the learner will be classified under level 0. In the same way, a learner with a weighted sum of 4 or 8 is also at level 0 because of skipping levels 1 and 2 for the weighted sum of 4 or levels 1, 2 and 3 for the weighted sum of 8.

3.7.1.4. Assignment of levels

Two methods were used to assign the learners into levels using the 3 out of 5 success criterion as follows:

- Classical or modified van Hiele levels: a learner's van Hiele level is defined to be the highest consecutive level he or she has mastered. If a learner satisfies the criterion at levels 1, 2 and 4, he or she will be assigned to level 2 (Atebe, 2008; Mateya, 2008).
- Forced van Hiele levels: according to this, "a learner is assigned to a level n if the learner meets the criterion at levels n and n – 1, but not one of n – 2 or

n – 3 or he or she meets the criterion at level n, all levels below n, but not at level n+1 yet also meets the criterion at one higher level" (Usiskin, 1982, p.34).

The main goal of the present research project was to develop and implement the instructional framework based on the levels assigned to the learners and evaluation of it. In this study only the first method, which is known as the classical or modified van Hiele level was used to assign the learners to different van Hiele levels and the same method was also used to evaluate the effectiveness of the instructional framework.

3.7.1.5. Analysis of the van Hiele Geometry Test

The initial analysis of the data was done using Microsoft Excel Spreadsheet. First it was analysed in terms of the percentage mean and then it was analysed in terms of the percentage number of learners in each level of the van Hiele levels according to the criterion developed by Usiskin (1982). An analysis of correct responses was also done using Microsoft Excel. The analysis is shown in Chapter 4.

A further analysis was done using IBM SPSS Version 19. It was used for the t-test to analyse whether there was a statistical difference between the performance of the control group and the experimental group and also for the comparison of these groups' performance in the pretest and the posttest.

The t-test is the most common statistical procedure for determining the existence of significance when two means are compared and it is a formula that generates a number, and this number is used to determine the probability level (p level) of rejecting the null hypothesis (McMillan & Schumacher, 2006).

In order for the above test to be carried out, the following hypotheses are constructed:

H_0: $\mu_0 = \mu_1$ against

H_1: $\mu_0 \neq \mu_1$

The level of significance was selected to be $\alpha = 0.05$

Here μ_0 and μ_1 are the sample means of the two groups and H_0 stands for the null hypothesis which assumes that the two groups have equal means and H_1 stands for the alternate hypothesis which assumes that the two groups have unequal means. For an obtained t-value to be significant, $|t| > 1.96$ at $\alpha = 0.05$ for a two-tailed independent samples test using the normal distribution (Gupta & Kapoor, 1983). If the obtained t-value is greater than the critical value, 1.96, the null hypothesis is rejected.

The independent samples t-test was used when the means of the experimental group and the control group were tested either in the pretest (see section 4.2.1.3) or in the posttest (see section 4.3.1.4) and the dependent samples t-test was used when each of the groups were tested for the effectiveness of the instructional framework. In other words, when the comparison of the means of the pretest and the posttest was done either for the experimental group or the control group, the dependent samples t-test was used (see sections 4.3.1.4.2 and 4.3.1.4.3).

The reason for choosing the t-test was because of the statistical principle of the T random variable converting itself to the normal random variable when the sample size is greater than 30. The t-distribution is essentially a corrected version of the normal distribution in which the population variance is unknown and hence is estimated by the sample standard deviation (student's t-test, retrieved from internet on 04/02/2012). The t-test is generally used when the population standard deviation is unknown (Struwig & Stead, 2001).

Even though it has been traditionally accepted as one should have a sample size of at least 30 to ensure that the sampling distribution of the mean is approximately normal, according to Weaver (2011), when we use statistical software to perform t-tests, we get a p-value computed using the appropriate t-distribution, regardless of the sample size and that the distinction between small and large sample t-tests is no longer relevant. Therefore, in the analysis of the statistical difference between the performance of the control group and the experimental group, t-test using IBM SPSS Version 19 was used in this study.

3.7.1.6. Validity and reliability of the van Hiele Geometry Test

The test items in the van Hiele Geometry Test were adopted with permission from the recent research done in Grahamstown area by Atebe (2008), which was adapted from Usiskin (1982). In his study, the split-half method was used to check the reliability of the test instruments. The split-half method required the construction of a single test consisting of a number of items and these items were then divided into parallel haves and the learners' scores from these halves were then correlated using the Spearman-Brown formula. The Spearman-Brown reliability coefficient (r) calculated for the van Hiele Geometry Test as reported by Atebe (2008) was 0.25 and the value of the reliability coefficient ranges between -1and 1. According to Atebe, the comparatively low reliability coefficient calculated for the van Hiele Geometry Test was a result of the fewness of the number of items in the test. Moreover, Atebe validated his test by consulting the geometry curriculum and the grade 10 mathematics textbooks. After constructing the test items he consulted two experts, one in geometry and the other in geometry education for cross checking. To ensure it further that the contents selected were within the prescribed syllabus, an educator questionnaire was administered on the educators and their responses indicated that the contents of the test fall within the prescribed syllabus. A pilot test was conducted and this also helped to refine the instruments. Thus the content validity was assured for the van Hiele Geometry Test.

After the pretest was done, I went back to the schools and showed the participants their scores and discussed their level of thinking. It was appreciated by the learners and their educators and they agreed that the scores assigned to them accurately represent their abilities. The reliability of the data obtained from the learners was enhanced and validated by a process of triangulation of data from multiple sources as explained in section 3.5.

3.8. The qualitative study

3.8.1. The interview tasks

Interview studies use personal contact and interaction to gather data necessary to address the questions being studied (Drew, Hardman & Hosp, 2008). Interviews are essentially vocal questionnaires. The interview involves direct interaction between individuals. The interview is a flexible tool for data collection, enabling multi-sensory channels to be used such as verbal, non-verbal, spoken and heard (Cohen, Manion & Morrison, 2007).

The purpose of the interview and the administration of it were discussed under Phase 6 in the research design.

The interview consisted of giving the learners seven open ended tasks dealing with geometric shapes, developed by Burger and Shaughnessy (1986), which were designed to reflect the descriptions of the van Hiele levels that were available in the literature. The tasks involved drawing triangles and quadrilaterals, identifying and defining shapes, sorting shapes and engaging in informal and formal reasoning about geometric shapes. These tasks were expected to draw out the characterisations of van Hiele levels 1 to 3 (Burger & Shaughnessy, 1986). Additional indicators were added for level 0 by the researcher as they were not available from the literature (see Chapter 7). No attempt was made to investigate van Hiele level 4 as none of the learners in the entire sample were at level 4. Two sets of drawing, identifying and sorting tasks were administered, one set for triangle shapes and one set for quadrilateral shapes. This interview tasks were administered after piloting it.

Examples of one set of tasks for triangles are described below. The tasks for quadrilaterals were similar.

Drawing Task

The learner was asked to draw a triangle, and to draw another triangle that was different from the first one in some way and to draw another that was different from the first two in some way and so on as long as the questions proved fruitful. Then the learner was asked how the figures differed and how many different triangles he/she could draw. The task investigated the properties that the learners varied to make different figures and explored whether they could draw infinite or only a few numbers of triangles.

Identifying and defining Tasks

Given a sheet of triangles the learner was asked to put a T on each triangle and was asked to justify his/her marking and if necessary, why some of the other figures had been omitted. And further to elicit properties the student perceives as necessary for a figure to be a triangle, the learner was asked, "What is the shortest list of things you tell someone to look for to pick out all the triangles on a sheet of figures?" the same was repeated for different quadrilaterals, namely square, rectangle, parallelogram and rhombus by asking to put an S on all the squares, R on all rectangles, P on all parallelograms and so on. Thus the activity explored the learners' definitions and class inclusions.

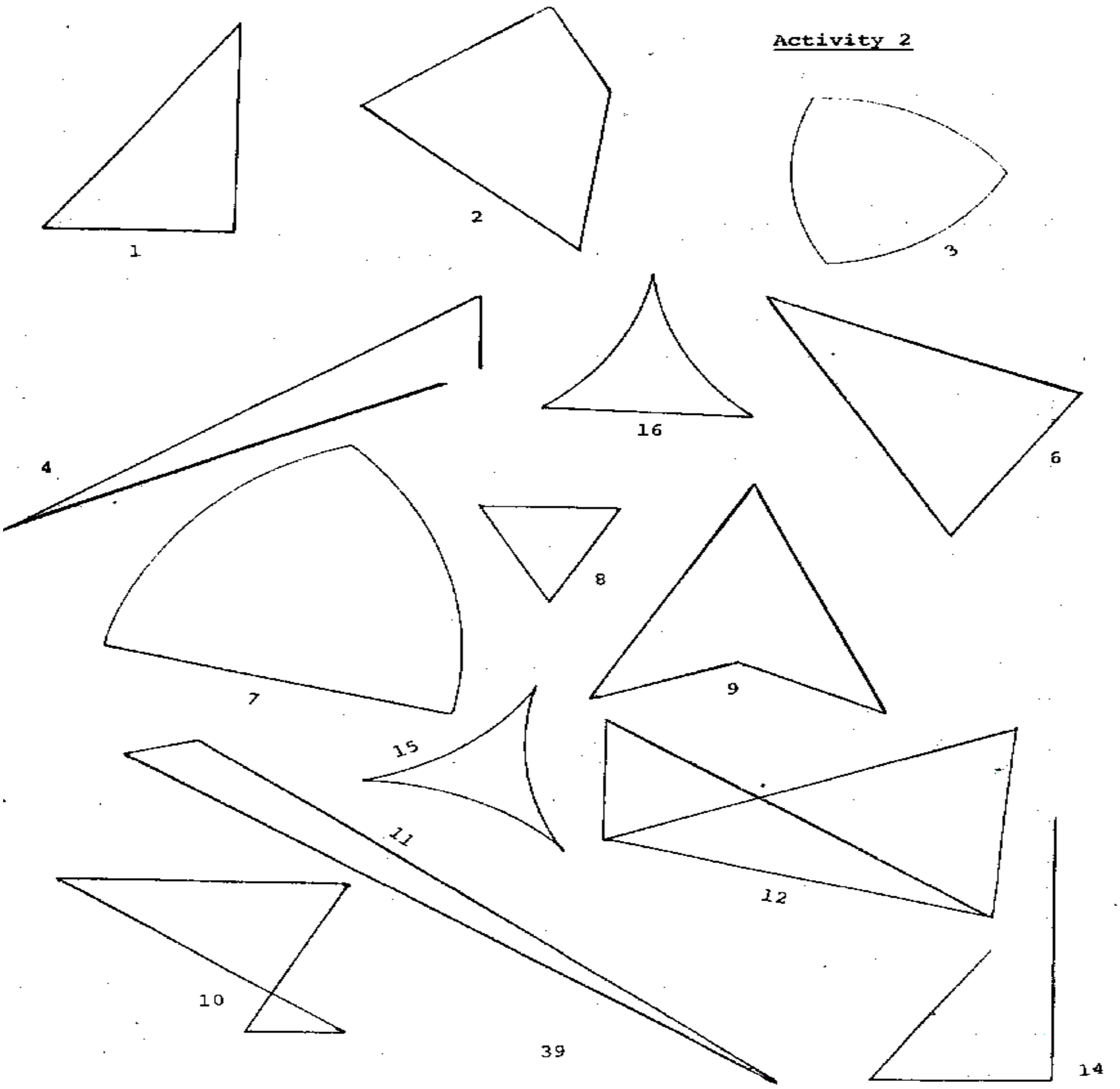


Figure 3.6: Activity 2A – Triangles

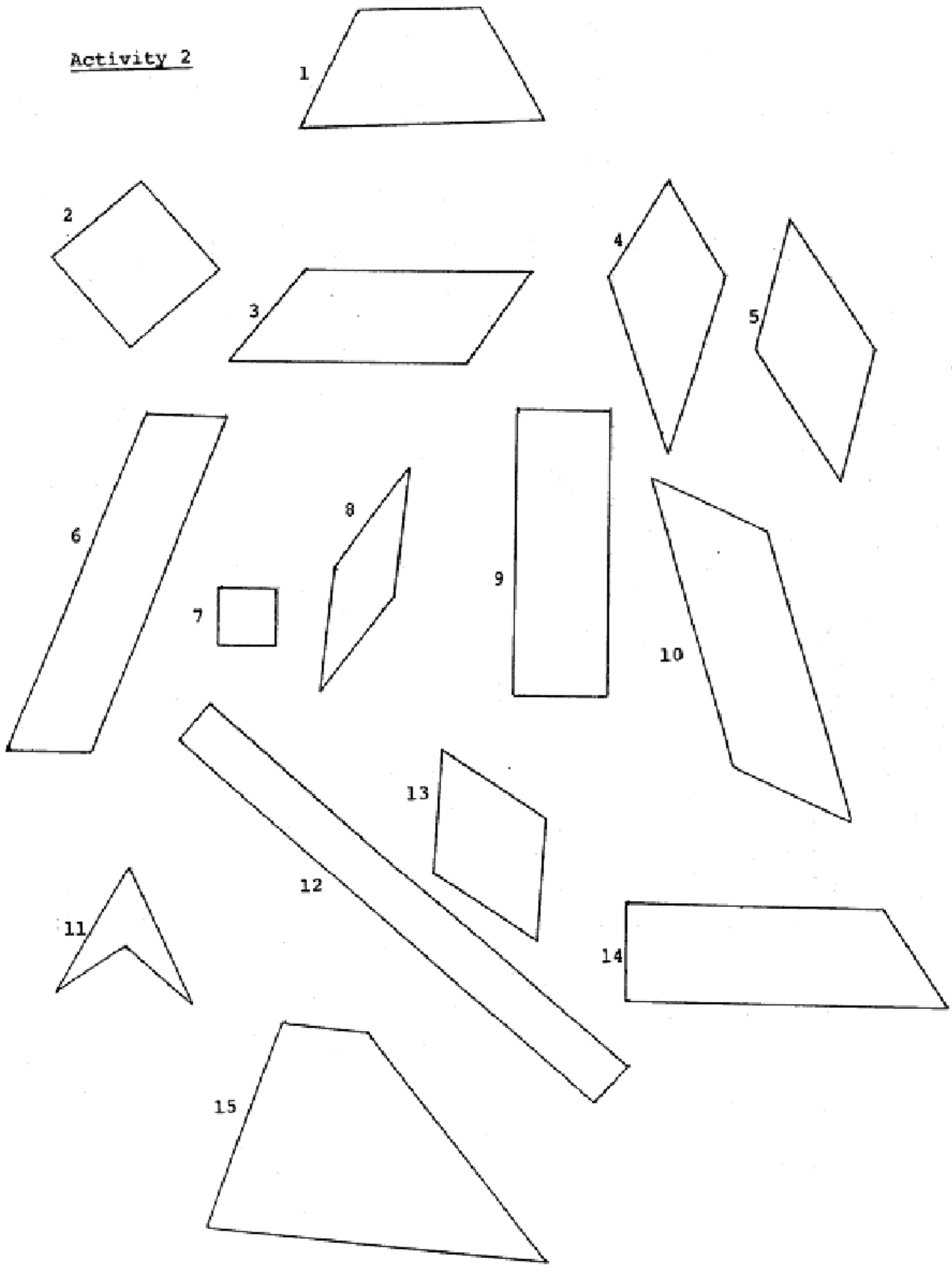


Figure 3.7: Activity 2B – Quadrilaterals

Sorting Task

A set of cut out triangles was spread out on the Table. The learner was asked, "Can you put some of these together that they are alike in some way? How are they alike?" These kinds of questions were repeated until he or she could come up with new sorting properties. The same was repeated for quadrilaterals also. To further determine the student's ability to distinguish common properties of pre selected triangles. The interviewer selected a set of triangles that have some common property: all isosceles, all right triangles, all obtuse etc and the learner was asked "All of these shapes are alike in some way. How are they alike?" The same was repeated for all the quadrilaterals.

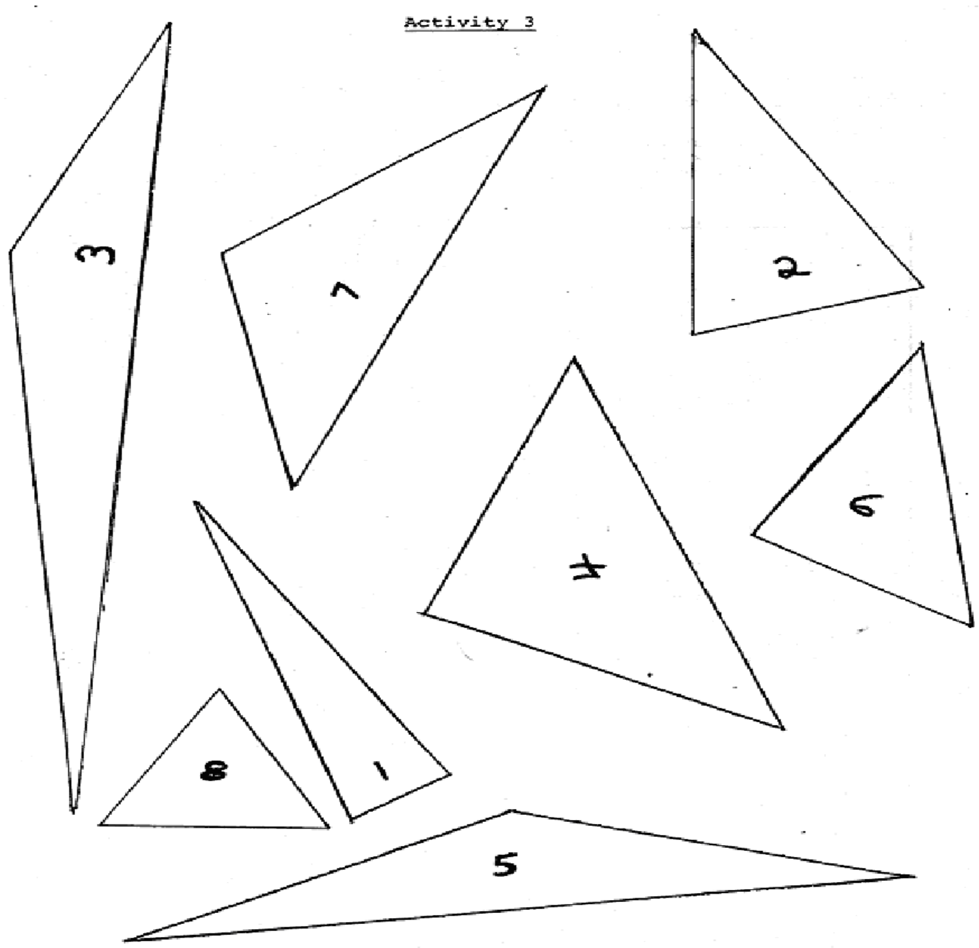


Figure 3:8: Activity 3A – Triangles

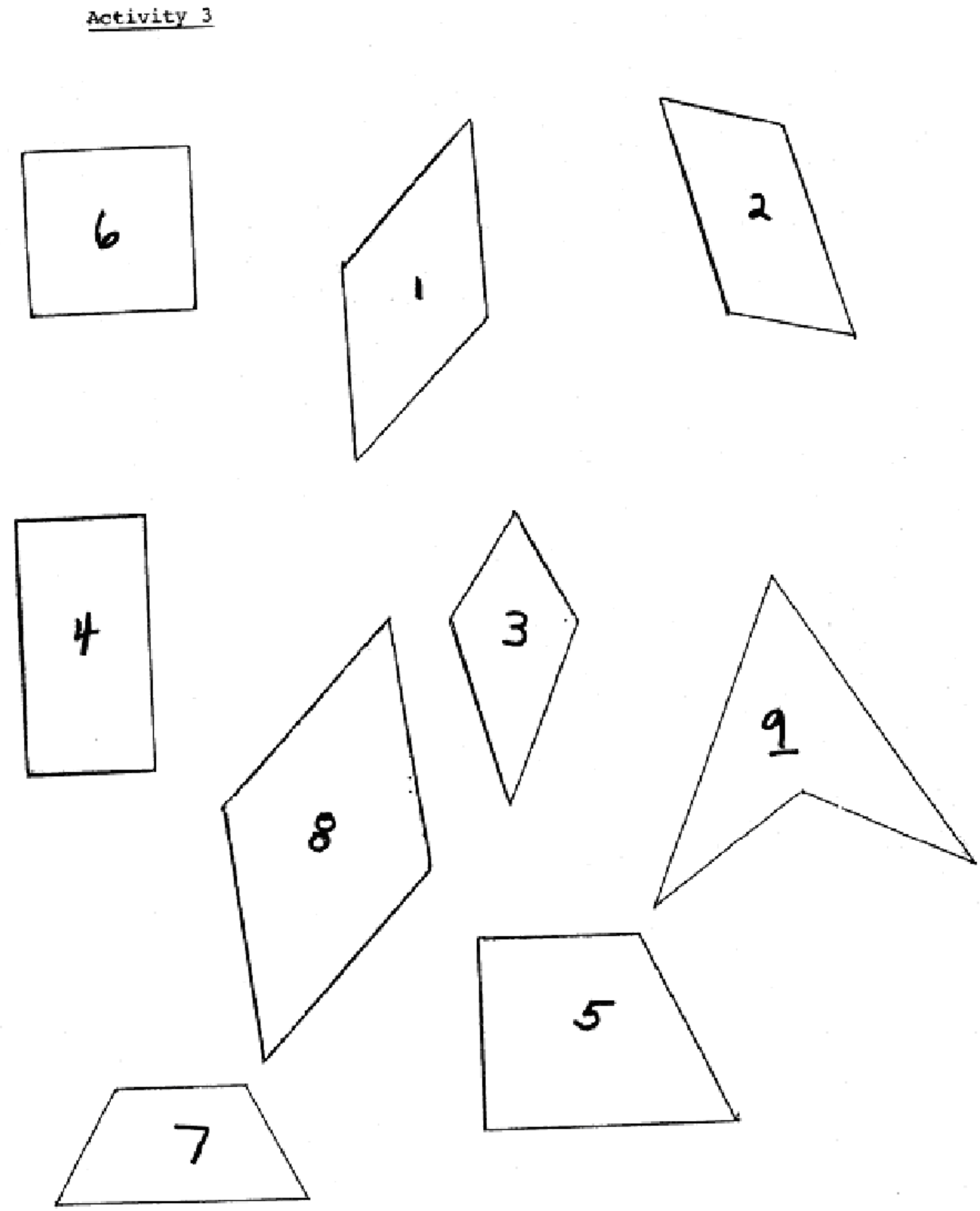


Figure 3.9: Activity 3B – Quadrilaterals

Mystery shape Task

This was a task to find the mysterious shape. The interviewer showed the learner a sheet of paper with some clues about certain shapes. The interviewer uncovered the clues one at a time and asked the learner to stop her when he /she has just enough clues to know for sure what type of shape it is and ask for another clue if he/she wants one. The learner was free to make a drawing of the shape if he/she wanted to. The learner was asked to think out loud if he/she wanted to and tell the interviewer what he/she was thinking about. This task elicited the role of necessary verses sufficient conditions to determine a shape. A sample is given below:

1. It is a closed figure with 4 straight sides.
2. All the sides are of the same length
3. One of the angles is 60^0
4. One of the angles is 120^0
5. Another angle is 60^0
6. Another angle is 120^0
7. Two sides are parallel
8. Two other sides are parallel
9. The diagonals are perpendicular
10. The diagonals bisect each other.

3.8.2. The reliability and validity of the interview tasks

The tasks that were used in the study were developed by Burger and Shaughnessy (1986), which was used to assess learners' geometric understanding at a specific van Hiele level. According to Burger and Shaughnessy, these tasks were developed to evaluate learners' basic geometric skills. The tasks were open ended and were designed to provide interpretation at several different van Hiele levels as the learners are at different levels of geometric understanding according to the van Hiele theory (Genz, 2006). There were three triangle tasks and four quadrilateral tasks which consisted of drawing shapes, identifying and defining shapes, sorting shapes, and logical reasoning about geometric shapes. The same tasks were used by Genz (2006) in her study on finding the difference between the geometrical thinking levels of standards based curriculum students and non standards based curriculum students in Utah. The present study adapted the same tasks with the consultation of the supervisor and Atebe, who is a researcher in van Hiele theory. This ensured the reliability and validity of the tasks. The tasks are attached in the Appendix D.

3.9. Chapter summary

This chapter discussed the whole research methodology of the present study. It started with the explanation of the framework and the activities that were developed for the instructional framework. The research design with its different phases involved in the study, the research sites, the participants and the way in which they were selected were also discussed in detail. The research instrument, the van Hiele Geometry Test, its rationale, its validity and reliability were also described in this chapter. The interviews schedule, its tasks and rationale were also discussed. In the next chapter, the data collected is presented and analysed.

CHAPTER 4

DATA ANALYSIS AND RESULTS – THE VAN HIELE GEOMETRY TEST – QUANTITATIVE DATA

"Analysis tries to make sense of data and one of the purposes of analysis is to find explanations which 'fit' our understanding".

(Altrichter, Posch & Somekh, 1995, p.120)

4.1. Introduction

In this chapter, the analysis of learners' performance in the van Hiele Geometry Test is presented. The study was undertaken mainly to find the effectiveness of a van Hiele-based instructional framework in grade 10. There were two research questions. The first research question of the study concerns the determination of the van Hiele levels of the learners in the study. The chapter begins with the analysis of the learners' performance by examining the percentage mean scores of the test and then allocating them into van Hiele levels. The second research question concerns the effectiveness of the van Hiele-based instructional framework. The second part of the chapter provides information on the effectiveness of the van Hiele-based instruction by comparing the percentage mean scores of the experimental and control groups from each of the five schools for the pretest and the posttest and comparing the number of learners in each of the van Hiele levels.

4.2. Focus one

What are the geometrical thinking levels of the learners in the sample?

The learners' performance in the van Hiele Geometry Test which was administered as a pretest was taken as the measure to find the geometrical thinking level of the learners. First it was analysed in terms of the percentage mean and then it was analysed in terms of the percentage number of learners in each level of the van Hiele levels according to the criterion developed by Usiskin (1982).

4.2.1. Analysis of the current van Hiele level of the learners in the van Hiele Geometry Test (pretest) according to percentage means

In the analysis that follows, the performance of the learners is provided by examining the percentage mean scores in the pretest. It is discussed under the sections 4.2.1.1 to 4.2.1.4.

4.2.1.1. Overall performance of the participants in the pretest

An analysis of the mean scores percentage was carried out to check the performance of all the learners who participated in the study.

Table 4.1: Overall performance of the participants in the pretest

School	Number	Mean score out of 20	Percentage mean scores (%)
School A	78	6.88	34.38
School B	107	6.35	31.73
School C	57	5.63	26.94
School D	65	7.86	39.29
School E	52	6.66	33.37
Total	359	6.68	33.14

Table 4.1 showed that the performance of the learners in the pretest from the different schools was almost the same even though they were relatively low percentages.

The learners in the sample obtained the mean score of 6.68 out of the maximum of 20 marks. This figure represents an overall percentage mean score of 33.14%. However, learners from different schools performed differently and hence some schools were closer to the overall percentage mean score. For example, School D had the highest percentage (39.29%) and School C obtained the lowest percentage (26.94%). Schools A, B and E had 34.38%, 31.37% and 33.37% respectively and were spread closer to the overall percentage mean scores.

4.2.1.2. Analysis of the overall percentage mean scores of all participants in the pretest according to experimental group and control group

Further analysis was carried out to check on the performance of the experimental group and the control group in the pretest in the entire study sample.

Table 4.2: Percentage mean scores of all learners in the pretest according to experimental group and control group

Overall percentage mean scores of all participants in the pretest	
Experimental group	32.74
Control group	33.54

Table 4.2 showed that the performance of all the learners in the pretest according to experimental group and control group was almost the same even though they were relatively low percentages.

Further analysis was carried out to determine the performance of learners in each school according to the experimental group and control groups.

4.2.1.3. Analysis of the learners' performance in the pretest according to experimental group and control group per school

Table 4.3: Learners' performance in the pretest according to experimental group and control group per school

School	Total number	Group	Number	Percentage mean score
School A	78	Experimental group	45	35.11
		Control group	33	33.64
School B	107	Experimental group	55	28.64
		Control group	52	34.81
School C	57	Experimental group	32	26.88
		Control group	25	27.0
School D	65	Experimental group	29	41.9
		Control group	36	36.67
School E	52	Experimental group	34	31.18
		Control group	18	35.56

Table 4.3 showed that the experimental group in School D had the highest percentage mean score (41.9%) and School C's experimental group had the lowest percentage mean score (26.88%).

In School A, learners from the experimental group obtained a percentage mean score of 35.11% which was slightly higher than the learners in the control group which scored 33.64%. The classes seemed to be similar in their performance.

In School B, the experimental group learners obtained a percentage mean score of 28.64% which was slightly lower than the learners in the control group which scored 34.81%. The learners in the control group performed better than the learners in the experimental group.

In School C, the experimental group learners obtained a percentage mean score of 35.11% which was slightly higher than the learners in the control group which scored 33.64%. The classes seemed to be similar in their performance.

In School D, the experimental group learners obtained a percentage mean score of 41.9% which was slightly higher than the learners in the control group which scored 36.67%. The experimental group's performance was better than that of the control group's.

In School E, the experimental group learners obtained a percentage mean score of 31.17% and the learners in the control group scored the percentage mean score of 35.56%. The learners in the control group performed better than the learners in the experimental group.

Further analysis involved checking whether there was a statistical difference in the percentage mean scores of the learners between the experimental group and control group in each school.

4.2.1.4. Statistical comparison of the learners' performance in the pretest according to experimental group and control group per school using t-test

The statistical analysis was compiled by use of IBM SPSS Version 19 and it was explained in Chapter 3 under the section 3.7.1.5. The independent samples 2-tailed t-test was used here to determine whether there was a statistically significant difference between the two classes taken from each school so as to verify whether the learners were of equal ability. The t-test scores were used to compare the attainment of levels for each group from all the five schools.

Table: 4.4: Learners' performance according to experimental group and control group per school in the pretest

School	Group	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
School A	Experimental group	45	35.11	9.32	76	0.694	0.245
	Control group	33	33.64	9.21			
School B	Experimental group	55	28.64	10.65	105	2.97	0.002
	Control group	52	34.81	10.84			
School C	Experimental group	32	26.88	11.34	55	0.043	0.483
	Control group	25	27.00	10.51			
School D	Experimental group	29	41.9	12.42	63	1.761	0.042
	Control group	36	36.67	11.46			
School E	Experimental group	34	31,18	12.80	50	1.394	0.085
	Control group	18	35,56	9.53			

As shown in Table 4.4, the test of significance indicated that the difference in the percentage mean scores between the experimental group and control group of School A, School C, School D and School E were not statistically significant at the 0.05 level of significance for a 2-tailed t-test while that of school B was statistically significant. This showed that the learners in the experimental group and the control group of School A, School C, School D and School E were of equivalent ability in terms of their performance in the test. In other words the different classes in the majority of the schools were equivalent in their performance in the van Hiele Geometry Test (pretest).

4.2.2. Analysis of the participants' performance in the pretest by gender

Further analysis was carried out to check whether there was a difference in the performance by gender.

4.2.2.1 Overall participants' performance in the pretest by gender

Learners' performance in the pretest was further analysed for a possible gender difference in the entire study sample.

Table 4.5: Overall participants' performance in the pretest by gender

School	Percentage mean score in the pretest	
	Male	Female
School A	34.00	34.79
School B	30.97	32.56
School C	26.60	27.19
School D	42.50	37.44
School E	31.92	33.46
Overall percentage mean	32.42	33.47

Table 4.5 showed that in the entire sample, there was a slight difference in the performance in the pretest in favour of the female learners. Male learners obtained a percentage mean score of 32.42% and female learners obtained a percentage mean score of 33.47%.

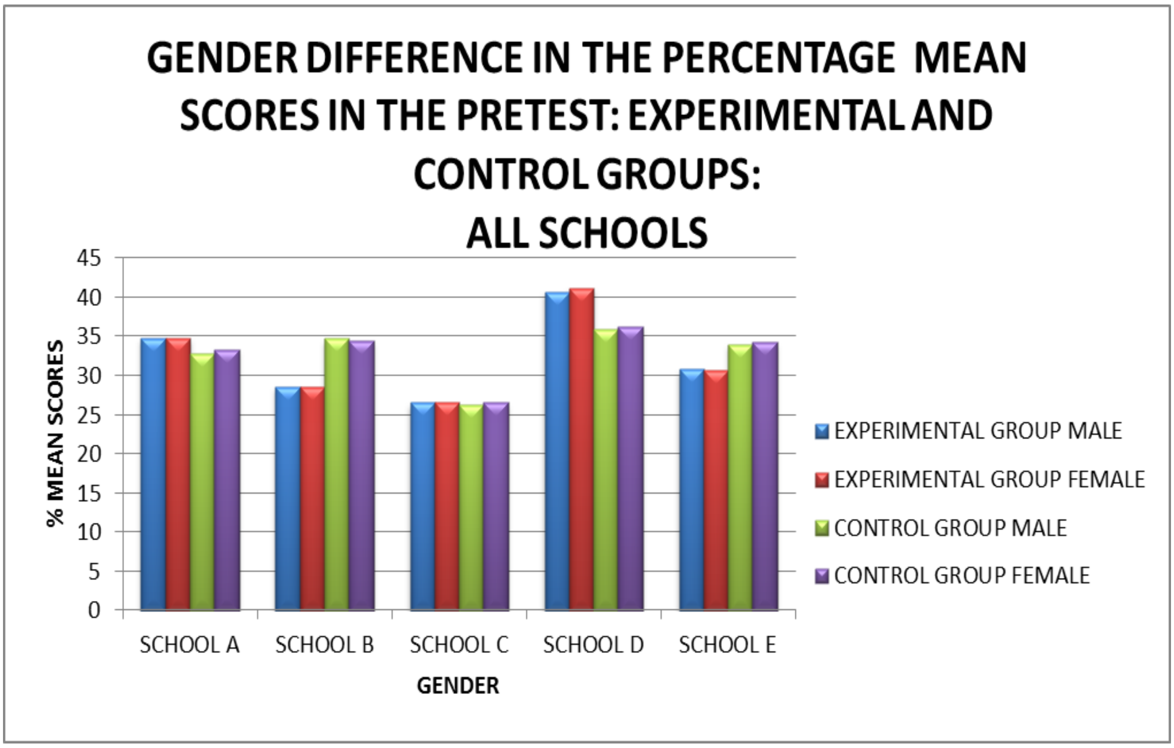


Chart 4.1: Gender difference in the percentage mean scores in the pretest: Experimental and control groups: All schools

Chart 4.1 also showed that, the percentage mean scores of the learners in the experimental group and control group according to gender were almost the same in all schools.

Further analysis was carried out to check whether there was a statistical difference in the percentage means scores of the learners by gender in each school.

4.2.2.2. Statistical comparison of the participants' performance in the pretest according to gender per school

Table 4.6: Learners' performance in the pretest according to gender in School A

School	Gender	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
School A N = 78	Male	30	34.00	8.85	76	0.366	0.358
	Female	48	34.79	9.56			

This test of significance indicated that the difference in the percentage mean scores between the male learners and female learners of School A was not statistically significant. This means that the male learners and female learners in School A were similar in their performance.

Table 4.7: Learners' performance in the pretest according to gender in School B

School	Gender	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
School B N=107	Male	62	30.97	10.90	105	0.727	0.235
	Female	45	32.56	11.51			

This test of significance indicated that the difference in the percentage mean scores between the male learners and female learners of School B was not statistically significant. This means that the male learners and female learners in School B were similar in their performance.

Table 4.8: Learners' performance in the pretest according to gender in School C

School	Gender	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
School C N=57	Male	25	26.6	11.34	55	0.200	0.421
	Female	32	27.19	10.7			

This test of significance indicated that the difference in the percentage mean scores between the male learners and female learners of School C was not statistically significant. It means that the male learners and female learners in School C were similar in their performance.

Table 4.9: Learners' performance in the pretest according to gender in School D

School	Gender	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
School D N=65	Male	20	42.50	11.52	63	1.573	0.061
	Female	45	37.44	12.14			

This test of significance indicated that the difference in the percentage mean scores between the male learners and female learners of School D was not statistically significant. This means that the male learners and female learners in School D were similar in their performance.

Table 4.10: Learners' performance in the pretest according to gender in School E

School	Gender	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
School E N=52	Male	26	31.92	13.79	50	0.464	0.323
	Female	26	33.46	9.77			

This test of significance indicated that the difference in the percentage mean scores between the male learners and female learners of School E was not statistically significant. This means that the male learners and female learners in School E were similar in their performance.

Table 4.11: Learners' performance in the pretest according to gender in all schools combined

Gender	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
Male	163	32.42	11.91	357	0.855	0.197
Female	196	33.47	11.24			

This test of significance indicated that the difference in the percentage mean scores between the male learners and female learners of the entire study sample was not statistically significant. This means that the male learners and female learners in the entire study sample were similar in their performance.

It can be assumed that the overall achievement in the van Hiele Geometry Test (pretest) was independent of gender or gender did not play a role in the performance of the entire sample.

4.2.3. Analysis of the learners' performance in the pretest according to the van Hiele levels

The van Hiele Geometry Test consisted of 4 subtests, with each subtest being made up of 5 items testing learners' attainment of a specific van Hiele level (see section 3.7.1 in Chapter 3). In the test, items 1 – 5 tested learners' attainment of level 1, items 6 – 10 tested attainment of level 2, items 11 – 15 tested attainment of level 3 and items 16 – 20 tested attainment of level 4. In the following sections, learners' performance is presented in two ways. The first section discusses the performance in terms of the mean score percentages and the second section discusses the assignment of learners into levels based on the '3 of 5 correct" success criterion as suggested by Usiskin (1982, p.22).

4.2.3.1. Mean scores of learners at each van Hiele level in the van Hiele Geometry Test

The tests were scored using Microsoft Excel by a computer assistant where the researcher is working and were later verified by the researcher and a grade 10 mathematics educator. Each correct response was given 1 mark. Hence each learner's total score ranges in between 1 – 20 in the van Hiele Geometry Test. The percentage scores were calculated using Microsoft Excel.

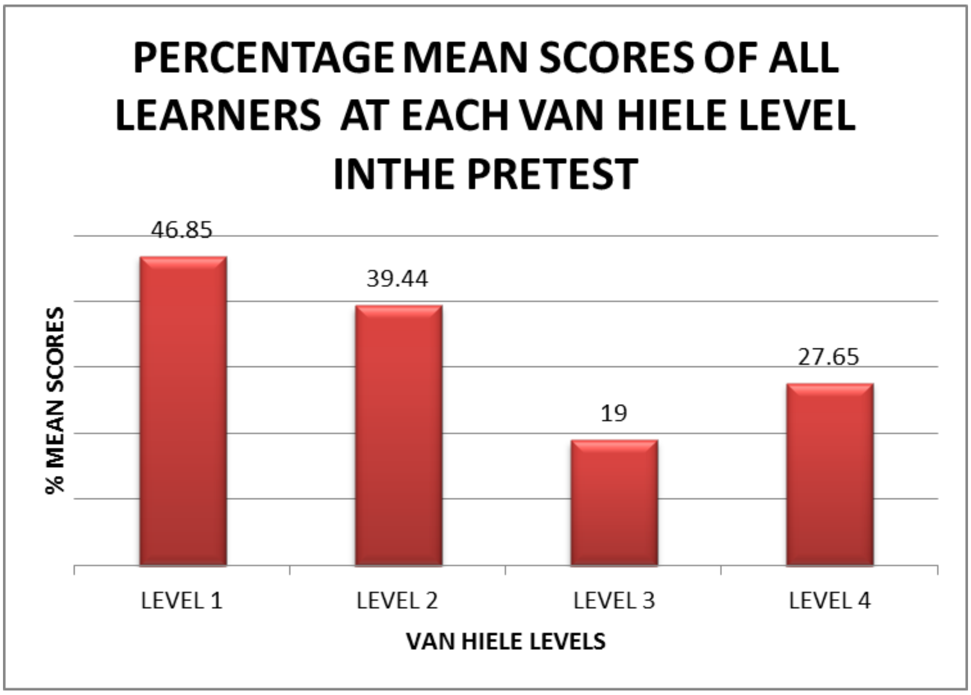


Chart 4.2: Percentage mean scores of all learners at each van Hiele level in the pretest

It was evident from Chart 4.2 that the highest percentage of mean score was at level 1 (46.85%) followed by 39.44% at level 2, 19% at level 3 and 27.65% at level 4. This showed a decrease in the percentage mean score in each successive higher level except at level 3 which was the lowest.

4.2.3.2. Overall percentage mean scores of learners at each van Hiele level in the van Hiele Geometry Test per school

A school wise analysis was done to find out the percentage of learners at each van Hiele level from each school.

Table 4.12: Percentage mean scores of learners at each van Hiele level: all schools

	Percentage mean scores of learners at each van Hiele level: All schools				
Van Hiele levels	School A	School B	School C	School D	School E
Level 1	50.51	41.41	37.65	65.08	39.61
Level 2	43.62	34.88	25.52	47.22	45.98
Level 3	15.72	17.91	21.59	20.36	19.41
Level 4	27.66	32.7	23.75	25.68	28.47

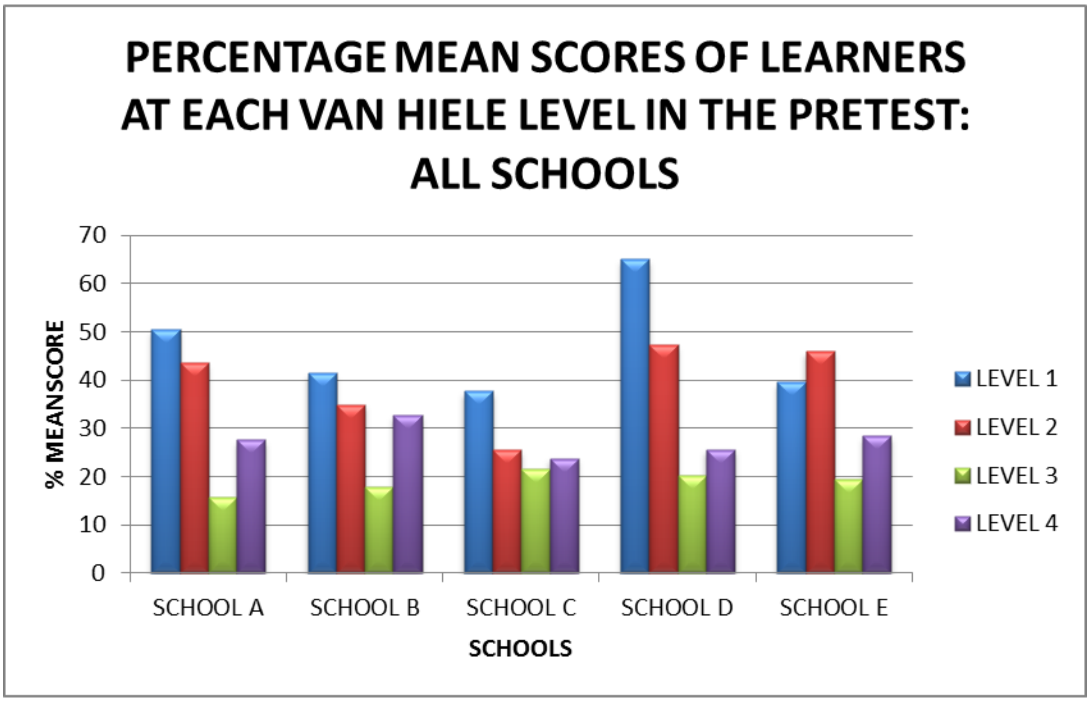


Chart 4.3: Percentage mean scores of learners at each van Hiele level in the pretest: All schools

As evident from Table 4.12 and Chart 4.3, School D had the highest percentage mean at level 1 and level 2 and School C's performance was the best at level 3 and School B's performance was the best at level 4. In all schools, percentage mean score was the lowest at level 3.

In School A, the percentage mean score of learners at levels 1, 2, 3 and 4 were 50.51%, 43.62%, 15.72% and 27.66% respectively. It was noted that the school had the highest percentage mean score at level 1 and the lowest at level 3.

In School B, the percentage mean score of learners at levels 1, 2, 3 and 4 were 41.41%, 34.88%, 17.91% and 32.7% respectively. It was noted that the school had the highest percentage mean score at level 1 and the lowest at level 3.

In School C, the percentage mean score of learners at levels 1, 2, 3 and 4 were 37.65%, 25.52%, 21.59% and 23.75% respectively. It was noted that the school had the highest percentage mean score at level 1 the lowest at level 3. School C's percentage score was lower than all the other schools in all levels except level 3.

In School D, the percentage mean score of learners at levels 1, 2, 3 and 4 were 65.08%, 47.22%, 20.36% and 25.68% respectively. It was noted that the school had the highest percentage mean score at level 1 and the lowest at level 3. School D's performance was better than the other schools at level 1 and level 2.

In School E, the percentage mean score of learners at levels 1, 2, 3 and 4 were 39.61%, 45.98%, 19.41% and 28.47% respectively. It was noted that the school had the highest percentage mean score at level 2 and the lowest at level 3.

The performance in all the levels in all the schools was similar.

4.2.3.3. Percentage mean scores of learners at each van Hiele level in the van Hiele Geometry Test in the experimental and control groups per school

A further analysis was carried out to determine the percentage mean scores of learners at each van Hiele level in the experimental and control groups in each school.

School A

Table: 4.13: Percentage mean scores of learners at each van Hiele level in School A

Percentage mean scores of learners at each van Hiele level			
Van Hiele levels	Description	Pretest	
		Experimental group (N = 45)	Control group (N = 33)
Level 1	Recognition	48.89	52.12
Level 2	Analysis	41.78	45.46
Level 3	Informal deduction	23.56	7.88
Level 4	Deduction	26.22	29.09

It was evident from Table 4.13 that in School A, the percentage mean scores of learners at levels 1, 2, 3 and 4 were 48.89%, 41.78%, 23.56% and 26.22% respectively for the experimental group and were 52.12%, 45.46%, 7.88% and 29.09% respectively for the control group. The experimental group performed better at level 3 only than that of the control group.

School B

Table: 4.14: Percentage mean scores of learners at each van Hiele level in School B

Percentage mean scores of learners at each van Hiele level			
Van Hiele levels	Description	Pretest	
		Experimental group (N = 55)	Control group (N = 52)
Level 1	Recognition	37.82	45
Level 2	Analysis	30.91	38.85
Level 3	Informal deduction	19.27	16.54
Level 4	Deduction	26.55	38.85

It was evident from Table 4.14 that in School B, the percentage mean scores of learners at levels 1, 2, 3 and 4 were 37.82%, 30.91%, 19.27% and 26.55% respectively for the experimental group and were 45%, 38.85%, 16.54% and 38.85% respectively for the control group. The experimental group performed better only at level 3.

School C

Table: 4.15: Percentage mean scores of learners at each van Hiele level in School C

Percentage mean scores of learners at each van Hiele level			
Van Hiele levels	Description	Pretest	
		Experimental group (N = 32)	Control group (N = 25)
Level 1	Recognition	42.5	32.8
Level 2	Analysis	20.63	30.4
Level 3	Informal deduction	19.98	23.2
Level 4	Deduction	25	22.4

It was evident from Table 4.15 that in School C, the percentage mean scores of learners at levels 1, 2, 3 and 4 were 42.5%, 20.63%, 19.98% and 25% respectively for the experimental group and were 32.8%, 30.4%, 23.2% and 22.4% respectively for the control group. The experimental group performed better at level 1 and level 4. It was noted that the performance at level 3 of the control group is better than that of the experimental group at level 4.

School D

Table 4.16: Percentage mean scores of learners at each van Hiele level in School D

Percentage mean scores of learners at each van Hiele level			
Van Hiele levels	Description	Pretest	
		Experimental group (N = 29)	Control group (N = 36)
Level 1	Recognition	77.93	52.22
Level 2	Analysis	44.14	50.29
Level 3	Informal deduction	17.93	22.78
Level 4	Deduction	28.57	22.78

It was evident from Table 4.16 that in School D, the percentage mean scores of learners at levels 1, 2, 3 and 4 were 77.93%, 44.14%, 17.93% and 28.57% respectively for the experimental group and were 52.22%, 50.29%, 22.78% and 22.78% respectively for the control group. The experimental group performed better at level 1 and level 4.

School E

Table 4.17: Percentage mean scores of learners at each van Hiele level in School E

Percentage mean scores of learners at each van Hiele level			
Van Hiele levels	Description	Pretest	
		Experimental group (N = 29)	Control group (N = 36)
Level 1	Recognition	35.88	43.33
Level 2	Analysis	45.29	46.67
Level 3	Informal deduction	18.82	20
Level 4	Deduction	24.71	32.22

It was evident from Table 4.17 that in School E, the percentage mean scores of learners at levels 1, 2, 3 and 4 were 35.88%, 45.29%, 18.82% and 24.71% respectively for the experimental group and were 43.33%, 46.67%, 20% and 32.22% respectively for the control group. The control group's performance was better in all levels.

4.2.4. Assignment of learners into different van Hiele levels of thinking

The grading of the van Hiele Geometry Test was done again using a second method which was based on the '3 of 5 correct" success criterion as suggested by Usiskin (1982, p.22) to assign learners into different van Hiele levels. It is explained in sections 3.7.1.3 and 3.7.1.4 in Chapter 3.

In this study only the classical/modified van Hiele level is considered as the study does not intend to compare the same learner in two different types of levels. The percentage number of learners, rounded off to the nearest whole number, at each van Hiele level was calculated for the entire sample in the study. The results are shown in Chart 4.4.

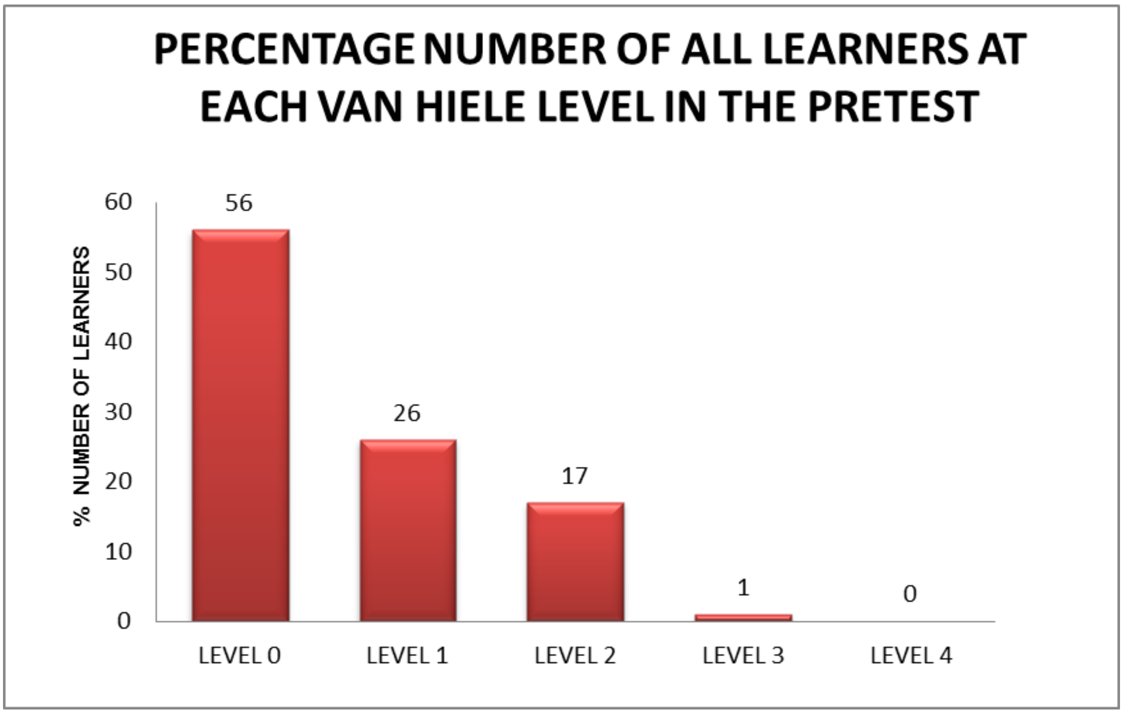


Chart 4.4: Percentage number of all learners at each van Hiele in the pretest

Chart 4.4 showed that the majority of the learners were at level 0 (56%). For the van Hiele levels 1, 2, 3, and 4, it was 26%, 17%, 1% and 0% respectively.

4.2.4.1. Analysis of percentage number of learners at each van Hiele level

A further analysis was done to find the percentage number of learners at each van Hiele level in the different schools. The analysis is presented in Table 4.18 and Chart 4.5.

Table 4.18: Percentage number of learners at each van Hiele Level: All schools

	Percentage number of learners at each van Hiele level : All schools				
Van Hiele levels	School A	School B	School C	School D	School E
Level 0	54	63	70	29	65
Level 1	23	27	26	34	14
Level 2	23	10	4	31	21
Level 3	0	0	0	6	0
Level 4	0	0	0	0	0

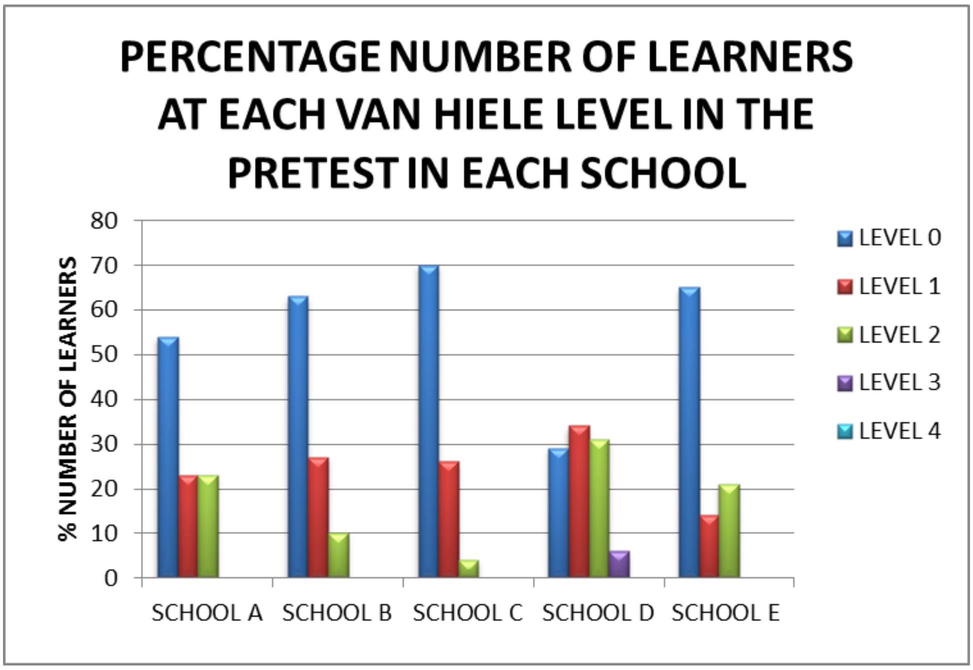


Chart 4.5: Percentage number of learners at each van Hiele level in the pretest in each school

As can be seen from Table 4.18 and Chart 4.5, the majority of the learners in all schools were at level 0 except for school D which had only 29% at level 0. School C had the highest number of learners at level 0 (70%) followed by School E (65%), School B (63%) and School A (54%). Level 3 was achieved by no learners in all the schools except by 6% of learners in School D. None of the schools had learners at level 4 thinking on the van Hiele scale.

4.2.4.2. Analysis of percentage number of learners at each van Hiele level per school

School A

Table 4.19: Percentage number of learners at each van Hiele level: School A

	Percentage number of learners at each van Hiele level	
Van Hiele levels	Experimental group	Control group
Level 0	60	46
Level 1	22	24
Level 2	18	30
Level 3	0	0
Level 4	0	0

It was evident from Table 4.19 that in School A, the majority of the learners were at level 0 (60% in the experimental group and 46% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 22%, 18%, 0% and 0% respectively in the experimental group and it was 24%, 30%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and the control group's performance was better than the experimental group. The school had no learners at level 3 and level 4.

School B

Table 4.20: Percentage number of learners at each van Hiele level: School B

	Percentage number of learners at each van Hiele level	
Van Hiele levels	Experimental group	Control group
Level 0	65	59
Level 1	24	31
Level 2	11	10
Level 3	0	0
Level 4	0	0

It was evident from Table 4.20 that in School B, the majority of the learners were at level 0 (65% in the experimental group and 59% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 24%, 11%, 0% and 0% respectively in the experimental group and it was 31%, 10%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and the control group's performance was better than the experimental group at level 1 and the experimental group had more learners at level 2. The school had no learners at level 3 and level 4.

School C

Table 4.21: Percentage number of learners at each van Hiele level: School C

	Percentage number of learners at each van Hiele level	
Van Hiele levels	Experimental group	Control group
Level 0	66	76
Level 1	34	16
Level 2	0	8
Level 3	0	0
Level 4	0	0

It is evident from Table 4.21 that in School C, the majority of the learners were at level 0 (66% in the experimental group and 76% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 34%, 0%, 0% and 0% respectively in the experimental group and it was 16%, 8%, 0% and 0% respectively in the control group. The experimental group had more learners at level 1 and had no learners at level 2. The control group's performance was lower than the experimental group. The school had no learners at level 3 and level 4.

School D

Table 4.22: Percentage number of learners at each van Hiele level: School D

	Percentage number of learners at each van Hiele level	
Van Hiele levels	Experimental group	Control group
Level 0	10	44
Level 1	45	25
Level 2	38	25
Level 3	7	6
Level 4	0	0

It was evident from Table 4.22 that in School D, for the van Hiele levels 0, 1, 2, 3, and 4, it was 10%, 45%, 38%, 7% and 0% respectively in the experimental group and it was 44%, 25%, 25%, 6% and 0% respectively in the control group. The experimental group had more learners at level 1, 2 and 3 and its performance was better than the control groups. The school had no learners at level 4.

School E

Table 4.23: Percentage number of learners at each van Hiele level: School E

	Percentage number of learners at each van Hiele level	
Van Hiele levels	Experimental group	Control group
Level 0	68	61
Level 1	9	22
Level 2	23	17
Level 3	0	0
Level 4	0	0

It was evident from Table 4.23 that in School, the majority of the learners were at level 0 (68% in the experimental group and 61% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 9%, 23%, 0% and 0% respectively in the experimental group and it was 22%, 17%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and at level 2. The school had no learners at level 3 and level 4.

The findings from the research focus one is discussed below as it forms the basis for the analysis of research focus two.

4.2.5. Findings and implications for teaching from the analysis of research focus one

The analysis of the levels of thinking showed that most of the learners were at level 0. This was an indication that the majority of learners had difficulty in recognising figures and figures in non-standard positions. Learners must understand that geometric shapes are defined by their properties and not by their orientation in space.

The low achievement at level 3 shows that the learners are not ready for formal proof in Euclidean geometry as it used to be the level expected of senior secondary school learners.

These findings lead to the importance on the delivery of instruction that is appropriate to learners' level of thinking. As explained in Chapter 2 (see section 2.6) many studies conducted in different parts of the world (Hoffer, 1981; Usiskin, 1982; Senk, 1985; Shaughnessy & Burger, 1985; Fuys, et al., 1988; Clements & Battista, 1992; King, 2003; Atebe, 2008) highlighted that learners' poor performance in geometry holds account for geometry classroom teaching.

In many western countries, the van Hiele theory has become the most influential factor in their geometry curriculum (Fuys, et al., 1988; van de Walle, 2004). Malloy

(2002) states that in implementing instruction based on the van Hiele framework, teachers need to recognise and understand the van Hiele levels of their students, and they need to help their students' progress through these levels in preparation for the axiomatic, deductive reasoning that is required in high school geometry. This was also supported by NCTM (2000).

Van Hiele (1999) points out that high school learners lack the prerequisite understandings about geometry and this lack creates a gap between their level of thinking and that required for the geometry that they are expected to learn.

As explained in section 3.2.1. in Chapter 3, geometric experience is the greatest single factor that influences the advancement through the levels. Activities that permit children to explore talk about and interact with content at the next level, while increasing their experiences at their current level, have the best chance of advancing the level of thoughts for those children (van de Walle, 2004).

Instruction intended to foster development should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and concluding in summary activities that help learners assimilate what they have learned into what they already know. Rich and stimulating instruction in geometry can be provided through playful activities with mosaics and tangram puzzles etc (van Hiele, 1999).

Promoting the transition from one level to the next should follow a five phase sequence of activities. As mentioned earlier in Chapter 2 (see section 2.4), throughout these phases, the teacher has to plan tasks, direct children's attention to geometric qualities of shapes, introduce terminology and engage children in discussions using these terms and encourage explanations and problem solving approaches that make use of children's descriptive thinking about shapes. Cycling through these five phases with materials like the mosaic puzzle enables children to build a rich background in visual and descriptive thinking that involves various shapes and their properties (van Hiele 1999).

This present study looked into the possibilities of improving the geometry education by introducing van Hiele-based instruction after determining the level of geometric thinking and it was implemented in the five schools under the quasi-experimental design. In the next section, the effectiveness of the framework is analysed.

4.3. Focus two

Can the researcher's developed instructional framework improve the geometrical thinking levels of the learners in the sample?

One of the objectives of the study was to develop an instructional framework in line with the van Hiele levels after determining the present level of thinking of the learners. The analysis that was presented in the previous section has indicated that most of the learners in the sample (56%) were at level 0 followed by 26% at level 1, 17% at level 2 and 1% at level 3 and no one at level 4.

Originally, when the study was planned, it was assumed that the learners might have been evenly distributed at level 3. But after answering research question 1, it was discovered that the assumption was challenged by the data. The majority of the learners were on level 0 and the instructional framework was tuned to cater for the majority. It would have been ideal to cater for each level separately. Nonetheless, resources, time and logistics involved with the timeframes of instruction at the schools participating in the study did not favour differentiated instruction to such heterogeneous group levels. In order to move on with the study and complete it, all the learners in the experimental group were given the same instruction.

Based on the above analysis the study developed the instructional framework to cater for the majority and implemented it in the five schools. One class was chosen by the respective mathematics teachers as an experimental group for the implementation. The other class was taught by the same teachers where they did not use the framework for teaching. The framework, the methodology and the research design are all explained in detail in Chapter 3.

To achieve the main purpose of the study as whether the researcher's developed framework made any improvement in the levels of geometric thinking, it is imperative to measure the shift in performance of the experimental group before and after the intervention with the framework. The same test was administered as a posttest on all the learners in the sample and a statistical analysis was conducted. For that a paired samples two-tailed t-test was used for comparison and the analysis is presented in the following sections.

4.3.1.1. Analysis of the overall percentage mean scores of all the learners in the pretest and the posttest for the van Hiele Geometry Test

Table 4.24: Overall percentage mean scores of all the learners in the pretest and the posttest

Overall percentage mean scores of all the learners in the pretest and the posttest	
Pretest	Posttest
32.99	39.97

Table 4.24 showed that the overall percentage mean of all the learners in the posttest was found to be 39.97% which was higher than the overall percentage mean of 32.99% in the pretest. This showed an improvement in the performance of the learners in the posttest.

4.3.1.2. Overall percentage mean scores of all learners in the pretest and posttest according to experimental and control groups

A further analysis was carried out to determine the performance of the experimental group and the control group in entire study sample.

Table 4.25: Percentage mean scores of all learners in the pretest and posttest according to experimental and control groups

Percentage mean scores	Experimental group		Control group	
	Pretest	Posttest	Pretest	Posttest
	32.74	42.7	33.54	36.99

It was evident from Table 4.25 that there was an increase in the percentage mean score of both groups in the posttest. The experimental group's percentage mean score increased from 32.74% to 42.7% and the control group's percentage mean score increased from 33.54% to 36.99%. The percentage increase in the experimental group was higher than that of the control group.

4.3.1.3. Percentage mean scores of all learners in the pretest and posttest according to experimental and control groups in each school

A further analysis was carried out to check on the performance of the experimental group and the control group in each of the schools.

Table 4.26: Percentage mean scores of experimental group and control group in each of the schools

Percentage mean scores	Experimental group		Control group	
	Pretest	Posttest	Pretest	Posttest
School A	35.11	44.67	33.64	38.18
School B	28.64	38.73	34.81	38.37
School C	26.88	35.94	27.00	30.20
School D	41.90	52.41	36.67	39.03
School E	31.17	41.76	35.56	39.17

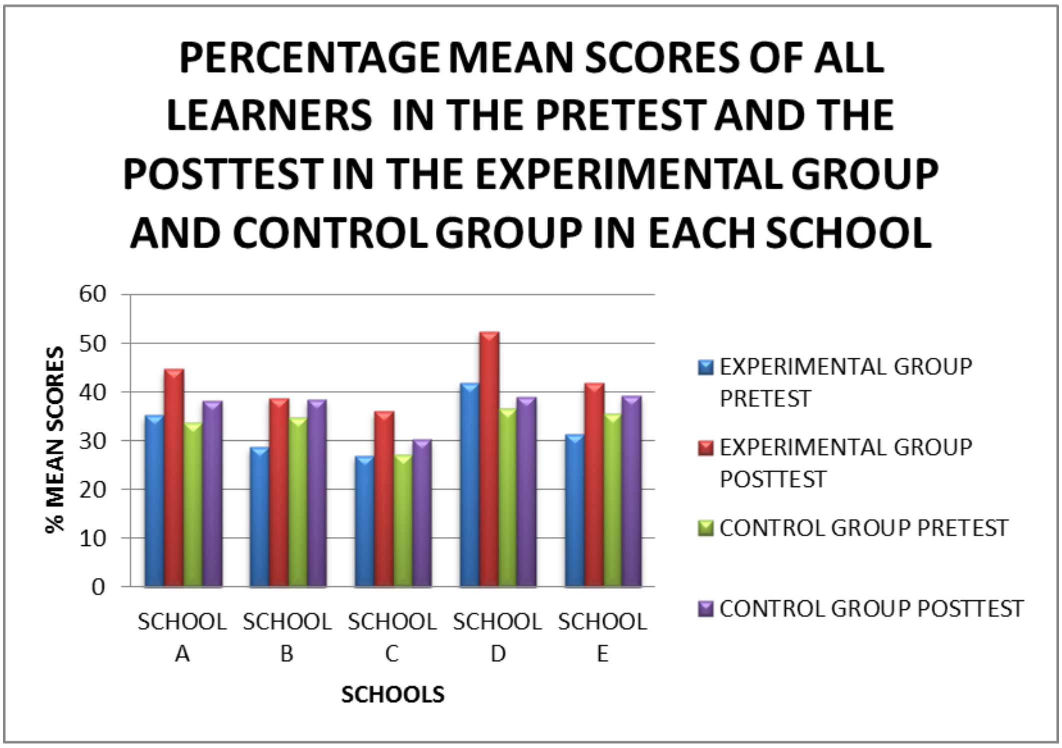


Chart 4.6: Percentage mean scores of all learners in the pretest and the posttest in the experimental group and control group in each school

Table 4.26 and the Chart 4.6 showed that the performance of the learners in the posttest from the different schools for both experimental and control group were higher than their performance in the pretest even though they were relatively low percentages. The percentage increase in the experimental group was higher than that of the control group.

Table 4.27 below showed a further separate analysis of experimental and control groups and the discussion is given in detail separately.

Table 4.27: Percentage mean scores of the pretest and the posttest of experimental group in each school

Percentage mean scores	Experimental group	
	Pretest	Posttest
School A	35.11	44.44
School B	28.64	38.73
School C	26.88	35.94
School D	41.9	52.41
School E	31.17	41.76

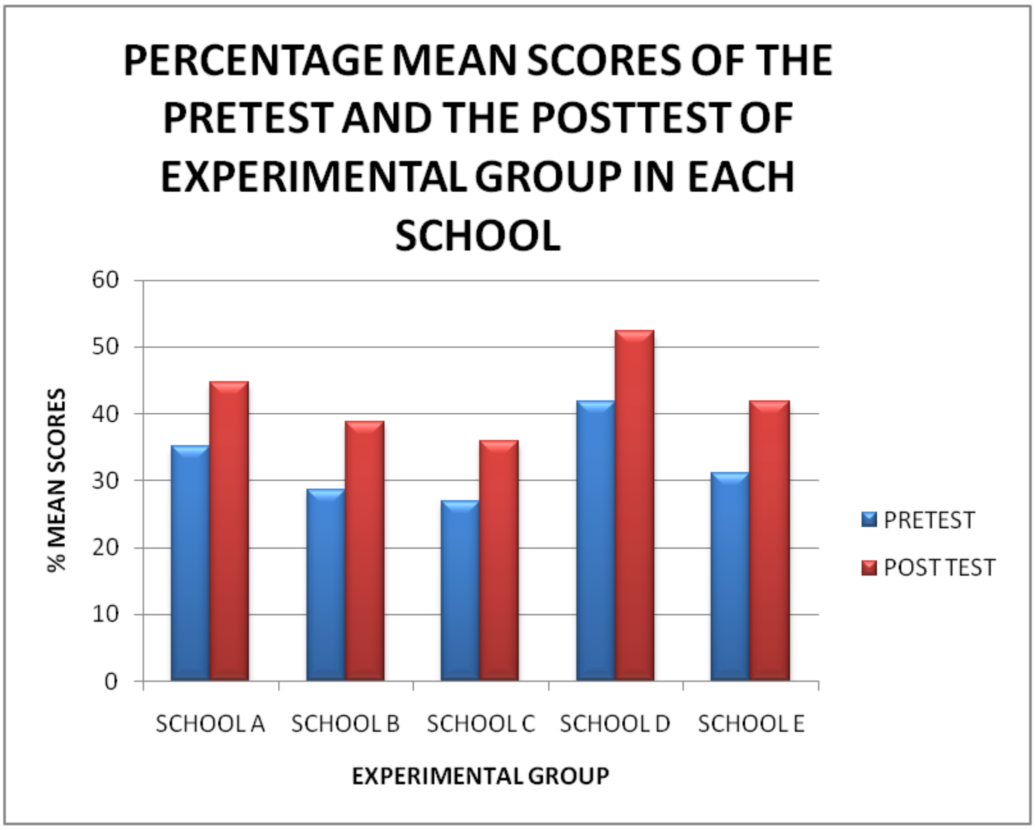


Chart 4.7: Percentage mean scores of the pretest and the posttest of experimental group in each school

It was evident from Table 4.27 and Chart 4.7 that there was a substantial increase in the percentage mean scores of the experimental group in all schools with the highest difference noticed in School D. School A's percentage increased from 35.11% to 44.67%, School B's percentage increased from 28.64% to 38.73%, School C's percentage increased from 26.88% to 35.94%, School D's percentage increased from 41.9% to 52.41% and School E's percentage increased from 31.17% to 41.76%.

It can be taken that the van Hiele-based instruction had a positive effect on the performance of the learners in the experimental group although other factors like maturation and history could also have played a part.

The analysis of the percentage mean scores of the learners in the control group in each school is shown below.

Table 4.28: Percentage mean scores of the learners in the control group in each school

Percentage mean scores	Control group	
	Pretest	Posttest
School A	33.64	38.18
School B	34.81	38.37
School C	27.00	30.20
School D	36.67	39.03
School E	35.56	39.17

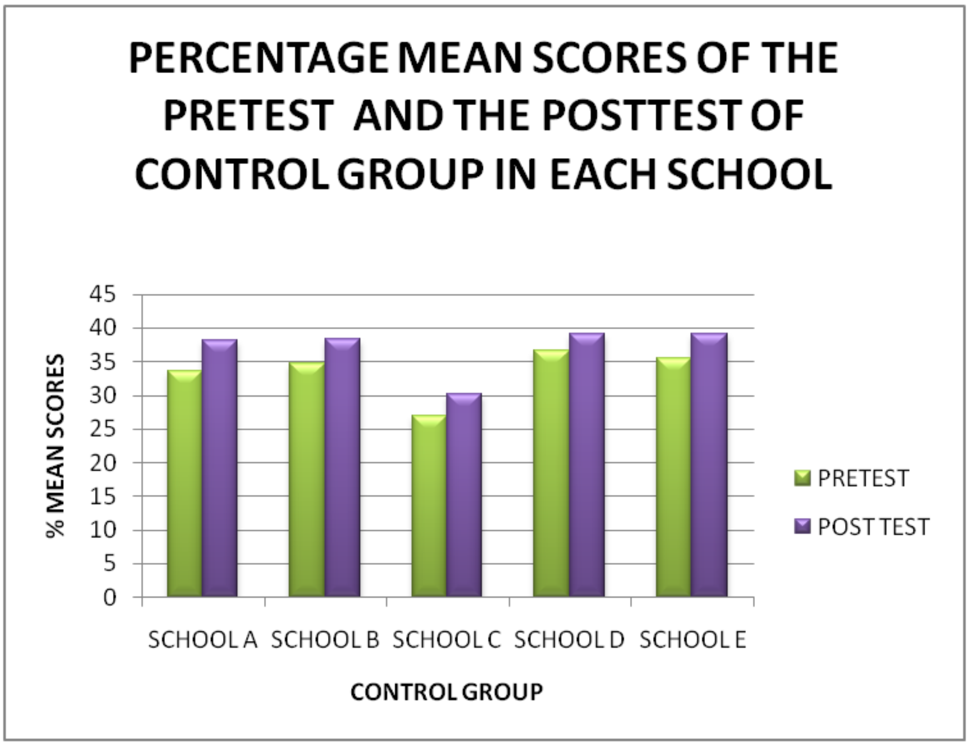


Chart 4.8: Percentage mean scores of the pretest and the posttest of control group in each school

It was evident from Table 4.28 and Chart 4.8 that there was an increase in the percentage mean scores of the control group in all schools with the highest difference noticed in School A. School A's percentage increased from 33.64% to 38.18%, School

B's percentage increased from 34.81% to 38.37%, School C's percentage increased from 27% to 30.2%, School D's percentage increased from 36.67% to 39.03% and School E's percentage increased from 35.56% to 39.17%.
This showed that the traditional method of teaching also had a positive effect even though it was not as considerable as the difference seen in the experimental groups in all the schools who were taught with the van Hiele-based framework.

A further statistical analysis was done to the test results between the experimental group and the control group from all the schools in terms of percentage means for statistical difference.

4.3.1.4. Statistical analysis of the test results between the experimental group and the control group from all the schools in terms of percentage mean scores

The 2-tailed paired sample t-test is appropriate in this study as it is testing the statistical significance in the difference between an experimental group and a control group mean in a pretest-posttest (McMillan & Schumacher, 2006).

The different statistical analysis are shown in sections 4.3.1.4.1 to 4.3.1.4.4.

4.3.1.4.1. Analysis of the learners' performance in the pretest according to experimental group and control group per school in the pretest

This analysis was carried out to find the answer to the question whether there was a statistically significant difference between van Hiele geometry test scores of the participants in the experimental group and control group per school in the pretest.

As shown in Table 4.4 in section 4.2.1.3, the difference between the percentage mean scores of the experimental groups and the percentage mean scores of the control groups was compared by means of t-test and was found that it was not significant for all schools except School B. This showed that the learners in the control group and the experimental group in all schools except School B were equivalent in their performance in the pretest.

4.3.1.4.2. Analysis of the learners' performance in the pretest – posttest according to experimental group per school

This analysis was carried out to find the answer to the question whether there was a statistically significant difference between van Hiele Geometry Test scores of the participants in the experimental group per school in the pretest and posttest.

The paired samples t-test was used here as the comparison was on the numerical information obtained from the same subjects under two surveys namely pretest and the posttest.

Table 4.29: Learners' performance in the pretest – posttest according to experimental group per school

School	Group	No	test	Percentage mean score	Standard Deviation	*df*	t-value	p-value
School A	Experimental group	45	pretest	35.11	9.32	44	5.179	0.000
			Posttest	44.67	8.69			
School B	Experimental group	55	pretest	28.64	10.65	54	7.366	0.000
			Posttest	38.73	9.63			
School C	Experimental group	32	pretest	26.88	11.34	31	4.446	0.000
			Posttest	35.94	9.02			
School D	Experimental group	29	pretest	41.9	12.42	28	4.231	0.000
			Posttest	52.41	10.58			
School E	Experimental group	34	pretest	31.17	12.8	33	4.583	0.000
			Posttest	41.76	11.34			

As seen in the Table 4.29, the difference between the percentage mean scores of the experimental group in the pretest and posttest was statistically significant and that the instructional frame work had a positive effect in all schools.

It is possible to say that the instructional framework might be responsible for this achievement, although other extraneous variables like maturation and history might be operating.

4.3.1.4.3. Analysis of the learners' performance in the pretest – posttest according to control group per school

This analysis was carried out to find the answer to the question whether there was a significant difference between van Hiele Geometry Test scores of the participants in the experimental group per school in the pretest and posttest.

Table 4.30: Learners' performance in the pretest – posttest according to control group per school

School	Group	No	test	Mean	Standard deviation	*df*	t-value	p-value
School A	Control Group	33	pretest	33.64	9.21	32	2.33	0.013
			Posttest	38.18	11.51			
School B	Control Group	52	pretest	34.81	10.85	51	1.974	0.027
			Posttest	38.37	10.23			
School C	Control Group	25	pretest	27	10.51	24	1.225	0.116
			Posttest	30.2	11.94			
School D	Control Group	36	pretest	36.67	11.46	35	1.004	0.161
			Posttest	39.03	13.25			
School E	Control Group	18	pretest	35.56	9.53	17	0.964	0.174
			Posttest	39.17	15.27			

As seen in Table 4.30, the difference between the average mean scores of the experimental group and the average mean scores of the control group in the pretest and posttest was compared by means of t-test and was found that it was not significant with the exception of School A.

4.3.1.4.4. Analysis of the participants' performance according to experimental group and control group per school in the posttest

This analysis was carried out to find the answer to the question whether there was a statistically significant difference between van Hiele Geometry Test scores of the participants in both groups per school in the posttest.

Table 4.31: Learners' performance according to experimental group and control group per school in the posttest

School	Group	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
School A	Experimental group	45	44.67	8.69	76	2.718	0.0045
	Control group	33	38.18	11.51			
School B	Experimental group	55	38.73	9.63	105	0.188	0.426
	Control group	52	38.37	10.23			
School C	Experimental group	32	35.94	9.02	55	1.998	0.026
	Control Group	25	30.2	11.94			
School D	Experimental group	29	52.41	10.58	63	4.531	0.000
	Control group	36	39.03	13.25			
School E	Experimental group	34	41.76	11.34	50	0.635	0.266
	Control group	18	39.17	15.27			

Table 4.31 showed that there was a statistical difference in the percentage mean of the learners in the posttest between the experimental group and control group in Schools A and D.

4.3.2. Overall participants' performance in the posttest by gender

Learners' performance in the pretest was further analysed for a possible gender difference in the entire study sample.

Table 4.32: Overall participants' performance in the van Hiele Geometry Test (posttest) by gender

School	Percentage mean scores in the posttest	
	Male	Female
School A	44.17	40.52
School B	38.47	38.67
School C	33.6	33.28
School D	45.25	44.88
School E	39.81	41.92
Overall percentage mean	39.82	40.10

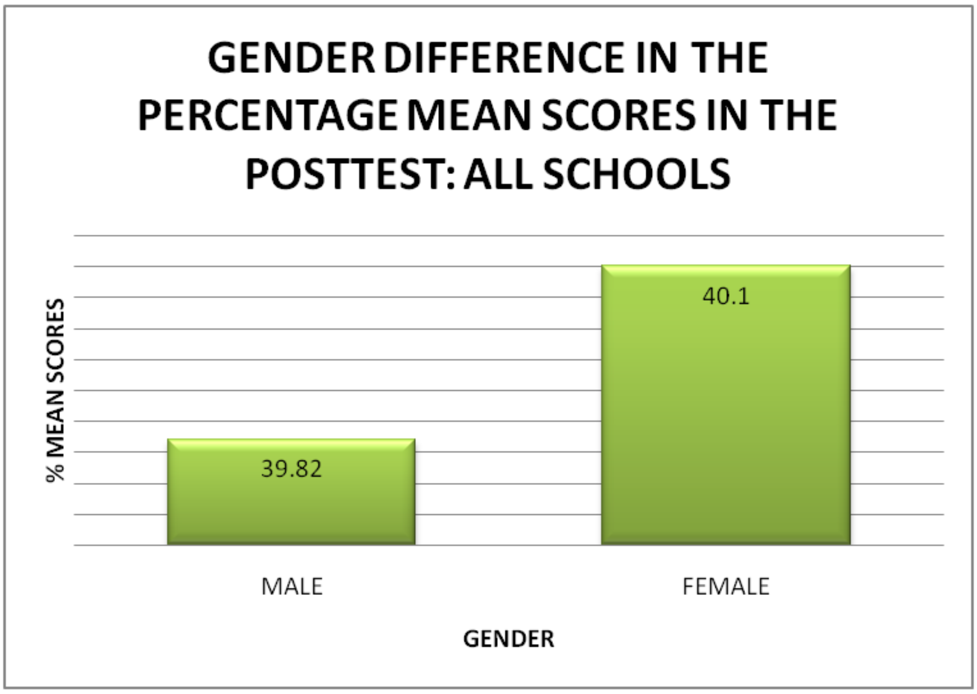


Chart 4.9: Gender difference in the percentage mean scores in the posttest

Table 4.32 and Chart 4.9 showed that in the entire sample, there was a slight difference in the performance in the posttest in favour of the female learners.

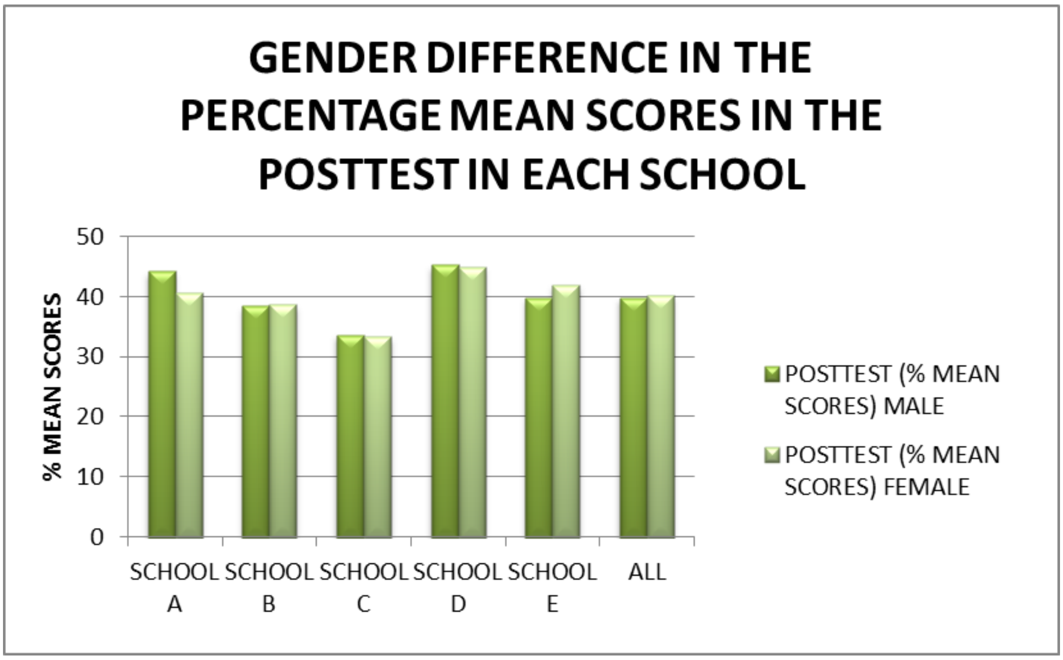


Chart 4.10: Gender difference in the percentage mean scores in the posttest in each school

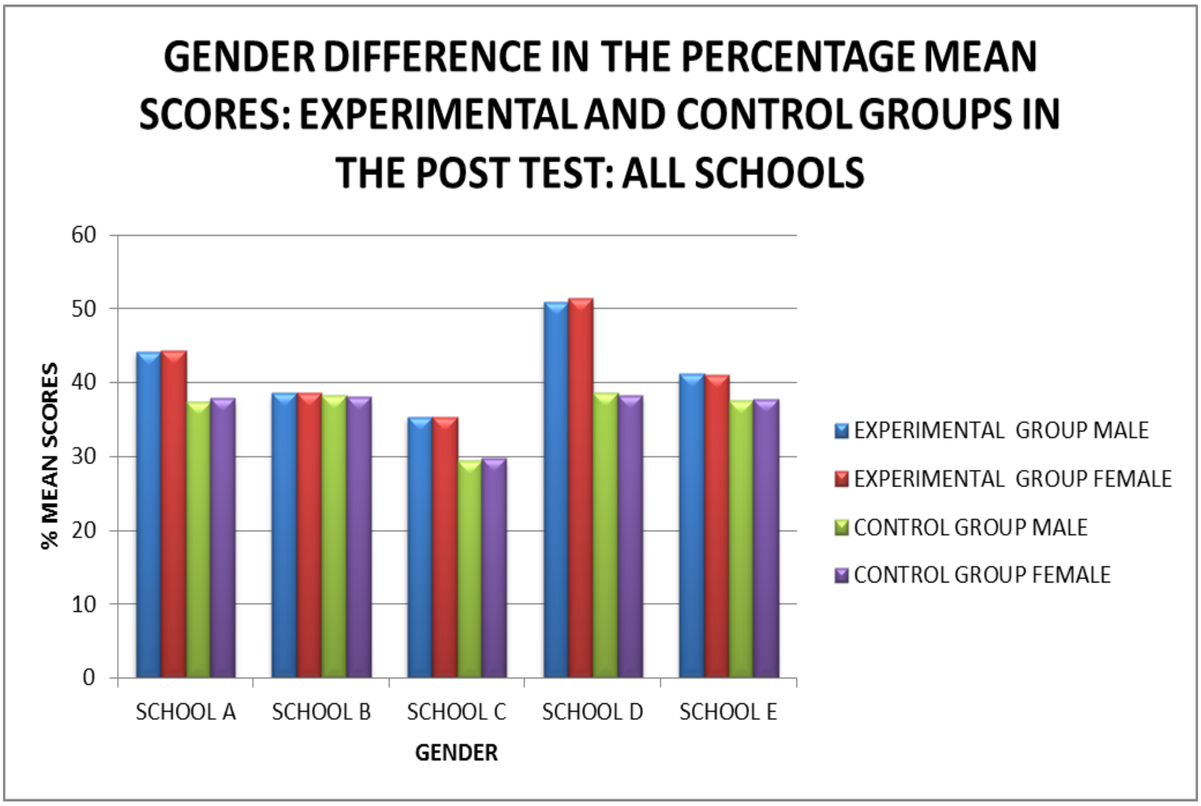


Chart 4.11: Gender difference in the percentage mean scores: Experimental and control groups in the posttest: All schools

The Charts 4.10 and 4.11 also showed that in all schools difference in the percentage mean was negligible according to gender.

In Schools A and B, male learners performed slightly better than the female learners. And in the other schools, Schools C, D and E, the female learners' performance was slightly better.

Further analysis was carried out to check whether there was a significant statistical difference in the percentage means scores of the learners by gender in each school.

4.3.2.1. Statistical comparison of the participants' performance in the posttest according to gender per school

Table 4.33: Learners' performance in the posttest according to gender in all schools

School	Gender	Number	Percentage Mean score	Standard deviation	*df*	t-value	p-value
School A N = 78	Male	30	44.17	9.01	76	1.590	0.067
	Female	48	40.52	11.07			
School B N=107	Male	62	38.47	8.85	105	0.099	0.461
	Female	45	38.67	11.25			
School C N=57	Male	25	33.6	10.85	55	0.111	0.456
	Female	32	33.28	10.75			
School D N=65	Male	20	45.25	10.19	63	0.097	0.462
	Female	45	44.88	15.21			
School E N=52	Male	26	39.81	11.79	50	0.595	0.278
	Female	26	41.92	13.79			

Table 4.34: Learners' performance in the posttest according to gender in all schools combined

Gender	Number	Percentage mean score	Standard deviation	*df*	t-value	p-value
Male	163	39.82	10.44	357	0.232	0.404
Female	196	40.10	12.92			

This test of significance indicated that the difference in the percentage mean scores between the male learners and female learners in each school and all the schools combined were not statistically significant. It can be assumed that the achievement in the posttest was independent of gender. Gender did not play a role in the performance of the sample.

4.3.3. Analysis of the learners' performance in the posttest according to the van Hiele levels

As mentioned in the analysis of the pretest, in the following sections, learners' performance in the posttest is presented in two ways. The first section discusses the performance in terms of the mean score percentages and the second section discusses the assignment of learners into the van Hiele levels based on the '3 of 5 correct" success criterion as suggested by Usiskin (1982, p.22).

4.3.3.1. Overall percentage mean scores of learners at each van Hiele level in the posttest

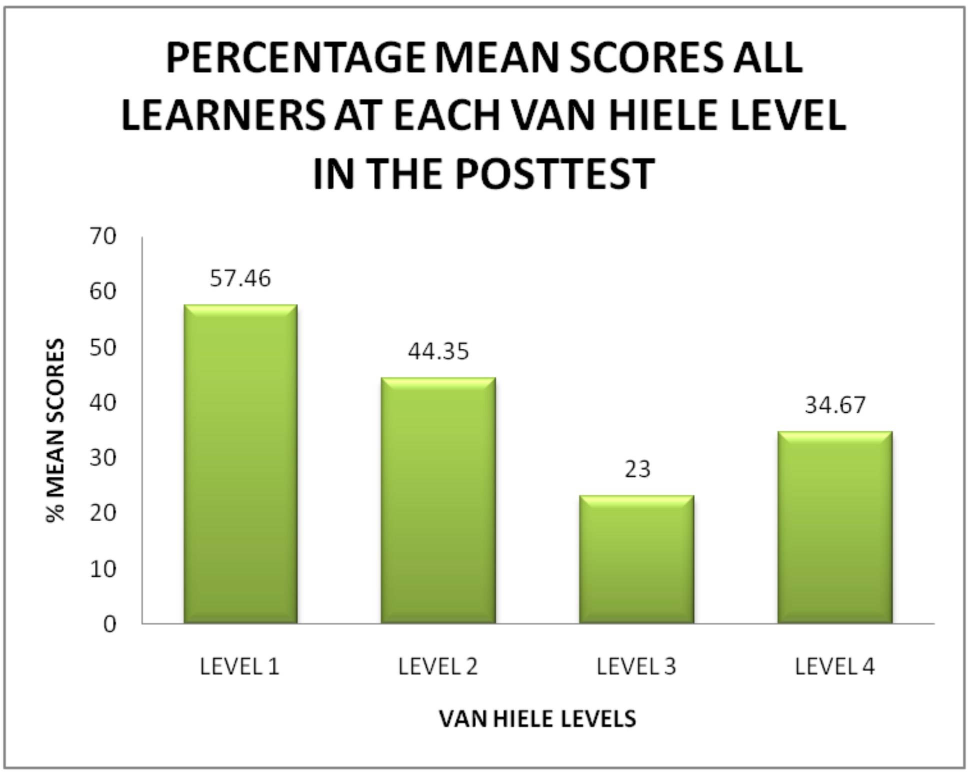


Chart 4.12: Percentage mean scores of all learners at each van Hiele level in the posttest

As evident from Chart 4.12 the percentage mean scores of learners at each van Hiele level in the posttest for level 1, 2, 3 and 4 were 57.46%, 44.35%, 23%, and 34.67% respectively. This was higher than the percentage mean scores of learners at each van Hiele level in the pretest which were 46.85%, 39.44%, 19%, and 27.65% respectively. This showed that more learners got more correct answers in each van Hiele level in the posttest.

A further analysis was carried out on the percentage mean scores of learners at each van Hiele level in all schools.

4.3.3.2. Analysis of the percentage mean posttest scores of learners at each van Hiele level in each school

Table 4.35: Percentage mean posttest scores of learners at each van Hiele level: All schools

	Percentage mean posttest scores of learners at each van Hiele level in each school				
Van Hiele levels	School A	School B	School C	School D	School E
Level 1	61.06	55.53	44.6	70.07	56.05
Level 2	46.3	39.42	36.94	55.13	43.99
Level 3	19.96	24.5	20.94	25.04	24.58
Level 4	38.42	35.22	29.8	32.65	37.26

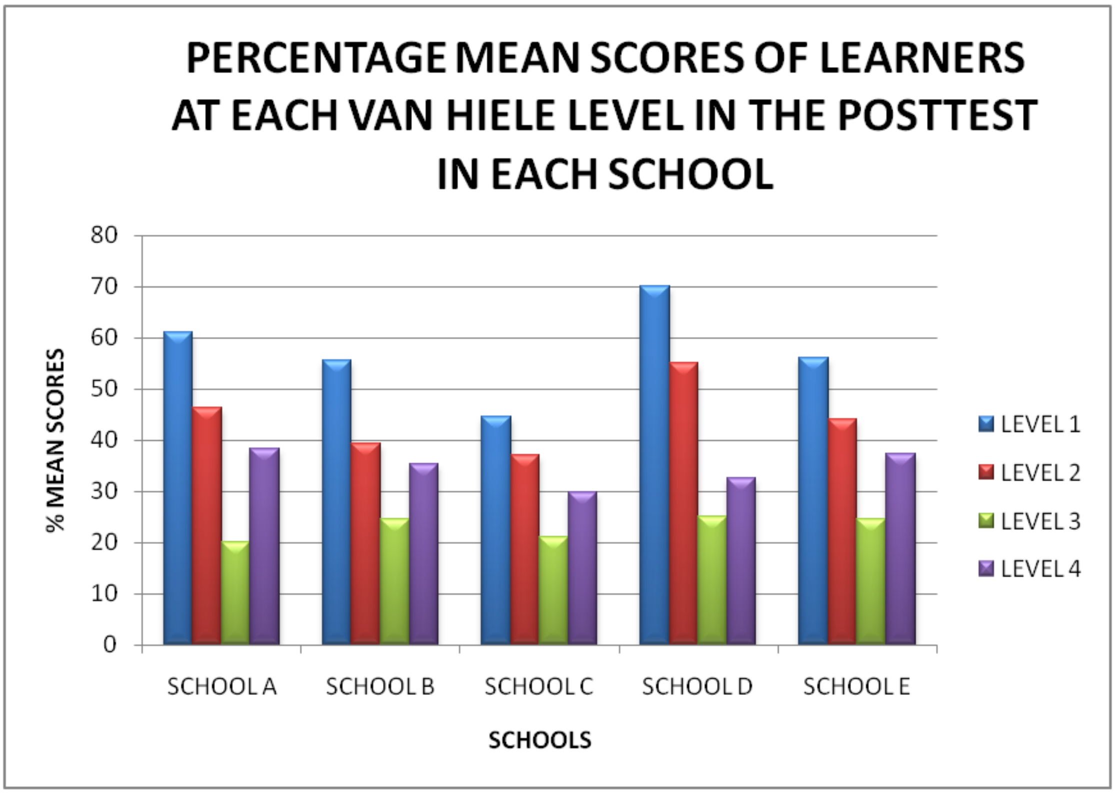


Chart 4.13: Percentage mean scores of learners at each van Hiele level in the posttest in each school

As evident from Table 4.35 and Chart 4.13, School D had the highest percentage mean score at level1 and level 2 and 3 and School C's performance was the lowest in all the levels except at level 3 and School A's performance is the highest at level 4. In all schools, percentage mean score was the lowest at level 3. This was consistent with their performance in the pretest.

In School A, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 61.06%, 46.3%, 19.96% and 38.42% respectively. The percentage mean score of learners in the pretest at levels 1, 2, 3 and 4 were 50.51%, 43.62%, 15.72% and 27.66% respectively. It was noted that the school had the highest percentage mean score at level 4. The percentage increase in all the levels in the posttest was similar.

In School B, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 55.53%, 39.42%, 24.5% and 35.22% respectively. The percentage mean score of learners in the pretest at levels 1, 2, 3 and 4 were 41.41%, 34.88%, 17.91% and 32.7% respectively. The percentage increase in all the levels in the posttest was similar.

In School C, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 44.6%, 36.94%, 20.94% and 29.8% respectively. The percentage mean score of learners in the pretest at levels 1, 2, 3 and 4 were 37.65%, 25.52%, 21.59% and 23.75% respectively. The percentage increase in all the levels in the posttest was consistent.

In School D, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 70.07%, 55.13%, 25.04% and 32.65% respectively. The percentage mean score of learners in the pretest in levels 1, 2, 3 and 4 were 65.08%, 47.22%, 20.36% and 25.68% respectively. It was noted that the school had the highest percentage mean scores at level 1, level 2 and level 3. School D's performance was better than the other schools at level 1 and level 2 in the pretest. The percentage increase in all the levels in the posttest was consistent.

In School E, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 56.05%, 43.99%, 24.58% and 37.26% respectively. The percentage mean score of learners in the pretest at levels 1, 2, 3 and 4 were 39.61%, 45.98%, 19.41% and 28.47% respectively. It was noted that the school had a decline in the percentage mean score at level 2. The percentage increase in all the other levels in the posttest was similar.

A further analysis was carried out for checking the mean scores percentage of learners at each van Hiele level in all the schools for the experimental group and control group.

School A

Table 4.36: Percentage mean scores of learners at each van Hiele level for posttest in School A

Van Hiele levels	description	Posttest	
		Experimental group (N = 45)	Control group (N = 33)
Level 1	Recognition	67.56	54.55
Level 2	Analysis	53.8	38.8
Level 3	Informal deduction	16.89	23.03
Level 4	Deduction	40.44	36.4

It was evident from Table 4.36 that in School A, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 67.56%, 53.8%, 16.89% and 40.44% respectively for the experimental group and were 54.55%, 38.8%, 23.03% and 36.4% respectively for the control group. In the pretest the percentage mean score of learners at levels 1, 2, 3 and 4 were 48.89 %, 41.78%, 23.56% and 26.22% respectively for the experimental group and were 52.12%, 45.46%, 7.88% and 29.09% respectively for the control group. The experimental group performed better at level 3 only than that of the control group in the posttest. The percentage increase in all the levels in the posttest was consistent.

School B

Table 4.37: Percentage mean scores of learners at each van Hiele level for posttest in School B

Van Hiele levels	description	Posttest	
		Experimental group (N = 55)	Control group (N = 52)
Level 1	Recognition	59.91	51.15
Level 2	Analysis	39.64	39.2
Level 3	Informal deduction	24	25
Level 4	Deduction	32.36	38.08

It was evident from Table 4.37 that in School B, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 59.91%, 39.64%, 24% and 32.36% respectively for the experimental group and were 51.15%, 39.2%, 25% and 38.08% respectively for the control group. In the pretest the percentage mean score of learners at levels 1, 2, 3 and 4 were 37.82%, 30.91%, 19.27% and 26.55% respectively for the experimental group and were 45%, 38.85%, 16.54% and 38.85% respectively for the control group. The experimental group performed better only at level 1 in the posttest. The percentage increase in all the levels in the posttest was consistent.

School C

Table 4.38: Percentage mean scores of learners at each van Hiele level for posttest in School C

Van Hiele levels	description	Posttest	
		Experimental group (N = 32)	Control group (N = 25)
Level 1	Recognition	50	39.2
Level 2	Analysis	41.88	32
Level 3	Informal deduction	21.88	20
Level 4	Deduction	30	29.6

It was evident from Table 4.38 that in School C, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 50%, 41.88%, 21.88% and 30% respectively for the experimental group and were 39.2%, 32%, 20% and 29.6% respectively for the control group. In the pretest, the percentage mean score of learners at level 1, 2, 3 and 4 were 42.5%, 20.63%, 19.98% and 25% respectively for the experimental group and were 32.8%, 30.4%, 23.2% and 22.4% respectively for the control group. The experimental group performed better at level 1, 2, 3 and level 4 in the posttest.

School D

Table 4.39: Percentage mean scores of learners at each van Hiele level for posttest in School D

Van Hiele levels	description	Posttest	
		Experimental group (N = 29)	Control group (N = 36)
Level 1	Recognition	80.69	59.44
Level 2	Analysis	64.14	46.11
Level 3	Informal deduction	28.97	21.11
Level 4	Deduction	35.86	29.44

It was evident from Table 4.39 that in School D, the percentage mean scores of learners in the posttest at level 1, 2, 3 and 4 were 80.69%, 64.14%, 28.97% and 35.86% respectively for the experimental group and were 59.44%, 46.11%, 21.11% and 29.44% respectively for the control group. In the pretest the percentage mean score of learners at levels 1, 2, 3 and 4 were 77.93%, 44.14%, 17.93% and 28.57% respectively for the experimental group and were 52.22%, 50.29%, 22.78% and 22.78% respectively for the control group. The experimental group performed better in all the levels in the posttest.

School E

Table 4.40: Percentage mean scores of learners at each van Hiele level for posttest in School E

Van Hiele levels	description	Posttest	
		Experimental group (N = 34)	Control group (N = 18)
Level 1	Recognition	57.65	54.44
Level 2	Analysis	43.53	44.44
Level 3	Informal deduction	24.71	24.44
Level 4	Deduction	41.18	33.33

It was evident from Table 4.40 that in School E, the percentage mean score of learners in the posttest at levels 1, 2, 3 and 4 were 57.65%, 43.53%, 24.71% and 41.18% respectively for the experimental group and were 54.44%, 44.44%, 24.44% and 33.33% respectively for the control group. In the pretest the percentage mean score of learners at levels 1, 2, 3 and 4 were 35.88%, 45.29%, 18.82% and 24.71% respectively for the experimental group and were 43.33%, 46.67%, 20% and 32.22% respectively for the control group. The experimental group's performance was better at level 1, 3 and 4 in the posttest.

4.3.4. Assignment of learners into different van Hiele levels of thinking for the posttest

The grading of the van Hiele Geometry Test was done again using the same method as discussed earlier in section 4.2.4. It was based on the "3 of 5 correct" success criterion as suggested by Usiskin (1982, p.22) to assign learners into different van Hiele levels.

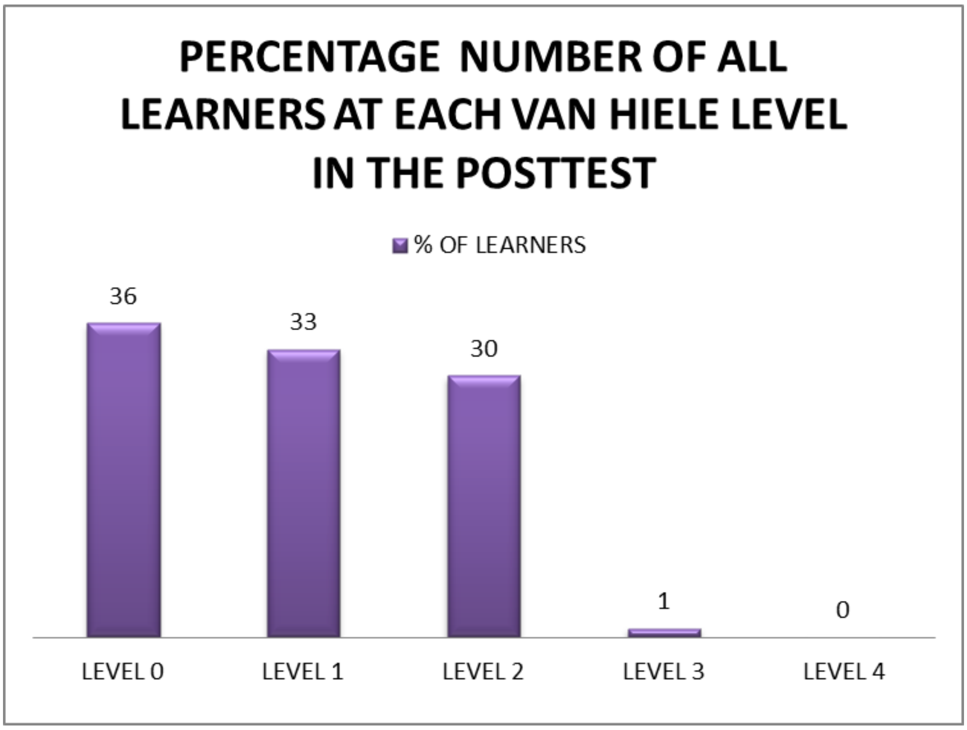


Chart 4.14: Percentage number of learners at each van Hiele level in the posttest in all schools combined

Chart 4.14 showed that the majority of the learners were at level 0 (36%). For the van Hiele levels 1, 2, 3, and 4, it was 33%, 30%, 1% and 0% respectively in the posttest. But in the pretest the majority of the learners were at level 0 (56%). For the van Hiele levels 1, 2, 3, and 4, it was 26%, 17%, 1% and 0% respectively.

It was evident from the comparison that the number of learners in each level had increased at levels 1 and 2, stayed the same at levels 3 and 4 and considerably decreased at level 0.

4.3.4.1. Analysis of percentage number of learners at each van Hiele level in each school for the posttest

A further analysis was carried out to find the percentage number of learners at each van Hiele level in the different schools. The analysis is presented in Table 4.41 and Chart 4.15 which are given below:

Table 4.41: Percentage number of learners at each van Hiele level in the posttest in all schools

	Percentage number of learners at each van Hiele level in all schools				
Van Hiele levels	School A	School B	School C	School D	School E
Level 0	24	44	51	20	38
Level 1	26	39	40	31	29
Level 2	50	17	9	46	31
Level 3	0	0	0	3	2
Level 4	0	0	0	0	0

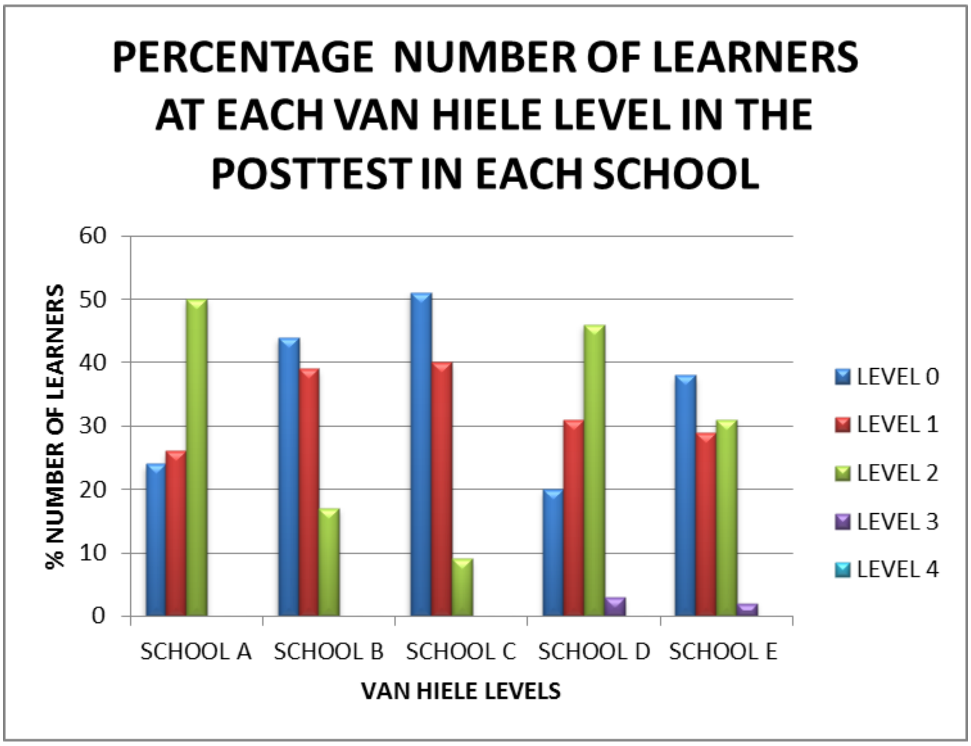


Chart 4.15: Percentage number of learners at each van Hiele level in the posttest in each school

As can be seen from Table 4.41 and Chart 4.15, the majority of the learners in all schools were at level 0 except for school D which had only 20% at level 0. School C had the highest number of learners at level 0 (51%) followed by School B (44%), School E (38%), and School A (24%). Level 3 was achieved only by learners in School D and E where the percentages were 3% and 2% respectively. None of the schools had learners at level 4 thinking on the van Hiele scale indicating that the learners were not ready for formal geometric proofs in grade 10.

In the pretest, the majority of the learners in all schools were at level 0 except for school D which had only 29% at level 0. School C had the highest number of learners at level 0 (70%) followed by School E (65%), School B (63%) and School A (54%). Level 3 was achieved by no learners in all the schools except by 6% of learners in School D. None of the schools had learners at level 4 thinking on the van Hiele scale indicating that the learners were not ready for formal geometric proofs in grade 10.

4.3.4.2. Analysis of percentage number of learners at each van Hiele level per school for the posttest

School A

Table 4.42: Percentage number of learners at each van Hiele level in the posttest in School A

	Percentage number of learners in each van Hiele level in School A	
Van Hiele levels	Experimental group	Control group
Level 0	13	39
Level 1	18	27
Level 2	69	24
Level 3	0	0
Level 4	0	0

It was evident from Table 4.42 that in School A, the majority of the learners in the experimental group were at level 2 (60%). In the control group, the majority of the learners were at level 0 (39%). For the van Hiele levels 0, 1, 3, and 4, it was 13%, 18%, 0% and 0% respectively in the experimental group and it was 27%, 24%, 0% and 0% respectively at level 1, level 2, level 3 and level 4 in the control group. The experimental group had more learners at level 2 and the experimental group's performance was better than the control group. The school had no learners at level 3 and level 4.

In the pretest, in School A, the majority of the learners were at level 0 (60% in the experimental group and 46% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 22%, 18%, 0% and 0% respectively in the experimental group and it was 24%, 30%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and the control group's performance was better than the experimental group. The school had no learners at level 3 and level 4.

School B

Table 4.43: Percentage number of learners at each van Hiele level in the posttest in School B

	Percentage number of learners in each van Hiele Level in School B	
Van Hiele levels	Experimental group	Control group
Level 0	36	52
Level 1	44	35
Level 2	20	13
Level 3	0	0
Level 4	0	0

It was evident from Table 4.43 that in School B, the majority of the learners were at level 1 (44%) in the experimental group and the control group had the majority of learners (52%) at level 0. For the van Hiele levels 0, 2, 3, and 4, it was 36%, 20%, 0% and 0% respectively in the experimental group and it was 35%, 13%, 0% and 0% respectively at levels 1, 2, 3 and 4 in the control group. The school had no learners at level 3 and level 4. The experimental group's performance was better than the control group in terms of having more learners at level 1 than at level 0.

In the pretest, in School B, the majority of the learners were at level 0 (65% in the experimental group and 59% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 24%, 11%, 0% and 0% respectively in the experimental group and it was 31%, 10%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and the control group's performance was better than the experimental group at level 1 and the experimental group had more learners at level 2. The school had no learners at level 3 and level 4.

School C

Table 4.44: Percentage number of learners at each van Hiele level in the posttest in School C

	Percentage number of learners in each van Hiele level in School C	
Van Hiele levels	Experimental group	Control group
Level 0	38	68
Level 1	50	28
Level 2	12	4
Level 3	0	0
Level 4	0	0

It was evident from Table 4.44 that in School C, the majority of the learners were at level 1 (50%) in the experimental group and the majority of the learners in the control group were at level 0 (68%). For the van Hiele levels 0, 2, 3, and 4, it was 38%, 12%, 0% and 0% respectively in the experimental group and it was 28%, 4%, 0% and 0% respectively at levels 1, 2, 3 and 4 in the control group. The school had no learners at level 3 and level 4. The experimental group's performance was better than the control group in terms of having more learners at level 1 than at level 0.

In the pretest, in School C, the majority of the learners were at level 0 (66% in the experimental group and 76% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 34%, 0%, 0% and 0% respectively in the experimental group and it was 16%, 8%, 0% and 0% respectively in the control group. The experimental group had more learners at level 1 and had no learners at level 2. The control group's performance was lower than the experimental group. The school had no learners at level 3 and level 4.

School D

Table 4.45: Percentage number of learners at each van Hiele level in the posttest in School D

	Percentage number of learners in each van Hiele Level in School D	
Van Hiele levels	Experimental group	Control group
Level 0	3	33
Level 1	31	31
Level 2	62	33
Level 3	4	3
Level 4	0	0

It was evident from Table 4.45 that in School D, the majority of the learners were at level 2 (62%) in the experimental group and the control group had equal number of learners at level 0 and level 3 (33%). For the van Hiele levels 0, 1, 3, and 4, it was 3%, 31%, 4%, and 0% respectively in the experimental group and it was 31%, 3% and 0% respectively in the control group. The experimental group had more learners at levels 1, 2 and 3 and its performance was better than the control group's. The school had no learners at level 4.

In the pretest, in School D, for the van Hiele levels 0, 1, 2, 3, and 4, it was 10%, 45%, 38%, 7% and 0% respectively in the experimental group and it was 44%, 25%, 25%, 6% and 0% respectively in the control group. The experimental group had more learners at levels 1, 2 and 3 and its performance was better than the control group's. The school had no learners at level 4.

School E

Table 4.46: Percentage number of learners at each van Hiele level in the posttest in School E

	Percentage number of learners in each van Hiele Level in School E	
Van Hiele levels	Experimental group	Control group
Level 0	35	44
Level 1	32	22
Level 2	33	28
Level 3	0	6
Level 4	0	0

It was evident from Table 4.46 that in School E, the majority of the learners were at level 0 (35% in the experimental group and 44% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 32%, 33%, 0% and 0% respectively in the experimental group and it was 22%, 28%, 6% and 0% respectively in the control group. The experimental group had more learners at level 1 and at level 2. The school had 6% of learners in the control group level 3 and no learners at level 4.

In the pretest, in School E, the majority of the learners were at level 0 (68% in the experimental group and 61% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 9%, 23%, 0% and 0% respectively in the experimental group and it was 22%, 17%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and at level 2. The school had no learners at level 3 and level 4.

The percentage number of learners at level 0 had been reduced considerably in both groups in the posttest.

4.3.5. Analysis of the percentage number of learners with correct responses for each item in the van Hiele Geometry Test

An analysis was carried out for the overall performance and for each school to find out the number of learners who obtained the correct answers for each of the concepts in the pretest and the posttest. This was to get an idea of which concept was the "worst attempted" and the "best attempted". The percentages of learners who made the correct choice for each question are given in the Table 4.47. As mentioned in section 3.7 in Chapter 3, items 1–5 were level 1 questions , items 6–10 were level 2 questions, items 11–15 were level 3 questions and items 16–20 were level 4 questions.

4.3.5.1. Analysis of correct response of the van Hiele Geometry Test for all schools - pretest and posttest

Table: 4.47: Analysis of correct responses by learners from all schools - pretest and posttest

			ALL LEARNERS (N=359)	
Item	Item description	Correct answer	Pretest percentage with choice	Posttest percentage with choice
1	Triangle recognition	**E**	73	79
2	Rectangle recognition	**A**	61	71
3	Square recognition	**C**	45	57
4	Quadrilateral recognition	**C**	40	51
5	Parallelogram recognition	**E**	22	27
6	Rectangle properties	**B**	49	46
7	Rectangle properties	**C**	55	61
8	Circle properties	**C**	31	36

9	Equilateral triangle properties	**E**	23	25
10	Square Properties	**A**	37	45
11	Connection between rectangle and parallelograms	**B**	18	23
12	Connection between rectangle and parallelograms	**D**	9	10
13	Connection between parts of a circle	**C**	16	18
14	Connection between right angled triangles and complementary angles	**C**	25	29
15	Connection between congruent and similar figures	**B**	27	34
16	Deduction- perpendicular lines	**D**	31	36
17	Deduction – parallel and perpendicular lines	**A**	31	37
18	Deduction- parallelogram and square	**C**	10	13
19	Deduction - angles	**B**	41	55
20	Deduction – properties of figures – squares and rectangles	**C**	18	22

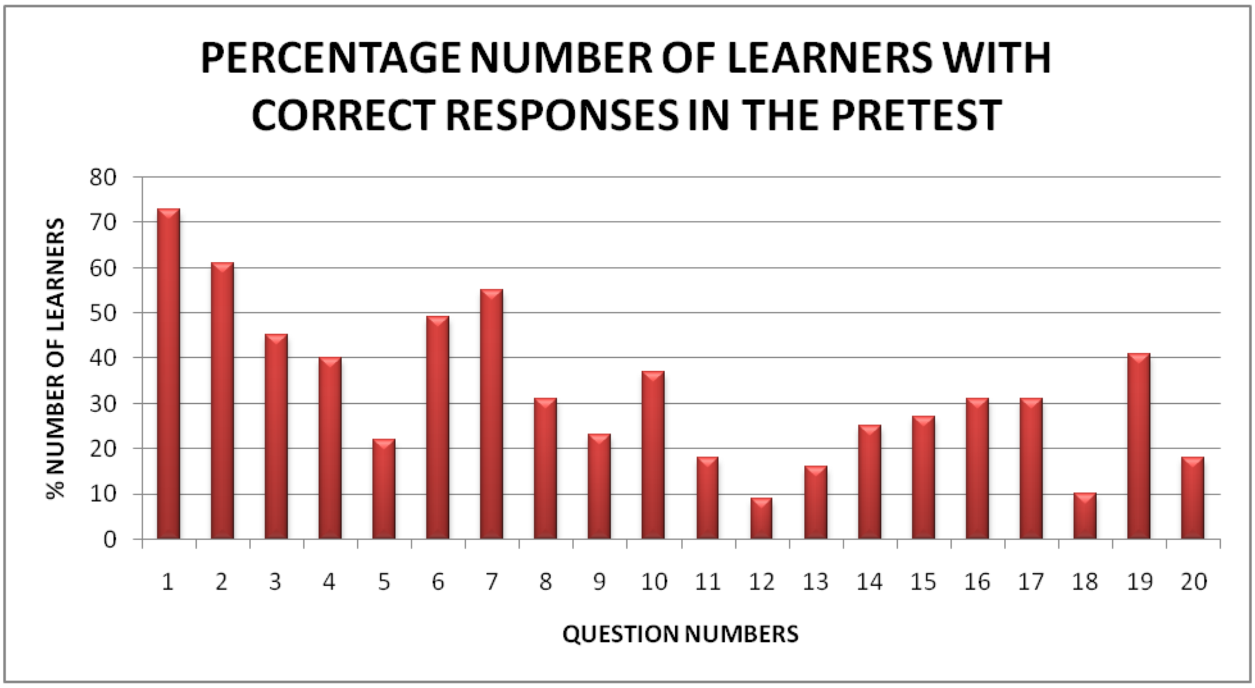


Chart 4.16: Percentage number of learners with correct responses in the pretest

As seen in Table 4.47 and Chart 4.16, in the pretest, at level 1 (items 1 –5), item No.1 was the one which had the highest percentage of number of correct responses (73%) and item No.5 was the one which had the lowest percentage number of correct responses (22%). At level 2 (items 6 –10), item No.7 was the one which had the highest percentage number of correct responses (55%) and item No.9 is the one which had the lowest percentage number of correct responses (23%). At level 3 (items 11 –15), item No.15 was the one which had the highest percentage number of correct responses (27%) and item No.12 was the one which has the lowest percentage number of correct responses (9%). At level 4 (items 16 –20), item No.19 was the one which has the highest percentage number of correct responses (41%) and item No.18 was the one which had the lowest percentage number of correct responses (10%). In overall, in all the items, item No.1 was the one which has the highest percentage number of correct responses (73%) and No.12 was the one which had the lowest percentage number of correct responses (9%). In other words, the items that got the highest percentage of correct responses were items No.1, No.7, No.15 and No.19 and the items that got the lowest percentage of correct responses were items No.5, No.9, No.12 and No.18 at levels 1, 2, 3 and 4 respectively.

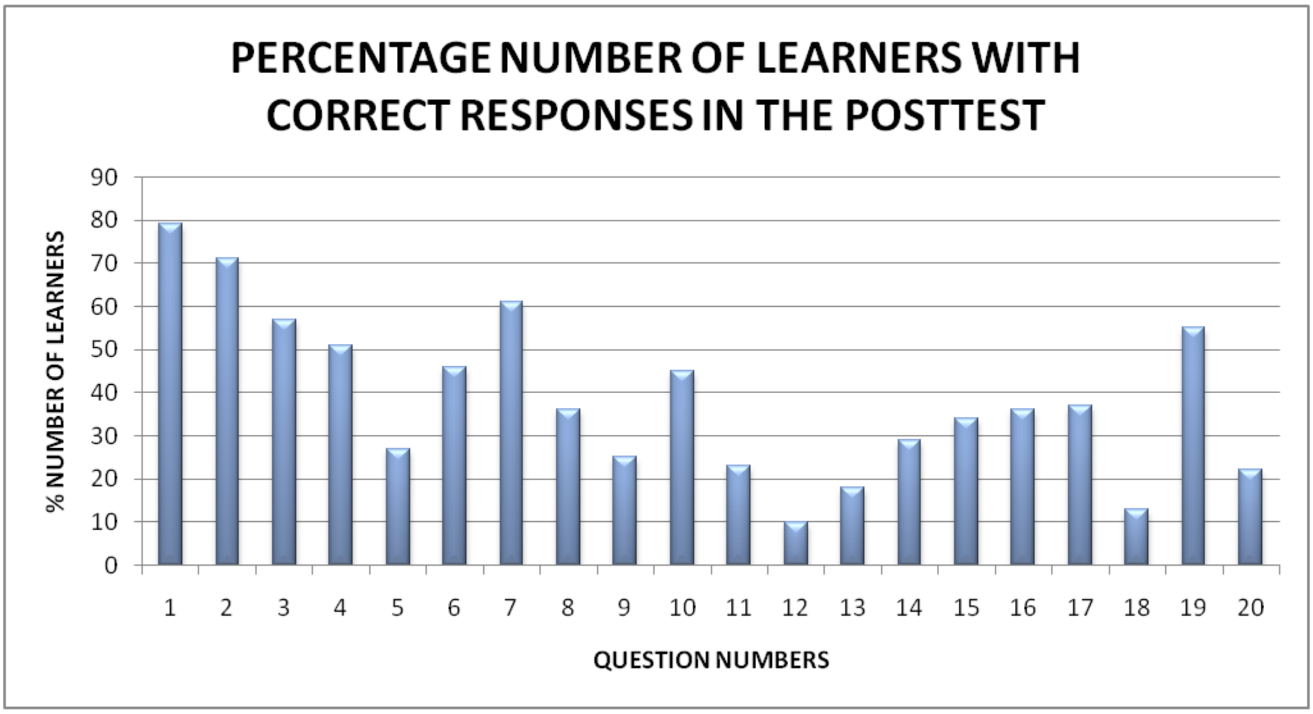


Chart 4.17: Percentage number of learners with correct responses in the posttest

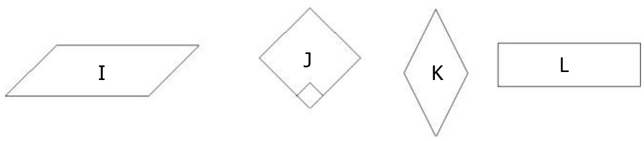
As seen in Table 4.47 and Chart 4.17, in the posttest, at level 1 (items 1 –5), item No.1 was the one which had the highest percentage of number of correct responses (79%) and item No.5 was the one which had the lowest percentage number of correct responses (27%). At level 2 (items 6 –10), item No.7 was the one which had the highest percentage number of correct responses (61%) and item No.9 was the one which had the lowest percentage number of correct responses (25%). At level 3 (items 11 –15), item No.15 was the one which had the highest percentage number of correct responses (34%) and item No.12 was the one which had the lowest percentage number of correct responses (10%). At level 4 (items 16 –20), item No.19 was the one which had the highest percentage number of correct responses (55%) and item No.18 was the one which had the lowest percentage number of correct responses (13%). In overall, in all the items, item No.1 was the one which had the highest percentage number of correct responses (79%) and No.12 was the one which had the lowest percentage number of correct responses (10%). In other words, the items that got the highest percentage of correct responses were items No.1, No.7, No.15 and No.19 and the items that got the lowest percentage of correct responses were items No.5, No.9, No.12 and No.18 at levels 1, 2, 3 and 4 respectively. These were the same items that got the highest and the lowest percentage of correct responses.

The items that got the lowest percentage of correct responses were items No.5, No.9, No.12 and No.18 at levels 1, 2, 3 and 4 respectively. To get clarification on these particular items, these items were again asked in the interviews. The responses of three learners are included in the analysis of interviews in Chapter 6.

These items are shown below:

Item 5

Which of these are parallelograms?



A. I only

B. L only

C. I and K only

D. J and L only

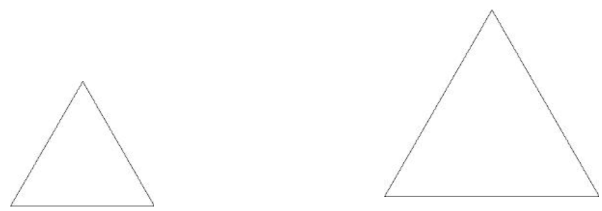
E. All are parallelograms

Figure 4.1: Item 5

The correct answer is "E". Most of the learners' response was "A".

Item 9

An equilateral triangle is a triangle with all the three sides equal in length. Two examples are given below.



Which of (A) – (D) is **true in every** equilateral triangle?

A. Each angle is an acute angle.

B. The measure of each angle must be 60°.

C. Each angle bisector is a line of symmetry.

D. Each angle bisector must also bisect the opposite side perpendicularly.

E. All of (A) – (D) are true.

Figure 4.2: Item 9

The correct answer is "E". Most of the learners' response was "A".

Item 12

Which is **true**?

A. All properties of rectangles are properties of all parallelograms.

B. All properties of squares are properties of all rectangles.

C. All properties of squares are properties of all parallelograms.

D. All properties of rectangles are properties of all squares.

E. None of (A) – (D) is true.

The correct answer is "D". As also noticed from the responses, most of the learners wrote "E" as the correct answer.

Item 18

Given a parallelogram, P from which of the following can we be sure that P is a square?

A. P has four sides all of which are equal in length.
B. P has diagonals of equal length.
C. P has diagonals of equal length which bisect each other at right angles.
D. P has four right angles and diagonals of equal measure.
E. P has diagonals that intersect at right angles and also has four equal sides.

The correct answer is "C". As also noticed from the responses, most of the learners wrote "A" as the correct answer.

A further analysis was done to find the learners' performance per item in the pretest and the posttest in each school.

4.3.5.2. Analysis of correct responses for each school's experimental and control group's performance in the pretest and the posttest

Each school's analysis was done separately for each school's experimental and control group's performance in the pretest and the posttest.

School A

Table 4.48: Analysis of correct responses for School A's experimental and control group's performance in the pretest and the posttest

Item	Item description	Correct answer	Experimental group (N=45) Percentage with choice		Control group (N= 33) percentage with choice	
			Pretest	Posttest	Pretest	Posttest
1	Triangle recognition	**E**	82	89	85	79
2	Rectangle recognition	**A**	58	96	82	70
3	Square recognition	**C**	53	71	36	55
4	Quadrilateral recognition	**C**	36	47	61	55
5	Parallelogram recognition	**E**	18	36	0	15
6	Rectangle properties	**B**	40	69	73	42
7	Rectangle properties	**C**	62	42	55	58
8	Circle properties	**C**	38	64	67	45
9	Equilateral triangle properties	**E**	27	18	12	15
10	Square Properties	**A**	44	76	21	33
11	Connection between rectangle and parallelograms	**B**	24	24	12	30
12	Connection between rectangle and parallelograms	**D**	11	13	0	12
13	Connection between parts of a circle	**C**	7	13	15	12
14	Connection between right angled triangles and complementary angles	**C**	31	20	18	18
15	Connection between congruent and similar figures	**B**	44	13	12	42
16	Deduction- perpendicular lines	**D**	29	22	36	42

17	Deduction – parallel and perpendicular lines	**A**	22	67	33	33
18	Deduction- parallelogram and square	**C**	9	16	12	12
19	Deduction - angles	**B**	44	84	61	67
20	Deduction – properties of figures – squares and rectangles	**C**	27	13	3	27

It was evident from the Table 4.48 and the analysis of correct responses sheet attached in Appendix G, for Item No.15, School A's experimental group in the pretest scored the highest percentage of correct responses (44%) along with the control group in School C in the posttest. School A's experimental group scored the highest percentage in Item 2 (96%), Item 3 (71%), Item 10 (67%) and Item 17 (67%) in the posttest. For Item No.6 and Item 8, School A's control group in the pretest scored the highest percentage of correct responses. For Item No.5 and Item No.12, School A's control group in the pretest scored the lowest percentage (0%). For Item 15 (12%) and Item No.20 (3%), also School A's control group in the pretest scored the lowest percentage.

School B

Table 4.49: Analysis of correct responses for School B's experimental and control group's performance in the pretest and the posttest.

			Experimental group (N=45)		Control group (N= 33)	
Item	Item description	Correct answer	Percentage with choice		percentage with choice	
			Pretest	Posttest	Pretest	Posttest
1	Triangle recognition	**E**	69	87	79	77
2	Rectangle recognition	**A**	49	69	60	63

3	Square recognition	**C**	36	65	37	63
4	Quadrilateral recognition	**C**	20	45	35	46
5	Parallelogram recognition	**E**	15	27	15	8
6	Rectangle properties	**B**	33	49	46	35
7	Rectangle properties	**C**	49	55	56	62
8	Circle properties	**C**	31	31	29	35
9	Equilateral triangle properties	**E**	11	22	15	21
10	Square Properties	**A**	31	40	48	44
11	Connection between rectangle and parallelograms	**B**	15	18	17	27
12	Connection between rectangle and parallelograms	**D**	11	9	10	8
13	Connection between parts of a circle	**C**	22	20	0	23
14	Connection between right angled triangles and complementary angles	**C**	22	35	25	27
15	Connection between congruent and similar figures	**B**	31	40	31	40
16	Deduction- perpendicular lines	**D**	31	31	44	40
17	Deduction – parallel and perpendicular lines	**A**	31	47	35	50
18	Deduction- parallelogram and square	**C**	5	11	17	10
19	Deduction - angles	**B**	47	45	71	67
20	Deduction – properties of figures – squares and rectangles	**C**	20	24	27	23

It was evident from the Table 4.49 and the analysis of correct responses sheet attached in Appendix G, for Item No.13, School B's control group in the pretest scored the lowest percentage of correct responses (0%). This means that none of the students in the control group answered the question on circles correctly. Moreover this was one of the questions that only 20% of the entire school consistently got correct in the pretest and posttest).

School C

Table 4.50: Analysis of correct responses for School A's experimental and control group's performance in the pretest and the posttest

			Experimental group (N=45)		Control group (N= 33)	
Item	Item description	Correct answer	Percentage with choice		percentage with choice	
			Pretest	Posttest	Pretest	Posttest
1	Triangle recognition	**E**	59	56	52	72
2	Rectangle recognition	**A**	47	34	44	56
3	Square recognition	**C**	25	47	40	36
4	Quadrilateral recognition	**C**	19	44	16	20
5	Parallelogram recognition	**E**	13	3	56	48
6	Rectangle properties	**B**	31	25	20	28
7	Rectangle properties	**C**	25	44	44	69
8	Circle properties	**C**	9	9	20	16
9	Equilateral triangle properties	**E**	3	19	24	20
10	Square Properties	**A**	9	34	44	32
11	Connection between rectangle and parallelograms	**B**	13	31	28	8
12	Connection between rectangle and parallelograms	**D**	3	9	20	4

13	Connection between parts of a circle	**C**	9	16	24	20
14	Connection between right angled triangles and complementary angles	**C**	22	6	24	28
15	Connection between congruent and similar figures	**B**	19	25	20	44
16	Deduction- perpendicular lines	**D**	25	25	24	20
17	Deduction – parallel and perpendicular lines	**A**	31	13	24	28
18	Deduction- parallelogram and square	**C**	9	22	8	16
19	Deduction - angles	**B**	9	31	12	12
20	Deduction – properties of figures – squares and rectangles	**C**	19	9	16	24

It was evident from Table 4.50 and the analysis of correct responses sheet attached in appendix G, that for Item No.3 (25%), No.7 (25%), No.8 (9%), No.9 (3%), No.10 (9%) and No.19 (9%), School C's experimental group in the pretest scored the lowest percentage of correct responses. In the posttest, for Item No.2 (34%), Item No.14 (6%) and for Item No.17 (13%), School C's experimental group scored the lowest percentage of correct responses.

School D

Table 4.51: Analysis of correct responses for School D's experimental and control group's performance in the pretest and the posttest

Item	Item description	Correct answer	Experimental group (N=45) Percentage with choice		Control group (N= 33) percentage with choice	
			Pretest	Posttest	Pretest	Posttest
1	Triangle recognition	**E**	86	93	94	81
2	Rectangle recognition	**A**	90	93	75	81
3	Square recognition	**C**	69	59	64	53
4	Quadrilateral recognition	**C**	93	97	69	69
5	Parallelogram recognition	**E**	52	62	25	17
6	Rectangle properties	**B**	59	72	72	42
7	Rectangle properties	**C**	55	93	69	61
8	Circle properties	**C**	31	66	33	44
9	Equilateral triangle properties	**E**	41	41	31	33
10	Square Properties	**A**	34	41	36	50
11	Connection between rectangle and parallelograms	**B**	17	38	17	6
12	Connection between rectangle and parallelograms	**D**	3	14	6	11
13	Connection between parts of a circle	**C**	17	17	28	14
14	Connection between right angled triangles and complementary angles	**C**	24	55	28	39
15	Connection between congruent and similar figures	**B**	28	21	33	36

16	Deduction- perpendicular lines	**D**	45	62	33	33
17	Deduction – parallel and perpendicular lines	**A**	17	21	22	22
18	Deduction- parallelogram and square	**C**	3	0	11	25
19	Deduction - angles	**B**	62	86	36	44
20	Deduction – properties of figures – squares and rectangles	**C**	7	7	11	25

It was evident from Table 4.51 and the analysis of correct responses sheet attached in appendix G, that for Item No.4 (97%), No.5 (62%), No.7 (93%), No.9 (41%), No.11 (38%) and No.19 (86%), School D's experimental group in the posttest scored the highest percentage of correct responses. In the posttest, for Item No.11 (6%) and Item No.14 (39%) School D's control group scored the lowest percentage of correct responses. Again, School D's control group scored the highest percentage of correct responses in Item No.1 (94%).

School E

Table 4.52: Analysis of correct responses for School E's experimental and control group's performance in the pretest and the posttest

			Experimental group (N=45)		Control group (N= 33)	
Item	Item description	Correct answer	Percentage with choice		percentage with choice	
			Pretest	Posttest	Pretest	Posttest
1	Triangle recognition	**E**	59	82	67	72
2	Rectangle recognition	**A**	38	79	67	67
3	Square recognition	**C**	44	59	44	61

4	Quadrilateral recognition	**C**	24	44	28	44
5	Parallelogram recognition	**E**	15	24	11	28
6	Rectangle properties	**B**	41	47	72	50
7	Rectangle properties	**C**	65	65	72	67
8	Circle properties	**C**	32	26	17	28
9	Equilateral triangle properties	**E**	29	21	33	39
10	Square Properties	**A**	59	59	39	39
11	Connection between rectangle and parallelograms	**B**	12	26	22	22
12	Connection between rectangle and parallelograms	**D**	18	18	11	6
13	Connection between parts of a circle	**C**	24	18	11	22
14	Connection between right angled triangles and complementary angles	**C**	24	24	28	33
15	Connection between congruent and similar figures	**B**	18	38	33	39
16	Deduction- perpendicular lines	**D**	12	38	33	50
17	Deduction – parallel and perpendicular lines	**A**	35	53	61	33
18	Deduction- parallelogram and square	**C**	15	12	6	6
19	Deduction - angles	**B**	44	62	28	50
20	Deduction – properties of figures – squares and rectangles	**C**	18	41	33	28

It was evident from Table 4.52 and the analysis of correct responses sheet attached in Appendix G, that for Item No.16 (12%) School E's experimental group in the pretest scored the lowest percentage of correct responses. In the posttest, for Item No.16 (50%) School E's control group scored the highest percentage of correct responses.

4.4. Chapter summary

In this chapter, the analysis of the van Hiele Geometry Test, which was written as a pretest before the intervention with the van Hiele levels-based instruction and as a posttest after the intervention to verify the effectiveness of the intervention was explained. In the first part of this chapter, the analysis of the pretest was done in detail first to answer the first research question. In the second part of the chapter the analysis of the posttest was done in comparison with the pretest to answer the second research question. An analysis of correct responses was carried out to check the percentage number of correct responses for each item.

In the next chapter, the analysis of the interviews with learners and educators is presented.

CHAPTER 5

DATA ANALYSIS AND RESULTS: THE INTERVIEWS – QUALITATIVE DATA

5.1. Introduction

The data for the learners' interviews consisted of the learners' drawings, the interviewer's field notes and the audio taped interviews. Learners' pencil and paper recordings from the various tasks were written or drawn responses to the interview questions. The field notes for each interview contained annotations about surprising or unexpected responses and indicators about learners' confidence. The audio recording of each interview provided further data about the learners' conversation. The learners' responses and the explanation for each task and the discussion between the learners and the interviewer were analysed using the same process developed by Burger and Shaughnessy (1986), in conjunction with the level indicators which was also used by Genz (2006). Each interview was audio taped and lasted approximately forty to sixty minutes. Three interviews are analysed below and another one is transcribed and is shown in Appendix E. The interviews consisted of giving the learners seven open ended activities dealing with geometric shapes, developed by Burger and Shaughnessy (1986), which were designed to reflect the descriptions of the van Hiele levels that were available in the literature. The activities involved drawing triangles and quadrilaterals, identifying and defining shapes, sorting shapes and engaging in informal and formal reasoning about geometric shapes. These tasks were expected to draw out the characterisations of van Hiele levels 1 to 3 (Burger & Shaughnessy, 1986). No attempt was made to investigate van Hiele level 4 as none of the learners in the entire sample were at level 4 (except for an explanation of question at level 4 from the van Hiele Geometry Test). Two sets of drawings and identifying and sorting tasks were administered, one set for triangle shapes and one set for quadrilateral shapes.

Some interesting patterns in the answering of the van Hiele Geometry Test were noticed from the analysis of the test. The average percentage of correct responses for

Questions 5, 9, 12, and 18 were found to be the lowest from each of van Hiele level subtest questions. Some questions on further clarifications on these items were also asked at the end of the interview session. Learners were given mathematics sets, pencils, pens, papers and erasers to use.

The interviews took place in the learners' classrooms after school hours. The learners were well informed in advance of what was expected of them and they were willing to co-operate. Throughout the interview a pleasant and co-operative atmosphere was dominant as the learners were not intimidated at any stage of the interview. As from the researcher's side, the subject matter in the script was well prepared in order to conduct an informed conversation. The learners were allowed to take their time and answer in their own way as suggested by Cohen, Manion and Morrison (2007).

Educators from the five schools were also interviewed to comment on the activities and the evaluation of the framework as a whole. The interview schedule and the sample from one educator are attached in Appendix F.

Similar responses from all the interviewees are summarised in section 5.3.

5.2. The analysis of the interviews with the learners

During the analysis, the audio data and the learners' written responses and the researcher's notes were reviewed often to find the relevant dialogue and examples that reflected the findings and to check the accuracy of the findings.

Even though 30 learners were interviewed, only three were chosen for inclusion in this report and in order to ensure anonymity, they will be named as Andiswa (18 year old female), Mila (16 year old male) and Nana (16 year old female). Each represents one level of van Hiele thinking. These interviews yielded a number of particularly interesting aspects of the van Hiele theory and it seemed to be impossible to separate the analysis and the discussion with the literature. Therefore a major part of the interpretation of the analysis is shown with its discussions in Chapter 6.

5.2.1. The analysis of the interviews

The analysis of the interviews is done under two major headings namely, triangle activities and quadrilateral activities. The responses of the three learners are shown under each heading as Learner 1 (Andiswa), Learner 2 (Mila) and Learner 3 (Nana).

5.2.1.1. Triangle Activities

Learner 1: Andiswa

Andiswa was classified as being on the pre-recognition level (level 0) from the van Hiele Geometry Test. It appeared that drawing triangles other than the one with usual names that she was familiar with was of concern to her. When required to draw as many different triangles as possible, she provided four triangles, of which two triangles were similar in shape and orientation.

Figure 5.1: Andiswa's triangles

The following conversation took place on the triangles drawn by her:

Researcher: *If I ask you to draw more triangles, can you draw more?*

Andiswa: *I am not sure....*

Researcher: *Why?*

Andiswa: *I know only four types ...*

Researcher: *Can you tell me how is No.2 different from No.1?*

Andiswa: *No.1 is a square* (both sides are equal)

Researcher: *And No.2?*

Andiswa: *Right angled triangle.*

Researcher: *And No.3?*

Andiswa: *Isosceles triangle.*

Researcher: *And No.4?*

Andiswa: *Scalene triangle.*

She could draw the last three of them correctly and gave the correct name to it.

Identifying and naming triangles was a problem for her. When asked to mark triangles from a sheet with some figures, she marked it like this:

Figure 5.2: Andiswa: Activity 2A – Triangles

On the activity of identifying and naming triangles, the following conversation took place:

Researcher: *Andiswa, Why did you put a "T" on No. 16?*

Andiswa: *Because... it is a 'quadrilateral triangle'*

Researcher: *Why do you say so?*

Andiswa: *Because "both sides" are equal.*

Researcher: *Why didn't you put a "T" for No. 3 and No.7?*

Andiswa: *They do not "look like" triangles*.

This showed that Andiswa could not identify certain triangles and she did not use the properties when she focused on identifying them (e.g., No.16). This showed that she had not reached visual, analysis and informal deduction levels of thinking.

To elicit the properties that the learner perceived as necessary for a figure to be a triangle, the following question was asked:

Researcher: *If you want your little sister to look for a triangle from this paper, what will you tell her to look for?*

Andiswa: *She should look for 'a figure with 3 sides'.*

Researcher: *What if she picks No.3 and No.7 also?*

Andiswa: Oh...ja...(giggling ... , thinking for a while), *she must look for a 'triangle with 3 sides'* (by pointing to the figures she marked in the paper).

For her, the properties that she perceived as necessary for a figure to be a triangle was not clear. Anything that 'looks like a triangle' was a triangle for her. No.16 was a 'quadrilateral triangle' because 'both sides were equal'.

When she was asked to sort triangles by putting triangles that had something in common, to find out what properties that she would concentrate on when comparing triangles, the following question were asked:

Researcher: *I am going to give you some triangle cut-outs. Some of them have something in common. Group them in such a way that they have something in common.*

Andiswa: (Long silence.... Not sure what to do...)

Researcher: *Some of them have something in common, they can be put together.*

Andiswa: *No.7, No.6 and No.4 can be together. Because, they are 'quadrilateral triangles'. No.... No.7 is 'isosceles' and No.6 is 'scalene'. No.1 and No.7 have two sides equal, and No.2 is 'isosceles' because two sides are equal and No.8 is a 'quadrilateral' triangle because 'both sides are not equal'.*

For her '*both sides are equal'* and *'two sides are equal'* had different meanings. She was not sure about the difference of 'e*quilateral'* and *'quadrilateral'* and used it in the wrong contexts.

Researcher: *Can you group these cut-outs in a different way?*

Andiswa: *No...Mam...*

Researcher: *I am going to put No.3, No.5 and No.7 together. Can you find something in common?*

Andiswa: *No.7 is isosceles, because two equal sides, Mam.*

Researcher: *What about 'No.3?*

Andiswa: ...*is a scalene triangle, Mam.*

Researcher: *But Andiswa... I put them together because they have something in common...*

Andiswa: *Ummm... (No answer)*

Researcher: *They are obtuse angled triangles.*

Andiswa: *Yes, Mam...*

She could not perceive that they had something in common. She gave a correct answer for putting No.1 and No.6 together by indicating that they were '*right angled triangles*'. Class inclusion was a problem for her.

She knew about right angled triangles in terms of how they looked and used the correct terminology for it.

Learner 2: Mila

Mila was classified as being on the recognition level (i.e., van Hiele level 1) from the van Hiele Geometry Test.

When required to draw as many different triangles as possible, Mila drew four of them which were similar in size but different in orientations.

Figure 5.3: Mila's triangles

The following conversation took place on the triangles that were drawn by him:

Researcher: *If I ask you to draw more triangles, can you draw more?*

Mila: *Umm..... I think so.*

Researcher: *Tell me, how are you planning to do it?*

Mila: *I can ... by changing the angles or by making different angles than what I have done*.

Researcher: *Tell me ... How is No.2 different from No.1?*

Mila: *In No.2 ...two angles are equal and one angle is smaller than them and has 'one long side'. In No.1, 'all sides are equal' and No.3 has a 'long length this side'* (by pointing to the base) *and two angles equal.*

Researcher: *And No. 4?*

Mila: *No.4 is same as No.1... but different in the way it is standing. No.3 has one short side and one side is longer.*

Identifying and naming triangles was not a problem for him. The use of precise language was a problem for him. When asked to mark triangles from a sheet with some figures, he marked it like this:

Figure 5.4: Mila: Activity 2A – Triangles

The following conversation took place:

Researcher: *Mila, why did you put a "T" on No.4, and No.6?*

Mila: Mam... *Because they have 'two sides equal' and 'one side not equal'...they have three sides.*

Researcher: *Why didn't you put a "T" on No.15 and No. 16?*

Mila: *They are not triangles.*

Researcher: *Why is No. 3 not a triangle?*

Mila: *Oh... Mam...I think... Mam...It is not a triangle because 'it looks circular shape... but not pointy'.*

Researcher: *How is No.3 different from No.1?*

Mila: *No.1 'has 90^0 angle and sharp corners than No.3'.*

This showed that he could identify certain triangles and he did not use the properties when he focused on identifying them. This indicated that he had not reached the analysis level of thinking and informal deduction level of thinking.

To elicit the properties that the learner perceived as necessary for a figure to be a triangle, the following question was asked:

Researcher: *If you want your little brother to look for a triangle from this paper, what will you tell him to look for?*

Mila: *I will give him an example of a triangle...tell him to look for 'angle with 3 lengths or sides' and has 'two sides equal'*

Researcher: *Are you saying that in all triangles two sides are equal?*

Mila: *No... Mam...not in all triangles...* (After thinking for a while), *'the shape that has 3 sides and they can.... add to 180^0'. That is... That will be the way...*

It appeared that Mila knew the properties of a triangle, but was not sure how to explain it. The use of correct terminology was a problem for him.

When he was asked to sort triangles by putting triangles that had something in common, to find out what properties he would concentrate on when comparing triangles, the following conversation took place:

Researcher: *I am going to give you some triangle cut-outs. Group them in such a way that they have something in common*.

Mila: (He put No.4 and No.8 together) *because 'they are equal size, but No.4 is smaller than No.8.* (He put No.2 and No.6 together) *No 2 and No.6 are same because No.6 is 'turned in a way that the shape is changed' and* (he put No.5 and No.7 together), *both have equal sides, No.5 is 'reduced' and No.7 is 'bigger' and for No.1 and in No.3 none of the sides are equal.*

It looked like he was referring to 'same shape' as 'equal size'. When asked whether he could group them in a different way, after thinking he said he could do it by putting No.1 and No.2 together by claiming that 'both have small bases' and 'heights' are equal. 'No.4 and No.7 have same shape', but No.7 is 'thinner'. 'No.5 and No.3 have different angles'. When I put No.3, No.5 and No.7 together and asked what both had in common, he said 'bases are longer' and he did not know that it was because of the obtuse angles. The use of descriptive, imprecise language like "long", "thinner", "heights" and "longer" showed that he was looking at the visual characteristics of the figures.

Learner 3: Nana

Nana was classified as being on the analysis level (i.e., van Hiele level 2) from the van Hiele Geometry Test.

When required to draw as many different triangles as possible, Nana provided four of them in which she used a ruler to measure the sides as she was drawing and indicating the equal sides. They were all in different sizes. She could draw all of them correctly and gave the correct name to it.

Figure: 5.5: Nana's triangles

The following conversation took place on the triangles she drew:

Researcher: *If I ask you to draw more triangles, can you draw more?*

Nana: Yes. By c*hanging the lengths.*

Researcher: *Can you tell me how is No.2 different from No.1?*

Nana: *No.1 is an equilateral triangle*.

Researcher: *And No.2?*

Nana: *Right angled isosceles triangle, with two sides equal.*

Researcher: *And No.3?*

Nana: *Scalene triangle.*

Researcher: *And No.4?*

Nana: *An isosceles triangle.*

She used very precise, short sentences to explain her triangles.

Identifying and naming triangles was not a problem for her. When asked to mark triangles from a sheet with some figures, she marked it the following way:

Figure 5.6: Nana: Activity 2A –Triangles

The following conversation took place:

Researcher: *Nana, why did you put a "T" on No.4, and No.6?*

Nana: *They have 3 sides.*

Researcher: *Why didn't you put a "T" on No. 16?*

Nana: *They are not triangles, because 'sides are not equal'*

Researcher: *Why is No. 3 not a triangle?*

Nana: *Its lines are curved.*

Researcher: *Why is No.2 not a triangle?*

Nana: *No.2 has 4 sides and therefore not a triangle.*

This revealed that she could identify triangles and used the properties when she focused on identifying them. This indicated that she had reached analysis level of thinking for the concept of triangles.

To elicit the properties that the learner perceived as necessary for a figure to be a triangle, the following question was asked.

Researcher: *If you want your little sister to look for a triangle from this paper, what will you tell her to look for?*

Nana: *I will tell her to look for a figure with 3 straight sides.*

This indicated that Nana knew the necessary condition for a figure to be a triangle.

When she was asked to sort triangles by putting triangles that have something in common, to find out what properties that she would concentrate on when comparing triangles, the following conversation took place:

Researcher: *I am going to give you some triangle cut-outs. Group them in such a way that they have something in common*.

Nana: (She put Nos.1, 2, 4 and 5 together) *because "they have two sides equal, but Nos.6, 7, and 3 are together* (after putting them together) *because all sides are different"0.*

When asked whether she could group them in a different way, after thinking she said that she could not, but when I put Nos.3, 5, and 7 together she said their angles were obtuse.

It was noted that Nana used the correct terminology and precise language in her explanations.

5.2.1.2. Quadrilateral activities

Learner1. Andiswa

In quadrilaterals, when asked to draw as many quadrilaterals as possible, she drew five different quadrilaterals and is shown below:

Figure 5.7: Andiswa's quadrilaterals

The following conversation took place based on the quadrilaterals that were drawn by her:

Researcher:	*Andiswa, can you please tell me how is No.2 different from No.1?*
Andiswa:	*No.1 is a parallelogram and No.2 is a rhombus.*
Researcher:	*What about No.3?*
Andiswa:	*No.3 is....*
Researcher:	*If you cannot tell me the name, can you please tell me how is No.3 different from No.1 and No.2 in terms of the sides?*
Andiswa:	*It had two 'straight sides that are parallel' and 'those ones are equal and those ones are not equal'* (by pointing opposite sides).
Researcher:	*O.k., what do you call No.4?*
Andiswa:	*No.4 is a parallelogram because it had 'two sides are equal'.*
Researcher:	*What about No.5?*
Andiswa:	*I don't know...But 'No. 5 is different', because 'it has no straight sides'.*

It appeared that her figures, the names of the figures and the explanations for them were not corresponding to each other.

When asked for identifying quadrilaterals, she said she knew squares, rectangles, parallelograms, kite, trapezium and rhombus, and when asked to put an S on all squares and R on rectangles, P on parallelograms and K on kite and so on, she marked them like this.

Figure 5.8: Andiswa – Activity 2B – Quadrilaterals

Researcher: *Why do you say No.2 and No.7 are squares?*

Andiswa: *No.2 and No.7 are squares because "they are squares". "Four sides are equal",*

Researcher: *Why do you say No.6 is also a square?*

Andiswa: *"because have two sides that are equal"...Oh... No.6 is a parallelogram*.

Researcher: *O.k. If you want to change your mind do so.*

(Andiswa changed her mind and marked No.6 as a parallelogram).

Researcher: *Why do you say so?*

Andiswa: "*Not sure why it is called so".*

Researcher: *Why do you say No.10 and No.13 are parallelograms?*

Andiswa: *No.10 and No.13 are parallelograms because they have 'equal non straight sides'.*

(Andiswa changed No.13 to a square when mentioned about equal sides)

Researcher: *Why do you say No.1 and No.15 are rhombi?*

Andiswa: *No.1 and No.15 are rhombi, "but they are not the same, but sometimes two sides, see …"* (could not complete the sentence)

It appeared that she had serious problems identifying quadrilaterals. At times she seemed to understand a square, but got confused between rectangles and parallelograms due to "two sides being equal".

To elicit the properties that the learner perceives as necessary for a figure to be a quadrilateral, the following conversation took place:

Researcher: *How will you ask your little sister to pick a square from these shapes?*

Andiswa: *She needs to look for 'four sides which are equal'.*

Researcher: *What about for a rectangle?*

Andiswa: *She should look... oh... I am not sure...*

Researcher: *What about parallelograms?*

Andiswa: *O.k. ... 'two sides which are equal and two sides which are not equal, which are not straight'.*

It looked like Andiswa did not know the necessary condition for a figure to be square, a rectangle and a parallelogram.

In the activity of grouping quadrilaterals, she was not sure how to group them and after a while, she put No.4 and No.6 together and said "they are squares because they have ‘four straight lines, two sides are same, and two sides are parallel”. No. 7 and No. 5 were rhombus but she was not sure why. She called No.8 and No.3 as kites but was not sure why they were called kites.

When she was asked to reshuffle to put them in a different way, she could not reshuffle them to do grouping again.

I arranged No.1 and No.2 together and asked her why she said they can be together, she said it was because “two sides are not straight and the other two sides are not straight”.

When given clues to find the mystery shape, it took a long while for her, but said ‘square is a closed figure with 4 straight sides’, when it has ‘2 long and short sides it is a parallelogram’ and when angles were given, she said they were ‘isosceles’. She could not come up with a name for the clues for the mystery shape A. For shape B also she struggled the same way and could not complete the same for C.

For her, parallelogram was sometimes ‘two long sides and two short sides’ (mentioned it two times) and another time it was ‘two sides which are equal and two sides which are not equal, which are not straight’. A square is a figure with ‘four straight sides’, which she mentioned it also two times.

‘All sides are same length’ is a ‘quadrilateral’ for her – which she used when she drew the triangles also.

The listing of properties in those items designed to assess analysis and informal deduction level thinking did not make sense and Andiswa used inappropriate vocabulary in almost all the tasks. It looked like, according to her, ‘quadrilateral’ stands for ‘all sides equal ‘whether it was a triangle or a quadrilateral, and she used it instead of “equilateral” and ‘all sides are straight’ was a square for her. Such a response suggested that this vocabulary did not have any meaning to her but that she had simply memorised the words without the conceptual understanding.

Her use of vocabulary and properties was owing to memory rather than understanding.

Learner 2: Mila

In quadrilaterals, when asked to draw as many quadrilaterals as possible, Mila drew five different quadrilaterals.

Figure 5.9: Mila’s Quadrilaterals

Researcher:	*O.k., Now tell me, if I ask you to draw more quadrilaterals, will you be able to draw more?*
Mila:	*yes, Mam...I can*
Researcher:	*How?*
Mila:	*I will change to a shape with non equal sides, or make a shape with three equal sides or all sides equal etc.*
Researcher:	*Mila, tell me how is No.1 different from No.2?*
Mila:	*No.1 has four sides that are equal, No.2 has two equal sides.*
Researcher:	*What about No.3?*
Mila:	*No.3 has two opposite sides that are equal, Mam.*
Researcher:	*And about No.4?*
Mila:	*No.4 has two sides that are equal, Mam.*
Researcher:	*What about No.5?*
Mila:	*No.5 has none of the sides are equal, Mam.*

It was noted that while talking about the quadrilaterals that were drawn by him, Mila did not talk about the specific name of the quadrilaterals; rather he used the lengths of the sides to differentiate them.

When asked to identify quadrilaterals, he said he knew squares, rectangles, parallelograms and trapezium, and when asked to put an S on all squares and R on rectangles, P on parallelograms and T on trapeziums and so on, he marked as follows:

Figure 5.10: Mila: Activity 2B – Quadrilaterals

Researcher: *Why do you say No.2 and No.7 are squares?*

Mila: *Because...Mam,... 'they have four sides that are equal', Mam.*

Researcher: *Why do you call Nos. 5, 6, 9, 10 and No.12 as rectangles?*

Mila: *They have 'two equal sides and two sides that are parallel to each other', Mam.*

Researcher: *Why do you say Nos.1, 3, 4 are parallelograms?*

Mila: *They have 'two sides, mam, 'that are equal', No.1 has 'two sides that are equal' and 'other sides that are not equal', No.4, 'all four sides equal', Mam, and for No.3, 'two sides that are parallel to each other', Mam.*

It looked like his explanations were rather messy and long to express a particular figure.

To elicit the properties that the learner perceives as necessary for a figure to be a quadrilateral, the following question was asked:

Researcher: *How will you tell your little brother to look for a square from these shapes?*

Mila: *He must look for 'four equal sides'.*

Researcher: *What about a rectangle?*

Mila: *'Two opposite sides must be equal'*

Researcher: *How will you ask him to choose parallelograms?*

Mila: *'Two equal sides opposite to each other'*

Researcher: *And what about rhombus?*

Mila: *Umm ... I don't know... Mam.*

Here also it was noted that Mila was not sure of the necessary properties of many quadrilaterals.

When it was asked about marking trapezium in the sheet of figures, he asked me whether I could give the definition of a parallelogram so that he could think of trapezium. I did mention it, but even after thinking for a while he could not mark any trapeziums. This showed that he resorts to rote learning when defining shapes. When it was asked about rhombus, he said he had forgotten the definition of rhombus and rechecked and confirmed that he did not mark any rhombus in the given set of quadrilaterals.

The following questions were asked about the class inclusion of shapes:

Researcher: *Can No.2 be a rectangle?*

Mila: *No...Mam, because 'a rectangle is made up of two sides are equal and because a rectangle cannot have four sides equal, a square cannot be called a rectangle'.*

Researcher: *Can No.9 be a parallelogram?*

Mila: *Yes, Mam, because 'a parallelogram is made up of two opposite sides that are equal to each other'.*

Researcher: *Can No.7 be a rhombus?*

Mila: *No. Mam, No.7 could not be a rhombus because 'it has all four sides equal to each other, but rhombus cannot'.* (Then got confused)*...'Yes'...Mam, but...I forgot the definition, Mam.*

Class inclusion seemed to be a bit of a problem to him as he could not remember most of the definitions. Rote learning could have been the underlying reason behind this.

In grouping quadrilaterals from the cut outs, No.4 and No.6 were grouped together because '*they have* 90^0 *angles* and in No.3 and in No.9, they have two sides that are equal to each other'. '*No.2 and No.8 have* 45^0 *angles'.* 'No*s.7 and 5 have three sides are equal to each other. No.1 has two sides equal to each other'.* Any angle that was less than 90^0 was 45^0 for him.

When asked whether it was possible to group again, it was almost the same except 'for No.1 and No.8 were given as same' because 'they are called kites with 4 equal sides'. 'No.2 and No.4 have two equal sides' and are called rectangles. It appeared that the definitions or the names he got for different shapes were all mixed-up. Rote learning could have been a reason for this kind of confusion.

When given clues to find the mystery shape, it took a long while for him, but said 'square is a closed figure with 4 straight sides' and, 'when it has two long and two short sides it is a rectangle' and he could finally come up with a name for the clues for the mystery shape B as a parallelogram. For shape C also he struggled the same way and could come up as Shape C as a rectangle.

The listing of properties in those items designed to assess analysis and informal deduction level thinking did not make sense and Mila used inappropriate definitions for quadrilaterals like parallelogram and rectangles. It was also evident that even though he was at level 1 for the concepts of triangles he had not mastered the level for quadrilaterals.

Learner 3: Nana

In quadrilaterals, when asked to draw as many quadrilaterals as possible, she drew five different quadrilaterals in which she used a ruler to measure the sides as she was drawing and indicating equal sides. They were all in different sizes.

Figure 5.11: Nana's Quadrilaterals

The following conversation took place on the quadrilaterals she drew:

Researcher: *If I ask you to draw more quadrilaterals, can you draw more?*

Nana: *Yes, by changing the lengths of the sides.*

Researcher: *Can you tell me how is No.2 different from No.1?*

Nana: *In No.2, opposite sides are equal and parallel and has all angles 90^0.*

Researcher: *And No.1?*

Nana: *In No. 1, 'all sides are equal and all angles are 90^0.*

Researcher: *And No.3?*

Nana: *In No.3, diagonals meet at 90^0.*

Researcher: *And No.4?*

Nana: *No.4, has two sides that are equal but one pair of opposite sides are parallel.*

Researcher: *What about No.5?*

Nana: *Opposite sides are equal, but the angles are not 90^0.*

It was noted that Nana spoke about her quadrilaterals in terms of the sides and angles. The term 'diagonal' was also mentioned in her explanation about the 3rd figure.

When asked about identifying quadrilaterals, she said that she knew squares, rectangles, parallelograms, kite, trapezium and rhombus, and when asked to put an S on all squares and R on rectangles, P on parallelograms and K on kites and so on, she marked like this:

Figure 5.12: Nana: Activity 2B – Quadrilaterals

On questions about the marking on the figures, the following conversation took place:

Researcher:	*Why do you say No.2 and No.7 are squares?*
Nana:	*They are squares because "all sides are equal and all angles are 90^0".*
Researcher:	*Why do you say No. 3, 6, and 10 are parallelograms?*
Nana:	*They are parallelograms because 'opposite sides are equal and parallel and they do not have 90^0 angles'*

Researcher:	*Why do you say Nos.9 and 12 are rectangles?*
Nana:	*They are rectangles because 'opposite sides are equal and parallel and all angles are 90⁰'.*
Researcher:	*What about Nos. 1, 14 and 15?*
Nana:	*They are trapeziums because 'they have only two sides that are parallel'.*

It also appeared that her definitions were very precise in terms of the sides and angles.

To elicit the properties that the learner perceives as necessary for a figure to be a quadrilateral, the following question was asked:

Researcher:	*How will you tell someone to pick a square from these shapes?*
Nana:	*That person should look for four sides which are equal and all angles 90^0.*
Researcher:	*What about a rectangle?*
Nana:	*'Opposite sides must be equal and parallel and all angles to be 90^0'.*
Researcher:	*How will you ask that person to choose parallelograms?*
Nana:	*'He/she should look for opposite sides to be equal and parallel and not 90^0 angles'.*
Researcher:	*And for a rhombus?*
Nana:	*'All sides should be equal, all angles equal and not 90^0'.*

It also appeared that her definitions were very precise.

The following questions were asked about the class inclusion of shapes:

Researcher: *Can No.2 could be a rectangle?*

Nana: *'Yes', because 'opposite sides are equal and angles are 90^0'.*

Researcher: *Can No.9 be a parallelogram?*

Nana. *'Yes', because 'opposite sides are equal and opposite sides are parallel'.*

Researcher: *Can No.7 be a rhombus?*

Nana: *'Yes', because 'all sides are equal all angles are equal'.*

It appeared that class inclusion was not a problem for her.

In grouping quadrilaterals, 'No.5 and No.7 are trapeziums', 'No.1 and No.8 have all sides equal and angles equal but angles are not 90^0'. For No.6, 'all sides are equal and angle are 90^0' but for 'No.2 all sides are equal but angles are not 90^0'.

When asked whether it was possible to group in a different way she said 'yes' by putting Nos.1, 8 and 2 together by saying that 'they have same angles and Nos.6 and 4 can go together as they have 90^0'.

When given clues to find the mystery shape, she could come up with a name for the clues for the mystery shape A as a rhombus, B as a trapezium and C as a parallelogram.

The listing of properties in those items designed to assess analysis and informal deduction level thinking did make sense and Nana used appropriate definitions and vocabulary that matched to van Hiele level 2.

5.2.1.3. Questions on the van Hiele Geometry Test

One question from each level that was answered wrongly by most of the learners was asked to each interviewee. The questions that were asked were question 5; question 9; question 12 and question 18 (see section 4.3.5 in Chapter 4).

Learner 1: Andiswa

Here again Andiswa gave a wrong answer to question 5 by saying that I is the only parallelogram because 'it has 2 sides equal which are not straight'. Question 9, 12 and 18 were also answered wrongly because of lack of understanding of relationship between figures.

Learner 2: Mila

Mila gave the correct answer for question 5 by saying that all of them are parallelograms. For question 9 he gave a wrong answer. For question 12 he gave a wrong answer. Question 18 was also answered wrongly because of lack of understanding of relationship between figures.

Learner 3: Nana

Nana gave the correct answer for question 5 by saying that all of them were parallelograms. For question 9, 12, and 18, she gave the correct answers.

5. 3. Analysis of the interviews with educators

Educators from the five schools were interviewed to comment on the activities and the evaluation of the framework as a whole. The interview schedule and the sample from one educator are attached in Appendix E.

Similar responses from all the interviewees are summarised. The following are some of the comments:

Comments on the activities in the instructional framework:

Table 5.1: Comments on the activities in the instructional framework

Activity Number	Description of the activity	Comments
1	Introductory game	Three out of the five educators commented that it helped the learners to identify and name figures. One educator commented that the comparison will help learners to know properties of shapes. One educator mentioned that it was too easy for Grade 10 learners
2	Shapes in pictures	All the educators commented that it helped the understanding of shapes.
3	Hidden geometric shapes	All the educators commented that it was an interesting activity and it tested the ability to use their knowledge on geometric shapes.
4	Tangram puzzles	Helped the learners to visualise shapes making up irregular shapes. This was an interesting activity to improve visual abilities. (similar responses from all educators)
5	Geometric item sorting activity	This was a good activity to find the similarities and differences of geometrical shapes (similar responses from all educators)

6	The family of quadrilaterals	Helped the learners with the connection between shapes. Helped with the classification of quadrilaterals. (similar responses from all educators)
7	Discovering with folding	This was another way of making the learners understand the properties of shapes. Learners enjoyed this activity. (similar responses from all educators)
8	Drawing and construction-circle	Helped the learners in drawing. Helped to learn the concept of diameter. (similar responses from all educators)
9	Conjecturing in plane geometry	Helped in knowing the properties of shapes. It was difficult for some learners.

2. Evaluation of the instructional framework

All the educators gave similar comments on the evaluation of each criterion.

Table 5.2: Evaluation of the instructional framework

No	Criteria	Comment
1	Practicability	practicable
2	Usefulness	useful
3	Suitability	Suitable and relevant
4	Time allocated for each activity	Appropriate
5	Beneficial for learners	beneficial
6	Self improving for educators	Great improvement
7	Interesting for learners	interesting
8	Participation from learners	good

3. How do you compare the instructional module with your own teaching method?

Some of the comments that were noticeable were:

- It was well prepared
- Well structured questions
- It helped learners to understand concepts
- More advanced than usual classroom teaching.
- Visual activities are very good for the learners to remember the shapes and their proper names
- It took a long time to complete a small topic

4. Overall impression on the instructional framework:

Some of the comments were:

- It had well structured activities that helped both educators and learners
- It was interesting and helped the learners well
- Learners were able to identify the geometric shapes and they could learn the properties of shapes
- Learners and educators could integrate different concepts in geometry
- It will be useful to teachers who were not exposed to geometry at tertiary level.
- Very good activity for grade 10 learners. Whoever is using this activity in the classroom is helping the learners in the best way to overcome the challenges associated with basic geometrical shapes and their properties.
- It is well laid out, that learners will not even require a teacher to explain. It is self explanatory.

From the comments it was clear that the activities were good for learners and the teachers appreciated that it was good in helping the learners to overcome the challenges associated with basic geometrical shapes and their properties.

5.4. Chapter summary

In this chapter, the interviews with three learners in three different van Hiele levels of thinking were analysed. The analysis of the interviews with the educators was also given.

In the next chapter, both the quantitative and qualitative data are discussed.

CHAPTER 6
DISCUSSIONS – QUANTITATIVE DATA AND QUALITATIVE DATA

6.1. Introduction

In Chapter 4, the analysis of the quantitative data from the written test and in Chapter 5, the analysis of the qualitative data from the interviews of both learners and teachers were presented. In this chapter, the findings from both the qualitative and quantitative data are discussed. The first part of this chapter discusses the quantitative data and the second part of the chapter discusses the qualitative data of the study.

The findings from the quantitative data of the current study are mainly compared against the results from the study of Usiskin (1982) and that of Atebe (2008). Usiskin's (1982) American study is still considered as a ground breaking study as it is one of the major studies conducted on van Hiele levels and it involved 2700 students from 99 classes in 13 high schools in five states in the United States of America. Atebe's (2008) study was the one major study conducted in two African countries, South Africa and Nigeria on the levels of thinking of South African and Nigerian senior secondary school learners based on the van Hiele theory (see section 2.6.6 in Chapter 2). By comparing the results presented here with these two studies, this study will have the benefit of checking its results against its counterparts internationally. Both these studies used paper and pen tests to elicit the level of thinking of the learners. Moreover the instrument that was used in this study to test the learners in the written test was adopted from that of Atebe (2008).

6.2. Discussion on the quantitative data – the van Hiele Geometry Test

In this section, the discussion on the analysis of learners' performance in the van Hiele Geometry Test is presented. The study was undertaken mainly to find the effectiveness of a van Hiele-based instructional framework in grade 10. There were two research questions. The first research question of the study was concerned about the determination of the van Hiele levels of the learners in the study. This chapter begins with the discussion of the learners' performance in terms of the percentage mean scores of the test and then in terms of their allocation into van Hiele levels. The second research question was concerned about the effectiveness of the van Hiele-based instructional framework. The second part of the chapter provides information on the effectiveness of the van Hiele-based instruction by comparing the percentage mean scores in the experimental and control groups. In doing this, the pretest and posttest from each of the five schools were compared. Further, a comparison of learners at each van Hiele level was made.

6.3. Focus one

What are the geometrical thinking levels of the learners in the sample?

The present geometrical thinking levels of the learners were obtained from the van Hiele Geometry Test that was administered as the pretest. First it was analysed in terms of the percentage mean and then it was analysed in terms of the percentage number of learners at each van Hiele levels according to the criterion developed by Usiskin (1982). The results from the analysis are discussed in the following sections.

6.3.1. Discussion on the current van Hiele level of learners as determined by the VHGT(pretest) according to percentage means

In the discussion that follows the performance of the learners is provided by examining the percentage mean scores in the pretest. It is discussed under the sections 6.3.1.1. to 6.3.1.3.

6.3.1.1. Overall performance of the participants in the pretest

From the analysis of the percentage mean scores of all the learners who participated in the study, the results are discussed below.

Table 4.1 in Chapter 4 showed that the performance of the learners in the pretest from the different schools was almost the same even though they were relatively low percentages.

The relatively low percentage mean scores obtained by the learners (33.14%) were found to be consistent with the findings of Atebe (2008), where the overall percentage mean scores of the 144 South African and Nigerian learners in grades 10, 11 and 12 in his study was 35.68%. The Nigerian subsample had a mean score of 31.84% and the South African subsample had a percentage mean score of 39.37%. In particular, the grade 10 learners in the Nigerian subsample obtained 24% and South African subsample of grade 10 learners obtained 39%. The South African subsample in Atebe's (2008) study performed slightly better. It was noticeable that the Nigerian subsample's performance was significantly lower than the performance of the learners in this study.

This was also consistent with the American study in 13 schools by Usiskin (1982), where the highest percentage score was 11.90% according to the grading method used by this author. Atebe (2008) asserts that the learners in Usiskin's study performed better than the sample in his study as the grading used by Usiskin was different (see Usiskin, 1982) and if he had used the same grading system, his sample's percentage mean would have been 6.09%.

A further discussion is carried out on the analysis to determine the performance of learners in each school according the experimental group and control groups.

6.3.1.2. Discussion of the learners' performance in the pretest according to experimental group and control group per school

Table 4.3 in Chapter 4 showed that the experimental group in School D had the highest percentage mean (41.9%) and School C's experimental group had the lowest percentage mean (26.88%).

In Schools A, C, D, and E, learners in the experimental group obtained slightly higher percentage mean scores than that of the learners in the control group. In School B, the experimental group learners obtained slightly lower percentage mean scores than that of the learners in the control group.

The low percentage mean of the majority of learners in the study indicated that learners had a weak geometrical conceptual understanding. The test items tested were mainly the basic concepts in triangles and quadrilaterals. Most of the learners could only identify the common shapes and they failed to compare shapes by means of their properties. This was evident from the cluster of correct answers in the first subset of items in the test which tested the lowest level of the geometrical thinking. This is also consistent with the study of Atebe (2008).

The following discussion is based on the analysis meant to determine whether there was a statistically significant difference in the percentage means scores of the learners between the experimental group and the control group in each school.

6.3.1.3. Statistical comparison of the learners' performance in the pretest according to experimental group and control group per school using t-test

The t-test analysis was used to compare the attainment of levels for each group from all the five schools.

Table 4.4 in Chapter 4 indicated the learners' performance in the pretest according to experimental group and control group in Schools A, B, C, D and E. It showed that the learners in the experimental group and the control group of School A, School C, School D and School E were of comparable ability in terms of their performance in the pretest except for School B where it showed a statistically significant difference. In other words, the two groups in the majority of the schools were similar in their performance in the van Hiele geometry Test (pretest).

6.3.2. Discussion of the overall participants' performance in pretest by gender

Learners' performance in the pretest was further analysed for a possible gender difference in the entire study sample.

Table 4.5 in Chapter 4 showed that in the entire sample, there was a slight difference in the performance in the pretest in favour of the female learners. In School D, male learners performed slightly better than the female learners. And in School A, School B, School C and School E, the female learners' performances were slightly better. The test of significance indicated that the difference in the percentage mean scores between the male learners and female learners in each school and all schools combined were not statistically significant at the 0.05 level of significance. It can be assumed that the overall achievement in the van Hiele Geometry Test (pretest) was independent of gender. Gender did not play a role in the performance of the entire sample.

This negligible difference in favour of female learners was not consistent with the previous studies as Atebe (2008) noticed a marginal difference of 4% in favour of male learners who obtained a percentage mean score of 38%, while the female learners obtained only a percentage mean score of 34%. In a study by Halat (2006) in Turkey, whereby a sample of 150 learners (66 boys and 84 girls) from grade 6 were tested on the acquisition of van Hiele levels, found that there was no statistically significant difference detected between boys and girls. This was also consistent with Usiskin's (1982) study on American students, where there was no statistically significant difference in the fall (test carried out at the beginning of the school year) result. But the spring (test carried out towards the end of the school year) results were in favour of the boys.

6.3.3. Discussion of the learners' performance in the pretest according to percentage mean scores in the van Hiele levels

It is evident from Chart 4.2 in Chapter 4 that the highest percentage mean score was at level 1 (46.85%) followed by 39.44% at level 2, 19% at level 3 and 27.65% at level 4. This showed a decrease in the percentage mean score in each successive higher van Hiele level except at level 3 which was the lowest.

This was consistent with that of Atebe (2008) where the learners obtained 47% at level 1, 44% at level 2, 20% at level 3 and 32% at level 4. Atebe's (2008) sample performed slightly better at each level than that of the learners in the present study.

This provided evidence for the hierarchical nature of the van Hiele levels. This was also mentioned in the section 4.2.3.1, where the cluster of correct answers was at level 1. However it is to be noted that the lowest percentage mean was at level 3 in both studies. It appears that many learners experience problems at level 3 thinking. More particularly item No.12 was found to be the hardest of all. A detailed explanation of it is given in the section of discussion of correct responses later in this chapter. Usiskin (1982) also reported the same situation where the items at level 5 turned to be easier than the items at level 3 and level 4.

Learners obtaining the lowest percentage at level 3 are a cause of concern as it is the level expected of the learners starting senior secondary school geometry. As mentioned earlier in section 2.2.1 in Chapter 2 under the description of each van Hiele level, the learners at this level are supposed to recognise that a property of a figure proceeds or follows from other properties and they should also understand the relationship between figures. Class inclusions are supposed to be understood at this level. At this level properties are logically ordered. It seems that the learners in the senior secondary schools across the world are battling to attain this level. It can be taken that as the junior secondary school geometry curriculum is not preparing the learners well enough to face the challenges in the senior secondary school. Teppo (1991) also suggests that systematic geometry instruction in the middle grades is necessary to prevent students from entering high school at low levels of geometric concept development.

As evident from Table 4.12 and Chart 4.3 in Chapter 4, School D had the highest percentage mean at level 1 and level 2 and School C's performance was the best at level 3 and School B's performance was the best at level 4. In all schools, the percentage mean scores were the highest at level 1 and the lowest were at level 3. It also appeared that there was a wider gap between level 2 and level 3 thinking in most of the schools. It was also consistent with the earlier studies of Usiskin (1982), Siyepu (2005) and Atebe (2008).

6.3.4. Discussion of the percentage number of learners at different van Hiele levels of thinking

The grading of the van Hiele Geometry Test was done again using a second method which was based on the '3 of 5 correct" success criterion as suggested by Usiskin (1982, p.22) to assign learners into different van Hiele levels.

Chart 4.4 in Chapter 4 showed that the majority of the learners were at level 0 (56%). For van Hiele levels 1, 2, 3, and 4, it was 26%, 17%, 1% and 0% respectively. This was consistent with the study of Atebe (2008), which found that grade 10 learners in the Nigerian sub-sample, the percentage number of learners were 75%, 17%, 8%, 0% and 0% at levels 0, 1, 2, 3 and 4. Whereas, the South African subsample of grade 10 showed a percentage of 38%, 33%, 17%, 0% and 8% respectively at levels 0, 1, 2, 3 and 4. The performance of the South African subsample of Atebe (2008) performed slightly better than the learners in the present study.

It was evident from Table 4.18 and Chart 4.5 in Chapter 4 that the majority of the learners in all schools were at level 0 except for school D which had only 29% at level 0. School C had the highest number of learners at level 0 (70%) followed by School E (65%), School B (63%) and School A (54%). Level 3 was achieved by no learners in all the schools except by 6% of learners in School D. None of the schools had learners at level 4 thinking on the van Hiele scale indicating that the learners were not ready for formal geometric proofs in grade 10.

The deep concern in all the groups presented here is the number of learners at level 3 and level 4. Only 1% of the learners in the entire sample of 359 learners in the present study were at level 3 and no one in Atebe's (2008) entire sample was on level 3 except for the South African subsample with 8% of them at level 4. Usiskin (1982) also noticed that 70% of his sample was operating at levels 1, 2 and 3.

The majority of learners in the study (56%) at level 0 indicated that learners had a weak knowledge in geometrical concepts. The main issue here is that most of the learners could not identify common figures and they could not recognise figures in non standard positions. This was also evident from the interviews conducted of which the details are provided later in section 6.5.2 in this chapter.

The only 1% at level 3 and no one at level 4 indicated that learners in this study had difficulty in class inclusion of shapes, relationships between different shapes and properties of shapes. This was also consistent with the study of Mayberry (1983), where similar problems were noticed with the 19 pre-service elementary teachers in her study. Wirszup (1976) also had claimed that the majority of the high school learners were in the first level of development (level 1) while the course they took demanded level 4 thinking. It was evident that the majority of the learners in the study were also not reaching the level set by the curriculum, which expected the learners to be operating at level 3.

It was evident that the learners in School D had more conceptual base than the learners in all the other schools. It was also evident that the learners in different schools involved in the study had varied exposure to geometric figures and their characteristics.

6.3.5. Discussion on the implications for teaching from the findings of research focus one

The implications for teaching from the findings of research focus one was thoroughly discussed in Chapter 4 under the section 4.2.5 in preparation for the analysis of research focus two in section 4.3.

The delivery of instruction that is appropriate to learners' level of thinking is very important. As explained in section 2.6 in Chapter 2 and in section 3.2.1 in Chapter 3, researchers in different parts of the world (Hoffer, 1981; Usiskin, 1982; Senk, 1985; Shaughnessy & Burger, 1985; van Hiele, 1986; Fuys, et al., 1988; Clements & Battista, 1992; King, 2003; Atebe, 2008) concluded that learners' poor performance in geometry holds account for geometry classroom teaching and learning and the main cause of learners' difficulty with geometry is teachers' failure to deliver instruction that is appropriate to the learners' geometric level of thinking.

In many western countries, the van Hiele theory has become the most influential factor in their geometry curriculum (Fuys, et al., 1988; van de Walle, 2004). The present study looked into the possibilities of improving the geometry education by introducing van Hiele-based instruction after determining the level of geometric thinking and it was implemented in the five schools under the quasi-experimental design. In the next section, the effectiveness of the framework is discussed.

6.4. Focus two

Can the researcher's developed instructional framework improve the geometrical thinking levels of the learners in the sample?

One of the objectives of the study was to develop an instructional framework using the van Hiele levels after determining the present level of thinking of the learners. The discussion that was presented in the previous section had indicated that most of the learners in the sample (56%) were at level 0 followed by 26% at level 1, 17% at level 2 and 1% at level 3 and no one at level 4.

Based on the above information, the study developed an instructional framework and implemented it in the five schools. To achieve the main purpose of the study as whether the researcher's developed framework made any improvement in the levels of geometric thinking, the same test was administered as a posttest on all the learners in the sample and a statistical analysis was conducted. For that a paired sample t-test was used for comparison and the analysis was presented in Chapter 4 and the discussion is presented in the following sections.

6.4.1. Discussion on the analysis of the overall percentage mean scores of all the learners in the pretest and the posttest

Table 4.24 in Chapter 4 showed that the overall percentage mean of all the learners in the posttest was 39.97% which was marginally higher than the overall percentage mean of 32.99% in the pretest. This showed an improvement in the performance of the learners in the posttest.

This was consistent with the earlier studies as King (2003), Halat (2007) and Erdogan and Durmus (2009) who reported that there was a positive improvement on the performance after the intervention. In Usiskin's study (1982), after a yearlong course in geometry, about a third of the students stayed at the same level or went down, about a third went up one level and about a third went up two or more levels.

It was evident from Table 4.25 in Chapter 4 that there was an increase in the percentage mean scores of both groups in the posttest. The experimental group's percentage mean scores increased from 32.74% to 42.7% and the control group's percentage mean scores increased from 33.54% to 36.99%. The percentage increase in the experimental group was greater than that of the control group. Table 4.26 and Chart 4.6 in Chapter 4 showed that the performance of the learners in the posttest from the different schools for both experimental and control group were higher than their performance in the pretest even though they were relatively low percentages. As seen in Table 4.29 in Chapter 4, the difference between the percentage mean scores of the experimental group in the pretest and posttest were compared by means of paired samples t-test and in all schools. It was found that, the difference between the percentage mean scores of the experimental group in the pretest and posttest was statistically significant at 0.05 level of significance and that the instructional framework had a positive effect in all schools, although other extraneous variables might be operating.

It can be taken that the van Hiele-based instruction had a positive effect on the performance of the learners in the experimental group. The traditional method of teaching also had a positive effect even though it was not as considerable as the difference seen in the experimental groups in all the schools who were taught with the van Hiele-based framework. Table 4.30 in Chapter 4 showed that the traditional method of teaching did not cause a statistically significant difference in the performance of the control group learners in the schools except in School A.

This was consistent with the earlier studies as Usiskin (1982) reported that there was a change at level from the fall to the spring result. Halat's (2007) study on Grade 6 learners also showed some improvement on the performance after the intervention, even though it was not statistically significant. Billstein and Williamson (2003) and

Chapbell (2003) as cited by Halat (2007), agree that standards based curricula have positive impact on students' performance and motivation in mathematics. The study by Erdogan and Durmus (2009) on pre-service elementary teachers in Turkey showed statistically significant improvement on the performance of the experimental group. The study of King (2003) in the Eastern Cape on Grade 6 learners also showed statistically significant improvement on the performance of the experimental group.

6.4.2. Discussion on the overall participants' performance in the posttest by gender

Table 4.32 and Chart 4.9 in Chapter 4 showed that in the entire sample, there was a slight difference in the performance in the posttest in favour of the female learners. Table 4.33 and Table 4.43 in Chapter 4 also showed that in all schools, there was no statistically significant difference in the percentage mean scores according to gender.

This was not consistent with Usiskin's (1982) study on American students in the Spring (test carried out towards the end of the school year) results where boys outperformed the girls after a yearlong geometry course, where they started off in Fall at the beginning of the year with no statistically significant difference.

6.4.3. Discussion on the overall percentage mean scores of learners and the percentage number of learners at each van Hiele level in the posttest

As evident from Chart 4.12 in Chapter 4, the percentage mean scores of learners at each van Hiele level in the posttest for levels 1, 2, 3 and 4 were 57.46%, 44.35%, 23%, and 34.67% respectively. This was higher than the percentage mean scores of learners at each van Hiele level in the pretest which were 46.85%, 39.44%, 19%, and 27.65% respectively. This showed that more learners got more correct answers at each van Hiele level in the posttest.

Chart 4.14 in Chapter 4 showed that the majority of the learners were at level 0 (36%). For the van Hiele levels 1, 2, 3, and 4, the percentage number of learners was 33%, 30%, 1% and 0% respectively in the posttest. But in the pretest the majority of the learners were at level 0 (56%) and for the van Hiele levels 1, 2, 3, and 4, it was 26%, 17%, 1% and 0% respectively. It was evident from the comparison that the number of learners at each level had increased at levels 1 and 2, stayed the same at levels 3 and 4 and considerably decreased at level 0.

The significant improvement in the performance of the experimental group in School A having more learners at level 2 than at level 0 and level 1 suggest that the van Hiele-based instruction had a positive effect. The significant improvement in the performance of the experimental group in School B having more learners at level 1 than at level 0 suggests that the van Hiele-based instruction had a positive effect. The improvement in the performance of the experimental group in School C having more learners at level 1 than at level 0 suggests that the van Hiele-based instruction had a positive effect. School D's performance was consistently the best performance out of the five schools. It can be assumed that the geometrical experience they had could be one of the factors that contributed to this performance. In School E, The percentage number of learners at level 0 had been reduced considerably in both groups in the posttest. The history and cultural background as shown in the description of the schools might also have contributed to it.

It was also consistent with the study of Usiskin (1982) which stated that about a third of the students stayed at the same level or went down, about a third went up one level and about a third went up two or more levels.

The majority of learners in the study (36%) at level 0 in the posttest, even though it had been considerably reduced from 56% of that in the pretest, still indicate that learners are having conceptual difficulties. The increase in the percentage number of learners at levels 1 and 2 from 26% to 33% and from 17% to 30% gives some hope that a structured programme can raise the level of thinking.

In general, it can be assumed that all schools benefitted out of the instructional framework through the considerable reduction of the percentage number of learners and the statistical inference that there was a significant increase in the percentage mean scores in the experimental group. Other extraneous variables like history and maturation also might have contributed to this positive change. A detailed list of findings is given at the end of the chapter in the summary of findings.

6.4.4. Discussion on the analysis of correct responses

The items that got the lowest percentage of correct responses were items No.5, No.9, No.12 and No.18 at levels 1, 2, 3 and 4 respectively. To get clarification on these particular items, these items were again asked in the interviews. The responses of three learners are included in the discussion of interviews later in this chapter. To have the complete discussion of each item, some anecdotes from the interviews are also discussed along with the discussion from the analysis of each item.

These items and some clarifications that emerged from the analysis and the interviews are discussed below:

Item 5

The correct answer was "E". Most of the learners' response was "A". For many of them "A" was the correct answer because "I" was the only one that "looks like a parallelogram" or "I" was the only parallelogram because "it has 2 sides equal which are not straight". The typical response of students at level 1 as described in Chapter 2 was that they recognise a figure by its appearance (or shape/form). It is the appearance of the shape that defines it for the student (van Hiele, 1986). Since the appearance is dominant at this level, appearances can overpower properties of a shape (van de Walle, 2001). The students reason about basic geometric concepts such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components (Burger & Shaughnessy, 1986). Properties of a figure play no explicit role in its identification (Pegg & Davey, 1998).

It was also evident from the interviews (refer Chapter 5, section 5.2.1.3 on questions on the van Hiele Geometry Test) that the learners' concept image of parallelogram is that not all angles or sides are allowed to be equal. Even though the learners may have been taught and they are able to recite, the standard definition of a parallelogram as a quadrilateral with opposite sides parallel, the learners may still not consider rectangle, square and rhombus as parallelograms (De Villiers, 2010).

The responses from the interviews suggest that many learners are only attending to the visual characteristics of the shapes. "They look like triangles", "they look like squares" commonly occurred in many conversations with the learners who are at levels 0 and 1.

This kind of responses lead to the conclusion that most of the learners in grade 10 were operating at level 0 or level 1 due to the fact that they only attended to the visual prototypes to characterise shapes and sort shapes. The use of imprecise properties to compare shapes was very prevalent in many learners. This is consistent with earlier studies where they defined the characteristics of level 1 thinking (van Hiele, 1986; Burger & Shaughnessy, 1986; Pegg & Davey, 1998; van de Walle, 2001).

Item 9

The correct answer was "E". Most of the learners' response was "A". This means that they attend to only a single attribute of figures and they do not attend to all the properties of figures. The typical response of students at level 1 as described in Chapter 2 states that students at this level identify a figure by its properties, which are seen as independent of one another (Pegg & Davey, 1998). The properties are seen as separate entities that cannot be combined together to describe a particular figure. As an example, Pegg (1995, p.90) notes, "an isosceles triangle can have two equal sides, two equal angles and an axis of symmetry, but no property implies another”. The students have not yet mastered which properties are necessary and which are sufficient to describe a geometric shape (Mason, 1998).

The responses from the interviews (refer Chapter 5, section 5.2.1.3 on questions on the van Hiele Geometry Test) suggest that many learners are only attending to a single property of figures and not all the properties of figures. This is consistent with earlier studies where they defined the characteristics of level 1 thinking (van Hiele, 1959/1986; Burger & Shaughnessy, 1986; Pegg, 1995; Mason, 1998; van de Walle, 2001).

Item 12

The correct answer was “D”. As also noticed from the responses, most of the learners wrote “E” as the correct answer. This is due to the inability of the learners to order the properties of concepts, form abstract definitions, and the inability to distinguish between the necessary and sufficiency of a set of properties in determining a concept (Burger & Shaughnessy, 1986). Class inclusions are supposed to be understood at this level (van Hiele, 1999). Learners are meant to engage in “if-then” reasoning, and shapes can be classified using only minimum characteristics. They may be able to follow and appreciate an informal deductive argument about shapes and their properties (van de Walle, 2001, p.310). Since most of the learners have not achieved this level of thinking, so is the lowest percentage of correct answers.

The responses from the interviews (refer Chapter 5, section 5.2.1.3 on questions on the van Hiele Geometry Test) also suggest that many learners have difficulty with the ordering of the properties of simple geometric shapes. It is also noticed that this particular question is the one with the lowest percentage of correct responses out of all the 20 items. This is consistent with the earlier studies of Usiskin (1982) and Atebe (2008).

Item 18

The correct answer was “C”. As also noticed from the responses, most of the learners wrote “A” as the correct answer. This leads to the thinking, which was also evident in the interviews that “P is a square, because the four sides are equal”. None of the learners in the entire sample are at level 4 due to the fact that they lack the ability to work with abstract statements about geometric properties and make conclusions based more on logic than intuition.

The learners in this level are supposed to use the concept of necessary and sufficient conditions and can develop proofs rather than learning by rote. They are meant to be able to work with abstract statements about geometric properties and make conclusions based more on logic than intuition. They are supposed to clearly observe that for a rectangle, the diagonals bisect each other just like a student operating at level 3, but there is an appreciation of the need to prove this using a series of deductive arguments (van de Walle, 2001). Since most of the learners have not achieved this level of thinking, so is the lowest percentage of correct answers.

The discussion of each item based on the performance of learners in each school is discussed below:

School A

It was evident from Table 4.48 that the learners in School A has got good conceptual understanding in rectangle recognition, square recognition, properties of squares, rectangles and circles, connection between congruent and similar figures and deduction in parallel and perpendicular lines. Their understanding is the lowest in parallelogram recognition, connection between rectangles and parallelograms and deduction of properties of squares and rectangles.

The above findings from the analysis of correct responses confirm that learners were in different levels for different concepts (good in rectangles and squares, but having problems in parallelograms) and in different levels for the same concept (can recognise rectangles and squares, but having problems in higher thinking levels in deduction of properties of squares and rectangles). This is consistent with earlier studies (Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988).

School B

It was evident from Table 4.49 and the analysis of correct responses sheet attached in Appendix G, for Item No.13, School B's control group in the pretest scored the lowest percentage of correct responses (0%). This means that none of the students in the control group answered the question on circles correctly. Moreover this was one of the questions that only 20% of the entire school consistently got correct in the pretest and posttest). This showed that they have a weak understanding in the concept of circles. But rather higher percentages of correct responses in item No.1 and No.2 show that they had fairly good understanding in triangle and rectangle recognition. The consistently lowest percentages of correct responses in Item No.9 showed difficulties in higher order thinking in the concept of triangles.

The above findings from the analysis of correct responses confirm that learners were in different levels for different concepts (good in rectangles and squares, but having problems in circles) and in different levels for the same concept (can recognise triangles, but having problems in higher thinking levels in deduction of properties of triangles). This is consistent with earlier studies (Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988).

School C

In general, School C's learners had a weak conceptual understanding in recognition of shapes, properties of shapes, connection between shapes and deduction on angles. This gives the inference that learners from the experimental group, even after doing all the 9 activities, still showed the same kind of thinking pattern as they had in the pretest. This was also noticed by De Villiers (1994) that if given opportunity, the learners still prefer to stick on to what they have learnt before. As mentioned earlier in

Chapter 2, van Hiele (1986) also mentioned that a class of learners who have started homogeneously may not pass through all the levels at the same time. This gives some idea that why the instructional framework did not produce the desired marginal difference in the experimental group of School C.

School D

In general, learners in School D's had a better conceptual understanding in recognition of shapes, properties of shapes, connection between shapes and deduction on angles. It can also be concluded that the learners were at different levels for different concepts. This is consistent with earlier studies (Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988).

School E

In general, the performance of the learners in School E showed that as they moved from lower levels to the higher levels of thinking, the percentage of correct responses were on a steady decline. This shows the hierarchical nature of the van Hiele levels and learners being in different levels for the same concept. This is consistent with earlier studies (Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988; Atebe, 2008).

An in-depth comparison of the inference of this with earlier studies is given below:

6.4.5. Discussion on the overall findings from the analysis of correct responses

The above findings from the analysis of correct responses discussed under each school confirm that learners were in different levels for different concepts (good in rectangles and squares, but having problems in parallelograms) and in different levels for the same concept (can recognise rectangles and squares, but having problems in higher thinking levels in deduction of properties of squares and rectangles). This is consistent with earlier studies (Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988).

The above inference strongly supports the criticism on van Hiele levels whether students reason at the same levels across topics. As mentioned in Chapter 2 under the section on criticism on the van Hiele levels, many researchers like Mayberry (1983), Burger and Shaughnessy (1986), Fuys, et al. (1988), Mason (1998) and Clements and Battista (1992) also reported that students were on different levels on different tasks, oscillation from one level to another on the same task, the levels can be dynamic rather than static and of a more continuous nature than their discrete.

It is also consistent with those of Usiskin (1982, p.70), who observed in the American students that although many could identify rectangles, "over two-thirds think that a square is not a rectangle" and De Villiers (2010), who observed that a student may still not consider rectangle, square and rhombus as parallelograms since the learners' concept image of parallelogram is that not all angles or sides are allowed to be equal even after being taught.

For the pretest, the average percentage of the correct responses to each level is again analysed and it was as follows: Level 1 – 48%, level 2 – 39%, level 3 – 19% and level 4 – 26%. For the posttest, the average percentage of the correct responses to each level was as follows: Level 1 – 65%, level 2 – 43%, level 3 – 23% and level 4 – 33%. This shows the hierarchical property of the van Hiele levels as more learners have answered the items correctly at level 1 and it is on a decline as it moves to the higher levels. But for both tests, level 3 items scored the lowest percentage of correct responses, it is an indication that item No.12 which scored the lowest percentage is causing the problem and it has been consistent with the studies of Usiskin (1982) and

Atebe (2008) that the van Hiele level 3 items were more difficult than level 4 for the American, Nigerian and South African learners.

In the next section, the discussion on the analysis of the interviews is presented.

6.5. Discussion on the analysis of the interviews – qualitative data

6.5.1. Introduction

As mentioned in section 5.1 in Chapter 5, the data for the learners' interviews consisted of the learners' drawings, the interviewer's field notes and the audio taped interviews. The learners' responses and the explanation for each task and the discussion between the learners and the interviewer were analysed using the same process developed by Burger and Shaughnessy (1986), in conjunction with the level indicators which was also used by Genz (2006). The interviews consisted of giving the learners seven open ended activities dealing with geometric shapes, developed by Burger and Shaughnessy (1986), which were designed to reflect the descriptions of the van Hiele levels that were available in the literature. The activities involved drawing triangles and quadrilaterals, identifying and defining shapes, sorting shapes and engaging in informal and formal reasoning about geometric shapes. These tasks were expected to draw out the characterisations of van Hiele levels 1 to 3 (Burger & Shaughnessy, 1986). Two sets of drawings and identifying and sorting tasks were administered, one set for triangle shapes and one set for quadrilateral shapes.

Some interesting patterns in the answering of the van Hiele Geometry Test was noticed from the analysis of the test. The average percentage of correct responses for Question Numbers 5, 9, 12, and 18 were found to be the lowest from each of van Hiele level subtest questions. Some questions on further clarifications on these items in van Hiele Geometry Test were also asked at the end of the interview session. Learners were given maths set, pencils, pens, papers and an eraser for them to use.

During the analysis, the audio data and the learners' written responses and the researcher's notes were reviewed often to find the relevant dialogue and examples that reflected the findings and to check the accuracy of the findings.

As mentioned in section 5.1 in Chapter 5, three learners' interviews were chosen to discuss here as they represent the learners of three different levels of thinking. Andiswa was classified as being on the pre-recognition level (level 0), Mila was classified as being on the recognition level (level 1) and Nana was classified as being on the analysis level (level 2) from the van Hiele Geometry Test. These interviews yielded a number of particularly interesting aspects of the van Hiele theory and are discussed below.

6.5.2. The discussion on the analysis of the interviews

The following noticeable characteristics of the van Hiele levels were evident in the interviews and are described here.

Language

Andiswa was classified as being on the pre-recognition level (level 0) from the van Hiele geometry Test. During the interview I felt that Andiswa was unaware of many characteristics and features of figures she drew and many other concepts that were linked to geometry learning that should be predominant to a grade 10 learner. Initially it felt to me as if we were speaking different languages. I had to adapt the level of my language accordingly and could only speak about the shapes with which she was familiar and got used to the wrong terminology (language) that she had put for each concept. She used imprecise language in most of her explanations. This conversation seemed to confirm van Hiele's (1986) suggestion that each level has its 'own language' and people need to be speaking the same language in order to understand one another. Each level has its own linguistic symbol and its own system of relations connecting these signs. A relation which is "correct" at one level can reveal itself to be incorrect at another level (van Hiele, 1986). Earlier studies also noticed the same findings about language in different levels (e.g. Mayberry, 1983; Burger & Shaughnessy, 1986; De Villiers & Njisane, 1987; Fuys, et al., 1988; Senk, 1989; Genz, 2006).

Mila was classified as being on the recognition level from the van Hiele Geometry Test. When we started off the interview I felt that he was operating at level 1 as he could draw triangles in different orientations and talked about making more triangles by changing the angles and sides. For him, concerning the concept of triangles, he had the language ability of a level 1 learner. But as we progressed he had problems with the naming and identification of most of the quadrilaterals except squares. He used descriptive imprecise language in most of his explanations. I had to adapt the level of my language accordingly and could only speak about the shapes with which he was familiar and I got used to the wrong terminology (language) that he was putting for each concept. This also seemed to confirm van Hiele's suggestion that each level has its 'own language' and people need to be speaking the same language in order to understand one another (1986). I thus encountered a similar problem to that experienced with Andiswa. Although Mila's use of the properties of shapes suggested level 1 thinking, his use of vocabulary indicated that he might still be in transition to this level. Earlier studies also noticed the same findings about language in different levels (e.g. Mayberry, 1983; Burger & Shaughnessy, 1986; De Villiers & Njisane, 1987; Fuys et al., 1988; Senk, 1989; Genz, 2006).

Nana was classified as being on the analysis level (level 2) from the van Hiele Geometry Test. It was evident from the interview that Nana was operating at level 2 even though she did not draw triangles in different orientations. I could see her confidence when she used a ruler to draw the triangles, to make sure that she drew what she meant. Concerning the concept of triangles, she had the language ability of a level 2 learner. At this level, language is important for describing shapes. During the interview I felt that Nana was aware of many characteristics and features of figures she drew and many other concepts that are linked to geometry learning that should be dominant to a grade 10 learner. It was felt to me as if we were speaking the same language. It was easy for me to adapt to the level of her language as she could speak about all the shapes which she was familiar with the correct terminology (language) that she had put for each concept. She used precise language in most of her explanations. This conversation seemed to confirm van Hiele's suggestion that each level has its 'own language' and people need to be speaking the same language in order to understand one another (1986). Earlier studies also noticed the same findings about language in different levels (e.g. Mayberry, 1983; Burger & Shaughnessy, 1986; De Villiers & Njisane, 1987; Fuys, et al., 1988; Senk, 1989; Genz, 2006).

Misconceptions

Instead of saying 'two sides equal', in a triangle, Andiswa used 'both sides are equal' when talking about triangles. She called a triangle (even though she identified it wrongly as a triangle), "quadrilateral triangle", when all the sides seemed to be equal. "Should look for a figure with three sides" suggested that Andiswa had misconceptions about the definition of triangles. For Andiswa, when given clues to find the mystery shape of quadrilaterals, 'square' was 'a closed figure with 4 straight sides', 'when it has two long and short sides' it is a parallelogram' and when talked about two angles of the same size in a quadrilateral, she said they were 'isosceles'. This kind of reasoning was evident in the research by Atebe and Schafer (2008, p.58) also. They reported that it is common among the majority of the learners. International studies like Burger and Shaughnessy (1986) and Renne (2004) and South African studies like Feza and Webb (1995) and King (2003) pointed out that many learners in the middle school had severe misconceptions concerning some important geometric ideas.

Mila's misconceptions were becoming evident as we started off with the quadrilateral activities. His understanding of the concepts of square was quite that of level 1 thinking. He had serious misconceptions in terms of the definitions of rectangles and parallelograms. Most of his definitions were incomplete. He mixed the characteristics of rectangles and parallelograms as he said rectangles had "two equal sides and two sides that are parallel to each other" and parallelograms had "two sides that are equal and other sides that are not equal", and again to elicit the properties that he perceived as necessary for a figure to be a rectangle, he mentioned as "two opposite sides must be equal" and for a parallelogram, it was "two equal sides opposite to each other". His idea of parallelogram and rectangle was quite mixed. It was evident from his marking of No.4 (Activity 2B, Quadrilaterals) as a "parallelogram". His thinking as "two sides that are equal and other sides that are not equal" did not specifically say where these equal sides could be. It was the same with his marking for No.5 as "rectangle" because rectangle was having "two equal sides and two sides that are parallel to each other". In the grouping activity of quadrilaterals he mentioned that 'No.2 and No.8 have 45^0 angles'. It seemed that any angle that was less than 90^0 was 45^0 for him.

This kind of reasoning was evident in the research by Atebe and Schafer (2008, p.58) also. They reported that it was common among the majority of the learners. International studies like Burger and Shaughnessy (1986) and Renne (2004) and South African Studies like Feza and Webb (1995) and King (2003) pointed out that many learners in the middle school had severe misconceptions concerning some important geometric ideas.

No misconceptions were evident right through the interview with Nana. She had no misconceptions in terms of the definitions of all the types of figures presented to her. Most of her definitions were complete. She never mixed the characteristics of any shapes.

Visual prototype

Andiswa, like many other learners was not attending even to the visual characteristics of the shapes. "They do not look like triangles" and "they look like triangles" and "they look like squares" commonly occurred in her conversation. This is typical of a learner operating at van Hiele level 1 (recognition/visual level). It appeared that Andiswa was starting to recognise some familiar shapes, although she had not yet learned the required vocabulary for these shapes and the language used was still not to be on a 'visual level'. It looked as if she had begun the transition to visual level. As stated in Chapter 2, past research had documented a lot of inferences on this issue as Mayberry's (1983) study on pre-service elementary teachers revealed that participants in her study were on different levels for different concepts. Burger and Shaughnessy (1986) also reported that students were on different levels on different tasks and some even oscillated from one level to another on the same task. Therefore, this study also supports the notion suggested by Clements and Battista (1992) that the levels can be dynamic rather than static and of a more continuous nature than their discrete descriptions. Fuys et al. (1988) also found that a significant number of participants in their study made some progress towards level 2 with familiar shapes such as squares and rectangles, but encountered difficulties with unfamiliar figures. This made them conclude that progress was marked by frequent instability and oscillation between levels. Research has supported this argument as Clements et al. (2001) suggest that different types of reasoning can coexist in an individual and develop simultaneously but at different rates and along different paths, where each path leads to slightly different combinations of multiple types of knowledge.

Mila never mentioned anything of the sort "looks like" when mentioning about figures and their appearance. This is unusual of a learner operating at van Hiele level 1 (recognition/ visual level). It appeared that he was starting to recognise some familiar shapes, although he had not yet learned the required vocabulary for these shapes and the language used was still to be on a 'visual level'. It looks as if he has begun the transition to analysis level. "It looks circular shape but not pointy" and "has 90^0 angle and sharp corners" showed some evidence of visual prototype thinking.

Nana also never mentioned anything of the sort "looks like" when mentioning about figures about their appearance.

Inadequate vocabulary

Andiswa used the word 'square' for a triangle with two equal sides. She even called 'all sides are same length' a 'quadrilateral'. When she drew the triangles and when dealing with the mystery shape and when mentioned about a shape (with 4 straight sides) with all the sides with the same length, she mentioned it as 'quadrilateral'. She called a triangle (even though she identified it wrongly as a triangle), "quadrilateral triangle", when 'all the sides seemed to be equal'. It looked like she hadn't developed the vocabulary or she was using the terminology in the wrong context. When asked to draw triangles and quadrilaterals, it was found that Andiswa had no problem drawing them. But for many figures, she could not find the correct name associated with those figures. A mismatch between the figures and the corresponding terminology associated with the figures was very evident. This was noticed by Renne (2004, p.258) as "… students could readily identify various two dimensional shapes such as squares, triangles and rectangles; however, they were unable to use mathematical terms and concepts to describe the shapes". The study of Rowan (1990) also noted the same inference, where a group of fifth grade learners knew the word rectangle but could not sort rectangles from sets with other quadrilaterals, even though they could name a rectangle when that was the only shape presented.

When asked to draw triangles and quadrilaterals, it was found that Mila had no problem drawing them. But for many figures, he could not find the correct name associated with those figures. A mismatch between the figures and the corresponding terminology associated with the figures was very evident. He had a sort of "I know what it is, but I have forgotten it" explanation to many questions, more especially unfamiliar shapes like trapezium and rhombus.

When asked to draw triangles and quadrilaterals, it was found that Nana had no problem drawing them. For all the figures, she could give the correct name associated with those figures. No mismatch between the figures and the corresponding terminology were evident. She could give appropriate and precise terminology associated with the shapes.

Orientation

'Squares have four straight sides and parallelograms have non straight sides' as indicated by Andiswa suggested that for her, lines that were not vertical/horizontal was not straight for her. It suggested that her spatial orientation was incorrect. This was supported by Atebe (2008, p.183) that in his study also a learner mentioned about a square being 'straight' means "two sides pointing up and two sides pointing this way" (using his hands to indicate horizontal parallel lines). But spatial orientation was not a problem for Mila and Nana as suggested by their drawings.

Rote learning

It was also evident from the interview that Andiswa had resorted to memorisation when answering questions at the visual, descriptive and informal deductive levels. That was when she was asked about identifying figures by their properties and relationship between shapes. When asked questions from the van Hiele Geometry Test, she could not answer any of them correctly. It was clear from the analysis mentioned that she had not yet progressed to these levels even though she was sort of familiar with the few shapes and it was highly unlikely that her responses on items using other figures could have made sense to her. Her drawings confirmed this lack of understanding. This also provided evidence on the hierarchical nature of the van Hiele levels (Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988; Atebe, 2008).

It was also evident from the interview that Mila had resorted to memorisation when answering questions at the visual, descriptive and informal deductive levels. That was when he was asked about identifying figures by their properties and relationship between shapes. He had memorised much of the terminology and had little understanding of the concepts being studied. When asked about trapezium, he asked me whether I could give the definition of a parallelogram so that he could think of trapezium. But he could not give an answer after thinking for a while and said he could not because he had forgotten it. This means that he knew that connections were possible between figures but rote learning was a kind of a barrier to his thinking.

This gave some evidence on him progressing towards a higher level of thinking but lack of understanding in basic concepts was hindering his progression.

No evidence of rote learning was present in the interview with Nana. She could identify figures by their properties and she could explain the relationship between shapes.

Measurement

It appeared that drawing triangles other than the one with usual names that Andiswa was familiar with was of concern to her. When required to draw as many different triangles as possible, she provided two triangles which were similar in size and said that she could not draw more than 4 triangles because she was "not sure" whether there were more triangles because she knew only four types. No attempt was made to measure the sides or angles of the figures whether it is drawn by her or given to her. She completely forgot to talk about angles in any figures. Not even once she tried to measure the sides of any shapes even though the protractor and the ruler were provided.

For Mila, even though no attempt was made to measure the sides or angles of the figures whether it is drawn by him or given to him, in grouping quadrilaterals from the cut-outs, shapes No.4 and No.6 were grouped together because 'they have 90^0 angles'. He mentioned about angle measurement a few times.

It was noted that while drawing triangles and quadrilaterals, Nana made every effort to measure and draw the figures according to their definitions.

The use of measurement in describing the properties of figures does not appear explicitly in the literature. Bennie (1998b) also raised a similar concern in her study on grade 9 learners.

Level of thinking

As mentioned earlier in Chapter 2, "thinking at the second level is not possible without that of the first level; thinking at the third level is not possible without thinking at the second level" (van Hiele, 1986, p.51). The van Hiele theory is hierarchical in that a student cannot operate with understanding on one level without having been through the previous levels. Mayberry (1983), Senk (1989) and Pegg (1995) confirm that a student who has not attained level *n* may not understand thinking of level *n +1* or higher. Therefore, as Hoffer (1981) suggested, for Andiswa to function adequately at one of the advanced levels in the van Hiele hierarchy, she must have mastered large portions of the lower levels.

When challenged with tasks set at level 2 thinking, Mila showed some transition towards level 2 thinking but definitely not higher than that. When asked whether a square can be a rectangle, he said it is not possible because 'a rectangle is made up of two sides that are equal and because a rectangle cannot have four sides equal, a square cannot be called a rectangle'. This showed that even though he could list some of the properties of squares and rectangles, he could not see that these were sub-classes of one another which were one of the characteristics of level 2 thinking. When asked questions from the van Hiele test, he could answer only the one from level 1 and could not answer any of the other ones correctly. This also gives more evidence that Mila's thinking was that of more appropriate to level 1. This also provides evidence on the hierarchical nature of the van Hiele levels. Therefore, as mentioned in the case of Andiswa, for Mila also, to function adequately at one of the advanced levels in the van Hiele hierarchy, he must have mastered large portions of the lower levels.

The van Hiele theory is hierarchical in that a student can operate with understanding on one level if he/she has been through the previous levels. On the same grounds as Hoffer (1981) suggested, Nana could function adequately at one of the advanced levels in the van Hiele hierarchy as she had mastered large portions of the lower levels. When asked whether a square could be a rectangle, the answer given was 'yes' because 'opposite sides are equal and angles are 90^0'. A rectangle could be a parallelogram because 'opposite sides are equal and opposite sides are parallel'. Also a square could be a rhombus because 'all sides are equal all angles are equal'. This kind of thinking was that of level 3 thinking as she could give the relationships between figures or it could be assumed that class inclusion could be possible at level 2.

Findings from Andiswa's interview

The above interpretations of the van Hiele theory would suggest that Andiswa was operating in the pre-recognition level for most of the concepts in triangles and quadrilaterals. In certain concepts it looked like she was in transition from pre- recognition to recognition level.

At this point it was noticeable that even though Andiswa had been taught with the instructional framework which was in line with the van Hiele levels, she could not be raised to a level of thinking that is expected at the secondary school level. Van Hiele also speaks of an unavoidable situation in class, where we find that a group of learners having started homogeneously do not pass the next level of thinking at the same time and that half of the class might speak a language which the other half may not understand (van Hiele, 1986). "The pupils might accept the explanations of the teacher, but it might not sink into their minds" (van Hiele, 1958, p.75 as cited by Fuys, et al., 1988). In stressing the importance of language, van Hiele notes that many failures in teaching geometry result from a language barrier- "the teacher using the language of a higher level than is understood by students" (Fuys, et.al, 1988, p.7). Andiswa's repeated mention of "quadrilateral triangle" also points out the fact that she might have learned (heard) it from her teacher. Atebe (2010) suggests that during teaching, teachers may use certain words in a sense peculiar to the subject that may not be precisely understood by the learners and the learners' proficiency in the teaching language is important for learning geometry specifically and mathematics in general. The findings from a few researchers on implications on language are already discussed under section 2.3 on the linguistic characteristics of the van Hiele theory.

Findings from Mila's interview

The interpretations that were discussed of the van Hiele theory would suggest that Mila was operating in the recognition (visual) level for most of the concepts in quadrilaterals even though some key indicators such as visual prototype to charactarise shapes were not dominant in him. He could recognise a square by its form and a square seemed different to him than a rectangle. Even though he could identify, name, compare and operate geometric shapes such as triangle and square, he could not operate well on trapezium, rhombus and kite and he could not explicitly identify the properties of these shapes. He could operate on common shapes such as rectangle and parallelogram to a certain extent without explicitly regarding to properties of its components such as angles being 90^0 or equal angles. He could sort and classify shapes based on some characteristics other than their appearances. In certain concepts it looked like he was in transition from recognition level to analysis level. It could be concluded that he had made some progress towards level 2 with familiar shapes such as triangles and squares but encountered difficulties with unfamiliar figures and that progress was marked by frequent instability and oscillation between levels.

It was also noticeable at this point that Mila's level of thinking was also not that expected at the secondary school level. As explained by other researchers like Teppo (1991) and Genz (2006), that systematic geometry instruction in the middle grades is necessary to prevent students from entering high school at low levels of geometric concept development.

Findings from Nana's interview

From the discussions, it could be concluded that Nana's level of thinking was the thinking level that was expected of a learner at the secondary school level.

The interpretations that were discussed of the van Hiele theory would suggest that Nana was operating at the analytic level for most of the concepts in quadrilaterals. She could identify, name, compare and operate geometric shapes such as triangles and quadrilaterals and she could explicitly identify the properties of these shapes. She could sort and classify shapes based on characteristics other than their appearances.

In certain concepts it looked like she was in transition from analytic level to informal deductive level as she could identify class inclusion and gave abstract definitions to all the shapes that were presented.

6.6. Summary of findings based on the interviews

In general it was found that the interviews were useful in analysing learners' responses. It might not be possible to conclude that the features that emerged in the interviews were the result of the nature of the activities, the result of the particular interpretation used, or features of the van Hiele theory itself. Several features of the levels emerged during the course of the interviews.

The inability of Andiswa to identify common shapes, the misconceptions of Mila, the language of Nana were typical of levels 0, 1 and 2 thinking respectively.

Even though the learners were classified into discrete van Hiele levels (see section 4.2.4), as from the discussion of the interviews of these learners under levels of thinking, it has been noted that the levels are not discrete, more continuous than static, and the learners seem to move back and forth between levels and they were in different levels for different concepts. Occasionally, they seemed to be attaining a higher level than their predominant level. Previous research studies such as international studies like Usiskin (1982); Mayberry (1983); Burger and Shaughnessy (1986), Fuys et al. (1988), and Wu and Ma (2006) have also noticed the same behavioural patterns in their studies.

The interviews from this study also support the claim of Mayberry (1983) that high school learners do not perceive the properties of shapes. It also supports Burger and Shaughnessy (1986) who stated that a number of secondary school learners in their study were not sufficiently grounded in basic geometric concepts and relations. The interviews also support the claim of Burger (1985), that many learners rely on imprecise qualities to identify shapes (like, 'pointy triangles' and 'slanted squares').

The present study found that learners attempting definitions of concepts were influenced by their levels of understanding. Learners at level 1 gave *visual* definitions and learners at level 2 gave correct definitions. This is consistent with the study by

Govender and De Villiers (2004). This study also found that many learners included irrelevant attributes when identifying and describing shapes, like orientation of the shape on the page (like 'turning a square to make a rhombus'). It was consistent with the studies of Burger (1985).

Findings from the study also suggest that language competency in general is a barrier to the attainment of higher levels of understanding. Coupled with the language feature of the van Hiele level as discussed in section 2.3 in Chapter 2, and as noticed in the learners, it can be assured that language is a barrier for learners who speak English as a second language. Setati and Barwell (2006) point out the use of the learners' main languages as a support whilst learners continue to develop proficiency in the language of learning and teaching at the same time as they learn mathematics.

According to the van Hiele theory people at different levels speak, use and understand geometrical terms differently. Wirszup (1976) noted that most of the terms used by teachers can only be understood by learners who have progressed to the third or fourth van Hiele level. Therefore, in a class while the teacher is trying to explain, he or she might be completely misunderstood. This was particularly noted in the interviews with Andiswa and Mila. Thus it is very important that teachers investigate the levels of the learners to provide meaningful instruction in the classroom.

As mentioned earlier, Pegg and Davey (1998) suggest that the descriptions of the levels are content specific and the levels are actually stages of cognitive development. Van Hiele (1986, p.41) asserts that "the levels are situated not in the subject matter but in the thinking of man". Progression from one level to the next is not the result of maturation or natural development. It is not age dependent as the stages of Piaget. It was also evident from the interviews. Andiswa who was 18 years old, the oldest in the group which was mentioned here, was at the lowest level and even though Mila and Nana were of the same age (16 years), they were also not operating at the same level even though all these learners had been through the same subject matter.

Previous research studies such as international studies like Usiskin (1982), Mayberry (1983), Burger (1985), Burger and Shaughnessy (1986), Fuys et al. (1988), Renne (2004), Genz (2006) and South African Studies like Feza and Webb (2005) and King (2003) point out that many learners in the middle school have severe misconceptions concerning some important geometric ideas. In South Africa, studies like, De Villiers and Njisane (1987); Siyepu (2005) and Atebe (2008) indicate that high school learners in general and more especially, Grade 12 learners are functioning below the levels that are expected of them, i.e., they are at concrete and visual levels than at abstract level in geometry. This was noted predominantly at level 0 and level 1 thinkers in the interviews. De Villiers points out that this may be due to the fact that the transition from concrete to the abstract level of thinking poses "specific problem to second language speakers" and success in geometry involves the acquisition of the technical terminology (1987). It is essential that connections between relationships of mathematical concepts and terminology should be established.

According to de Villiers (2010), in traditional teaching, learners are introduced to rectangles, parallelograms and other geometric figures as *static geometric objects* – as an example, a rectangle might be introduced by comparison to a shape of a door or a static picture in a book, but the door or a picture in a book cannot be transformed into a square unless parts are cut off. Thus the concept of rectangle is completely disjoint from the concept of a square. This was very evident in the present interviews.

When given a set of quadrilaterals and when the learners were asked to mark all the parallelograms, they marked only the parallelogram, simply not knowing or realising the intention of the question that all special cases (e.g., rectangles, squares and rhombi) had to be marked as well. This is in line with the finding of Mayberry (1983) where only 3 out of the 19 pre-service mathematics teachers indicated squares are also rectangles.

The sample from the experimental group, even after doing all the 9 activities, still showed the same kind of thinking pattern that are mentioned above. De Villiers (1994) also corroborates the same view as he mentions that even after doing the activities, if given opportunity, they still prefer to define a parallelogram as a quadrilateral with both pairs of opposite sides are parallel, but not all angles or sides equal. This gives some idea that why the instructional framework did not produce the desired marginal difference in the experimental group.

All the aspects that are discussed for each learner are of importance to instruction as it is a big concern which affects the understanding of mathematics in general and geometry in particular. It also appeared that the learners in different schools involved in the study had varied exposure to geometric figures and their characteristics.

Just knowing the definition of a concept does not at all guarantee the understanding of the concept. De Villiers (2010), observed that although a student may have been taught and he is able to recite, the standard definition of a parallelogram as a quadrilateral with opposite sides parallel, the student may still not consider rectangle, square and rhombus as parallelograms since the learners' concept image of parallelogram is that not all angles or sides are allowed to be equal. The present research also observed the same in the interviews.

The language competency in general is a barrier to the attainment of higher levels of understanding. Atebe (2010) points out that learners' proficiency in the teaching language is important for learning mathematics generally and geometry specifically. Language is important for learning and thinking and that the ability to communicate mathematically is central to learning and teaching school mathematics (Setati, 2008). The present research is strengthened by these earlier studies which inferred the same conclusion through their studies.

According to the van Hiele theory, understanding of formal textbook definitions only develop at level 3, and that the direct provision of such definitions to learners at lower levels would be doomed to failure (De Villiers, 2010). On the other hand, if we think of the constructivist theory also, learners ought to be engaging in the activity of defining and be allowed to choose their own definitions. According to De Villiers (2010), the meaningful definition of a rectangle for a learner operating at level 1 can be called visual definition, where a rectangle is a figure which looks like this (draws or identifies a quadrilateral with all angles 90^0 and two long and two short sides) and that of a level 2 thinker is called uneconomical definition, where a rectangle is a quadrilateral with opposite sides parallel and equal, all angles 90^0, equal diagonals, two long sides and two short sides as that of a level 3 thinker, is called correct, economical definitions , where a rectangle is a quadrilateral with two axes of symmetry through opposite sides. The present interviews clearly substantiated the above kind of definitions.

It appears that class inclusion is difficult to accomplish with geometric figures. The learners' spontaneous definition at van Hiele levels 1 and 2 as shown above, would also tend to be in such a way that they would not allow the inclusion of squares among the rectangles, because rectangles have two long and two short sides. On the contrary, level 3 thinker will allow the inclusion of squares among the rectangles. This also validates De Villiers' (2010) claim.

6.7. Discussion on the analysis of the interviews with educators

Educators from the five schools were interviewed to comment on the activities and the evaluation of the framework as a whole.

From the comments shown in Tables 5.1 and 5.2 in Chapter 5, it was clear that the activities were good for learners and the educators appreciated that it was good in helping the learners to overcome the challenges associated with basic geometrical shapes and their properties.

6.8. Chapter summary

In this chapter, the interviews with the three learners in the three different van Hiele levels of thinking were discussed. Findings from the interviews were also given. The discussions on the analysis of the educators were also given.

In the next chapter, conclusions from the research project are given.

CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

7.1. Introduction

This research originated from a concern with the teaching and learning of geometry in South African senior secondary schools. It was conducted to determine whether an instructional framework based on the van Hiele levels might improve the level of geometric thinking of grade 10 learners. For this, participants' level of thinking was determined before and after the instruction with the van Hiele-based instructional framework using van Hiele Geometry Test.

The study was conducted in five purposively selected senior secondary schools in Mthatha district in the Eastern Cape Province. Two intact classes of grade 10 were selected from each school. The study involved a total of 359 learners and five mathematics educators.

A combination of quantitative and qualitative research procedures was used to collect data whilst a quasi-experimental design guided the research process. One class from each school was selected as the experimental group to be compared with the second class as the control group, but the participants were not randomly selected and assigned to the groups. The experimental group was instructed with the van Hiele instructional framework, while the control group was instructed with the traditional method.

The research was done in six phases in addressing the objectives. Phase 1 was concerned determining the present geometrical thinking levels of the participating learners. It was established through a van Hiele Geometry Test and in phase 2, based on the levels of the majority of learners, an instructional framework was developed with 9 activities. Phase 3 was about giving the workshop for the educators of the five schools. In Phase 4, the instructional framework was implemented for the experimental group in all the five schools. In Phase 5, the evaluation of the instructional framework was done through a posttest that was administered on all the participating learners. Interviews with educators were also conducted to collect their opinions on the instructional framework. In Phase 6, interviews with selected learners from the five schools on their levels of thinking were done to enrich the study by giving it a qualitative approach.

Many of the findings in the study were discussed in Chapter 6 and related to the literature review in Chapter 2. A 'chapter summary' which gives a summary of what has been discussed in each chapter is given at the end of each chapter.

This chapter provides an overall summary of all the findings as follows:

- Research objectives;
- Research questions;
- A summary of findings which are significant from the study;
- Significance of the study;
- Limitations of the study;
- Implications and recommendations for teaching and learning;
- Areas of future research;
- A personal reflection.

7.2. Research objectives

The first objective – The determination of the van Hiele levels of the selected Grade 10 learners in the participating schools

The first objective was addressed in the first phase of the study. In the first phase, a paper and pen test called the van Hiele Geometry Test was administered to the learners. The first objective was solely achieved by administering the van Hiele Geometry Test. The test was adopted from Atebe (2008) and Atebe adapted the tests from Usiskin's (1982) CDASSG project. The findings from the test were discussed in

Chapter 6. The findings were consistent with those of Usiskin (1982) and Atebe (2008).

The second objective – The development of the instructional framework and the implementation of it in the participating schools

This objective was addressed in phases 2, 3 and 4 of this study. A total of 9 activities were developed and compiled in the form of a booklet was implemented as the instructional framework and is given in Appendix D. This was done based on the levels of the majority of the learners in the sample and it was also done to suit the wider application of it. The majority of our South African learners are in the underprivileged rural areas. The second objective tried to address problems of curricular, textual and instructional factors by looking at the learners' cognition in geometry and used that knowledge in developing an instructional framework to enhance the geometry instruction. Phase 3 was about giving the workshop for the educators of the five schools. The qualified and experienced educators in the five schools were given workshops on two different dates. During the workshops the activities were discussed and modified. These five educators were instrumental in modifying and implementing the framework. In Phase 4, the instructional framework was implemented for the experimental group in all the five schools. The educators implemented the instructional framework for about a month in their schools in their mathematics lessons. The experimental group was taught with the instructional framework and the control group was taught with the traditional method. Each class had 5 hours of lessons per week as per the normal time Table, which gave the educators 20 hours of normal school hours. A detailed description of the implementation program was discussed in section 3.6.4 in Chapter 3.

The third objective – Assessment of the effectiveness of the instructional framework

This objective was achieved through Phase 5. In Phase 5, the evaluation of the instructional framework was done through a posttest that was administered on all the participating learners. The test was marked according to the same criteria as that of the pretest. The data were analysed the same way as the pretest and the data analysis is shown in Chapter 4. The initial analysis using Microsoft Excel showed that the experimental groups' mean scores were higher than the control groups' mean scores and that the instructional framework had a significant effect. A statistical measure, t-test using IBM SPSS Version 19 confirmed the statistical difference. After the whole program, interviews with educators were also conducted to collect their opinions, attitudes and suggestions regarding the implementation and effectiveness of the van Hiele framework.

7.3. Research questions

The first question – What are the van Hiele levels of geometric thinking of the learners participated in the study?

The purpose of the first question was to address the first objective of the study. This was achieved through the administration and the analysis of the van Hiele Geometry Test (see Chapter 4). The findings from the van Hiele Geometry test provided the levels at which the learners were operating. The instructional framework was developed based on the levels of the majority of the learners. Thus the findings from the first question helped to achieve the first two objectives of the study.

The second question – Can a researcher-designed instructional framework in line with the van Hiele levels improve the levels of geometric thinking of the participating learners?

The second question was pertaining to the third objective of the study. This was answered through the implementation of the framework and the posttest of the van Hiele Geometry Test (see Chapter 4). The effectiveness was assessed by checking whether there was a significant difference between the average scores of the experimental group and the average scores of the control group in the pretest and posttest and it was compared by means of t-test.

In order to support the quantitative data obtained from the tests, interviews with learners were conducted. Analysis of the responses on the written test proved more difficult than expected and some interesting features emerged as a result. In cases where there appeared to be certain trends in a learner's understanding, it was decided to conduct structured interviews with the learners in order to explore these features in greater detail and to obtain clarification on his/her understanding in geometric shapes. The analysis shed some insight into the levels of thinking of the participating learners (see Chapter 6).

Learning mathematics with understanding is the vision of school mathematics recommended by the NCTM (2000). If our learners are not given proper exposure to learning mathematics even from lower grades, they will find it very difficult to follow it in the high schools. Therefore, effective classroom practices in mathematics should be provided by educators in the lower grades.

7.4. A summary of findings

Based on the literature reviewed in Chapter 2 and the analysis, results and the discussions in Chapters 4, 5 and 6, quite a few findings emerged as significant. These findings are presented in this chapter, organised according to the two research questions of the study.

7.4.1. Summary of findings relating to the first research question

The summary of findings presented here is in terms of the learners' performance in the van Hiele Geometry Test. In general, the van Hiele geometry written test was found to be useful in classifying the learners into different levels of thinking and got confirmed with the interviews of which the details are given in the next section. The test items were formulated in the notion that items 1-5 test van Hiele level 1 thinking, items 5-10 test level 2 thinking, items 11-15 test level 3 thinking and items 16-20 test level 4 thinking. According to the criteria ('3 of 5 success' criteria) used, a learner is said to be operating in a level only if he or she has satisfied the criteria for the previous levels, it was necessary to classify learners into the lower level than that of the basic level. Therefore learners were assigned into level 0 also.

The entire study sample

- An overall low percentage mean score of 33.14% obtained in the pretest for the van Hiele Geometry Test indicated that the majority of the learners in the study were at low van Hiele levels of thinking which were mainly level 0 and level 1. The results of the present study is strengthened by the earlier studies by Usiskin (1982) on the American high school learners and Atebe (2008) on Nigerian and South African senior secondary school learners which inferred the same conclusions through their studies. The study supports Clements and Battista (1992), on the existence of the level 0 called pre-recognition.

- It was evident from the mean scores that learners' performance in the tests decreased progressively at each successive higher van Hiele level.

- The results confirm the hierarchical nature of the van Hiele levels as more learners had answered the items correctly at level 1 and it was on a decline as it moves to the higher levels. The learners in this study obtained the lowest percentage mean at level 3 which is consistent with the earlier studies of Usiskin (1982) and Atebe (2008), which concluded that the learners experienced more difficulty in attempting level 3 items than level 4 items.

- The percentage number of learners in each level proved to be the highest at level 0, followed by level 1 and level 2. There were no learners at level 3 and level 4 in all schools except in School D. These results are strengthened by the earlier studies of Usiskin (1982) and Atebe (2008) which inferred the same results. Our senior secondary school learners are not ready to do the formal proof that demands a thinking level of 4.

- The significantly low percentages of learners at the higher levels of the van Hiele suggest that the learners experience a lot of difficulties in identifying and classifying shapes, properties of shapes and proof writing. This has also been noticed by past researchers like Usiskin (1982); Fuys, et al. (1988); Clements and Battista (1992); Siyepu (2005) and Atebe (2008).

- In the analysis of correct responses it was found that item 1 was the "best attempted" with the highest percentage of correct responses and item 12 was the "worst attempted" with the lowest percentage of correct responses.

School differences

- In the participating schools, School D obtained the highest percentage mean score of 39.29% in the pretest, which was significantly higher than their peers in School C which obtained the lowest mean score percentage of 26.94%. The percentages of other schools in the ascending order are School B (31.73%), School E (33.37%) and School A (34.38%).

- In the participating schools, in the pretest, in the experimental groups, the overall percentage mean scores was 32.74% and the experimental group of School D obtained the highest percentage mean scores of 41.9% and which was significantly higher than their peers in the experimental group in School C which obtained the lowest mean scores percentage of 26.89%. The percentages of other schools in the ascending order are School B (28.64%), School E (31.17%) and School A (35.11%).

- In the participating schools, in the pretest, in the control groups, the overall percentage mean score was 33.54% and the control group of School D obtained the highest percentage mean score of 36.67% and which was significantly higher than their peers in the control group in School C which obtained the lowest mean score percentage of 27%. The percentages of other schools in the ascending order are School A (33.64%), School B (34.81%) and School E (35. 56%).

- In all the schools, in the pretest, there was no statistical difference in the percentage mean scores between the control group and experimental groups except in that of School B. The test of significance ensured that the two groups of learners taken from the same school are of equal ability so that the results of the posttest are not affected by their difference in ability.

- In all the schools, there was no statistical difference in the percentage mean scores of the male and female learners in the experimental and control groups in all schools.

- At each van Hiele level, in the pretest, the overall percentage mean was highest at level 1 (46.85%) followed by level 2 (39.44%), level 4 (27.65%) and level 3 (19%). This is consistent with the earlier studies of Usiskin (1982) and Atebe (2008).

- In all the schools, School D obtained the highest percentage mean scores of 65.08 % at level 1 in the pretest, which was significantly higher than their peers in School C which obtained the lowest mean scores percentage of 37.65%. The percentages of other schools at level 1 in the ascending order are School E (39.61%), School B (41.41%) and School A (50.51%).

- In all the schools, School D obtained the highest percentage mean scores of 47.22 % at level 2 in the pretest, which was significantly higher than their peers in School C which obtained the lowest mean scores percentage of 25.52%. The percentages of other schools at level 2 in the ascending order are School B (34.88%), School A (43.62%) and School E (45.98%).

- In all the schools, School C obtained the highest percentage mean scores of 21.59 % at level 3 in the pretest, which was higher than their peers in School A which obtained the lowest mean score percentage of 15.72%. The percentages of other schools at level 3 in the ascending order are School B (17.91%), School E (19.41%) and School D (20.36%).

- In all the schools, School B obtained the highest percentage mean score of 32.7 % at level 4 in the pretest, which was significantly higher than their peers in School C which obtained the lowest mean score percentage of 23.75%. The percentages of other schools at level 4 in the ascending order are School D (25.68%), School A (27.66%) and School E (28.47%).

- One particular school was consistently performing as the 'best' in the test and one particular school was consistently performing as the 'worst' in the test. It could be assumed that their geometrical experience was playing a role in this performance. As seen from the description of the schools, the conditions might have contributed to this difference.

Gender differences

- There was no significant difference between the overall percentage mean score of male learners (32.42%) and female learners (33.47%) in the pretest. This was consistent with the earlier studies of Usiskin (1982), Halat (2006) and Atebe (2008). For the entire study sample and for each of the participating schools, the gender difference did not play a role in the performance. There was no statistical difference in the performance of male learners and female learners.
- The results obtained from the posttest were similar in terms of gender differences, but the percentage mean scores were higher than that of the pretest. In other words, the majority of the learners performed better in the posttest.

Comparison between the experimental group and control group

- In all the schools, in the pretest, there was no statistical difference in the percentage mean scores between the control groups and experimental groups except in that of School B. The test of significance ensured that the two groups of learners taken from the same school are of equal ability in all the schools except in School B.

7.4.2. Summary of findings relating to the second research question

The entire study sample

- This study supports King (2003) who found that there was significant difference in the performance of grade 6 learners after the intervention of the structured geometry course to the experimental group.

- Results from this study contradicts Genz (2006) and Halat (2007) who concluded that there was no difference detected in the acquisition of levels in schools using a curriculum based on the van Hiele theory (standards based curriculum) and schools using traditional curriculum (non standards based curriculum).

- The acquisition of the levels of thinking is not age dependent.

- Van Hiele speaks of the levels as the levels as situated in the thinking of the man, not in the subject matter. The same subject matter was taught to the learners, but the acquisition of the level depends on each learner's thinking.

School differences

- There was a substantial increase in the percentage mean scores of the experimental group in all schools with the highest difference in School D. School A' percentage mean scores increased from 35.11% to 44.67%, School B's percentage mean scores increased from 28.64% to 38.73%, School C's percentage mean scores increased from 26.88% to 35.94%, School D's percentage mean scores increased from 41.9% to 52.41% and School E's percentage increased from 31.17% to 41.76%. It can be taken that the van Hiele-based instruction had a positive effect on the performance of the learners in the experimental group.

- In the pretest, in School A, the majority of the learners were at level 0 (60% in the experimental group and 46% in the control group). For the van Hiele levels 1, 2, 3, and 4, the percentages of number of learners were 22%, 18%, 0% and 0% respectively in the experimental group and it was 24%, 30%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and the control group's performance was better than the experimental group. The school had no learners at level 3 and level 4. In the posttest, in School A, the majority of the learners in the experimental group were at level 2

(60%). In the control group, the majority of the learners are at level 0 (39%). For the van Hiele levels 0, 1, 3, and 4, it was 13%, 18%, 0% and 0% respectively in the experimental group and it was 27%, 24%, 0% and 0% respectively at level 1, level 2, level 3 and level 4 in the control group. The experimental group had more learners at level 2 and the experimental group's performance was better than the control group. The school had no learners at level 3 and level 4. The significant improvement in the performance of the experimental group having more learners at level 2 than at level 0 and level 1 suggests that the van Hiele-based instruction had a positive effect.

- In the pretest, in School B, the majority of the learners were at level 0 (65% in the experimental group and 59% in the control group). For the van Hiele levels 1, 2, 3, and 4, the percentage number of learners were 24%, 11%, 0% and 0% respectively in the experimental group and it was 31%, 10%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and the control group's performance was better than the experimental group at level 1 and the experimental group had more learners at level 2. The school had no learners at level 3 and level 4. In the posttest, in School B, the majority of the learners were at level 1 (44%) in the experimental group and the control group had the majority of learners (52%) at level 0. For the van Hiele levels 0, 2, 3, and 4, it was 36%, 20%, 0% and 0% respectively in the experimental group and it was 35%, 13%, 0% and 0% respectively at levels 1, 2, 3 and 4 in the control group. The school had no learners at level 3 and level 4. The experimental group's performance was better than the control group in terms of having more learners at level 1 than at level 0. The significant improvement in the performance of the experimental group having more learners at level 1 than at level 0 suggests that the van Hiele-based instruction had a positive effect.

- In the pretest, in School C, the majority of the learners were at level 0 (66% in the experimental group and 76% in the control group). For the van Hiele levels

1, 2, 3, and 4, it was 34%, 0%, 0% and 0% respectively in the experimental group and it was 16%, 8%, 0% and 0% respectively in the control group. The experimental group had more learners at level 1 and had no learners at level 2. The control group's performance was lower than the experimental group. The school had no learners at level 3 and level 4. In the posttest, in School C, the majority of the learners were at level 1 (50%) in the experimental group and the majority of the learners in the control group were at level 0 (68%). For the van Hiele levels 0, 2, 3, and 4, it was 38%, 12%, 0% and 0% respectively in the experimental group and it was 28%, 4%, 0% and 0% respectively at levels 1, 2, 3 and 4 in the control group. The school had no learners at level 3 and level 4. The experimental group's performance was better than the control group in terms of having more learners at level 1 than at level 0. The improvement in the performance of the experimental group having more learners at level 1 than at level 0 suggests that the van Hiele-based instruction had a positive effect.

- In the pretest, in School D, for the van Hiele levels 0, 1, 2, 3, and 4, the percentages of learners were 10%, 45%, 38%, 7% and 0% respectively in the experimental group and it was 44%, 25%, 25%, 6% and 0% respectively in the control group. The experimental group had more learners at level 1,2and 3 and its performance was better than the control group's. The school had no learners at level 4. In the posttest, in School D, the majority of the learners were at level 2 (62%) in the experimental group and the control group had equal number of learners at level 0 and level 3 (33%). For the van Hiele levels 0, 1, 3, and 4, it was 3%, 31%, 4%, and 0% respectively in the experimental group and it was 31%, 3% and 0% respectively in the control group. The experimental group had more learners at level 1, 2 and 3 and its performance was better than the control group's. The school had no learners at level 4. School D's performance was consistently the best performance out of the five schools. It can be assumed that the geometrical experience they have can be one of the factors that contributed to this performance. The history and cultural background as shown in the description of the school might also have contributed to it.

- In the pretest, in School E, the majority of the learners were at level 0 (68% in the experimental group and 61% in the control group). For the van Hiele levels 1, 2, 3, and 4, the percentages of learners were 9%, 23%, 0% and 0% respectively in the experimental group and it was 22%, 17%, 0% and 0% respectively in the control group. The experimental group had more learners at level 0 and at level 2. The school had no learners at level 3 and level 4. In the posttest in School E, the majority of the learners were at level 0 (35% in the experimental group and 44% in the control group). For the van Hiele levels 1, 2, 3, and 4, it was 32%, 33%, 0% and 0% respectively in the experimental group and it was 22%, 28%, 6% and 0% respectively in the control group. The experimental group had more learners at level 1 and at level 2. The school had 6% of learners in the control group level 3 and no learners at level 4. The Percentage number of learners at level 0 had been reduced considerably in both groups in the posttest.

- In general, it could be assumed that all schools benefitted out of the instructional framework through the considerable reduction of the percentage of the number of learners at level 0 and the statistical inference that there was a significant increase in the percentage mean scores of the experimental groups. Other extraneous variables like history and maturation also might have contributed to this positive change.

Comparison between the experimental group and control group

- The statistical difference in the percentage mean scores of the experiment groups between the pretest and the posttest indicate that the intervention might have had a positive effect in the performance of the learners.

- Even though all the learners in the experimental group were taught with the instructional framework and there was a significant increase in their overall performance in the posttest, the performance in different schools and in individual learners were not the same. Many of them could not be raised to a level of thinking that is expected at the secondary school level. Van Hiele also speaks of an unavoidable situation in class, where we find that a group of learners having started homogeneously do not pass the next level of thinking at the same time.

- The difference in the percentage mean scores of the control groups between the pretest and the posttest also show that there was an improvement in the performance. This is also due to the instruction given to the learners in the traditional method. The improvement was not as significant as that of the experimental groups.

- Maturation and history of the learners also might have played a role in the increase in the scores of the learners in both experimental and control groups.

Gender differences

For the entire study sample and for each of the participating schools, the gender difference did not significantly play a role in the performance. There was no statistical difference in the performance of male and female learners.

7.4.3. Summary of findings from the interviews

This qualitative aspect of the study supported the research questions.

In general, it was found that the interviews were useful in analysing learners' responses. It might not be possible to conclude that the features that emerged in the interviews were the result of the nature of the activities, the result of the particular interpretation used, or features of the van Hiele theory itself. It is felt, however, that a number of interesting features of the van Hiele theory and its interpretations have emerged, for example,

- The existence of prerecognition level;
- The transition between levels;
- Learners at different levels for different concepts of basic figures;
- The need to identify misconceptions and rote learning;
- The importance of the linguistic property.

All the aspects that are discussed for each learner are of importance to instruction as it is a big concern which affects the understanding of mathematics in general and geometry in particular. It also appeared that the learners in different schools involved in the study had varied exposure to geometric figures and their characteristics.

7.4.3.1. Summary of findings relating to first research question from the interviews

- The interviews were found to be useful in analysing learners' levels of thinking. It generally supported the learners' levels from the van Hiele Geometry Test.

- Triangle tasks were found to be easier and in the case of quadrilaterals, providing information for unfamiliar shapes such as rhombus, kite and trapezium was found be a problem for many learners.

7.4.3.2. Summary of findings relating to second research question from the interviews

- As with some learners in the experimental group it was evident that there was some improvement in their understanding and levels of thinking as a result of the intervention. Most of the learners responded positively to the activities in the framework. The learners who were operating at level 2 found that the first three activities in the instructional framework were a bit 'dragging' or 'rather too easy' and the learners who were at level 0 found that the last activity 'a bit demanding'. This might have affected their performance in one way or other.

- Van Hiele (1986) speaks of an unavoidable situation whereby, a group of learners having started homogeneously do not pass the next levels of thinking at the same time. At times, it happens that half of the class will speak a language that the other half is unable to understand. This research also found the unavoidability of such a situation, where quite a few of the learners could not attain the targeted level. This was evident in the interviews. For some learners, the new short learning experience that was acquired did not sink into their minds to change the thinking patterns and ideas (or misconceptions) they had learned already. This gives an idea as to why the instructional framework did not produce the desired marginal difference in all the learners in the experimental group.

- The language competency in general is a barrier to the attainment of higher levels of understanding. The learners' competency in the teaching language is important for learning mathematics generally and geometry specifically.

- Learners were found to be at different levels for different concepts and at different levels for different tasks even after being taught with the van Hiele-based instruction.

7.5. Recommendations from the interviews

7.5.1. General recommendations from the interviews

In light of the evidence from the interviews, it is recommended that:

- Knowing the definition of a concept alone does not guarantee the understanding of the concept.

- The language competency in general is a barrier to the attainment of higher levels of understanding. Learners' proficiency in the teaching language is important for learning mathematics generally and geometry specifically.

- Understanding of formal textbook definitions is not easy for all learners. Learners should be engaged in the activity of defining and be allowed to choose their own definitions and educators then need to lead them to the correct definitions with understanding.

- Junior secondary school curriculum should enable the learners to develop visual skills related to common two and three dimensional figures.

7.5.2. Level indicators for level 0 (pre-recognition level) as affirmed by the present study

Burger and Shaughnessy had suggested level indicators for level 1 to level 4 based on their research in 1986 and had been widely used as indicators for interview tasks. A recent research done by Genz (2006) also made use of these tasks and indicators.

Clements and Battista (1992, p.429) introduced level 0 called pre-recognition as an addition to the van Hiele levels in 1992.

In the present study, it was found that the majority of its learners were at level 0 (i.e., in the pre-recognition level) from the van Hiele Geometry Test which was used for assigning the levels. To further confirm these levels as whether they indicate the actual levels, a structured one on one interview was conducted for a sample of 30 purposively selected learners taken from the five schools.

This present study's interviews also made use of Burger and Shaughnessy's (1986) tasks as activities for the interviews. The responses from the learners were checked against the indicators for level 1 and 2.

The interviews with the learners at level 0 had showed some common characteristics. These level indicators which are given below are confirmatory findings when compared with those of Burger and Shaughnessy (1986) and Clements and Battista (1992).

Level indicators for pre-recognition level

1. Inability to identify many common shapes.
2. Attending to only a subset of the shapes' visual characteristics.
3. Groups of figures recognised as the same shape (e.g. Inability to differentiate squares and rectangles in their visual form).
4. Drawing similar sizes to make variety of the same shapes.
5. Inability to conceive infinite variety of shapes other than the usual common varieties of the shapes.
6. Use of incorrect terminology (e.g. using the word 'non straight' for lines that are 'not vertical').
7. Inability to see common properties of shapes even when they are put together.
8. Switching of key words - 'equal' for 'parallel' and vice versa.
9. Different meanings for the same statement for different shapes (e.g. 'both sides are equal' and 'two sides are equal' have different meanings).
10. Inability to use mathematical terms and concepts to describe common shapes.
11. Inability to guess the shape in the mystery shape task.
12. Ability to distinguish only between curvilinear and rectilinear shapes (e.g. can differentiate between a circle and a rectangle).

7.6. Significance of the study

Along with the points as stated in Chapter 1 earlier, this study is significant and novel in many ways.

This study was the first of its kind to attempt to use van Hiele levels to develop an instructional framework to introduce geometry in senior secondary schools in South Africa, more particularly in the Eastern Cape.

Along with the studies conducted in different parts of the world (Hoffer, 1981; Usiskin, 1982; Senk, 1985; Shaughnessy & Burger, 1985; Fuys et al., 1988; Clements & Battista, 1992) and in South Africa (King, 2003; Siyepu, 2005; Atebe, 2008) this study also contributed towards the teaching and learning interface of geometry education in particular and mathematics in general by coming up with the instructional framework that suits the South African context.

One of the major reasons for learners' poor performance in senior secondary school is identified as their limited exposure to geometry due to the lack of rich and well sequenced geometry curriculum in the primary school level (Clements & Battista, 1992; De Villiers, 1997; Siyepu, 2005). The instructional framework will hopefully close the gap between these two curricula as it is sequenced in such a way that it has integrated informal activities as a starting point.

According to the principles of NCS, learners are meant to achieve learning outcomes based on the knowledge, skills and values that are specific for that outcome. No particular textbook is prescribed by the Department of Education and educators are meant to choose from textbooks that are available to them to look for content that are relevant to achieve the learning outcomes, and it has often been taken as a difficult task for educators. The instructional framework has provided some solutions to this problem.

The curriculum reforms and changes that have been implemented in South Africa within the past 10 years and the latest addition, Continuous Assessment and Policy Statement (CAPS) have added major changes to the curriculum. Schools are sitting with financial burden of buying new textbooks with changes 'here and there' while textbook authors and publishers could have made those changes available as 'addendums'. The five teachers who tried the instructional framework in the booklet form are quite confident that they can use the booklet in future as to start off their geometry lessons. The training given to the educators will hopefully empower them for the effectiveness of their teaching.

Finally, the 359 learners from the five schools who participated in the study provided an in-depth and comprehensive idea of the levels of thinking of learners entering senior secondary school phase. The findings from the study can be utilised by mathematics educators and curriculum developers for their attempt to improve the instructional strategies in our schools.

7.7. Limitations of the study

Even though this study has its strength that it has developed, implemented and evaluated a framework that was useful in improving the geometrical thinking of learners in the sample in general, the study had a few limitations. Some of the limitations were mentioned in Chapter 1 in Section 1.11 under the limitations of the study. The following limitations were noted as the study completed:

The learners who were operating at level 2 found that the first three activities in the instructional framework were a bit 'dragging' or 'rather too easy' and the learners who were at level 0 found that the last activity 'a bit demanding'. Even though this can be viewed as a limitation, in the global picture, the instructional framework was meant to lead from lower levels to higher levels of thinking.

An attitude that was noticed from the learners at the beginning as reported by the educators was that 'we are too old' for this kind of activities, but later they found that it was necessary for them to start off like that. A similar attitude was also noticed in the interviews as I started off by asking to draw different triangles, they felt that it was 'too easy' and later they admitted that it was right for their age.

The results described in Chapters 4, 5 and 6 should be viewed in the light of possible limitations imposed by the features of the research design.

Finally, the interpretation of the interviews and the suggestions thereof were subject to my understanding of the van Hiele theory. The bias of the researcher and how the responses of the learners were interpreted should be considered. Therefore, any knowledge claims made in this study should be read in the light of these limitations, even though every aspect of the study was thoroughly verified through discussions with my supervisor, a researcher in geometry education and my colleagues.

7.8. Implications and recommendations for teaching and learning

With reference to the research questions investigated in this study, it appears that the framework based on the van Hiele levels has a positive effect. Based on this the following implications and recommendations are suggested for teaching and learning:

- The geometric thinking level of the learners should be identified before the teaching program. It is recommended that for effective teaching in geometry to happen, we should start at the learners' level of thinking. For this, it is important that the levels are identified before commencing the teaching program.

- Educators' method of teaching has an effect. In comparison with the traditional way of teaching, the teaching method adopted with the instructional framework had a more tangible effect.

- To improve geometry teaching, educators need to develop tasks or activities that help them better understand the nature of their learners' geometric reasoning and they also should have an understanding about research concerning such reasoning.

- Sufficient teaching and learning resources enhance the effectiveness of teaching and learning. The worksheets that were used in the instructional framework did have a positive effect on the teaching and learning as it provided enough teaching and learning resources.

- Structured programme can improve the teaching. As evident from the interviews with the educators, the structured sequence of activities in the framework could improve the teaching.

- Levels should be identified in earlier grades and appropriate experiences should be given in order to have better achievement in geometry in senior secondary schools.

- Junior secondary school curriculum should enable learners to develop visual skills related to common two and three dimensional figures and to learn properties of such figures.

- While framing the curriculum, care should be given to arrange the contents in such a way that it should develop the geometric thinking from one level to the higher level.

- Changes in the instructional practices need to be coupled with the changes in the curriculum to observe the effects on learner achievement.

- Constructive activities should be encouraged. The activities in the instructional framework involved a lot of constructive activities, which were enjoyable by the learners.

- Appropriate exploration tasks can be used to create a classroom environment that promotes meaningful justifications and is beneficial to building learners' understanding of proof.

- Learners should be made familiar with the techniques of drawing and folding for enhancing their geometric thinking.

- Higher levels of geometric thinking can be attained by the implementation of educator guided, learner centered, hands on instructional programme. It was evident from the higher Percentage number of learners in the experimental group in the posttest.

- The process of gradually moving from the concrete and active to abstract and more passive learning under the guidance of the educators would make the learning of geometry more relevant and enjoyable for our learners within our limited financial circumstances.

- Educators' main objectives should be to help gain an insight and an understanding of the subject matter and consolidate their conceptual understanding.

- Conducting interviews in obtaining information on learner understanding is very useful. The interviews gave more in-depth understanding of the level of thinking of the learners than the paper and pen test. It was evident from the interviews that one-on-one, casual, informal way of presenting geometrical ideas has an impact on improved geometric thinking. Friendly atmosphere also can elicit better output from learners.

- Learners' cultural background and their specific use of words in their vernacular should be taken into consideration by educators.

- Learners should move gradually from an informal investigation of geometry to a more proof oriented focus. Introduction of informal investigative approach to geometry can lead to a more formal proof oriented geometry.

- It is necessary to design appropriate experiences for pre-service and in-service educators to familiarise themselves with the van Hiele theory so as to design and use appropriate material for instruction according to the levels. Van Hiele theory should be introduced into the curriculum of mathematics education. The majority of the present in-service educators are not familiar with the van Hiele theory.

- The current initiatives by the Department of Education to improve the matric mathematics pass rate should be extended and broadened also to the General Education and Training Band. If learners are better prepared in their junior secondary school, it will ensure success in their efforts in senior secondary schools.

- The initiatives aimed at revitalising teacher education and learner performance must also include efforts to improve classroom practices. While focussing on upgrading the qualifications of educators, their conceptual knowledge and skills also should be strengthened and reinforced.

7.9. Areas of further research

The study presented here is of its first kind in South Africa to develop an instructional framework using manipulatives to increase the van Hiele levels of geometrical thinking of grade 10 learners. The findings of the study can therefore be taken as tentative due to the fact that it is tested only on one specific grade and a relatively small sample. Further research can be done with the same variables to larger sample in other grades. This will help to make more general statement about learners' geometric thinking. It can also be researched in various levels of the schooling system such as primary and junior secondary levels so that curriculum developers and textbook writers can align their work according to the levels of thinking as proposed by the van Hiele theory.

Past research and this study also found that learners can attain different levels in different topics and they perform at different levels on different tasks on triangles and quadrilaterals. Research can also be done in schools with other curriculum with more advanced topics.

The study used only 30 learners from the five different schools for the audio taped interviews. I felt that the interviews gave me a more in-depth understanding of the levels of geometric understanding. Further research can be done with larger sample and a video study might reveal more details of the learners' geometrical thinking.

The learners in the study who were at different levels of thinking used different language. Future research into the relationship between language use and the van Hiele theory might be useful in giving solutions to understand this phenomenon.

In the South African context, formal proof is going to be part of the Euclidean geometry taught at schools. More research is needed with learners who can write proofs. This can provide guidance for developing effective teaching materials and methods for teaching secondary school geometry.

7.10. Personal reflection

As a mathematics educator who has only recently been exposed to research in the field, this study has certainly highlighted a number of aspects of research that are valuable in preparing me as a better mathematics educator. The whole research process was a fascinating and enriching experience in terms of academic knowledge, human relationships and personal values. I was welcomed everywhere with open arms and the cooperation that I received from the Principals, teachers,

learners, my colleagues and also the support from friends and family at every step of the study are beyond words.

When I visited the libraries of Rhodes, Nelson Mandela Metropolitan University, University of KwaZulu-Natal, and my own home library, Walter Sisulu University, in search of literature, the cooperation was amazing! Everyone I met was so supportive!

My Supervisor, all the Professors and PhD students in the academia that I contacted while attending conferences and SAARMSTE Research School were all down to earth people. They were always approachable, understanding, kind and contributing! They did everything to dispel the feeling that PhD is a hard nut to crack! Their humility made me even more humble.

Yes, I had challenges! – reading, writing, rewriting, and running around to get things done on time while on a full time job at my school - from the proposal stage to the finalising of the thesis. But the grace of God Almighty, encouragement and support in every step of the study from my supervisor, and the patience, support and the sacrifice from my husband and my two kids made me to overcome all the challenges and difficulties with a smile!

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