THE EFFECTIVENESS OF THE CONCRETE/ SEMI-CONCRETE/ ABSTRACT (CSA) APPROACH AND DRILL-PRACTICE ON GRADE 10 LEARNERS' ABILITY TO SIMPLIFY ADDITION AND SUBTRACTION ALGEBRAIC FRACTIONS

BY

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ABSTRACT

This study was conducted in one of the education districts in the Eastern Cape Province of South Africa. The purpose was to analyse the effectiveness of the concrete/semi-concrete/abstract (CSA) approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. The following two objectives were set. First, to identify the learners' challenges in studying addition and subtraction of algebraic fractions in grade 10; and second to analyse the effectiveness of the CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. Both threshold concepts and troublesome knowledge, Polya's problem-solving techniques, CSA Approach theory and Drill-practice theory were all pertinent as a theoretical framework for the study. Positivism research paradigm was adopted for the study and it afforded the researcher opportunity to employ quantitative research approach. Based on the research question of this study, an experimental design was chosen as a suitable descriptive design. Purposive sampling method was used to select three schools which involved 135 grade 10 mathematics learners. Stratified random sampling method was thereafter employed to select 45 learners from each school for the study. The learners were grouped in each school as top, average and weak based on their performance in Algebra in term one. Pre-questionnaire and post-questionnaire were used to obtain data regarding challenges learners experience in simplifying addition and subtraction of algebraic fractions. Ethical clearance from the relevant school and university authorities were obtained. On the first two days, the researcher briefed the school authorities and learners and explained to them the purpose and details of the study. Day three was used to administer the pre-questionnaire test, thereafter, the next ten days were used to teach addition and subtraction of both numeric and algebraic fractions with same and different numerators and denominators. The next two days were used for revision and the last day was used to administer the post-questionnaire test out 25 marks. The respondent rate was 98.5%. The data collected were analysed by using SPSS version 16.10. Both descriptive and inferential statistics were used to analyse the data. The pre-questionnaire scores revealed that majority of the learners' perceived fractions as two separate entities and as a result add or subtract numerator to numerator and denominator to denominator. It was also discovered that learners had a challenge in finding LCM of algebraic fractions. A t-Test for independent means was used to test the following hypotheses at $\alpha = 0.05$: $\mathbf{H_0}$: The CSA approach and drill-practice intervention has no significant effect on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions; $\mathbf{H_1}$: The CSA approach and drill-practice will significantly enhance Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. The t-Test revealed a p-value of 0.139 which was statistically significant at $\alpha = 0.05$. Therefore, the researcher rejected the null hypothesis and concluded that the CSA approach and drill-practice have significantly enhanced the Grade 10 learners' ability to simplify algebraic fractions.

DECLARATION

I, Bernard Prince Awuah, hereby declare that **THE EFFECTIVENESS OF THE CONCRETE/SEMI-CONCRETE/ABSTRACT APPROACH AND DRILL-PRACTICE ON GRADE 10 LEARNERS' ABILITY TO SIMPLIFY ALGEBRAIC FRACTIONS** is my original work except for the references to other researchers' or authors' work, which have been dully acknowledged.

Signature____________________ Date: 25/08/2016

Student number: 201515251

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DEDICATION

I dedicate this dissertation to my wife, Augusta Awuah (Mrs) and daughter, Nyantachiwaa Akosua Awuah whose presence provided the source of inspiration and comfort needed throughout this study.

LIST OF ACRONYMS

CDE: Centre for Development in Education

CSA: Concrete/Semi-concrete/Abstract

DBE: Department of Basic Education

EC: Eastern Cape

NMAP: National Mathematics Advisory Panel

SMT: School Management Team

TIMSS: The International Mathematics and Science Study

USA: United States of America

DECLARATION OF PLAGIARISM

"I Bernard Prince Awuah student number 201515251 hereby declare that I am fully aware of the University of Fort Hare's policy on plagiarism and I have taken every precaution to comply with the regulations.

Signature:………………………………………………………………………………"

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CHAPTER 1: INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

The interest in this study was motivated by the researcher's concern with challenges which learners face with algebraic fractions in mathematics which he personally faced in day-to-day experiences in classrooms. Several researchers have reported on learners' difficulties with fractions (Gould, Outhred & Mitchelmore, 2006; Duzenli-Gokalp & Sharma, 2010; Gabriel, Coche, Szucs, Carette, Rey & Content, 2013; Booth, Newton & Twiss-Garrity, 2014).

The data from both the South African National School Certificate results (Department of Basic Education (DBE), 2013) and the Annual National Assessment (DBE, 2014) paint a bleak picture of the learners' performance in Mathematics as a subject and fractions in particular.

This study sought to analyse the effectiveness of concrete/semi-concrete/abstract (CSA) approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions as an important component in the Grade 10 South African Mathematics curriculum. It was hoped that the results would assist to improve Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions.

1.2 BACKGROUND

Globally, mathematics is a fundamental part of human thought and logic and is a prerequisite for scientific, technological and economic progress of any country (Lynn & Brocado, 2009; Umameh, 2011). The world's technological advances today demand a resilient mathematical background which leads to job opportunities (Steen, 2001; Kahn, 2001). Kurt and Somchai (2002), Pandor (2006) and Tella (2008) refer to research data and posit that the importance of acquiring a solid background in Mathematics as a subject is well recognised as a gateway to a variety of future professions.

1.2.1 Concern with poor learner performance in Mathematics

Mathematics is crucial in ensuring continuing the production of highly trained persons needed by industry, science and technology (House, 2006; Ojose, 2011). Steen (2001) emphasises that mathematics does not only empower people with the capacity to control their lives, but also provides a firm basis for theories and guarantees society a vigorous economy.

Mathematics offers an effective way of structuring mental discipline and increases logical reasoning and mental rigour (Skemp, 2008). This subject is crucial not only for academic qualification at school level, but it prepares the learners for their future as well, irrespective of which walk of life they choose to be a part of (Davis & Hersh, 2012).

Despite the importance of mathematics highlighted above, learners continue to fail the subject (Feza-Piyose, 2012). According to Fonseca and Conboy (2006), the rate of leaners' low performance in Mathematics at school level is a cause of great concern across the globe. Good grounding in mathematics is essential to understanding science. Consequently some countries, like the

United States and Tanzania, have already begun influencing all citizens, with special emphasis on school, industrial, government and science communities, to protest vigorously against poor performance in mathematical and science subjects in high schools to ensure that future results would be more encouraging (Roach, 2005; Yussuf, 2007).

It is perturbing to note that consistent trends of low achievement in Mathematics have also been recorded in South Africa (Makgato, 2007; Govender, 2009). In South Africa, learners usually learn operations on fractions through training and drill in the use of appropriate algorithms applicable to specific operations (Maharaj, Brijlall, & Molebale, 2007). It has been asserted that procedural knowledge, such as algorithms for operations, is often taught without context or conceptual understanding, implying that algorithms are multifarious and only mastered through memorisation (Sharp, Garofalo & Adams, 2002).

Makgato and Mji (2006) cite several studies pointing to the high failure rate in Mathematics in South Africa in comparison to other countries. Examples of such studies include: Howie (2001, 2003), Centre for Development in Education (CDE) (2004), Naidoo (2004), Reddy (2004) and UNESCO/UNICEF: Monitoring Learning Achievement Project (2005). While South African learners in general are not performing well in Mathematics, the situation is even worse among black South African learners (Brodie, 2004).

It may therefore be logical to argue that the aforementioned decline in the Mathematics pass rates reflects learners' inability to succeed in Mathematics at secondary school level (Fonseca & Conboy, 2006). Fonseca and Conboy emphasise that it is a world-wide phenomenon. Presumably, this could be one of the reasons which led to the skill shortages in science-related fields, particularly in developing countries of Africa (Madibeng, 2006).

This is the reason why most learners, when deciding on careers, tend to choose the so called 'soft' options. These are professions such as teaching, social work, policing and others which have very little to do with mathematics. It is a well-known fact that the number of scientists, accountants and others who need to have Mathematics as part of their training is fairly small in developing countries. A huge percentage of the population becomes teachers and social workers. In addition, the majority tends to opt for subjects like History instead of Mathematics. This is the root cause of the skills shortages in developing countries which Madibeng (2006) refers to.

Therefore, it is clear, based on the fore-going discussions that South Africa's situation regarding Mathematics needs to be improved. In order for the learners to understand the subject, the concepts need to be taught thoroughly. Hence, a concerted effort is needed to assist the learners to improve their understanding of the subject.

The tables below represent the analysis of the mathematical results nationally and provincially, and those of the district focused on in the current study. Since the study focuses on fractions, the analysis of algebraic fractions results in the district being studied is also presented. These tables are presented to highlight the mathematics performance in the country in general and the district under the current study in particular.

Table 1.1 represents the analysis of Annual National Assessment Results (2012-2014) (mathematics) in South Africa. Table 1.2 represents the Mathematics pass rate for the National Senior Certificate (2012-2014) in South Africa. Table 1.3 displays candidates' performance in Mathematics by province (2012 to 2014)

Table 1.4 represents the analysis of (2013) matric Mathematics results of 10 selected schools from the district being studied. Table 1.5 shows the analysis of (2014) Grade 10 Mathematics final examinations of 10 selected schools from the district under the study. Finally, Table 1.6 presents the analysis of (2015) March common controlled test for three Grade 10 classes from three schools (algebraic fractions).

Table 1.1: Analysis of Annual National Assessment Results 2012-2014 (Mathematics) in South Africa

Grade	2012	2013	2014
1	68%	60%	68%
2	57%	59%	62%
3	41%	53%	56%
4	37%	37%	37%
5	30%	33%	37%
6	27%	39%	43%
9	13%	14%	11%

Extracted from DBE (2014) analysis of results for ANA

The Table 1.1 above presents the analysis of (2012-2014) Annual National Assessment Results (Mathematics) in South Africa. It can be observed from the table that the Grade 9 Mathematics scores remained at a low level as was the case in 2012 and 2013. Nonetheless, the data for the Grades 1 to 6 point towards an improvement of test scores but there is still much to be done. Learners at this level need to acquire the requisite skills and knowledge in the Mathematics so that when they reach high school level (from grades 10 to 12) they can cope with the syllabus of the subject without any struggle.

Mathematics learners who reach Grade 10 usually have to be retaught the mathematics basics they should have acquired in Grades 8 to 9. Because of these learners' challenges, they do not show any interest in the subject when they come to the high school. Therefore, this is a problem that needs urgent attention of mathematics educators.

Table 1.2: Mathematics pass rate for the National Senior Certificate (2012-2014) in South Africa

YEAR	No. WROTE	No. ACHIEVED 30% AND ABOVE	No. ACHIEVED 40% AND ABOVE	PASS RATE OF 30% AND ABOVE	PASS RATE OF 40% AND ABOVE
2012	225874	121970	80716	54.0	35.7
2013	241509	142666	97790	59.1	40.5
2014	225458	120523	79050	53.5	35.1

Adapted from DBE, 2014 analysis of results.

Table 1.2 above displays the overall pass rate in Mathematics in South Africa in general from 2012 to 2014 (National Senior Certificate Schools Mathematics pass rate (NSCSR), 2014). This report indicates an increase in performance from 2012 to 2013 but a decrease in 2014. It also shows that the pass rate of 40% and above ranges between 35.1% and 40.5%. There is enough evidence to emphasise that more than 50% of the learners who wrote mathematics from 2012 to 2014 had levels 1 and 2. Level 1 is a score which ranges from 0% to 29% and level 2 a score ranges from 30% to 39%.

Based on South African's Universities' admissions criteria, a learner who wants to pursue a degree in a mathematics-related course must at least have a level 4 in Mathematics. Level 4 is a score which ranges from 50% to 59%. Table 1.2 shows that over 50% of the learners did not qualify to study for a bachelor's degree in any mathematics-related course. There is a clear indication that learners' performance in mathematics is poor. Therefore, special attention is needed since Mathematics is one of the critical subjects in South Africa.

Table 1.3: Candidates' performance in Mathematics by province (2012 to 2014)

PROVINCE	2012 PASS RATE, 30% AND ABOVE	2013 PASS RATE, 30% AND ABOVE	2014 PASS RATE, 30% AND ABOVE	2012 PASS RATE, 40% AND ABOVE	2013 PASS RATE, 40% AND ABOVE	2014 PASS RATE, 40% AND ABOVE
Eastern cape	38.1	43.4	42.0	21.9	26.4	25.1
Free state	64.8	71.1	65.8	43.3	49.8	44.5
Gauteng	71.0	73.6	69.3	52.4	54.4	50.7
Kwazulu-Natal	48.1	53.6	40.7	29.6	35.4	24.3
Limpopo	52.4	59.3	56.9	34.0	40.0	35.8
Mpumalanga	53.1	58.3	56.6	34.7	39.8	35.6
North west	59.6	67.4	61.7	37.7	44.8	40.3
Northern cape	54.9	57.7	63.4	36.5	38.2	42.4
Western cape	75.3	73.3	73.9	56.9	56.8	56.6

Adapted from DBE, 2014 analysis of results.

Table 1.3 above presents the analysis of candidates' performance in Mathematics with regard to the provinces in South Africa. It can be observed that there is a difference in performance among the provinces. It can be observed that the Eastern Cape Province is struggling to attain a 50% pass rate. It can also be noticed that in 2012, 2013 and 2014 the pass rate above 30% was 38.1%, 43.4% and 42.0% respectively. The results from 2012 to 2014 indicate that among all the provinces, Eastern Cape learners' performances are not inspiring. In addition, the pass rate of 40% and above is less than 30% which shows that much needs to be done to improve learners' understanding of Mathematics.

Table 1.4: Analysis of (2013) matric Mathematics results of 10 selected schools from the district being studied

School	NO. WROTE	Pass (%) 30% and above
A	275	28.0
B	31	35.5
C	96	15.6
D	53	24.5
E	29	24.1
F	21	14.3
G	156	52.6
H	9	0.0
I	109	16.5
J	87	35.6

Adapted from (DBE) 2013 analysis of results. School names are not indicated for ethical reasons.

Table 1.4 represents the average pass rate of Mathematics for 10 selected schools in the district where the present research was carried out. It is apparent that out of the 10 schools selected 9 of them got less than 50.0% pass rate in

Mathematics. The pass rate for the 10 schools ranges from 0.0% to 52.6% (DBE, 2013).

Furthermore, the overall pass rates of the national Mathematics paper in 2013 and 2014 in the district currently studied were 34.3% and 32.2% respectively (DBE, 2014). It can be observed that the Mathematics performance in the district in 2013 and 2014 were not even up to 40%. These results are a clear indication that the district needs interventions which could assist to improve the learners' understanding of Mathematics in general.

Table 1.5: Analysis of (2014) Grade 10 Mathematics final examinations of 10 selected schools from the district under the study

SCHOOL	NO. WROTE	NO. PASSED (30% and above)	% PASSED
A	330	162	49
B	41	0	0
C	201	26	13
D	204	82	40
E	183	48	26
F	407	137	34
G	100	5	5
H	326	42	13
I	423	111	26
J	136	80	59

Adapted from the District 2014 analysis of results.

Table 1.5 above indicates the 2014 final examinations analysis of Grade 10 Mathematics results in the district in which the current study was carried out. The ten schools were selected from the schools in Table 1.4 and named as school A up to school J. It can be observed from the table that of the 10 selected schools only three schools got a pass rate of 40% and above. The pass rate of the rest of the seven schools ranges from 0% to 34%. Looking at school B, forty-one (41) learners sat for the 2014 final examinations in Mathematics. None of the forty-one learners passed. In school H, 326 learners sat for the Mathematics examinations and only 13% of the learners passed which is a disaster. Also in school I, 423 learners sat for the Mathematics examinations and only 26% of the learners passed. These results show that much needs to be done.

1.2.2 A focus on algebraic fractions

Algebraic fractions are an important element of algebra which is a component of mathematics. An understanding of algebraic fractions contributes to success in algebra which is a component of mathematics as alluded to by other researchers (Carpenter, et al., 1980; Lipkus, et al., 2001; Jigyel & Afamasaga-Fuata'i, 2007; Reyna & Brainerd, 2008).

Educators and researchers describe learning of fractions as a challenging area of the Mathematics curriculum (National Assessment of Educational Progress (NAEP), 2005; Jigyel & Afamasaga-Fuata'I, 2007; Siegler, Thompson & Schneider, 2011; Nortno & Boyce, 2013). According to Gould et al. (2006), the understanding of part-whole relationships and the notations are difficult for learners.

Reports of learners' performance in Mathematics in the United States of America indicate that there is emphasis on identifying effective instructional strategies to improve learners' understanding of fractions (National Mathematics Advisory Panel (NMAP), 2008; Siegler, Carpenter, Fennel, Geary, Lewis, Thompson & Wray, 2010). The NMAP (2008) stresses the need for learners' to acquire algebraic skills prior to high school as they are essential for learners to understand higher level mathematics such as trigonometry and probability.

A study conducted by NAEP in (1990) in the United States of America also indicated that only 46% of 12th Grade learners displayed success in fractions. A study indicates that understanding of fractions is a challenging area of mathematics for North American learners (NAEP, 2005). Learners also appear to have challenges in retaining fraction concepts (Lipkus, Samsa & Rimer, 2001; Reyna & Brainerd, 2008). Learners need to understand the concepts so that they could recall when they need.

A report by the NMAP (2008) also highlights the significance of using fractions as a skill that is fundamental and essential to algebra, as well as prerequisite to overall post-high-school success. Of immediate concern to this panel were the diverse reports of declining learners' performance in algebra as they proceeded to higher grades, marked by less than 40 percent meeting the expected ability levels (NMAP, 2008). Furthermore, the results indicated that around 40 percent to 50 percent of high school learners significantly struggle with the content of basic elementary level fractional mathematics (NMAP, 2008).

According to Das and Zajonc (2008), similar difficulties with the learning of algebraic fractions have also been reported among learners in Saudi Arabia. A study conducted by the World Bank ranked the overall performance of Saudi Arabian learners at 43rd out of 51 countries that participated in the Trend in Instructional Science and Mathematics (Das & Zojonc, 2008).

In addition, at the Annual Conference workshop of Mathematics Association of Ghana (MAG) (2002), it was stated that the teaching of basic concepts of fractions to learners was one of the problems at the basic level. The Chief Examiner's Report of the Basic Education Certificate Examination (BECE) (2002) in Ghana also indicated a number of problems Grade 9 candidates encountered in answering questions that involved fractions:

a) Subtracting mixed fractions from whole numbers.
b) Adding and subtracting fractions having different denominators.
c) The misapplication of the least common multiples (LCM).

Reports from the BECE (1998-2001) were further studied by the MAG and similar evidence of difficulties were cited. Based on the above opinions, there is evidence to say that learners' challenges with fractions are not peculiar to only one country.

Table 1.6: Analysis of (2015) March common controlled test for three Grade 10 classes from three schools (algebraic fractions)

Marks obtained out of 8 marks	Learners (School A)	Learners (School B)	Learners (School C)
0(0%)	39 (45.3%)	23 (33.3%)	41 (40.6%)
1(12.5%)	16 (18.6%)	22 (31.9%)	25 (24.7%)
2(25%)	7 (8.1%)	11 (15.9%)	16 (15.8%)
3(37.5%)	13 (15.1%)	9 (13.0%)	3 (3.0%)
4(50%)	5 (5.8%)	2 (2.9%)	8 (7.9%)
5(62.5%)	5 (5.8%)	1 (1.4%)	6 (5.9%)
6(75%)	0 (0.0%)	1 (1.4%)	2 (2.0%)
7(87.5%)	1 (1.2%)	0 (0.0%)	0 (0.0%)
8(100%)	0 (0.0%)	0 (0.0%)	0 (0.0%)
Total (Learners)	86 (100.0%)	69 (100.0%)	101 (100.0%)

Adapted from the District, 2015 analysis of Grade 10 results

Table 1.6 above presents results of a test of algebraic fractions of three classes from three selected schools in the district under the current study. This table shows that 87.1% of the learners from school A, 94.1% from school B and 84.1% from school C scored less than 50%.

It is clear that learners need intervention with algebraic fractions to assist their understating of Algebra. Also, poor performance in algebraic fractions and related tasks is recognised by fellow Mathematics educators in the district, and learners mention that fractions are hard to comprehend. Therefore, the focus in this study is to analyse the effectiveness of the CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions.

Jamilah and John (2010) posit that teaching and learning of fractions is both important and challenging at the lower levels of education. They argue that for that reason educators need to provide experiences that involve other mathematical concepts. These experiences need to be set in meaningful situations to which learners can relate learning of algebraic fractions.

Hansen (2006) argues that, in spite of these experiences, learners continuously face difficulties because educators tend to forget to analyse the errors exhibited by the learners. These errors can be utilised as a source of information in teaching (Hansen, 2006). Suhrit and Roma (2010) also suggest that understanding learners' mistakes can enhance learners' performance in teaching and learning of algebraic fractions.

A study conducted by Yetkiner and Capraro (2009) argue that educators are required to possess a conceptual understanding of fractional operations to deliver a sense-making curriculum.

Based on the aforementioned discussions, there is a sufficient indication to emphasise that educators need to identify the learners' mistakes in addition and subtraction of algebraic fractions. It is assumed that learners' mistakes can give the educators insight about their challenges and suggest ways to offer assistance to address their difficulties. Moreover, educators themselves need to understand the concepts of fractions.

Several researchers (Charalambous & Pitta-Pantazi, 2005; Jigyel & Afamasaga-Fuataí, 2007; Watanabe, 2012) have dealt with fractions and found that learners perceive fractions as separate whole numbers. This results in them adding numerator to numerator and denominator to denominator when they are asked to add or subtract fractions. Carpenter, Corbitt, Kepner, Lindquist and Reys in (1980) reported that learners reason that $\frac{2}{5}$ is bigger than $\frac{2}{3}$ since the number 5 is bigger than 3. As a result of these, learners apply whole number reasoning when dealing with fractions.

A particular interpretation of fractions as part-whole also results in learners struggling to build an understanding of and work with improper fractions (Charalambous & Pitta-Pantazi, 2005; Watanabe, 2006). Kieran (1994) and Watanabe (2012) identify that learners' hold on to the representations they are exposed to as the foundation for their conceptual understanding. In this case, it makes sense to select precise representations that have longevity and power. Jigyel and Afamasaga-Fuataí (2007) also find in their study that it is a challenge for many learners to explain how a fraction wall demonstrates equivalency.

This lack of understanding results in learners relying on memorised algorithms and making frequent errors in the application of these algorithms (Brown & Quinn, 2006). Kieran, as stated in Huinker (2002) and referred to in Petit, Laird and Marsden (2010) established that untimely experiences with formal procedures may lead to symbolic knowledge that is not based on understanding. According to Suh, Moyer and Heo (2005), many learners at the high school level in the USA struggle with fractions even after they have learnt fractions for a number of years in school.

In this regard, several studies (National Council of Teachers (NCT), 2000; Suh et al., 2005; Van Eck, 2006; Nieves, 2009) have recommended strategies that can be utilised to improve learners' understanding of algebraic fractions. According to NCT in the USA (2000), acquiring the basic number combinations, such as single digit multiplication, division and computational fluency is an element to future success in other areas of mathematics.

A study conducted by Suh et al. (2005) emphasise that the concrete/semi-concrete/abstract (CSA) approach is suitable for teaching and learning of fractions. They state that developing visual models for fractions is an important factor influencing understanding of fraction computation. Nieves (2009) emphasises that it enables learners to gain understanding of concepts and fluency in computation. Van Eck (2006) states that noncompetition games do not create more positive learners' attitudes towards mathematics. The presence of a coach, mentor or advisor in combination with competition can make learners function to their maximum ability.

As a continuation of the background to this study, one of the key objectives of learning fractions in Grade 10 is to manipulate algebraic expressions by "… multiplying, dividing, adding and subtracting algebraic fractions" (Department of Education (DoE), 2011:13). Educators and researchers agree that fraction concepts are among the universal, multifarious and significant mathematical ideas that learners encounter before reaching high school (Behr, Lesh, Post, 1992; Mack, 1993).

However, even with this early introduction, learners continuously exhibit trouble conceptualising fractions, possibly forming one of the most critical barriers to the mathematical maturation of learners (Verschaffel, Greer &

Torbeyns, 2006; Anthony & Walshaw, 2007; Lamon, 2007; Young-Loveridge, Taylor, Hawera & Sharma, 2007).

Wu (2001: 11) emphasises "that no matter how much algebraic thinking is introduced in the early grades, and no matter how useful this might be, the failure rate in algebra will continue unless the teaching of fractions and decimals is thoroughly revamped. The proper study of fractions offers a rise that leads learners smoothly from whole number arithmetic up to algebra". The wide use of fractions in everyday life makes information about fractions necessary as early as elementary grades.

It is against the background of poor performance in Mathematics and the challenges learners face in algebraic fractions that the researcher decided to narrow the field of study by focusing on the effectiveness of CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions.

1.3 RATIONALE

The motivation for choosing this topic is that learners need to understand the concept of algebraic fractions since it lays the foundation for other mathematical topics such as algebra. Fellow educators' concern about the learners' performance and the researcher's ten years teaching experience in one of the public high schools within South Africa has also been the reason for this study.

It is hoped that if the problem is addressed, it can assist in improving performance of algebra, trigonometry and probability as mentioned earlier on. The studies by Lipkus, et al. (2001) and Reyna and Brainerd (2008) with the inference that learners have difficulties in fractions, were not done in the education district where this study was carried out and the researcher did not locate any study of this nature in the district. This rationale leads to the statement of the problem.

1.4 STATEMENT OF THE PROBLEM

As explained in the foregoing sections, the problem is that the learners do not only have difficulties to understand mathematics generally, but also algebraic fractions. As such, research on the problem emanating from a gap as shown in the rationale above regarding learners' challenges on addition and subtraction of algebraic fractions in an Eastern Cape education district and exploring ways to overcome them to empower learners are important.

1.5 THE PURPOSE OF THE STUDY

The purpose of this study was to analyse the effectiveness of the CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions.

1.6 RESEARCH QUESTIONS

Based on the foregoing discussions, the main research question of this study was: How can Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions be improved?

The following sub-research questions were postulated for the current study.

- What challenges do Grade 10 learners face in addition and subtraction of algebraic fractions?
- Can the CSA approach and drill-practice instructional strategies be used to lessen the difficulties in addition and subtraction of algebraic fractions in Grade 10?

1.6.1 Research Objectives

The research objectives are to:

- Identify the challenges facing learning of addition and subtraction of algebraic fractions in Grade 10; and
- Analyse the effectiveness of the CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions.

1.6.2 Research Hypotheses

Based on the background, statement of the problem and the research questions, the following hypotheses have been postulated:

- $\mathbf{H_0}$: The CSA approach and drill-practice intervention has no significant effect on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions.
- $\mathbf{H_1}$: The CSA approach and drill-practice will significantly enhance Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions.

1.7 THEORETICAL FRAMEWORK

This study was guided by the following theoretical framework, namely, threshold concepts and troublesome knowledge, and Polya's problem-solving techniques.

1.7.1 Threshold Concepts and Troublesome Knowledge

Meyer and Land (2006: 2) state that a “threshold concept can be considered as akin to a gateway opening up a new and previously inaccessible way of thinking about something. It represents a transformed way of understanding, or interpreting, or viewing something without which the learner cannot progress”. It changes the way learners perceive things, and the level of conceptual understanding is also improved. The acquisition of knowledge arises through a process of gathering key concepts per particular subject. The concepts are rooted in learners’ conceptual understanding, consequently assisting in problem solving.

“A core concept is a conceptual ‘building block’ that progresses understanding of the subject; it has to be understood but it does not necessarily lead to a qualitative view of subject matter” (Meyer & Land, 2006: 4). Threshold concepts have the following characteristics which are briefly enumerated and explained (Meyer & Land, 2006):

1.7.1.1 Transformative

Transformation brings change in an individual’s conceptual thinking. The more the threshold concept develops, the more it brings a new perspective about things around an individual.

1.7.1.2 Irreversible

Once a learner understands the concept it is not likely to be forgotten. That means that knowledge acquired is not easy to forget.

1.7.1.3 Integrative

Once a concept is grasped, it is easy to link it to the already existing concepts and that would enable easy retention of the concepts as they are linked.

1.7.1.4 Bounded

A threshold concept is likely to be bounded in that "any conceptual space will have terminal boundaries, bordering with thresholds into a new conceptual area" (Meyer & Land, 2006: 6).

1.7.1.5 Troublesome

Mastery of a threshold concept might involve 'common sense' and intuitive understanding of it. It then becomes tough and uncomfortable to reverse learners' intuitive understanding.

Deficient mastery of prerequisite skills and concepts has been assumed as one of the understanding factors that contribute to learners' inability to simplify addition and subtraction of algebraic fractions.

Based on the aforementioned discussions, it is clear that, if threshold concepts could be developed in learners, the concepts could build their confidence and therefore reduce their difficulties in addition and subtraction of algebraic fractions.

1.7.2 Polya's Problem-Solving Techniques

Polya developed four fundamental principles that need to be considered during problem solving. Based on the principles, four steps were developed to be followed during problem-solving. The researcher identified a technique which sought to solve the particular problem of learners who continue to commit errors in addition and subtraction of algebraic fractions. This might be attributed to the learners' problem-solving techniques. Understanding of the afore-going theory can assist to improve Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions.

Polya's problem-solving theory describes four steps to be followed during problem-solving in mathematics education. According to Polya (1945: 11), the following are the principles to be considered during problem-solving:

1.7.2.1 Understanding the Problem

Learners might appear to be incompetent but perhaps do not understand the question fully. Polya states that educators need to ask learners the following questions:

- Do you understand all the words used in the problem statement?
- What are you asked to find?
- Can you rewrite the question in your own words?
- Think of a picture or diagram that might help you to understand the problem.
- Is there enough information to enable you to find a solution?

1.7.2.1 Devise a Plan

There are many reasonable strategies to be employed in order to solve a particular problem. Polya (1945: 13) states that the skill of choosing a suitable strategy is the best thing to solve problems. According to Polya (1945: 13), the following are the strategies that a learner can choose in order to solve a particular problem:

- Guess and check
- Make an orderly list
- Use direct reasoning
- Solve a simpler problem than the complex one
- Work backward
- Use a formula

1.7.2.3 Carry out the Plan

This is considered to be easier than devising a plan as it only requires the patience of implementing the devised plan. This will include for instance, correct substitution if the chosen strategy was the use of a formula. According to Polya (1945: 14), "Consistency throughout the algorithms employed to arrive at the final answer is of the highest importance in this step".

1.7.2.4 Look Back

Taking time to reflect on your work enables you to predict the pertinent strategies for solving a future problem. If the devised plan does not work, you will have to discard it and use another one until you arrive at the correct answer.

The study sought to answer the question: can the CSA approach and drill-practice instructional strategies improve Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. The objectives of this study are based on Polya's theory on problem-solving techniques. The study sought answers to the aforementioned research questions and provide a solution for an identified problem.

In view of the above discussions, it is possible that, if the teaching and learning process at schools could be influenced by the foregoing theory, both learners and educators guided by the four-step principle could improve learners' ability to simplify addition and subtraction of algebraic fractions in Grade 10.

According to Siegler et al. (2010) in USA indicates that CSA approach involves learners learning through multiple modalities. It allows the learners to develop a fundamental understanding of fractions through representations. In their study, they discovered that CSA approach is an instructional strategy to effectively teach learners who struggle with algebraic fractions. Also, they posit in their study that learners who use concrete materials develop more precise and more comprehensive mental representations.

Also, according to Anderson (2008) and Mary, Jill and Sara (2016), drill-practice greatly increases the possibility that learners will permanently acquire new information. In drill-practice, learners are provided with a clear strategy to cope with the task and guided in step-by-step manner through the steps of the task (Wicken & McCarley, 2008). Because of the conceptual understanding which can be improved by the CSA approach and problem solving techniques which can also be ensured by drill-practice instructional strategies, both CSA approach and drill-practice instructional strategies are pertinent for the theoretical framework.

1.8 SIGNIFICANCE OF THE STUDY

The findings of this research would serve as a springboard for further study that could offer awareness of high school Mathematics learners' understanding of addition and subtraction of algebraic fractions.

The findings of the current study would also provide valuable information for educators on the effectiveness of the CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. It would have an influence on the development of the learners' mathematical thinking.

Finally, the study would offer strategies that can assist in improving learners' performance in addition and subtraction of algebraic fractions. It is required for learners to have understanding of rational expressions, especially learners who anticipate to successfully study majors in science, technology, engineering, and mathematics (STEM).

1.9 OPERATIONAL DEFINITION OF TERMS

For the purpose of this study, the following key concepts within the context of the research have to be explained. Namely, fractions, senior phase and further education and training.

1.9.1 CSA approach

CSA approach is a systematic method used to effectively teach significant connections from hands-on manipulatives to representational pictures, to abstract concepts and symbols (NMAP, 2008; Nieves, 2009).

1.9.2 Cuisenaire rods

Cuisenaire rods are coloured wooden or plastic rods that have values from one to ten and are coloured by the number they represent: white rod = 1cm, red rod = 2 cm, light green rod = 3 cm, Lavender rod = 4 cm, yellow rod = 5 cm, dark green rod = 6 cm, black rod = 7 cm, brown rod = 8 cm, blue rod = 9 cm and orange rod = 10 cm (Heddens, 1997).

1.9.3 Drill-practice

Drill-practice is described as a method of instruction characterised by systematic repetition of concepts, examples and practice. In drill-practice, learners are provided with a clear strategy to cope with the task and guided in step-by-step manner through the steps of the task (Anderson, 2008; Wicken & McCarley, 2008).

1.9.4 Fractions

A number on the number line that can be located between two whole numbers. A rational number representing $\frac{m}{n}$ where $m \neq 0$. A "part of a whole", a ration, quotient of a division problem, or part of a set" (Gersten, Clark, & Witzel, 2009).

1.10 SUMMARY OF LITERATURE REVIEW

The literature review is divided into two sections. The first section discusses challenges Grade 10 learners' face in studying algebraic fractions. These challenges are: relationship between numerator and denominator; fractions as part/whole relationship; vague representations; numeric-symbolic procedure; and wrong conception and inability to find least common multiples (LCM).

The second section discusses strategies to improve learners' ability to simplify addition and subtraction of algebraic fractions in Grade 10. These strategies are: concrete/semi-concrete/abstract (CSA) approach; drill-practice; the essence of error analysis in acquiring knowledge; and importance of games and competition on attitudes and learning.

Based on the theoretical framework of the study, the researcher adopted CSA approach and drill-practice instructional strategies as suitable for the current study.

1.11 SUMMARY OF RESEARCH DESIGN AND METHODOLOGY

This section explains the research design and methodology that was used to address the research questions and the objectives of the current study. Particular attention was focused on data collection, data collection methods, data analysis.

A positivism research paradigm was adopted for the study and it afforded the researcher opportunity to employ a quantitative research approach. The research design employed was descriptive. The study focused on Grade 10 Mathematics learners in public high schools which involved 135 learners from three selected schools.

Purposive sampling was used to select the three schools. Thereafter, stratified random sampling was used to select 45 learners from each of the three schools. A pilot study was conducted to ensure validity of the instrumentation. Data were collected by means of a pre- and post-questionnaire. The pre-questionnaire was used to establish baseline information on the learners' ability to simplify addition and subtraction of algebraic fractions. The post-questionnaire was used to assess the effectiveness of the CSA approach and drill-practice instructional strategies on the learners' ability to simplify addition and subtraction of algebraic fractions in Grade 10.

1.12 DATA ANALYSIS

According to Houser (2008) and van Zyl (2012), once the data has been collected, it is essential to make sense of it by organising and coding the data to accelerate the analysis thereof. The raw data gathered was processed using the Statistical Package for Social Science (SPSS) Version 16.10.

Descriptive statistics describing the distribution of scores, the relationship among variables and variability through the use of frequencies and means were used to analyse the data. In effect the descriptive statistics allowed the researcher to get an accurate first impression of "what the data look like" (van Zyl, 2012).

In addition, a t-test for independent means was used for the hypotheses testing. The t-test was used based of the fact that only one group received the treatment. The means were different because they were averages computed from different groups. The level of significance at which the research hypotheses were tested was 0.05.

In testing the hypotheses, the following steps were taking into consideration:

- Statement of the null hypothesis. In this case, the null hypothesis was as follows: the CSA approach and drill-practice intervention has no significant effect on Grade 10 learners' ability to simplify algebraic fractions.
- Establishing the level of significance or risk or Type I error associated with the null hypothesis.
- Selection of the appropriate test statistic. The appropriate test statistic for the null hypothesis was t-test for independent means.
- Computation of the test statistic. In this case the SPSS Version 16.10 was used to calculate the test statistic.
- Determination of the critical value.
- Comparison of the obtained and the critical value.

- If the obtained value is more extreme than the critical value, the null hypothesis cannot be accepted.
- If the obtained value is less extreme than the critical value, the null hypothesis is the most attractive explanation (van Zyl, 2012: 185).

1.12.1 Management of TYPE I and II Errors in data analysis

TYPE I error is a type of error that occurs in the data analysis stage when a researcher rejects the null hypothesis when in fact it is true, whereas TYPE II error is committed where a researcher accepts the null hypothesis when it is in fact not true. These errors directly affect the validity of the study. The researcher has to be mindful of the place and significance of the tests, not forgetting the problem of the Hawthorne effect operating negatively or positively on learners who have to undertake the tests (Cohen, Marion & Morrison, 2007).

The Hawthorne effect is the phenomenon in which participants alter their behaviour as a result of being part of the study. The researcher ensured standardised procedure in administering a test. In the data analysis stage, the researcher would avoid TYPE I and or TYPE II error by presenting the data without misrepresenting meaning. By a pilot study the researcher ensured that invalidity was minimised as much as possible throughout the study.

The validity of the study cannot be achieved through tests only but when the results of different tools are used, they should be analysed concurrently. According to (Cohen et al., 2007: 117) “For research to be reliable it must be carried out on a similar group of respondents in a similar context, then similar results would be found”. To test the reliability and validity of the instruments the pre-questionnaire and post-questionnaire were developed and administered as a pilot study.

1.13 ETHICAL CONSIDERATIONS

For transparency, efficacy, validity and reliability of research results, the researcher must consider ethical values for the research participants (Chaska, 2008). Ethical considerations, according to Creswell and Clark (2011), are obligations that relate to the researcher's respect for the rights, needs, values and desires of the research participants. Creswell and Clark (2011) state that researchers need to have experience and the requisite knowledge on how to conduct research. This comprises the ethical rules, personal responsibilities, and methods to be used in selecting participants.

1.13.1 Confidentiality and anonymity

The research participants were assured that all information regarding their identity would be handled as confidentially as possible. The responses given by the research participants were treated with confidentiality. In addition, the research participants were given full assurance that their privacy and identity would be withheld and secured from public exposure during and after the study to ensure anonymity.

1.13.2 Informed consent

The researcher obtained permission from the Department of Basic Education, the principals of the selected schools and participants before the study was carried out (see appendix D, E, F, G and H). The research participants were informed about when the study would commence and also come to an end. This made them aware of how much of their valuable time would be needed (Chaka, 2008). The respondents were made to understand that participation was voluntary and they could choose to withdraw their consent at any time when they did not want to be part of the current study.

1.14 DELIMITATIONS OF THE STUDY

The findings of this study cannot be generalised since the sample of the study consisted of selected Grade 10 Mathematics learners. In other words, this study did not include all the grade 10 Mathematics learners in the district.

Moreover, this study was narrowed to only one education district in the Eastern Cape Province and it also involved only three schools; therefore, the study will have a limited generalisability as mentioned above.

1.15 ORGANISATION OF THE DISSERTATION

This section presents the organisation of the dissertation. This dissertation is presented in five chapters.

Chapter one: This chapter presents the introduction, background, statement of the problem, rationale, the purpose, significance, the research questions and objectives. The definition of terms, summary of the literature review, summary of the research design and methodology and data analysis are highlighted. The ethical considerations are also described. Finally, the delimitation of the study is explained briefly in this chapter.

Chapter two: Chapter two critically reviews five learners' challenges in studying addition and subtraction of algebraic fractions and two instructional strategies that are suitable for the current study.

Chapter three: This chapter considers the research design and methodology used in conducting the study. This includes research paradigm, approach, design, population, sample, instrumentation and procedure for data collection.

Chapter four: This chapter deals with the data analysis.

Chapter five: Discussions, conclusion and recommendations are provided in this chapter. References and appendices are presented at the end.

1.16 SUMMARY

The chapter highlighted the background and rationale as well as the statement of the problem. It was discovered that mathematics in general has become a serious challenge to learners across the globe. In South Africa, the problem is critical and needs to be addressed.

This study was motivated by the decline of leaners' interest in mathematics resulting in the high failure rate among learners in the Eastern Cape Province of South Africa. Based on the researcher's experience as a Mathematics educator, simplifying algebraic fractions poses a challenge to learners and might adversely impact on their pass rate in Algebra.

In view of the above problems, the study employed threshold concepts and troublesome knowledge, and Polya's problem-solving techniques as theoretical framework.

The next chapter describes the literature review for this study.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

The literature review is structured as follows. The first section discusses various challenges learners encounter with addition and subtraction of algebraic fractions calculations. The second section discusses strategies to improve learners' ability to simplify algebraic fractions. The final section provides a summary of the chapter.

2.2 CHALLENGES IN STUDYING ALGEBRAIC FRACTIONS

This section provides an account of the literature reviewed on learners' challenges in studying addition and subtraction of algebraic fractions in Grade 10. Five key factors have been identified in the literature as the challenges in studying addition and subtraction of algebraic fractions in Grade 10. These challenges are listed and discussed under the following subheadings.

2.2.1 Relationship between numerator and denominator

The first challenge in studying addition and subtraction of algebraic fractions is the relationship between numerator and denominator. This challenge emanates from numerical fractions which is also applicable to algebraic fractions. A study conducted in Australia found that learners normally perceive fractions as two separate whole numbers (Jigyel & Afamasaga-Fuata'i, 2007). Consequently they apply whole number reasoning when working with algebraic fractions. For example, the majority of Grade 10 learners, when asked to estimate the sum and difference of $\frac{11x}{12}+\frac{7x}{8}$ and $\frac{3x}{8}-\frac{x}{5}$ choose $\frac{18x}{20}$ and $\frac{2x}{3}$ as the answers in a multiple-choice format. Learners also reason that $\frac{y}{4}$ is bigger than $\frac{y}{3}$ since the number 4 is bigger than 3.

A study conducted by Huinker (2002) in the United States of America (USA), as cited in Petit et al. (2010: 146) states that 'learners who can translate between various fraction representations "are more likely to reason with fraction symbols as quantities and not as two whole numbers" when solving problems'.

This means that learners need to understand the meaning of fractions before the application of the operations such as addition, subtraction, multiplication and division of fractions. For example, the algebraic fraction $\frac{x}{2}$ needs to be explained to learners as "a half of x number of items". When the learners grasp the concept, it would be easier to apply according to Petit et al. (2010).

In dealing with algebraic fractions, learners are required to acquire a good understanding of the multiple constructs of fractions. Without this, learners may not understand the possible meanings of the numerator and the denominator of algebraic fractions, and the distinctions between them (Petit et al., 2010; Empson & Levi, 2011).

Additional confusion about the role of the numerator and denominator arises with an early introduction to fraction notation and the careless use of unclear language. Describing one-third as 'one over three' or 'one out of three' leads to learners conceptualising each as a separate whole number, rather than recognising the multiplicative relationship that is inherent in the notation (that is to say that one-third is one one-third unit or that it refers to one one-third of a whole) (Lamon, 2005).

Based on the arguments above, educators need to use appropriate language such as one-third of x instead of 'x over three' to represent an algebraic fraction like $\frac{x}{3}$. This could be one of the reasons why learners perceive fractions as two separate entities, and consequently apply whole number reasoning when adding or subtracting algebraic fractions. This is a clear indication that learners do not understand that fractions are symbols of quantities. They recognise numerator and denominator as two separate entities.

2.2.2 Fractions as part-whole relationships

The second challenge in studying addition and subtraction of algebraic fractions relates to ‘fraction as part of a whole relationship’. There is a general agreement that the singular interpretation of fractions as part-whole explanation results in learners’ struggling to build an understanding of and work with improper fractions (Thompson & Saldanha, 2003; Lamon, 2005; Watanabe, 2006; Charalambous & Pitta-Pantazi, 2007). Research has recognised an overemphasis of fractions as exclusively part-whole relationships in North America. The instructions confine learners’ understanding of fractions as quantity, leading to a number of consistent misunderstandings.

According to Simon (2002), learners do not understand the equivalence of pieces of congruent figures that have been divided in half; this points to an understanding of fractions as an arrangement rather than a quantity.
Researchers in Mathematics education stress that learners must have an understanding of fraction as measure, in addition to part-whole in order to assist their development of an understanding of addition and subtraction of fractions (Lamon, 2005; Watanabe, 2006; Moseley & Okamoto, 2008).

Watanabe (2006) further stresses the different ways of understanding fractions in his discussion of the Japanese emphasis on the unit fraction to define a fraction. That is to say that a fraction may also be considered to be a multiple of a unit fraction. For example, “ $\frac{2}{3}$ would be considered as 2 times the unit obtained by partitioning 1 into 3 equal parts” (Watanabe, 2006 & 2012). We can name this as “two one-third units”. If we are adding fractions such as $\frac{1}{3}$ and

$\frac{2}{3}$ we can say "one one-third and two more one-thirds gives us three one-third units."

In view of the above discussions, there is a clear indication that the single interpretation of fractions as part-whole relationship pose a challenge to learners' when dealing with fractions. This is the reason why learners become confused when they are dealing with improper fractions. For instance, they do not understand why the numerator should be bigger than the denominator when they are dealing with improper fractions.

2.2.3 Vague Representations

A vague representation is identified as the third challenge in studying fractions. Circle representations are difficult to divide equally, leading learners to focus more on the number of partitions and less on the congruency. These partitions confuse learners about whether partitions must be congruent or not. A research study conducted by Watanabe in Japan (2006) posits that this count-wise approach to pieces of a circle as a whole number (one piece) does not account for the importance of equal area. It does not also account for the importance of a whole in relation to the pieces.

In Ontario, it is particularly confusing when learners in primary grades use circle illustrations when studying fractions (Watanabe, 2012). The concept of area of a circle is not formally addressed until intermediate grades. This creates an interesting situation in which learners are required to use the concept of area of a circle to produce equal partitions, though, they have not been formally introduced to the properties of the area of the circle (Watanabe, 2012).

There is substantive documentation of learners failing when they attempt to partition circles evenly unless they are considering halves and fourths. Fractions other than halves and fourths including thirds, fifths, sixths, ninths, etc., appear to be highly problematic (Watanabe, 2012). These representations are used consistently and with the purpose of developing learners' understanding of fraction as quantity.

It is therefore necessary to emphasise the underlying concepts of expressing all fractions as a multiple of a unit fraction. This can be done by comparisons based on like-units, and identification of the whole (Watanabe, 2012). This set of representations strongly supports moving from understanding the different meanings of fractions to operations with fractions.

The effectiveness of the consistent use of representations is supported by the findings of researchers Watanabe (2007) and Charalambous, Delaney, Hsu, and Mesa, (2010). They found that learners hold on to the representations they are initially exposed to as the foundation for their conceptual understanding. Therefore, it is important to ensure that representations are chosen to fit the problem context. The fact that fractions are singular quantities needs to be emphasised (Gould et al., 2006; Watanabe, 2007; Charalambous et al., 2010).

Studies in this area provides evidence to suggest that some of the many representations are potentially distracting. They do not help learners build deep understanding. Therefore, it makes sense to select precise representations that have longevity and power to enhance learners' conceptual understanding of fractions.

2.2.4 Numeric-Symbolic Procedures

Kiernan, as cited in Huinker (2002) and referenced in Petit et al. (2010), found that premature experiences with formal procedures may lead to symbolic knowledge that is not based on understanding. This is further compounded by the progressive removal of the use of models of fractions to privilege symbol notation. This has the potential to impede learners in developing fluency across the different representations of fractions (Gould et al., 2006).

Jigyel and Afamasaga-Fuata'i (2007) found in their research study in Australia that many of the Year 8 learners could not explain how fraction bars demonstrated equivalency. This lack of understanding fractions results in learners relying on memorised algorithms and making frequent errors in the application of these algorithms (Brown & Quinn, 2006). Saxe, Taylor, McIntosh, and Gearhart (2005) suggest monitoring understanding of fraction notation separately from the understanding of fraction concepts as learners develop these two domains somewhat independently.

Moss and Case (2001) found similar evidence of two independent processes: a global structure for proportional evaluation; and a numeric structure for splitting or doubling. In their study, coordination of these two structures did not occur until approximately ages 11 and 12, leading the leaner to understanding of semi-abstract concepts of relative proportion and simple fractions and percentages such as one half (50 percent) and three fourths (75 percent).

Based on these observations, Moss and Case (2001) developed an innovative instructional lesson sequence, beginning with a beaker of water. The learners began using general terms to describe the beaker as nearly full, nearly empty, etc. The lessons then introduced percentage such as "100% full," linking to learners pre-existing knowledge and schema, as well as their familiarity with real contexts and familiar representations. Next, the lesson sequence introduced decimals, and finally connected these forms of describing amounts to fractions.

2.2.5 Wrong Conceptions and inability to find Least Common Multiple (LCM)

According to Gabriel, Coche, Szucs, Carette, Rey and Content, (2013), the complicated nature of fractions poses a conceptual challenge for learners. They posit that it is necessary to explain thoroughly the following concepts to the learners: ratio, operator, quotient and measure. In this context of the current study, ratio of females to males, 2:5, meaning $\frac{2}{5}$ of the group; operator $\frac{2}{5}$ stands for 2 divided by 5 or $\frac{1}{5}$ multiplied by 2; quotient is the result of division; measure explains fractions as numbers and intervals.

Another notion revealed by Gabriel et al. (2013), is the part-whole nature of fractions used in sharing. For instance, when a litre of cold drink is shared equally amongst 5 learners, the litre should be divided into 5 equal portions. The notion of operator for division may be used. Gabriel et al. (2013) maintains that the conceptual and procedural nature of knowledge that fractions require pose a challenge to learners' choice of the correct notation that needs to be applied to a specific problem.

A study conducted by Watanabe (2012) discloses that learners apply easy methods or "short cuts" which are easier alternatives and then they end up making mistakes. He emphasises that it is essential to study these alternatives and correct learners at an early stage.

There is enough evidence to support the fact that the basic concepts of fractions need to be addressed to bring about understanding. In view of the argument put forward by Watanabe (2012), this could be one of the reasons why learners add numerator to numerator and denominator to denominator without finding the LCM. The learners seem to be confused and hence they result to any 'short cut' method that comes to mind.

According to Hecht Vagi (2012), the learners' inability to add or subtract fractions is a result of lack of conceptual knowledge that governs this domain. Hallet, Nunes, Bryant, Thorpe (2012) also discovered that learners often have the following imbalances in procedural and conceptual knowledge of fractions: more conceptual, more procedural, equally good on both, or equally poor on both. These challenges are as a result of prior knowledge of fractions. Siegler, Fazio, Bailey and Zhou (2013) state that fractions are difficult for many learners because learners assume that algorithms, procedures and properties of whole numbers are also properties of all other numbers.

This is a clear indication that learners need to understand the difference between whole numbers and fractions. This could assist them (learners) to realise that algorithms, procedures and properties of whole numbers are not properties of all other numbers including fractions. According to Bush and Karp (2013), learners do not pay special attention to procedures of fractions such as sharing and ordering which comprise the identification of different forms of numbers. In addition, they state that fractions are rational numbers and not whole numbers. So, learners are required to understand that there are infinite rational numbers between 0 and 1.

Based on the above discussions with regards to the various challenges learners face in studying algebraic fractions, the current study focused on two of the challenges which appeared to be suitable. The two principal challenges are the relationship between the numerator and the denominator, and the wrong conceptions and inability to find the LCM.

The researcher chose to study the relationship between numerator and denominator and learners' misconceptions and inability to find the LCM based on the influence those two challenges have on mathematical topics such as probability, trigonometry and calculus. Also, based on the researcher's judgement as an educator and a marker, these two challenges are the common ones which occur frequently when analysing learners' solutions to mathematics problems. For instance, when Grade 12 learners are asked to differentiate $f(x) = \frac{1}{x}$ from first principles, they find it difficult to simplify the following expression $\frac{1}{x+h} - \frac{1}{x}$. This simplification needs the concept of LCM.

2.3 STRATEGIES TO IMPROVE LEARNERS' ABILITY TO SIMPLIFY ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS

According to Star, Caronongan, Foegen, Furgeson, Keating, Larson, Lyskawa, McCallum, Porath, & Zbiek (2015), algebra is important in the mathematics domain and it requires abstract thinking. It goes beyond the emphasis of mathematics operations and dwells much on the use of symbols to denote numbers and express mathematical relationships.

Algebra is an important topic in mathematics for future studies in advanced mathematics like calculus, probability and trigonometry. Therefore, it is a key for success in mathematics (Star et al., 2015).

Strategy is a general approach for solving a problem that may involve a sequence of steps to be implemented, as well as the rationale behind the use and effectiveness of these steps (Star et al., 2015).

According to McLead and Newmarch (2006), most high school learners acknowledge that the concept of fractions is a mathematical topic which is problematic, regardless of the constant use of the concept related to sharing. Educators have to look into the vital question of why learners perceive fractions to be difficult.

The key answer might be the standard means of the notation of fractions. Alternatively, the reason might be the language used which is often formal. In addition, fractions might be quite confusing since most fractions do not necessarily act like the standard number. At times, they represent a given quantity, which can be visualised, while at other times a given operation. Effective instructional strategies to teaching fractions are required to give high school learners a chance to appropriately explain the manner in which they view fractions (McLead & Newmarch, 2006).

In view of the discussions above, the following were identified in the literature as instructional strategies that can assist to improve Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. These strategies are enumerated and discussed under the next subheadings.

2.3.1 Concrete/Semi-Concrete/Abstract (CSA) Approach

The first instructional strategy adopted by the researcher based on the research problem and the theoretical framework for this study was the CSA approach. This strategy is a systematic method utilised to effectively introduce various mathematics concepts, such as algebraic fractions, to learners. This instructional strategy essentially targets the fundamental conceptual understanding of the discipline of mathematics (NMAP) (2008).

Studies conducted by the American Institute for Research (2007) reveal that the CSA approach is an effective means for appropriate mathematics instruction which can considerably enhance the overall performance of high school learners who struggle with mathematics. Since algebraic fractions is a component of mathematics and has been identified as a problem area, this study focuses on a CSA approach which can enhance the learners' ability to simplify addition and subtraction of algebraic fractions in Grade 10.

According to the NMAP (2008) in Washington DC in the USA, the CSA approach can enhance learners' conceptual understanding of addition and subtraction of algebraic fractions. This approach can be used to teach significant connections from hands-on manipulatives to representational pictures, to abstract concepts and symbols.

A study investigating the overall effectiveness of the CSA approach has effectively examined diverse content areas such as addition, subtraction, multiplication, division, fractions and algebra (Butler, Miller, Crehan, Babbitt & Pierce, 2003).

A CSA approach might be duly implemented for individual purposes, small group purposes, or even for the entire class. Educators can make provision for multiple opportunities for demonstration and practice to assist learners in the achievement of mastery of mathematical concepts. The educators can actually prompt learners with certain tasks at each given step of the overall practice. If a learner is actively solving a defined problem, the educator can then read it aloud and subsequently summarise whatever the learner completed as he or she moves in a sequential manner through the defined stages.

The concrete stage is the “doing” stage of using concrete objects to model problems. This stage consists of visual, tactile and kinaesthetic modalities. At this stage, concrete materials are used to help learners with a first-hand experience (Witzel, Riccomini & Schineider, 2008).

The semi-concrete stage is the “seeing” stage. This is the stage where learners are introduced to representations of concrete materials. Here the educator transforms the concrete models on a representational level, which may involve drawing pictures, using circles or rectangles to imprint pictures for counting.

The abstract stage is the “symbolic” stage of using abstract symbols to model problems. At this stage, the educator models the mathematics concept at a symbolic level, using only numbers, notation and mathematical symbols to represent the number of rectangles or groups of rectangles. The educator uses operations to indicate addition, subtraction, multiplication and division (Access Centre, 2004). This is the stage where educators need to ensure that learners are participating.

Nieves (2009) emphasises that a CSA approach can enhance learners' understanding of algebraic fractions concepts and their computation. This approach is also supported by other research findings in the USA, which state that developing visual models for the teaching and learning of fractions is a significant influential factor in building understanding for fraction computation (Witzel, Riccomini & Schineider, 2008; Siegler, et al., 2010).

According to Siegler et al., (2013), visual representations can be used to help illustrate the need for common denominators when adding and subtracting fractions. For example, an educator can demonstrate addition by using fractions of an object $\frac{1}{3}$ of a rectangle and $\frac{1}{2}$ of a rectangle. By placing the $\frac{1}{3}$ of the rectangle and the $\frac{1}{2}$ rectangle together inside a third rectangle, the educator can show the approximate sum. He or she can then show that $\frac{1}{3}$ of the rectangle equals $\frac{2}{6}$, and that $\frac{1}{2}$ equals $\frac{3}{6}$, and that the sum is exactly $\frac{5}{6}$ of the rectangle. This type of concrete demonstration can assist learners to understand why common denominators are necessary in addition and subtraction of algebraic fractions (Fazio & Siegler, 2011).

A study conducted in Australia by Swan and Marshall (2010: 14) states that mathematics manipulative material includes any "object can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered". Finding a range of suitable real world models as context for teaching mathematical ideas is recognised as part of good teaching. Research-based study shows that learners who use concrete materials develop more precise and more comprehensive mental representations. It motivates learners to understand mathematical ideas, and better apply these ideas in real life situations (Siegler et al., 2010).

According to a research study conducted in the USA by Siegler et al. (2010), it was discovered that a CSA approach involves learners learning through multiple modalities. It allows the learners to develop a fundamental understanding of fractions through representations. In their study, they found CSA approach as effective instructional strategy to teach algebraic fractions to learners who struggle with mathematics.

Based on the above discussions and the theoretical framework for this study, the researcher considered a CSA approach a suitable instructional strategy for the current study. The researcher taught the learners using cardboard, Cuisenaire rods and strips of papers to facilitate the conceptual understanding of algebraic fractions. The researcher is of the opinion that a CSA instructional strategy can enhance Grade 10 learners' understanding of addition and subtraction of algebraic fractions.

2.3.2 Drill-Practice

Drill-practice is described as a method of instruction characterised by systematic repetition of concepts, examples, and practice. Drill-practice is a discipline and repetitious exercise, used in teaching to perfect a skill or procedures. As an instructional strategy, drill and practice promotes acquisition of knowledge which could facilitate the Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. Drill-practice greatly increases the possibility that learners will permanently acquire new information (Moss & Case, 2001; Anderson, 2008; Mary, Jill & Sara, 2016)

This instructional strategy can aid performance by automating a task in order to free resources for another task (Wicken & McCarley, 2008). Immediately one task has been automised, attention resources can be applied to other tasks. In the first place interference between tasks depends on the demands of the tasks for a limited supply of mental resources. Second, the resource demand of a task decreases with practice until resource-free automaticity is reached (Wicken & McCarley, 2008).

Learners are required to make links with the prior procedural, or computational knowledge. In fact, the absence of either computational or structural understanding at various stages of learning actually delays further development (Nieves, 2009). Timed practice drills provide an effective traditional means of developing automaticity when combined with untimed practice of facts for mastery.

According to the National Council of Teachers of Mathematics in the USA (2000), knowing the basic number combinations, such as single digit multiplication and division, and having computational fluency or automaticity, are essential and fundamental to future success in other areas of mathematics. Unfortunately, high school learners acquire mathematics skills without synchronising the development of automaticity because they do not have to memorise basic mathematics facts or formulae due to the availability of reference sheets and calculators for standardised assessments (Krudwig, 2003).

In drill and practice, learners are provided with a clear strategy to cope with the task and guided in a step-by-step manner through the steps of the task. Thereby a learner's attention is guided to the accurate execution of the task steps instead of having the learner divide attention between identifying a strategy and performing the task at the same time (Wicken & McCarley, 2008).

The aforementioned discussions point to the fact that, when the learners acquire a skill, there is a need for educators to drill the learners to practise so that they will not forget the skill they have developed. Since learners have knowledge about factors and multiples from the junior grades, educators can build on this to handle the addition and subtraction of fractions. When the learners are drilled on the multiples of numbers, they may not have challenges in finding the LCM of common fractions.

Therefore, there is an indication that drill-practice are suitable strategies or skills that can enhance teaching and learning of addition and subtraction of algebraic fractions since repetition strengthens impression. Hence the researcher perceives this strategy to be relevant for the current study.

2.3.3 The Essence of Error in Acquiring Knowledge

Melis (2008: 1) argues that in order for learners to shift from routine and factual knowledge to, "more emphasis on developing competence such as solving Mathematics-related problems, reasoning, and communicating mathematically, learners need to be motivated to explore, verbalise ideas, and build confidence in them, in learning Mathematics". This can be done by accommodating their mistakes. If learners' mistakes are considered to be a source of their learning, such an understanding ought to improve learning and learners' performance in algebraic fractions because research shows that learners learn best if they are involved in the construction of their knowledge (Jonassen, 2000; Suffolk, 2008).

According to Luneta (2008), mistakes are genuine attempts to comprehend mathematics. This can manifest when learners vigorously attempt to make sense of their knowledge by trying to associate everyday information and school knowledge. Research conducted by Suffolk (2008) supports the idea that knowledge of common mathematical errors and misconceptions of learners can provide educators with a focus for teaching and learning in general and algebraic fractions in particular.

Hansen (2006: 16) argues that "the most effective teachers ... cultivate an ethos where learners do not mind making mistakes, because errors are seen as part of learning." This presumes that Mathematics educators need to avoid learners' mistakes, at some stage but they need to minimise them. Educators need to be careful on how they handle the mistakes. Hansen (2006) further suggests that when learners are given opportunity to identify mathematical mistakes it can result in greater openness on the side of the learners to explore and discuss their own mistakes. Hansen (2006) continues to advise that, an educator must become a guide to facilitate learning in classroom situations.

This study recommends that an educator needs to slowly but deliberately move from the centre stage into an invisible position in the classroom in order to facilitate conditions in which learners take charge of their own learning. Thus, with the support of the educator as a facilitator, learners' understanding of algebraic fractions is likely to be developed when they are in a position to compare their thinking with that of fellow learners in the context of error analysis.

According to the above arguments, learners' mistakes and misconceptions can be used to facilitate teaching and learning. This strategy can be beneficial in the classroom situation. The aforementioned discussion supplies enough evidence that learners' mistakes can be used as a platform to assist them.

2.3.4 Importance of Games and Competition on Attitudes and Learning

Some research finds that games improve or reduce the negative effects of the wrong attitudes and lack of motivation, thereby increasing student performance (Nieves, 2009) although Van Eck (2006) found that noncompetition games do not create more positive student attitudes towards mathematics. The presence of a coach, mentor, or advisor in conjunction with competition can make learners function to their maximum ability. This person can increase the positive effects of competition, such as self-efficacy and positive attitude, and simultaneously decrease the stress of competition and mathematics anxiety (Nieves, 2009).

In view of the above arguments regarding the various strategies to improve the learners' understanding of addition and subtraction of algebraic fractions, the current study adopted the CSA approach and drill-practice.

The choices were made based on the guided theoretical framework. According to Meyer and Land (2006), threshold concepts and troublesome knowledge changes the way learners perceive things, and the level of conceptual understanding is also enhanced. The concepts are rooted in learners' conceptual understanding, consequently assisting in problem solving. Based on the fore-going discussions CSA approach can assist learners to gain understanding of concepts of algebraic fractions and their computation (Nieves, 2009).

Also, drill-practice on the other hand increase the possibility that learners will permanently remember new information (Moss & Case, 2001; Anderson, 2008; Mary, Jill, & Sara, 2016). Hence, the researcher employed the CSA approach and drill-practice instructional strategies as a basis for long-term conceptual change among the learners, with regard to their ability to understand addition and subtraction of algebraic fractions.

2.4 SUMMARY

The chapter looked at the challenges learners face in studying fractions relevant to the study. In addition, instructional strategies that can improve learners' ability to simplify algebraic fractions were also discussed. The literature review in this chapter established that factors which cause poor performance in algebraic fractions are common in both developed and under-developed countries but are more critical in under-developed countries, including South Africa. Based on the research objectives and the theoretical framework of this current study, the researcher would be addressing the following two challenges: The learners' perception of fractions as two separate entities and the wrong conception and inability to find the LCM. The afore-going challenges were addressed using these instructional strategies found to be relevant to the study. They are: CSA approach and drill-practice.

The next chapter describes the research design and methodology adopted to address the research objectives and the questions for this current study.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

Researchers have different beliefs and ways of viewing and interacting with their surroundings. Therefore, the way in which research studies are conducted vary, although, there are certain standards and rules that guide a researcher's actions and beliefs. Such standards or principles can be described as a paradigm. To gain a better understanding of why and how the researcher chose the methodological approach in this study, an initial discussion will be completed about the paradigm that best fits the focus of this study.

Following a discussion about the research paradigm, the focus of this chapter is to discuss the research design and methodology used to analyse the effect of CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify algebraic fractions in one Education District in the Eastern Cape Province. It includes details of the research design, the description of the population, the sample, the data collection tools, content of the instruments, validity and reliability of the instruments, the pilot study, information about the data analysis procedures and ethical issues.

3.2 RESEARCH PARADIGM

A research paradigm is defined as a world view, general perspective, and a way of breaking down the complexity of the real world when doing research (Taylor et al., 2006). Additionally, Weaver and Olson's (2006: 460) definition of research paradigm reveals how research could be affected and guided by a certain paradigm by saying "paradigms are patterns of beliefs and practices that regulate inquiry within a discipline by providing lenses, frames and processes through which investigation is accomplished".

Weaver and Olson (2006) and Grix, (2010) identified three research paradigms, namely, positivism, interpretivism and post-positivism. These paradigms are discussed in the following subsections.

3.2.1 Positivism

Positivism is based on a realist, foundationalist epistemology which views the world as existing independently of our knowledge of it. Positivists believe that there are patterns and regularities, causes and consequences, in the social world just as there are in the natural world (Grix, 2010). A positivist believes in the possibility of making causal statements. Therefore, many seek to employ scientific methods to analyse the social world.

Positivists seek "objectivity" in research (Weaver & Olson, 2006). Positivism places an emphasis on empirical theory in the production of knowledge; it rejects normatic questions (for instance, questions of values and trust) and believes that social science can be value-free.

3.2.2 Interpretivism

Interpretivism describes the second research paradigm. Interpretivist positions, in contrast to positivism and realism, are based on an anti-foundationalist epistemology, and interpretivists subscribe to the view that the world does not exist independently of our knowledge of it (Grix, 2010). The world is socially constructed through the interaction of individuals and the separation of "fact" and "value" is not as clear-cut as the positivists claim.

The emphasis in this paradigm is on understanding as opposed to explanation, as interpretativists do not believe in relying on mere observation for understanding social phenomena. In contrast to positivism, this position sees the social and natural sciences as being distinct from one another. Interpretivists, in general, do not strive to establish causal explanations in the social world.

Researchers who subscribe to this paradigm tend to place emphasis on meaning in the study of social life and emphasise the role language plays in constructing "reality" (Weaver & Olson, 2006; Grix, 2010).

3.2.3 Post-positivism

The post-positivism represents the third research paradigm discussed in the current study. The post-positivism approach is also described as critical realism (Weaver & Olson, 2006; Grix, 2010). Critical realism is on both sides of positivists and interpretivists paradigms, sharing a foundationalist epistemology with positivism and allowing for interpretations in research.

Critical realists approach believes that while social science can use the same methods as natural science regarding causal explanations (positivism), it also needs to move away from them by adopting an interpretive understanding. Critical realists, unlike interpretativists, generally seek not only to understand but also to explain the social world (Grix, 2010).

Based on the above arguments regarding the various research paradigms as well as their underpinning research approaches, the current study adopted the positivism paradigm. The choice was made because it was suitable for the research problem.

3.3 RESEARCH APPROACH

The research approach adopted for this study was quantitative. A quantitative approach is a scientific approach that collects and analyses numerical data which are concerned with the relationship between one set of facts and another (Bell, 2005; McMillan & Schumacher, 2006; Gay, Mills & Airasian, 2006). It usually condenses measurement into numbers (Johnson & Christensen, 2008).

In other words, it describes relationships in phenomena as the degree of influence one factor has over another in terms of reciprocal influences (Gay et al., 2006). In view of the discussions above, the quantitative research approach embraces the assumption that individuals inhabit a relatively stable, uniform, and coherent world that can be measured, understood and generalised by striving to establish a relationship between two or more variables (Bell, 2005; Gay et al., 2006).

The aim of this research was to analyse the effectiveness of the CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. Based on the aim of this study, primary data was sourced from the respondents and therefore a quantitative research approach was appropriate for the study. Moreover, the choice of quantitative approach was suitable based on the research question of this study.

3.4 RESEARCH DESIGN

According to McMillan and Schumacher (2006), a research design describes the plan for choosing the subjects, research sites, data collection methods, data analysis approaches and how the data is to be presented to answer research question(s). The main purpose for the research design is to provide results that are judged to be credible.

Based on the purpose of this study, a quantitative research approach using a descriptive design was followed. In order for the researcher to gain different perspectives and draw attention to different challenges facing the understanding of addition and subtraction of algebraic fractions and their operations in Grade 10, a descriptive research design was chosen to be suitable for the current study.

According to Gay et al. (2006), this research approach necessitates data gathering from respondents who are directly involved in a research study. Hence this research design was appropriate since the respondents were directly involved in the study.

3.4.1 Descriptive Design

Descriptive research aims at providing a clear and accurate description of individuals, phenomena or processes (Gay et al., 2006). In addition, a descriptive study provides a picture of the specific details of a situation, a social setting, a relationship (Neuman, 2011) or a picture of a phenomenon as it naturally occurs (Bickman & Rog, 2009).

According to Bickman & Rog (2009), descriptive design is typically concerned with determining the frequency with which something occurs. A rich description of the experiences of Grade 10 Mathematics learners was seen as an ideal process for the current study. The data was used to describe the phenomenon, through emerging patterns, to ensure the descriptive nature of this research.

Based on the research question of this study, an experimental design was chosen. Experimental design is a research plan in which the researcher directly manipulates or controls one or more independent variables and assesses their effect on the dependent variables (Hair, Black, Babin & Anderson, 2010).

3.5 POPULATION

Population is a group of potential participants to whom the researcher anticipates to generalise the results of the study (van Zyl, 2012). According to McMillan and Schumacher (2006) and Johnson and Christensen (2008), population can also be described as the entire group of individuals having the characteristics that interest the researcher. In view of the above explanations, population can be defined as a complete collection of observations that the researcher intends to study.

According to the Education Management Information System (EMIS) statistics, the Eastern Cape Province in South Africa is the second largest in terms of the number of educators and learners involved. These schools are grouped into 30 districts (Matomela, 2006).

The target population for the current study consisted of Grade 10 Mathematics learners in all public high schools in one education district in the Eastern Cape Province in South Africa. The researcher regarded the identified population as the relevant group about whom generalisations can be made (van Zyl, 2012).

3.6 SAMPLE

A sample is the number of individuals selected from the target population for the study (van Zyl, 2012). In this study, “purposive sampling” was used to select three schools which involved 135 Grade 10 mathematics learners. The researcher had easy access to the three schools selected.

The schools were considered appropriate by the researcher to collect the necessary information for the study. All three schools offered Mathematics as a subject and their class sizes were quite large, comprising more than seventy learners in a class. Due to the large number of learners in each class, the researcher used stratified random sampling to select 45 learners from each school comprising: 15 top, 15 average and 15 weak learners, based on their performance in algebra.

The selection of the 135 learners as the research sample for this study was mainly purposive. This sample size was adequate and large enough to answer the research questions. The sample was considered suitable to safely minimise sampling error. The larger the sample, the smaller the sampling error and the more representative and accurate conclusions and generalisations (McMillan & Schumacher, 2006; Robson, 2007; Johnson & Christensen, 2008).

3.6.1 Sampling

As mentioned in the above discussions, a stratified random sampling method was adopted to select the participants for the current study. Stratified random sampling is a technique in which a population is divided into subpopulations, called strata. Then, simple random sampling (SRS) is used to select the respondents from each stratum, and the learners in the strata are selected independently. The strata are often subgroups of interest to the investigator (Lohr, 2009).

Stratified random sampling was used based on the following reasons:

a) Stratified random sampling often gives more precise estimates for population means and totals.
b) It is more convenient to administer and may result in lower cost for the data collection process.
c) It can protect the researcher from the possibility of obtaining an unsuitable sample (Lohr, 2009).

The stratified random sampling which was employed in selecting the sample of the subjects involved the following processes as proposed by McMillan and Schumacher (2006) and Johnson and Christensen (2008):

• Mark lists of all Grade 10 Mathematics learners were collected (on day one of visiting the schools),

• Names of top, average and weak learners were written on separate sheets of papers,

• Numbers were assigned to the names of the learners in each category,

• In a raffle format, 15 learners were selected from each category in order to offset group bias.

As a result, 45 learners were selected from each school to give a total sample of 135 learners.

3.7 INSTRUMENTATION

According to McMillan & Schumacher (2006), research instruments are described as tools used for data collection needed to find solutions to the problem under investigation. The main data-collection instruments utilised in this study were the pre-questionnaire and post- questionnaire.

According to Johnson and Christensen (2008), a questionnaire may be described as a self-report data-collection instrument that each research respondent completes as part of a research study. Although there are different research tools that could have been used for this study, the pre-questionnaire and post-questionnaire were considered based on the research design. The following reasons for a questionnaire were also considered:

- It is the most widely used technique for obtaining data from subjects (McMillan & Schumacher, 2006),

- It is considered to be relatively economical and therefore was assumed to be convenient for collecting data from a large sample of Grade 10 Mathematics learners. In addition, since it is simple and requires less time, it would not take much time of learners who are already overloaded with school work,

- It is normally treated confidentially and safeguards anonymity, and therefore could result in more honest responses,

- Statements or questions are phrased the same for all. This was expected to eliminate bias that normally occurs in interviews,

- This instrument could provide a substantial amount of data that could be quantified, summarised and reported to all stakeholders (learners, educators, principals, parents and Department of Education). Though the questionnaire was considered to be the best research instrument for this study, it was very important to have well-designed questionnaires in order to elicit in-depth and accurate data (Gay et al., 2006).

This was achieved by spending much time on planning and developing both questionnaires, asking skilled people to evaluate them as well as piloting them in order to avoid lengthy, disordered questionnaires with ambiguous questions or statements since such factors may discourage respondents (Johnson & Christensen, 2008).

3.7.1 Design of Pre-Questionnaire and Post-Questionnaire

As mentioned earlier, the researcher used pre-questionnaire and post-questionnaire tests to gather the required information for the current study. The pre-questionnaire and post-questionnaire designs consisted of five questions on both addition and subtraction of algebraic fractions.

The pre-questionnaire and post-questionnaire were divided into two sections, A and B. Section A was used to source biographical data such as age and gender. Section B consisted of five questions. These items in section B include: Addition of fractions with algebraic numerator; addition of fractions with algebraic denominator; subtraction of fractions with algebraic numerator; subtraction of fractions with algebraic denominator; and addition and subtraction of algebraic fractions combined.

The pre-questionnaire items were designed to establish baseline information on Grade 10 learners' perception on fractions as two separate entities and their inability to determine LCM. The post-questionnaire items were designed to analyse the effectiveness of the CSA approach and drill-practice instructional strategies on the learners' ability to simplify addition and subtraction of algebraic fractions.

3.8 RELIABILITY AND VALIDITY OF INSTRUMENTS

Reliability and validity have always been seen as the most crucial criteria for evaluating quantitative research instruments such as questionnaires if the researcher's interpretation of data are to be valuable (Gay et al., 2006; Wiid & Diggenes, 2009; Lohr, 2009).

3.8.1 Reliability

Reliability is an assessment of the degree of consistency between multiple measurements of variables. One form of reliability is test-retest, by which consistency is measured between the responses for an individual at two points in time. The objective is to ensure that responses are not too varied across time periods so that a measurement taken at any point in time is reliable (Hair, Black, Babin & Anderson, 2010).

This is confirmed by McMillan and Schumacher (2006), who state that reliability refers to the consistency of measurement, the extent to which the scores are similar over different forms of the same data instrument, or occasions of data collection. In other words, data collection is reliable if a researcher gets essentially the same data from observation to observation during any measuring instance or that varied from time to time for a given unit of analysis measured twice or more by the same instrument (Robson, 2007).

However, Robson (2007) goes further by stating that it is usually impossible to get an exact repetition of a measurement when working with people. Therefore, to ensure reliability in this study, the Cronbach alpha was used. The Cronbach alpha value obtained was 0.865. The Cronbach alpha obtained showed that the research instruments were reliable.

Cronbach alpha is a measure of reliability that ranges from 0 to 1, with values of 0.6 to 0.7 deemed the lower limit of acceptability. The generally agreed upon limit for Cronbach's alpha is 0.7, although it may decrease to 0.6 in exploratory research (Hair et al., 2010). One issue in assessing Cronbach's alpha is its positive relationship to the number of items in the scale (Hair et al., 2010).

3.8.2 Validity

Validity is the extent to which a scale or set of measures accurately represent the concept of interest (Hair et al., 2010). According to Johnson and Christensen (2008) and McMillan and Schumacher (2006), validity can be described as whether or not something actually measures what it claims to measure for particular people in a particular context and that the interpretations made on the basis of the test scores are correct.

In this study, it was very important to consider both content and construct validity of the measuring instrument. Johnson and Christensen (2008) describes construct validity as the one that involves relating a measuring instrument to a general theoretical framework in order to determine whether the instrument is tied to the concepts and theoretical assumptions that are employed. Content validity is described as the degree to which a measuring instrument measures an intended content area.

To ensure validity, after drafting the measuring instruments they were given to fellow mathematics educators, experts, experienced researchers and to the supervisor to check the validity of the instruments before administering them. They assessed the inclusiveness, content and relevancy of the questions to the subject under study.

3.9 PILOT STUDY

To enhance both validity and reliability of the research instruments, a pilot study was carried out. Bell (2005), and McMillan and Schumacher (2006) emphasise that data gathering instruments need be piloted in order to:

- Guarantee validity and reliability,
- Guarantee that the questions mean the same to all respondents,
- Approximate how long it takes the respondents to complete the questions,
- Check that all the questions and instruments are concise and clear,
- Check ambiguity,
- Check biased items,
- Check problems that have been experienced so that the researcher can remove any items which do not yield usable data and ensure that the respondents experience no difficulties in completing the questionnaires, and
- Finally, have direction.

Based on the aforementioned explanations, the researcher conducted a pilot study which involved nine (9) learners. The 9 learners who took part in the pilot study were excluded from the main study. It was found that the questionnaires contained valid responses. These results were analysed by the researcher by using the Statistical Package for the Social Sciences (SPSS), Version 16.10.

As mentioned earlier, the Cronbach alpha coefficient obtained was 0.865. This value is greater than the 0.7 which is deemed as an acceptable reliability Cronbach alpha coefficient. The value of the Cronbach alpha obtained suggested that the data gathering instrument was reliable. Hence the pre-questionnaire and the post-questionnaire were used to collect the data for the study.

Table 3.1 below illustrates the marks obtained by the nine leaners. It can be observed from Table 3.1 that both marks are consistent. For example, learner 'I' got 2 marks for the pre-questionnaire and 3 marks for the post-questionnaire. Leaner 'IV' got 9 for the pre-questionnaire and 9 for the post-questionnaire. Hence, these marks also support the reliability of the instruments.

Table 3.1: Analysis of the pilot study results

LEARNER	MARKS (1)	MARKS (2)
I	02	03
II	05	07
III	12	10
IV	9	9
V	02	00
VI	05	07
VII	05	08
VIII	06	05
IX	00	02

Source: Researcher's Database

The pre-questionnaire and the post-questionnaire tests were given to fellow mathematics educators and experts to check how valid they were. Afterwards, the nine learners' completed the questionnaire within a range of 20 to 25 minutes. The learners were entreated to comment on the time they spent to complete the questionnaires and whether there were questions which were not clear and difficult to answer. They all indicated that the questions were clear and there was no problem with the time as well.

3.10 DATA COLLECTION PROCEDURE

Primary data collection procedures were used in the current study to allow the researcher to tailor the data collection method to suit the specific needs of the study, and identify the specific tools to be used (Houser, 2008). Primary data are data collected by the researcher using a range of collection tools such as interviews, observation and questionnaires rather than simply relying on existing data sources (Wilson, 2012).

In this study, a paper-based self-administered pre-questionnaire and a post-questionnaire were adopted to collect the empirical data. A self-administered questionnaire is one which a respondent completes on his or her own: there is no agent administering the questionnaire (Burns & Bush, 2006).

According to Gay et al. (2006), the higher the percentage of the questionnaires returned, the better the data. Robson (2007) strongly warns that many questionnaires suffer from poor response. Therefore, to safeguard against this, and to obtain a high percent response rate, the pre-questionnaire and post-questionnaire tests were administered by the researcher to collect the data.

The Department of Basic Education and the principals of the selected schools granted the researcher permission to use three weeks for the intervention. The researcher used the first day at each school to introduce himself to the head of Mathematics department and Mathematics educators and explain the purpose of the research.

The researcher collected the term 1 mark lists of Grade 10 mathematics learners from the schools. The learners were grouped as top, average and weak based on their performance in Algebra in term 1, the term during which fractions are taught. Then 15 learners each were randomly selected from the groups.

On day two, the researcher familiarised himself with the learners and also explained to them the purpose and how the study would be conducted. Day three was used to administer the pre-questionnaire test out of 25 marks by the researcher. The pre-questionnaire test consisted of sections A and B as explained earlier on.

The pre-questionnaire test was written in a classroom which was offered to the researcher by the principals of the schools to carry out the study. Day four was used to teach addition of numerical fractions with same denominators. Day five was used to teach addition of numerical fractions with different denominators. Day six was used to teach subtraction of numerical fractions with same denominators, and day seven was used to teach subtraction of numerical fractions with different denominators. The numerical fractions were taught to revise the learners' relevant previous knowledge. The researcher wanted the learners' to see how the understanding of the numerical fractions could be applied in the algebraic fractions.

On day eight, the researcher taught addition of fractions with same algebraic denominators, and day nine was used to teach addition of fractions with different algebraic denominators. On day ten, subtraction of fractions with same algebraic denominators was taught. Day eleven was used to teach subtraction of fractions with different algebraic denominators. Day twelve and day thirteen were used to teach addition and subtraction of algebraic fractions combined.

The researcher encouraged the learners to meet him on day fourteen for revision. Finally, day fifteen was used to administer the post-questionnaire test on both addition and subtraction of algebraic fractions.

The post-questionnaire test was also out of 25 marks. The response rate was 98.5%. Since the researcher administered the pre- and post-questionnaires himself, a very good rapport was established between the learners and the researcher and also between fellow educators in the selected schools. Furthermore, it gave the researcher a chance to judge the seriousness with which the respondents took the whole exercise.

3.11 IMPLEMENTATION OF THE CSA APPROACH AND DRILL-PRACTICE STRATEGIES

In this section, the researcher outlines how the CSA approach and drill-practice instructional strategies were used to analyse the effectiveness on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. The implementation of the approaches took the form of going to a classroom with the suitable teaching and learning materials (TLM). Some of the prepared TLMs were strips of cardboard, a fraction board and Cuisenaire rods.

The lessons were structured as follows: Activity-based teaching for the learners to experience actions of dividing concrete materials into equal parts to represent algebraic fractions, and the use of words and symbols to represent actions and objects. As mention earlier, the following two strategies were adopted as suitable by the researcher to analyse their effectiveness on Grade 10 learners' ability to solve algebraic fractions for the current study, namely, a concrete/semi-concrete/ abstract approach and drill-practice.

3.11.1 Concrete/Semi-concrete/Abstract approach

This section explains how the researcher used CSA approach to help Grade 10 learners' understanding of addition and subtraction of algebraic fractions. To begin with, the researcher selected appropriate concrete materials such as Cuisenaire rods. The learners were encouraged to familiarised themselves with the materials based on the lengths of each rod. The learners were made to compare one orange rod with five red rods. They realised that one orange rod was the same as the five red rods putting together. They were also made to compare one yellow rod with five white rods. The learners recognised that one yellow rod was equal to the five white rods putting together.

When the learners mastered the concrete level of performance, the researcher introduced appropriate drawing procedures. The learners were made to draw simple representations of the Cuisenaire rods they previously used. Finally, when the learners mastered the semi-concrete or representative level, they were assisted to use symbols to model problems.

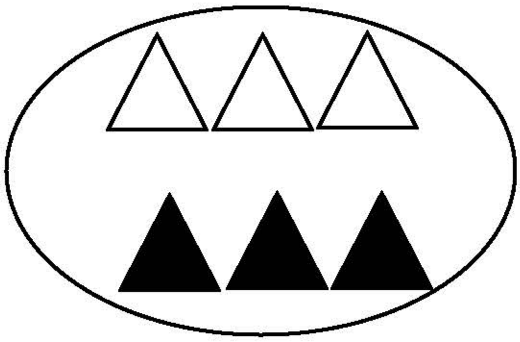
The activities embarked upon with the learners to address the research questions are discussed below: Learners were made to understand that $\frac{x}{2}, \frac{2x}{3}$, "a half of x" and "two thirds of x" are symbols and words representing the concept of particular algebraic fractions. The researcher explained through activities that "a half" is obtained when an object or item is divided into two equal parts.

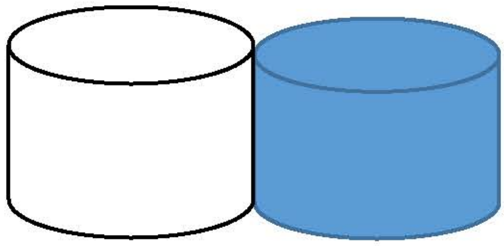
Similarly, "a third" and "a quarter" can be attained by dividing an object into three equal parts and four equal parts respectively. Also, "two thirds" can be achieved by dividing into three equal parts and taking two out of the three equal parts. The researcher explained to the learners that "a half" depends upon what we started with as a whole.

The "whole" is not just for a single object but it can be a set of objects. For example, in figure 3.1 all the shaded triangles below indicate a half of the set of triangles and in figure 3.2, the shaded cylinder represents a half of the set of cylinders.

Figure 3.1 and 3.2: Representation of a half

Figure 3.1 **Figure 3.2**





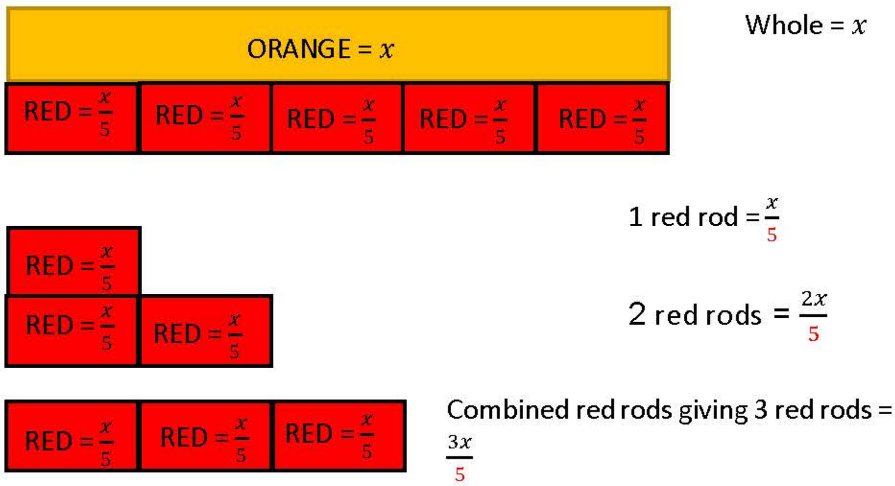
Source: Created by the researcher

The researcher then through activities introduced the learners to using symbols to represent these actions and pointed out that these symbols can be treated as objects. The learners were made to understand the meaning of denominator and numerator. For instance, $\frac{4x}{5}$; the denominator means the "whole" which is x has been divided into five equal parts and the numerator also means that four of the parts are under consideration. Therefore, $\frac{4x}{5}$ is four-fifths of the whole. Similarly, $\frac{2y}{7}$ means that the "whole" which is y has been divided into seven equal parts and two of the parts are under consideration.

3.11.1.1 Addition of algebraic fractions

For example, $\frac{x}{5}+\frac{2x}{5}$ (Same denominators), the learners were asked to take orange rod as a whole and red rod which indicates "one-fifth" of the orange rod which represents $\frac{x}{5}$. Then, they again took two red rods which indicate "two-fifth" which denotes $\frac{2x}{5}$ of the orange rod. Learners were told to join the red rods together giving three red rods, which is "three-fifth" of the orange. The answer then becomes $\frac{3x}{5}$. At this point the learners saw that when the denominators of the fractions are the same, one can maintain the denominator and add only the numerators. Afterwards, more examples exercises were given to the learners to do.

3.11.1.2 *Presentation addition of fractions (Same denominator)*



$$\therefore \frac{x}{5} + \frac{2x}{5} = \frac{3x}{5}$$

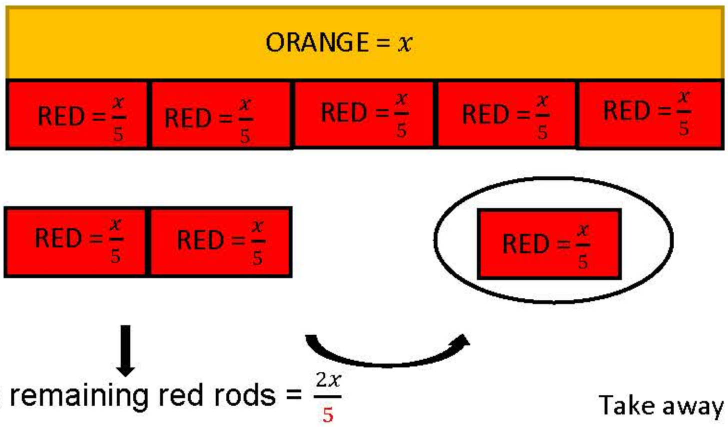
Materials used: Cuisenaire rods

3.11.1.3 *Subtraction of algebraic fractions*

In an example to deal with subtraction, $\frac{3x}{5} - \frac{x}{5}$ (same denominators), the learners selected the orange rod as a whole, and three red rods which indicate three-fifths of the orange rod. They were asked to take away one red rod which indicates one-fifth of the orange rod from the three-fifths selected earlier on. They realised that the remaining red rods were “two-fifths”. Similarly, the learners were made to understand that when the denominators are the same, one only needs to subtract the right-hand side numerator from the left-hand side one. They were also made to understand that whether the right-hand side number is bigger or smaller, since the negative sign affects it, one needs to subtract it from the other.

3.11.1.4 Presentation on subtraction of fractions

(Same denominator)



$$\therefore \frac{3x}{5} - \frac{x}{5} = \frac{2x}{5}$$

Materials used: Cuisenaire rods

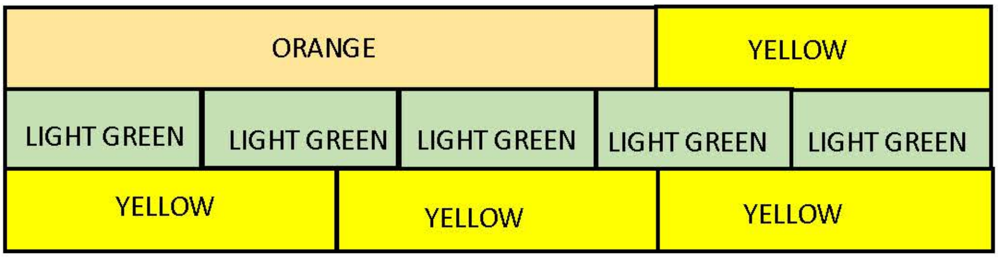
3.11.1.5 Addition of algebraic fractions (different denominators)

The example, $\frac{y}{3} + \frac{y}{5}$ (Different denominators) was used; the researcher discussed with the learners the addition and subtraction of different denominators. Two rods of which one can be divided into five equal parts and the other into three equal parts were selected. Light green and yellow were chosen and divided into five and three equal parts respectively.

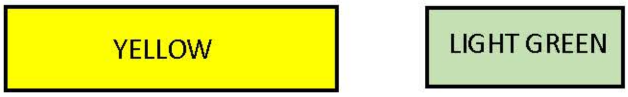
The following fractions $\frac{y}{3}$ and $\frac{y}{5}$ were formed with the rods. The researcher guided the learners to change the yellow rod for five whites representing $\frac{5y}{15}$

and the light green rod for three whites denoting $\frac{3y}{15}$. Then, the researchers asked the learners to join together the whites to get eight whites representing $\frac{8y}{15}$. The answer then becomes $\frac{8y}{15}$.

3.11.1.6 Presentation on addition of fractions (different denominators)



Take 1 yellow and 1 light green

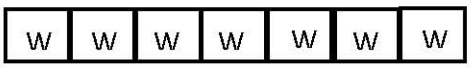


$\frac{y}{3} + \frac{y}{5} = ?$

Change yellow for 5 whites and light green for 3 whites

$\frac{5y}{15}$ $\frac{3y}{15}$

Join the whites together



$\frac{8y}{15}$

$\therefore \frac{y}{3} + \frac{y}{5} = \frac{5y+3y}{15} = \frac{8y}{15}$

The learners realised that addition and subtraction of algebraic fractions with different denominators requires finding the least common multiple (LCM) of the denominators.

3.11.2 Drill-practice

This section describes how the researcher used the drill-practice strategy to assist the learners in understanding addition and subtraction of algebraic fractions. To start with, the learners were asked to list multiples of numbers such as 3; 6; 7; 11; 13 etc. Thereafter, the numbers were given in pairs to determine the common multiples. They were made to understand that the smallest number in the common multiples was the LCM.

The following fractions were given to the learners to determine the LCM. $\frac{2}{5}+\frac{1}{5}$; $\frac{7}{9}+\frac{4}{3}$ and $\frac{5}{6}+\frac{1}{4}$. The majority of the learners were able to present the correct LCM as 5; 9 and 12 respectively. The learners were made to discover that when the denominators are the same, the LCM is the denominator.

Also, it was discovered that when the bigger denominator is a multiple of the smaller denominator, the bigger number of the denominator is the LCM. Finally, the learners found that when the denominators are not the same or the bigger denominator is not a multiple of the smaller denominator, the product of the denominators is the LCM.

The researcher drilled and practised with the learners the multiples of algebraic expressions. For instance, the multiples of x are x; x^2; x^3; x^4; ... and the multiples of xy are xy; x^2y^2; x^3y^3; x^4y^4; ... The learners were made to realise that the same approach as explained earlier with the numerals is applicable to the algebraic expressions. Thereafter, the researcher discussed with the learners the relationship between numerator and denominator. The learners were made to recognise that denominator is the number of equal sections a whole is divided into and the numerator is the section(s) under consideration. For example, if a whole represents x and it is divided into five equal parts, one-fifth will be equal to $\frac{x}{5}$.

The researcher drilled and practised with learners the following fractions with algebraic numerators and denominators.

(a) $\frac{x}{5}+\frac{2x}{5}=\frac{x+2x}{5}=\frac{3x}{5}$

(b) $\frac{5y}{9}-\frac{2y}{9}=\frac{5y-2y}{9}=\frac{3y}{9}$

(c) $\frac{8w}{7}+\frac{4w}{7}=\frac{8w+4w}{7}=\frac{12w}{7}$

(d) $\frac{5}{x}+\frac{3}{x}=\frac{5+3}{x}=\frac{8}{x}$

(e) $\frac{5x}{7}-\frac{3}{5x}+\frac{x}{2}=\frac{10x(5x)-14(3)+35x(x)}{70x}=\frac{50x^2-42+35x^2}{70x}=\frac{85x^2-42}{70x}$

The researcher encouraged the learners to use the questions below for revision.

(f) $\frac{7}{x^2}-\frac{9}{x^2}=\frac{7-9}{x^2}=\frac{-2}{x^2}$

(g) $\frac{x}{4}+\frac{2x}{5}=\frac{5(x)+4(2x)}{20}=\frac{5x+8x}{20}=\frac{13x}{20}$

(h) $\frac{3x}{8}-\frac{x}{5}=\frac{5(3x)-8(x)}{40}=\frac{15x-8x}{40}=\frac{7x}{40}$

(i) $\frac{7}{x}+\frac{11}{2x}=\frac{2(7)+1(11)}{2x}=\frac{14+11}{2x}=\frac{25}{2x}$

(j) $\frac{13}{2x}-\frac{2}{5x}=\frac{5(13)-2(2)}{10x}=\frac{65-4}{10x}=\frac{61}{10x}$

3.12 SUMMARY

This chapter explained the research design and methodology used to answer the research questions. The study included 135 learners from one Education District in the Eastern Cape Province in South Africa. Purposive sampling was used to select three schools and stratified random sampling was also employed to select 45 respondents from each school.

A pilot study was conducted to ensure reliability of pre-questionnaire-post-questionnaire tests which were used for the data collection. Two strategies namely, concrete/semi-concrete/abstract approach, and drill-practice were adopted during the intervention as the teaching strategies. How the intervention was carried out was also explained in this section.

The next chapter deals with the data analysis and presentation of the results obtained from both the pre-test and the post-test.

CHAPTER 4: DATA ANALYSIS AND PRESENTATION

4.1 INTRODUCTION

This chapter focuses on the analysis and presentation of the data. It presents the responses of respondents regarding Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions. Based on the background of the literature review, the responses of the respondents, as they are reflected in answers from the pre-questionnaire and post-questionnaire that directed the study, are analysed, summarised, organised and presented.

4.1.1 Procedure and Data Analysis and Presentation

Questions 1, 2, 3, 4 and 5 of both pre-questionnaire and post-questionnaire were set for Grade 10 Mathematics learners. Therefore, data analysis was done according to the responses of the Grade 10 mathematics learners. Responses were analysed statistically and results were presented either as tables, bar graphs or pie charts.

It is from this data analysis that the Grade 10 learners' challenges with regard to the relationship between numerator and denominator and their wrong conceptions and inability to find LCM to simplify addition and subtraction of algebraic fractions are presented.

4.2 RESPONSE RATE AND RESPONDENTS' PROFILE

In this section the response rate attained for the study is reported followed by data regarding the respondents' profiles.

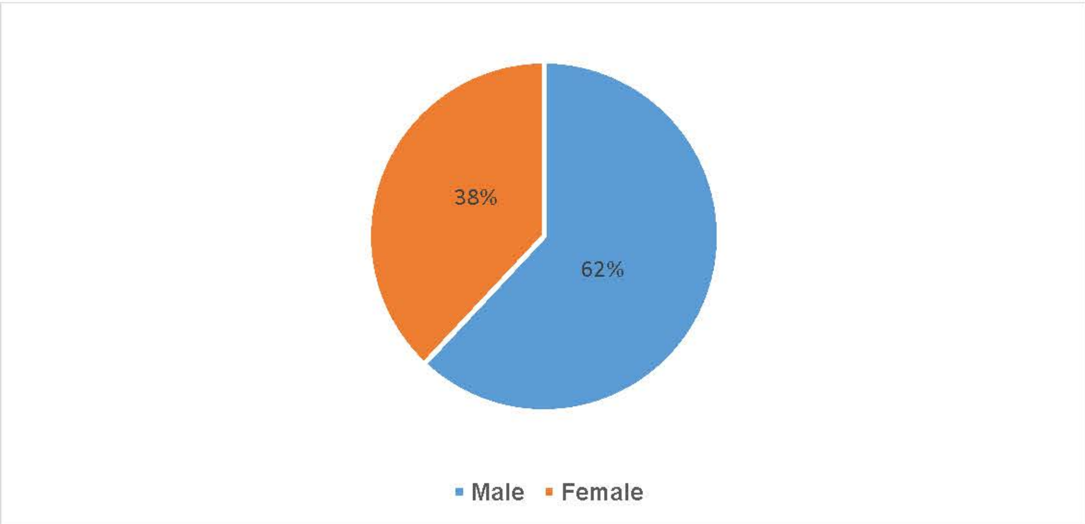
4.2.1 Response rate

As mentioned in the previous chapter, a stratified random sampling method was used to select 135 learners for the study. The analysis of the data is based on 133 leaners who answered all the research questions. This represents the response rate of 98.5%.

4.2.2 Profile of respondents'

The learners selected for the study were requested to indicate their gender and age. The results are shown in Figure 4.1 and 4.2 respectively.

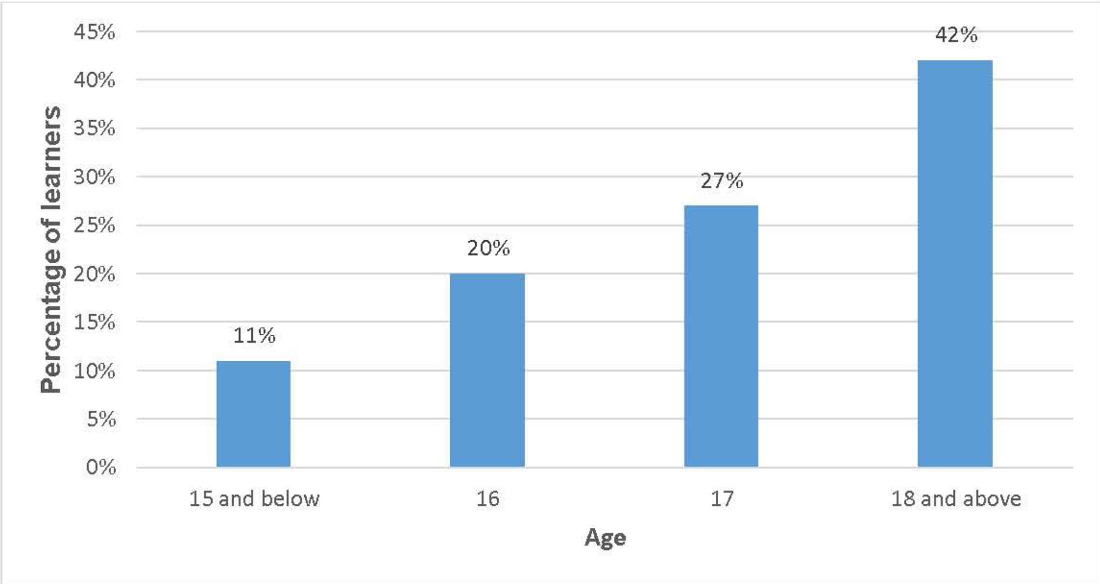
Figure 4.1 Respondents' Gender



Source: Researcher's Database

Figure 4.1 depicts that 62% of the respondents were males while 38% were females. This shows that male respondents were more than their female counterparts' for this study.

Figure 4.2 Respondents' Age



Source: Researcher's Database

Figure 4.2 indicates that 31% (11%+20%) of respondents were 16 years and below. The majority of the respondents comprising 42% were 18 years and older. It can be observed that few learners were fifteen years and younger. The learners who were fifteen years and younger were only 11%. Practically, it is observed that most of the learners exceeded the age of 16 years. This could probably mean that majority of the learners were not supposed to be in Grade 10 class at the age of more than 16 years.

The questions on both gender and age were included in order to clarify gender and age disparities of the sample as they do not form part of the research questions but are vital in the sample description.

4.3 PRE-QUESTIONNAIRE AND POST-QUESTIONNAIRE RESULTS

As emphasised in Chapter 3 of this study, five questions were formulated to analyse learners' challenges regarding the relationship between numerator and denominator and their wrong conceptions and inability to determine LCM to simplify algebraic fractions. The aim of this section was to present the findings from the analysis. This section starts off by analysing the responses obtained from question 1: *Addition of two fractions with algebraic numerators*. This was followed by the analysis obtained from question 2: *Addition of two fractions with algebraic denominators*. This was also followed by the analysis of question 3: *Subtraction of two fractions with algebraic numerators*. Then, question 4: *Subtraction of two fractions with algebraic denominators*. Finally question 5: Addition and subtraction of algebraic fractions combined.

Referring to the exercise as a 'challenge' implies that learners encounter problems when they perceive numerator and denominator of fractions as two separate entities. It also indicates learners' wrong conceptions and their inability to find the LCM to simplify algebraic fractions.

4.3.1 Analysis of pre-questionnaire results

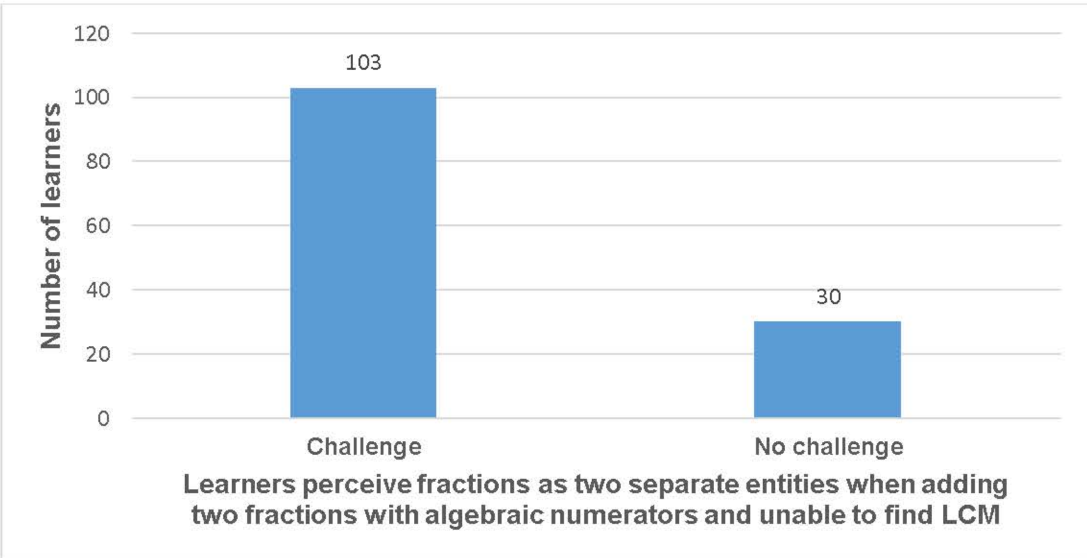
This section presents the analysis of the pre-questionnaire results on learners' perceiving numerator and denominator of algebraic fractions as separate entities, and their inability to find the LCM to simplify algebraic fractions. This analysis includes all the selected schools for the current study. The responses to the 5 questions on addition and subtraction of algebraic fractions are analysed and presented based on the research questions.

4.3.1.1 Question 1: Addition of two fractions with algebraic numerators

Table 4.1: Analysis Result of pre-questionnaire Question 1

Addition of algebraic fractions	Number of learners	Percentage
Challenge	103	77
No Challenge	30	23
Total	133	100

Figure 4.3: Analysis Result of pre-questionnaire Question 1



Source: Researcher's Database

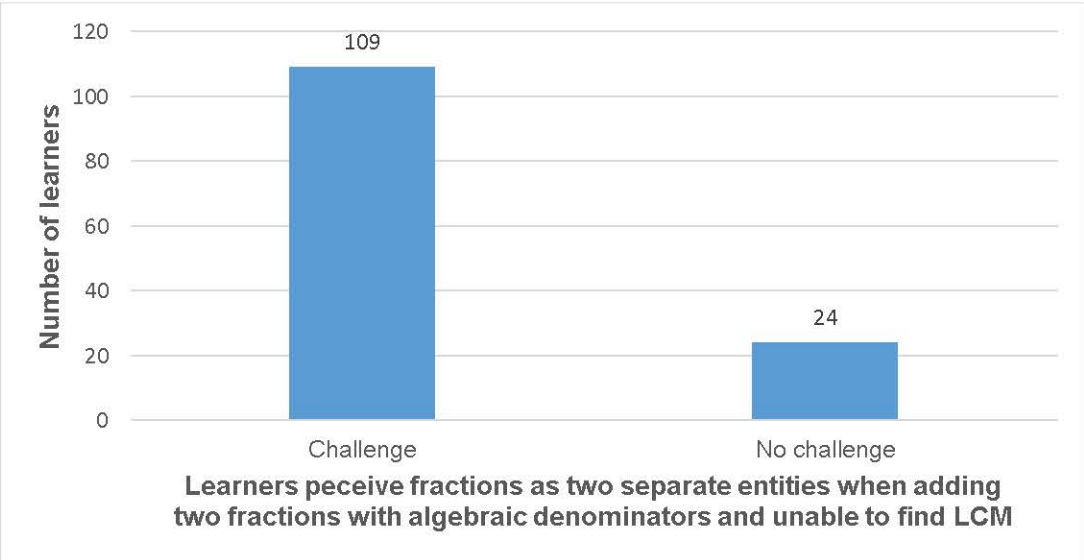
Table 4.1 indicates that 77% of the 133 respondents perceived numerator and denominator as separate entities when they were asked to simplify addition of two fractions with *algebraic numerators*. They were unable to find the LCM to simplify the algebraic fractions. Figure 4.3 also displays that greater number of the learners perceived numerator and denominator as separate entities when they were asked to simplify the addition of two fractions with algebraic numerators. They were not capable to find the LCM to simplify the algebraic fractions.

4.3.1.2 Question 2: Addition of two fractions with algebraic denominators

Table 4.2 Analysis Result of pre-questionnaire Question 2

Addition of algebraic fractions	Number of learners	Percentage
Challenge	109	82
No Challenge	24	18
Total	133	100

Figure 4.4: Analysis Result of pre-questionnaire Question 2



Source: Researcher's Database

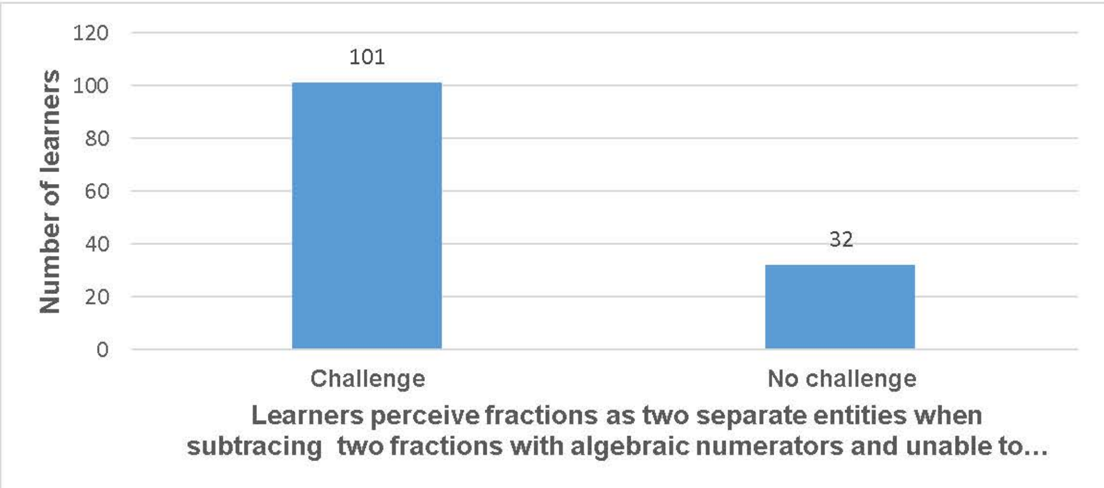
Table 4.2 indicates that 82% of the 133 respondents perceived numerator and denominator as separate entities when they were asked to simplify the addition of two fractions with *algebraic denominators*. They were not able to find the LCM to simplify the algebraic fractions. Figure 4.4 also shows that the majority of the learners perceived numerator and denominator as separate entities when they were asked to simplify the addition of two fractions with *algebraic denominators.*

4.3.1.3 Question 3: Subtraction of two fractions with algebraic numerators

Table 4.3: Analysis Result of pre-questionnaire Question 3

	Subtraction of algebraic fractions	Number of learners	Percentage
	Challenge	101	76
	No Challenge	32	24
	Total	133	100

Figure 4.5: Analysis Result of pre-questionnaire Question 3



Source: Researcher’s Database

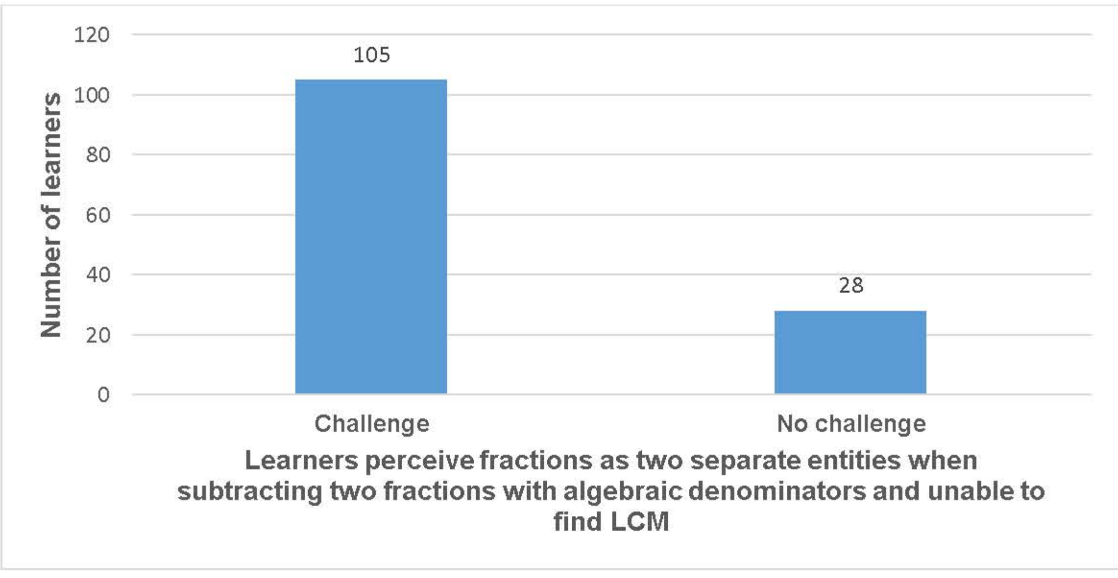
Table 4.3 illustrates that 76% of the 133 respondents' perceived numerator and denominator as two separate entities when they were asked to simplify subtraction of two fractions with *algebraic numerators*. Figure 4.5 also depicts that the majority of the learners perceived numerators and denominator as two separate entities when they were asked to simplify subtraction of two fractions with *algebraic numerators.* They were unable to find the LCM to simplify the algebraic fractions.

4.3.1.4 Question 4: Subtraction of two fractions with algebraic denominators

Table 4.4: Analysis Result of pre-questionnaire Question 4

	Subtraction of algebraic fractions	Number of learners	Percentage
	Challenge	105	78
	No Challenge	28	22
	Total	133	100

Figure 4.6: Analysis Result of pre-questionnaire Question 4



Source: Researcher's Database

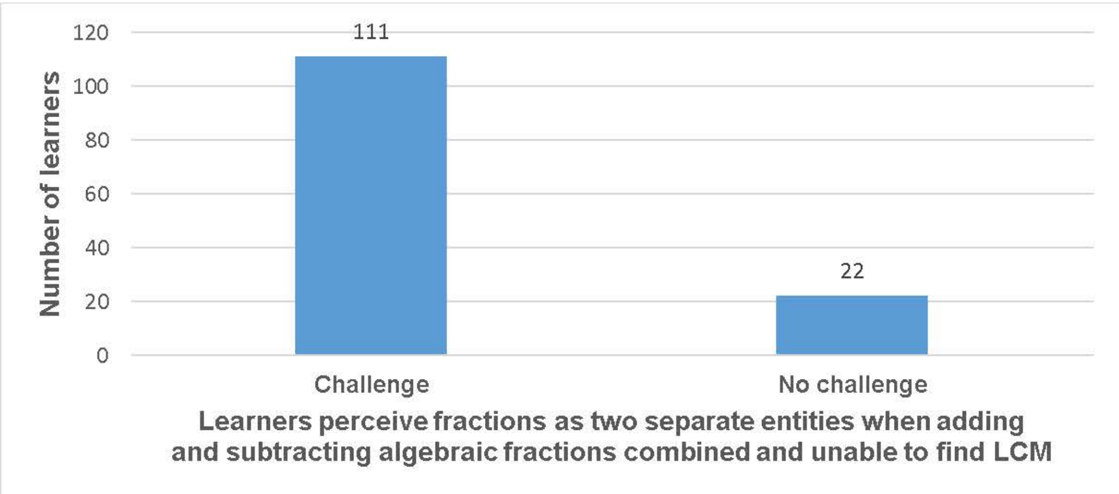
Table 4.4 shows that 78% of the 133 respondents perceived numerator and denominator as two separate entities when they were asked to simplify subtraction of two fractions with *algebraic denominators*. They were not able to find the LCM to simplify the algebraic fractions. Figure 4.6 also indicates that majority of the learners' perceived numerator and denominator as separate entities when they were asked to simplify subtraction of two fractions with *algebraic denominators*.

4.3.1.5 Question 5: Addition and subtraction of algebraic fractions combined

Table 4.5: Analysis Result of pre-questionnaire Question 5

Addition & Subtraction of algebraic fractions	Number of learners	Percentage
Challenge	111	83
No Challenge	22	17
Total	133	100

Figure 4.7: Analysis Result of pre-questionnaire Question 5



Source: Researcher's Database

Table 4.5 indicates that 83% of the 133 respondents perceived numerator and denominator as two separate entities when they were asked to simplify algebraic fractions with addition and subtraction combined. They could not find the LCM to simplify the algebraic fractions.

Figure 4.7 also displays that the majority of the learners had the same perception that numerator and denominator were two separate entities. They could not determine the LCM to simplify the algebraic fractions.

4.3.2 Analysis of post-questionnaire results

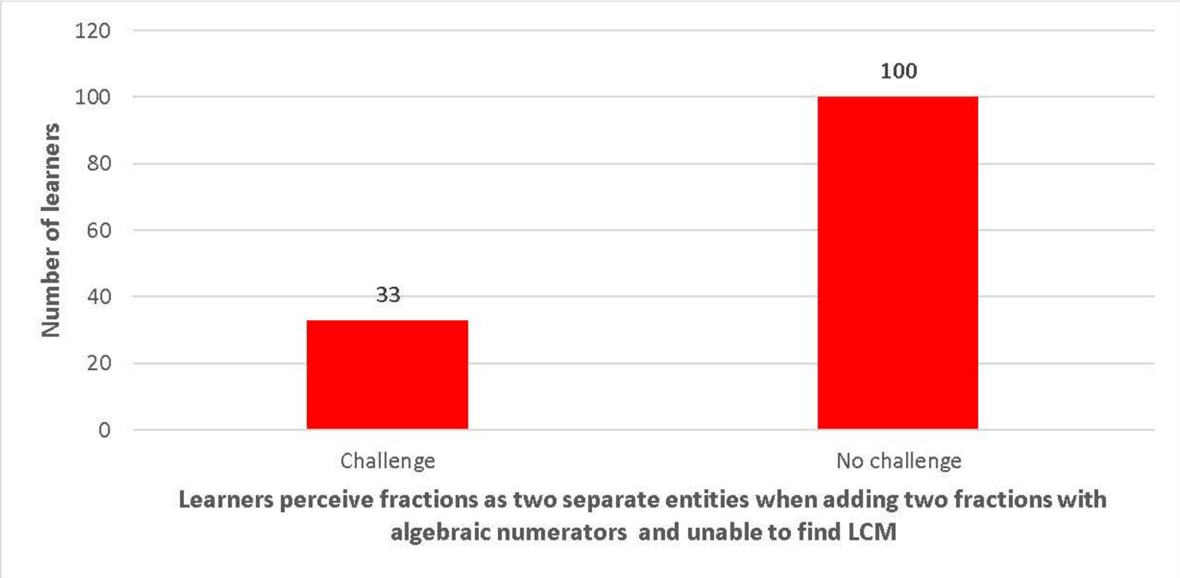
This section presents the analysis of the results obtained from the post-questionnaire conducted in the three schools. The analyses are presented based on the research questions in line with the literature review of this study. The main focus was on the relationship between numerator and denominator and the learners' inability to find the LCM to simplify algebraic fractions.

4.3.2.1 Question 1: Addition of two fractions with algebraic numerators

Table 4.6 Analysis Result of post-questionnaire Question 1

Addition of algebraic fractions	Number of learners	Percentage
Challenge	33	25
No Challenge	100	75
Total	133	100

Figure 4.8 Analysis Result of post-questionnaire Question 1



Source: Researcher's Database

Table 4.6 presents the results of question 1. It can be observed that 25% of the 133 respondents perceived numerator and denominator as two separate entities when they were asked to simplify addition of two fractions with *algebraic numerators.*

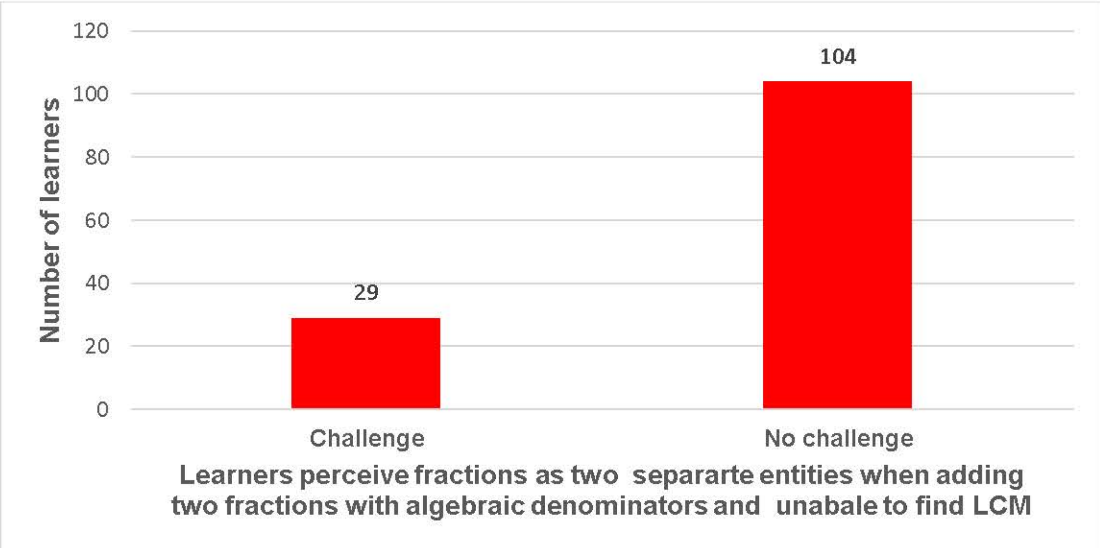
They could not determine the LCM to solve the algebraic fractions. Figure 4.8 also shows that fewer learners perceived numerator and denominator as two separate entities and were not able to simplify the algebraic fractions.

4.3.2.2 *Question 2: Addition of two fractions with algebraic denominators*

Table 4.7 Analysis Result of post-questionnaire Question 2

	Addition of algebraic fractions	Number of learners	Percentage
	Challenge	29	22
	No Challenge	104	78
	Total	133	100

Figure 4.9 Analysis Result of post-questionnaire Question 2



Source: Researcher's Database

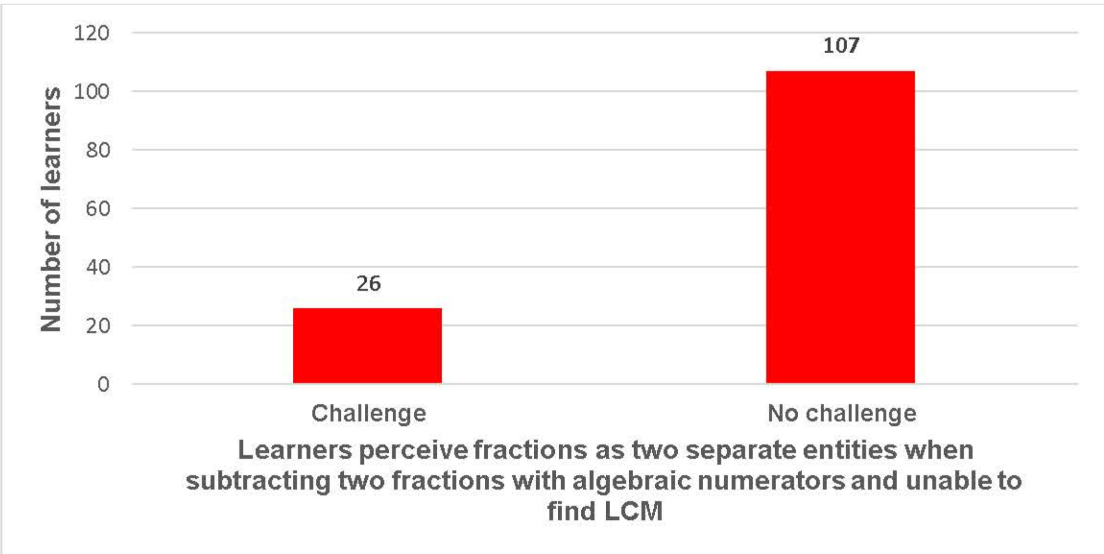
Table 4.7 indicates that 22% of the 133 respondents perceived numerator and denominator as two separate entities when they were asked to simplify the addition of two fractions with *algebraic denominators*. Those learners were not able to find the LCM to simplify the algebraic fractions. Figure 4.9 also depicts that fewer respondents perceive numerator and denominator as two separate entities and were not able to simplify the algebraic fractions.

4.3.2.3 Question 3: Subtraction of two fractions with algebraic numerators

Table 4.8 Analysis Result of post-questionnaire Question 3

	Subtraction of algebraic fractions	Number of learners	Percentage
	Challenge	26	20
	No Challenge	107	80
	Total	133	100

Figure 4.10 Analysis Result of post-questionnaire Question 3



Source: Researcher's Database

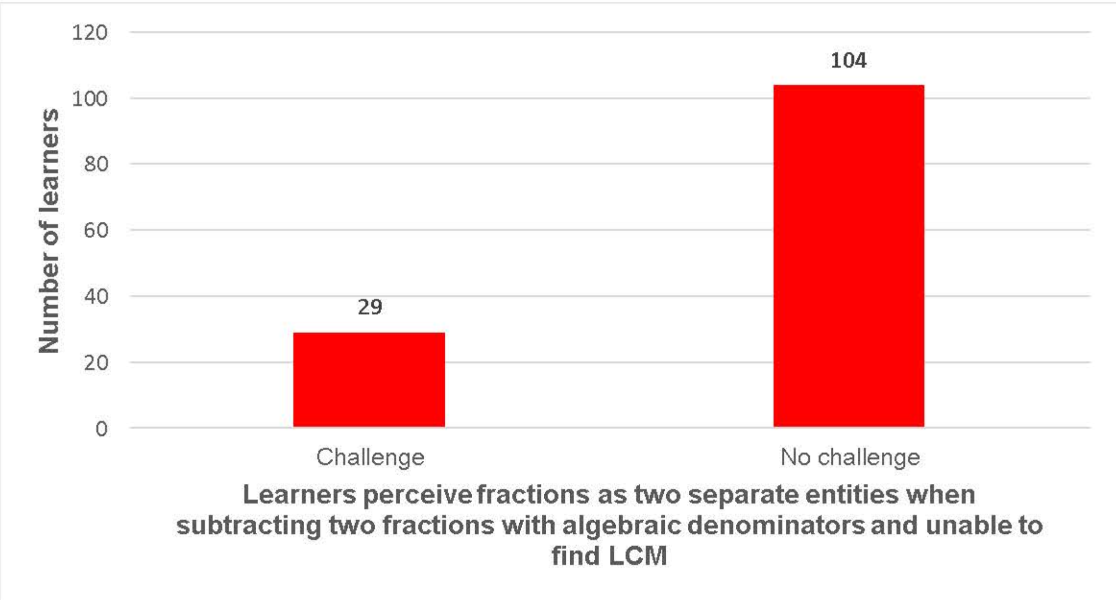
Table 4.8 indicates that 20% of the 133 respondents perceived numerator and denominator as two separate entities when they were asked to simplify subtraction of two fractions with *algebraic numerators*. Those learners were not able to find the LCM to simplify algebraic fractions. Figure 4.10 also indicates that fewer respondents perceive numerator and denominator as separate entities and were unable to simplify the algebraic fractions.

4.3.2.4 Question 4: Subtraction of two fractions with algebraic denominators

Table 4.9 Analysis Result of post-questionnaire Question 4

	Subtraction of algebraic fractions	Number of learners	Percentage
	Challenge	29	22
	No Challenge	104	78
	Total	133	100

Figure 4.11 Analysis Result of post-questionnaire Question 4



Source: Researcher's Database

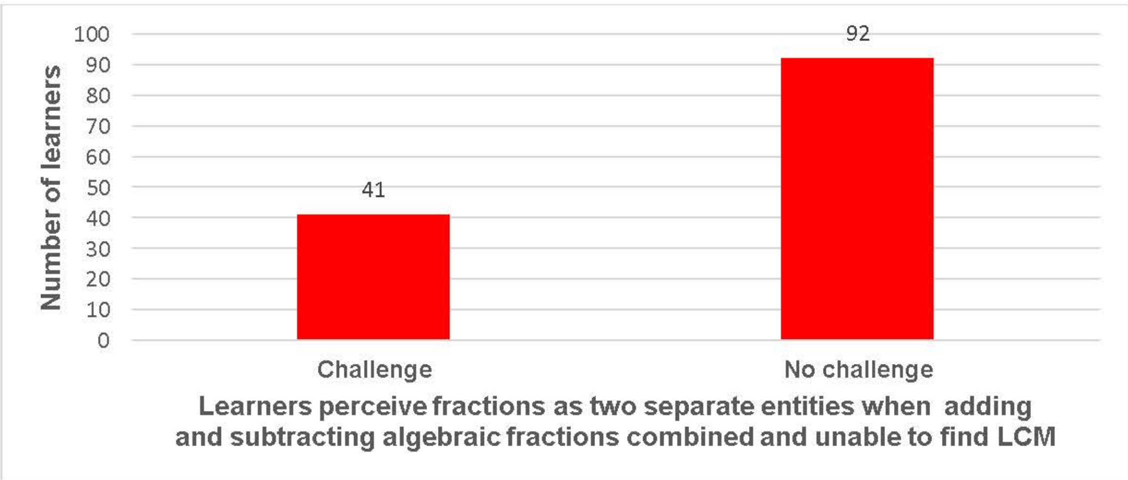
Table 4.9 shows that 22% of the 133 respondents perceived numerator and denominator as two separate entities when they were asked to simplify subtraction of two fractions with *algebraic denominators.* Those respondents were not able to determine the LCM to simplify the algebraic fraction. Figure 4.11 also indicates that fewer respondents perceive numerator and denominator as two separate entities and were not able to simplify the algebraic fractions.

4.3.2.5 Question 5: Addition and subtraction of algebraic fractions combined

Table 4.10 Analysis Result of post-questionnaire Question 5

	Addition & Subtraction of algebraic fractions	Number of learners	Percentage
	Challenge	41	31
	No Challenge	92	69
	Total	133	100

Figure 4.12 Analysis Result of post-questionnaire Question 5



Source: Researcher's Database

Table 4.10 indicates that 31% of 133 respondents perceived numerator and denominator as two separate entities when they were asked to simplify addition and subtraction of algebraic fractions combined. Those respondents were not able to solve the algebraic fractions. Figure 4.12 also shows that fewer of the 133 respondents perceive numerator and denominator as two separate entities and were unable to simplify addition and subtraction of algebraic fractions combined.

Table 4.11 Analysis Result of Mean, median and mode

Averages	Pre-Questionnaire	Post-Questionnaire
Mean	3.33	14.20
Median	2.00	13.00
Mode	1.00	8.00

Source: Researcher's Database

Table 4.11 presents the mean, median and mode calculated using the pre-questionnaire and post-questionnaire scores of the 133 respondents. It can be observed that the pre-questionnaire mean score is 3.33, the median is 2.00 and the modal score is 1.00. Also, the table indicates that the post-questionnaire mean score is 14.20, the median is 13.00 and that of the modal score is 8.00.

Table 4.12 Analysis Result of t-Test for Independent Means

p-value	0.139

Source: Researcher's Database

Table 4.12 represents a t-test for independent means on both pre-questionnaire and the post-questionnaire scores. It can be observed that the p-value is 0.139. The p-value indicates statistically significant at $\alpha = 0.05$.

4.4 SUMMARY

This chapter dealt with the data analysis and presentation. The gender and the age of the learners were presented in a pie chart and bar graph respectively. Both pre-questionnaire and post-questionnaire results were analysed and presented. The mean, median and modal scores of the respondents for the pre-questionnaire and post-questionnaire were also presented in Table 4.11.

Finally, Table 4.12 presented t-test for independent means for the analysis of the effectiveness of the CSA approach and drill-practice instructional approaches. The table indicated a p-value of 0.139 which is statistically significant at $\alpha = 0.05$.

The next Chapter presents discussions, conclusions and recommendations based on the data above.

CHAPTER 5: DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter is the final chapter of this study and it is dedicated to the discussions of the major findings and conclusions. In addition, recommendations arising from the findings are provided. The chapter addresses the research questions taking into consideration the literature review and the theoretical framework for this study.

5.2 DISCUSSIONS AND CONCLUSIONS

This section addresses the discussions and conclusions of the findings focussed on the research questions with regards to the literature review and the theoretical framework. Based on the theoretical framework, the following challenges were relevant for the study: relationship between numerator and denominator; and wrong conceptions and inability to find the LCM as mentioned in the literature.

This section is classified according to the questions and learners' responses to the tasks as addition of two fractions with *algebraic numerators* and *algebraic denominators*; subtraction of two fractions with *algebraic numerators* and *algebraic denominators*; and combination of addition and subtraction of three algebraic fractions.

5.2.1 Addition of two fractions with Algebraic numerators and denominators

The questions on addition of two fractions with algebraic numerators and algebraic denominators were used to ascertain learners' challenges in addition of algebraic fractions relevant to the current study. Table 4.1 indicated that 77% of the learners perceive numerator and denominator as two separate entities when they were asked to simplify fractions with algebraic numerators. In view of their perceptions about fractions as two separate entities they could not find the LCM to simplify addition of two fractions with algebraic numerators.

In Table 4.2, it was noticed that 82% of the learners perceive numerator and denominator as two separate entities and were not able to simplify addition of two fractions with algebraic denominators. It was established from the current study that learners do not know the relationship between the numerator and denominator of fractions with algebraic numerators and denominators. For this reason, they added numerator to numerator and denominator to denominator.

This result supports the studies conducted by Jigyel and Afamasaga-Fuata'I (2007) who reported that learners perceive fractions as two separate whole numbers and consequently apply whole number reasoning when working with them. The study also supports Siegler et al. (2013), who found that learners presumed that algorithms, procedures and properties of whole numbers are also properties of all other numbers.

In addition, the current study is in agreement with the study conducted by Huinker (2002) in the United States of America (USA), as cited in Petit et al. (2010: 146), who reported that learners who could translate between various fraction representations "are more likely to reason with fraction symbols as quantities and not as two whole numbers" when solving problems.

Another challenge identified was the learners' inability to find the LCM. They took any of the denominators as the LCM even if they were not. Hecht Vagi (2012) also relates the learners' inability to find the LCM of adding algebraic fractions to a lack of conceptual knowledge that governs this field. A study by Hallet et al. (2012) also found that learners frequently have disparities in procedural and conceptual knowledge of fractions and hence they find it very difficult to understand fractions.

In view of the challenges discovered based on the pre-questionnaire results, the researcher undertook an intervention using a concrete/semi-concrete/abstract approach, and drill-practice strategies. After the intervention, a post-questionnaire was administered to investigate the effectiveness of the strategies employed.

The following results were obtained and shown as challenges with regard to the learners' perceptions of algebraic fractions as two separate entities and their inability to find the LCM. In Tables 4.6 and 4.7 it can be observed that 25% and 22% of the respondents perceived algebraic fractions as two separate entities and were unable to find the LCM to solve algebraic fractions. Based on the information in Tables 4.6 and 4.7, it can be observed that there was an improvement in learners' ability to simplify addition of fractions with algebraic numerators and denominators.

This result supports the studies conducted by the American Institute for Research (2007) which revealed that CSA approach is an effective means for appropriate mathematics instruction which can significantly improve the overall performance of high school learners who struggle with mathematics. Also, this study supports the study conducted by Butler et al. (2003) who reported that CSA approach is an effective instructional strategy on content areas such as addition, subtraction, multiplication, division, fractions and algebra.

5.2.2 Subtraction of two fractions with algebraic numerators and denominators

Since the purpose of the current study was to analyse the effectiveness of the CSA approach and drill-practice instructional strategies on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions, it was advisable for the researcher to investigate the challenges in subtraction of algebraic fractions as well. The learners were given questions on subtraction of fractions with algebraic numerators and algebraic denominators.

It can be observed in Table 4.3 that 76% could not simplify subtraction of two fractions with algebraic numerators. It was discovered that the learners' subtracted the numerator from numerator and denominator from denominator. Also Table 4.4 shows the number of learners who had similar challenges in subtraction of two fractions with algebraic denominators were 78%. The researcher discovered that learners displayed such challenges because they decided to use a 'short-cut' which did not lead them to the correct answers.

This supports the study conducted by Watanabe (2012) which states that learners apply easy methods or 'short-cuts' as alternatives and end up making errors. He postulates that it is necessary to study these alternatives and correct them as they will become ingrained previous knowledge.

Fractions are rational numbers and do not hold place value as whole numbers do. The current study shows that the learners failed to apply the algorithm for addition and subtraction of fractions which relates to finding the LCM. Based on the discussions above, there is enough evidence for the researcher to conclude that when adding fractions with algebraic numerators and algebraic denominators, learners add the numerators and denominators separately. Also they are unable to determine the LCM for algebraic fractions. Hence this current study supports the study conducted by Jigyel and Afamasaga-Fuata'I

(2007) which emphasises that learners perceive fractions as separate whole numbers and consequently apply whole number reasoning when dealing with addition of fractions.

In order to see the effectiveness of the intervention embarked on to assist learners on subtraction of algebraic fractions, a post-questionnaire was conducted. The following results were displayed. In Tables 4.8 and 4.9 it can be noticed that 20% and 22% of the respondents perceived subtraction of two fractions with algebraic numerators and denominators as two separate entities. Based on the information in Tables 4.3 and 4.4, it can be concluded that learners' challenges with regard to their inability to subtract fractions with algebraic numerators and denominators had been reduced.

This study is in agreement with the study conducted by Wicken and McCarley (2008) who reported that drill-practice provide learners with clear strategy to cope with the task and guided in step-by-step manner through the steps of the task. Hence, a learner's attention is guided to the accurate execution of the task steps instead of having the learner divide attention between identifying a strategy and performing the task at the same time.

5.2.3 Addition and subtraction of algebraic fractions combined

In order to gather enough information, the researcher finally used a question on addition and subtraction of algebraic fractions combined to investigate the challenges regarding the relationship between numerator and denominator; and the wrong conceptions and inability to determine the LCM. It can be concluded from the pre-questionnaire results in Table 4.5 that 83% of the learners perceive algebraic fractions as two separate entities. This indicates that majority of the learners were unable to simplify addition and subtraction of algebraic fractions combined.

As pointed out by Siegler et al. (2013), learners use algorithms, procedures and properties of whole numbers when dealing with fractions. It was discovered by the researcher that learners need to understand the relationship between numerators and denominators. In addition, they need to be taught properly on how to find the LCM.

In Table 4.10, it can be observed that the post-questionnaire result of addition and subtraction combined was 30.8%. With reference to Table 4.5 which illustrates the analysis of the pre-questionnaire results, it can be observed that the learners' inclination to perceive numerator and denominator as separate entities had been reduced. This is due to the fact that the majority of learners were able to simplify addition and subtraction of algebraic fractions combined.

This result supports the study conducted by Suh et al. (2005) who reported that CSA approach is suitable for teaching and learning of fractions. They state that developing visual models for fractions is an important factor influencing the conceptual understanding of fraction computation.

5.2.4 Mean, median and mode

The information in Table 4.12 highlighted the mean, median and the mode (measures of central tendency) of pre-questionnaire and the post-questionnaire scores of 133 respondents for the current study. It was found that the measures of central tendency of the post-questionnaire scores were greater than that of the pre-questionnaire scores.

Therefore, the researcher is of the view that learners' ability to simplify algebraic fractions has been enhanced. Hence, this result supports the use of the CSA approach and drill-practice instructional strategies to improve Grade

10 learners’ ability to simplify algebraic fractions (Suh et al., 2005; Nieves, 2009; Mary et al., 2016)

5.2.5 t-Test

In view of the t-Test for independent means presented in Table 4.12, it can be observed that the p-value is 0.139. This value indicates that it is statistically significant to reject the null hypothesis at $\alpha = 0.05$. Therefore, the researcher can conclude that CSA approach and drill-practice have significantly enhanced Grade 10 learners’ ability to simplify addition and subtraction of algebraic fractions.

This result supports the studies conducted by Nieves (2009) who reported in her studies that CSA approach is basically an intervention approach that allows learners to gain understanding of concepts and fluency in computation by gradually moving through the three phases. This result also supports the study conducted by Suh et al., (2005) who emphasised that CSA approach is suitable for teaching and learning of fractions. They also highlighted that developing visual models for fractions is a noteworthy influential factor in building understanding for fraction computation.

In addition, this result supports the studies conducted by Mary et al. (2016) who reported that drill-practice is an instructional method characterised by systematic repetition of concepts. It represents concise teaching and perfecting a skill or procedures. Also, the result of this study supports the studies conducted by Anderson, (2008) and Nieves, (2009) who reported that drill-practice promotes acquisition of knowledge or skill through repetitive practice.

5.3 RECOMMENDATIONS

Evidence from the current study indicates that learners perceive fractions as separate whole numbers and therefore add or subtract numerators and denominators separately. In addition, learners' inability to determine LCM consequently results in "a shortcut" way of dealing with simplification of algebraic fractions.

Consequently, in this section broad areas of recommendations are provided, together with possible strategies that may be utilised.

- The first instructional strategy recommended is the concrete/semi-concrete/abstract (CSA) approach. CSA approach is basically an intervention approach that allows learners to gain understanding of concepts and fluency in computation (Nieves, 2009). Visual models are often employed in this approach for teaching fractions (Suh et al., 2005, Siegler et al., 2010). Based on the impact of this strategy during the intervention, a CSA approach is recommended to enhance the teaching and learning of algebraic fractions.
- The second strategy recommended to effectively teach algebraic fractions is drilling and practice. This is an instructional method characterised by systematic repetition of concepts, examples, and practice problems. It represents concise teaching and perfecting a skill or procedures (Mary, Jill & Sara, 2016). As an instructional strategy, it promotes acquisition of knowledge or skill through repetitive practice. In addition, this strategy increases the likelihood that learners will permanently remember new information (Moss & Case, 2001; Anderson, 2008). Consistent with previous studies (e.g. Anderson, 2008; Nieves, 2009) and the positive impact of this strategy, educators are encouraged to drill learners with different questions ranging from simple to complex.
- Learners need to be taught the concept of addition and subtraction of algebraic fractions from simple to complex operations. That is, teaching

needs to start with simple objects and shapes to explain the concepts, and to let learners do simple addition and subtraction of common fractions with same denominator; and move to common fractions with different denominators.

- Educators need to have the relevant content knowledge and relevant pedagogical expertise for teaching content to learners, as it plays a vital role in the teaching process. The skills to impact knowledge can assist to reduce learners' challenges in addition and subtraction of algebraic fractions and can also assist in learners' understanding.

5.4 LIMITATIONS OF THE STUDY AND AREAS FOR FUTURE RESEARCH

This study adds an important contribution to literature and knowledge but it also has limitations which can serve as future research foci. First, this study focused on only one education district in the Eastern Cape Province (EC) of South Africa and was confined to three schools within the district. The researcher acknowledges that there are many districts in EC. Therefore, this study should be generalised with caution. Future researchers should investigate other districts within the EC in a quest to generalise the findings.

The second limitation relates to the methodology adopted for the study. The quantitative approach was followed. Future researchers can use a qualitative or a mixed method approach to understand learners' opinions regarding the whole concept of algebraic fractions and the experience in the classroom. The third limitation relates to the number of challenges investigated in this study. In the current study only two challenges namely, relationship between the numerator and denominator and wrong conception and inability to find the LCM were investigated.

The researcher is convinced that other factors could serve as a challenge in teaching and learning of algebraic fractions. Future researchers' can investigate other challenges such as vague representations of fractions and numeric and symbolic procedures. The final limitation relates to the various strategies identified. Two strategies were adopted for the current study namely, a concrete/semi-concrete/abstract (CSA) approach, and drill-practice. Future researchers can investigate other matters such as importance of games and competition on attitudes and learning, and the essence of error in acquiring knowledge.

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APPENDICES

Appendix A: Pre-Questionnaire

MARKS: 25

DURATION: 30 MINUTES

INSTRUCTIONS

- ***Please answer all the questions***
- ***In Section A, indicate with a tick (X) where appropriate***
- ***In section B, all working must be shown in the spaces provided***

SECTION A: BIOGRAPHICAL DATA

Gender

Male	1
Female	2

Age

15 years and below	1
16 years	2
17 years	3
18 years and above	4

SECTION B: ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS

Simplify the following:

1. $\frac{3x}{5} + \frac{2x}{3}$

(5)

2. $\frac{2}{5x} + \frac{3}{x}$

(5)

3. $\frac{3x}{7} - \frac{x}{2}$

(5)

4. $\frac{7}{2x} - \frac{2}{5x}$

(5)

5. $\frac{9x}{2} + \frac{2}{5x} - \frac{5x}{3}$

(5)

/25/

THANK YOU

Appendix B: Post-Questionnaire

MARKS: 25

DURATION: 30 MINUTES

INSTRUCTIONS

- ***Please answer all the questions***
- ***In Section A, indicate with a tick (X) where appropriate***
- ***In section B, all working must be shown in the spaces provided***

SECTION A: BIOGRAPHICAL DATA

Gender

Male	1
Female	2

Age

15 years and below	1
16 years	2
17 years	3
18 years and above	4

SECTION B: ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS

Simplify the following:

1). $\frac{3x}{11}+\frac{5x}{3}$

(5)

2). $\frac{2}{7x}+\frac{3}{2x}$

(5)

3). $\frac{5x}{7}-\frac{x}{5}$

(5)

4). $\frac{7}{15x} - \frac{2}{5x}$

(5)

5). $\frac{9x}{2} + \frac{2}{5x} - \frac{5x}{3}$

(5)

THANK YOU **/25/**

APPENDIX C: LETTER TO THE PRINCIPALS

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN YOUR SCHOOL IN FULFILMENT FOR A MASTERS DEGREE IN EDUCATION WITH UNIVERSITY OF FORT HARE

I am Bernard Prince Awuah, a mathematics educator in one of the districts in the Eastern Cape Province and a student pursuing Master's degree in Education (MEd) at University of Fort Hare.

I am currently working on an M. Ed dissertation which is entitled "Exploring strategies to improve the understanding of algebraic fractions in grade 10. This study is under the supervision of Professor K. J. Mammen in Education Department in the University. The study will include 45 grade 10 Mathematics learners. The learners will be writing pre-test. After that, they will be taught and write post-test as means of gathering information.

Participation will be voluntary. Anonymity and confidentiality will be ensured to all participants. Furthermore, the research process will not disrupt any lesson or any schedule activities in the school.

I am suitably qualified to conduct this study in accordance with the ethical principles.

Enclosed, please find copies of the ethical clearance certificate, a consent letter from the Department of Education and sample of the pre-test and the post-test questions.

The documents are submitted for your perusal and approval to undertake the research study. A written approval would be appreciated.

Thank you.

Yours faithfully,

…………………………………….

(Bernard Prince Awuah)

APPENDIX D: CONSENT LETTER FROM DEPARTMENT OF EDUCATION

Province of the
EASTERN CAPE
EDUCATION

College of Education • R61 Flagstaff Main Road • Private Bag x1010 • Lusikisiki • 4820 • REPUBLIC OF SOUTH AFRICA • Tel: +27 (0)39 253 1946 / 039 253 6647 • Fax: +27 (0)39 253 1422/ 086 514 3697 • Website: www.ecdoe.gov.za

Enquiries; V.E. Matwasa   Cell: 083 275 0709   Email: ernomatwasa@gmail.com

TO: **BERNARD PRINCE AWUAH – MED RESEARCH STUDY**

FROM: **CHIEF EDUCATION SPECIALIST – INSTITUTIONAL DEVELOPMENT SUPPORT AND GOVERNANCE (IDS & G)**

SUBJECT: **CONSENT TO CONDUCT RESEARCH**

DATE: **20 JANUARY 2016**

In response to your letter dated the 24 August 2015, the Education Office at Lusikisiki gives consent to you to conduct the research towards the MED qualification at the University of Fort Hare and look forward to the outcomes of the study to assist us improve the District performance in Mathematics. We really take pride at the academic advancement people like you aspire for.

We wish you all the success in your study.

V.E. Matwasa

CES – IDS & G

DEPARTMENT OF EDUCATION
DISTRICT DIRECTOR

2016 -01- 20

SIGNATURE.........................

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EASTERN CAPE
2004 - 2014

Ikamva eliqaqambileyo!

APPENDIX E: CONSENT LETTER FROM SCHOOL A

TO: BP AWUAH

16/ 02/ 2016

RE: PERMISSION TO CONDUCT RESEARCH

Referring to your request to conduct a research study dated on the 2/02/2016. Permission is hereby granted based on the following: Your research study must be voluntary; it must not disturb our school activities and finally, research ethics must be ensured. The school will be interested to have a copy of your study.

Good luck!

Yours faithfully,

DEPARTMENT OF EDUCATION
EASTERN CAPE

.....................2016 -02- 16

P. O. BOX 419 LUSIKISIKI 4820
EMIS NO: 501 (Principal) NO: 083 353 6350
PRINCIPAL ....

APPENDIX F: CONSENT LETTER FROM SCHOOL B

22 – 02 - 2016

TO: BERNARD PRINCE AWUAH

RE: PERMISSION TO CONDUCT RESEARCH STUDY

In respond to your letter received which was dated on the 2nd of February, 2016. We are grateful that you have chosen our school as your research field. Permission is hereby granted for you to conduct your research on the following basis:

- Learners should take part on voluntary basis
- Ethical issues relating to the research must be honoured.
- Your research should not disrupt academic activities in the school.

All the best in your studies

Yours faithfully

DEPARTMENT OF EDUCATION

(Principal)

2016 -02- 22

SIGNATURE

APPENDIX G: CONSENT LETTER FROM SCHOOL C

To: Mr. Awuah

18/ 02/ 2016

RE: PERMISSION TO CONDUCT RESEARCH

Referring to your letter received on the 02/02/ 2016. Permission is hereby granted for you to conduct your research on the following conditions:

1. Your research must be conducted on voluntary basis
2. All ethical issues relating to research must be obeyed
3. Your research is subjected to the rules of our school, this includes the curricular activities and its code of conducts and must not disturb the daily activities of the school

I wish you the best in your research study

Yours faithfully

DEPT. PRINCIPAL
SIGNATURE:.............
DATE:...18/02/2016

....

(Principal)

APPENDIX H: ETHICAL CLEARANCE CERTIFICATE FROM UNIVERSTY OF FORT HARE

University of Fort Hare
Together in Excellence

ETHICAL CLEARANCE CERTIFICATE
REC-270710-028-RA Level 01

Certificate Reference Number:	MAM071SAWU01
Project title:	**Exploring strategies to improve the understanding of algebraic fractions in grade 10.**
Nature of Project:	Masters
Principal Researcher:	Bernard Prince Awuah
Sub-Investigator:	
Supervisor:	Prof K.J Mammen
Co-supervisor:	

On behalf of the University of Fort Hare's Research Ethics Committee (UREC) I hereby give ethical approval in respect of the undertakings contained in the above-mentioned project and research instrument(s). Should any other instruments be used, these require separate authorization. The Researcher may therefore commence with the research as from the date of this certificate, using the reference number indicated above.

Please note that the UREC must be informed immediately of

- Any material change in the conditions or undertakings mentioned in the document
- Any material breaches of ethical undertakings or events that impact upon the ethical conduct of the research

- The Principal Researcher must report to the UREC in the prescribed format, where applicable, annually, and at the end of the project, in respect of ethical compliance.

Special conditions: Research that includes children as per the official regulations of the act must take the following into account:

Note: The UREC is aware of the provisions of s71 of the National Health Act 61 of 2003 and that matters pertaining to obtaining the Minister's consent are under discussion and remain unresolved. Nonetheless, as was decided at a meeting between the National Health Research Ethics Committee and stakeholders on 6 June 2013, university ethics committees may continue to grant ethical clearance for research involving children without the Minister's consent, provided that the prescripts of the previous rules have been met. This certificate is granted in terms of this agreement.

The UREC retains the right to

- Withdraw or amend this Ethical Clearance Certificate if
  - Any unethical principal or practices are revealed or suspected
  - Relevant information has been withheld or misrepresented
  - Regulatory changes of whatsoever nature so require
  - The conditions contained in the Certificate have not been adhered to

- Request access to any information or data at any time during the course or after completion of the project.

- In addition to the need to comply with the highest level of ethical conduct principle investigators must report back annually as an evaluation and monitoring mechanism on the progress being made by the research. Such a report must be sent to the Dean of Research's office

The Ethics Committee wished you well in your research.

Yours sincerely

Professor Gideon de Wet
Dean of Research

02 November 2015

APPENDIX I: REQUEST FOR PARENTS' CONSENT

Dear Parent/ Guardian

My name is Bernard Prince Awuah and I am a student at University of Fort Hare, East London. The name of my supervisor is Professor K. J. Mammen. The purpose of this study is to explore strategies to improve the understanding of addition and subtraction of algebraic fractions in grade 10.

Your child along with others have been selected for the study. The participants will be given a pre-test after, they will be taught and a post-test will be given to them. Participation is voluntary, the child may decide not to answer some questions. Their names will not be written on the test papers.

If you have any questions about this research, please contact me on 0731875082.

Yours Sincerely,

______________________ I have read the procedures described above

______________________ I voluntarily give my consent for my child

______________________ to participate in B. P. Awuah's study of exploring strategies to improve the understanding of addition and subtraction of algebraic fractions in grade 10

________________________ ____________________

Parent/ Guardian Date

APPENDIX J: CERTIFICATE OF LANGUAGE EDITING

Carina Barnard

Editing/Translation

6 Villa Monte Verde 082 558 9993

Lynnwood Manor 012 804 5802

0081 carina.barnard

@absamail.co.za

Declaration

To whom it may concern:

I hereby confirm that I edited the thesis

The effectiveness of concrete/semi-concrete/abstract (CSA) approach and drill-practice on Grade 10 learners' ability to simplify addition and subtraction of algebraic fractions

by

Bernard Prince Awuah

Barnard

CJ Barnard

12 August 2016

APPENDIX K: TURNITIN REPORT

turnitin

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This receipt acknowledges that Turnitin received your paper. Below you will find the receipt information regarding your submission.

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CHAPTER 1

1.1 INTRODUCTION

The interest in this study is motivated by the researcher's concern with challenges which learners face in fractions in mathematics (Gould, Outhred & Mitchelmore, 2006; Duzenli-Gokalp & Sharma, 2010; Gabriel, Coche, Szucs, Carette, Rey & Content, 2013; Booth, Newton & Twiss-Garrity, 2014). The data from both the South African National Matric results (Department of Education (DoE), 2013) and the Annual National Assessment (DoE, 2014), paint a bleak picture of the learners' performance in Mathematics as a subject and fractions in particular.

This study seeks to explore the strategies that could help to improve the understanding of addition and subtraction of algebraic fractions as an important component in the Grade 10 South African school mathematics curriculum. It is hoped that the results would assist to improve Grade 10 learners' understanding of addition and subtraction of algebraic fractions.

The research was carried out in one of the education districts in the Eastern Cape Province in South Africa. The study included 135 learners from three public senior secondary schools. A stratified random sampling was used to select 45 learners from each school.

1.2 BACKGROUND

1

- preferences

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