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Comparison of some reliability estimation methods for Laplace distribution using simulations

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ABSTRACT

In this paper, we derived an estimator of reliability function for Laplace distribution with two parameters using Bayes method with square error loss function, Jeffery's formula and conditional probability random variable of observation. The main objective of this study is to find the efficiency of the derived Bayesian estimator compared to the maximum likelihood of this function and moment method using simulation technique by Monte Carlo method under different Laplace distribution parameters and sample sizes. The consequences have shown that Bayes estimator has been more efficient than the maximum likelihood estimator and moment estimator in all sample sizes.

Keywords: Laplace distribution, Reliability function, Bayes estimator, Maximum likelihood estimator, Jeffery formula

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1. Introduction

Laplace distribution stands for an important probability distribution as usable model for estimating the reliability of electronic systems, particularly, in the field of communication engineering. It was highly adopted design engineers for these systems to investigate the durability, quality and diversity of the performance as well as speed of completion of the tasks assigned to these systems [1]. Laplace distribution has two parameters (*a*, *b*). Accordingly, *a* is the shift parameter and *b* is the scale parameter in which $a \in R$ and $b \in R^{+}$ for each of them respectively. Laplace distribution is a member of continuous probability distribution and considered as a model for failure time distribution. This distribution is one of the earliest in probability theory, and it was introduced by the French scientist Pierre Simon Laplace in 1774.
In 2004, Nadarajah has presented several Laplace distribution and derived the corresponding forms for the reliability with R=pr(x_2<x1), when .x_1 and x_2 are independent random variables belong to the univariate family of distributions. The calculation involved the use of special functions [2].
In [3], several Bayesian estimators of the scale parameter for Laplace distribution have derived under diverse loss function involving the squared log error loss function, quadratic loss function and entropy loss function. In view of that, several classical estimators have been acquired like maximum likelihood estimator, uniformly minimum variance unbiased estimator and minimum mean squared error estimator [4].
A study about investigating properties and estimators for reliability of double exponential distribution has been presented in [5]. It was dealt with stress and strength with deriving the forms for the estimation methods (maximum likelihood, moment and shrinkage). A comparison of them has been detailed using simulation and comparative statistical criterion based on mean square error (MSE) [5]. Li, in 2017, has derived Bayes estimator of the shape parameter of Laplace distribution using Bayesian technique under a new loss function. It has been a compound function of LINEX function. The Bayes estimator of the parameter has been derived under a prior distribution of the parameter based on Gamma prior distribution [6]. Onwukwe and Agu, in 2019, have applied the modified Laplace distribution based on two life datasets with simulated data. Parameters of the distributions

were estimated using method of maximum likelihood estimation. The study has given comparison about the modified Laplace distribution with Laplace distribution and generalized error distribution using Schwartz Criteria (SC) measure of fitness. The results obtained revealed that the modified Laplace distribution has a better fit than the Laplace and generalized error distributions [7].In our study, we derived new estimator based on reliability function of Laplace distribution with two parameters (*a*, *b*) using Bayes method, Jeffery's formula and conditional probability random variable of observation $f(t_1,t_2,...,t_n/a,b)$. As a result, the posterior distribution for the parameters is:

$$h\left(a,b\setminus t_1,.....,t_n\right)=\frac{f\left(t_1,...,t_n\setminus a,b\right)g\left(a,b\right)}{\iint\limits_{\forall a\forall b} f\left(t_1,...,t_n\setminus a,b\right)g\left(a,b\right)da\ db}$$

We used $L(\hat{R},R)=(\hat{R}-R)^2$ so that Bayes estimator $\overset{*}{R}(t)$ for Reliability $R(t)$ is:

$$\overset{*}{R}(t)=E[R(t)/x_1,....,x_n]$$

$$\overset{*}{R}(t)=\iint\limits_{\forall a\forall b} R(t)\,h(a,b/x_1,....,x_n)da\ db$$

We used Monte Carlo simulation and compared it with the maximum likelihood reliability function and moment reliability of Laplace distribution based on several values for the parameters of Laplace distribution and sizes. Simulation results have shown that Bayes estimator is the best method. Section 2 contains the derived Bayes estimator for reliability of Laplace distribution. Section 3 presents the theoretical part which explains the probability density function, cumulative, moments of Laplace distribution with two parameters. As well, some properties of this distribution are given with derived Bayes estimator, invariant maximum likelihood estimator and moment estimator for the reliability function R(t). Section 4 provides a simulation study using MATLAB simulator. Section 5 presents the major conclusions of this study.

2. Material and methods

The probability density function for Laplace distribution (p.d.f) with two parameters (a,b) is [2]:

$$f(t)=\begin{cases}\frac{1}{2b}\exp(-\frac{|t-a|}{b}) \quad , \quad -\infty<t<\infty\end{cases} \qquad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

a and b are the shift and scale parameters respectively $(\ -\infty<a<\infty \quad , \quad b>0\ )$. The cumulative distribution function of Laplace's distribution (c.d.f) is [3]:

$$F(t)=\begin{cases}\frac{1}{2}\exp\left[-\left(\frac{a-t}{b}\right)\right] \quad when \quad t<a\end{cases} \qquad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$F\left(t\right)=1-\frac{1}{2}\exp\left[-\left(\frac{t-a}{b}\right)\right] \quad when \quad t\geq a \qquad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

Therefore, the reliability function is:

$$R\left(t\right)=1-\frac{1}{2}\exp\left[-\left(\frac{a-t}{b}\right)\right] \quad when \quad t<a \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$R\left(t\right)=\frac{1}{2}\exp\left[-\left(\frac{t-a}{b}\right)\right] \quad when \quad t\geq a \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

It is known that the cumulative distribution function is complementary to the reliability function based on:

$$R(t)=1-F(t)$$

Since; $\lim\limits_{t\to a^+} R(t)=\lim\limits_{t\to a^-} R(t)$

Then, R (t) is continuous function at t=a

The moment method is a commonly used method for estimating parameters. The random variable $X = X_1 - X_2$ is standard Laplace distribution with X_1, X_2 that are random variables. They are both based on exponential distribution with scale parameter equal to one. It can be expressed as:

$$M_X(t) = E\left(e^{tX}\right)$$

$$M_X(t) = E\left(e^{t(X_1 - X_2)}\right)$$

$$M_X(t) = E\left(e^{tX_1}\right) \,.\, E\left(e^{-tX_2}\right)$$

$$M_X(t) = \frac{1}{1-t} \,.\, \frac{1}{1+t} \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

Based on linear transformation, we get:

$$z = a + bx$$

$$f(x) = \frac{1}{2}\exp(-|x|)$$

$$g(z) = f\left(\frac{z-a}{b}\right) .|J|$$

$$x = \frac{z-a}{b} \Rightarrow J = \frac{dx}{dz} = \frac{1}{b}$$

This makes z as a random variable with two parameters (*a*, *b*) as follows:

$$g(z) = \frac{1}{2}\exp\left(\frac{|z-a|}{b}\right) \cdot \frac{1}{b}$$

$$g(z) = \frac{1}{2b}\exp\left(\frac{|z-a|}{b}\right)$$

The moment generating is as following:

$$M_z(t) = E\left(e^{t(a+bx)}\right)$$

$$M_z(t) = e^{at}.E\left(e^{btx}\right)$$

$$M_z(t) = \frac{e^{at}}{1-b^2t^2}$$

The variance can be expressed by the following formulas:

$$Var(z) = E\left(z^2\right) - \left[E(z)\right]^2$$

$$Var(z) = a^2 + 2b^2 - a^2 = 2b^2$$

$$Var(z) = s = 2b^2$$

$$\tilde{b} = \frac{s}{\sqrt{2}}$$

Accordingly, the mean is unbiased estimator for shift parameter a:

$$E(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n} E(x_i)$$

$$E(\bar{X}) = \frac{1}{n}.na = a$$

$$\tilde{a} = \bar{x}$$

Therefore, we can use the arithmetic mean to estimate the unknown parameter a, we usually use the median and both of them a measure of central tendency.

$$\tilde{R}(t)=1-\frac{1}{2}\exp\left[-\left(\frac{\tilde{a}-t}{\tilde{b}}\right)\right] \quad , \quad t<a \quad ....(7)$$

$$\tilde{R}(t)=\frac{1}{2}\exp\left[-\left(\frac{t-\tilde{a}}{\tilde{b}}\right)\right] \quad , \quad t\geq a \quad .... \quad (8)$$

The likelihood function for a sample of size n from (*a*, *b*) for reliability function for Laplace distribution is feasibly derived by [8]:

$$f\left(t_1,.....,t_n;a,b\right)=\prod_{i=1}^{n}\frac{1}{2b}\exp(-\frac{|t-a|}{b})$$

$$f\left(t_1,.....,t_n;a,b\right)=\left(\frac{1}{2b}\right)^n\exp\left[-b^{-1}\sum_{i=1}^{n}|t_i-a|\right] \quad .... \quad (9)$$

$$\ln f\left(t_1,.....,t_n;a,b\right)=-nln2-nlnb-b^{-1}\sum_{i=1}^{n}|t_i-a|$$

$$\frac{\partial\ln f\left(t_1,.....,t_n;a,b\right)}{\partial b}=-\frac{n}{\hat{b}}+\frac{1}{\hat{b}^2}\sum_{i=1}^{n}|t_i-\hat{a}|=0$$

$$\hat{b}=\frac{1}{n}\sum_{i=1}^{n}|t_i-\hat{a}| \quad .... \quad (10)$$

And in the same way concerning *a*, we will get the following:

$$\frac{\partial\ln f\left(t_1,.....,t_n;a,b\right)}{\partial a}=\frac{1}{b}\sum_{i=1}^{n}|t_i-\hat{a}|=0$$

$$\hat{a}=\bar{t} \quad .... \quad (11)$$

Therefore, the property of invariant the maximum likelihood estimator for reliability function is given by:

$$\hat{R}(t)=1-\frac{1}{2}\exp\left[-\left(\frac{\hat{a}-t}{\hat{b}}\right)\right] \quad , \quad t<a \quad ....(12)$$

$$\hat{R}(t)=\frac{1}{2}\exp\left[-\left(\frac{t-\hat{a}}{\hat{b}}\right)\right] \quad , \quad t\geq a \quad .... \quad (13)$$

Our suggested Bayes estimator was derived as follows:
Let $(t_1,.....,t_n)$ be a random sample of size an independent observation from a Laplace distribution having p.d.f, we will get:

$$f\left(t/a,b\right)=\left(\frac{1}{2b}\right)\exp\left[-\frac{|t-a|}{b}\right] \quad ....-\infty<x<\infty$$

The likelihood function is based on the following:

$$f\left(t_1,...,t_n/a,b\right)=\prod_{i=1}^{n}f(t_i/a,b)$$

$$f\left(t_1,...,t_n/a,b\right)=\left(\frac{1}{2b}\right)^n\exp\left[-\frac{1}{b}\sum_{i=1}^{n}|t_i-a|\right]$$

Using Jeffrey's variant prior, we will have [3]:

$$g_1(a)\propto\frac{1}{2k} \quad , \quad -k<a<k \quad where \quad k\in R$$

$$g_2(b) \propto \frac{1}{b} \quad , \quad b > 0$$

$$g(a,b) \propto g_1(a) \,.\, g_2(b)$$

$$g(a,b) \propto \frac{1}{2kb}$$

The posterior probability density function can be expressed as:

$$h(a,b \backslash t_1,...,t_n) \, \alpha \, \frac{2^{-n}}{b^{n+1}} \exp\left[-\frac{1}{b}\sum_{i=1}^{n}|t_i - a|\right]$$

$$h(a,b \backslash t_1,...,t_n) = \frac{2^{-n}c}{b^{n+1}} \exp\left[-\frac{1}{b}\sum_{i=1}^{n}|t_i - a|\right] \qquad \cdots\cdots\cdots\cdots(14)$$

Where,

$$c^{-1} = 2^{-n}\int_{-k}^{k}\left[\int_{0}^{\infty}\frac{1}{b^{n+1}}\exp\left\{-\frac{1}{b}\sum|t_i - a|\right\}db\right]da$$

Let $w = \frac{1}{b}\sum_{i=1}^{n}|t_i - a|$

Then, $\Gamma(n) = \int_{0}^{\infty} w^{\,n-1}e^{-w}\,dw$ and $db = -\frac{b^2 dw}{\sum_{i=1}^{n}|x_i - a|}$

$$c^{-1} = 2^{-n}\int_{-k}^{k}\left[\int_{0}^{\infty} w^{n-1}e^{-w}dw\right]\frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^n}$$

$$c^{-1} = 2^{-n}\,\Gamma(n)\int_{-k}^{k}\frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^n}$$

Consequently, equation (14) becomes in the following form:

$$h(a,b/t_1,...,t_n) = \frac{\frac{1}{b^{n+1}}\,e^{-\frac{1}{b}\sum_{i=1}^{n}|t_i - a|}}{\Gamma(n)\int_{-k}^{k}\frac{da}{\left\{\sum_{i=1}^{n}|x_i - a|\right\}^n}} \qquad \cdots\cdots\cdots \quad (15)$$

Marginally, we can get simplify equation (15) as follows:

$$h_1(a/t_1,...,t_n) = \int_{0}^{\infty} h(a,b/t_1,...,t_n)\,db$$

$$= \int \frac{\frac{1}{b^{n+1}}\,e^{-\frac{1}{b}\sum_{i=1}^{n}|t_i - a|}}{\Gamma(n)\int_{-k}^{k}\frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^n}}\,db$$

$$h_1(a/t_1,...,t_n)=\frac{\dfrac{\Gamma(n)}{\left\{\sum_{i=1}^{n}|t_i-a|\right\}^n}}{\Gamma(n)\displaystyle\int_{-k}^{k}\frac{da}{\left\{\sum_{i=1}^{n}|t_i-a|\right\}^n}} \quad h_2(b/t_1,...,t_n)=\int_{-k}^{k}h(a,b/t_1,...,t_n)\,da$$

$$h_1(a/t_1,...,t_n)=\frac{\dfrac{1}{\left\{\sum_{i=1}^{n}|t_i-a|\right\}^n}}{\displaystyle\int_{-k}^{k}\frac{da}{\left\{\sum_{i=1}^{n}|t_i-a|\right\}^n}} \qquad \cdots\cdots\cdots\cdots(16)$$

$$h_2(b/t_1,...,t_n)=\frac{\dfrac{1}{b^{n+1}}\displaystyle\int_{-k}^{k}e^{-\frac{1}{b}\sum_{i=1}^{n}|t_i-a|}da}{\Gamma(n)\displaystyle\int_{-k}^{k}\frac{da}{\left\{\sum_{i=1}^{n}|t_i-a|\right\}^n}}$$

The parameters a and b are taken into consideration as follows:

$$a^*=E\left[a\backslash x_1,....,x_n\right]$$

$$a^*=\int_{-k}^{k}a\;\; h_1(a\backslash t_1,....,t_n)\,da$$

$$a^*=\int_{-k}^{k}a\frac{\dfrac{da}{\left\{\sum_{i=1}^{n}|t_i-a|\right\}^n}}{\displaystyle\int_{-k}^{k}\frac{da}{\left\{\sum_{i=1}^{n}|t_i-a|\right\}^n}} \qquad \cdots\cdots\cdots\cdots\cdots(17)$$

$$b^*=E\left[b\backslash t_1,....,t_n\right]$$

$$b^*=\int_{0}^{\infty}b\;\; h_2(b\backslash t_1,....,t_n)\,db$$

$$b^*=\int_0^\infty b\, h_2(b/t_1,...,t_n)\,db=\int_0^\infty b\frac{\dfrac{1}{b^{n+1}}\displaystyle\int_{-k}^{k}e^{-\frac{1}{b}\sum_{i=1}^{n}|t_i-a|}da}{\Gamma(n)\displaystyle\int_{-k}^{k}\frac{da}{\left\{\sum_{i=1}^{n}|t_i-a|\right\}^n}}db$$

$$b* = \int_0^\infty b^n \frac{\int\limits_{-k}^{k} e^{-\frac{1}{b}\sum_{i=1}^{n}|t_i - a|} da}{\Gamma(n) \int\limits_{-k}^{k} \frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^n}} db$$

$$b* = \frac{\Gamma(n-1) \int\limits_{-k}^{k} \frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^{n-1}}}{\Gamma(n) \int\limits_{-k}^{k} \frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^n}}$$

$$b* = \frac{1}{n-1} \frac{\int\limits_{-k}^{k} \frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^{n-1}}}{\int\limits_{-k}^{k} \frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^n}} \quad \cdots\cdots\cdots\cdots \quad (18)$$

The Bayes estimator $R^*(t)$ for R (t) using squared error loss function stands for the Posterior mean [9] and given by:

$$R^*(t) = E\left[R(t) \backslash t_1, \ldots., t_n\right]$$

$$R^*(t) = \int\limits_{-k}^{k} \int\limits_{0}^{\infty} R(t)\, h(a, b \backslash t_1, \ldots, t_n)\, db\, da$$

$$R^*(t) = \int\limits_{-k}^{k} \int\limits_{0}^{\infty} \frac{1}{2} \exp\left(-\left(\frac{a-t}{b}\right)\right) h(a, b \backslash t_1, \ldots, t_n)\, db\, da$$

$$R^*(t) = \int\limits_{-k}^{k} \int\limits_{0}^{\infty} \frac{1}{2} e^{-\left(\frac{a-t}{b}\right)} \frac{\frac{1}{b^{n+1}} e^{-\frac{1}{b}\sum_{i=1}^{n}|t_i - a|}}{\Gamma(n) \int\limits_{-k}^{k} \frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^n}} db\, da$$

$$R^*(t) = \frac{1}{2\Gamma(n) \int\limits_{-k}^{k} \frac{da}{\left\{\sum_{i=1}^{n}|t_i - a|\right\}^n}} \int\limits_{-K}^{K} \int\limits_{0}^{\infty} \frac{1}{b^{n+1}} e^{-\frac{1}{b}\left(a - t + \sum_{i=1}^{n}|t_i - a|\right)} db\, da$$

Let *v* and *db* as follows:

$$v = \frac{1}{b}\left(a - t + \sum_{i=1}^{n}|t_i - a|\right)$$

$$db = \frac{-b^2 \; dv}{a - t + \sum_{i=1}^{n} |t_i - a|}$$

We get $R^*(t)$ as:

$$R^*(t) = \frac{1}{2} \; \frac{\int_{-k}^{k} \frac{da}{\left(a - t + \sum_{i=1}^{n} |t_i - a|\right)^n}}{\int_{-k}^{k} \frac{da}{\left[\sum_{i=1}^{n} |t_i - a|\right]^n}} \qquad \ldots\ldots\ldots\ldots t \ge a \quad (19)$$

Similarly, when $t < a$, we can get more simplification as follows:

$$R^*(t) = \int_{-k}^{k}\int_{0}^{\infty} 1 - \frac{1}{2}\exp\left(-\left(\frac{a-t}{b}\right)\right) h\left(a, b \setminus t_1, \ldots, t_n\right) db \; da$$

$$R^*(t) = 1 - \frac{1}{2}\int_{-k}^{k}\int_{0}^{\infty} \exp\left(-\left(\frac{a-t}{b}\right)\right) h\left(a, b \setminus t_1, \ldots, t_n\right) db \; da$$

$$R^*(t) = 1 - \frac{1}{2} \; \frac{\int_{-k}^{k} \frac{da}{\left(a - t + \sum_{i=1}^{n} |a - t_i|\right)^n}}{\int_{-k}^{k} \frac{da}{\left[\sum_{i=1}^{n} |a - t_i|\right]^n}} \qquad \ldots\ldots\ldots\ldots \quad t < a \quad \ldots\ldots\ldots\ldots \quad (20)$$

3. Simulation

Monte Carlo simulation has been employed as recurrent sampling to get the statistical properties of several phenomenoa. A primary variant of the simulation can be seen in the Buffon's needle experiment. In the 1930s, Enrico Fermi firstly experimented the Monte Carlo technique while studying neutron diffusion, but he did not publish anything on it [10]. Recently, many researchers have used simulation for generating Laplace distribution [5, 11]. In simulation, we adopted r=1000 where r is the replications. Arbitrary values of parameters shift (a =-1, 0,1) and scale (b= 1, 2, 3) parameters respectively.
Arbitrary values of sample sizes as (n =110, 20, 50, 75, 100) from Laplace distribution were selected by using MATLAB simulator version 2017.

Laplace distribution has generated based on:

$$u = 1 - \frac{1}{2}\exp\left\{-\left(\frac{t}{b}\right)\right\} \qquad t = -b\ln\left\{2(1-u)\right\} \qquad for \quad t < 0$$

$$t = b\ln(2U) \quad for \quad , t \ge 0$$

Then, the values of reliability $\tilde{R}(t)$ of the moment have computed according to equations (7) and (8).

Maximum likelihood $\hat{R}(t)$ has been according to equation (12) and (13), while the reliability of Bayes estimator $R^*(t)$ has been according to the equations (19) and (20).

Finally, we computed the efficiency of the two estimators using Mean Square Error (MSE):

$$MSE\left(\tilde{R}\right)=\frac{1}{r}\sum_{i=1}^{r}\left(\tilde{R}_i-R\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

Where,

R is the real value of reliability.

$\tilde{R}$ is the estimator of $R(t)$ according to the method.

4. The results and discussions

The results of the estimator under different samples sizes are depicted in details in Tables 1-9 as follows:

Table 1. The values of MSE for estimators when (a = 0, b = 1) with different sample sizes

n \ Methods	Moment method	M.L.E method	Bayes method	Best method
10	0.33111	0.423289	0.004232	Bayes method
20	0.29222	0.350076	0.002178	Bayes method
50	0.28116	0.214541	0.001089	Bayes method
75	0.19967	0.188881	0.000851	Bayes method
100	0.01496	0.101254	0.000659	Bayes method

From Table 1 when (a =0, b = 1) with different samples sizes (n = 10, 20, 50, 75 and 100), the best method is Bayes method because of the smallest value of MSE.

Table 2. The values of the MSE for estimators when (a = 0, b = 2) with different samples sizes

n \ Method	Moment method	M.L.E method	Bayes method	Best method
10	0.520113	0.551416	0.005736	Bayes method
20	0.510569	0.528407	0.001354	Bayes method
50	0.434581	0.160337	0.001128	Bayes method
75	0.390127	0.125205	0.000840	Bayes method
100	0.340123	0.118359	0.000716	Bayes method

From Table 2 when (a = 0, b = 2) with different sample sizes (n = 10, 20, 50, 75 and 100), the best method is Bayes method due to its minimum value of MSE.

Table 3. The values of the MSE for estimators when (a = 0, b = 3) with different samples sizes

n \ Method	Moment method	M.L.E method	Bayes method	Best method
10	4.998450	5.852958	0.002495	Bayes method
20	3.987643	3.021546	0.002060	Bayes method
50	3.500789	2.901644	0.001175	Bayes method
75	2.987643	2.092493	0.000929	Bayes method
100	1.997863	0.249968	0.000643	Bayes method

We notice from Table 3 when (a = -0, b = 3) with different sample sizes (n = 10, 20, 50, 75 and 100) that the best method is Bayes method because of the smallest value of MSE.

Table 4. The values of the MSE for estimators when (a = 1 and b = 1) with different samples sizes

n \ Method	Moment method	M.L.E method	Bayes method	Best method
10	2.997845	3.596532	0.007365	Bayes method
20	2.456872	3.368286	0.005589	Bayes method
50	1.998764	2.003696	0.002171	Bayes method
75	0.998780	0.320198	0.000524	Bayes method
100	0.898754	0.214941	0.000116	Bayes method

We notice from Table 4 when (a = -1, b = 1) with different samples sizes (n = 10, 20, 50, 75 and 100) that the finest method is Bayes method because of smallest value of MSE.

Table 5. The values of the MSE for estimators when (a = 1, b = 2) with different sample sizes

n \ Method	Moment method	M.L.E method	Bayes method	Best method
10	4.167892	4.796259	0.002367	Bayes method
20	3.065543	3.625585	0.001743	Bayes method
50	3.041905	3.447336	0.000965	Bayes method
75	1.999876	1.458064	0.000547	Bayes method
100	0.943289	0.772277	0.000165	Bayes method

Based on Table 5 when (a = 1, b = 2) with different samples sizes (n = 10, 20, 50, 75 and 100), the best method is Bayes method because of smallest value of MSE.

Table 6. The values of the MSE for estimators when (a = 1, b = 3) with different sample sizes

method \ n	Moment method	M.L.E method	Bayes method	Best method
10	2.021345	2.390146	0.025873	Bayes method
20	1.999456	2.012150	0.005742	Bayes method
50	1.689324	1.119627	0.002219	Bayes method
75	0.876526	0.524271	0.001115	Bayes method
100	0.123459	0.394955	0.000236	Bayes method

We perceive from Table 6 in the case of (a = 1, b = 3) with different sample sizes (n = 10, 20, 50, 75 and 100) that the best method is Bayes method because of smallest value of MSE.

Table 7. The values of the MSE for estimators when (a = -1, b = 1) with different sample sizes

n \ Method	Moment method	M.L.E method	Bayes method	Best method
10	0.153452	0.194804	0.001977	Bayes method
20	0.100834	0.115557	0.000117	Bayes method
50	0.119659	0.112913	0.000113	Bayes method
75	0.091932	0.071003	0.000071	Bayes method
100	0.089652	0.064394	0.000064	Bayes method

We notice from Table 7 when *(a* = -1, *b* = 1*)* with different sample sizes (n = 10, 20, 50, 75 and 100) the best method is Bayes method because of smallest value of MSE.

Table 8. The values of the MSE for estimators when (a = -1, b = 2) with different samples sizes

n \ Method	Moment method	M.L.E method	Bayes method	Best method
10	0.193987	0.303912	0.000319	Bayes method
20	0.189378	0.270925	0.000103	Bayes method
50	0.103481	0.085614	0.000085	Bayes method
75	0.097652	0.078553	0.000083	Bayes method
100	0.083219	0.060191	0.000060	Bayes method

We notice from Table 8 when (a = -1, b = 2) with different sample sizes(n=10,20,50,75,100) that the finest method is Bayes method because of the lowest value of MSE.

Table 9. The values of the MSE for estimators when (a = -1, b = 3) with different samples sizes

Method \ n	Moment method	M.L.E method	Bayes method	Best method
10	0.239765	0.335462	0.000392	Bayes method
20	0.207652	0.223917	0.000231	Bayes method
50	0.119321	0.117482	0.000117	Bayes method
75	0.108432	0.093257	0.000065	Bayes method
100	0.087651	0.061206	0.000061	Bayes method

We observe from Table 9 when *(a* = -1, *b* = 3*)* with different samples sizes (n=10,20,50,75, and 100) that the best method is Bayes method owing to the smallest value of MSE.

5. Conclusions

- The Bayes estimators of reliability function of the Laplace distribution $R^*(t)$ is more efficient than the maximum likelihood estimator of the reliability $\hat{R}(t)$ in all samples size because of smallest value of the MSE.
- The moment estimator of reliability function for Laplace distribution $\tilde{R}(t)$ is more effectual than the maximum likelihood estimator of the reliability $\hat{R}(t)$ in sample sizes (10 and 20) because of the smallest value of the MSE.
- The maximum likelihood estimator of the Reliability $\hat{R}(t)$ is more effectual than the moment estimator of reliability function for Laplace distribution $\tilde{R}(t)$ in sample sizes (50, 75, and 100) because of smallest value of the MSE.
- We can use Bayesian estimator for reliability function of Laplace distribution with two parameters via Jeffery model with applications in speech recognition to model priors on DFT coefficients and in JPEG image compression to model AC coefficients, generated by the DCT.

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