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Concordia University–Portland

College of Education

Doctorate of Education Program

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The Impact of Teacher Self-Efficacy on Methodology and the Use of Graphing Technology in Teaching Factoring Quadratic Functions: Perspectives of International Introductory Algebra Teachers

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Concordia University–Portland

College of Education

Dissertation submitted to the Faculty of the College of Education

in partial fulfillment of the requirements for the degree of

Doctor of Education in

Professional Inquiry, Leadership, and Transformation

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2019

Abstract

The primary purpose of the study was to determine whether there is a relationship between self-efficacy of international algebra teachers and their level of incorporating technology in teaching factoring quadratic functions to introductory algebra students. The secondary purpose of the study was to examine the influence of self-efficacy on the perspective of international teachers with respect to the methods they use to teach factoring quadratic functions to introductory algebra students. The participants, 54 mathematics educators form 15 countries on five continents, replied to the UVGIA survey instrument. Quantitative analysis of data brought two results. There is a strong positive relationship between the level of self-efficacy of teachers and their level of implementations of technology regardless of country of origin. The second result shows that the level of self-efficacy of math teachers is statistically different in individualistic countries versus collectivistic countries, revealing higher self-efficacy in collectivistic countries. However, their level of implementation of technology is not statistically different. Qualitative analysis of open-ended questions showed teachers' perspectives on teaching and learning factoring quadratic functions to introductory algebra students. Teachers identify students' lack of basic mathematical skills, their lack of understanding graphs, difficulties with identifying the purpose, and difficulties factoring when the leading coefficient is different than 1. Teachers recommend incorporating meaningful applications into mathematical methods with real-life contexts, graphs and visualizations, and systematic reviews of background knowledge. They suggest removing automatic procedures in favor of conceptual understanding and eliminating some methods of factoring.

Keywords: self-efficacy, factoring quadratic functions, implementation of educational technology, introductory algebra, collectivistic and individualistic countries, teachers' recommendations.

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Chapter 1: Introduction

Mathematical knowledge is a critical part of a student's education because it impacts the social, personal, and cultural aspects of a person's life (RAND Mathematics Study Panel, 2003). Algebra, in particular, is integral to all other branches of mathematics, and has become a nationwide requirement for high school graduation (Council of Chief State School Officers, 2010). A key component of algebra is the ability to factor quadratic functions.

Factoring quadratic functions is classified as a very challenging and complex algebraic topic to teach, especially when introduced to students for the first time (Annette & Kitt, 2000; Kotsopoulos, 2007; Leitze & Kitt, 2000). Such difficulties are noticed not only in the United States but also internationally (Didiş, Baş, & Erbaş, 2011; Vaiyavutjamai, Ellerton, & Clements, 2005). Therefore, algebra teachers are constantly seeking different approaches to more effectively introduce factoring to their students. Many mathematical methods have been developed and various teaching techniques have been used to increase students' learning of algebraic concepts. Technological advances offer new instructional options which could be incorporated into teaching mathematics. These new approaches include simulations, computer gaming environments, adaptive learning, and graphing tools. Technology has become an important part of education since students internationally have become technologically skilled and they use it in their everyday life.

In this study, the researcher attempted to find how international teachers strategically teach factoring quadratic functions to introductory algebra students, specifically their preferences of teaching methods. Special attention was given to incorporating graphing technology to mathematical methods. Technology was viewed as a new opportunity and additional mathematical representation to improve students learning.

Teacher self-efficacy has been linked to willingness to adopt instructional changes (Achurra & Villardón, 2013) and to student achievement (Knoblauch & Hoy, 2008; Muijs & Reynolds, 2005; Ross, 1992; Skaalvik & Skaalvik, 2007). This study further explored the relationship between teacher self-efficacy and teacher preferences of using mathematical methods, including incorporating technology, in teaching factoring quadratic functions in introductory algebra courses.

In addition, the researcher aimed to compare teachers' perceptions of self-efficacy and their level of incorporating technology. The division of countries as collectivistic and individualistic was based on recommendations from Hofstede (1983) through the dimensions of cultural diversity.

Conceptual Framework

A significant part of any algebra curriculum is factoring polynomials which is a process employed to solve algebraic equations (Donnell, 2010). However, it is one of the most challenging and complex topics to teach (Kotsopoulos, 2007), particularly to introductory algebra students. One of the difficulties comes from insufficient preparation of students for introductory algebra classes. Students struggle with basic arithmetic and algebraic manipulations (Boulton-Lewis, Cooper, & Wills, 2001; Didis & Erbas, 2015; Kotsopoulos, 2007; Martinez, Bragelman, & Stoelinga, 2016; Nielsen, 2015; Sells, 1973; Zakaria & Maat, 2010), exponents (French, 2002; Nielsen, 2015), and understanding the significance of the equal sign (Kieran, 1981; Martinez et al., 2016). Frequently, students have problems with mathematical symbols and letters, which Maredi and Oosthuizen (1995) called the "language barrier" (p. 245).

However, the situation with teaching factoring quadratic functions is more complex than improving the preparation of students. In the past, factoring had many interpretations related to the historical stages of algebra (Katz & Barton, 2007) and each of the previous approaches still impacts, to varying degrees, the current view and the methods of teaching it. Even more, teachers and educators keep creating new procedures, hoping to produce a better and easier method for students to understand the concept. There is no agreement on the best approach and therefore, no unified methodology used in schools to teach factoring quadratic functions (Gray & Tall, 1994; Lima, 2008; NCTM, 1989; Sönnerhed, 2009).

These apparent teaching challenges inspired this study. Vygotsky (1978) stressed the importance of introducing different interpretations of a concept when constructing students' knowledge: "if one changes the tools of thinking available to a child, his mind will have a radically different structure" (p. 126). Many educators support using multiple representations to enhance students' learning (Ainsworth, 2006; Cabahug, 2012; Ogbonnaya, Mogari, & Machisi, 2013). According to Ainsworth (1999) "a common justification for using more than one representation is that this is more likely to capture a learner's interest and, in so doing, play an important role in promoting conditions for effective learning" (p. 131). All the functions of multiple representations apply to graphing technology, a modern tool in mathematics classrooms. In particular, incorporating graphing calculators to teach factoring quadratic functions as early as possible to introductory algebra students could play an essential role in the teaching-learning processes. Graphs of parabolas could be used to establish factorability of quadratic trinomials (existence of real solutions), to estimate possible solutions, and to verify answers calculated by algebraic or computational methods.

Using technology in math education is widely encouraged. The National Council of Teachers of Mathematics (NCTM) (2000) pointed out that technology "influences the mathematics that is taught and enhances students' learning" (p. 24) as it has been "widely recognized as a significant teaching and learning tool" (Thach & Norman, 2008, p. 152). Many studies have shown benefits of incorporating graphing calculators into teaching algebra (Cheung & Slavin, 2013; Dreiling, 2007; Drijvers, Monaghan, Thomas, & Trouche, 2014; Ellington, 2006; Kyungsoon, 1999; Wilkins, 1995). However, technology requires changes in methodology (Drijvers et al., 2014; Kastberg & Leatham, 2005; Kyungsoon, 1999), classroom organization (Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2012; Hivon, Pean, & Trouche, 2008.; Hoyles et al., 2010; Kendal, Stacey, & Pierce, 2005), and teacher-student interactions (Doerr & Zangor, 2000; Geiger, Faragher, Redmond, & Lowe, 2008; Goos & Bennison, 2008; Rivera, 2011). Overall, technology use requires teachers to alter their pedagogy.

According to Achurra and Villardón (2013), predisposition to incorporating changes in curriculum, instructional methods, and classroom organization is related to teachers' self-efficacy. At the same time, many studies linked self-efficacy of teachers to students' achievements (Achurra & Villardón, 2013; Ashton & Webb, 1986; Caprara, Barbaranelli, Steca, & Malone, 2006). Therefore, this study was focused on teachers' level of self-efficacy and its influence on their preferences in teaching methods, ways of incorporating graphing technology, and the perceived influence on student learning when teaching factoring quadratic functions to introductory algebra students. The study included teachers from a wide variety of countries and therefore was able to compare the results obtained from collectivistic and individualistic countries as defined by Hofstede (1983).

Problem Statement

There is a universal challenge to teach factoring quadratic functions in introductory algebra courses. The problem is that teacher self-efficacy and other variables may negatively influence teachers' ability and willingness to use technology-based graphing tools to more effectively teach factoring quadratic functions to introductory algebra classes.

Research Questions and Hypothesis

The following research questions were closely examined in the study:

1. What is the relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?
2. How does the intersection of self-efficacy of teachers and teaching factoring quadratic functions to introductory algebra students, influence teacher preferences in mathematical methods of instruction?
3. What differences exist between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, and (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?

Below, there are hypotheses related to the research questions that were analyzed using statistical tools.

$H1_0$. There is no relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.

$H1_a$. There is a relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.

$H3_0$. There are no differences between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.

$H3_a$. There are differences between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra.

Purpose of the Study

The purpose of the study was to determine whether there is a relationship between self-efficacy of international algebra teachers and their level of incorporating technology in teaching factoring quadratic functions to introductory algebra students. Secondly, the researcher aimed to examine the influence of self-efficacy on the perspective of international teachers with respect to the methods they use to teach factoring quadratic functions to introductory algebra students.

Definition of Terms

This study used the following definitions:

Self-Efficacy: A person's belief in his or her capacity to achieve a goal (Bandura, 1977b, 1989, 1997).

Factoring Quadratic Functions: A process of representing the quadratic function as a product of two linear factors (Factoring, n.d.).

Introductory Algebra: A course of mathematics which introduces mathematical abstractions for the first time (Dumbauld, 2018).

Individualism: Defined by Hofstede (n.d.) as the degree in which people view themselves as independent instead of being simply a member of a larger whole. It refers to the individual's ability and expectation to make personal choices and decisions.

Collectivism: Defined by Hofstede (n.d.) as the degree to which a person is aware of and content with their role in society which is determined by the society itself.

Since countries fall on a continuum, for the purposes of this study, respondents who scored less than 50 on Hofstede's scale were termed *collectivistic* and those scoring 50 or above others were termed *individualistic* (Hofstede, 2001).

Assumptions

The following assumptions undergird this study:

1. It was assumed that the participating teachers had experience in teaching factoring quadratic functions to introductory algebra students.
2. It was assumed that the participating teachers were honest in reporting their perceptions on their teaching practice.
3. It was assumed the evaluator of the survey was not biased or prejudiced in the rating of respondent data.

Limitations

The researcher recognized that there were certain limitations inherent in conducting this research study. The limitations were as follows:

1. Some responses may have been impacted by the limited access to educational technology in schools in certain geographic areas.

2. The responses may have been impacted by participants' level of understanding and communicating in English. Mathematics researchers from over 25 countries attended the conferences and therefore translating into all languages was not feasible.

Delimitations

The following delimitations existed in the study:

1. The sample was delimited to the participants of the two international conferences: The International Conference of Mathematics Educators in Hungary in 2017 and The Conference in Contemporary Mathematics Education in 2018. The attendees of the conferences and their networks were the only potential participants of the study.
2. The study was delimited to the countries of the attendees and their networks.

Significance of the Study

The results of the study may provide educators insights on the role of technological tools such as graphing calculators as an educational tool in teacher factoring quadratic functions. It could also help mathematics curriculum developers to offer teachers' preferences in choosing mathematical methods to teach factoring quadratic functions to introductory algebra students.

Summary and Transitions

The challenges with teaching factoring quadratic functions to introductory algebra students have been experienced not only in the United States but also internationally. Various mathematical methods and teaching techniques have been developed to address this challenge. Meanwhile, technology has become an important part of education, providing new options and teaching tools. This study examined teachers' preferences in choosing mathematical methods and ways to incorporate graphing technology to teach factoring quadratic function to introductory algebra students. Those preferences were compared to teachers' self-efficacy, as self-efficacy has been linked to teachers' willingness to adopt changes (Achurra & Villardón, 2013) and to students' mathematical achievement (Knoblauch & Hoy, 2008; Muijs & Reynolds, 2005; Ross, 1992; Skaalvik & Skaalvik, 2007).

In addition, the researcher aimed to compare, internationally, teachers' perceptions of their level of self-efficacy and their level of incorporating technology. The division of countries that was used in this study was specified by Hofstede (1983) through the dimensions of cultural diversity as collectivistic and individualistic countries.

Chapter 2 reviews the literature related to the study: the importance of teaching algebra, the past and present approaches to teaching factoring quadratic functions, the challenges with teaching factoring quadratic functions, the role of multiple representations with special focus on graphing technology in algebra class, and the role of teachers in teaching-learning processes with a special attention to the role of teachers' self-efficacy. Chapter 3 is a comprehensive overview of the methodology used for this study. Chapter 4 is a presentation of the data and its subsequent analysis. Chapter 5 is a discussion of the study results, implications, and conclusions.

Chapter 2: Literature Review

This chapter includes the review of research publications related to the purpose of this study, which is to investigate teachers' perspective on effectiveness of using graphing technology in teaching factoring quadratic functions to introductory algebra students. The researcher divided this review into the six following topics: (a) the importance of teaching algebra, (b) past and present methods of teaching factoring quadratic functions (c) challenges with teaching factoring quadratic equations, (d) multiple representations, (e) graphing technology in algebra class, and (f) the role of teachers.

The subject of this research, teaching factoring quadratic functions, is one of the most essential topics in algebra curriculum, and algebra is a fundamental branch of mathematics (RAND Mathematics Study Panel, 2003). This chapter discusses the significance of algebra and the importance of factoring quadratic equations in algebra curriculum. It also deliberates the historical views and interpretations on factoring trinomials and its impact on the currently used methodologies in mathematics classrooms. The presented research revealed various difficulties that teachers and introductory algebra students encounter when teaching and learning factoring trinomials. The main difficulties are the abstract nature of the subject, insufficient preparation of students, insufficient preparation of teachers, and the deficiency of a unified methodology. A search for ways to address the challenges pointed to the concept of multiple representations and its educational role in algebra class. Technological tools, the modern type of representation, offers new possibilities for teachers and students to incorporate. Therefore, the main focus of this study was to investigate factors influencing teachers' use of graphing devices when the concept of factoring quadratic functions is introduced for the first time. The visual feature of graphing technology, the different representation of quadratic functions, and the additional assistance with the basic calculations could be a supplementary support for struggling students in introductory algebra classes.

Incorporating technology requires constant enhancement of teachers' knowledge, skills, and classroom practice. Many researchers link The willingness of using technology has been linked to high level of self-efficacy (Achurra & Villardón, 2013), and similarly self-efficacy is related to students' achievements (Knoblauch & Hoy, 2008; Muijs & Reynolds, 2005; Ross, 1992; Skaalvik & Skaalvik, 2007). This study examines the relationship between the level of implementation of visual technology in teaching factoring quadratic functions to introductory algebra students and the level of self-efficacy of the teachers. The second goal of the study is to examine the relationship between the level of self-efficacy of the teachers and the teachers' perceived effect of their instruction on student learning when teaching factoring quadratic functions to introductory algebra students.

The Importance of Teaching Algebra

In 2003, a RAND Mathematics Study Panel commissioned by U.S. government identified three areas in a person's life that involve mathematical knowledge: social, personal, and cultural. In a social context, it offers the ability to understand, reason, and make "judgments on public issues and policies of a technical nature" (p. 31). In a personal context, it "extends the options available in one's career as well as in one's daily life" (p. 32). In a cultural context, "mathematics constitutes one of humanity's most ancient and noble intellectual traditions" (p.32), language of science and technology, and "powerful tools for analytical thought and the concepts and language for creating precise quantitative descriptions of the world" (p. 32). A mathematical knowledge of algebra provides benefits in several contexts.

Yet, in order for students to be successful in high school level mathematics, they must master algebra, the tool to all branches of mathematics (Ma & Wilkins, 2007; Martinez, Bragelman, & Stoelinga, 2016; Moses, 2001; RAND Mathematics Study Panel, 2003). The RAND Mathematics Study Panel (2003) listed language and structure of algebra as necessary tools "for representing and analyzing quantitative relationships, for modeling situations, for solving problems, and for stating and proving generalizations" (p. 21). Algebra has become a gatekeeper course to more advanced mathematics and science classes, and, consequently, to college (Adelman, 2006; Ma & Wilkins, 2007; Martinez, et al., 2016; Moses, 2001; NCTM, 2000; RAND Mathematics Study Panel, 2003; Schachter, 2013). Thus, for students to be adequately prepared for college and professional success, high school graduation requirements have been expanded (College Board, 2000; Council of Chief State School Officers, 2010; Kober, et al., 2006; Martinez, et al., 2016; RAND Mathematics Study Panel, 2003). For example, before 2007, students in Oregon needed to complete only two years of mathematics to graduate from high school, and after 2007 it was raised to three years (Oregon Department of Education, 2006). Since 2010, all three years of mathematics instruction must be on the level of introductory algebra and above (Oregon Department of Education, 2017). Currently, over 40 states and the District of Columbia have adopted the Common Core State Standards for Mathematics, with the same high school requirements of at least three years of high mathematics courses at algebra level or higher (Common Core State Standards Initiative, 2019).

Historically in the United States, algebra fluctuated from not being part of the high school or college curriculum, to being a requirement for high school graduation (Chazan, 2008; Kilpatrick & Izsák, 2008). Since reforms in 2000, students are expected to learn algebra before the tenth grade (College Board, 2000). The "global competitiveness of the United States, equitable opportunities for students, the incorporation of algebraic thinking in the K—12 mathematics curriculum, and high-stake assessments" (Eddy, et al., 2015, p. 62) are supporting reasons for the Algebra for All movement. (American Diploma Project, 2004; College Board, 2000; No Child Left Behind Act, 2001). The effort of the movement is to help all high school students in the United State to pass algebra level classes (Silver, 1995).

Past and Present Teaching of Factoring Quadratic Functions

A significant part of any algebra curriculum is the ability to factor polynomials. Students in an introductory level of algebra learn how to factor quadratic functions, the second degree of polynomials. This skill is used to solve algebraic equations (Donnell, 2010; Roebuck, 1997) to simplify rational expressions, and solve rational equations. The ability to factor polynomials is also useful in more advanced mathematics courses. For example, factoring polynomials is used to solve linear differential equations, as well as in calculus, physics, and numerical analysis.

Short history of quadratic equations. Over the years, there have been various methods developed of solving quadratic equations and a variety of methods to teach this skill. Katz and Barton (2007) identified four main historical periods in the development of algebraic methodology:

> The conceptual stages [of algebra] are the geometric stage, where most of the concepts of algebra are geometric; the static equation-solving stage, where the goal is to find numbers satisfying certain relationship; the dynamic function stage, where motion seems to be the underlying idea; and finally, the abstract stage, where structure is the goal (p.186).

The view on factoring and solving quadratic equations has changed in every historical stage. At first, solving equations was interpreted as geometric problems, like in Babylonian algebra (2000–1700 BCE) and ancient Greece (Katz & Barton, 2007). Then, in about the ninth century, al Khwarizmi used algorithms to find solutions to certain quadratic equations. However, he still justified it by geometry, which is called "cut-and-paste" geometry (Katz & Barton, 2007). Later, European and Arabic mathematicians came up with algorithmic methods to solve quadratic equations; it was the static equation-solving stage (Katz & Barton, 2007). In Europe during the 17th century, an improvement of mathematical notation over the years allowed for the use of equations in the sciences, such as motion, and it started the dynamic function stage of algebra. In 19th and 20th century, "algebra had become less about finding solutions to equations and more about looking for common structures in many diverse mathematical objects" (Katz & Barton, 2007, p. 197). This period is called the abstract stage.

Currently used methods of solving quadratic equations. There are several basic methods to solve quadratic equations that are used presently in schools.

Table 1

Factoring Trinomials Using the Quadratic Formula

Name	Example
Quadratic Formula	Factor $6x^2 - 19x + 10$. We can use the quadratic formula to find the solutions to the quadratic equation: $x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{19-\sqrt{(-19)^2-4\cdot 6\cdot 10}}{2\cdot 6} = \frac{19-11}{12} = \frac{2}{3}$ $x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{19+\sqrt{(-19)^2-4\cdot 6\cdot 10}}{2\cdot 6} = \frac{19+11}{12} = \frac{5}{2}$ Therefore, $(x-\frac{2}{3})(x-\frac{5}{2}) = 0$ Our leading coefficient is 6, so we need to multiply 6, or 3×2: $6\cdot(x-\frac{2}{3})(x-\frac{5}{2}) = 0\cdot 6$, or $3\cdot(x-\frac{2}{3})\cdot 2\cdot(x-\frac{5}{2}) = 0$ After distributing 3 and 2, we got: $(2x-5)(3x-2) = 0$ Therefore, $6x^2-19x+10 = (2x-5)(3x-2)$

Sönnerhed (2009)

The main are: the quadratic formula (see Table 1), completing the squares (see Table 2), and incorporating zero property of multiplication, i.e., factoring of underlying polynomials.

Table 2

Factoring Trinomials by Completing the Square

Name	Example
Completing the Square	Factor $6x^2 - 19x + 10$. We can use the completing the square method to find the solutions to the quadratic equation: $6x^2 - 19x + 10 = 0$ $6\left\{x^2 - \frac{19}{6}x + \left(-\frac{19}{12}\right)^2\right\} - 6\left(-\frac{19}{12}\right)^2 + 10 = 0$ $6\left(x - \frac{19}{12}\right)^2 - \frac{361}{24} + \frac{240}{24} = 0 \quad \rightarrow \quad 6\left(x - \frac{19}{12}\right)^2 - \frac{121}{24} = 0$ Divide both sides by 6: $\left(x - \frac{19}{12}\right)^2 - \frac{121}{144} = 0$ $(x - \frac{19}{12})^2 = \frac{121}{144}$ $x - \frac{19}{12} = \pm\sqrt{\frac{121}{144}}$, $x - \frac{19}{12} = \pm\frac{11}{12}$ $x_1 - \frac{19}{12} = \frac{11}{12}$ or $x_2 - \frac{19}{12} = -\frac{11}{12}$ $x_1 = \frac{11}{12} + \frac{19}{12} = \frac{30}{12} = \frac{5}{2}$ $\quad x_2 = -\frac{11}{12} + \frac{19}{12} = \frac{8}{12} = \frac{2}{3}$ So, $(x - \frac{2}{3})(x - \frac{5}{2}) = 0$ Our leading coefficient is 6, so we need to multiply both sides of the equation by 6, or 3×2: $6 \cdot (x - \frac{2}{3})(x - \frac{5}{2}) = 0 \cdot 6$ therefore $3 \cdot (x - \frac{2}{3}) \cdot 2 \cdot (x - \frac{5}{2}) = 0$ After distributing 3 and 2, we got: $(2x - 5)(3x - 2) = 0$, Therefore, $6x^2 - 19x + 10 = (2x - 5)(3x - 2)$ Bosse & Nandakumar (2005)

These techniques are part of Common Core Mathematics Standards (Common Core State Standards Initiative, 2019). The first one, the quadratic formula, is the most powerful method that can be applied to factor all trinomials (Bosse & Nandakumar, 2005; Roebuck, 1997). The second one, completing the square, is also a method that could be used for all second-degree polynomials, and it has applications later in calculus classes (Bosse & Nandakumar, 2005; Flax, 1982; Moore, 1978; New York State Education Department, 1965; Sönnerhed, 2009). This method requires algebraic manipulations to form a perfect square and then solving the simplest case of quadratic equations: $ax^2 = b$. It always follows the same steps, and therefore, it is easier to teach and use.

Table 3

Factoring Special Cases of Trinomials

Name	Example
The Square of a Binomial $a^2 \pm 2ab + b^2 = (a \pm b)^2$	Factor $x^2 + 12x + 36$ Since x^2 is the square of x and 36 is the square of 6 and $12x$ is twice the product of x and 6, Then, $x^2 + 12x + 36 = (x+6)^2$ Mathematics 9th year (1966)
The Difference of Two Squares $a^2 - b^2 = (a+b)(a-b)$	Factor $x^2 - 9$ Since x^2 is the square of x and 9 is the square of 3 Then, $x^2 - 9 = (x+3)(x-3)$ Mathematics 9th year (1966, p. 195)
Greatest Common Factor	Factor $6x^2 - 9x$ The greatest common factor is $3x$, and using the distributive property: $6x^2 - 9x = 3x(2x-3)$ Mathematics 9th Year (1966)

The third technique, polynomial factorization, can be a long and complicated process. The success depends on the level of mastering many different formulas. Each of those formulas works only in specific circumstances. Students must identify which algorithm or series of algorithms to use, and then correctly apply them. There are three basic cases: finding the greatest common factor, the difference of two squares, and the square of a binomial (see Table 3).

In addition, there are more advanced methods, where the most commonly used are by "grouping" (Foerster, 1994; Lemon, 2004; McLean, 1959; New York State Education Department, 1965) (see Table 4) and "guess and check" (Larson, Boswell, Kanold, & Stiff, 2001; Lemon, 2004) (see Table 5).

Table 4

Factoring Trinomials Using "Grouping" Method

<table>
<tr><th>Name</th><th>Example</th></tr>
<tr><td>"Grouping"
or
"Long" Method</td><td>
Factor $6x^2 - 19x + 10$ (Using integers only)
We have to make the right decision about splitting the middle term.
Since 6×10=60, so the product of the two splitting coefficients must be also 60. At the same time the sum of those two coefficients must be -19
All the possibilities:
<table>
<tr><th>Splitting Coefficients</th><th></th><th>Product</th></tr>
<tr><td>– 1</td><td>– 60</td><td>60</td></tr>
<tr><td>– 2</td><td>– 30</td><td>60</td></tr>
<tr><td>– 3</td><td>– 20</td><td>60</td></tr>
<tr><td>– 4</td><td>– 15</td><td>60</td></tr>
<tr><td>– 5</td><td>– 12</td><td>60</td></tr>
<tr><td>– 6</td><td>– 10</td><td>60</td></tr>
</table>
The only way to split the middle term is $-19x = -4x - 15x$.
Therefore, $6x^2 - 19x + 10 = 6x^2 - 15x - 4x + 10 = 3x(2x-5) - 2(2x-5) = (2x-5)(3x-2)$
So, $6x^2 - 19x + 10 = (2x-5)(3x-2)$
Lemon (2004)
</td></tr>
</table>

Table 5

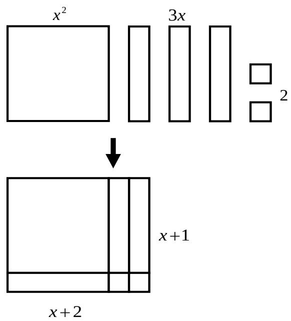
Factoring Trinomials by "Guess and Check" Method

Name	Example
"Guess and Check"	Factor $6x^2 - 19x + 10$ (Using integers only) 1. Since a leading coefficient is 6, therefore, there are two possibilities: $(1x - p)(6x - q)$ or $(2x - p)(3x - q)$ 2. The constant is 10, so the product of *p* and *q* is 10, which gives us two possibilities: 1×10 or 2×5 The list of all possibilities and outcomes $(1x-1)(6x-10) = 6x^2 - 16x + 10$ $(2x-1)(3x-10) = 6x^2 - 23x + 10$ $(1x-10)(6x-1) = 6x^2 - 61x + 10$ $(2x-10)(3x-1) = 6x^2 - 32x + 10$ $(1x-2)(6x-5) = 6x^2 - 17x + 10$ $(2x-2)(3x-5) = 6x^2 - 16x + 10$ $(1x-5)(6x-2) = 6x^2 - 32x + 10$ $(2x-5)(3x-2) = 6x^2 - 19x + 10$ Only the last combination produced the giving trinomial. Therefore, $6x^2 - 19x + 10 = (2x-5)(3x-2)$ Lemon (2004)

Furthermore, some teachers use physical models to teach factoring trinomials, like algebra tiles (Gibb, 1974; Hirsch, 1982; Sönnerhed, 2009) or factoring puzzles (Hollingsworth & Dean, 1975) (see Figure 1).

Algebra Tiles

Factor $x^2 + 3x + 2$



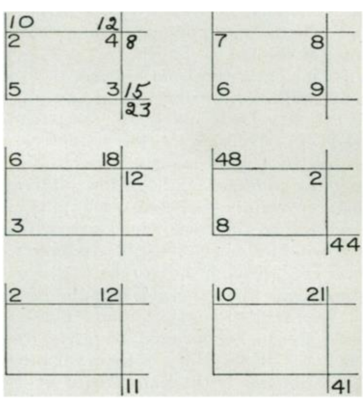
Therefore, $x^2 + 3x + 2 = (x+2)(x+1)$

Hirsch (1982, p. 388)

Factoring Puzzles

Factor $10x^2 + 23x + 12$

The first puzzle corresponds to factorization of this trinomial.



Therefore,

$$10x^2 + 23x + 12 = (2x+3)(5x+4)$$

Hollingsworth & Dean (1975, p. 428)

Figure 1. Factoring trinomials using physical models.

Also, there are several methods of factoring trinomials, very often created by teachers and only used locally. Autrey's & Austin's method (Autrey & Austin, 1979; Sönnerhed, 2009), Baker's method (Baker, 1969) (see Table 6), Savage's method (Savage, 1989) (see Table 8), and Moskol's method (Moskol, 1979) (see Table 7) are all examples of teacher-generated methods.

Table 6

Additional Methods of Factoring Trinomials

Name	Example
Autrey's & Austin's Method	Factor $8x^2+10x+3$ First write $(8x\quad)(8x\quad)$ leaving space to write other numbers after each 8*x*. Consider 8×3, where 8 is the coefficient of x^2 and 3 is the constant. We want to find two integers whose product is 24 and whose sum is 10, such as 4 and 6. Write these integers after 8x to get $(8x+4)(8x+6)$ Now, divide each parenthesis by their greatest common factor. Here 8 and 4 are divided by 4, and 8 and 6 are divided by 2: $(2x+1)(4x+3)$ Therefore, $8x^2+10x+3=(2x+1)(4x+3)$ Autrey & Austin (1979, p. 127)
Baker's Method	Factor $6x^2-41x+63$ $6x^2-41x+63=6x^2+ax+bx+63$ Where: $-41=a+b$, $\frac{6}{b}=\frac{a}{63}\Rightarrow ab=6\cdot 63=378$, and $(a-b)^2=(a+b)^2-4ab$ So, $(a-b)^2=(-41)^2-4\cdot 378=1681-1512=169$. Therefore, $a-b=\pm 13$ If $\begin{cases} a-b=13 \\ a+b=-41 \end{cases}$ So $\begin{array}{l} 2a=-28 \\ a=-14 \end{array}$ and $\begin{array}{l} 2b=-54 \\ b=-27 \end{array}$ $6x^2-14x-27x+63=2x(3x-7)-9(3x-7)=(3x-7)(2x-9)$ Therefore, $6x^2-41x+63=(3x-7)(2x-9)$ Baker (1969, p. 631)

Table 7

Additional Methods of Factoring Trinomials: Moskol's Method

Name	Example
Moskol's Method	Factor $6x^2 - 19x + 10$

Draw 3×3 box:

$6x^2$	10	$60x^2$

Because $6x^2 \times 10 = 60x^2$

Find two numbers with the product of $60x^2$ and a sum of $-19x$.

Put those numbers in the last column.

$6x^2$	10	$60x^2$
		$-4x$
		$-15x$

Because
$(-4x) \cdot (-15x) = 60x^2$ and $(-4x) + (-15x) = -19x$

Find a factor of $6x^2$ and a factor of 10 whose product is $-4x$. Put those numbers into second row.

<u>By convention, the linear factor must always be positive.</u>

$6x^2$	10	$60x^2$
$2x$	-2	$-4x$
		$-15x$

Find a factor of $6x^2$ and a factor of 10 whose product is $-15x$. Put those numbers into second row.

$2x$	-2	$-4x$
$3x$	-5	$-15x$

Therefore, $6x^2 - 19x + 10 = (2x - 5)(3x - 2)$

Moskol (1979, p. 676)

Table 8

Additional Methods of Factoring Trinomials: Savage's Method

Name	Example
Savage's Method	Factor $x^2 + 14x - 207$ Since the sum of the numbers must be 14, let the numbers be $(7+a)$ and $(7-a)$. Therefore, $(7+a)(7-a) = -207$ $49 - a^2 = -207$, so $a^2 = 256 \Rightarrow a = \pm 16$ The numbers required are $7+16$ and $7-16$. Therefore, $x^2 + 14x - 207 = (x+23)(x-9)$ Savage (1989, p.35)

Currently, the factoring methods used at schools are: the slip-slide method (Donnell, 2010; Steckroth, 2015), the X method (Lemon, 2004), and the Box method (see Table 9, 10, 11).

Table 9

Additional Methods of Factoring Trinomials: Diamond Method

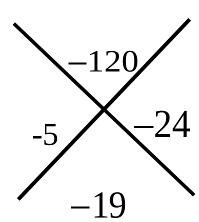
Name	Example
X-Method or Diamond Method	Factor $6x^2 - 38x - 80$ $6x^2 - 38x - 80 = 2(3x^2 - 19x - 40)$ Thus, a = 3, b = – 19, c = – 40 And,  Therefore, $2(3x+5)(3x-24)$ Divide each factor by its GCF: $2\left(\frac{3}{1}x + \frac{5}{1}\right)\left(\frac{3}{1}x - \frac{24}{3}\right) = 2(3x+5)(x-8)$ Therefore, $6x^2 - 38x - 80 = 2(3x+5)(x-8)$ Lemon (2004, p. 35)

Table 10

Additional Methods of Factoring Trinomials: Box Method

Name	Example
Box Method	Factor $10x^2+31x+15$

1. Factor GCF
2. Draw 2 $\times$ 2 box
3. Put the ax^2 term in the top left box and the c term in the bottom right box
4. Multiply this diagonal. The other blank diagonal has to multiply to be the same product
5. Find the positive or negative factors of this product to get the *bx* term. Those two factors will go in the blank boxes. Don't forget the variable!
6. Find the CCF from the top row and "solve" the box.
7. Write these factors using parenthesis. Don't forget the GCF from step 1!

10x²	
	15

$10x^2 \cdot 15 = 150x^2$

150=1$\times$150 150=3$\times$50 150=6$\times$25

150=2$\times$75 150=5$\times$30 150=10$\times$15

Therefore:

	5x	**3**
2x	*10x²*	*6x*
5	*25x*	*15*

The answer is:
$10x^2+31x+15=(5x+3)(2x+5)$

Harbin, D. (2013)

Table 11

Additional Methods of Factoring Trinomials

Name	Example
Slip-Slide Method	Factor $6x^2 - 19x + 10$ (Using integers only) Let's multiply the leading coefficient by the constant: $x^2 - 19x + 60$ We are looking for two numbers with a sum of –19 and a product of 60. Both numbers must be negative: –15 and –4. So, $(x-15)(x-4)$. Next, "slide" back the leading coefficient: $\left(x-\frac{15}{6}\right)\left(x-\frac{4}{6}\right)=\left(x-\frac{5}{2}\right)\left(x-\frac{2}{3}\right)$ Next, "slide" back the denominators in front of each x: $(2x-5)(3x-2)$ Therefore, $6x^2 - 19x + 10 = (2x-5)(3x-2)$ Steckroth (2015)

There is a debate regarding what method should be used in schools. The quadratic formula and completing the squares have wide applications and are based on specific algorithms that make them easier to teach than factoring. Therefore, in 1989, the National Council of Teachers of Mathematics' [NCTM's] Curriculum and Evaluation Standards for School Mathematics suggested teachers focus on using the quadratic formula and eliminating factorization tricks, which require a long time to teach and do not always provide an answer (NCTM, 1989). However, the quadratic formula cannot be used for higher degrees polynomial equations, but factorization could. Hence the factoring method is more universal. Therefore, for students to be prepared to factor higher degree polynomials, they should be exposed to and have experience with factorization earlier in introductory algebra class.

Similar to factoring, solving polynomial equations has different interpretations corresponding to historical stages of algebra. Katz and Barton (2007) argued that many algebraic concepts were born as geometric and, therefore, it would be logical to use geometric interpretation when teaching algebra. Many mathematics educators incorporate such an approach (Fachrudin & Putri, 2014; Flax, 1982; Hess & Hess, 1978; Moore, 1978). Hirsch (1982) and Gibb (1974) introduced physical models that could be used to solve quadratic equations. However, it has not been proven that visual geometric models improve students' understanding (Bunch, 1972) Additionally, all physical models have many limitations. Gibb (1974) pointed out that all physical models are positive, and therefore, the models could represent only polynomials with positive coefficients, usually natural numbers. In addition, Gibb (1974) mentioned the difficulties with finding physical models for polynomials with real coefficients because it requires dealing with lengths of radicals. Such models are not easy illustrations of factoring anymore.

The other physical models, which became more popular at the end of the 20th century, are manipulatives. Leitze and Kitt (2000) believed that "manipulating algebra tiles combines an algebraic and geometric approach to algebraic concepts" (p. 463). It is a way to make students understand the concept instead of memorizing the steps (Annette & Kitt, 2000; Leitze & Kitt, 2000). Some studies found evidence to support using hands-on learning as a tool to help struggling students (Annette & Kitt, 2000; Cruse, 2012; Goldsby, 1994; Patterson, 1997). Patterson (1997) suggested that manipulatives could be viewed as "the opportunity to bridge the gap to algebraic thinking" (p. 240). However, many research studies conducted in regular algebra classes were not able to establish the effectiveness of using manipulatives as an instructional tool (Goldsby, 1994; Li, 2007; Stockdale, 1980).

Challenges With Teaching Factoring Quadratic Functions

Factoring polynomials is one of the most challenging and complex topics to teach (Kotsopoulos, 2007). The difficulties come from the complexity of the procedures and the number of possible methods to choose form. A beginning algebra course is the first time students are introduced to the symbolic notation and abstract thinking in mathematics. They are required to decide which formula could work and perform multi-level calculations in their heads. Overwhelmed students often view memorization as a possible way to succeed in algebra instead of truly understanding it (Annette & Kitt, 2000; Leitze & Kitt, 2000).

In addition, mastering factorization requires prior knowledge and skills (Bransford, Brown, & Cocking, 2000; Glaser, 1984; Rauff, 1994). Therefore, if students did not fully learn all operations on real numbers they may struggle with performing factorization. Many researchers show that, in general, students in introductory high school level courses are not ready to learn algebra. They struggle with basic arithmetic, algebraic manipulations (Boulton-Lewis et al., 2001; Didis & Erbas, 2015; Kotsopoulos, 2007; Martinez et al., 2016; Nielsen, 2015; Sells, 1973; Zakaria & Maat, 2010), and understanding the equal sign (Kieran, 1981; Martinez et al., 2016). Frequently, students have problems with mathematical symbols and letters, which Maredi and Oosthuizen (1995) called "the language barrier" (p. 245). For example, some students believe that alphabetic order matters and, therefore, the letter that comes first must represent a smaller number then the letter that comes later (Wagner, 1981), or they assume that x on different sides of the same equation represents two different numbers (Nielsen, 2015).

The other potential challenge could come from misunderstanding the exponents (Lima, 2008). Solving quadratic equations requires mastering not only natural exponents, but also fractional ones, such as square roots. The confusion could easily be illustrated by incorrect way of calculating the square of binomial: $(a+b)^2 = a^2 + b^2$ (French, 2002), or by focusing on only one solution of quadratic equation instead of possibly two (Nielsen, 2015).

Another identified challenge is related to the introductory algebra curriculum. Solving quadratic equations comes after studying linear equations. Therefore, students automatically try to use the same steps they have learned not long ago (Lima, 2008). Unfortunately, the method of "isolating a variable" does not work anymore. It needs to be replaced by leaving zero on one side of an equation (when factoring or using the quadratic formula) or by leaving a constant number on one side (when using the completing the square).

Some studies focused on teachers' contribution to ineffectiveness of teaching algebra. For example, Duarte (2010) conducted a study to examine prospective middle school teachers' confidence and knowledge about quadratic functions. Fifty-two prospective teachers and two instructors were observed, tested, filling questionnaires, and five of them were interviewed. The study took place within a period of five months. The results showed that "there were mismatches between prospective teachers' confidence and their actual mathematical understanding" (p. 1). Similarly, the study of Li (2007) of 72 middle school and high school algebra teachers in Texas showed some weaknesses of understanding algebraic equations: "(a) the balancing method, (b) the concept of equivalent equations, and (c) the properties of linear equations in their general forms" (p.1). However, mathematical knowledge seems not to be enough. Kotsopoulos (2007) pointed out that algebra teachers need to possess an understanding of how students learn: "It is necessary for teachers to have insight on how the brain creates memory and pedagogical directions might be enhanced with that knowledge" (p. 19).

Another problem could be related to the current methodology of teaching, and/or a curriculum itself. Challenges with factoring trinomials provoked teachers and educators to come up with many different methods that supposed to be easier for students to learn. Are the outcomes what educators hoped for? Is it possible that there are too many methods that confuse students? A 30-year-old suggestion of NCTM (1989) to focus only on the quadratic formula represents an opposite view. The idea of removing from introductory algebra courses all different ways of solving second degree polynomials keeps coming back as a possible solution to the overcomplicated current methodology. However, while this method may bring comfort to students, it does not prepare them for factoring polynomials in the following algebra classes (Gray & Tall, 1994; Lima, 2008; Sönnerhed, 2009).

International Study. It is important to examine some international research of teaching factoring trinomials as a comparison to the U.S. system. It seems like everywhere factoring is not an easy topic to teach, and therefore some educators do not include it in the introductory mathematics courses, or just include certain topics only. For example, Sönnerhed's (2009) study that investigated the pedagogical structure of eight textbooks for Swedish upper-secondary mathematics reveled that in all books factorization takes the least amount of time. The program included four teaching methods of solving trinomial equations: the quadratic formula, the completing the square, square root method for simple quadratic equation, and factorization. The main focus was put on the quadratic formula and the completing the square method.

Similarly, Lima (2008) showed that mathematics teachers in Brazil taught the quadratic formula only "to guarantee success for their students" (p. 7) and dismissed all other ways of solving quadratic equations, including factoring. The results pointed out the lack of students' understanding of algebraic principles and focus on memorialization. Lima (2008) stressed the importance of challenging students in order to develop their proper conceptual understanding and pointed out that “being able to choose a suitable method for each situation is not a matter of memorizing them all” (p. 7).

The same problems with students memorizing formulas and procedures without understanding the concept was reported in Turkey. Didiş et al. (2011) conducted a study of 113 high school students. It showed that factoring trinomials was difficult for them. Students knew some formulas but struggled with making decision which one should be used. The results reveled “lack of both instrumental and relational understanding the associated mathematics” (p. 2).

The interesting study was conducted by Vaiyavutjamai et al. (2005) to compare abilities to solve the same quadratic equations by ninth grade students in Thailand, tenth grade students in Brunei Darussalam, and second-year university students in the United States. Overall, 465 students participated in the study. The results reviled that “most of the students in the three subsamples acquired neither instrumental nor relational understanding of elementary quadratic equations” (p. 735). The authors called for new international mathematics education research to “guide teachers about how students think about quadratic equations, and especially about what can be done to help teachers to improve their students’ concepts of a variable in that context” (p. 742).

Multiple Representations

Deficiencies of students’ conceptual understanding of factoring polynomials reported by numerous studies (Vaiyavutjamai & Clements, 2006; Vaiyavutjamai et al., 2005; Zakaria & Maat, 2010) call for search for more effective methodologies. One possibility would be to improve effectiveness of using multiple representations of quadratic functions as early as possible; even to introductory algebra students. According to Ainsworth (1999) “a common justification for using more than one representation is that this is more likely to capture a learner's interest and, in so doing, play an important role in promoting conditions for effective learning" (p. 131). Some of the research suggested exposing students to different strategies and interpretations, like visual, hands-on, and abstract (Ainsworth, 2006; Cabahug, 2012; Ogbonnaya et al., 2013), as it is believed that "when learners can interact with an appropriate representation their performance is enhanced" (Ainsworth, 2006, p. 183).

What are the benefits of using multiple representations? According to Ainsworth (1999) it has three main functions: "complementary, constrain, and construct" (p. 134). Teachers use different representations to achieve specific goals, like to expose students to *complementary* information, to *constrain* students to different interpretations, or to *construct* a deeper understanding. It is crucial, according to Ainsworth (1999), to be aware of educational goals of a teacher before measuring outcomes of using specific representations.

Zazkis and Liljedahl (2004) viewed the role of multiple representations differently. They believed that multiple representations could be used as "tools for manipulation and communication and tools for conceptual understanding" (p. 167). The first one, serves two functions: communicating ideas and communicating between people about given representations. Zazkis and Liljedahl (2004) stressed the importance for students to possess abilities to communicate effectively their own thoughts as well as to "recognize and interpret what ideas are being communicated by the symbols" (p. 167). The second role is related to forming or constructing knowledge by being exposed to representations.

Nevertheless, all research agreed that "learners find translating between representations difficult" (Ainsworth, 1999, p. 132), and even more, for some students it seems to be impossible to transfer from one representation to the other (Molenje, 2012). If students cannot see connections between representations then they cannot benefit from multiple representations

(Ainsworth, Bibby, & Wood, 1998; Moschkovich, Schoenfield, & Arcavi, 1993). Therefore, the use of multiple representations must be carefully selected based on students' abilities, mathematical topic, and educational goals.

Graphing Technology in Algebra Class

Effectiveness of using different methodologies to teach algebra concepts is a subject of deliberation of mathematics educators. An interesting perception was suggested by Katz and Barton (2007). They pointed out at the history of algebra as a "potential direction that methodology of teaching algebra should aim, as they believed in the value of the pedagogical implications of the history of algebra" (p. 197). Thus, they suggested that abstract concepts should be left to teach at the end, after mastering numbers, geometry, and "real" algebra. In contrast, Kastberg and Leatham (2005) focused on importance of methods chosen to teach a specific concept besides the progression of topic in math curriculum. They stressed that large-scale curriculum is not the only option for effective teaching, though, "the method of instruction was a critical factor in determining student learning" (p. 30). The other suggestions pointed out at technology and its role to enhance methodologies of teaching algebra (Gillespie, 2013; Pan, 1998). It is an opportunity of current times that should be taken into consideration by mathematics educators. All the features of multiple representations apply to educational technology. Therefore, technology, should be incorporated into teaching processes based on the pedagogical goals of a teacher. Kastberg and Leatham (2005) reported that "although developing more than one representation for a mathematical object can take a substantial amount of time, effective use of graphing calculators allows quick and easy development of and translation between representations" (p. 28).

It seems natural to expect technology, so dominant in every aspect of people's lives, to be included in mathematics classroom. Over 20 years ago, Pan (1998) emphasized: "the robots of the future are waiting" (p. 69) and "speedy computer algorithms offer new answers" (p. 62). Educators and researchers expected technology to dramatically impact and reform teaching and learning mathematics (Barrett & Goebel, 1990; Demana & Waits, 1990). Almost 30 years later, it seems like technology in fact keeps impacting mathematics classroom, however, not always in the most effective ways. Gillespie (2013) reported the point of view of Salman Kahn, the founder of the Kahn Academy, who stressed the general misuse of technology in K–12 schools; technology that could be a foundation to the "radically transform education" (p. 38). Salmon Kahn's vision included incorporating technology into effective self-paced learning, based on students' interactions.

So far, the impact of technology on mathematics education has taken its own pace and direction. In addition to offering new teaching methods, it has changed the access to information, the way data is stored or displayed, the method of communication, assessment, and more. Overall, "technology has been widely recognized as a significant teaching and learning tool" (Thach & Norman, 2008, p. 152), especially in mathematics. National Council of Teachers of Mathematics (NCTM) (2000) pointed out that technology "influences the mathematics that is taught and enhances students' learning" (p. 24). Furthermore, Thach and Norman (2008) stressed that technology not only reinforces skills and strategies but also can facilitate open-ended exploratory experiences that deepen mathematical understanding (p. 157) and allows students to study and inspect algebra and statistics, explore their own ideas and interpretations (Bessier, 2006; Hyun & Davis, 2005). Students use technology to present ideas, represent and explore concepts, manipulate variables, graphs, objects, and mathematical models (Thach & Norman, 2008).

However, according to Thach and Norman (2008), in order for technology to be effective, it must be “a tool for facilitating learning across the curriculum” (p. 152) which is still challenging. The need for such integration was addressed by the International Society for Technology in Education (ISTE) (2007) in the ISTE Standards by putting more stress on *developing-skills-and-strategies-through-the-use-of* technology, instead of *learning-to-use* technology.

In algebra classrooms, computers and graphing calculators are two main types of technology that could be incorporated in the curriculum. Each of them has advantages and disadvantages. Both of them could be used as a supplementary tool in teaching factoring quadratic functions.

Graphing calculators. One of the most accessible piece of technology in algebra classroom is a graphing calculator. Laughbaum (2003) called it “teaching enhancement” that should become permanent tools in every algebra class (p. 301). The National Council of Teachers of Mathematics (NCTM) and the National Research Council (NRC) encouraged using graphing calculators as a supplementary tool in mathematics classroom (Dreiling, 2007; NCTM, 1989). Three decades ago, NCTM (1989) listed the conditions that each student should be provided at school:

(a) appropriate calculators would be available at all times

(b) a computer would be available in every classroom for demonstration purposes

(c) every student would have access to a computer for individual and group work

(d) students would learn to use the computer as a tool for processing information

(e) students would learn to perform calculations to investigate and solve problems (p. 8)

The role of incorporating graphing calculator in teaching mathematics was outlined by Laughbaum (2003):

> The graphing calculator is not only a teaching tool in the classroom in the hands of the teacher; it is also a teaching tool in the hands of students when they are given investigations, concept development and guided discovery exercises, open-ended homework exercises, and summative modeling projects. In addition to these teaching tools, the introduction of calculator applications has opened the door for many new kinds of activities. (p. 301)

Numerous research studies pointed to visual technology as an “opportunity to explore representations and their connections” (Drijvers et al., 2014, p.17) in algebra class (Hong & Thomas, 2006; Kidron, 2001; Thomas, Monaghan, & Pierce, 2006). Dreiling (2007) reported that viewing graphing calculators as a visual representation, like a physical model or manipulatives, could improve students’ understanding. Similarly, Cheung and Slavin (2013) concluded that using graphing calculators increases understanding of mathematical concepts and abilities in problem solving, which could be verified by students’ improved test scores. However, “the type and extent of the gains are a function of how the technology is used in the teaching of mathematics” (Drijvers et al., 2014, p.15). Also, Wilkins’s (1995) study of a small group of eighth grade students concluded that using graphing calculators as an enhancement to regular instructional pedagogy in basic algebra class benefited students, especially female students. They scored significantly higher in problem solving involving factoring polynomials. Also, a meta-analysis of 54 research studies “revealed that students’ operational skills and problem-solving improved when calculators were an integral part of testing and instruction” (Ellington, 2006, p.

433). However, the study suggested a further research to measure the retention level of the acquired algebra knowledge and skills.

Using graphing calculators was expected to have huge positive impact on learning mathematics. However, “educational technology is making a modest difference in learning of mathematics. It is a help, but not a breakthrough” (Cheung & Slavin, 2013, p. 20). Using graphing calculators has limitations of which mathematics teachers as well as students need to be aware (Mitchelmore & Cavanagh, 2000). The study of 25 high school students conducted by Mitchelmore and Cavanagh (2000) showed students tend to use graphs carelessly, without true understanding and connection with mathematical concepts. Mitchelmore and Cavanagh (2000) identified four basic explanations for students’ ineffectiveness of using graphing calculators: “a tendency to accept the graphic image uncritically, without attempting to relate it to other symbolic or numerical information; a poor understanding of the concept of scale; an inadequate grasp of accuracy and approximation; and a limited grasp of the processes used by the calculator to display graphs” (in Drijvers et al., 2014, p. 15). Furthermore, Hong, Thomas, and Kiernan (2001) reported that incorporating graphing calculators could compromise understanding of algebra concepts by students with learning difficulties, which is parallel to findings of Kendal and Stacey (2000) who concluded that only the most talented students can benefit from being exposed to multiple representation: numerical, graphical, and symbolic.

Lack of consistency in effectiveness of using technology reported by research studies could be linked to the fact that incorporating technology into teaching methods must be influenced by the educational goals of teachers. *When* and *how* to use technology in algebra class remains unclear. Which mathematic topic should be taught using traditional methods, which one with technology only, and which integrated way is a subject of debate (Kendal et al., 2005).

Obviously, “different areas of the curriculum have their own needs” (Drijvers et al., 2014, p. 18) as well as each algebra topic.

The Role of Teachers

Teachers are a central part of mathematics education. The RAND Mathematics Study Panel (1999) pointed out that “the quality of mathematics teaching and learning depends on what teachers *do* with their students, and what teachers *can do* depends on their knowledge of mathematics” (pp. 16-17). This is especially true for the quality and success of incorporating technology in mathematics education (Hoyles et al., 2010). The NCTM (2000) stated that “effective use of technology in the mathematics classroom depends on the teacher” (p. 25). Therefore, knowledge, attitudes and beliefs of teachers about using technology are fundamental to the successful learning experience of students.

Incorporating technology into mathematics classroom requires many changes to assure its effectiveness. Kyungsoon (1999) researched the relationship between using graphing calculators and teachers’ practice. The author noticed that over time, teachers used gradually more graphing calculators in their lesson plans. Also, they adjusted the content and their pedagogy to the particular use of technology. Kyungsoon (1999) reported that each teacher used graphing calculators differently, based on their interpretations. Karadeniz (2015) conducted a study of 11 teachers in a high school precalculus class. The results showed that teachers believed in importance of using graphing calculators to explore math, solve problems, check work, and look for additional solutions. At the same time, they expressed concerns about students relying too much on graphing calculators. Teachers pointed out that students’ arithmetic skills decreased when students started using technology for all calculations.

According to Drijvers et al. (2014), there are five basic factors related to the success of using technology: teachers' orientations, the instrumental genesis (Artigue, 2002; Kendal et al., 2005; Rabardel, 1995; Verillon & Rabardel, 1995), the orchestration of technological tools (Drijvers & Trouche, 2008; Trouche, 2005), teachers' perception of the nature of mathematical knowledge and how it should be learned (Zbiek & Hollebrands, 2008), their mathematical content knowledge; and their mathematical knowledge for teaching (Hill & Ball, 2004; MKT-Ball, Hill & Bass, 2005; Zbiek, Heid, Blume, & Dick, 2007). Based on those factors, teachers invest in their knowledge and skills in technology, restructure their classroom to match the new educational conditions (orchestration of classroom), and refine their role as teachers.

Teachers' knowledge. The main factor of teachers' professional success is their understanding of the subject matter and their abilities to incorporate various methodologies to teach it. Unfortunately, some research studies revealed insufficient teachers' content knowledge in algebra (Caswell, 2009; Darley & Leapard, 2010; Duarte, 2010; Li, 2007; Jones, 2010). Incorporating professional development to improve teachers' understanding were suggested by Steele, Johnson, Otten, Herbel-Eisenmann, and Carver (2015). However, the findings of Li's (2007) study did not demonstrate usefulness of professional development but pointed out at teachers' academic backgrounds as being more significant. Seventy-two algebra teachers were given an assignment, and the teachers who majored in mathematics overperformed others. The results showed that the length of experience in teaching or the number of courses taken in mathematical education did not make a significant difference in the teachers' mathematical knowledge and skills.

Besides understanding the subject matter, in order to effectively incorporate technology, teachers have to "understand information technology broadly enough to apply it productively at work" and "to recognize when information technology can assist or impede the achievement of a goal and continually to adapt to changes in information technology" (Drijvers et al., 2014, p. 10). Subject matter was divided into three types by La Rocque Palis (2010): technological, pedagogical, and content knowledge. All three are necessary to design successful lesson plans where technology joins other methods to focus together on reaching the teachers' educational goals. Unfortunately, many teachers need to improve their knowledge and practical skills before they can successfully use technology in their classrooms. (Zbiek & Hollebrands, 2008).

Teacher-student relationships. One of the main implications of using technology in the mathematics classroom is the change it creates in the traditional teacher-student relationship. Technology initiatives somewhat change to the role of a teacher to a mediator between different approaches to mathematic concepts (Doerr & Zangor, 2000; Geiger et al., 2008; Rivera, 2011; Goos & Bennison, 2008). Heid, Sheets, and Matras (1990) referred to teachers as students' facilitators, connectors, and integrators, and Hoyles and Lagrange (2010) referred to them as facilitators, gatherers and validators. The different roles of teachers and their relationship with students are critical to embrace, since each "teacher is a key to the successful use of digital technology in the mathematics classroom" (Drijvers et al., 2014, p. 10). Ruthven, Deaney, and Hennessy (2009) also underlined the importance for teachers to be "pre-structure and shape of technology-and-task-mediated student activity" (Drijvers et al., 2014, p. 12).

Classroom orchestration. Using technology implies changes in classroom management and organization. The goal is to adapt to the "proliferation and increasing complexity of technological resources" (Drijvers et al., 2014, p. 11) in the classroom and engage students to learn. Ruthven (2012) recognized five structuring aspects of classroom practice: working environment, resource system, activity format, curriculum script, and time economy (Drijvers et al., 2014, p. 12). The study of Doorman et al. (2012) researched the relationship between learning arrangement and understanding of functions. The results were inconclusive, because it is difficult to measure students' learning outcomes based on the classroom management and organization. However, many other research studies concluded that adjusting classroom orchestration is crucial for learning outcomes (Doorman et al., 2012; Hivon et al., 2008; Hoyles et al., 2010; Kendal et al., 2005).

Teachers are unsure about technology. Not all teachers are willing to change their teaching styles because of technology. Dreiling (2007) studied 157 high school mathematics teachers and showed that graphing calculators were more often incorporated in higher-level classes and they were used because of the visual representation the provided. However, teachers did not modify their ways of teaching because of the technology. Instead, "teachers use graphing calculators as an extension of their existing teaching style" (p. 1).

Also, not all educators believe that technology has a place in mathematics classrooms (Forgasz, 2006; Goos & Bennison, 2008; Thomas, 2006). Laughbaum (2003) found "many argue that every piece of technology used to teach mathematics takes away from teaching time" (p. 301). Overloaded curricula and standardized tests put a lot of pressure on teachers and limit their instructional time (Kastberg & Leatham, 2005). Incorporating technology requires additional training and based on mixed results of research, teachers' efforts may not be rewarded with improved test scores (Kendal & Stacey, 1999).

Taking advantage of technology. There is a consensus among educators that mathematics teachers do not take full advantage of technology even if readily available (Quesada & Dunlap, 2011). For example, Fleener's (1995) study showed that over 70% of 94 middle school and secondary mathematics teachers did not incorporate calculators that were available to them. In addition, more than 70% of students had access to their own calculators but were not allowed to use them. The same study showed that 90% of the teachers agreed that calculators are motivational by allowing students to solve more complicated problems and making mathematics more fun (Fleener, 1995).

Similarly, Quesada and Dunlap (2011) conducted research focused on whether mathematics teachers "taking full advantage of the capabilities that hand-held graphing technology offers, without Computer Algebra Systems (CAS)" (p. 1). One hundred and twenty teachers took a test and a survey about their level of knowledge of using hand-held graphing technology in different pre-calculus topics. The results of the test were very low, and teachers admitted to using a mostly traditional teaching style. According to Laughbaum (2003), "A national survey in 1998 confirmed that the majority of developmental mathematics educators are not using hand-held graphing technology in mathematics classrooms" (p. 301). Rajan's (2013) study showed that teachers and students in introductory mathematics believed in using graphing calculators for complex computational formulas only.

Self-efficacy of teachers. Effective use of technology requires not only new knowledge and skills, but also refining methodology, classroom orchestration, and the role of teachers. According to Achurra and Villardón (2013), "teachers with high self-efficacy levels are more open to new ideas, show greater willingness to try new teaching methods, design and organize their classes better, and are more enthusiastic and satisfied with their teaching" (p. 367). Therefore, teachers with a high level of self-efficacy are most likely pursue learning about technology and experiment with incorporating technology in their lesson plans to more effectively meet student learning needs (Tschannen-Moran & Hoy, 2001). Chen (2010) identified self-efficacy as one of two main factors influencing teacher's use of technology. The second category was training and value. Conversely, many researchers identified a deficiency of self-efficacy as a cause for teachers to underutilize technology (Kellenberger, & Hendricks, 2003).

Self-efficacy was defined by Bandura (1977a) as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (p. 3). Teachers' self-efficacy has been linked to student achievement by numerous research studies (Achurra & Villardón, 2013; Ashton & Webb, 1986; Gibson & Dembo, 1984; Knoblauch & Hoy, 2008; Muijs & Reynolds, 2005; Ross, 1992; Skaalvik & Skaalvik, 2007). In particular, the study of Achurra and Villardón (2013) of 71 teachers and over 200 students from the University of Deusto and the Catholic University of Temuco revealed significant a relationship between teachers' self-efficacy and students' perceived learning. Interestingly, the study showed that "nationality marks significant differences in teachers' self-efficacy levels" (p.374).

Theory. The research is based on Vygotsky's sociocultural theory of learning which stresses the importance of social interaction during processes of developing scientific concepts. According to Vygotsky (1978), learning a mathematical concept, which is a scientific concept, and requires a human and symbolic mediation. Kozulin (2003) distinguished between those two types of mediations as "another human being and learning activity" (p. 17).

The critical role of a human mediator in students' learning processes comes from the belief that "each psychological function appears twice in development, once in the form of actual interaction between people, and the second time as an inner internalized form of this function" (Kozulin, 2003, p. 19). Vygotsky's theory put teachers in a central place in student's learning. Teachers use language and mathematical symbols to communicate and to mediate the development of mathematical concepts. Therefore, “in this way, the individual’s mathematical knowledge is both cognitively and socially constituted” (Berger, 2005, p. 153).

The symbolic mediator could be any learning activity tool. According to Verenikina (2010) “tools can be either external (physical, technical) such as artifacts, instruments and machines or internal (psychological) such as laws, signs, procedures, methods and language” (p. 21). Computers and all kinds of technology are types of physical tools (Verenikina, 2010). However, not all physical and psychological tools are learning mediators. Kozulin (2003) stressed that:

> The symbolic tool fulfills its role only if it is appropriated and internalized as a generalized instrument, that is, a psychological tool capable of organizing individual cognitive and learning functions in different contexts and in application to different tasks. (p. 26)

Based on Vygotsky’s theory, using technology to teach factoring quadratic functions could become a symbolic mediator if it organizes students’ knowledge and adds supplementary structure.

Summary

Teaching factoring quadratic functions is challenging, especially to students who are learning it for the first time. Based on the literature review presented in this chapter, the problems are recorded not only in U.S. classrooms but also around the world. Many deliberations among mathematics educators have focused on developing the most effective curriculum and styles of teaching. However, there is no consensus about what methods of factoring trinomials should be included in introductory algebra courses. This curriculum dilemma is linked to the question “how?” Using technology to enhance instructional pedagogy seems to be very strongly supported by research. Particularly visual technology, like graphing calculator, could supplement teachers' regular practice and help with visualization, interpretation of graphs, and conceptual understanding, as well as to provide students with a way to check or to estimate answers in the 'guess and check' method.

The use of technology requires a constant enhancement of knowledge, skills, methodology, and classroom management and organization. It also can alter the role of the teacher. It is linked to teachers' level of self-efficacy, which describes the level that teachers believe in their abilities to perform a task. This study examined the relationship between the level of implementation of visual technology, such as graphic calculators, in teaching factoring quadratic functions to introductory algebra students and the level of self-efficacy of the teachers. The second goal of the study was to examine the relationship between the level of self-efficacy of the teachers and the teachers' perceived impact of their instruction on student learning when teaching factoring quadratic functions to introductory algebra students.

Chapter 3 introduces the research design appropriate for responding to the research questions. It includes sampling methods and the survey instrument used to gather data, as well as procedures executed to analyze data.

Chapter 3: Methodology

The challenges with teaching factoring quadratic functions to introductory algebra students are widely recognized among mathematics educators internationally. Consequently, algebra teachers are constantly learning and changing pedagogy to effectively introduce factoring to their students. Since the level of self-efficacy of teachers has been linked to a willingness to adopt changes (Achurra & Villardón, 2013) and to students' achievement (Knoblauch & Hoy, 2008; Muijs & Reynolds, 2005; Ross, 1992; Skaalvik & Skaalvik, 2007), this study explored the relationship between self-efficacy of the teachers and their preferences of using mathematical methods in teaching factoring quadratic functions to introductory algebra students and the teachers' self-perceived impact of their instruction on student learning.

In this chapter, the researcher introduces the research design appropriate for responding to the research questions. It includes sampling methods and the survey instrument used to gather data, as well as procedures executed to analyze data.

Purpose of the Study

The purpose of the study was to determine whether there is a relationship between self-efficacy of international algebra teachers and their level of incorporating technology in teaching factoring quadratic functions to introductory algebra students. Secondly, the researcher aimed to examine the influence of self-efficacy on the perspective of international algebra teachers with respect to the methods they use to teach factoring quadratic functions to introductory algebra students.

Research Questions

The following research questions were examined in the study:

1. What is the relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?
2. How does the intersection of self-efficacy of teachers and teaching factoring quadratic functions to introductory algebra students, influence teacher preferences in mathematical methods of instruction?
3. What differences exist between collectivistic and individualistic countries with respect to perceptions of algebra teachers self-efficacy in teaching, and (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?

Hypotheses

$H1_0$. There is no relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.

H1a. There is a relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.

$H3_0$. There are no differences between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students

H3a. There are differences between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students

Research Design

The study used the ex post facto design. The ex post facto design entailed looking at existing conditions without any direct manipulation of the independent variable; analyzing and comparing distinct independent and dependent variables. The existing condition in this study was the challenge of teaching factoring quadratic functions to introductory algebra students. The distinct independent and dependent variables were: the self-efficacy of the algebra teachers (the independent variable – IV) and the level of incorporating graphing technology in teaching factoring quadratic functions (dependent variable – DV), and the choice of mathematical methods when teaching factoring quadratic functions to introductory algebra students (dependent variables – DV) respectively. The self-efficacy (IV) occurred without researcher manipulation and the researcher was not responsible for imposing it; hence, it was an *experience* rather than a manipulated treatment. In this study, the researcher searched to determine whether the level of self-efficacy (IV) of the algebra teachers was related to the level of implementation of graphing technology in teaching factoring quadratic functions (DV_1), and the choice of mathematical methods when teaching factoring quadratic functions to introductory algebra students (DV_2).

Target Population, Sampling Method (Power) and Related Procedures

The target population was international algebra teachers. It was comprised of mathematics teachers from the United States and from different countries, experienced in teaching factoring quadratic functions. The target population was teachers who attended the International Conference of Mathematics Educators in Hungary in 2017 (ICME) and the Conference in Contemporary Mathematics Education in Poland (CCME) in July 2018. The

ICME in Hungary had 128 teachers and researchers from 22 countries (see Table 12). The CCME in Poland had 78 participants from 15 countries (see Table 13).

Table 12

Countries Represented at the ICME in Hungary in 2017

	Country	Continent	Number of Participants
1.	Ghana	Africa	1
2.	Namibia	Africa	1
3.	South Africa	Africa	6
4.	**USA**	**North America**	**32**
5.	Peru	South America	2
6.	India	Asia	6
7.	Japan	Asia	1
8.	Israel	Asia / Middle East	11
9.	Iran	Asia / Middle East	2
10.	Jordan	Asia / Middle East	2
11.	Lebanon	Asia / Middle East	2
12.	Australia	Australia	16
13.	Czech Republic	Europe	2
14.	Germany	Europe	6
15.	Greece	Europe	3
16.	Hungary	Europe	8
17.	Ireland	Europe	8
18.	Italy	Europe	3
19.	Poland	Europe	2
20.	Slovenia	Europe	1
21.	Spain	Europe	6
22.	UK	Europe	7
		Total	128

In addition, each of the participants were asked to share a link of the study's survey with other mathematics teachers in their countries who might participate in the study. The primary sampling method was a convenience sampling with a secondary snowball sampling. In the

United States, 32 algebra teachers took part in the study. The international countries included Ghana, Namibia, South Africa, Peru, India, Japan, Israel, Iran, Jordan, Lebanon, the Czech Republic, Germany, Greece, Hungary, Ireland, Italy, Kazakhstan, the Netherlands, Poland, Portugal, the Russian Federation, Slovenia, Spain, the United Kingdom, and Australia. The countries were categorized as individualistic or collectivist as defined by Hofstede (1983, and n.d.).

Table 13

Countries Represented at the CCME in Poland in 2018

	Country	Continent		Number of Participants
1.	Czech Republic	Europe		6
2.	Denmark	Europe		1
3.	Germany	Europe		3
4.	Greece	Europe		6
5.	Hungary	Europe		2
6.	Israel	Asia / Middle East		4
7.	Italy	Europe		6
8.	Kazakhstan	Europe		1
9.	Latvia	Europe		1
10.	Netherlands	Europe		3
11.	Poland	Europe		39
12.	Portugal	Europe		2
13.	Russian Federation	Europe		1
14.	Slovakia	Europe		1
15.	USA	North America		2
			Total	78

Prior to conducting this study, a power analysis indicated that at least 84 mathematics educators should complete the survey for the researcher to achieve a minimum power level of 0.80, with alpha = 0.05, and a moderate effect size for the study.

Instrumentation

The researcher used a survey instrument, the Using Visual Graphing in Introductory Algebra survey instrument (UVGIA). The UVGIA was specifically created to measure three variables, the independent variable and two dependent variables, and the demographics of the participants (see Appendix A). The final version of the survey contained four parts: background information, teachers' sense of self-efficacy scale, role of technology in teaching factoring quadratic functions, and teaching factoring trinomials. The first part collected the demographics of the participants and each of the remaining survey sections measured a different variable per section. The structure of the first part (see Appendix A, Part 1) was based on a survey created by Molenje (2012) who put all the background information of the participants at the beginning of his survey. This section gathered information about participants' origins, preparation and experience in teaching mathematics. The following questions were also added: the name of the country (1), preparation for teaching mathematics (4) and (5), the teaching experience (6 and 7), and the current teaching assignment (8). Overall, the backgroundiInformation comprised eight questions (see Appendix A, Part 1). The data collected in this part was categorical.

The second part, with 12 items, measured the level of teachers' self-efficacy (see Appendix A, Part 2). This part was adopted from Tschannen-Moran & Hoy's (2001) Teachers' Sense of Efficacy Scale (TSES). The study was granted the permission from both authors to incorporate the TSES survey instrument (see Appendix B). Each item of the survey focused on specific teaching ability and used a 9-point Likert scale. The TSES tested three dimensions: student engagement, instructional strategies, and classroom management. According to Tschannen-Moran and Hoy (2001), the TSES has been verified as a reliable and valid instrument to measure self-efficacy of teachers. The Cronbach Alpha reliability coefficient was 0.90 for the

TSES. In each dimension, Cronbach Alpha was determined as 0.81 for student engagement, 0.86 for instructional strategies, and for classroom management (see Appendix C). A few items included on this survey were "How much can you do to control disruptive behavior in the classroom?" and "How much can you do to motivate students who show low interest in school work?"

The level of self-efficacy was measured by an average of a person's scores in this section. The data collected in this part was continuous, as the average level of self-efficacy could be any number from 1 to 9, where 1 represented the lowest level and 9 the highest level of self-efficacy. The second reason the data was considered continuous is the survey was designed to measure the construct of self-efficacy and so the items were not analyzed individually.

The third part of the survey measured the role of technology in teaching factoring quadratic functions and contained two sections. The first one focused on teachers' beliefs and knowledge and the second on the use of visual technology in the process of teaching factoring quadratic functions (see Appendix A, Part 3). Each item of the second section identified a different way and goal of incorporating graphing tools to teach factoring. For example, question 5 focused on computational abilities of graphing technology, question 7 used graphs to check the answers, and question 8 assessed solutions to quadratic equations. The third part of the survey contained 12 questions.

In the third part, the section entitled "Teacher Beliefs" used a five-point scale: Strongly Disagree; Disagree; Neutral; Agree; Strongly Agree, with the corresponding numerical values of 1 to 5. The second section, "Use of Graphing Technology in Teaching Factoring Quadratic Functions," used a different five-point scale: To No Extent; To a Little Extent; Moderately; To Great Extent; and To an Extreme, with the corresponding numerical values of 1 to 5. The level of incorporating technology was measured by an average of a person's scores in this section. The data collected in this part was continuous, as the average level of incorporating technology could be any number from 1 to 5, where 1 represented the lowest level and 5 the highest level of incorporating technology in teaching factoring quadratic functions. The second reason the data was considered continuous is the survey was designed to measure the construct of incorporating technology and so the items were not be analyzed individually.

Part 3 of the survey has been verified as a reliable and valid instrument to measure the level of implementation of technology in teaching factoring quadratic functions. Cronbach Alpha reliability coefficient was 0.90 for the Part 3 of the survey (see Table 14).

Table 14

Cronbach Alpha Reliability Coefficient of Part 3: Implementation of Technology

	Survey Items	Alpha
Implementation of Technology	31, 34, 35, 36, 38, 39, 40, 42, 43, 44, 45, 46	.900
Beliefs about Technology	31, 34, 35, 36	.715
Use of Technology	38, 39, 40, 42, 43, 44, 45, 46	.906

The fourth and the final part of the survey, "Teaching Factoring Quadratic Functions: Teacher Perspective," gave participants the opportunity to express their opinions about methodology and curriculum of teaching factoring quadratic functions freely, as all three questions were open-ended. The first item referred to students' challenges with understanding factoring that are the most noticeable by teachers. The last two questions were adapted from Karadeniz's (2015) interview protocol. Karadeniz (2015) examined teachers' perspectives on using graphing calculators in high school precalculus classes. He used an interview as his main instrument to collect data. Karadeniz's (2015) last two questions focused on teachers' perspective on the positive and negative impacts of using technology on students' learning. Those two questions were adopted to this survey as open-ended to allow teachers to provide complete and thoughtful answers (see Appendix A, Part 5).

Data Collections

Before collecting data, the study was approved by the Institutional Review Board (IRB#: 1217381-1) at Concordia University. The researcher started with getting the list of email addresses of all participants of the International Conference of Mathematics Educators in Hungary in 2017 and the Conference of Contemporary Mathematics Education in Poland in 2018. The individuals on the list were prospective participants. Emails were sent to the prospective participants on the list to introduce the study and provide a link to the survey (see Appendix E). In the emails, the prospective participants were encouraged to forward the survey link and the consent form to other mathematic teachers who may be interested in participating in the study but who did not attend the conference.

Operationalization of Variables

In the study, the level of self-efficacy of algebra served as an independent variable. Self-efficacy was an abstract construct that was measured using Hoy and Woolfolk's (1993) way of operationalization. According to Hoy and Woolfolk (1993), teachers with a high level of self-efficacy exceed in classroom management and with solving practical problems. They are intentional in managing classroom behavior, systems and thinking and "that nurtures gradual movement toward independent thought and action in teaching" (Hoy & Woolfolk, 1993, p. 369). Therefore, the study was used to measure the level of self-efficacy of algebra teachers by their abilities to control their classroom in different circumstances, to motivate various types of students to learn, and to teach using a variety of strategies.

The first dependent variable was the level of implementing graphing technology in teaching factoring quadratic functions to introductory algebra students. Teachers with a high level of implementation of graphing technology in their classrooms displayed knowledge and beliefs in effectiveness of incorporating technology as a teaching tool. In addition, they had clear educational goals in mind when using technology. The level of implementation of graphing technology was measured by both the level of teachers' knowledge and beliefs and the level of consistency of using graphing technology to complete a specific teaching goal. Those goals include checking the answers, guessing the possible answers, understanding different representations of functions, performing computations correctly, and understanding the concept of graphs.

The second dependent variable was the choice of mathematical methods used to teach factoring quadratic functions to introductory algebra students. Teachers shared their opinions on students' main challenges with learning factoring quadratic functions. They also included their suggestions of adding and removing from the current introductory algebra methods of teaching factoring to make students more successful.

Data Analysis Procedures

To answer the first research question about the relationship between the level of self-efficacy of the teachers and the level of implementation technology in teaching factoring quadratic functions to introductory algebra students, the researcher used Pearson's Correlation Coefficient. The Pearson's correlation coefficient formula established the existence of a correlation between the two variables (IV and DV_1) and expressed the strength of such linear correlation as a real number between -1 and +1.

To answer the second research question about the influence of teachers' self-efficacy on their preferences in mathematical methods of instruction selected to teach factoring quadratic functions to introductory algebra students, qualitative analysis was used. The answers to the open-ended questions (Part 4 of the survey) provided data that was coded using Saldaña's (2016) coding methods and entered into a computer program to detect patterns and to provide visual representations.

An independent-sample *t*-test was used to answer the third research question about the differences between collectivistic and individualistic countries with respect to (a) self-efficacy in teaching, (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students. A t-test utilized statistical examination to analyze means ((a) and (b)) of two populations (collectivistic and individualistic countries). To insure accuracy through the process of gathering, reviewing, coding, and analyzing the data, the researcher stayed an unbiased witness as suggested by Creswell (2012).

Limitations and Delimitations of the Research Design

The main limitation of the study was related to the access to international mathematics teachers who speak English and have ability to attend international conferences. The convenience sampling followed by snowball sampling had little chances to produce random sample representing algebra teachers with all levels of self-efficacy.

The study was delimited to using two recent international conferences in Hungary and Poland as a base to recruit participants for the study. The attendees of the conferences and their networks were the only potential participants of the study.

Internal and External Validity

To assess external validity of the study, and therefore the degree to which the results of the study could be extrapolated beyond the study, it was essential to examine the population validity and ecological validity. The main threat to internal validity of the study was selection bias. The study used a convenience sample of the attendees of the two international conferences and their networks. The algebra teachers who participate in international conferences are often unique mathematics educators and researchers who are highly educated, aware of the different methods of teaching mathematics, and have access to and experience with incorporating technology into teaching methods. However, their network could be average teachers, with the daily successes and struggles. To reduce the threats to population validity, the researcher extended the collecting data to two international conferences. It gave the opportunity to increase sample size and thus possibly the number of the participants in the study coming from the networks.

Internal validation of the context of the survey can be evaluated by the degree to which "the instrument actually reflects the abstract construct being examined" (Burns & Grove, 2001, p. 814). It means to what extent the four parts of the survey measured or described the four variables of the study. The research used the TSES, well-documented as a variable and reliable instrument to measure the level of self-efficacy of teachers and three self-constructed parts of survey to measure three dependent variables. I piloted the parts of the instruments that I created to assure the validity and reliability of the survey.

There was no threat to external validity of the study, because of the ex post facto design. The study did not require any direct manipulation of the independent variable and was only based on reports of algebra teachers about existing conditions: the way they view themselves as teachers, the way they incorporate graphing technology in teaching factoring quadratic functions to introductory algebra teachers, the way they perceived their impact on student learning, and their preferences in choosing mathematical methods to teach factoring.

Summary

The purpose of the study was to determine whether there was a relationship between self-efficacy of international algebra teachers and their level of incorporating technology in teaching factoring quadratic functions to introductory algebra. Secondly, the researcher aimed to examine the influence of international teachers' self-efficacy on their perspective on the mathematical methods used to teach factoring quadratic functions to introductory algebra students.

The participants of the study were attendees of two international conferences. The total of 208 potential participants were asked to share a link to the study's survey with other mathematics teachers in their countries, who might want to participate in the study. The primary sampling method was a convenience sampling with a secondary snowball sampling. A specially designed survey instrument, created by the researcher, was used to measure the variables of interest.

Chapter 4: Data Analysis and Results

This chapter includes the description of data obtained from the UVGIA survey instrument specially created for the needs of the study, as well as the findings related to the three research questions. The data analysis and research findings in this chapter consists of five parts: (a) description of the sample, (b) summary of the results, (c) data analysis, (d) ancillary findings, and (e) summary.

The purpose of the study was to determine whether there was a relationship between self-efficacy of international algebra teachers and their level of incorporating technology in teaching factoring quadratic functions to introductory algebra students. Secondly, the researcher aimed to examine the influence of self-efficacy on the perspective of international teachers with respect to the methods they use to teach factoring quadratic functions to introductory algebra students.

Description of the Sample

The researcher planned to use the list of email addresses of the participants of the International Conference of Mathematics Educators in Hungary in 2017 (128 participants), and the Conference of Contemporary Mathematics Education in Poland in 2018 (78 participants). The goal of using emails was to not only recruit the participants of the conferences, but also to connect with other mathematics teachers who would be interested in participating in the study but who did not attend the conferences. However, a new personal data distribution law, called the General Data Protection Regulation (GDPR), took effect in Europe and defined the terms of gathering personal and research data as well as conditions of using and distributing it (GDPR Key Change, n.d.). All of the organizations operating within the European Union, mathematics conferences included, have been obligated to comply with the GDPR. Under the regulations, the research had limited access to the prospective participants. A total of 75 surveys were received, only 54 were completed and useable and therefore analyzed for the study.

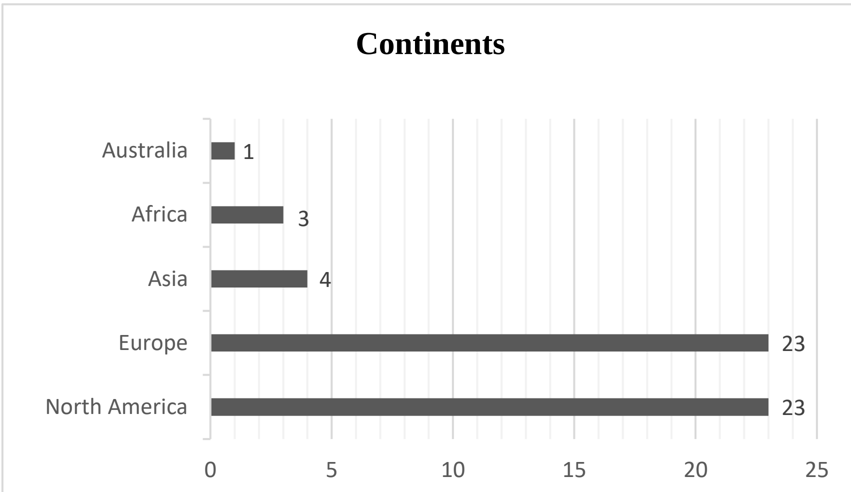


Figure 2. The distribution of continents of the participants.

Table 15

Countries of the Participants of the Study

	Country	Continent	Number of Participants
1.	Australia	Australia	1
2.	Denmark	Europe	1
3.	France	Europe	1
4.	Hungary	Europe	3
5.	India	Asia	2
6.	Kenya	Africa	1
7.	Kuwait	Asia/ Middle East	1
8.	Morocco	Africa	1
9.	Netherlands	Europe	1
10.	Nigeria	Africa	1
11.	Poland	Europe	14
12.	Russia	Europe	1
13.	United Arab Emirates	Asia/ Middle East	1
14.	United Kingdom	Europe	2
15.	USA	North America	23
		Total	54

The participants were form 15 countries and five continents, including North America, Europe, Africa, Asia, and Australia (see Figure 2 and Table 15). Note that participants from Europe and North America formed an evenly split majority, while Africa, Asia, and Australia were represented by very small samples. The gender ratio of the participants was almost 1:1, with 48% females and 52% males. Note that the researcher also had a uniform age distribution (see Figure 3). There were the same number of participants aged 30–39 years old, 40–49 years old, 50–59 years old, and 60 and older. Each of the age group had 22% of the participants. The smallest group was the youngest 20–29, but it is not surprising, as often teachers enter the workforce at the age of about 25.

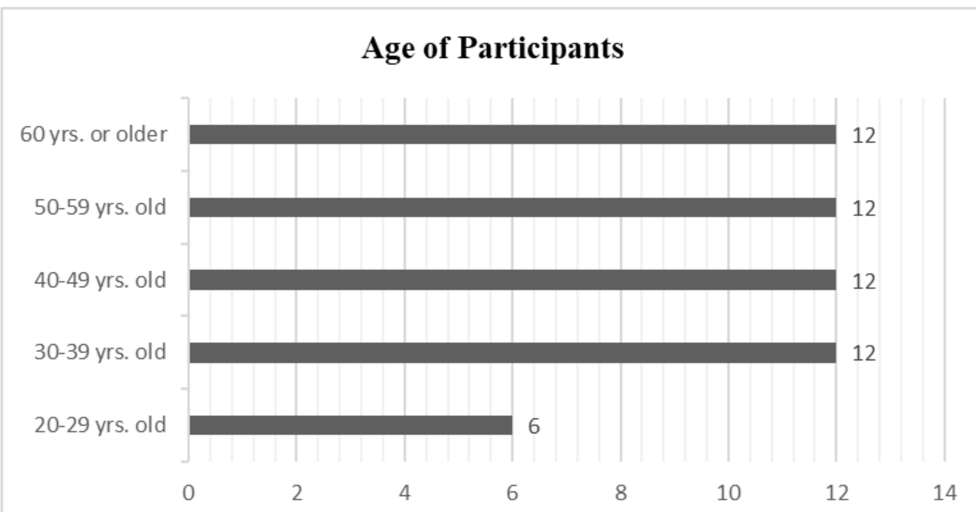


Figure 3. The distribution of age of the participants.

As expected, the participants of the study were highly educated. Most of the teachers (80%) had taken a college abstract algebra course and at least one college calculus class (see Figure 5). Almost half of the participants (44%) have earned a Ph.D. or Doctorate in Education degree, and 32% had a Master of Art or Master of Science in Mathematics degree. Only 7% of the participants had a teaching certificate or credential as their highest qualification (see Figure 4).

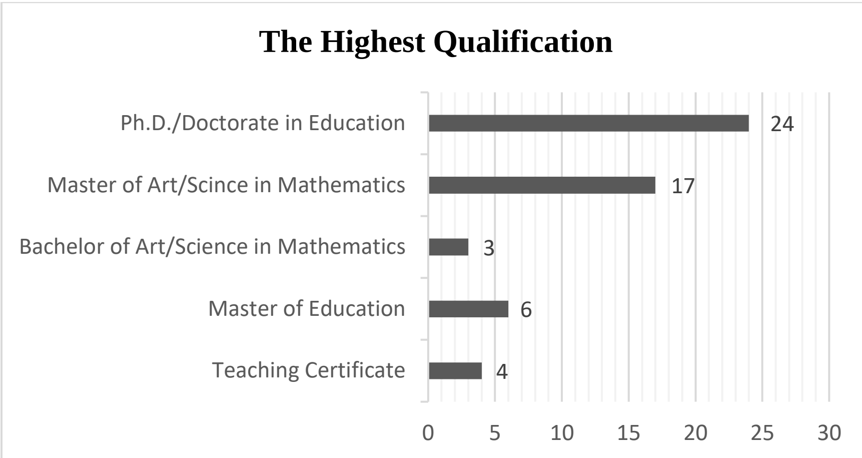


Figure 4. The highest qualifications of the participants.

Interestingly, 38% of participants took at least one educational technology course. Note that almost 50% of those teachers were Polish and only 9% came from the U.S. Unfortunately, only about 70% of the participants took at least one math/methods/pedagogy/didactic class. This is quite surprising, because approximately one third of teachers never addressed pedagogy during their preparation training. This ratio is even lower for the American participants: only 43% of them took at least one math/methods/pedagogy/didactic class.

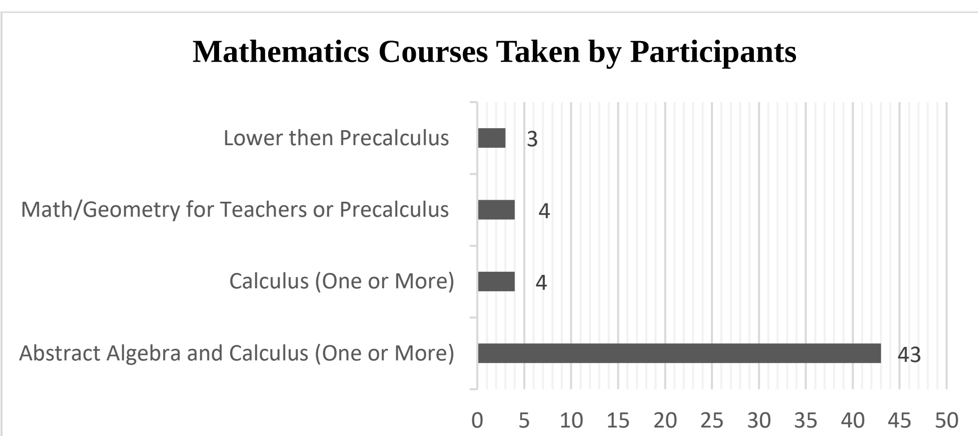


Figure 5. Mathematics courses taken by the participants.

The participants of the study were teachers with different lengths of teaching experience. Each decade of the length of experience was represented in the study (see Figure 6). While the group of teachers with 21–25 years of experience was underrepresented in this study, other group were quite homogenous with 8–12 participants.

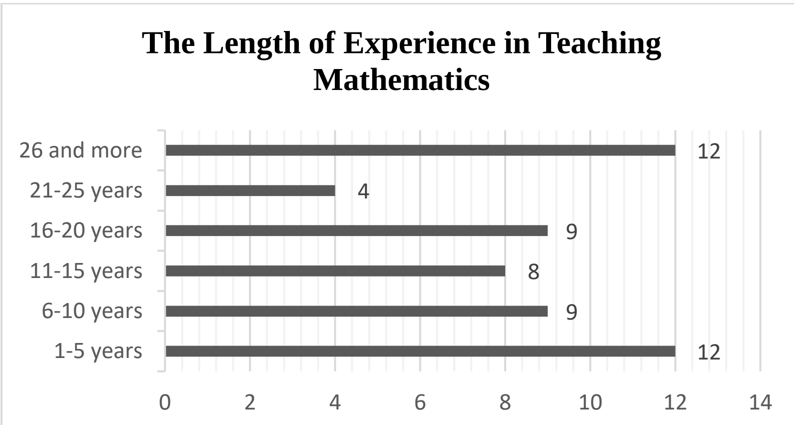


Figure 6. Experience in teaching mathematics (in years).

The teachers taught different age groups. However, most of the participants of the study had experience teaching mathematics to college freshman (72%) and 17% had exclusively taught at college level only (see Figure 7). Note that majority of the participants had experience with junior and high school students, where introductory algebra is usually required for the first time.

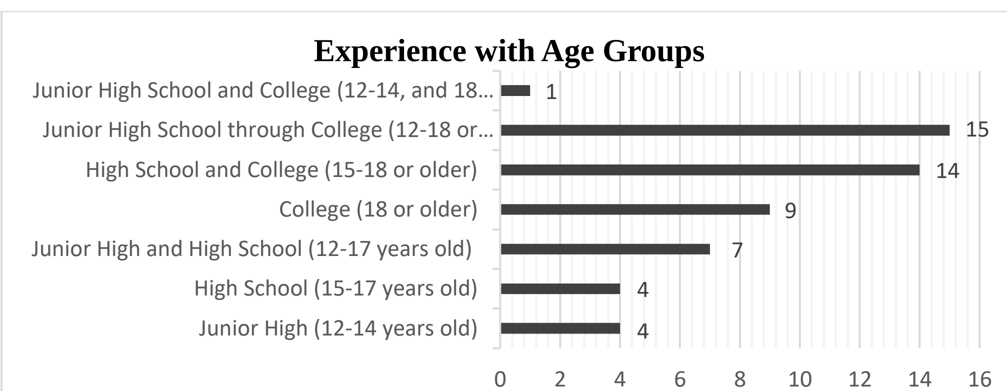


Figure 7. Experience with age groups of the participants.

Summary of the Results

The researcher considered the following research questions:

1. *What is the relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?*

   The analysis of data showed that there is a strong positive relationship between the level of self-efficacy of teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions.

2. *How does the intersection of self-efficacy of teachers and teaching factoring quadratic functions to introductory algebra students, influence teacher preferences in mathematical methods of instruction?*

   To evaluate this research question, the data from three open-ended questions were collected and analyzed. The first open-ended question was related to the challenges with learning factoring quadratic functions. The most frequent response was lack of basic mathematical skills of the learners. Teachers also listed insufficient understanding of graphs, hard to understand the purpose, complexity of factoring quadratic functions, and challenging case when leading coefficient is different than

   The second open-ended question asked for recommendations for additions to mathematics methods of teaching that could improve students' understanding of factoring quadratic functions. The most frequent response was to include more examples and real-life applications. Teachers also indicated adding more graphs and visualizations, background knowledge, factoring by grouping , as well as technology and the use of graphing calculators. 10% of teachers believed that nothing should be added to algebra method to make students more successful in factoring trinomials as the curriculum is already overloaded.

The third open-ended question asked to recommend removals from mathematics methods of teaching that could improve students' understanding of factoring quadratic functions. Almost half of the responders stressed the need of reducing the number of methods of factoring quadratic functions. However, teachers did not agree what method needs to be eliminated and which ones need to stay. Teachers also pointed out the necessity of eliminating "drill and kill practice" and more focusing on conceptual understanding instead. Almost 30% of teachers did not see anything that needed to be removed from algebra methods of teaching in order to improve students' comprehension in factoring quadratic functions in introductory algebra classes.

3. *What differences exist between collectivistic and individualistic countries with respect to perceptions of algebra teachers self-efficacy in teaching, and (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?*

Data analysis shows that there is a difference in the level of self-efficacy of teachers between collectivistic and individualistic countries. The mean of self-efficacy scores of collectivistic countries is significantly greater than the mean of self-efficacy scores in individualistic countries. Interestingly, the data from this study shows that there is no difference in the level of implementation of technology in teaching factoring quadratic functions between collectivistic and individualistic countries.

The UVGIA survey instrument was used to collect data that was analyzed to answer the three research questions. The level of self-efficacy was selected as an independent variable and was expressed by the Self-Efficacy Score. The Self-Efficacy Score was calculated by the average of the answers chosen by a participant in Teachers' Sense of Efficacy Scale 1 based on Tschannen-Moran & Hoy (2001). The minimum value of the Self Efficacy Score was 1 and the maximum value was 9. The dependent variable, the level of incorporating technology, was expressed by the Incorporating Technology Score. The Score was calculated by the average of the answers chosen by a participant in Role of Technology in Teaching Factoring Quadratic Functions. The minimum value of the Incorporating Technology Score was 1 and the maximum value is 5.

The statistical means of the variables, the standard deviations, minimum and maximum values are presented in Table 16.

Table 16

Statistics of the Self-Efficacy Score and Incorporating Technology Score

Measure	*n*	*M*	*SD*	*Min*	*Max*
Self-Efficacy Score	54	6.79	1.20	4.5	9.00
Incorporating Technology Score	54	3.51	0.89	1.75	5.00

Detail Analysis

The results of quantitative analysis will be introduced first, followed by the outcomes from qualitative approach. Therefore, findings associated with research question 2 will be reported after findings associated with research questions 1 and 3.

Findings associated with research question 1.

Research question 1. *What is the relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?*

$H1_0$. *There is no relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.*

H1a. *There is a relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.*

A correlational analysis was conducted to investigate the relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students. The correlation coefficient, *r*, computed revealed a statistically significant relationship between the Self-Efficacy Scores and the Implementation of Technology Scores, $r\,(54)=0.529$, $p < 0.01$. This indicates a strong positive relationship between the level of self-efficacy of teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions (see Figure 8). There is enough evidence to reject the null hypothesis. This means that teachers with a higher level of self-efficacy implement more technology-based tools to teach factoring quadratic functions to introductory algebra students, and consequently, teachers with lower level of self-efficacy use less technology-based tools to teach factoring quadratic functions.

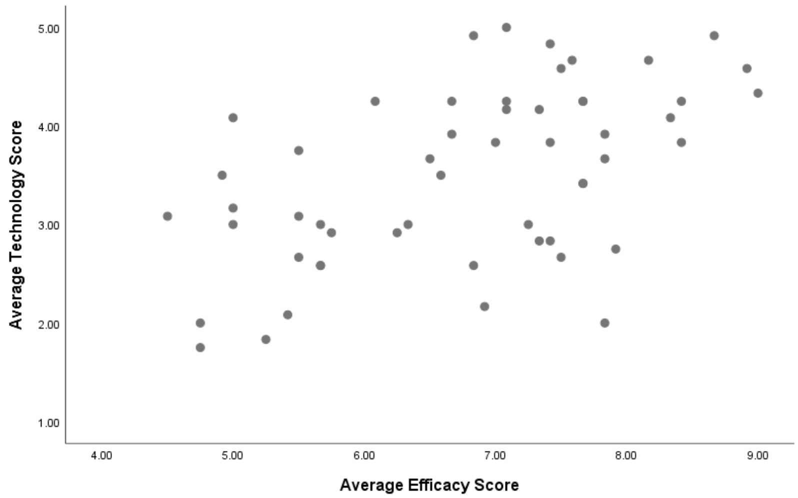


Figure 8. Participants' self-efficacy scores and implementation of technology scores.

Findings associated with research question 3.

Research Question 3. *What differences exist between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?*

$H3_0$. *There are no differences between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, and (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.*

$H3_a$. *There are differences between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, and (b) teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra.*

There was a small sample of collectivistic countries (10) and 44 individualistic countries (see Table 14). However, according to Winter (2013), independent-sample t-test is a robust statistical approach that can still provide accurate responses even with such differences in sample sizes.

Self-efficacy. An independent-sample t-test was conducted to test the difference between collectivistic and individualistic countries with respect to perceptions of algebra teachers' self-efficacy in teaching. The analysis revealed a statistically significant difference in the Self-Efficacy Scores between collectivistic and individualistic countries, $t(52) = -2.904$, $p < 0.01$, η^2=0.73. The average Self-Efficacy Scores of collectivistic countries ($M = 7.72$, $SD = 1.154$) was higher than the average Self-Efficacy Scores of individualistic countries ($M = 6.58$, $SD = 0.95$). The 95% confidence interval for the difference in means ranges from -1.92929 to -0.35252. The use of η^2 revealed a large effect size. The η^2 index indicated that 73% of the variance of the Efficacy Score was accounted for affiliation with country type i.e. whether it is collectivistic or individualistic country of participants. Hence, there is enough evidence to reject the null hypothesis that there would be no difference in the Efficacy Score between collectivistic and individualistic countries.

Table 17

Means and Standard Deviations of Self-Efficacy Scores

	n	*M*	*SD*
Individualistic Countries	44	6.58	1.16
Collectivistic Countries	10	7.72	0.95

Mathematics teachers from collectivistic countries have a greater perception of their self-efficacy than mathematics teachers from individualistic countries. The statistical means of the Efficacy Scores and the standard deviations of the participants affiliated with individualistic and collectivistic countries are presented in Table 17.

Implementation of technology. An independent-sample *t*-test was conducted to test the difference between collectivistic and individualistic countries with respect to algebra teachers' level of implementation of technology in teaching factoring quadratic functions. The analysis revealed no statistically significant difference between collectivistic and individualistic countries in respect to the Implementation of Technology Score, $t(52) = -0.740$ and $p = 0.463$. The mean of the Implementation of Technology Score of collectivistic countries ($M = 3.69$, $SD = 0.90$) was slightly higher than the average the Implementation of Technology Score of individualistic countries ($M = 3.46$, $SD = 0.88$).

Table 18

Means and Standard Deviations of the Implementation of Technology Scores

	n	*M*	*SD*
Individualistic Countries	44	3.46	0.88
Collectivistic Countries	10	3.69	0.90

The null hypothesis that there would be no difference in the level of the implementation of technology between collectivistic and individualistic countries cannot be rejected. The statistical means of the Technology Scores and the standard deviations of the participants affiliated with individualistic and collectivistic countries are presented in Table 18.

Findings associated with the research question 2.

Research Question 2: *How does the intersection of self-efficacy of teachers and teaching factoring quadratic functions to introductory algebra students, influence teacher preferences in mathematical methods of instruction?*

To answer the second research question, data obtained from three open-ended questions was coded and categorized. It resulted in 19 themes. Theme 1–8 was related to the first open-ended question about challenges with learning factoring quadratic functions, theme 9–15 to the second open-ended question about additions to methods of teaching to improve students' understanding of factoring, and theme 16–18 to the last open-ended question about removals from methods of teaching to improve student' understanding of factoring.

Challenges with learning factoring trinomials.

Open-ended question 1: *What do you think is the most challenging part of factoring trinomials for students to understand when you teach it?*

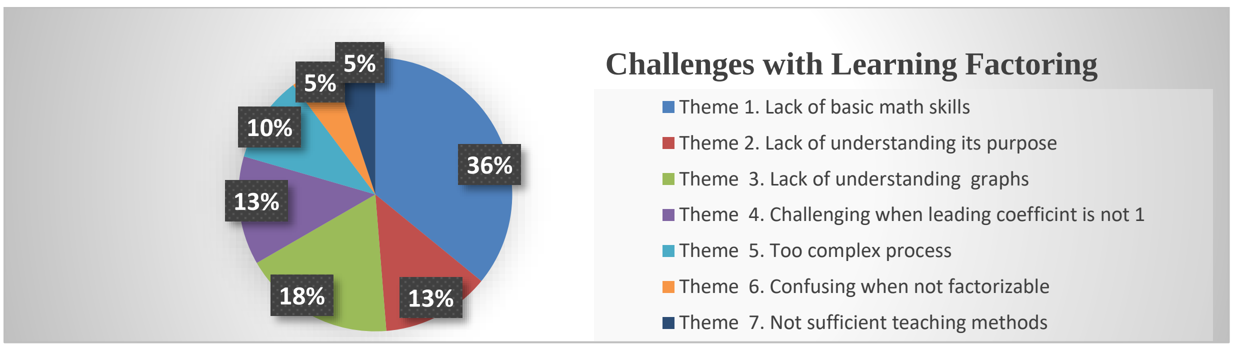


Figure 9. Challenges with learning factoring, teachers' perspective.

The responses to the first open-ended questions was coded and divided into 7 themes (see Table 19 and Figure 9). The frequencies of each theme is shown in Figure 8. Examples of the participants' responses for each theme is presented in Table 19.

Table 19

Challenges with Learning Factoring Quadratic Functions

Theme Number	Themes Representing Challenges with Learning Factoring	Examples of Teachers' Responses Associated with a Theme
Theme 1	Lack of basic math skills	1. Difficulties with knowing factors of a number 2. Problems with reverse thinking 3. Lack of algebra skills 4. Problems with basic computations
Theme 2	Lack of understanding its purpose	1. Students don't know how to apply factoring in real world 2. Hard to understand the purpose of the activity 3. I don't have a good answer to the WHY DO WE NEED THIS IN LIFE
Theme 3	Lack of understanding graphs	1. Students cannot find the link between the equation and the graph 2. They don't see that solutions depend on the coefficients 3. Students don't understand that they are finding the x-intercepts of a parabola 4. They struggle with the connection between the figurative (graphical) representation of the function and the analytical
Theme 4	Challenging, when leading coefficient is not 1	1. Very difficult case when leading coefficient is greater than 1 2. Challenging to factor when the polynomial isn't monic 3. . The students don't understand where we obtain the middle terms from in order to complete the problem.
Theme 5	Too complex process	1. Challenging with too many methods 2. Hard to understand how all the different pieces work together -too many moving parts/options/steps 3. It is difficult to demonstrate using a model or teaching aid
Theme 6	Confusing when not factorizable	1. Students do not understand that a factorized form is another form of the trinomial and that some trinomials cannot be factored. 2. Challenging when sometimes we can't factorize trinomials
Theme 7	Ineffective teaching methods	1. Most people have never learned factoring before, people never learned it well. 2. It's usually poorly explained as being a magic formula

The most frequently mentioned challenge that students need to overcome when learning factoring quadratic functions was their lack of basic mathematical skills (36%). It was followed by students' lack of understanding of graphs (18%). Teachers stressed that students cannot identify the purpose of learning factoring (13%) and they struggle with factoring when leading coefficient is not 1 (13%). The common challenge is the fact that factoring is a very complex process (10%).

Figure 10 demonstrates the means of the Self-Efficacy Scores calculated for teachers within the same theme. It is noticeable that the lowest mean of self-efficacy had teachers that pointed out at the challenges with leading coefficient different than 1 (M = 5.87), possibly due to the fact that they had difficulties explaining this topic to their students. The highest mean of self-efficacy had teachers who picked lack of basic mathematics skills (M = 7.37), which indicates that sometimes expectations about student preparation is too high.

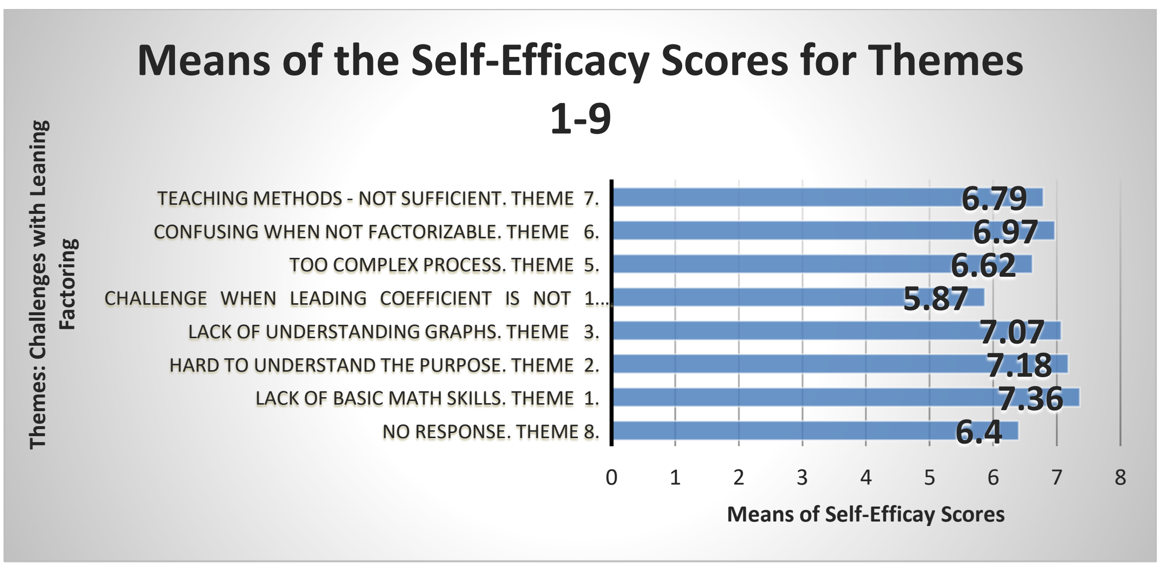


Figure 10. Means of self-efficacy scores for teachers representing the same theme 1-8.

Additions to mathematics methods of teaching factoring trinomials suggested by teachers.

Open-ended question 2: What would you ADD to introductory algebra methods of teaching to make students more successful in factoring trinomials?

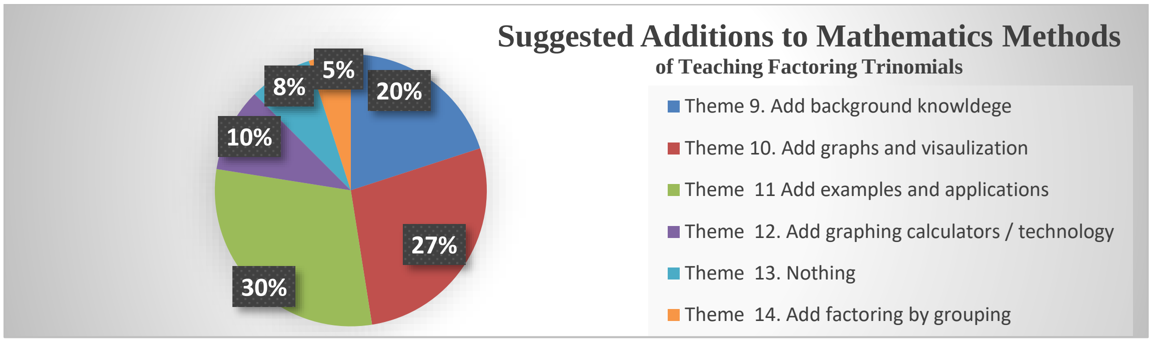


Figure 11. Suggested additions to introductory algebra methods of teaching factoring trinomials.

The responses received from the second open-ended question was coded and divided into seven themes (see Figure 11 and Table 20). The most often recommended addition to the introductory algebra method of teaching was including more examples, meaningful application, and the real-life contexts with more interactive activities (30% of responses). Examples and real-life applications engage learners and make the topic more concrete for students. Teachers also indicated adding more graphs and visualization (27.5% of responses). Adding background knowledge was pointed out by 20% of teachers. For example, teachers suggested reviewing the operations on integers, FOIL method, like terms, and the concept of a function. 10% of teachers pointed out at technology and graphing calculator to visualize, and to enhance understanding (see Figure 11).

Interestingly, 5% of teachers suggested spending more time on factoring by grouping, which is an advanced way of factoring, that requires creativity and true understanding of the material. 10% of teachers believed that nothing should be added to the current algebra methods to make students more successful in factoring trinomials as the curriculum is already overloaded.

Table 20

Suggested Additions to Introductory Algebra Methods of Teaching Factoring Trinomials

Theme Number	Themes Representing Suggested Additions to Math Methods	Examples of Teachers' Responses Associated with a Theme
Theme 9	Add Background knowledge	1. Revision of operations on integers 2. Add the concept of the function as input-output, plot and draw trinomial and understand factoring 3. Adding more accommodations for those that struggle with basic computation 4. Include some exercises on products in different contexts including geometry 5. Review FOIL 6. Review like terms
Theme 10	Add graphs and visualization	1. Add many graphs 2. Add graphing parabolas before solving equations 3. Include the use of graphing techniques 4. To acquaint students with the analytical, graphical and vital representation of quadratic functions
Theme 11	Add examples and applications	1. Stress application more than memorization 2. I'm always looking for ways to engage students with meaningful application. 3. Add real-life contexts
Theme 12	Add graphing calculators / technology	1. Students should also be able to incorporate other methods, for example to incorporate technology, so add it. 2. Incorporate graphing calculators 3. Use technology like GeoGebra
Theme 13	Nothing	1. Nothing, there's too many of them so far 2. Nothing
Theme 14	Add factoring by grouping	1. Put more emphasis on factoring by grouping 2. More time on factoring by grouping algorithm

Figure 12 demonstrates the means of the Self-Efficacy Scores calculated for the teachers within the same theme. It is noticeable that the lowest mean of self-efficacy had teachers that with no response (*M* = 6.15) and the highest mean of self-efficacy had teachers who wanted to add graphing calculators/technology (*M* = 7.79).

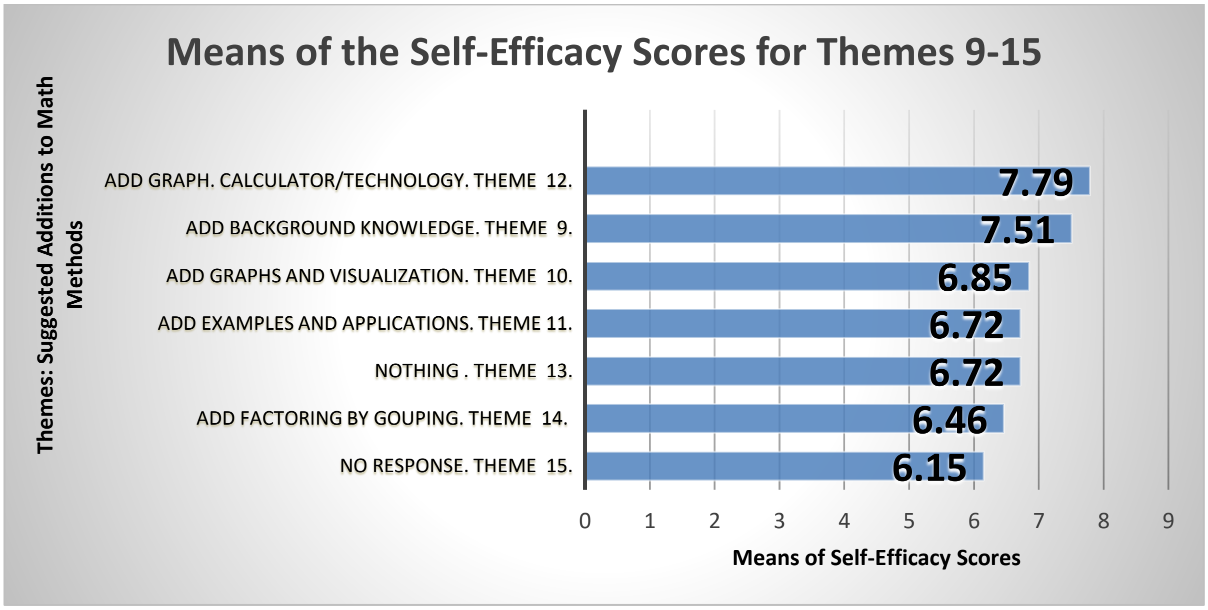


Figure 12. Means of self-efficacy scores for teachers representing the same theme 9-15.

Teachers' suggestions for removals from mathematics methods of teaching factoring.

Open-ended question 3*: What would you REMOVE from introductory algebra methods of teaching to make students more successful in factoring trinomials?*

The responses received from the third open-ended question was coded and divided into 3 themes (see Table 21 and Figure 13.).

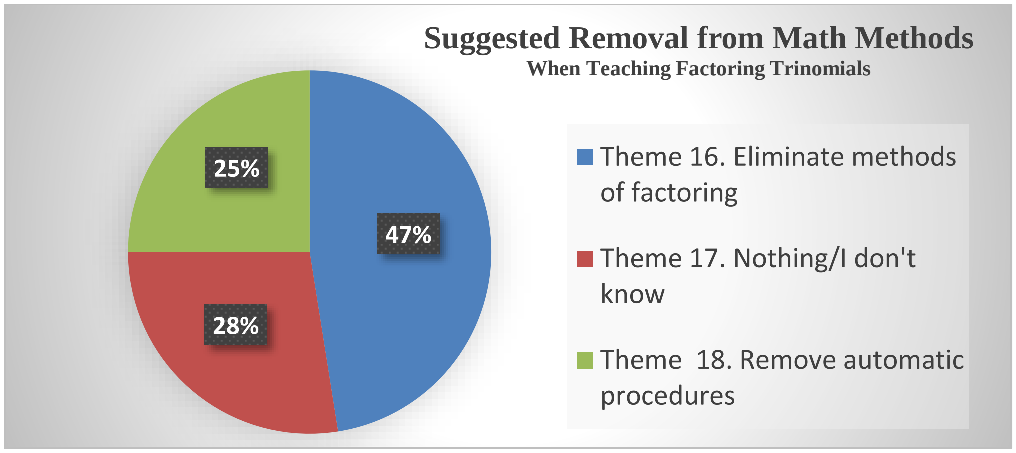


Figure 13. Suggested removal from introductory algebra methods of teaching factoring trinomials.

Table 21

Suggested Eliminations from Introductory Algebra Methods of Teaching Factoring Trinomials

Theme Number	Themes Representing Suggested Removal from Math Methods	Teachers' Responses Associated with a Theme
Theme 16	Remove methods of factoring	1. Remove all these methods that confuse students 2. Remove everything and leave standard procedures, only one method to factorize. 3. At first, focus on less amount of methods and remove the rest
Theme 17	Nothing / I don't know	1. Nothing 2. I don't know
Theme 18	Remove Automatic Procedures Focus on Concepts	1. Stop over-emphasis on procedures 2. It would be great if students had the time to discover the concepts vs. a one-size fits all method shoved down their throats so they can practice it enough to get fluent 3. Remove drill and kill practice

Interestingly, almost half of the suggestions (47.5%) stressed the need of reducing a number of methods of factoring quadratic functions that students need to learn (Theme 16). Unfortunately, teachers did not agree with methods to be eliminated (see Figure 14).

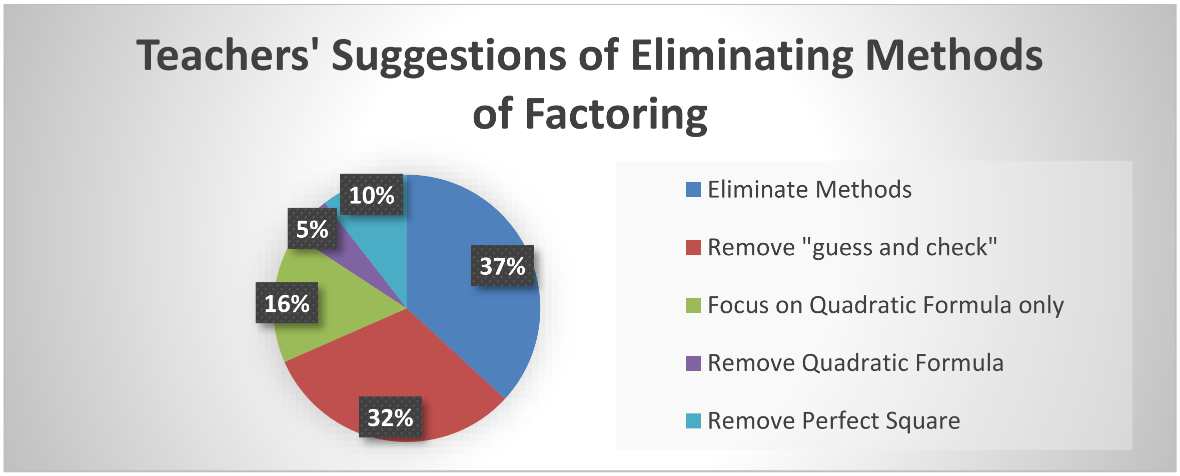


Figure 14. Teachers' suggestions of eliminating methods of factoring (theme 16).

37% of teachers who believed in reducing the methods of factoring, did not identify the method/s at all. 32% decided on the "guess and check" method to be removed.16% of teachers specified the "quadratic formula" as the only method of solving quadratic equations that supposed to be used in introductory algebra class. Interestingly, 5% completely disagreed, suggesting removing the "quadratic formula" as one of the factoring methods, which is surprising as this is the universal method for solving all quadratic equations. Unexpectedly, 10% of teachers believed that the "Perfect Square" formula is to be removed, even though, it is not very difficult and is useful later in calculus classes (see Figure 14).

The number of methods imposed on introductory algebra students often implied the use of automatic procedures. Therefore, 25% of teachers stressed the importance of reducing "drill and kill practice" and focusing on conceptual understanding instead. Almost 30% of teachers did not see anything that needed to be removed in order to improve students' comprehension in factoring quadratic functions in introductory algebra classes.

Figure 15 demonstrates the means of the Self-Efficacy Scores calculated for the teachers within the same theme. It is noticeable, that the lowest mean of self-efficacy had teachers that did not respond to the question (M = 6.31) and the highest mean of self-efficacy, the ones who believed that they do not know what needs to be removed or who want nothing to be removed (*M* = 7.51).

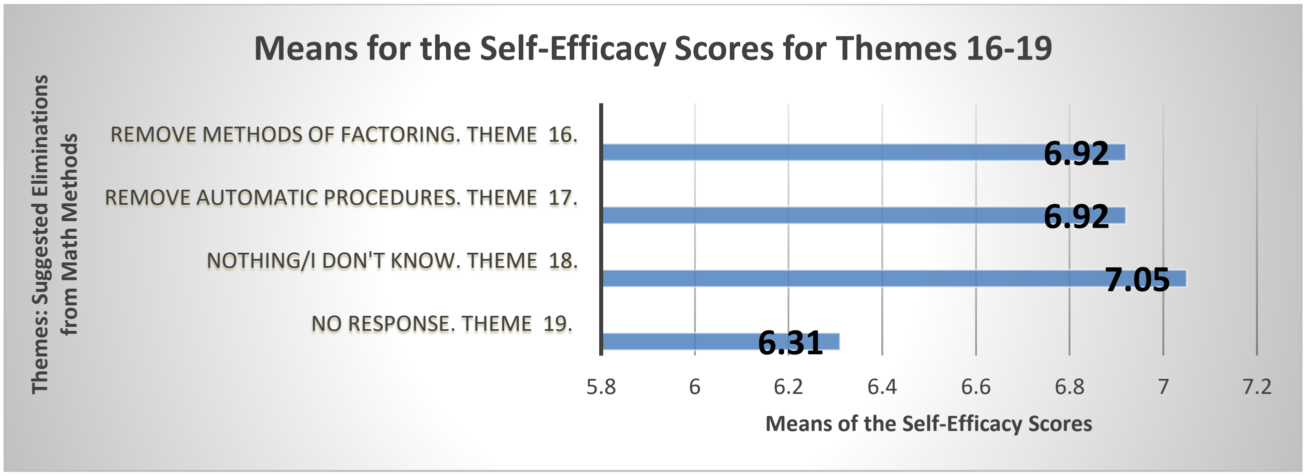


Figure 15. Means of self-efficacy scores for teachers representing the same theme 16-19.

Ancillary Findings

The impact of gender. Interestingly, there was no significant differences between female and male responders in respect to the level of self-efficacy as well as to the level of incorporating technology when teaching factoring quadratic functions (see Table 22).

Table 22

Means and Standard Deviations of the Efficacy Score and Technology Score by Females and Males

	n	*Efficacy Score*		*Technology Score*	
		M	*SD*	*M*	*SD*
Female	26	6.75	1.20	3.47	0.94
Male	28	6.82	1.20	3.54	0.84

The impact of teaching experience. Based on intuitive beliefs the length of teaching experience should have an impact on teachers' level of self-efficacy as well as their level of implementation of technology. The results of the survey exposed unusual relationship between those variables.

Table 23

Statistics of the Self-Efficacy Score by Years of Teaching Experience

Measure		*n*		*M*	*SD*	*Min*	*Max*
1-5 yrs.		12		6.31	.94	4.75	7.67
6-10 yrs.		9		7.01	1.71	4.92	9.00
11-15 yrs.		8		7.06	.92	5.50	8.33
16-20 yrs.		9		6.16	1.36	4.50	8.42
21-25 yrs.		4		7.15	.364	6.67	7.5
26 and more		12		7.27	1.00	4.75	8.67
	Total:	51	Mean:	6.79	1.198	4.50	9.00

The level of self-efficacy dropped for teachers with 16—20 years of teaching experience (see Table 23 and Figure 16), i.e. around the age of 40.

In order to test that the length of experience had an effect on the level of self-efficacy of teachers, a one-way ANOVA was performed.

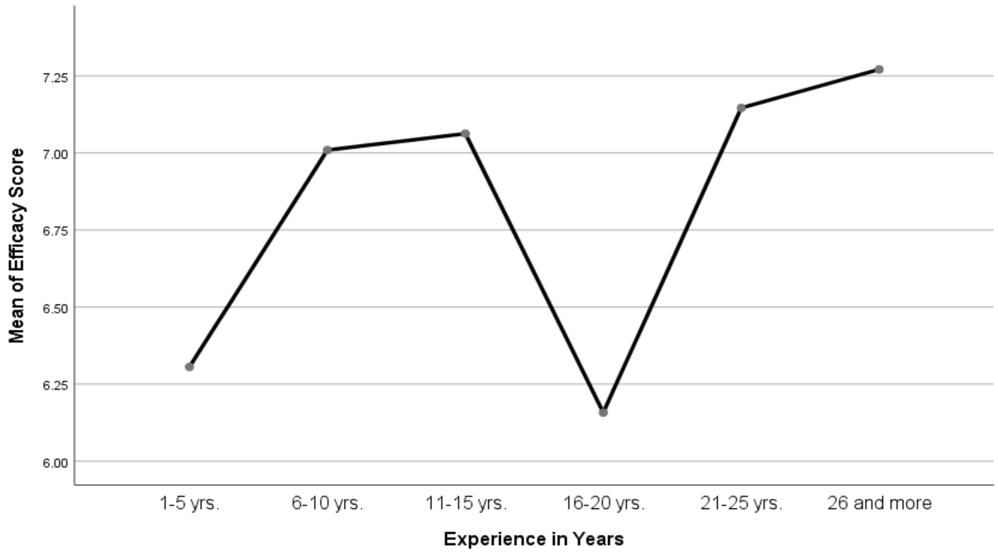


Figure 16. Mean of efficacy score by years of teaching experience.

The results of the ANOVA are presented in Table 24. The researcher concludes that there were no statistically significant differences between group means as determined by one-way ANOVA (F(5,48) = 1.58, p = .184).

Table 24

Summary of the results of the One-way Analysis of Variance

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups (Discipline type)	5	10.72	2.14	1.58	.185
Within Groups (Error)	48	65.28	1.36		
Total	53	76.01			

Chapter 4 Summary

The study confirmed the existence of challenges in teaching factoring quadratic functions to introductory algebra students. The participating teachers identified basic difficulties to overcome when learning factoring. They also listed suggestions of changes (removing or adding) the methods of teaching to improve students understanding. The main voice of teachers focused on improving students' understanding of mathematical basics and to include visualization and applications of factoring.

The study also concluded a strong positive relationship between the level of self-efficacy of teachers and their level of implementation of technology in teaching factoring quadratic functions. The analysis of data showed a statistically significant difference in the level of self-efficacy of teachers between collectivistic and individualistic countries. However, there was no significant differences between collectivistic and individualistic countries in respect to the level of implementation of technology in teaching factoring quadratic functions to introductory algebra students.

Chapter 5 includes discussion on these results of the study and their relation to the community of practice, to the literature, and to the community of scholars.

Chapter 5: Discussion and Conclusion

The challenges with teaching factoring quadratic functions to introductory algebra students are widely recognized among mathematics international educators. Therefore, algebra teachers are constantly seeking new ways and different approaches to more effectively introduce factoring to their students. Since the level of self-efficacy of teachers has been linked to willingness to adopt changes (Achurra & Villardón, 2013) and to student achievement (Hoy & Woolfolk, 1993; Muijs & Reynolds, 2005; Ross, 1992; Skaalvik & Skaalvik, 2007), this study further explored this relationship and teachers' preferences of using mathematical methods in teaching factoring quadratic functions in introductory algebra courses. Special attention was given to incorporating graphing technology to mathematical methods and its relationship with self-efficacy of teachers. Technology was viewed as a new opportunity and additional mathematical representation to improve students learning. The study was conducted internationally. The participants were from 15 countries and five continents.

In this chapter, the researcher discusses the results of the study and how they relate to the community of practice, to the literature, and to the community of scholars. Chapter 5 is divided into six parts: (a) summary of the results, (b) discussion of the results in relation to the literature, (c) limitations; implications of the results for practice, policy and theory, (d) recommendations for further research, and (e) conclusions.

Summary of the Results

The researcher considered the following research questions:

1. *What is the relationship between the level of self-efficacy of the teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?*

The analysis of the data showed that there is a strong positive relationship between the level of self-efficacy of teachers and the teachers' level of implementation of technology in teaching factoring quadratic functions.

2. *How does the intersection of self-efficacy of teachers and teaching factoring quadratic functions to introductory algebra students, influence teacher preferences in mathematical methods of instruction?*

To evaluate this research question, the researcher collected data from three open-ended questions. The first open-ended question was related to the challenges with learning factoring quadratic functions. The most frequent response was lack of basic mathematical skills of the learners. Teachers also listed: insufficient understanding of graphs, hard to understand the purpose, complexity of factoring quadratic functions, and challenging case when leading coefficient is different than 1.

The second open-ended question asked for recommendations for additions to mathematics methods of teaching that could improve students' understanding of factoring quadratic functions. The most frequent response was to include more examples and real-life applications. Teachers also indicated adding more graphs and visualizations, background knowledge, factoring by grouping , as well as technology and graphing calculator. Among the teachers, 10% believed that nothing should be added to algebra method to make students more successful in factoring trinomials, as the curriculum is already overloaded.

The third open-ended question asked to recommend removals from mathematics methods of teaching that could improve students' understanding of factoring quadratic functions. Almost half of the responders stressed the need of reducing a number of methods of factoring quadratic functions. Unfortunately, teachers did not agree what method needs to be eliminated and which one needs to stay. Teachers did point out the necessity of eliminating “drill and kill practice,” with more focus on conceptual understanding.” Almost 30% of teachers did not see anything that needed to be removed from algebra methods of teaching in order to improve students’ comprehension in factoring quadratic functions in introductory algebra classes.

3. *What differences exist between collectivistic and individualistic countries with respect to perceptions of algebra teachers (a) self-efficacy in teaching, and (b) teachers’ level of implementation of technology in teaching factoring quadratic functions to introductory algebra students?*

The data analysis shows that there is a difference in the level of self-efficacy of teachers between collectivistic and individualistic countries. The mean of self-efficacy scores of collectivistic countries is significantly greater than the mean of self-efficacy scores in individualistic countries.

Interestingly, data shows that there is no difference in the level of implementation of technology in teaching factoring quadratic functions between collectivistic and individualistic countries.

Discussion of The Results in Relation to the Literature

Teachers’ self-efficacy and technology use. The analysis of the data showed that there is a strong positive relationship between the level of self-efficacy and the teachers’ level of implementation of technology in teaching factoring quadratic functions. These results are fully aligned with many studies that concluded that self-efficacy of teachers has a direct impact on the level of implementation of technology (Abbitt, 2011; Albion, 1999; Albion, 2001; Curts, Tanguma, & Peña, 2008; Marcinkiewicz, 1993; Teo, 2009). According to Minshew and Anderson (2015), there are “three factors that the best predicted how a teacher integrated technology: time commitment, willingness to change, and amount of technology training" (p. 336). The willingness to change is a component of self-efficacy, and therefore, the results of this study fully supported the conclusions of Koehler, Mishra, Kereluik, Shin, and Graham (2014)

However, some studies show that many teachers are unsure and unconvinced of the role of technology in their mathematics classrooms (Greaves, Hayes, Wilson, Gielniak, & Peterson, 2012). Mishne (2012) found that "teachers more self-efficacious in instructional strategies use technology less in their classrooms and are less proficient in technology than teachers who believe they are less self-efficacious" (p. 70). Such a relationship could be interpreted as a negative correlation. The research study of Mishne (2012) involved very experienced elementary teachers, where almost 75% of participants had over 10 years of experience (p. 54). The factor that could influence the results of Mishne's (2012) study was the fact that the participants were only elementary school teachers (K–6) who typically cover elementary topics in mathematics. In addition, according to Mishne (2012), the teachers in his study "did not consider technology a necessary component of effective instruction" (p. 80) at this level.

Additionally, findings of Kim, Kim, Lee, Spector, & DeMeester (2013) concluded that pedagogical beliefs of teachers are substantial in incorporating technology into teaching methods. Teachers with student-centered pedagogical beliefs are more willing to use technology, than teachers with teacher-directed pedagogical beliefs. Since elementary school teachers, especially in the lower grades, often represent teacher-directed pedagogical beliefs (Kim et al., 2013), and therefore they may not be as willing to use educational technology as teachers in junior and high school.

Teachers' self-efficacy in individualistic and collectivistic countries. According to Bandura, "strong efficacy beliefs lead to greater persistence in the face of difficulties, reduce fear of failure, improve problem-focused analytic thinking, and raise aspirations" (Oettingen, 1995, p. 169). The important question is, if those advantages of self-efficacy are universal, or they are influenced by the culture? From one point of view, self-efficacy is a characteristic "founded in basic psychological principles and mechanisms common to human agency in general" (Oettingen, 1995, p. 169). Therefore, self-efficacy could be viewed as a universal attribute of a person. However, self-efficacy relates to personal goals, and goals are born, implemented, kept, or dropped within specific cultural values. Consequently, according to Oettingen (1995), the concept of self-efficacy is strictly linked to the culture and should be examined in relation to the culture. The division of countries, that represents different cultures, was specified by Hofstede (1983) through the dimensions of cultural diversity. Many studies concluded that the level of self-efficacy is different in individualistic countries versus collectivistic counties (Achurra & Villardón, 2013; Aslan, 2015; Efe, & Yücel, 2016; Li & Kirkup, 2007; Oettingen, 1995; Olafsson & Macdonald, 2012). The results of this study aligned with the above conclusion that there is a difference in the level of self-efficacy of mathematics teachers between collectivistic and individualistic countries. Specifically, this study concluded that the level of self-efficacy of teachers in collectivistic countries was much higher than in individualistic countries. This result aligned with Earley (1993) and Randhawa and Gupta (2000). However, researchers investigating self-efficacy through cross-cultural comparison obtained mixed results. Some identified individualistic countries with higher level of self-efficacy than collectivistic countries (Leung, 2001; Mau, 2000; Scholz, Gutiérrez-Doña, Sud, & Schwarzer, 2002; Schwarzer & Born, 1997).

There are many reasons for such discrepancies. According to Klassen (2004) self-efficacy has many interpretations and the results of a research study depends on a selected definition and a chosen instrument to measure the level of self-efficacy. In addition, self-efficacy changes when people immigrate to different culture (Salili, Chiu, & Lai, 2001). The participants of this study, though they came from different countries, were not necessarily typical representatives of their cultures. The mathematics teachers and researchers were highly educated and exposed to many different cultures through participating in international conferences. Some of them had experience teaching abroad. According to Salili et al. (2001), this could contribute to the different outcome of the study as related to self-efficacy.

Use of educational technology in individualistic and collectivistic countries. On the other hand, this study concluded that there is no significant difference between collectivistic and individualistic countries in respect to the level of implementation of technology in teaching factoring quadratic functions to introductory algebra students. The analysis of the data revealed differences in accessibility of educational technology as well as in perception on incorporating technology based on teachers' training, preparations, and expectations among different countries. Participants of the study addressed those issues in the additional comments. For example, one participant stated, "In the U.S. I had access to a class set of graphing calculators. However, in Kuwait it is not a school supply and students can buy them, but usually the school requires them for upper level mathematics courses." Another participant stated, "In India the teachers are not in practice of using graphing technology." In summary, comparison of the level of use of technology in mathematical classroom across countries encounters challenges, because beside personal preferences, the other factors needs to be taken into consideration, like economic situation and cultural perception. With the self-reported data, it is difficult to assess and compare the level of satisfaction of teachers with incorporating technology.

To further examine the differences in perception of implementation of educational technology in individualistic versus collectivistic countries, the researcher used data provided by the Teaching and Learning International Survey (TALIS) (OECD, 2008b).

Table 25

Percentage of Experienced Teachers who Reported High Professional Development Needs in ICT Teaching Skills (Organization for Economic Cooperation and Development, 2008b)

	Individualistic Index	% of Experienced Teachers from a Country Participating in *TALIS*
Turkey	37	15.1
Slovak Republic	52	15.2
Belgium	75	15.5
Korea	18	17.5
Iceland	60	17.9
Australia	90	18.5
Denmark	74	19.8
Poland	60	22.6
Hungary	80	23.8
Austria	55	24.2
Malta	59	24.2
Portugal	27	24.9
Mexico	30	25.8
Slovenia	27	25.8
Italy	76	25.8
Spain	51	26.5
Bulgaria	30	28.0
Estonia	60	28.5
Norway	69	29.6
Ireland	70	35.9
Lithuania	60	36.5
Brazil	38	36.6
Malaysia	26	43.3
TALLIS average		25.3

Notes: The shaded countries are individualistic
Based on: OECD, *TALIS Database Teaching and Learning International Survey 2008*.
Statlink: http://dx.doi.org/10.1787/888932578353

The goal was to identify the relationship between affiliation to individualistic or collectivistic country and the percentage of experienced teachers who reported high needs for additional professional development in using educational technology (in ICT teaching skills). Table 25 shows the list of countries participated in TALIS 2008 with the corresponding individualistic index. Shaded countries are identified as individualistic.

The list starts with the country having the smallest percentage of teachers reporting high professional development needs in ICT teaching skills, and it ends at the highest percentage. There is no noticeable pattern among the collectivistic and individualist countries in the list.

In addition, an independent-samples *t*-test revealed no difference in percentage of teachers reporting high professional development needs in ICT teaching skills between individualistic and collectivistic countries.

Teachers' views on methods of teaching factoring quadratic functions. The conceptual framework of this study was based on the observation that teaching and learning factoring quadratic functions in introductory algebra classes is challenging across the globe. The purpose of the open-ended questions of the UVGIA survey was to examine the current views of teachers on students' difficulties when learning factoring quadratic functions and gather teachers' perspective on the possible changes. Interestingly, 22% of the participants did not respond to the first open-ended question, and almost 26% of teachers did not address the next two. One could argue that the teachers got tired of answering questions, or their limited English abilities could prevent them from participating. Another possibility is that these teachers do not experience any difficulties with teaching factoring, or they are tired of expressing their opinions over and over and not seeing any improvement in the school curriculum.

Challenges with teaching factoring quadratic functions. More than one third of the teachers (with the highest mean of Self-Efficacy Score) identified lack of students' basic mathematical skills as a biggest challenge to overcome when presenting factoring quadratic functions in introductory algebra class. Among responses teachers acknowledged: students need to review "basic computation," "they are weak on algebra," they struggle with "knowing factors of a number," they have "problems with reverse thinking," they "lack algebra skills," and they make "simple multiplication and distribution errors." This outcome corresponds with many research studies (Boulton-Lewis et al., 2001; Didis & Erbas, 2015; Kotsopoulos, 2007; Martinez et al., 2016; Nielsen, 2015; Sells, 1973; Zakaria & Maat, 2010).

Insufficient understanding of graphs was mentioned by 17% of responding teachers (with high mean of the Self-Efficacy Scores of 7.07). For example, one of the answers pointed out: "the absence of a connection between the figurative (graphical) representation of the function and the analytical." Note, that currently there is no research related to the importance of graphs when teaching factoring quadratic functions.

Teachers (with low mean of the Self-Efficacy Scores of 6.62) stressed that factoring is a very complex process (14.6% of responses), it is "too abstract" and it has "the additional complexity - multiple steps." Also, teachers (with the lowest mean of the self-Efficacy Scores of 5.87) identified experiencing extra difficulties with the case of leading coefficient different than 1 (12% of responses); "for trinomials with leading coefficient greater than 1, the added complexity of finding the right pair of products that add to the middle coefficient seems to challenge my students. By the time they master the basic (LC =1) cases, they have a hard time adjusting for this new wrinkle." The outcome of the study aligned with Kotsopoulos (2007) that concluded that factoring polynomials is one of the most challenging and complex topics to teach.

In addition, teachers (with high mean of the Self-efficacy Scores of 7.18) pointed out the challenges with identifying the purpose of learning factoring (12%). One of the teachers wrote:

> I do not have a good answer to the "Why do we need this stuff in life" question, and that question is important to my students. Also, I am at the beginning of my teaching career and have not had time to research good answers to that question, because factoring trinomials is a less important skill to our schools' parents than other, more directly applicable skills, like numeracy and interpreting graphs of data.

Also, teachers with low mean of the Self-Efficacy Scores of 6.78 mentioned the insufficient teaching methods make factoring even more difficult to learn (about 7% of responses): "it's usually poorly explained as being a magic formula, instead of being rigorously explained as splitting up the middle term and pulling out common factors." The research studies of Duarte (2010), Kotsopoulos (2007), and Li (2007) to list a few, concluded the same problems with teachers' limited abilities to teach factoring quadratic functions to introductory algebra classes.

Suggested additions to mathematical methods of instruction. The most frequent recommended addition to the introductory algebra method of teaching was to incorporate more meaningful applications and real-life examples (30% of responders). Teachers suggested including "real-life contexts," "a good, true story about how it helped someone solve a real critical problem. Testimony from an expert or study about how factoring trinomials is related to skills employers' value," to "stress application more than memorization," and one teacher shared: "I am always looking for ways to engage students with meaningful application".

Teachers also indicated adding more graphs and visualization (27.5% of responses). They suggested to include "more methods/practice; use of graphing techniques," "to acquaint students with the analytical, graphical and vital representation of quadratic functions," to use "graphing parabolas before solving equations," and to include "the visualization of how/why this works without 'guess and check'. Even something structured like a method would be helpful." The need of incorporating visual representations (graphs and modeling) into mathematics classroom has been previously acknowledged. The results of this study aligned with the Rivera's (2011) vison of creating a Visually-Oriented School Mathematics Curriculum, the curriculum that allows to "transit personally-constructed visuals, both externally-drawn and internally derived, into more structured visual representations within the context of a sociocultural grounded mathematical activity" (p.1).

Consistently, to remediate basic mathematical knowledge, teachers proposed to add teaching or reviewing background knowledge (20% of responses). They, for example, wanted to add "revision of Operations on integers, L.C.M and G.C.D.(H.C.F.)," "multiplication of numbers offhand," they wanted to make sure students "understand the function as input-output, plot and draw trinomial," and they requested to put "more emphasis on the distinction between and identifying terms/like terms and factors/common factors". The group of teachers requesting adding background knowledge got the second highest mean of the Self-Efficacy Scores (7.51).

Only 10 % of teachers suggested adding technology or graphing calculators. Interestingly, this group of teachers got the highest mean of the Self-Efficacy Scores (7.79). Lemon & Garvis (2014, p. 389) pointed out the common problems with using educational technology identified by Department of Education, Employment and Workplace Relations [DEEWR] (2011) in Australia as teachers' lack of confidence in the role of technology in learning (Dawson, 2008); reluctance to change from more traditional teaching methods (Barak, 2007); and isolation of the knowledge of technology from pedagogical and discipline expertise. (Mishra & Koehler, 2006). The results supported the findings of DEEWR (2011).

Only 5% of teachers asked for adding factoring by grouping. Those teachers got a very low mean of the Self-Efficacy Scores (6.46). This result is quite unexpected since factoring by grouping is the most advanced way of factoring and requires true understanding of all mathematical concepts.

One tenth of teachers believed that nothing should be added to the algebra method to make students more successful in factoring trinomials, as the curriculum is already overloaded. This group of teachers got the same mean of the Self-Efficacy Scores as the teachers who choose adding examples and applications (6.72). Furthermore, the noticeable fact was that the 26% of teachers who did not respond to this question got the lower mean of the Self-Efficacy Scores.

Suggested eliminations from mathematical methods of instruction. Almost half of the teachers (48.7%) stressed the need of reducing a number of methods of factoring quadratic functions. Unfortunately, teachers did not agree with methods that need to be eliminated. 32% pointed out at the "guess and check" method that "should be taught later as a short cut." 16% of teachers specified the quadratic formula as the only method of solving quadratic equations that supposed to be used in introductory algebra class. Interestingly, 5% completely disagreed, suggesting removing the quadratic formula. Unfortunately, this idea is very unreasonable because it would eliminate the only method that works for every quadratic equations, also showing absence of real solutions. Unexpectedly, 10% of teachers believed that the perfect square formula should be removed, even though, it is not very difficult and become very useful in calculus classes: "I would eliminate the Factoring Perfect Square Trinomial formulas. The students do not need to add another formula to their list, and potentially becoming more confused. They should just practice the method that comes more natural for them."

The results of this part of the study exactly reflected the current ongoing debate what method of factoring quadratic functions should be introduced at introductory algebra level. With the agreement that students are asked to learn too much, there is no consensus about what to do, which method to teach first, and how to progress and built students' understanding and skills.

In addition to decreasing number of methods of factoring trinomials, 25% of teachers stressed the importance of reducing "drill and kill practice" and focus on conceptual understanding instead. For example, one teacher pointed out that "It would be great if students had the time to discover the concepts vs. a one-size fits all method shoved down their throats so they can practice it enough to get fluent. Discovery of the method puts understanding first to make fluency easier to achieve as opposed to going through the motions with no understanding." This result supported the findings of Annette & Kitt (2000) and Leitze & Kitt (2000) that students often view memorization as a possible way to comprehend algebra instead of truly understand it.

Limitations

The researcher recognizes that there were certain limitations inherent in conducting this research study. The first constraint was a new personal data distribution law (GDPR) that took effect in Europe after the research was planned and before conducting the UVGIA survey. The law limited the access to the teachers and led to the lower than anticipated number of foreign participants. Consequently, the sample size was limited to 54 participants, which was lower than anticipated (84 of participants) and therefore lower the power of the study.

Secondly, the responses were impacted by participants' level of understanding and communicating in English. Though, the mathematics educators were attendees of international conferences, where English was the official language, many of them displayed limited English proficiency. Consequently, some parts of the UVGIA survey, especially open-ended questions, were left with limited or blank responses.

In addition, a cross-cultural comparison of incorporating technology exposed economical differences among countries and various teacher training programs and expectations. All those factors impacted the responses of the participants.

Implications of the Results for Theory, Policy, and Practice

Based on the findings of this study and other related studies, I concluded a few implications for the theory, policy, and practice.

1. School districts should consider changes in students' processes of preparation before entering algebra classes. The teachers of the study stressed the lack of students' understanding the basic mathematical concepts, and therefore, many of those students are not conceptually ready to learn factoring quadratic functions. Those facts need acknowledgement and actions.
2. In addition, this study gathered teachers' suggestions that school districts could use to reshape the introductory algebra curriculum related to solving quadratic equations. Teaching solving quadratic equations is one of the most challenging and complex topics to teach (Kotsopoulos, 2007), and to simplify it, the teachers of this study recommended reducing the number of methods of factoring quadratic functions introduced to students. It would allow to focus on developing conceptual understanding of each introduced method. The teachers proposed to include more

real-life applications which would help students to identify the purpose of learning solving quadratic equations.

3. The main implication of the study was to link self-efficacy of teachers to their levels of implementation of technology when teaching factoring quadratic functions. In the 21st century, graphing technology should be expected to be a natural choice to incorporate as a supplementary tool in all mathematics courses. Using technology, especially graphing technology, is widely encouraged (Cheung & Slavin, 2013; Dreiling, 2007; Drijvers, Monaghan, Thomas, & Trouche, 2014; Ellington, 2006; Kyungsoon, 1999; NCTM, 2000, Wilkins, 1995). In teaching mathematics graphing technology becomes a new representation (Ainsworth, 2006; Cabahug, 2012; Ogbonnaya, Mogari, & Machisi, 2013), a variation of a concept, that is critical to construct students' knowledge (Vygotsky, 1978).

The teachers of this study viewed technology as a useful tool in mathematics education. They also pointed out that students who struggle with graphs and visualizations experience challenges with learning factoring quadratic functions. However, the same teachers underutilized graphing technology in their classrooms. Since this study linked self-efficacy of teachers to their levels of implementation of technology, this connection could be used by districts, school administrations, and policy makers to support teachers and provide additional training to improve their abilities to effectively incorporate educational technology.

Recommendation for Further Research

The results of the study identified the need of the further research in the following areas:

1. The first recommendation of the further study is a research to examine how 15 years of teaching experience impacts mathematics teachers' level of self-efficacy and/or level of implementation of technology.

When analyzing the data, the researcher noticed an unusual relationship between the length of teaching experience and the teachers' level of self-efficacy (see Figure 13). The level of self-efficacy increases with the length of experience except from the teachers with 16–25 years of experience where the self-efficacy score dropped even lower than the level of self-efficacy of new teachers (1–5 years of experience). The same tendency was established between the length of teaching experience and the level of implementation of technology when teaching factoring quadratic functions to introductory algebra students.

Perhaps those anomalies could explain the results concluded by Mishne (2012) that teaching experience is not a predictor of technology proficiency of teachers (p. 71). Overall, some studies supported the conclusion that age and experience are not the significant predictors of the level of self-efficacy nor computer self-efficacy (Adebowale Adediwura, & Bada, 2009; Efe et al., 2016; Mishne, 2012; Tuncer & Tanaş, 2011;). However, Elbitar (2015) showed that age (which, in most cases, expresses also experience) had significant effect on computer self-efficacy of teachers. And even more, Czaja et al. (2006) concluded that computer self-efficacy of older teachers (over 65-years-old) dropped in comparison to the younger teachers.

The TALIS 2008 included a section which compared self-efficacy of new teachers and experienced teachers. The results revealed significantly lower self-efficacy of new teachers in comparison to experienced teachers (see Appendix D). The conclusion of the TALIS 2008 study aligned with the conclusion of this study. Based on data of the

National Center of Education Statistics, the comparison of number of teachers with different teaching experience in four school years: 1999–2000, 2007–2008, 2011–2012, and 2015–2016 showed, that in two more current years, the number of teachers starts decreasing at age 30–39, and then, from age 40–49 the decrease is very sharp (see Figure 17).

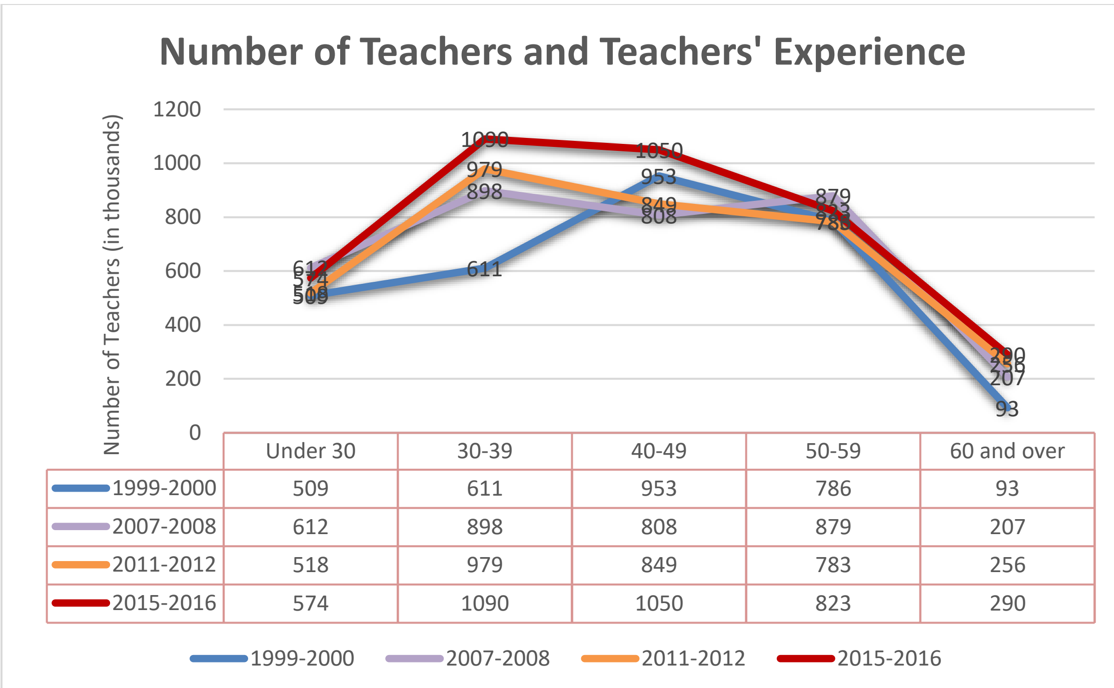


Figure 17. Number of teachers and teachers' experience (USA).
Based on U.S. Department of Education, National Center for Education Statistics, Schools and Staffing Survey (SASS),

The average teacher age of 40 corresponds, in average, with about 15 years of experience. It is possible, that this group of teachers experience a professional crisis? Their self-efficacy drops, level of implementation of technology drops, and the overall number of teachers starts decreasing. This interesting fact needs to be further examined.

2. The second area of further explorations should investigate still unclear impact of gender on self-efficacy and on computer self-efficacy. For mathematic teachers, is

gender a predictor of the level of self-efficacy, or the level of implementation of technology? Is it different in collectivistic and individualistic countries?

This study found no relationship between gender and teachers' level of self-efficacy, and no relationship between gender and teachers' level of implementation of technology in teaching factoring quadratic functions to introductory algebra students. The results are in agreement with research that reported no gender impact on self-efficacy of teachers nor on the level of implementation of technology into teaching methods (Amankwah, Jegede, & Sarfo, 2007; Efe, et al., 2016; Jegede, 2007; Loyd & Gressard, 1984; Pamuk & Peker, 2009; Roussos, 2007). However, some studies disagreed. The stereotypical view connects males with technology more than females (Brosnan & Lee, 1998; Durndell & Haag, 2002; Ozturk, Bozkurt, Kartal, Demir, & Ekici, 2011; Pamuk & Peker, 2009; Topkaya, 2010;). For example, Pamuk and Peker (2009) concluded, that "males had more positive attitude towards computer than females" (p. 455), and that male teachers have a significantly higher computer self-efficacy than the female. Yet, surprisingly Erdemir, Bakırcı, and Eyduran (2009) conjectured that female teachers have a higher level of self-efficacy than male teachers in using educational technology.

3. Another intriguing question that needs further investigation is: what prevents teachers from incorporating graphing technology? In general, teachers agreed with the urgent need of increasing visualization when teaching factoring quadratic functions. They also settled on the fact that educational technology is underutilized in mathematics classrooms. Therefore, why do they not use educational technology?
4. One of the goal of this study was to compare characteristics (self-efficacy and implementation of technology) between collectivistic and individualistic countries. I

recommend further research to explore the impact of a culture on self-efficacy of mathematics teachers involving more participants form more countries.

Conclusion

The fact that teaching factoring quadratic functions to introductory algebra students is challenging internationally was the main assumption of the conceptual framework of this study (see Figure 17). Results coming from survey of participating teachers supported and confirmed this assumption. Overcoming challenges with teaching factoring at school have been part of mathematics education for over 40 years and are well described in professional literature (Boulton-Lewis et al., 2001; Bransford, et al., 2000; Didis & Erbas, 2015; Duarte, 2010; French, 2002; Glaser, 1984; Kieran, 1981; Kotsopoulos, 2007; Lima, 2008; Martinez et al., 2016; Nielsen, 2015; Rauff, 1994; Sells, 1973; Wagner, 1981; Zakaria & Maat, 2010). The data analysis of this study confirms the list of the commonly mentioned challenges cited by teachers who present factoring in their classrooms. The list includes students' lack of basic mathematics skills, their lack of understanding graphs, too complex process, lack of understanding the purpose, challenges with leading coefficient different than 1, inadequate teaching methods, and confusing case when a function is not factorable. Teachers and educators for generations have been trying to juggle mathematical methods they use in their classrooms to improve students' comprehension in factoring quadratic functions in introductory algebra classes without much success. Adding or removing parts of mathematics curriculum have not brought the expected improvement to students' understanding.

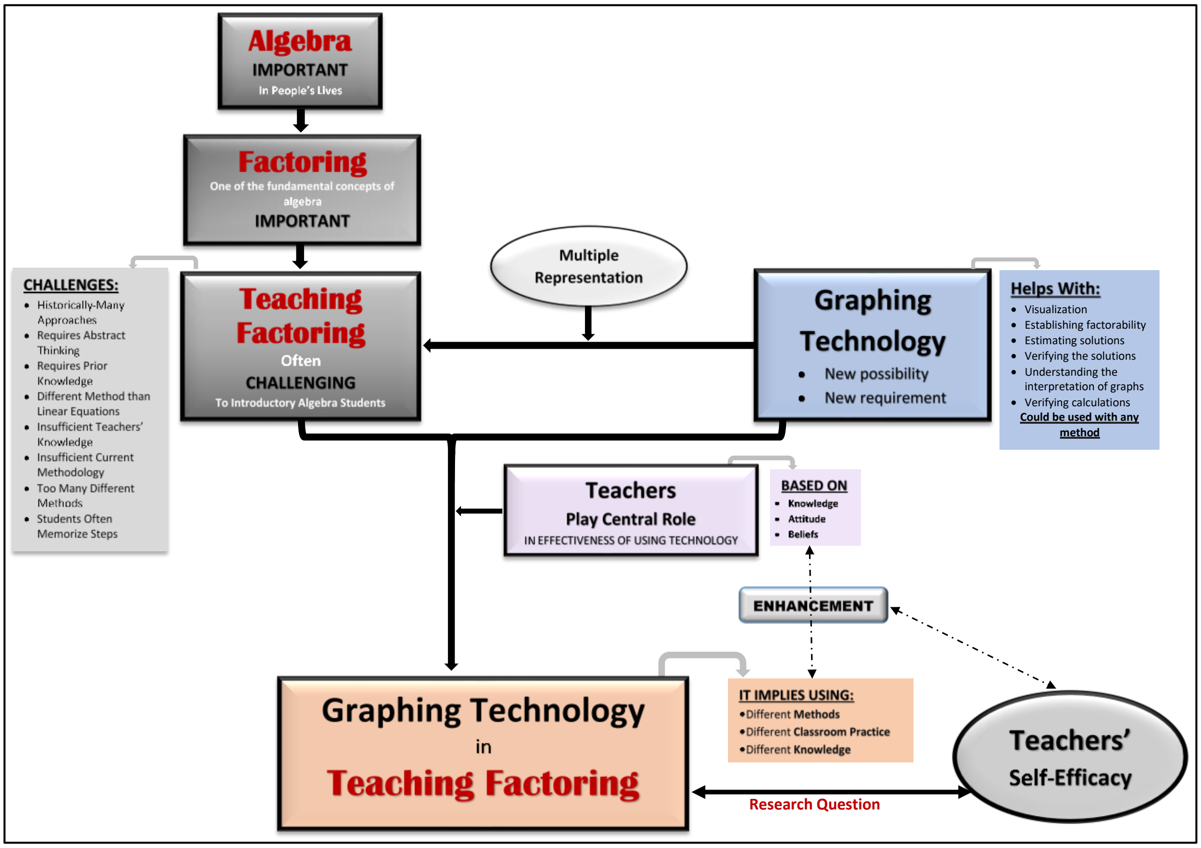


Figure 18. Conceptual framework of this study.

The conceptual framework schema (see Figure 18) illustrates the flow of teaching process and the central role of teachers' in decision making for classroom presentations. This study concentrated on assessing the influence of self-efficacy and technology use by algebra teachers, and the results shows that self-efficacy is strongly correlated to incorporating technology while discussing factoring of trinomials. Hence, this study adds an arrow from self-efficacy oval to the Graphing Technology box.

The analysis of the open-ended questions of the survey of this study confirmed the well-known recommendations for improving the instructions on factoring: adding background knowledge, adding graphs and visualizations, including more examples and applications, removing the number of methods and therefore automatic procedures and focusing on concepts.

In the conceptual framework, the researcher pointed out at graphing technology as a new possibility and requirement to enhance mathematical methods of teaching factoring quadratic functions (see Figure 17). This new suggestion was also mentioned by many participants. However, only 10% of responders recommended technology as a necessary addition to the currently used methods of teaching factoring trinomials. It shows, that technology is not taken advantage of in mathematical instructions. Few years ago, Lemon and Garvis (2014, p. 389) pointed out common problems with using educational technology identified by Department of Education, Employment and Workplace Relations (2011) in Australia: teachers' lack of confidence in the role of technology in learning (Dawson, 2008); reluctance to change from more traditional teaching methods (Barak, 2007); and isolation of the knowledge of technology from pedagogical and discipline expertise (Mishra & Koehler, 2006).

According to Ng, Nicholas, and Williams (2010), teachers are not going to change their behaviors unless they shift their beliefs. It means, self-efficacy of teachers has huge impact on teachers' goals and behaviors (Schunk & Meece, 2006). This study established existence of that impact in the context of using technology and preferences of mathematical methods of instruction. In addition, it confirmed the importance of self-efficacy on teachers' instructional choices. The findings of the study supported the relationship between self-efficacy of teachers and their behavior: their level of implementation of technology. The results concluded that mathematics teachers with the higher level of self-efficacy implemented more technology.

Tschannen-Moran and Hoy stated that "compelling evidence has been accumulating over the past three decades reveling the relationship of teacher's beliefs about their capability to impact students' motivation and achievement to important processes and outcomes in school" (2006, p. 944). While their study is more general, it confirms the underlining importance of self-efficacy of teachers, as it linked to the level of implementation of technology in mathematics classrooms.

In conclusion, the researcher recommends that all educational technology training and professional development should include intentional activities to improve teachers' self-efficacy to increase their willingness to incorporate new technology-based tools in introductory algebra courses. The results of this study clearly demonstrate a powerful connection between teacher self-efficacy, a willingness to adapt to new strategies, and the actual use of technology-based tools.

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Appendix A: The UVGIA Survey

Using Visual Graphing in Introductory Algebra Survey

Your participation in this survey on the relationship between visual graphing, student learning, and teacher perceptions is valuable. Your responses will remain confidential, so we kindly ask for honest current responses. The data will be used for research purposes only and the results will be used to inform introductory algebra instruction.

Please follow the instructions attached to each of the five parts of the survey.

Part 1. Background Information

1. In what country do you teach?					
2. What is your gender?					
Male		Female		Other	
3. How old are you? (Circle one)					
20-29 yrs. old	30-39 yrs. old	40-49 yrs. old	50-59 yrs. old	60 yrs. or older	
4. What is your highest qualification to teach mathematics? Choose everything that apply to you:					
☐ Bachelor of Science in Mathematics			☐ Bachelor of Art in Mathematics		
☐ Master of Science in Mathematics			☐ Master of Art in Mathematics		
☐ Master of Education			☐ Doctorate in Education		
☐ Ph.D.			☐ Teaching Certificate		
5. Which of the following university level courses did you take before becoming a mathematics teacher:					
☐ Precalculus			☐ Mathematics for Teacher		
☐ Calculus Differential			☐ Geometry for Teachers		
☐ Calculus Integral			☐ Math Methods/Pedagogy/Didactic Course		
☐ Calculus Multivariable			☐ History of Mathematics		
☐ Abstract Algebra			☐ Educational Technology		
6. How many years have you taught mathematics at school?					
1-5 yrs.	6-10 yrs.	11-15 yrs.	16-20 yrs.	21-25 yrs.	26 nd more
7. What age group have you taught mathematics? (check all that apply).					
12-14	15-17	18 and older	None		
8. What age group do you currently teach (check all that apply):					
12-14	15-17	18 and older	None		
9. To what age group have you introduced factoring quadratic functions (check all that apply)					
12-14	15-17	18 and older	None		

Part 2. Teachers' Sense of Efficacy Scale1 (short form)

(Based on Tschannen-Moran & Hoy, 2001)

	Teacher Beliefs	**How much can you do?**								
	Directions: This questionnaire is designed to help us gain a better understanding of the kinds of things that create difficulties for teachers in their school activities. Please indicate your opinion about each of the statements below. Your answers are confidential.	Nothing		Very Little		Some Influence		Quite a Bit		A Great Deal
1	How much can you do to control disruptive behavior in the classroom?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
2	How much can you do to motivate students who show low interest in school work?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
3	How much can you do to get students to believe they can do well in school work?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
4	How much can you do to help your students value learning?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
5	To what extent can you craft good questions for your students?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
6	How much can you do to get children to follow classroom rules?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
7	How much can you do to calm a student who is disruptive or noisy?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
8	How well can you establish a classroom management system with each group of students?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
9	How much can you use a variety of assessment strategies?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
10	To what extent can you provide an alternative explanation or example when students are confused?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
11	How much can you assist families in helping their children do well in school?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
12	How well can you implement alternative strategies in your classroom?	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

Additional comments, questions, or concerns: ____

Part 3. Role of Technology in Teaching Factoring Quadratic Functions

	Technology in Teaching Mathematics Teacher Beliefs					
	Directions: Please indicate your opinion about each of the statements below. Your answers are confidential.	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1	I am familiar with using technology in my classroom to teach factoring quadratic functions	(1)	(2)	(3)	(4)	(5)
2	I am confident in using technology in my classroom to teach factoring quadratic functions	(1)	(2)	(3)	(4)	(5)
3	I feel technology could make teaching factoring quadratic functions effective in my classroom.	(1)	(2)	(3)	(4)	(5)
4	I feel students engage with learning when technology is incorporated in my classroom activities.	(1)	(2)	(3)	(4)	(5)

	Use of Graphing Technology in Teaching Factoring Quadratic Functions					
	Directions: Please indicate your opinion about each of the statements below. Your answers are confidential.	To No Extend	To a Little Extend	Moderately	To Great Extend	To an Extreme
1	I have access to graphing technology	(1)	(2)	(3)	(4)	(5)
2	I use graphing technology in my teaching methods.	(1)	(2)	(3)	(4)	(5)
3	I expect students to use factoring programs to solve quadratic equations	(1)	(2)	(3)	(4)	(5)
4	I teach students to use graphs generated by graphing technology to solve quadratic equations.	(1)	(2)	(3)	(4)	(5)
5	I teach students to use graphing technology to perform computations	(1)	(2)	(3)	(4)	(5)
6	I teach students to use graphs generated by graphing technology for early introduction of different representations of functions	(1)	(2)	(3)	(4)	(5)
7	I teach students to use graphing technology to check their answers by graphing.	(1)	(2)	(3)	(4)	(5)
8	I teach students to use parabolas obtained from graphing technology to assess solutions to quadratic equations in ‘guess and check” method.	(1)	(2)	(3)	(4)	(5)

Additional comments, questions, or concerns: ____

Part 4. Teaching Factoring Quadratic Functions
Teacher Perspective

1. What do you think is the most challenging part of factoring trinomials for students to understand when you teach it?

   Explain_________________________

2. What would you **ADD** to introductory algebra methods of teaching to make students more successful in factoring trinomials?

   Explain_________________________

3. What would you **REMOVE** from the introductory algebra methods of teaching to make students more successful in mathematics?

   Explain_________________________

Additional comments, questions, or concerns: ____

____

Thank you for taking the time to respond to my survey!

Appendix B: Permission to Use the TSES

Email asking for the permission

June 20, 2018

Permissions Editor:
Megan Tschannen-Moran, College of William and Mary
Anita Woolfolk Hoy, the Ohio State University

Dear Mrs. Tschannen-Moran and Mrs. Woolfolk Hoy:

I am a doctoral student from Concordia University in Portland OR writing my dissertation tentatively titled: The impact of teacher self-efficacy on methodology of teaching factoring quadratic functions: Perspective of global and local teachers on effectiveness of using graphing technology in introductory algebra, under the direction of my dissertation committee chaired by Dr. Angela Owusu-Ansah.

I would like your permission to reproduce to use survey instrument (Teachers' Sense of Efficacy Scale, short form) in my research study. I would like to use your survey under the following conditions:

- I will use this survey only for my research study and will not sell or use it with any compensated or curriculum development activities.
- I will include the copyright statement on all copies of the instrument.
- I will send my research study and one copy of reports, articles, and the like that make use of these survey data promptly to your attention.

If these are acceptable terms and conditions, please indicate so by signing one copy of this letter and returning it to me either through postal mail or e-mail:

Malgorzata Mart
[contact information redacted]

Thank you very much,

Sincerely,

Malgorzata Mart
Doctoral Candidate

____

Signature:

____

Expected date of completion 05/2019

Permission #1 to Use the TSES:

William & Mary
School of Education

MEGAN TSCHANNEN-MORAN, PHD
PROFESSOR OF EDUCATIONAL LEADERSHIP

July 3, 2018

Malgorzata,

You have my permission to use the Teacher Sense of Efficacy Scale (formerly called the Ohio State Teacher Sense of Efficacy Scale), which I developed with Anita Woolfolk Hoy, in your research. You can find a copy of the measure and scoring directions on my web site at http://wmpeople.wm.edu/site/page/mxtsch . Please use the following as the proper citation:

> Tschannen-Moran, M & Hoy, A. W. (2001). Teacher efficacy: Capturing an elusive construct. *Teaching and Teacher Education, 17*, 783-805.

I will also attach directions you can follow to access my password protected web site, where you can find the supporting references for this measure as well as other articles I have written on this and related topics.

I would love to receive a brief summary of your results.

All the best,

Megan Tschannen-Moran
The College of William and Mary
School of Education

P.O. Box 8795 • Williamsburg, VA 23187-8795 • (757) 221-2187 • mxtsch@wm.edu

Permission #2 to Use the TSES:

You are welcome to use the TSES in your research as you describe below. This website might be helpful to you:

http://u.osu.edu/hoy.17/research/instruments/

Best wishes in your work

Anita

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Appendix C: Reliabilities of the TSES

In Tschannen-Moran, M., & Woolfolk Hoy, A. (2001). Teacher efficacy: Capturing and elusive construct. *Teaching and Teacher Education*, *17*, 783-805, the following were found:

	Long Form			**Short Form**		
	Mean	SD	alpha	Mean	SD	alpha
TSES (OSTES)	.1	.94	.94	7.1	.98	.90
Engagement	7.3	1.1	.87	7.2	1.2	.81
Instruction	7.3	1.1	.91	7.3	1.2	.86
Management	6.7	1.1	.90	6.7	1.2	.86

Because this instrument was developed at the Ohio State University, it is sometimes referred to as the *Ohio State Teacher Efficacy Scale* (OSTES). We prefer the name, *Teachers' Sense of Efficacy Scale (TSES)*.

Appendix D: TALIS 2008: Teachers' Perceived Self-Efficacy

	New teachers			Experienced teachers	
	Mean	(S.E.)		Mean	(S.E.)
Australia	0.20	0.14		0.32	0.03
Austria	0.39	0.08		0.24	0.02
Belgium (Fl.)	-0.10	0.05	*	0.07	0.02
Brazil	-0.10	0.09		-0.10	0.03
Bulgaria	-0.03	0.13		0.23	0.03
Denmark	-0.01	0.07	*	0.31	0.03
Estonia	-0.54	0.05	*	-0.39	0.01
Hungary	-0.49	0.08		-0.41	0.02
Iceland	0.18	0.07	*	0.37	0.03
Ireland	0.10	0.09	*	0.31	0.03
Italy	0.24	0.06		0.37	0.02
Korea	-1.05	0.07	*	-0.75	0.02
Lithuania	-0.09	0.08		0.06	0.02
Malaysia	-0.15	0.07	*	0.02	0.03
Malta	-0.27	0.08	*	-0.01	0.04
Mexico	0.11	0.11		0.08	0.03
Norway	0.34	0.07	*	0.52	0.03
Poland	-0.28	0.04	*	-0.12	0.02
Portugal	-0.12	0.09		-0.08	0.02
Slovak Republic	-0.57	0.07	*	-0.27	0.02
Slovenia	0.04	0.05		0.00	0.01
Spain	-0.46	0.07		-0.45	0.02
Turkey	-0.37	0.14	*	0.08	0.03
TALIS average	***-0.13***	***0.02***	*****	***0.02***	***0.01***

*Note***.** Statistically significant differences are marked with an *.
Source: *Teaching And Learning International Survey 2008*
Statlink: http://dx.doi.org/10.1787/888932578505

Appendix E: TALIS 2008: Email Used to Introduced the Study

Dear ______

I would like to invite you to participate in the study: Factoring Quadratic Functions in Introductory Algebra: Perspective of Global and Local Teachers on Effectiveness of Using Graphing Technology. The purpose of the study is to identify different ways of incorporating graphing technology that have been used to teach factoring in both, the United States and globally, the impact of using technology on students learning reported by teachers, as well as the teachers' perspectives on using different mathematical methods to teach factoring to introductory algebra students. All the findings will be related to the level of self-efficacy of the teachers.

The main instrument of the study is a survey. It contains five parts and has in total 41 items. The survey should take less than 30 minutes of your time. All the survey responses will be kept strictly confidential.

Your participation is greatly appreciated. We invite you to complete the survey as well as to share a link to the survey with other mathematics teachers in your country, who may wish to participate in the study. Please click the link below to start the survey:

____link_________________

Thank you very much for your time. Your experience, knowledge, and beliefs are very important for the study.

Sincerely,

Malgorzata Mart

Concordia University, USA

Appendix F: Statement of Original Work

The Concordia University Doctorate of Education Program is a collaborative community of scholar-practitioners, who seek to transform society by pursuing ethically-informed, rigorously- researched, inquiry-based projects that benefit professional, institutional, and local educational contexts. Each member of the community affirms throughout their program of study, adherence to the principles and standards outlined in the Concordia University Academic Integrity Policy. This policy states the following:

Statement of academic integrity.

As a member of the Concordia University community, I will neither engage in fraudulent or unauthorized behaviors in the presentation and completion of my work, nor will I provide unauthorized assistance to others.

Explanations:

What does "fraudulent" mean?

"Fraudulent" work is any material submitted for evaluation that is falsely or improperly presented as one's own. This includes, but is not limited to texts, graphics and other multi-media files appropriated from any source, including another individual, that are intentionally presented as all or part of a candidate's final work without full and complete documentation.

What is "unauthorized" assistance?

"Unauthorized assistance" refers to any support candidates solicit in the completion of their work, that has not been either explicitly specified as appropriate by the instructor, or any assistance that is understood in the class context as inappropriate. This can include, but is not limited to:

- Use of unauthorized notes or another's work during an online test
- Use of unauthorized notes or personal assistance in an online exam setting
- Inappropriate collaboration in preparation and/or completion of a project
- Unauthorized solicitation of professional resources for the completion of the work.

Statement of Original Work (Continued)

I attest that:

1. I have read, understood, and complied with all aspects of the Concordia University–Portland Academic Integrity Policy during the development and writing of this dissertation.

2. Where information and/or materials from outside sources has been used in the production of this dissertation, all information and/or materials from outside sources has been properly referenced and all permissions required for use of the information and/or materials have been obtained, in accordance with research standards outlined in the *Publication Manual of The American Psychological Association*

Malgorzata Mart

Digital Signature

Malgorzata Mart

Name (Typed)

December 10, 2018

Date