

Correction to: Material description of fluxes in terms of differential forms

Original

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Correction to:
“Material description of fluxes in terms of differential forms”

“Dedicated to Prof. David Steigmann in recognition of his contributions”

**Salvatore Federico · Alfio Grillo ·
Reuven Segev**

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- 4 **Correction to: Continuum Mech. Thermodyn. (2016) 28:379–390**
5 **<https://doi.org/10.1007/s00161-015-0437-2>**

6 Although the final result presented in Equation (34) of our work [?] is correct,
7 the proof of Equation (34) contains an error. The text starting immediately
8 after Equation (33) with “When a metric tensor \mathbf{g} ...” and ending immediately
9 before Equation (36) with “... in the alternative notation” should be replaced
10 with the text below.

11 **Correction to the Proof of Equation (34)**

12 Let us assume that the space \mathcal{S} is equipped with a metric tensor \mathbf{g} , i.e., a
13 symmetric and positive-definite tensor field valued in $[T\mathcal{S}]_2^0$, defining the scalar
14 product of two vectors \mathbf{u} and \mathbf{v} as $\mathbf{u}\cdot\mathbf{v} = \mathbf{g}(\mathbf{u}, \mathbf{v})$. The metric \mathbf{g} induces the

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15 *musical isomorphisms* $\flat : TS \rightarrow T^*S : \mathbf{v} \mapsto \flat(\mathbf{v}) \equiv \mathbf{v}^\flat$, which maps a vector \mathbf{v}
 16 with components v^c to a covector \mathbf{v}^\flat with components $g_{ac}v^c$, and its inverse
 17 $\sharp : T^*S \rightarrow TS : \boldsymbol{\alpha} \mapsto \sharp(\boldsymbol{\alpha}) \equiv \boldsymbol{\alpha}^\sharp$, which maps a covector $\boldsymbol{\alpha}$ with components α_c
 18 to a vector $\boldsymbol{\alpha}^\sharp$ with components $g^{ac}\alpha_c$, where g^{ac} are the components of the
 19 inverse of the matrix $\llbracket g_{ab} \rrbracket$ of \mathbf{g} . The isomorphism \sharp and the metric tensor \mathbf{g}
 20 induce the scalar product of covectors $\boldsymbol{\alpha} \cdot \boldsymbol{\beta} = \mathbf{g}(\boldsymbol{\alpha}^\sharp, \boldsymbol{\beta}^\sharp) = \boldsymbol{\alpha}(\boldsymbol{\beta}^\sharp)$.

21 The $(n-1)$ -dimensional tangent bundle Ts of the hypersurface s determines
 22 a 1-dimensional sub-bundle of T^*S containing the annihilators of Ts , i.e., the
 23 covectors $\boldsymbol{\nu}$ such that $\boldsymbol{\nu}\mathbf{u} \equiv \boldsymbol{\nu}(\mathbf{u}) = 0$, for every $\mathbf{u} \in Ts$. Moreover, using the
 24 scalar product of covectors, we can define the *unit normal covector* \mathbf{n} to the
 25 hypersurface s as the annihilating covector such that $\|\mathbf{n}\|^2 = \mathbf{n} \cdot \mathbf{n} = 1$.

26 The integral (33) of an $(n-1)$ -form $\boldsymbol{\omega}$ on the hypersurface s can be ex-
 27 pressed in terms of the axial vector field \mathbf{w} of $\boldsymbol{\omega}$ with respect to the volume form
 28 $\boldsymbol{\mu}$, i.e., \mathbf{w} is such that $\iota_{\mathbf{w}}\boldsymbol{\mu} = \boldsymbol{\omega}$. If we introduce the *axial projector* $\mathbf{a} = \mathbf{n}^\sharp \otimes \mathbf{n}$
 29 (in components, $a^a_b = n^a n_b$) and the *transverse projector* $\mathbf{t} = \mathbf{i} - \mathbf{n}^\sharp \otimes \mathbf{n}$ (in
 30 components, $t^a_b = \delta^a_b - n^a n_b$, where \mathbf{i} is the spatial identity tensor, it holds
 31 that $\mathbf{i} = \mathbf{a} + \mathbf{t}$ and that any vector field \mathbf{w} can be decomposed as

$$\mathbf{w} = \mathbf{i}\mathbf{w} = (\mathbf{a} + \mathbf{t})\mathbf{w} = \mathbf{a}\mathbf{w} + \mathbf{t}\mathbf{w} = \mathbf{w}_a + \mathbf{w}_t, \quad (\text{C1})$$

32 where $\mathbf{w}_a = \mathbf{a}\mathbf{w} = (\mathbf{n}\mathbf{w})\mathbf{n}^\sharp$ and $\mathbf{w}_t = \mathbf{t}\mathbf{w} = \mathbf{w} - (\mathbf{n}\mathbf{w})\mathbf{n}^\sharp$ are the axial and the
 33 transverse component of \mathbf{w} , respectively. By construction, \mathbf{w}_t is an element of
 34 the tangent bundle of the $(n-1)$ -dimensional manifold $s \subset S$. Hence, due to
 35 linearity, the $(n-1)$ -form $\boldsymbol{\omega} = \iota_{\mathbf{w}}\boldsymbol{\mu}$ can be written as

$$\boldsymbol{\omega} = \iota_{\mathbf{w}}\boldsymbol{\mu} = \iota_{(\mathbf{w}_a + \mathbf{w}_t)}\boldsymbol{\mu} = \iota_{\mathbf{w}_a}\boldsymbol{\mu} + \iota_{\mathbf{w}_t}\boldsymbol{\mu}. \quad (\text{C2})$$

36 Let now $\{\mathbf{u}_1, \dots, \mathbf{u}_{n-1}\} \subset Ts$ be a set of linearly independent vectors span-
 37 ning Ts . Since \mathbf{w}_t can be expressed as a linear combination of $\mathbf{u}_1, \dots, \mathbf{u}_{n-1}$,
 38 we obtain

$$(\iota_{\mathbf{w}_t}\boldsymbol{\mu})(\mathbf{u}_1, \dots, \mathbf{u}_{n-1}) = \boldsymbol{\mu}(\mathbf{w}_t, \mathbf{u}_1, \dots, \mathbf{u}_{n-1}) = 0. \quad (\text{C3})$$

39 Comparing Eq. (C3) with the definition of $\boldsymbol{\omega}$ in Eq. (C2), we find

$$\boldsymbol{\omega}(\mathbf{u}_1, \dots, \mathbf{u}_{n-1}) = (\iota_{\mathbf{w}}\boldsymbol{\mu})(\mathbf{u}_1, \dots, \mathbf{u}_{n-1}) = (\iota_{\mathbf{w}_a}\boldsymbol{\mu})(\mathbf{u}_1, \dots, \mathbf{u}_{n-1}), \quad (\text{C4})$$

40 and, since (C4) must hold true for all $(n-1)$ -tuples $\{\mathbf{u}_1, \dots, \mathbf{u}_{n-1}\} \subset Ts$, we
 41 can write

$$\boldsymbol{\omega} = \iota_{\mathbf{w}}\boldsymbol{\mu} \equiv \iota_{\mathbf{w}_a}\boldsymbol{\mu}, \quad (\text{C5})$$

42 i.e., only the axial component of \mathbf{w} , which is the component parallel to the vector
 43 \mathbf{n}^\sharp associated with the normal covector \mathbf{n} to the hypersurface s , contributes to
 44 $\boldsymbol{\omega}$. Finally, by exploiting the result $\mathbf{w}_a = (\mathbf{n}\mathbf{w})\mathbf{n}^\sharp = (\mathbf{w}\mathbf{n})\mathbf{n}^\sharp$ and the linearity
 45 of the interior product, Eq. (C5) becomes

$$\boldsymbol{\omega} = \iota_{\mathbf{w}}\boldsymbol{\mu} \equiv \iota_{\mathbf{w}_a}\boldsymbol{\mu} = (\mathbf{w}\mathbf{n})\iota_{\mathbf{n}^\sharp}\boldsymbol{\mu} = (\mathbf{w}\mathbf{n})\boldsymbol{\alpha}, \quad (34 \text{ corr.})$$

46 where

$$\boldsymbol{\alpha} = \iota_{\mathbf{n}} \boldsymbol{\mu} \quad (1)$$

47 is the $(n - 1)$ -form induced on the hypersurface s by the volume form $\boldsymbol{\mu}$ and
48 the metric \mathbf{g} . Therefore, on the basis of these results, the flux of an exten-
49 sive quantity q across the hypersurface s can be expressed in the alternative
50 notation [...]

51 References

- 52 1. S. Federico, A. Grillo, R. Segev, Material description of fluxes in terms of differ-
53 ential forms, *Continuum Mechanics and Thermodynamics*, **28**(1-2), 379-390 (2016)
54 <https://doi.org/10.1007/s00161-015-0437-2>