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New solution approaches for the Train Load Planning Problem

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Abstract The present paper faces the train load planning problem (TLPP) in container terminals. The problem consists of assigning containers to rail wagons while maximizing the total priority of the containers loaded and minimizing the number of rehandles executed in the terminal yard. Two different heuristic approaches, based on an innovative way to compute weight limitations and on two 0/1 integer programming models, are proposed and compared on the basis of specific key performance indicators (KPIs). The heuristic approaches are compared by using random generated instances based on real-world data. An extensive computational analysis has been performed.

Keywords .

Train load planning problem; optimization; 0/1 integer linear programming; slot weight limitations; load configurations

1 Introduction

Following the financial crisis of 2008, containerization transportation demand has started growing again; the current forecasting for future years indicates a continuous increase of containerized cargo transportation at a global level. This has a strong impact on both seaports and inland ports, which have to handle an increasing amount of goods while respecting parameters of efficiency and speed. In such a context, rail transportation should be strengthened, representing the most suitable transport mode to forward large volumes of freight

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inland. If properly organized, rail transportation is able to handle higher quantities of cargo more quickly than road transport (Alumur et al. , 2012) and to relieve road networks of heavy vehicles, bringing environmental benefits both in terms of pollution and congestion.

An important issue concerning the effectiveness of rail transport in container terminals is related to the Train Load Planning Problem (TLPP). The TLPP consists of assigning containers to train wagons while pursuing specific goals, such as minimizing the time needed for loading the train, optimizing the use of resources or, simply, minimizing the total loading costs. Particularly in small to medium sized terminals, but also in some larger ones, this planning activity is executed by rail planners based on their experience and on rail tables considering the features of containers, wagons and rail lines. This often results in non optimal train plans in terms of time and cost minimization, consequently affecting the competitiveness of both rail transport and terminals. The present paper considers the optimal loading of trains in container terminals, proposing two solution approaches to quickly solve the TLPP, thus helping rail planners performing at their best.

1.1 Literature review

In recent literature, a lot of research on container terminals concerns models for operations management, see for instance (Steenken et al. (2004), Stahlbock and Voss (2008), Vis and De Koster (2003)); on the other hand, numerous papers have been dedicated to different problems related to rail transportation (see, among others, Anghinolfi et al. (2011), Wanga and Yun (2013), Woxenius and Bergqvist (2011), Vaidyanathan et al. (2008), Bektas et al. (2009) and Luan et al. (2017)).

Ahuja et al. (2005) provide a survey about railroad planning and scheduling problems, including the railroad blocking problem, train scheduling and train dispatching problems, locomotive scheduling and crew scheduling problems and yard location problem. The main factors affecting the efficiency of rail transport in container terminals are discussed by Corry and Kozan (2008), who propose a simulation model combined with some heuristic rules. In Parola and Sciomachen (2009) the authors present a study of multimodal container flows passing through an Italian maritime terminal, focusing the attention on the performance of road and rail connections. The problem concerning the inter-terminal transportation flows is analyzed by Tierney et al. (2014).

Only in recent years the planning of train loading operations has obtained attention from both researchers and terminal operators. Some works related to this topic concern landside intermodal terminals while few recent papers focus on seaports. In Bostel and Dejax (1998) some models and heuristic methods for container allocation problems on trains in rail-rail terminals are presented, while Corry and Kozan (2006) propose an assignment model for the dynamic load planning of containers in road-rail terminals; more specifically, Corry and Kozan (2006) provide several techniques for the train loading problem in order

to assign containers to train slots while minimizing the container handling time and optimizing the weight distribution on trains; however, only one type of container is considered and weight restrictions for wagons are not included in the model. In a following work (Corry and Kozan (2008)), more types of containers are considered and the aim is to also minimize the train length.

In Lai et al. (2008) a loading assignment model for freight trains is provided with the final goal of generating more fuel-efficient trains.

A good contribution for planning train loading operations in intermodal terminals is provided in Bruns and Knust (2012): three different integer linear programming formulations are proposed for assigning containers to wagon slots with the goal of maximizing train utilization and minimizing both transportation costs for loading containers and set up costs for changing wagon configurations. Moreover, different types of containers are considered as well as various weight restrictions related to wagon configurations. A more recent paper (Bruns et al. (2014)) tackles uncertainty related to both train wagons and load units by using robust optimization; in addition, two different planning phases are distinguished: the “pre-loading” and “while-loading” planning. Another very recent contribution to the train load planning problem is given by Mantovani et al. (2017) that formulated and solved an ILP model for the single and double-stack planning problem with the aim of minimizing the resulting loading cost.

Ambrosino et al. (2011) provide the first work related to the TLPP in seaports and present two mathematical formulations and a heuristic approach for the TLPP with the goal of minimizing rehandling operations in the yard and maximizing train utilization, assuming that the train is loaded in a sequential way (i.e., the rail crane cannot perform back and forth movements when loading the train). Note that rehandles - also called reshuffles - are unproductive movements executed in the yard, necessary in order to take out all those containers that block the picking up of the required containers (as better explained in the next section). In the seaport context rehandles have a higher impact - in terms of time and cost - in respect to inland intermodal terminals, in which the available space is usually greater (Rodriguez et al. , 2012).

Ambrosino et al. (2013) present a comparison among different train loading policies (i.e., sequential loading, non sequential loading and intermediate combinations) by varying the stacking strategies applied in the terminal yard; Ambrosino and Siri (2015) provide a comparison of different solution approaches for the train load planning problem in a seaport container terminal based on a mathematical programming model. Anghinolfi and Paolucci (2014) propose a general purpose Lagrangian heuristic for solving the TLPP in seaport terminals for a sequence of trains having different destinations. In Anghinolfi and Paolucci (2014) the authors extend the model proposed in Ambrosino et al. (2011) for including more trains and for minimizing also the distance traveled by containers when they are transferred from the yard to trains.

The present paper focuses on the sequential loading planning problem in a seaport container terminal and proposes two heuristic approaches devoted to

reduce the problem complexity, quickly solve the TLPP and be easily applied in operational contexts. With respect to Bruns and Knust (2012) and Ambrosino and Siri (2015), the proposed heuristic approaches use only a weight condition with proper weight limitations for satisfying stability and structural constraints. This work also proposes different interpretations of wagon load configurations based on a new way to represent wagon slots with respect to the previous literature devoted to the TLPP. The TLPP here investigated has the aim of minimizing the number of unproductive movements in the yard (re-handles) and the sum of the priority of containers that can not be loaded on the train. Even if the problem presents two objectives, it has not been solved as a bi-objectives optimization problem but minimizing the weighted sum of the two previous mentioned objectives. In the recent literature, Ambrosino et al. (2016) formulated a particular TLPP when the train is loaded with two cranes as a multi-objective optimization problem (MOOP).

The effectiveness of the proposed approaches has been compared with real data and real train plans provided by some Italian container terminals.

The remainder of the paper is organized as follows. Section 2 describes in more detail the problem under investigation. In Section 3, the heuristic approaches are presented. Section 4 provides an extensive computational analysis; finally, some concluding remarks are reported in Section 5.

2 Problem description.

Figure 1 shows a typical process of train loading in a container terminal: containers that arrive in the terminal and have to continue their trip by rail, are stacked in the yard in specific blocks; from here, they are picked up by the terminal equipment - such as Rubber Tyred Gantry (RTG) cranes or reach stackers - and transported close to the rail park located inside the terminal to be loaded on rail wagons - usually by a Rail Mounted Gantry (RMG) crane.

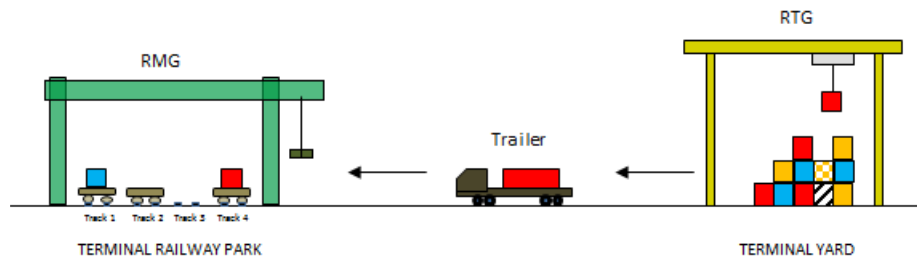


Fig. 1 A generic rail process in a container terminal.

The TLPP under investigation can be stated as follows: given a train composed of a set of empty wagons characterized by different weights, lengths and load configurations, and given a set of containers characterized by different

weights, lengths, priorities and stacking positions in the yard, the TLPP consists of assigning containers to wagons in order to satisfy weight and length constraints, while both minimizing rehandles in the stacking area and maximizing the priorities of the containers loaded on the train. The “priority” reflects the importance associated with each container and is related to one or more of the following parameters: customer significance, value of goods transported in the container and urgency of sending the load unit. On the other hand, rehandles are unproductive movements that must be minimized because they strongly affect time and cost associated with loading operations. The number of rehandles to execute in order to pick up a container is affected by the type of terminal equipment used. To make this point clearer, if the black striped container in the block under the RTG in Figure 1 has to be picked up before the container above it, thus one rehandle has to be executed. The situation is different if using a reach stacker (as shown in Figure 2), and assuming that - as occurs in reality - the whole block is operated from one side: four and seven rehandles are needed when approaching the yard block from the right side and the left side, respectively.

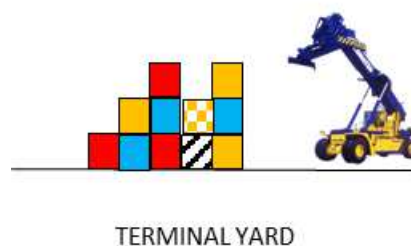


Fig. 2 Rehandles in case of yard handled with reach stackers.

In the TLPP here analyzed, it is assumed that the rail planner can choose between a set of possible containers usually respecting specific characteristics (such as the ship of origin or the final destination), instead of having a list of containers to load. Note that the considered case is more general and also includes the situation with a fixed list of containers.

It is also noted that the destinations of containers are not here explicitly analyzed, since a train having a given destination (i.e., a rail station, a company or an inland terminal) is here considered. In any case, the approaches presented in this paper can easily be extended to include also the container destination.

Considering the timing of trains, in the majority of container terminals the planning is applied to one train at a time. Given a particular yard configuration, the optimal load for a specific train is determined. After this planning, the train can be physically loaded, as stated in Bruns et al. (2014). Not considered in this work are all the possible changes regarding the yard configuration

(due to new container arrivals, housekeeping operations or rehandles) that may occur during the planning phase, during the loading process and between planning and loading operations, since the object of this paper is an “off-line” planning and not an “on line” one (i.e., physical loading). For the next train planning, a new yard configuration caused by all changes above mentioned is then taken into account.

Some other assumptions have been made for the TLPP here considered. As already highlighted, the sequential loading of the train is considered, meaning that the rail crane devoted to load containers on the train starts loading from the first wagon onwards. Note that this loading mode is assumed when it is preferable, both from an economic and temporal standpoint, to execute container rehandles instead of moving the rail crane backward and forward along its track (see Ambrosino et al. (2013) for more details). Some considerations about the effectiveness of the sequential loading method for solving the train loading problems have been proved in Ambrosino and Siri (2015).

We also assume that the terminal yard area, where the containers waiting to be loaded on trains are stored, is located very close to the terminal rail park; therefore distances from the yard to the rail park can be neglected. Besides, only 20 and 40 foot containers are considered because they represent the most used typologies among containerized load units.

More formally, given C containers in the stocking area, characterized by:

- a weight ω_i (expressed in tons), $i = 1, \dots, C$;
- a length λ_i (either 20' or 40') $i = 1, \dots, C$;
- a priority π_i (high, medium or low) $i = 1, \dots, C$;
- their relative position in the storage yard, $\gamma_{i,j}$, $i, j \in \{1, \dots, C\}$, $i \neq j$ indicating the position of a container i with respect to container j belonging to the same stack. More precisely, $\gamma_{i,j} = 1$ indicates that container i is located over j ; otherwise $\gamma_{i,j} = 0$;

given a train composed by W wagons characterized by:

- a weight capacity $\bar{\omega}_w$, $w = 1, \dots, W$;
- a set of possible slots \mathcal{S}_w , $w = 1, \dots, W$, having different length μ_s , $s \in \mathcal{S}_w$ (either 20' or 40') and different positions on the train ρ_s , $s \in \mathcal{S}_w$ (expressed in TEUs);

the problem consists in defining the assignment of containers to the wagons' slots in such a way to maximise the priorities of containers loaded and minimize the number of rehandles in the yard, while satisfying the train capacity and stability and structural constraints.

Note that TEU stands for Twenty Foot Equivalent Unit and a TEU corresponds to a Twenty-Foot (20') container.

Concerning the train capacity, it is necessary to check both weight capacity ($\bar{\Omega}$) and the TEU capacity (M).

2.1 Stability and structural constraints

When dealing with real loading problems, weight constraints on each wagon and on the train must be considered in a very strict manner to satisfy both the structural capacity and the stability of each type of wagon. So, the transversal (cross) weight distribution on each wagon must be checked. Figure 3 shows the cross equilibrium for a generic wagon.

Some other limitations are imposed by the train speed and by the features of the rail infrastructure network (such as bridges, tunnels, slopes, etc.).

Rail terminal planners utilize specific weight tables in order to determine feasible plans for loading containers onto wagons; these tables are elaborated to help them determining the allowable combinations of containers-wagons and respect both stability constraints and structural capacity conditions related to the wagons and rail network.

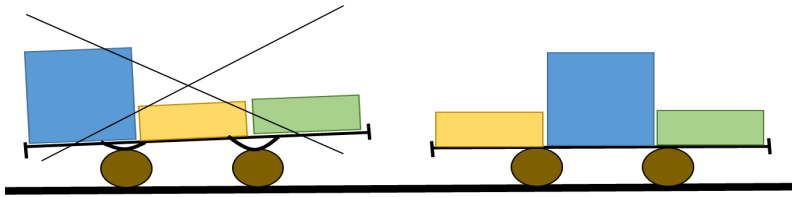


Fig. 3 Cross equilibrium for a generic wagon.

Figure 4 provides an example of a weight table describing the real load pattern for two types of wagons utilized in the Italian rail infrastructure and carrying 2 and 3 TEUs respectively.

Wagon w_1 of Figure 4 is characterized by two load configurations and three slots (i.e. s_1 , s_2 and s_3); two possible load configurations correspond to two 20' containers (k_1) and one 40' container (k_2). Wagon w_2 is fully described by 7 slots (i.e. from s_4 to s_{10}) and by three possible load configurations: three 20' containers (k_3), two 20' containers (k_4), and one 40' container with one 20' container (k_5).

A set of weight configurations is associated to each load configuration; for instance b_1 , b_2 and b_3 are the weight configurations associated with load configuration k_1 . For each weight configuration the maximum weight for each slot is indicated. As described above, these weight limits are established in order to respect the wagons' stability and structural constraints. For instance, if two 20' containers have to be loaded on the 2-TEUs wagon (as w_1 in Figure 4), the first load configuration k_1 of w_1 is selected together with a weight configuration (among the three permitted, b_1 , b_2 and b_3) depending on the weight of the containers to be loaded. If a container weighing 14 tons has to be loaded, weight configuration b_2 can be chosen and the container can be put into slot s_1 , while in the remaining slot s_2 it is possible to load a 20' container weighing less than 33 tons.

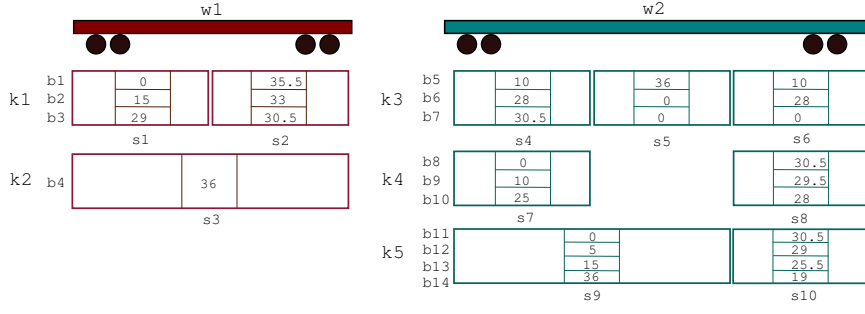


Fig. 4 Representation of a wagon by using load and weight configurations.

In Bruns and Knust (2012), for each load configuration, the authors start their analysis from the set of possible weight configurations as described in Figure 4. In order to reduce the set of possible configurations, they propose to use the load configurations and then to check the stability and structural constraints by using the following weight conditions $C1, C2, C3$:

- $C1$. the payload per bogie must be restricted according to the characteristics of the wagon (i.e. tare, length, weight capacity, distance between the bogies attachments) and of the rail line connecting the port with its inland;
- $C2$. the payload on a bogie must not be larger than three times the payload on the other bogie;
- $C3$. the payload for each slot on the wagon has to be limited.

When using conditions $C1, C2, C3$, the number of possible configurations to choose between drastically decreases, passing from 4 ($b1, \dots, b4$) to 2 ($k1, k2$) in case of wagon $w1$ and from 10 ($b5, \dots, b14$) to 3 ($k3, k4, k5$) in case of wagon $w2$.

The interested reader can find in Appendix 1 an explanation of how conditions $C1, C2, C3$ are derived by lever principles, while in the following a formal description of these conditions is reported.

Let us introduce the following useful additional notation:

- \mathcal{B}_w the set of load configurations for wagon $w = 1, \dots, W$;
- S_k the set of slots belonging to configuration k of wagon w , $k \in \mathcal{B}_w$, $w = 1, \dots, W$;
- $a_w \geq 0$ the payload of bogie a of wagon w , $w = 1, \dots, W$;
- $b_w \geq 0$ the payload of bogie b of wagon w , $w = 1, \dots, W$;
- d_w the distance between the bogies attachments of wagon w , $w = 1, \dots, W$;
- γ_w the maximum payload for each bogie of wagon w , $w = 1, \dots, W$;
- t_w the weight of wagon w (tare), $w = 1, \dots, W$;
- e_s the lever of slot s , $s \in S_k$, of configuration k of wagon w , $k \in \mathcal{B}_w$, $w = 1, \dots, W$;
- $\delta_{k,s} \geq 0$ the weight loaded in slot s of S_k , $k \in \mathcal{B}_w$, $w = 1, \dots, W$;
- $\hat{\delta}_{k,s} \geq 0$ maximum weight that can be loaded in slot s of S_k , $k \in \mathcal{B}_w$, $w = 1, \dots, W$;

The payload can be computed thanks to the following equations:

$$a_w = \sum_{s \in \mathcal{S}_k} \frac{d_w - e_s}{d_w} \delta_{k,s} + \frac{t_w}{2} \quad (1)$$

$$b_w = \sum_{s \in \mathcal{S}_k} \frac{e_s}{d_w} \delta_{k,s} + \frac{t_w}{2} \quad (2)$$

Conditions $C1, C2, C3$ follows: equations (3) and (4) are condition $C1$, (5) and (6) are condition $C2$, and finally $C3$ is verified by equation (7)

$$a_w \leq \gamma_w \quad (3)$$

$$b_w \leq \gamma_w \quad (4)$$

$$a_w - 3b_w \leq 0 \quad (5)$$

$$b_w - 3a_w \leq 0 \quad (6)$$

$$\delta_{k,s} \leq \hat{\delta}_{k,s} \quad (7)$$

The novel idea proposed in this paper is to define the weight limitations used in $C3$ (i.e., $\hat{\delta}_{k,s}$) in such a way to respect also conditions $C1$ and $C2$ and, then, to use only condition $C3$ for checking stability and structural constraints (see Section 3).

2.2 Representations of wagon configurations by even and odd slots enumeration system (EOS)

With the aim of reducing the size of the model for solving the TLPP, another way of representing the set of slots belonging to each wagon is proposed in this paper. Looking at Figure 4 it is evident that slots $s6, s8, s10$ represent the same physical slot, but it is necessary to enumerate it three times since this slot belongs to three different load configurations. The idea proposed in this paper is not to use loading configurations and to enumerate each slot only once as explained below.

In Figure 5 the new slots representation is reported. For each wagon, all the possible slots are identified and enumerated adopting the bays enumeration system used for ships, that is, 20 foot slots are identified by odd numbers, while 40 foot ones are associated with even ones. Thanks to this slot enumeration system, the number of slots necessary for describing the 3 TEUs wagons (i.e., $w2$) reduces, passing from seven slots - as depicted in Figure 4 - to four slots (i.e., $s5, s6, s7$ and $s9$ of Figure 5). For a 2-TEUs wagons (i.e., $w1$) the number of slots remains the same but their enumeration changes (i.e., $s2$ is now a 40' slot whereas in Figure 4 it is a 20' slot due to a progressive slots' enumeration).

Note that this enumeration system, derived by the container ships, can be adapted when containers of different size have to be loaded on trains (i.e., 45' containers).

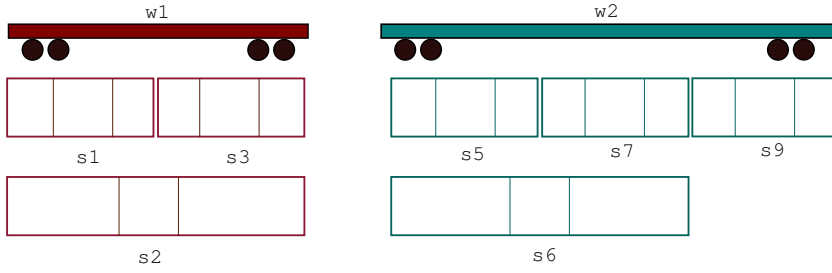


Fig. 5 Representation of a wagon by using the new slot enumeration.

3 The heuristic approaches

In this section, two heuristic approaches for solving the TLPP described above are introduced. Each of them is based on two steps as depicted in Figure 6.

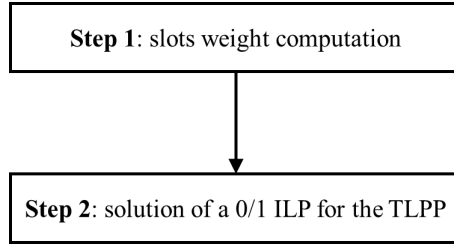


Fig. 6 The two steps of the heuristic approaches.

As already explained in Section 2.1, Bruns and Knust (2012) check stability and structural constraints through the load configurations of wagons and weight conditions $C1$ - $C3$. The novel idea proposed in this paper is to use only condition $C3$ for checking stability and structural constraints; this is possible if the weight limitations used in $C3$ is defined in such a way to respect also conditions $C1$ and $C2$.

The weight limitation for each slot of each wagon configuration is computed in *Step 1*. In *Step 2*, when solving the TLPP the weight limitations and the stability and structural constraints are verified thanks to the maximum weight that can be loaded on each slot of the train (condition $C3$), being the maximum weight of each slot derived by *Step 1*.

In the second step, the TLPP is solved thanks to a 0/1 integer linear programming (0/1 ILP) formulation using conditions $C3$ with the maximum weights calculated in *Step 1*.

The *LC – approach*, uses the load configurations for describing wagon slots while the second approach, the *EOS – approach*, uses the EOS enumeration system described in Section 2.2.

This approach can be considered as a decomposition approach that can not guarantee the optimality. In fact, when applying *Step 1* of the procedure, some

weight configurations may be lost and, consequently, they are not available when solving the *Step 2* of the TLPP. Anyway, as shown in Section 4, it performs very well.

3.1 Step 1: a procedure for determining slot weight limitations

The first step of each heuristic approach consists of a procedure for computing the weight limitation for each slot of each wagon to be used in the second step. In the current section the procedure to adopt when the load configurations are used is described; anyway it is easy to modify and use it when wagons are described by the even/odd enumeration system.

The computed weight limitations are able to:

- grant stability;
- satisfy all the structural constraints for both the wagons and the rail lines;
- satisfy structural constraints related to containers.

The procedure can be described in the following steps:

- *Step 1A*: a LP model (*Wsd*) is solved for determining the maximum weights for wagon slots in a such a way to satisfy conditions *C1* and *C2* and respect some operative bounds (i.e., in real cases, the maximum weight of 20' and 40' containers do not exceed 28 and 37 tons, respectively).
- *Step 1B* (weight check and redefinition): the weight limitations obtained at the end of *Step1A* are compared with the weights of the stored containers. If necessary, the model *Wsd* is solved again with new bounds.

Note that *Step1B* has been introduced in order to have a control on the capability of loading the heaviest containers stored in the yard. The output of these two steps are the slots' weights that permit to solve the TLPP with a lean formulation.

Before further explaining *Step 1A* and *Step 1B* (i.e., the *Wsd* model and the weight control), the following additional notation must be introduced:

- Υ_{20} and Υ_{40} the maximum weight of respectively 20' and 40' containers stored in the yard;
- Ψ_{20} and Ψ_{40} the average weights of respectively 20' and 40' containers stored in the yard.

The additional decision variables are the following:

- $\delta_{k,s}^H \geq 0$ indicates the maximum weight for slot $s \in S_w$, of wagon w , $w = 1, \dots, W$, in load configuration $k \in \mathcal{B}_w$ of wagon w .

3.1.1 Step 1A

For each wagon w , for each configuration k , $k \in B_w$, the set of weights associated with the slots of configuration k , $\delta_{k,s}^H$, $s \in S_k$ are computed by solving the following LP model.

Wsd - Weight slot definition

$$\max \sum_{s \in S_k} \delta_{k,s}^H \quad (8)$$

such that

(3) -(6)

$$a_w - \sum_{s \in S_k} \frac{d_w - e_s}{d_w} \delta_{k,s}^H = \frac{t_w}{2} \quad (9)$$

$$b_w - \sum_{s \in S_k} \frac{e_s}{d_w} \delta_{k,s}^H = \frac{t_w}{2} \quad (10)$$

$$\delta_{k,s}^H \leq 28 \quad s \in S_k : \mu_s = 20' \quad (11)$$

$$\delta_{k,s}^H \leq 37 \quad s \in S_k : \mu_s = 40' \quad (12)$$

$$\delta_{k,s}^H \geq \Psi_{20} \quad s \in S_k : \mu_s = 20' \quad (13)$$

$$\delta_{k,s}^H \geq \Psi_{40} \quad s \in S_k : \mu_s = 40' \quad (14)$$

$$a_w \geq 0 \quad (15)$$

$$b_w \geq 0 \quad (16)$$

$$\delta_{k,s}^H \geq 0 \quad s \in S_k \quad (17)$$

The objective function (8) maximizes the sum of the weights of the slots of the considered configuration k .

Constraints (9) and (10) compute the payloads of the bogies of wagon w . Each payload per bogie of wagon w must not be greater than a given maximum value that reflects the structural constraints for both the wagon and the rail network (condition C1), as required in (3) and (4). The payload on a bogie of wagon w must not be larger than three times the payload on the other bogie (condition C2), as required in (5) and (6).

As far as bounds are concerned, $\delta_{k,s}^H$ must not be greater than the maximum weight of a 20' or 40' container, i.e., 28 tons or 37 tons, respectively for 20' and 40' slots, as required in (11) and (12). Thanks to (13) and (14), $\delta_{k,s}^H$ is not lower than the average weights of 20' or 40' containers stored in the yard (i.e., Ψ_{20} and Ψ_{40}), respectively for 20' and 40' slots.

Finally, constraints (15)-(17) define the decision variables.

3.1.2 Step 1B

In order to be able to load any container that is stored in the yard, a weight control on $\delta_{k,s}^H$ is executed; in particular these weights, obtained by solving (8)-(17) for each configuration k , $k \in B_w$ of each wagon w , are compared with the weight of the heaviest containers stored.

After having computed the weight limitations for each slot $s \in \mathcal{S}_k$ of each configuration k , $k \in B_w$ of each wagon w , $w = 1, \dots, W$ (i.e., $\sum_{w=1}^W |B_w|$ Wsd models have been solved) the following steps are executed:

1. sort the 20' containers located in the yard in descending order in accordance with their weights;
2. sort the slots characterized by $\mu_s = 20'$ in descending order respect to their weights;
3. compute z_{20} the number of 20' containers having a weight greater than the maximum weight of the 20' train slots (i.e., check if there are some 20' containers that cannot be loaded on the train);
4. for R^{20} wagons, solve the model Wsd by adding constraint (18), in such a way to determine for R^{20} wagons at least a slot \hat{s} (such that $\mu_{\hat{s}} = 20'$) with a weight limitation greater than the maximum weight Υ_{20} .

$$\delta_{k,\hat{s}}^H \geq \Upsilon_{20} \quad (18)$$

R^{20} is fixed as follows: if $z_{20} \leq W$, then $R^{20} = z_{20}$, else (i.e. $z_{20} \geq W$) $R^{20} = W$.

The same steps are repeated for 40' containers; in particular, for R^{40} wagons, model Wsd is solved by adding constraint (19), in such a way to determine for R^{40} wagons at least a slot \hat{s}' (such that $\mu_{\hat{s}'} = 40'$) with a weight limitation greater than the maximum weight Υ_{40} .

$$\delta_{k,\hat{s}'}^H \geq \Upsilon_{40} \quad (19)$$

$MWsd$ denotes the modified version of model (8)-(17) obtained by adding constraints (18) and (19).

To better clarify the procedure proposed above, the reader should refer to the flow chart diagram presented in Figure 7.

A similar procedure is required when the enumeration system (EOS) is used to describe wagon slots; when EOS is adopted, at the end of *Step 1* the maximum weight for slot s (i.e., δ_s^H) is obtained.

For both approaches, at the end of *Step 1*, the computed weight limitations granting stability and respecting structural constraints can be used in the 0/1 ILP formulations for solving the TLPP (as shown in the following).

3.2 Step 2: 0/1 ILP formulations for TLPP

In the second step of the heuristic approaches proposed for the TLPP, a 0/1 ILP formulation is solved. In the following, the two formulations used are

provided; they differ only in the representation and enumeration of wagon slots.

3.2.1 Formulation based on load configuration (P1) used in LC-approach

The following formulation $P1$ is derived from the model proposed in Ambrosino and Siri (2015) for the sequential loading of a train. $P1$ is based on load configurations for describing wagon slots, but with respect to Ambrosino and Siri (2015) it is not based on weight configurations, since stability and structural constraints are verified by weight constraints for the slots, in which the weight limitations are those derived by the procedure described in Section 3.1. This difference reflects only in new constraints (24).

Note that this is a novelty with respect to the previous literature which was based on more complex wagon configurations (Ambrosino and Siri (2015)) or on conditions $C1 - C3$ (Bruns and Knust (2012)).

Anyway, the whole formulation of $P1$ is here reported for a better understanding of both $LC - approach$ and $EOS - approach$.

Before describing the mathematical formulation for solving the TLPP under investigation, let us introduce an additional notation for the problem decision variables:

- $x_{i,s} \in \{0, 1\}$, $i = 1, \dots, C$, $s = 1, \dots, S$: $\lambda_i = \mu_s$, equal to 1 if container i is assigned to slot s and 0 otherwise (note that only $x_{i,s}$ variables for slot-container pairs with the same length are generated);
- $f_{w,k} \in \{0, 1\}$, $w = 1, \dots, W$, $k \in \mathcal{B}_w$, equal to 1 if load configuration k is chosen for wagon w and 0 otherwise.
- $y_{i,j} \in \{0, 1\}$, $i, j \in \{1, \dots, C\}$: $\gamma_{i,j} = 1$, equal to 1 if container i is rehandled to load container j .

and α the unitary rehandling cost (i.e., the cost of one rehandling operation).

The resulting formulation $P1$ follows.

Model P1

$$\min \alpha \cdot \sum_{\substack{i,j \in \{1, \dots, C\}: \\ \gamma_{i,j}=1}} y_{i,j} + \sum_{i=1}^C \pi_i \cdot \left(1 - \sum_{s=1}^S x_{i,s} \right) \quad (20)$$

such that

$$\sum_{s=1}^S x_{i,s} \leq 1 \quad i = 1, \dots, C \quad (21)$$

$$\sum_{i=1}^C x_{i,s} \leq 1 \quad s = 1, \dots, S \quad (22)$$

$$\sum_{k \in \mathcal{B}_w} f_{w,k} = 1 \quad w = 1, \dots, W \quad (23)$$

$$\sum_{i=1}^C \omega_i \cdot x_{i,s} \leq \sum_{k \in \mathcal{B}_w} \delta_{k,s}^H \cdot f_{w,k} \quad w = 1, \dots, W \quad s \in \mathcal{S}_w \quad (24)$$

$$\sum_{i=1}^C \sum_{s \in \mathcal{S}_w} \omega_i \cdot x_{i,s} \leq \bar{\Omega}_w \quad w = 1, \dots, W \quad (25)$$

$$\sum_{i=1}^C \sum_{s=1}^S \omega_i \cdot x_{i,s} \leq \bar{\Omega} \quad (26)$$

$$\sum_{s=1}^S \rho_s \cdot x_{i,s} - \sum_{s=1}^S \rho_s \cdot x_{j,s} \leq M y_{i,j} + M \left(\sum_{s=1}^S x_{i,s} - \sum_{s=1}^S x_{j,s} \right) \quad \forall i, j \in \{1, \dots, C\} : \gamma_{i,j} = 1 \quad (27)$$

$$x_{i,s} \in \{0, 1\} \quad i = 1, \dots, C \quad s = 1, \dots, S \quad (28)$$

$$y_{i,j} \in \{0, 1\} \quad i, j \in \{1, \dots, C\} : \gamma_{i,j} = 1 \quad (29)$$

$$f_{w,k} \in \{0, 1\} \quad w = 1, \dots, W \quad k \in \mathcal{B}_w \quad (30)$$

The cost function (20) takes into account rehandling costs in the yard and penalty costs for containers not loaded, giving precedence to the loading of higher priority containers. Constraints (21) make sure that each container is assigned at most to one slot, whereas constraints (22) assure that no more than one container is loaded in each slot. A load configuration is chosen for each wagon thanks to constraints (23); constraints (24) assure that the weight of a container loaded on a wagon slot does not exceed the maximum weight for the slot associated to the specific load configuration chosen. $\delta_{k,s}^H$ is derived from the procedure described in the previous section. Note that the respect of weight limitations imposed in order to guarantee the stability of wagons is implicitly assured by constraints (24), thanks to $\delta_{k,s}^H$.

Constraints (25) are related to the maximum weight of each wagon, while not overcoming the maximum allowable weight for the whole train is assured by constraint (26). Rehandling movements are defined by constraints (27): for each couple of containers (i, j) such that $\gamma_{i,j} = 1$, if container j (located under container i in the same stack) is loaded before container i , it means that train position ρ_s of container i is higher than the one of container j and so the term $(\sum_{s=1}^S \rho_s \cdot x_{i,s} - \sum_{s=1}^S \rho_s \cdot x_{j,s})$ assumes a positive value; so, in this case, the variable $y_{i,j}$ has to assume a value equal to 1, that means that a rehandle is counted. It must be noted that the term $M(\sum_{s=1}^S x_{i,s} - \sum_{s=1}^S x_{j,s})$ is necessary in order to assure the validity of constraints (27) when only container i is loaded while container j remains in the yard, and viceversa when container i is not loaded at all.

Finally, constraints (28)-(30) define the decision variables of the problem.

3.2.2 Formulation based on the new slot enumeration (P2) used in the EOS-approach

Formulation *P2* is here presented in an attempt to reduce the number of variables and constraints generated in formulation *P1*. For this purpose the slot enumeration depicted in Figure 5 is used; for each wagon the set S_w of slots is partitioned into two subsets, the subset of oddslots (OS_w) and that of evenslots (ES_w).

The resulting formulation *P2* follows.

Model P2

Min (20)

such that (21)

$$\sum_{i=1}^C x_{i,s} + \sum_{i=1}^C x_{i,s+1} \leq 1 \quad w = 1, \dots, W \quad s \in ES_w \quad (31)$$

$$\sum_{i=1}^C x_{i,s} + \sum_{i=1}^C x_{i,s-1} \leq 1 \quad w = 1, \dots, W \quad s \in ES_w \quad (32)$$

$$\sum_{i=1}^C \omega_i \cdot x_{i,s} \leq \delta_s^H \quad s = 1, \dots, S \quad (33)$$

(25)-(29)

Model *P2* differs from model *P1*; in the fact that variables $f_{w,k}$ are no longer necessary and consequently constraints (23), which in *P1* enable choosing and assigning a load configuration, are no longer needed.

Constraints (22) of model *P1*, indicating that no more than one container can be assigned to one slot, are replaced by constraints (31) and (32) needed to define the use of even and odd slots. These constraints are necessary in order to make sure that, when an even slot is loaded (i.e. a 40' container is loaded), the corresponding odd slots are left vacant. For instance, making reference to Figure 5, if a container is loaded in slot $s2$, then both slots $s1$ and $s3$ must be left free.

Constraints (24) are replaced by the easier constraints (33). More specifically, constraints (33) express the fact that the weight of a container cannot exceed the maximum weight for the corresponding slot on which it is loaded.

As before, δ_s^H has been determined through *Step 1* of the proposed procedure.

4 Computational results

In order to validate the proposed approaches for the TLPP and to compare them, we have generated 8 sets of random instances based on real data provided by an Italian container terminal operator. These sets differ from each other in the number of wagons (W), ranging from 15 to 40, and in the number of containers stored in the yard (C), ranging from 60 to 200.

Each train is composed by wagons having a capacity of either 2 or 3 TEUs. The number of 2 and 3 TEUs wagons composing the train, and influencing the TEUs capacity of the train (M), is randomly generated.

The number of 20' and 40' containers composing the stacks in the yard and influencing the TEUs stored in the yard (T), is randomly generated on the basis of real data.

Summarizing, each instance is characterized by a train having a given number of wagons (W), a TEUs capacity (M) and is related to a certain number of containers (C) and TEUs in the yard (T), as shown in Table 1.

The priority assigned to each container is generated in a probabilistic way among three priority classes, "low" (having a priority value equal to 10 or 20, respectively for 20' and 40' containers), "medium" (having a priority value equal to 15 or 30), and "high" (having a priority value equal to 20 and 40). Thus, each instance is characterized by a total yard priority (P), which is the sum of the priorities assigned to all the containers in the yard. Note that the absolute values of the container priorities have been determined by executing some tests for calibrating them in accordance with the importance of rehandles and their weight: it is considered more relevant to fully load the train instead of minimizing container rehandles in the yard. This fact emerged from some interviews with the train planners of the Italian container terminals that we have contacted. The purpose of the interviews was to have an order among the objectives that guide the planners during their choices. As a consequence, the rehandling weight α has been set equal to 1.

Container weights are uniformly distributed between a minimum and a maximum value, defined in a different way for 20' and 40' containers.

Within each set, five instances have been generated. Table 2 details the five instances related to one of the sets reported in Table 1, i.e., set A. In Table 2, for each instance of set A, the number of 20' and 40' containers (# cnr 20' and # cnr 40') in the yard, the number of TEUs in the yard (T), the total yard priority (P) and average (Ψ) and maximum (\mathcal{Y}) weights of 20'/40' containers are reported, together with the TEU capacity of the train (M). These instances are available for the scientific community at the following link: <https://www.researchgate.net/project/Train-Loading-Planning-Optimization>.

The procedures and formulations of the heuristic approaches have been implemented in Visual Studio 2012 C# and solved by using Cplex 12.5 on a pc Intel(R) Core i5 CPU M520 2,40GHz Ram 6GB.

Since the first step is very similar for the two approaches, the comparison here presented is about the performance of formulations $P1$ and $P2$.

Table 1 Instance features

Set	W	C	T	M
A (15-60)	15	60	89	37
B (15-80)	15	80	123	37
C (20-60)	20	60	94	51
D (20-80)	20	80	122	50
E (30-150)	30	150	218	75
F (30-200)	30	200	290	77
G (40-150)	40	150	228	100
H (40-200)	40	200	294	99

Table 2 Set A features

Inst.	# cntr 20'	# cntr 40'	T	P	Ψ_{20}	Ψ_{40}	Υ_{20}	Υ_{40}	M
A1	36	24	84	2235	14.04	21.42	23.95	29.46	35
A2	40	20	80	2540	14.44	21.52	21.49	29.56	39
A3	40	20	80	2585	14.89	19.67	23.70	29.75	35
A4	20	40	100	2530	15.58	20.39	23.83	28.69	38
A5	20	40	100	2695	13.43	23.57	23.57	29.66	37
average	31.2	28.8	89	2517	14.48	21.31	23.31	29.42	37

More specifically, models $P1$ and $P2$ have been compared on the basis of the number of generated variables (# var.) and constraints (# const.), CPU time (expressed in seconds), value of the objective function (obj.), optimality gap (Gap), number of rehandles executed (R) and number of containers loaded on the train (L). Moreover, the following Key Performance Indicators (KPIs), determined on the basis of the interviews made with the Italian rail planners, have been proposed for comparing the obtained solutions:

- Π , i.e., the ratio - expressed as a percentage - between the sum of the priorities related to the containers loaded on the train and the total yard priority (P). This index reflects the value of the containers loaded in terms of their priority;
- τ , i.e., the ratio - expressed as a percentage - between the number of TEUs loaded on the train (U) and the train TEU capacity (M). It provides an indication of how much the train has been loaded. Note that the number of TEUs is calculated by summing the number of 20' containers with the double of the number of 40' containers.

Π and τ are expressed by equations (34) and (35), respectively.

$$\Pi = \frac{\sum_{i=1}^C \sum_{s=1}^S \pi_i x_{i,s}}{P} 100 \quad (34)$$

$$\tau = \frac{U}{M} 100 \quad (35)$$

Tables 3 and 4 present the results obtained by solving models $P1$ and $P2$ respectively, with a time limit of 10 minutes. The tables' numbers represent

the average of the 5 instances generated per each instance set. All instances have been solved up to optimality in few seconds.

When comparing models $P1$ and $P2$, it can be noted that the number of variables and constraints of model $P2$ are lower than model $P1$; this is due to the elimination of load configurations and to the use of odd and even slots (see Section 2.2).

Moreover, the CPU time is lower in the case of $P2$ (on average about 50 % less than model $P1$, even if in absolute terms the difference is less than 3 seconds). This difference is more significant for larger instances, as the case of sets G and H (see Tables 3, 4).

Both the proposed formulations can be effectively used for solving the TLPP, thanks to their shorter CPU times. The train is always fully loaded (i.e., $\tau = 100\%$), while the number of rehandles (R) grows with the number of containers in the yard (C).

Table 3 Average results for Model $P1$

Instance type	# var.	# const.	CPU time	obj.	Gap	R	L	Π	τ
A (15-60)	2349	333	1.54	855	0	15	25	69.4%	100%
B (15-80)	2869	373	0.88	1729	0	17	24	54.0%	100%
C (20-60)	2963	396	1.39	604	0	16	33	79.7%	100%
D (20-80)	3965	439	1.31	1306	0	20	33	66.1%	100%
E (30-150)	12182	736	5.61	2647	0	42	50	60.6%	100%
F (30-200)	16858	874	5.96	4379	0	36	51	49.3%	100%
G (40-150)	14764	851	9.68	2131	0	33	66	70.4%	100%
H (40-200)	20876	975	14.78	3502	0	50	68	61.0%	100%
<i>average</i>	<i>9652</i>	<i>622</i>	<i>5.14</i>	<i>2144</i>	<i>0</i>	<i>29</i>	<i>44</i>	<i>63.8%</i>	<i>100%</i>

Table 4 Average results for Model $P2$

Instance type	# var.	# const.	CPU time	obj.	Gap	R	L	Π	τ
A (15-60)	1674	286	0.94	855	0	15	25	69.4%	100%
B (15-80)	2142	334	0.70	1729	0	17	24	54.0%	100%
C (20-60)	2119	332	0.79	604	0	16	33	79.7%	100%
D (20-80)	2872	381	0.90	1306	0	20	33	66.1%	100%
E (30-150)	8465	644	2.46	2647	0	42	50	60.6%	100%
F (30-200)	11446	774	4.79	4379	0	36	51	49.3%	100%
G (40-150)	10561	733	5.14	2131	0	33	66	70.4%	100%
H (40-200)	14691	859	5.15	3502	0	50	68	61.0%	100%
<i>average</i>	<i>6746</i>	<i>543</i>	<i>2.61</i>	<i>2144</i>	<i>0</i>	<i>29</i>	<i>44</i>	<i>63.8%</i>	<i>100%</i>

4.1 A comparison with a model that uses conditions $C1, C2, C3$

To better evaluate the effectiveness of the approaches proposed, we have compared the solutions obtained by using both the *LC – approach* and *EOS – approach* with those obtained by solving model (17)-(27) provided in Ambrosino and Siri (2015) that consider load configuration and the conditions $C1 – C3$ necessary to verify stability and structural constrains (that will be later denoted as model $P3$).

Table 5 provides a clarification of the three different models compared. Formulation $P3$ has been solved with the same time limit of 10 minutes.

Table 5 Differences between models

Model name	conditions C1-C3	Type of wagons slot enumeration
P1	C3*	load configurations /progressive
P2	C3*	EOS system
P3	C1,C2,C3	load configurations /progressive

$C3^*$ indicates that the weight used in condition $C3$ is that derived from *Step 1* (see Section 3.1).

When comparing model $P3$ with models $P1$ and $P2$ in terms of number of variables and constraints (Figures 8 and 9), it is clear that $P3$ has the greatest number of constraints and the same number of variables of $P1$. Thus $P2$ is always the smallest in size.

Figures 10 and 11 represent the CPU time and the objective function value of $P1$, $P2$ and $P3$. Model $P3$ is not able to solve up to optimality all the considered instances when the time limit of 10 minutes is fixed. Model $P3$ solves up to optimality only the instances of set B .

Just to give an idea of the behaviour of the three models, Tables 6 and 7 report the detailed results for two sets of instances: set A and F. Models $P1$ and $P2$ are able to solve all the instances up to optimality in a few seconds, while by running model $P3$, only 2 of 5 instances of set A and no instance belonging to set F have been solved up to optimality within the imposed time limit. Anyway, the optimality gap (reported in column gap) is always very low; this gap is that furnished by the solver CPLEX. Note that the optimal solution of $P3$ for instance $A5$ is different, in terms of loaded priority and number of rehandles, from that obtained by $P1$ and $P2$, while the objective function value is the same.

Note that, for investigating the quality of the solutions obtained by *LC – approach* and *EOS – approach*, we compared them with the optimal solutions obtained by $P3$ (i.e., $P3$ running without a time limit). In all solved instances, the solutions obtained are the optimal ones.

These results highlight the effectiveness of *Step 1* of the proposed solution approaches; the elimination of conditions $C1 – C3$ allows to reduce CPU times.

Table 6 Set A results

Model	Instance	CPU time	gap	R	Π	τ
P1	A1	1.98	0	20	72.0%	100%
P1	A2	1.56	0	9	69.3%	100%
P1	A3	1.41	0	17	69.2%	100%
P1	A4	2.18	0	13	73.9%	100%
P1	A5	0.56	0	17	62.7%	100%
<i>average P1</i>	<i>A</i>	<i>1.54</i>	<i>0</i>	<i>15</i>	<i>69.4%</i>	<i>100%</i>
P2	A1	2.93	0	20	72.0%	100%
P2	A2	0.42	0	9	69.3%	100%
P2	A3	0.44	0	17	69.2%	100%
P2	A4	0.51	0	13	73.9%	100%
P2	A5	0.41	0	17	62.7%	100%
<i>average P2</i>	<i>A</i>	<i>0.94</i>	<i>0</i>	<i>15</i>	<i>69.4%</i>	<i>100%</i>
P3	A1	600.26	0.02	23	68.0%	91.9%
P3	A2	600.30	0.02	19	63.2%	89.5%
P3	A3	4.21	0.00	17	69.2%	100%
P3	A4	600.44	0.01	21	73.2%	100%
P3	A5	1.26	0.00	20	66.3%	100%
<i>average P3</i>	<i>A</i>	<i>361.09</i>	<i>0.01</i>	<i>20</i>	<i>68.0%</i>	<i>96.3%</i>

Table 7 Set F results

Model	Instance	CPU time	gap	R	Π	τ
P1	F1	7.81	0	45	48.9%	100%
P1	F2	4.96	0	30	49.3%	100%
P1	F3	6.34	0	34	50.4%	100%
P1	F4	6.10	0	27	47.4%	100%
P1	F5	4.56	0	45	50.4%	100%
<i>average P1</i>	<i>F</i>	<i>5.96</i>	<i>0</i>	<i>36</i>	<i>49.3%</i>	<i>100%</i>
P2	F1	7.28	0	45	48.9%	100%
P2	F2	3.54	0	30	49.3%	100%
P2	F3	3.99	0	34	50.4%	100%
P2	F4	5.33	0	27	47.4%	100%
P2	F5	6.11	0	45	50.4%	100%
<i>average P2</i>	<i>F</i>	<i>5.25</i>	<i>0</i>	<i>36</i>	<i>49.3%</i>	<i>100%</i>
P3	F1	600.60	0.01	60	48.3%	98.7%
P3	F2	600.73	0.03	35	46.7%	95.9%
P3	F3	600.12	0.03	43	46.8%	93.6%
P3	F4	600.91	0.04	29	44.1%	93.8%
P3	F5	600.79	0.02	54	46.9%	94.6%
<i>average P3</i>	<i>F</i>	<i>600.76</i>	<i>0.03</i>	<i>44</i>	<i>46.6%</i>	<i>95.3%</i>

4.2 Comparison with real train plans

Finally, in order to further validate the effectiveness of the proposed approaches, we have collected some typical real train plans from one of the Italian container terminals that we have interviewed. Table 8 provides the data related to these plans: the number of wagon W composing the train, the number of 20' and 40' containers loaded, the TEUs capacity of the train (M), the number of TEUs loaded on the train (U) and the related TEUs load percentage (τ).

It can be noted that, for all the real train plans provided, the train is not fully loaded (in fact, τ is far below 100 %).

We have solved instances t1-t5 by using models P1 and P2: the obtained results showed that the train is fully loaded (i.e., $\tau = 100\%$) in all cases t1-t5.

Finally it has to be pointed out that the average time required by a rail planner to plan a train loading is between half an hour and one hour, depending on different factors such as its experience and on the numerosity and typology of rail cars and containers to be loaded. This time is definitely not comparable with the few seconds required to solve our models.

Table 8 Real train plans of a Northern Italian container terminals

Train plan	W	# 20'cntr	# 40'cntr	M	U	τ
t1	17	18	18	63	54	85.71%
t2	17	21	17	63	55	87.30%
t3	18	0	21	56	42	75.00%
t4	11	2	16	49	34	69.39%
t5	20	49	3	60	55	91.67%

5 Conclusion

In this paper two different solution approaches for solving the TLPP have been presented. Differently from the previous literature, the proposed solution approaches use only a weight condition with proper weight limitations for satisfying stability and structural constraints. Moreover, this work also proposes different interpretations of wagon load configurations based on a new way to represent wagon slots.

An extensive computational analysis has been performed. The proposed solution approaches have been validated by using both random generated instances based on real-world data and real train plans.

Considering that in container terminals it is necessary to take quick and correct decisions, one of the main goal of the present research was to compare solution approaches in order to identify the most suitable one to be used in a specific container terminal context. Based on the results obtained, we can conclude that both the proposed approaches can be included in a decision support system to help container terminal operators in optimizing train loading operations.

Moreover, being the proposed approaches based on models that minimize the weighted sum of the number of re-handles and the sum of the priority of containers that can not be loaded on the train, the authors will investigate the multi-objective optimization approach in order to offer more than one solution to terminal operators.

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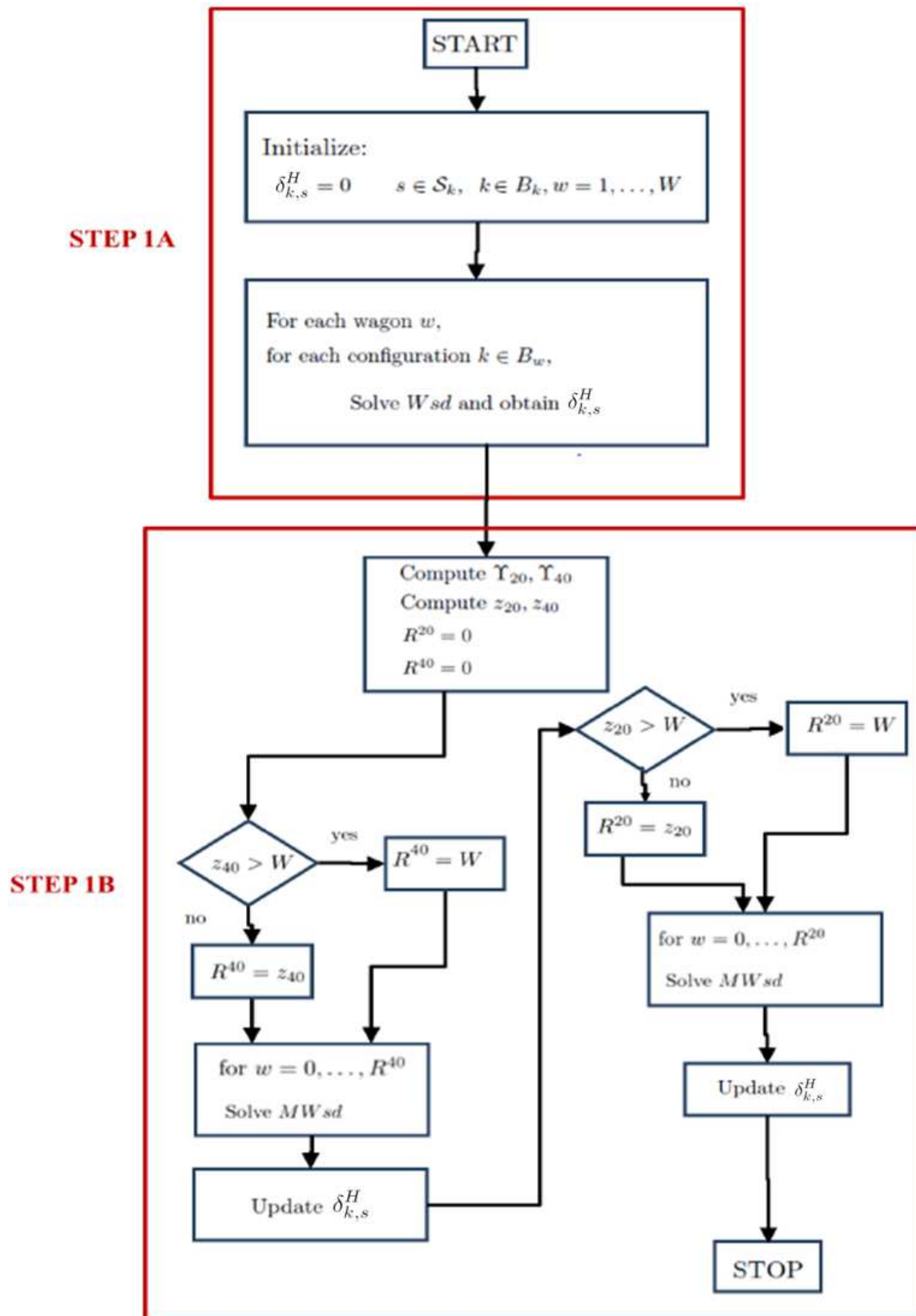
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Appendix 1. The lever principles

Stability conditions are derived from lever principles, which state that two unequal forces, when acting in opposite directions, arrive at an equilibrium when the product of the magnitude of a generic force \vec{F}_1 and its lever arm e_1 (the distance of its point of application from the fulcrum), is equal to the product of the magnitude of a second force \vec{F}_2 with its corresponding lever arm e_2 ($\vec{F}_1 \cdot e_1 = \vec{F}_2 \cdot e_2$). Note that a bogie - also called railroad truck or wheel truck - represents a structure underneath a train to which axles (and, hence, wheels) are attached through bearings.

To better clarify lever principles, refer to Figure 12; levers of containers c_1 and c_2 (e_1 and e_2 , respectively) are determined as the distance between their center of gravity - which should be in the middle of the container - and the attachment of one of the two bogies (note that containers' levers are calculated in reference to the same bogie). Moreover, the distance (d) between bogies is known; finally, it is assumed that the tare mass of the wagon is equally distributed on the two wagon bogies.

Fig. 7 Flow chart diagram of the procedure *Wsd*.

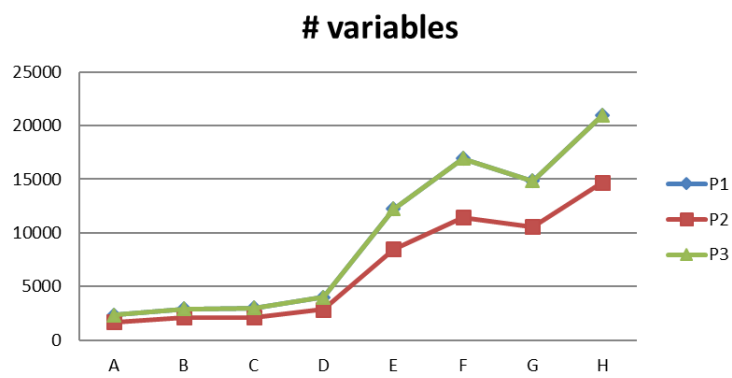


Fig. 8 Comparison between $P1$, $P2$ and $P3$ - Number of variables

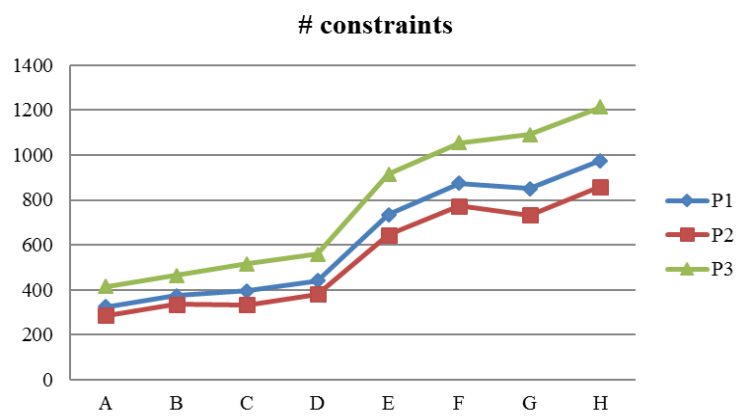


Fig. 9 Comparison between $P1$, $P2$ and $P3$ - Number of constraints

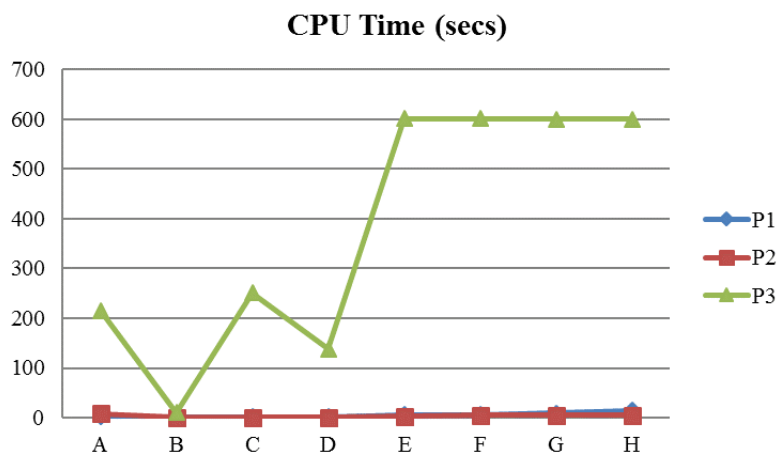


Fig. 10 Comparison between $P1$, $P2$ and $P3$ - CPU time (seconds)

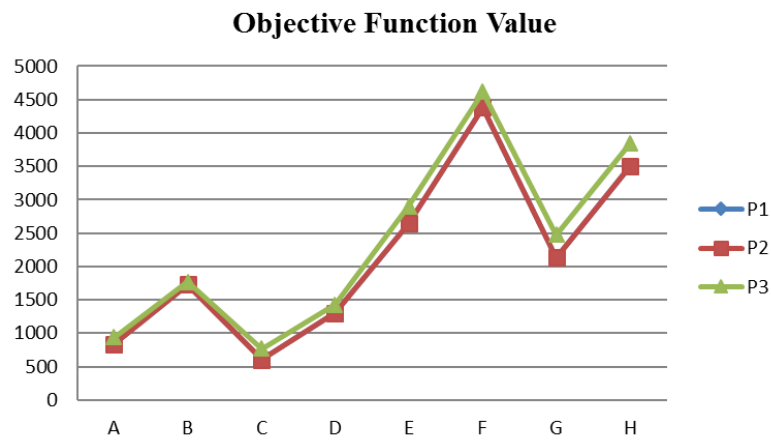


Fig. 11 Comparison between $P1$, $P2$ and $P3$ - Objective function

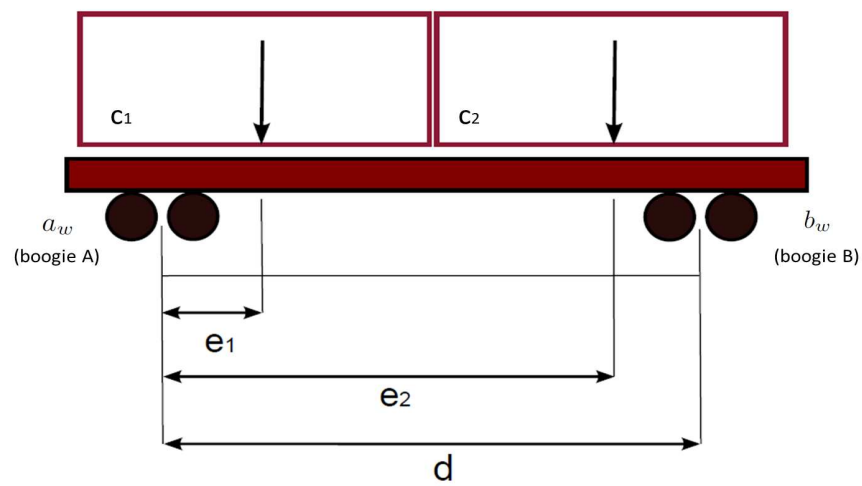


Fig. 12 Lever principle for weight distribution on wagon bogies.