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Original article

# Noise Properties of Two Mutually Coupled Spin-Transfer Nanooscillators in the Phase Locking Regime

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## Abstract

**Introduction.** Today, many research endeavors are devoted to the miniaturization of microwave sources. One of the promising approaches is the use of magnetic nanostructures (spintronics elements), providing a wide range of frequency tuning and low power consumption. The main disadvantage of spintronics generators (spin-transfer nanoscillators – STNO) is a low output power of generated oscillations (tens of nanowatts and less). A possible solution is to sum up the power of many STNOs in a mutual synchronization mode.

**Aim.** The investigation of noise properties of two connected STNOs with identical and non-identical parameters in a phase synchronization mode.

**Materials and methods.** A model was developed of two STNOs interconnected by spin waves taking into account thermal noises. Spectral power densities of the amplitude and phase noise were obtained by the method of effective linearization.

**Results.** Dependencies were obtained in a general form for attenuation coefficients of the amplitude and phase fluctuations of noise sources for each STNO. Three cases of synchronization were considered: completely identical STNOs, two identical STNOs but with different oscillation frequencies, and two non-identical STNOs, differing in an allowance of self-excitation by frequencies and amplitudes of the oscillations. It was possible to obtain a gain in the amplitude and phase noise for two identical STNOs. In this case, an increase in the allowance of self-excitation led to a decrease in the level of phase and amplitude noise.

**Conclusion.** This analysis of the attenuation coefficients for non-identical STNOs demonstrates the possibility of improving the noise properties of each of the generators. In this case, the best noise value is obtained for an STNO with greater stability in a stand-alone mode.

Keywords: spin-transfer nanooscillator, mutual phase locking, noise properties, spectral power density

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**Introduction.** Oscillation sources of microwave frequency range devices are based on either of the following: lumped elements (capacitors and inductors), delay lines, resonators on surface acoustic waves (SAWs), spin-wave devices, dielectric (including ceramic) cir-

cuits, waveguides, or other resonators [1]. An important task in the use of self-oscillators is to control their frequency stability. In schemes with lumped elements, this is most often solved by using varicaps [1]. In spin-wave devices, the dependence of the frequency of the

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ferromagnetic resonance on the magnitude of the constant magnetic field, which can easily change under the influence of direct current, is used.

Recently, special attention has been given to solid state physics, where nanoscale spin-wave devices - "spintronics" devices - are studied. Such devices are spin-transfer nanoscillators (STNO), which are multilayer nanostructures, most often cylindrical, made up of alternating magnetic and nonmagnetic layers [2–7]. Nowadays, using various configurations of nanolayers, it has become possible to achieve generation frequencies of more than 40 GHz [2]. The advantages of STNO over other well-known self-oscillators are: small size, wide frequency range: from hundreds of megahertz to tens of gigahertz with relative frequency tuning over an octave, integrability with the CMOS process, small operating voltages and currents (less than 0.3 V), short transition time process (nanosecond scale), and an extended section of the frequency's linear dependence on the control of the external direct current or external magnetic field. Already, options are proposed for using STNOs as microwave signal detectors [8] and in memory devices [9]. The possibility of generating radiation in the terahertz range is actively being studied [10, 11]. Note that an important property of STNO is non-isochronism, the dependence of the oscillation frequency on the amplitude.

One of the main characteristics of any oscillator is the level of phase noise. A low noise level of the oscillator is a prerequisite for the development of many radar and radio transmitting devices. For STNOs, this, along with a low output power, is the main drawback that limits their practical application. One way to reduce phase noise is to synchronize with an external force or mutually synchronize several oscillators. As an external force, external harmonic influence (EHI) or EHI usually acts in combination with a phase locked loop [12–15]. The purpose of synchronization is to impose the stability of a reference non-tunable oscillator on a frequency tunable oscillator. In addition, the synchronization systems studied in this work are used in communication, radar and radio navigation technology, control, measuring complexes, especially in frequency synthesizers, complex signal synthesizers, signal demodulators with angular modulation, signal phase and frequency meters, signal tracking devices of carrier frequencies of received signals, and inclock synchronization devices. Despite the large number of works on STNOs, the theoretical noise properties of interconnected STNO have not been previously studied. In this paper, we study the effect of synchronization of two coupled STNOs in phase synchronization mode on amplitude and phase noise.

Mathematical Model of Two Connected STNOs. We assume that STNOs are connected through a common ferromagnetic layer where spin waves propagate. The structure of coupled STNOs has been studied in a number of works (see, for example, [2]) and is not given here. Let us consider an assembly of two complex equations describing the dynamics of two connected STNOs [2]:

$$\begin{cases} \frac{dc_1}{dt} + j\omega_1(|c_1|^2)c_1 - \Delta\Gamma_1(|c_1|^2)c_1 = \Omega_1 e^{j\beta_1}c_2 + n_1(t); \\ \frac{dc_2}{dt} + j\omega_2(|c_2|^2)c_2 - \Delta\Gamma_2(|c_2|^2)c_2 = \Omega_2 e^{j\beta_2}c_1 + n_2(t), \end{cases}$$
(1)

where  $c_i$  (*i* = 1, 2) are the complex amplitudes of the spin waves of the first and second STNO;  $\omega_i = \omega_{0i} + N_i |c_i|^2$  is the dependence of the oscillation frequency on the amplitude ( $\omega_{0i}$  is the frequency of the ferromagnetic resonance of the ith STNO;  $N_i$  is the nonisochronism coefficient);  $\Delta \Gamma_i \left( |c_i|^2 \right) =$  $=\Gamma_{Gi}\left[\left(\zeta_{i}-1\right)-\left(\zeta_{i}+Q_{i}\right)\left|c_{i}\right|^{2}\right]\left(\Gamma_{Gi}=\alpha\omega_{i}; \alpha \text{ is the}\right)$ Gilbert damping constant;  $\zeta_i = I_i / I_{\text{th},i}$  is the self-excitation factor (supercriticality);  $I_i$  is the current through the *i*th STNO and  $I_{\text{th},i}$  is the critical current at which in the unconnected case oscillations arise in the STNO;  $Q_i$ is the phenomenological parameter [2]);  $\Omega_i$  is the connection coefficient of two STNOs;  $\beta_i$  is the phase delay in the synchronization system;  $n_i(t)$  is the additive noise addition caused by thermal fluctuations of the ferromagnetic material. Search for the solution (1) in the form:

$$c_i = U_i \exp\left[-j\omega_{\rm av}t - j\varphi_i(t)\right],\tag{2}$$

where  $U_i$ ,  $\varphi_i$  are the slowly varying amplitude and phase of the *i*-th oscillator, respectively;  $\omega_{av} = (\omega_1 + \omega_2)/2$  is the average frequency of two STNOs.

The derivative of the complex amplitude (2) has the form:

f works on STNOs, the theoretical noise proper-  
f interconnected STNO have not been previously
$$\frac{dc_i}{dt} = \left\{ \frac{dU_i}{dt} \frac{1}{U_i} + \left[ -j\omega_{av} - \frac{d\varphi_i(t)}{dt} \right] \right\} c_i, \quad (3)$$

Substitute (3) and (2) in (1). We get an assembly of two complex equations:

$$\begin{cases} \frac{dU_{1}}{dt} \frac{1}{U_{1}} + \left[ -j\omega_{av} - \frac{d\varphi_{1}(t)}{dt} \right] + j\omega_{1}(U_{1}) - \Delta\Gamma_{1}(U_{1}) = \\ = \frac{\Omega_{1}U_{2}}{U_{1}} e^{j\beta_{1}} e^{-j\left[\varphi_{2}(t) - \varphi_{1}(t)\right]} + \frac{n_{1}(t)}{U_{1}} e^{j\left[\omega_{av}t + \varphi_{1}(t)\right]}; \\ \frac{dU_{2}}{dt} \frac{1}{U_{2}} + \left[ -j\omega_{av} - \frac{d\varphi_{2}(t)}{dt} \right] + j\omega_{2}(U_{2}) - \Delta\Gamma_{2}(U_{2}) = \\ = \frac{\Omega_{2}U_{1}}{U_{2}} e^{j\beta_{2}} e^{-j\left[\varphi_{1}(t) - \varphi_{2}(t)\right]} + \frac{n_{2}(t)}{U_{2}} e^{j\left[\omega_{av}t + \varphi_{2}(t)\right]}. \end{cases}$$
(4)

We can now move from (4) to 4 valid equations for the amplitude and phase of each generator:

$$\begin{cases} \frac{dU_{1}}{dt} = U_{1}\Delta\Gamma_{1}(U_{1}) + \Omega_{1}U_{2}\cos(\varphi_{1} - \varphi_{2} + \beta_{1}) + \tilde{n}_{1}; \\ \frac{dU_{2}}{dt} = U_{2}\Delta\Gamma_{2}(U_{2}) + \Omega_{2}U_{1}\cos(\varphi_{2} - \varphi_{1} + \beta_{2}) + \tilde{n}_{2}; \\ \frac{d\varphi_{1}}{dt} = -\omega_{av} + \omega_{1}(U_{1}) - \frac{\Omega_{1}U_{2}}{U_{1}}\sin(\varphi_{1} - \varphi_{2} + \beta_{1}) + \frac{\tilde{n}_{1}}{U_{1}}; \\ \frac{d\varphi_{2}}{dt} = -\omega_{av} + \omega_{2}(U_{2}) - \frac{\Omega_{2}U_{1}}{U_{2}}\sin(\varphi_{2} - \varphi_{1} + \beta_{2}) + \frac{\tilde{n}_{2}}{U_{2}}, \\ \text{where } \tilde{n}_{1,2} = \operatorname{Re}\left\{\frac{n_{1,2}(t)}{U_{1,2}}e^{j\left[\omega_{av}t + \varphi_{1,2}(t)\right]}\right\}; \quad \omega_{i}(U_{i}) = \\ = \omega_{0i} + N_{i}U_{i}^{2}. \end{cases}$$

We calculate the stationary values of the amplitudes and phases of the oscillators  $U_1^0$ ,  $U_2^0$ ,  $\varphi_1^0$ ,  $\varphi_2^0$ . To do this, we equate the derivatives in (5) to zero. We get:

$$\begin{cases} 0 = U_{1}^{0}\Gamma_{G1}\left[\left(\zeta_{1}-1\right)-\left(\zeta_{1}+Q_{1}\right)\left(U_{1}^{0}\right)^{2}\right]+\\ +\Omega_{1}U_{2}^{0}\cos\left(\varphi_{1}^{0}-\varphi_{2}^{0}+\beta_{1}\right)=f_{1};\\ 0 = U_{2}^{0}\Gamma_{G2}\left[\left(\zeta_{2}-1\right)-\left(\zeta_{2}+Q_{2}\right)\left(U_{2}^{0}\right)^{2}\right]+\\ +\Omega_{2}U_{1}^{0}\cos\left(\varphi_{2}^{0}-\varphi_{1}^{0}+\beta_{2}\right)=f_{2};\\ 0 = -\omega_{av}+\omega_{01}+N_{1}\left(U_{1}^{0}\right)^{2}-\\ -\frac{\Omega_{1}U_{2}^{0}}{U_{1}^{0}}\sin\left(\varphi_{1}^{0}-\varphi_{2}^{0}+\beta_{1}\right)=f_{3};\\ 0 = -\omega_{av}+\omega_{02}+N_{2}\left(U_{2}^{0}\right)^{2}-\\ -\frac{\Omega_{2}U_{1}^{0}}{U_{2}^{0}}\sin\left(\varphi_{2}^{0}-\varphi_{1}^{0}+\beta_{2}\right)=f_{4}, \end{cases}$$
(6)

where  $f_1, ..., f_4$  are nonlinear functions of stationary states  $U_1^0, U_2^0, \varphi_1^0, \varphi_2^0$ . We can now move from (6) to the equations with the stationary phase difference  $\psi = \varphi_1 - \varphi_2$  since its value determines the stationary amplitudes of STNO oscillations:

$$\begin{cases} 0 = U_1^0 \Gamma_{G1} \Big[ (\zeta_1 - 1) - (\zeta_1 + Q_1) (U_1^0)^2 \Big] + \\ + \Omega_1 U_2^0 \cos(\psi^0 + \beta_1); \\ 0 = U_2^0 \Gamma_{G2} \Big[ (\zeta_1 - 1) - (\zeta_2 + Q_2) (U_2^0)^2 \Big] + \\ + \Omega_2 U_1^0 \cos(-\psi^0 + \beta_2); \\ 0 = \omega_{01} - \omega_{02} + N_1 (U_1^0)^2 - N_2 (U_2^0)^2 - \\ - \frac{\Omega_1 U_2^0}{U_1^0} \sin(\psi^0 + \beta_1) + \frac{\Omega_2 U_1^0}{U_2^0} \sin(-\psi^0 + \beta_2) \end{cases}$$

We can now move to the equations for small deviations  $\delta u_1$ ,  $\delta u_2$ ,  $\delta \phi_1$ ,  $\delta \phi_2$  with respect to the stable stationary mode. We get an assembly of equations in the following form:

$$\begin{cases} \frac{d\delta u_1}{dt} = \frac{\partial f_1}{\partial U_1} \delta u_1 + \frac{\partial f_1}{\partial U_2} \delta u_2 + \frac{\partial f_1}{\partial \varphi_1} \delta \varphi_1 + \frac{\partial f_1}{\partial \varphi_2} \delta \varphi_2 + \tilde{n}_1; \\ \frac{d\delta u_2}{dt} = \frac{\partial f_2}{\partial U_1} \delta u_1 + \frac{\partial f_2}{\partial U_2} \delta u_2 + \frac{\partial f_2}{\partial \varphi_1} \delta \varphi_1 + \frac{\partial f_2}{\partial \varphi_2} \delta \varphi_2 + \tilde{n}_2; \\ \frac{d\delta \varphi_1}{dt} = \frac{\partial f_3}{\partial U_1} \delta u_1 + \frac{\partial f_3}{\partial U_2} \delta u_2 + \frac{\partial f_3}{\partial \varphi_1} \delta \varphi_1 + \frac{\partial f_3}{\partial \varphi_2} \delta \varphi_2 + \frac{\tilde{n}_1}{U_1^0}; \\ \frac{d\delta \varphi_2}{dt} = \frac{\partial f_4}{\partial U_1} \delta u_1 + \frac{\partial f_4}{\partial U_2} \delta u_2 + \frac{\partial f_4}{\partial \varphi_1} \delta \varphi_1 + \frac{\partial f_4}{\partial \varphi_2} \delta \varphi_2 + \frac{\tilde{n}_2}{U_2^0}. \end{cases}$$

Using the spectral method  $\left(\frac{d}{dt} = j\omega\right)$ , we can now move to a linear inhomogeneous system of equations:

$$\begin{cases} \left(f_{1U_{1}} - j\omega\right)\delta u_{1} + f_{1U_{2}}\delta u_{2} + f_{1\phi_{1}}\delta\phi_{1} + f_{1\phi_{2}}\delta\phi_{2} = \tilde{n}_{1}; \\ f_{2U_{1}}\delta u_{1} + \left(f_{2U_{2}} - j\omega\right)\delta u_{2} + f_{2\phi_{1}}\delta\phi_{1} + f_{2\phi_{2}}\delta\phi_{2} = \tilde{n}_{2}; \\ f_{3U_{1}}\delta u_{1} + f_{3U_{2}}\delta u_{2} + \left(f_{3\phi_{1}} - j\omega\right)\delta\phi_{1} + f_{3\phi_{2}}\delta\phi_{2} = \frac{\tilde{n}_{1}}{U_{1}^{0}}; \\ f_{4U_{1}}\delta u_{1} + f_{4U_{2}}\delta u_{2} + f_{4\phi_{1}}\delta\phi_{1} + \left(f_{4\phi_{2}} - j\omega\right)\delta\phi_{2} = \frac{\tilde{n}_{2}}{U_{2}^{0}}, \end{cases}$$

where  $f_{iU_k} = \partial f_i / \partial U_k$ ,  $f_{i\varphi_k} = \partial f_i / \partial \varphi_k$ ; i = 1...4, k = 1...4.

Assembly (7) can be specified by the Carmer method in the following form:

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$$\begin{cases} \delta u_{1}(\omega) = \frac{\left[\Delta_{11}(\omega) + \frac{\Delta_{13}(\omega)}{U_{1}^{0}}\right]\tilde{n}_{1} + \left[\Delta_{12}(\omega) + \frac{\Delta_{14}(\omega)}{U_{2}^{0}}\right]\tilde{n}_{2}}{\Delta(\omega)}; \\ \delta u_{2}(\omega) = \frac{\left[\Delta_{21}(\omega) + \frac{\Delta_{23}(\omega)}{U_{1}^{0}}\right]\tilde{n}_{1} + \left[\Delta_{22}(\omega) + \frac{\Delta_{24}(\omega)}{U_{2}^{0}}\right]\tilde{n}_{2}}{\Delta(\omega)}; \\ \delta \phi_{1}(\omega) = \frac{\left[\Delta_{31}(\omega) + \frac{\Delta_{33}(\omega)}{U_{1}^{0}}\right]\tilde{n}_{1} + \left[\Delta_{32}(\omega) + \frac{\Delta_{34}(\omega)}{U_{2}^{0}}\right]\tilde{n}_{2}}{\Delta(\omega)}; \\ \delta \phi_{2}(\omega) = \frac{\left[\Delta_{41}(\omega) + \frac{\Delta_{43}(\omega)}{U_{1}^{0}}\right]\tilde{n}_{1} + \left[\Delta_{42}(\omega) + \frac{\Delta_{44}(\omega)}{U_{2}^{0}}\right]\tilde{n}_{2}}{\Delta(\omega)}; \end{cases}$$

where  $\Delta_{ii}(\omega)$  are the corresponding determinants of an assembly (7). In particular,

$$\Delta(\omega) = \begin{vmatrix} f_{1U_1} - j\omega & f_{1U_2} & f_{1\phi_1} & f_{1\phi_2} \\ f_{2U_1} & f_{2U_2} - j\omega & f_{2\phi_1} & f_{2\phi_2} \\ f_{3U_1} & f_{3U_2} & f_{3\phi_1} - j\omega & f_{3\phi_2} \\ f_{4U_1} & f_{4U_2} & f_{4\phi_1} & f_{4\phi_2} - j\omega \end{vmatrix}.$$

And the determinants  $\Delta_{ij}(\omega)$  are searched by replacing the *i*-th column with a column of constant terms, for example,

$$\Delta_{11}(\omega) = \begin{vmatrix} f_{2U_2} - j\omega & f_{2\varphi_1} & f_{2\varphi_2} \\ f_{3U_2} & f_{3\varphi_1} - j\omega & f_{3\varphi_2} \\ f_{4U_2} & f_{4\varphi_1} & f_{4\varphi_2} - j\omega \end{vmatrix},$$
  
$$\Delta_{12}(\omega) = -\begin{vmatrix} f_{1U_2} & f_{1\varphi_1} & f_{1\varphi_2} \\ f_{3U_2} & f_{3\varphi_1} - j\omega & f_{3\varphi_2} \\ f_{4U_2} & f_{4\varphi_1} & f_{4\varphi_2} - j\omega \end{vmatrix}.$$

Next, we can now move from (8) to spectral densities  $S_{\delta u_{1,2}}, S_{\delta \varphi_{1,2}}$  of amplitude and phase noises, respectively:

$$\begin{cases} S_{\delta u_{1}}(\omega) = \left| \frac{\Delta_{11}(\omega) + \frac{\Delta_{13}(\omega)}{U_{1}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}1} + \left| \frac{\Delta_{12}(\omega) + \frac{\Delta_{14}(\omega)}{U_{2}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}2}; \\ S_{\delta u_{2}}(\omega) = \left| \frac{\Delta_{21}(\omega) + \frac{\Delta_{23}(\omega)}{U_{1}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}1} + \left| \frac{\Delta_{22}(\omega) + \frac{\Delta_{24}(\omega)}{U_{2}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}2}; \\ S_{\delta u_{2}}(\omega) = \left| \frac{\Delta_{21}(\omega) + \frac{\Delta_{23}(\omega)}{U_{1}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}1} + \left| \frac{\Delta_{22}(\omega) + \frac{\Delta_{24}(\omega)}{U_{2}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}2}; \\ S_{\delta \phi_{1}}(\omega) = \left| \frac{\Delta_{31}(\omega) + \frac{\Delta_{33}(\omega)}{U_{1}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}1} + \left| \frac{\Delta_{32}(\omega) + \frac{\Delta_{34}(\omega)}{U_{2}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}2}; \\ S_{\delta \phi_{2}}(\omega) = \left| \frac{\Delta_{41}(\omega) + \frac{\Delta_{43}(\omega)}{U_{1}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}1} + \left| \frac{\Delta_{42}(\omega) + \frac{\Delta_{44}(\omega)}{U_{2}^{0}}}{\Delta(\omega)} \right|^{2} S_{\tilde{n}2}. \end{cases}$$

$$P = 0.1241 \text{ GHz}.$$

$$P = 0.66; \quad \sigma = 61.5 \text{ GHz/A}; \quad \alpha = 0.01; \quad \frac{\Gamma_{G}}{2\pi} = 0.1241 \text{ GHz}.$$

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Using the obtained expressions (9), it is possible to quantitatively study the level of spectral power densities of the amplitude and phase noise of two coupled STNOs.

Noise properties of two identical STNOs. Let us consider the case of two absolutely identical STNOs  $\omega_{\text{fm1}} + N_1 (U_1^0)^2 = \omega_{\text{fm2}} + N_2 (U_2^0)^2 = \omega_0; \omega_{\text{fm1}}, \omega_{\text{fm2}}$ are the ferromagnetic resonance frequencies.  $\Gamma_{G1} = \Gamma_{G2}; \quad \zeta_1 = \zeta_2; \quad Q_1 = Q_2; \quad \Omega_1 = \Omega_2.$  In this case, the time delay will be considered equal to zero  $\beta_1 = \beta_2 = 0$ . In general, the effect of phase delay reduces to the frequency of the obtained dependences. The amplitudes and phases of the STNO in this case will be equal to  $U_1^0 = U_2^0, \ \psi^0 = 0.$ 

In this case, we can now move to one equation for determining the stationary amplitude and drop subscripts 1 and 2:

$$0 = U^0 \Gamma_{\mathbf{G}} \left[ (\zeta - 1) - (\zeta + Q) (U^0)^2 \right] + \Omega U^0.$$

Then the stationary value of the amplitude will have the form:

$$U^{0} = \sqrt{\frac{\zeta - 1 + \frac{\Omega}{\Gamma_{G}}}{\zeta + Q}}$$

The oscillation frequency in this case will be equal to:

$$\omega_0 = \omega_{\rm fm} + N \frac{\zeta - 1 + \frac{\Omega}{\Gamma_{\rm G}}}{\zeta + Q} \,.$$

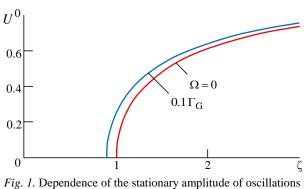
If  $\Omega \rightarrow 0$  the frequency and amplitude of the oscillations of the coupled STNOs tend toward the frequency and amplitude of the oscillations of the selfregulating STNO. Let the STNO parameters be equal

[2]: 
$$\frac{N}{2\pi} = 10.48 \text{ GHz}; \quad \frac{\omega_{\text{fm1}}}{2\pi} = 12.41 \text{ GHz}; \quad \zeta = 2;$$
  
 $Q = 0.66; \quad \sigma = 61.5 \text{ GHz/A}; \quad \alpha = 0.01;$   
 $\frac{\Gamma_{\text{G}}}{2\pi} = 0.1241 \text{ GHz}.$ 

The stationary values of the amplitude and frequency of oscillations, in this case, will depend on the margin of self-excitation  $\zeta$  and the connection coef-

ficient  $\Omega$  of two generators at  $\frac{\omega_0}{2\pi} = 18.84$  GHz. The dependence  $U(\zeta, \Omega)$  is shown in Fig. 1.

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at different values of the self-excitation margin

Let us compare the obtained spectral characteristics of the amplitude and phase noises with the characteristics of the self-regulating STNO. We will use the value of the coherence as a parameter  $\Omega$ . The peculiarity of the mutual synchronization assembly is the phenomenon of mutual assistance for the setting-in of oscillations. In this case, even with a margin of self-excitation less than unity, a stable state of oscillations is possible. A physical limit is the value of the coupling coefficient; it should be no more than  $\Gamma_G$  because the maximum realistically attainable values of the self-excitation margin are approximately  $\approx$ 3–4.

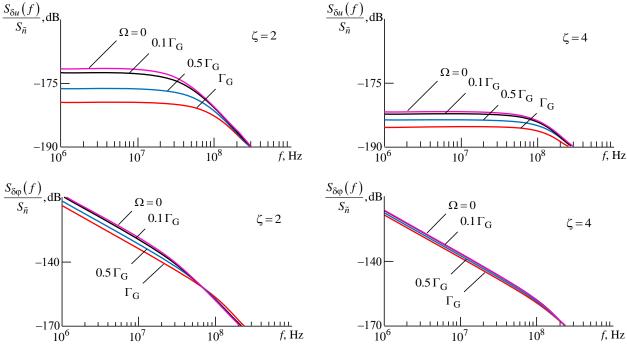
The dependence of the amplitude and phase noise at different values of the connection coefficient  $\Omega$  as well as two values of the self-excitation margin ( $\zeta = 2$  and  $\zeta = 4$ ) constructed according to formulas (11) are shown in Fig. 2

According to Fig. 2, the mutual synchronization of an ensemble of two STNOs leads to a decrease in the amplitude and phase noise of each of the oscillators. Moreover, an increase in the connection coefficient between STNOs leads to a decrease in the amplitude and phase noise. Also, an increase in the connection coefficient leads to an increase in the field of offsets from the frequency of the STNO oscillations, at which a gain in the amplitude and phase noise is ensured. The spectral density of phase noise far exceeds the spectral density of amplitude noise. This is typical for all self-excited oscillators.

An analysis of the expressions obtained shows that in order to improve the noisiness of STNOs, it is necessary to increase the self-excitation margin  $\zeta$ , to reduce losses in the equivalent oscillatory system  $\Gamma_G$ , and to reduce the nonisochronism coefficient *N*. Nonisochronism, being a mechanism for changing the oscillation frequency, leads to a significant deterioration in noisiness. However, a decrease *N* leads to a decrease in the possible frequency range of the oscillator.

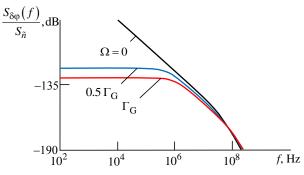
The obtained calculations of amplitude and phase noise make it possible to design a system of synchronized STNOs with a minimum level of phase and amplitude noise.

Noisiness of two non-identical STNOs. Let us consider the case of synchronization of two non-



*Fig. 2.* Dependence of amplitude and phase noise at different values of the connection coefficient  $\Omega$  and two values of the self-excitation margin  $\zeta = 2$  and  $\zeta = 4$ . Nonisochronism coefficient N = 0

Noise Properties of Two Mutually Coupled Spin-Transfer Nanooscillators in the Phase Locking Regime



*Fig. 3.* Phase noise of two STNOs with different selfexcitation margins and corresponding discomposure  $\Delta \omega = -9.77 \cdot 10^7$  rad/s for two connection coefficients ( $\Omega = 0.5\Gamma_G$ ;  $\Omega = \Gamma_G$ ) and noise of self-regulating STNO ( $\Omega = 0$ )

identical STNOs that differ in the self-excitation margin. For such an assembly, a gain in the phase noise level is obtained for both STNOs. In this case, the best noise value is achieved for the STNO which has a large self-excitation margin, in accordance with Fig. 3.

**Conclusion**. The dependences are obtained in a general form for each STNO's noise source's attenuation coefficients of the amplitude and phase fluctuations. Two cases of synchronization were

considered - completely identical and non-identical STNOs, differing by a self-excitation margin, frequencies, and amplitudes of oscillations. It is possible to obtain a gain in the level of amplitude and phase noise for two identical STNOs. In this case, an increase in the allowance of self-excitation leads to a decrease in the level of phase and amplitude noise. Nonisochronism, in its turn, leads to an increase in the level of amplitude and phase noise. In the second case, it is possible to obtain the best value of phase and amplitude noise. At the same time, in order to obtain a less noisy STNO, it is necessary to increase the connection coefficient of two STNOs and to increase the frequency mismatch of two STNOs while remaining within the system synchronism line. This is because the control action to STNOs in this model depends on the frequency difference between the generators. With equal frequencies, such an effect is minimal in accordance with the shortened equations. This analysis of attenuation coefficients for non-identical STNOs demonstrates the possibility of improving the noise properties of each of the generators. In this case, the best noise value is obtained for STNOs with greater stability in stand-alone mode.

#### Author's contributions

**A. A. Mitrofanov**, obtaining a model of two connected spin-transfer nanoscillators in the condition of noise. Obtaining and analysis of power spectral densities of two connected spin-transfer nano-oscillators in the condition of noise. Preparation of the text for the study.

A. R. Safin, supervision of the study. Preparation of the text for the study.

E. M. Torina, obtaining spectral power densities of two connected spin-transfer nanoscillators. Preparation of the text for the study.

N. N. Udalov, supervision of the study.

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