CHALLENGE JOURNAL OF CONCRETE RESEARCH LETTERS 11 (2) (2020) 39–45

Challenge Journal of Concrete Research Letters

Research Article

Investigation of optimal designs for concrete cantilever retaining walls in different soils

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ABSTRACT

In this paper, the investigation of the optimum designs for two types of concrete cantilever retaining walls was conducted utilizing the artificial bee colony algorithm. Stability conditions like safety factors of sliding, overturning and bearing capacity and some geometric instances due to inherent of the wall were considered as the design constraints. The effect of the existence of the key in wall design on the objective function was probed for changeable properties of foundation and backfill soils. In optimization analysis, the concrete of the wall, which directly affects parameters such as carbon dioxide emission and the cost, was considered as the objective function and analyzes were performed according to different discrete design variables. The optimum concrete cantilever retaining wall designs satisfying constraints of stability conditions and geometric instances were obtained for different soil cases. Optimum designs of concrete cantilever retaining wall with the key were attained in some soil cases which were not found the feasible optimum solution of the concrete cantilever retaining wall. Results illustrate that the artificial bee colony algorithm was a favorable metaheuristic optimization method to gain optimum designs of concrete cantilever retaining wall.

ARTICLE INFO

Article history:
Received 23 April 2020
Accepted 24 May 2020

Keywords:
Concrete cantilever retaining wall
Check for sliding, overturning, and bearing capacity
Optimum design
Discrete design variables
Artificial bee colony algorithm

1. Introduction

In geotechnical engineering, cantilever retaining wall is commonly employed for enduring lateral soil pressure occurred between two different soil levels. Cantilever retaining wall, which comes into existence by combining a base and thin stem, is manufactured utilizing materials like stone, concrete or concrete-reinforcement. Design of a cantilever retaining wall must not be only ensured stability conditions but also should have a low cost. While a designer is trying to meet the requirement of being safe and economical wall design, the effect of changing parameters on the wall design should also be deliberated. In the wall design, considering parameters, for instance, retained height, the existence of groundwater, the physical position of construction area, intended use of the structure, the completion time of the construction and soil properties have made the design process complex with many unknowns. That is why utilizing metaheuristic optimization methods in the solution of this kind of engineering problems has become quite popular in recent years. Metaheuristic optimization methods are algorithms that mimic the behaviours of creatures like the process of survival, foraging, and migration in nature. The metaheuristic optimization methods which does not guarantee the accurate solution are robust and effective by courtesy of approaching the feasible solution in a reasonable time.

Many metaheuristic optimization methods that provide optimum solutions for the complex engineering problems have been presented hitherto; the genetic algorithm (GA) by Goldberg (1989), the particle swarm optimization (PSO) by Kennedy and Eberhart (1995), the ant colony algorithm (ACO) by Dorigo and Gambardella

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ISSN: 2548-0928 / DOI: https://doi.org/10.20528/cjcrl.2020.02.003

(1997), the harmony search algorithm (HSA) by Geem et al. (2001), the artificial bee colony algorithm (ABC) by Karaboga (2005), the firefly algorithm (FA) by Yang (2009), and the cuckoo search algorithm (CS) by Rajabioun (2011). The study conducted by Sarıbaş and Erbatur (1996) has been one of the first examples for the optimum cantilever wall design investigated the optimum wall weight and the optimum cost. Since the metaheuristic algorithms are simple, effective, and easy to implement, using these algorithms to analyze the optimum design and cost of cantilever retaining walls have become widespread. Studies for the optimization analysis of cantilever retaining wall were carried out by Camp and Akın (2012), Gandomi (2015), Temur and Bekdaş (2016), and Uray et al. (2019).

In this study, the optimum designs of concrete cantilever retaining wall (CRW) and concrete cantilever retaining wall with key (CRWK) were investigated utilizing the artificial bee colony algorithm. For changing soil instances, obtained optimum designs of CRW and CRWK have been compared in terms of concrete wall weights.

2. Materials and Methods

2.1. Geotechnical design of the concrete cantilever retaining wall designs

The optimum wall design of the concrete cantilever retaining wall (CRW) and the concrete cantilever retaining wall design with key (CRWK) must be provided stability conditions like check for sliding, overturning, and bearing capacity with the allowable safety factors for the safe wall design. The wall dimensions and acting loads to the wall utilized for calculation of the safety factors of sliding, overturning, and bearing capacity in the design of CRW and CRWK were indicated respectively in Figs. 1 and 2.

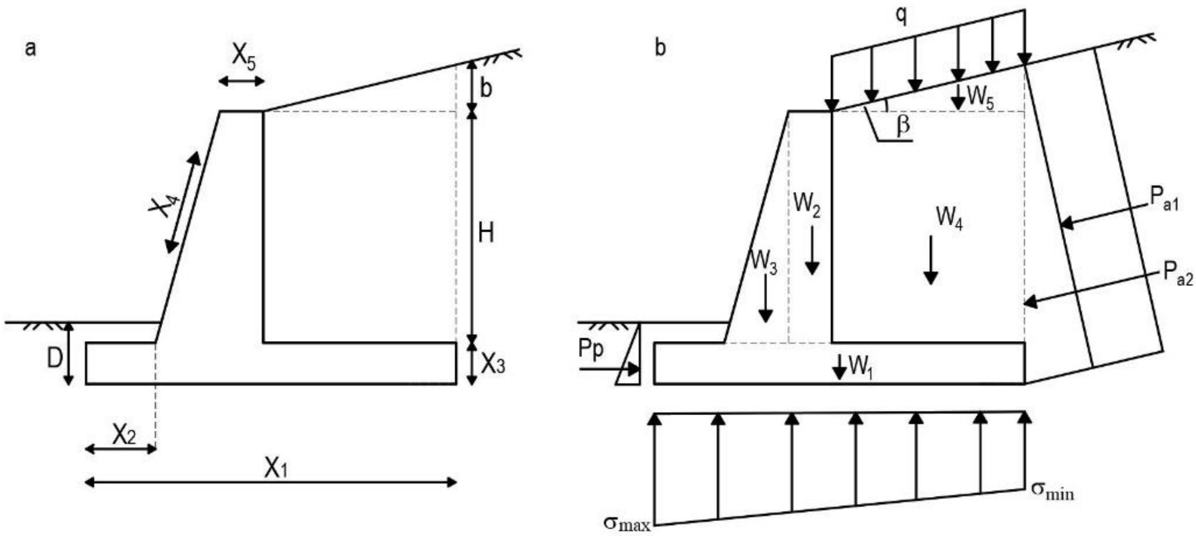


Fig. 1. Concrete cantilever retaining wall design: (a) wall dimensions; (b) acting loads to the wall.

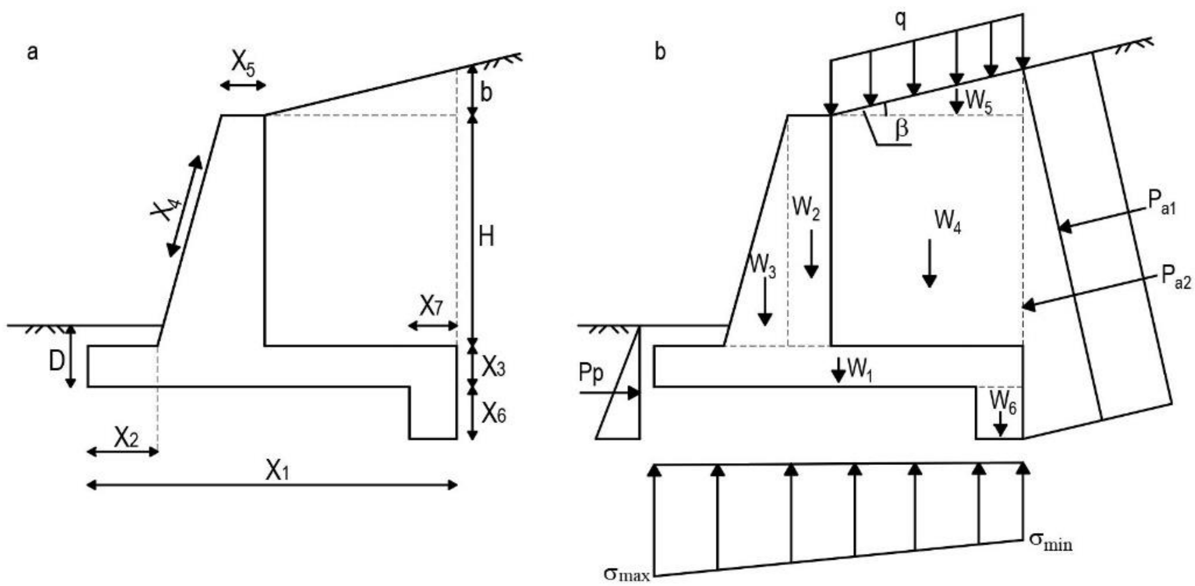


Fig. 2. Concrete cantilever retaining wall design with key: (a) wall dimensions; (b) acting loads to the wall.

While W_1, W_2, W_3, and W_6 correspond to the concrete weight of the wall for CRW or CRWK, W_4 and W_5 sign the soil weight above the back extension of the wall. P_a and P_p are active and passive soil pressure forces, respectively and P_a (Eq. (1)) is equal to the sum of P_{a1} and P_{a2} given in Figs. 1(b) and 2(b).

$$P_a = P_{a1} + P_{a2} = qK_a(b + H + X_3) + 0.5K_a\gamma_{bs}(b + H + X_3)^2 \quad (1)$$

The passive soil pressure force is calculated using Eq. (2) for the CRW design and Eq. (3) for CRWK design. Here, γ_{bs}, γ_{fs} and c_{fs} are the unit volume weights of backfill soil and foundation soil, and cohesion of foundation soil, in turn.

$$P_p = 0.5K_p\gamma_{fs}D^2 + 2c_{fs}\sqrt{K_p}\,D \quad (2)$$

$$P_p = 0.5K_p\gamma_{fs}(D + X_6)^2 + 2c_{fs}\sqrt{K_p}\,(D + X_6) \quad (3)$$

In the determination of active and passive soil pressure coefficients given in Eqs. (4) and (5) Rankine theory (1857) is utilized. In equations, β, $\emptyset_{fs}$, and $\emptyset_{bs}$ correspond to respectively the slope of the backfill soil, the angle of internal friction of the backfill soil and the angle of internal friction of the foundation soil.

$$K_a = \cos\beta\,\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\emptyset_{bs}}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\emptyset_{bs}}} \quad (4)$$

$$K_p = \tan^2\left(\frac{45 + \emptyset_{fs}}{2}\right) \quad (5)$$

In the wall designs, stability criteria which are safety factors of sliding ($F_s(s)$), overturning ($F_o(s)$), and bearing capacity ($F_s(bc)$), have been employed; given in respectively Eqs. (6), (7) and (8) with allowable the minimum and the maximum values of safety factors.

$$F_s(s,\min) \le F_s(s) = \frac{\sum V\tan\left(\frac{2}{3}\emptyset_{fs}\right) + \frac{2}{3}X_1c_{fs} + P_p}{P_a\cos\beta} \le F_s(s,\max) \quad (6)$$

$$F_s(o,\min) \le F_s(o) = \frac{\sum M_r}{\sum M_o} \le F_s(o,\max) \quad (7)$$

$$F_s(bc,\min) \le F_s(bc) = \frac{q_u}{q_{\max}} \le F_s(bc,\max) \quad (8)$$

Eq. (9) has determined the sum of vertical forces ($\sum V$) effective directly on the resistance to sliding for CRW and CRWK. To determine the overturning safety factor rotation effect of acting loads to the wall must be calculated. The total moment ($\sum M_r$) of forces which withstand overturning of the wall according to toe point of the wall base is given Eq. (10) for CRW or CRWK. The total moment ($\sum M_o$) which try to overturn the wall at the toe point is determined as M_o given in Eq. (11).

$$\sum V = \sum W_i + P_a\sin\beta \quad (9)$$

$$\sum M_r = \sum W_ix_i + \dots + W_6x_6 + P_a\sin\beta X_1 \quad (10)$$

$$\sum M_o = 0.5qK_a(b + H + X_3)^2\cos\beta + 0.5K_a\gamma_{bs}(b + H + X_3)^3/3 \quad (11)$$

The loads coming from the wall ($q_{\max}$) must be safely transferred to the soil by the foundation, and these loads must be carried by the soil (q_u). The minimum base pressure ($q_{\min}$) in the interface between the soil and the wall must be greater than the zero ($e < X_1/6$) as the soil cannot bear the tension. Expressions of $q_{\max}$, $q_{\min}$, and eccentricity (e) are given Eqs. (12) and (13), respectively. General bearing capacity expression suggested by Meyerhof (1963) have been utilized for the calculation of the ultimate bearing capacity (q_u) (Das, 2016).

$$q_{\substack{\min\\ \max}} = \frac{\sum V}{X_1}\left(1 \pm \frac{6e}{X_1}\right) \quad (12)$$

$$e = \frac{X_1}{2} - \frac{\sum M_r - \sum M_o}{\sum V} \quad (13)$$

2.2. Formulation of the optimization problem for the concrete cantilever retaining walls

In the optimization problems generally should have three basic concepts, including the design space, the design constraints and the objective function. The dimensions of CRW are the base width (X_1), the toe extension (X_2), the base thickness (X_3), the inclination of the wall front face (X_4), and the top thickness of the stem (X_5) given in Fig 1(a). The dimensions of CRWK given in Fig. 2(a) which are the base width (X_1), the toe extension (X_2), the base thickness (X_3), the inclination of the wall front face (X_4), and the top thickness of the stem (X_5), the height of key (X_6) and the thickness of key (X_7) were selected as the discrete design variables. To determinate of the lower-the upper bounds for the discrete design variables, wall dimensions suggested in the provisions of Building Code Requirements for Structural Concrete (ACI 318-08, 2008) and LRFD Bridge Design Specifications (AASHTO, 2010) were employed. The lower-the upper bounds of the discrete design variables with increments have been tabulated in Table 1.

The wall designs obtained by using different values of the design variables given in Table 1 must provide these basic four rules for the external stability of the wall: (i) Safety factor of sliding of the wall must be greater than its minimum acceptable value, (ii) Safety factor of overturning of the wall must be greater than its minimum allowable value, (iii) The pressure transferred from the base to the soil must be smaller than the ultimate bearing capacity of the soil, (iv) The eccentricity of resultant force at the base surface must be within in the core not to occur tension stress. Therefore, these rules were defined as design constraints for having values of minimum and maximum safety factors. The minimum safety factors values of sliding, overturning and bearing capacity have been taken as F_s (s, min) = 1.50, F_s (o, min) = 1.50, and F_s (bc, min) = 3.00, respectively (Das, 2016). Also, the maximum safety factors (F_s (s, max), F_s (o, max), F_s (bc, max)) whose changing values depend on soil properties were considered with the aim of obtaining more economical design. Besides, the eccentricity control in base width constraint and geometric design constraints due to the wall dimensions were taken into consideration too in the optimization analyses. Normalized expressions of all design constraints are designated in Table 2.

Table 1. Design variables and limit bounds for CRW and CRWK.

Design variables	Lower bound	Upper bound	Increment
X_1: Base width	$0.25H$	$1.0H$	$0.05H$
X_2: Toe extension	$0.15X_1$	$0.60X_1$	$0.05X_1$
X_3: Base thickness	$0.06H$	$0.15H$	$0.015H$
X_4: Inclination of wall front face (%)	0	6	1
X_5: Top thickness of stem (cm)	0.20	0.40	0.05
X_6: Height of key	$0.60X_3$	$1.20X_3$	$0.10X_3$
X_7: Thickness of key	$0.20X_1$	$0.40X_1$	$0.05X_1$

Table 2. Design constraints.

Design constraints	Normalized expression
The sliding safety factor lower bound	$g_x(1) = 1 - F_s(s)/F_s(s, \min)$
The sliding safety factor upper bound	$g_x(2) = F_s(s)/F_s(s, \max) - 1$
The overturning safety factor lower bound	$g_x(3) = 1 - F_s(o)/F_s(o, \min)$
The overturning safety factor upper bound	$g_x(4) = F_s(o)/F_s(o, \max) - 1$
The bearing capacity safety factor lower bound	$g_x(5) = 1 - F_s(bc)/F_s(bc, \min)$
The bearing capacity safety factor upper bound	$g_x(6) = F_s(bc)/F_s(bc, \max) - 1$
The eccentricity constraint	$g_x(7) = X_1/(6e)$
The geometric constraint 1	$g_x(8) = X_5/(HX_4 + X_5) -1$
The geometric constraint 2	$g_x(9) = (X_2 + HX_4 + X_5)/X_1/-1$

In this paper, the objective function of the optimization problem taken as concrete weight of CRW and CRWK. Wall weights of CRW and CRWK have been compared for different soil conditions. The mathematical expressions of the objective formulation for CRW and CRWK are given Eqs. (14) and (15), respectively.

$$f_{\min} = W_1 + W_2 + W_3 \quad (14)$$

$$f_{\min} = W_1 + W_2 + W_3 + W_6 \quad (15)$$

2.3. Artificial bee colony algorithm

The artificial bee colony algorithm (ABC) suggested by Karaboga (2005) is one of the metaheuristic optimization algorithms. It is inspired by swarm intelligence that is effective interpersonal communication for surviving as a swarm in nature and having basic life needs such as nutrition, defense and migration. General concepts and algorithm steps have given for the artificial bee colony algorithm based on bees' nutritional processes and behaviour in this chapter. Bees tried to find the best food source in the algorithm are divided into three groups; as the employed bees, the onlooker bees and the scout bees. The employed bees seek food source vicinity of the hive and evaluate the quality of a found food source. If this quality is better than the previously found quality of the food source, the location of the food source is kept in their mind. The onlooker bees observe the returned employed bees that dance to share the information like the amount of nectar, quality, and location about the found food source. Factors like the type of dance or time of dancing are influential in selecting which food source is worth to prefer by the onlooker bees. When the food sources are consumed, the process of the employed bee and the onlooker bee is fulfilled. The random seeking of the scout bee commences for the possible food sources around the hive. The scout bee that finds a food source turns into an employed bee. Only one scout bee is allowed, and the number of employed bees is equal to the number of food sources in the algorithm.

In this optimization problem of the wall design, designs of CRW and CRWK with the changeable values of design variables correspond to food sources, and the quality of the food sources are the weights of the wall. The main steps of the ABC algorithm for the optimum design of CRW and CRWK are as follows:

Step 1: The ABC algorithm parameters which are the number of employed bees (*NEB*), the number of onlooker bees (*NOB*), the number of the food source (*NFS*), the number of maximum iteration (*maxiter*), and limit (Akay and Karaboga, 2012) are defined and initial food source areas are formed by using Eq. (16).

$$x_{ij} = x_j^{\min} + \text{rand}(0,1)\left(x_j^{\max} - x_j^{\min}\right)$$
$$(i = 1, \dots, NFS, j = 1, \dots, N) \quad (16)$$

Here, N is the total number of design variables. In this study, N has been taken 5 and 7 for designs of CRW and CRWK, respectively. Rand (0,1) means a random number between 0 and 1. $x_j^{\min}$ and $x_j^{\max}$ are given as the lower and the upper bounds of the j^{th} design parameter, respectively. The food matrix (FM) corresponding the design space is formed by using values of design variables given in Table 1 and Eq. (16). For each row of FM, which states wall designs, values of the objective function are calculated.

Step 2: The employed bees determine a new food source and evaluate its quality. In determining a new food source which is neighbor of its current food source Eq.17 is used.

$$v_{ij} = \begin{cases} x_{ij} + \emptyset_{ij}\left(x_{ij} - x_{kj}\right), & \text{rand}(0,1) < MR \\ x_{ij}, & \text{rand}(0,1) \geq MR \end{cases}$$
$$(\emptyset_{ij} = [-1,1]) \quad (17)$$

Here, x_{ij} means the j^{th} design variable randomly selected of the i^{th} food source and k is a randomly chosen value between 1 and NFS. The modification rate, MR, is a control parameter for use in checking a new source is developed or not. The value of MR has been suggested to be between 0.30-0.80 (Akay and Karaboga, 2012). Fitness value for the appropriate new food source (v_{ij}) is calculated by using Eq. (18).

$$\text{fitness}_i = \begin{cases} 1/(1+f_i), f_i \geq 0 \\ 1 + \text{abs}(f_i), f_i < 0 \end{cases} \quad (18)$$

Here, f_i is the objective function value of the new food source. The selection process is performed between x_i and v_i by using Deb's Rules (Deb, 2000) which consider constraint violations of the obtained wall designs (Karaboga and Akay, 2011). If the penalty value of the new wall design is better than the worst penalty value one in mind, the worst wall design replaced with the new wall design. Otherwise, the worst one remains in mind.

Step 3: All employed bees fulfil their seeking in the vicinity of the hive for the food sources and keep in their mind the information about them. In the algorithm, it means new wall designs obtained. Employed bees share information about the food sources like the amount of nectar and location of the food sources on the dance area. To give an idea to the onlooker bees is determined probability values used in a probabilistic selection based on the information of the food sources. The onlooker bees evaluate the transferred information in proportion the calculated values of the fitness and constraint violations of the food sources. Probability of selection of the food source by the onlooker bees is defined in Eq. (19).

$$p_i = \frac{\text{fitness}_i}{\sum_{j=1}^{NFS} \text{fitness}_j} \quad (19)$$

Step 4: The onlooker bees select the food source area using the information provided by the employed bees in this step. If the produced random number within the range [0,1] is greater than the p_i (Eq. (19)), the onlooker bees produce a new food source like the employed bees by using Eq. (17). The new food source and the old food source are compared, and then the better one is selected by using Deb's rules. This process continues until all onlooker bees complete their search for the food sources.

Step 5: It is checked whether the nectar in a food source is exhausted or not when the employed and the onlooker bees complete the cycles. After abandoned food sources are determined by using the limit parameter, scout bee initializes the searching for a new food source by using Eq. (16). This cycle continues until the current iteration number reaches the maximum iteration number, and then the algorithm terminates.

3. Optimization Analyses and Results

The optimum weights of CRW design given in Fig. 1 and CRWK design given in Fig. 2 were obtained by using the ABC algorithm. In the optimization analyses, sixteen variable soil conditions presented in Table 3 which include two different values for the cohesion of foundation soil, two different values for the angle internal friction of the foundation soil, and four different values for the angle of internal friction of backfill soil were taken into consideration as example wall designs. Except for the three above mentioned soil parameters, the other input parameters have been taken the same for all example wall designs.

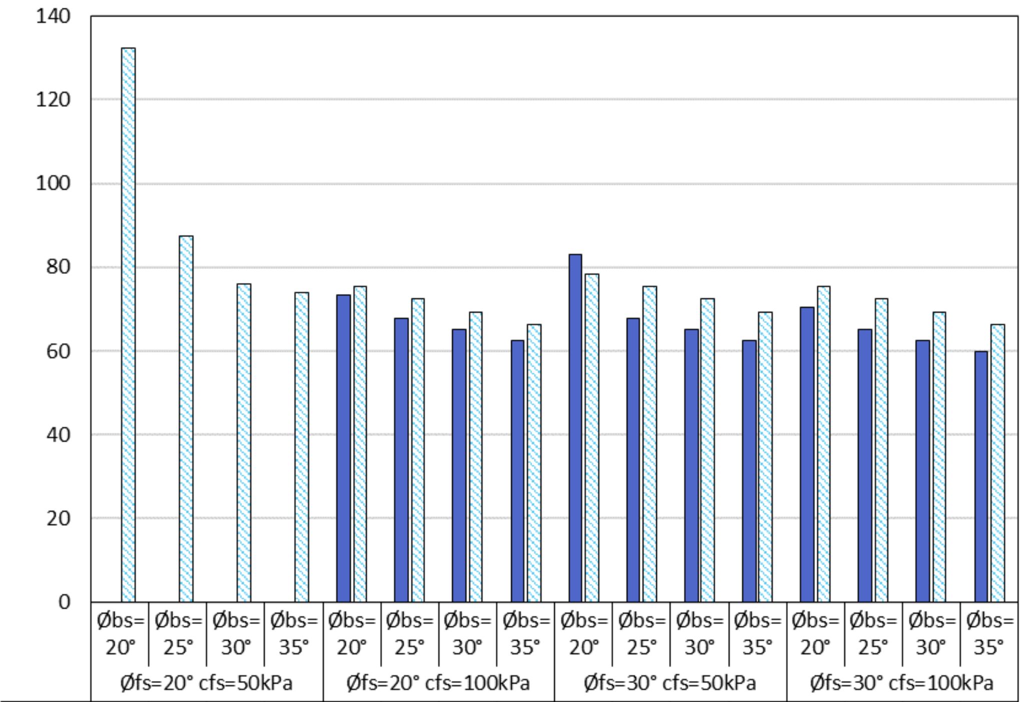
Initial food sources were formed by using Eq. (16) for the design variables demonstrated in Table 1. The values of objective functions by using Eqs. (14) or (15) and penalty values by using design constraints given in Table 2 have been calculated for the wall designs. The ABC algorithm by continuing iterations and cycles achieved the optimum wall design among all possible wall designs, which has the minimum penalty value with the minimum wall weight for current soil condition.

In this study, the algorithm parameters of modification rate, population size, number of the food source, limit and number of maximum iterations were taken as 0.40, 30, 15, NFSxN and 5000, respectively. For each soil condition, the algorithm has been operated in 500 times, by the number of maximum iterations and it also has been observed that the more minimum wall weight cannot be obtained anymore with continuing analysis of the cycles. The optimum wall weights are demonstrated in Fig. 3 for the various soil conditions.

Fig. 3 illustrates that the optimum wall weights decrease with the increasing the angle of internal friction of the backfill soil and the cohesion of the foundation soil for all angle of internal friction of the foundation soil ($\emptyset_{bs}$). The feasible wall the feasible wall design satisfied the design constraints was attained only for CRWK design in the soil condition $\emptyset_{bs}$=20-35°, $\emptyset_{fs}$=20° and c_{fs}=50 kPa. The CRW weight was smaller than the CRWK weight when the values of the cohesion of the foundation soil and the angle of internal friction of the backfill soil were the minima ($Ø_{bs}$=20°, $Ø_{fs}$=30°and c_{fs}=50kPa). There is no conclusion that CRWK designs are less costly than CRW designs for other soil conditions.

Table 3. Parameters of example wall designs.

Input parameter		Unit	Symbol	Value
Height of stem		m	H	6
Surcharge load		kPa	q	20
Backfill slope		°	β	10
Depth of foundation		m	D	0.50
Unit weight of foundation soil		kN/m³	γ_{fs}	19
Unit weight of backfill soil		kN/m³	γ_{bs}	18
Unit weight of concrete		kN/m³	γ_c	25
Cohesion of backfill soil		kPa	c_{bs}	0
Cohesion of foundation soil	Case 1	kPa	c_{fs}	50
	Case 2	kPa	c_{fs}	100
Internal friction angle of foundation soil	Case 1	°	$Ø_{fs}$	20
	Case 2	°	$Ø_{fs}$	30
Internal friction angle of backfill soil	Case 1	°	$Ø_{bs}$	20
	Case 2	°	$Ø_{bs}$	25
	Case 3	°	$Ø_{bs}$	30
	Case 4	°	$Ø_{bs}$	35



	Øfs=20° cfs=50kPa				Øfs=20° cfs=100kPa				Øfs=30° cfs=50kPa				Øfs=30° cfs=100kPa			
	Øbs=20°	Øbs=25°	Øbs=30°	Øbs=35°	Øbs=20°	Øbs=25°	Øbs=30°	Øbs=35°	Øbs=20°	Øbs=25°	Øbs=30°	Øbs=35°	Øbs=20°	Øbs=25°	Øbs=30°	Øbs=35°
CRW	0.00	0.00	0.00	0.00	73.20	67.80	65.10	62.40	83.10	67.80	65.10	62.40	70.50	65.10	62.40	59.70
CRWK	132.2	87.38	76.12	73.81	75.36	72.34	69.31	66.29	78.38	75.36	72.34	69.31	75.36	72.34	69.31	66.36

Fig. 3. Optimum wall weights for the various soil conditions.

In the investigation of the optimum wall designs, lower-upper bounds of the safety factors have been selected to obtain the safe and economical design. Safety factors of sliding, overturning and bearing capacity, for wall design examples are shown in Fig. 4. The lower (F_s (s, min), F_s (o, min), F_s (bc, min)) and the upper (F_s (s, max), F_s (o, max), F_s (bc, max)) bounds of safety factors for different soil conditions have been demonstrated at the same figure with the red lines.

It is evident from Fig. 4 while obtained optimum wall designs were satisfied with the lower bounds of safety factors.

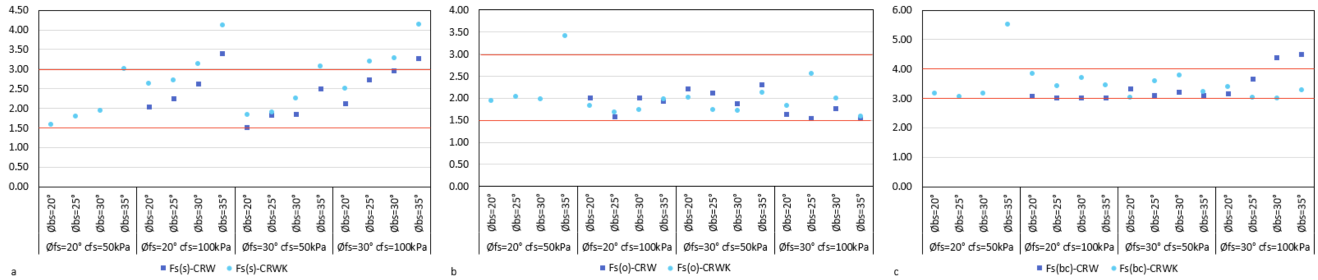


Fig. 4. Safety factors of example wall designs: a) sliding; b) overturning; c) bearing capacity.

4. Conclusions

In this study, the optimum designs of concrete cantilever retaining walls have been investigated using the artificial bee colony algorithm, an effective optimization technique that has been widely applied to engineering problems. The wall dimensions of concrete cantilever retaining wall (CRW) and the concrete cantilever retaining wall with the key (CRWK) satisfied stability conditions have been attained to find the minimum wall weight. According to the result of the optimization analysis, the costs of CRW and CRWK designs have growth when the angle of internal friction of the foundation soil is smaller than 25°. CRWK design is more economical than CRW design just for poor foundation soil. Adding a key to the concrete cantilever retaining wall is insignificant in terms of obtaining the more economical wall designs for quality foundation soil. Consequently, it is observed that the artificial bee colony algorithm can be effectually used in obtaining the optimum concrete retaining wall designs.

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