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Research Article

Use of trigonometric series functions in free vibration analysis of laminated composite beams

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ABSTRACT

In this study, free vibration analysis of layered composite beams is performed by using an analytical method based on trigonometric series. Based on the first-order shear deformation beam theory, the governing equations are derived from the Lagrange's equations. Appropriate trigonometric series functions are selected to satisfy the end conditions of the beam. Navier-type solution is used to obtain natural frequencies. Natural frequencies are calculated for different end conditions and lamina stacking. It was seen that the slenderness, E_{11}/E_{22} and fiber angle have a significant effect on natural frequency. The results of the study are quite compatible with the literature.

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1. Introduction

Laminated composites, which have become one of the important subjects of today, are light and corrosion resistant and have high strength. Commonly used laminated composites are used in beams, columns and plates that are structural elements, thus, it is quite essential to know and understand static and dynamic behavior of such structures.

There is an extensive research in the literature on the laminated composite beams. In these works, different analytical and numerical techniques were used. Reddy (1997) gave analytical and numerical solution procedures for bending, buckling and free vibration problems of laminated composite plates and beams considering the different lamination theories. In laminated beams, effect of shear deformation is highly important. The first-order shear deformation theory (FSDT) was thus developed to include the effect of shear. In this theory, a constant transverse shear strain through-the-thickness was assumed, and a shear correction factor must be used. However, FSDT is widely used in the analysis of laminated composite beams (Yuan and Miller, 1989; Teboub and Hajela, 1995; Banerjee, 1998; Chakraborty et al., 2002; Goyal and Kapania, 2007; Jafari-Talookolaei et al., 2012; Kahya, 2012).

The number of studies using higher-order theory is quite high in the literature. Song and Waas (1997) used the simple higher-order theory that assumes the cubic variation for the displacement field through the thickness in buckling and free vibration analyses of stepped laminated composite beams. Kant et al. (1998) presented analytical solution to the natural frequency analysis of composite and sandwich beams based on higher order refined theory. Karama et al. (1998) proposed a new laminated composite beam model based on the discrete layer theory for bending, buckling and free vibration problems of thin and thick beams. Aydogdu (2005) studied the vibration of cross-ply laminated beams subjected to different sets of boundary conditions by Ritz method is based on a three-degree-of-freedom shear deformable beam theory. Zhen and Wanji (2008) gave analytical solutions to vibration and stability problems of laminated composite and soft-core sandwich beams according to several displacement-based theories. Li et al. (2014) presented the free vibration analyses of laminated composite beams using refined higher-order shear deformation theory. Mantari and Canales (2016) provided an analytical solution for buckling and free vibration analysis of laminated beams using a refined and generalized shear deformation theory involving thickness

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expansion. Nguyen et al. (2017) developed a new trigonometric-series solution based on a higher-order theory for analysis of composite beams with arbitrary lay-ups. Kahya and Turan (2018) presented a higher-order finite element for free vibration and buckling of laminated composite and sandwich beams. Kahya et al. (2019) presented free vibration analysis of laminated composite beams including open transverse cracks by using a shear-deformable thirteen degrees-of-freedom finite element model.

This study presents an analytical method for free vibration of laminated composite beams. Free vibration analysis of layered composite beams is performed by using an analytical method based on trigonometric series. Based on the first-order shear deformation beam theory, the governing equations are derived from the Lagrange's equations. Natural frequencies are calculated for different end conditions and lamina stacking. The results of the study are quite compatible with the literature.

2. Material and Method

Consider a laminated beam as shown in Fig. 1. The displacement field of first-order shear deformation theory is given by:

$$u(x,z,t)=u^0(x,t)-z\phi^0(x,t),\quad w(x,z,t)=w^0(x,t) \tag{1}$$

where u^0, w^0 and ϕ^0 are the axial and transversal displacements, and cross-sectional rotation, respectively. t is time. The strain-displacement relations are given by

$$\varepsilon_{xx}=u^0_{,x}-z\phi^0_{,x},\qquad \gamma_{xz}=w^0_{,x}-\phi^0 \tag{2}$$

where ε_{xx} and γ_{xz} are the normal and shear strains, respectively. $(\cdot)_{,x}$ denotes the derivative with respect to x. The constitutive relations for an orthotropic ply configuration are given by

$$\sigma_{xx}=\bar{C}_{11}\varepsilon_{xx},\quad \tau_{xz}=K\bar{C}_{55}\gamma_{xz} \tag{3}$$

where σ_{xx} and τ_{xz} are the normal and shear stresses, respectively. K is shear correction factor which is taken as 5/6 for a rectangular cross-section. $\bar{C}_{11}$ and $\bar{C}_{55}$ are the transformed material constants which are given by

$$\bar{C}_{11}=C_{11}\cos^4\theta+2(C_{12}+2C_{66})\cos^2\theta\sin^2\theta+C_{22}\sin^4\theta,\quad \bar{C}_{55}=C_{55}\cos^2\theta+C_{44}\sin^2\theta \tag{4}$$

where θ is the fiber angle measured from the positive x-axis in counter clockwise direction. C_{ij} terms are

$$C_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}},\quad C_{12}=\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}},\quad C_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}},\quad C_{44}=G_{23},\quad C_{55}=G_{13},\quad C_{66}=G_{12} \tag{5}$$

where E_{ij} and G_{ij} denote Young's and shear modulus, respectively, ν_{ij} is Poisson's ratio.

The governing equations of motion can be obtained by Lagrange's equations which is given by

$$\frac{d}{dt}\left(\frac{\partial\Pi}{\partial\dot{q}_i}\right)-\frac{\partial\Pi}{\partial q_i}=0 \tag{6}$$

where $\Pi=T-(U+V)$ is the Langragian functional, q_i denotes the generalized coordinates corresponding to nodal displacements. The strain and kinetic energy of the beam can be given by

$$\begin{aligned}U&=\frac{1}{2}\int_0^L\int_A(\sigma_{xx}\varepsilon_{xx}+\tau_{xz}\gamma_{xz})dAdx=\frac{1}{2}\int_0^L\Big\{A_0\left(u^0_{,x}\right)^2-2A_1u^0_{,x}\phi^0_{,x}\\&+A_2\left(\phi^0_{,x}\right)^2+B_0\left(\left(\phi^0\right)^2-2\phi^0w^0_{,x}+\left(w^0_{,x}\right)^2\right)\Big\}dx\\T&=\frac{1}{2}\int_0^L\int_A\rho(z)(\dot{u}^2+\dot{w}^2)dAdx=\frac{1}{2}\int_0^L\Big\{I_0\left(\dot{u}^0\right)^2\\&-2I_1\dot{u}^0\dot{\phi}^0+I_2\left(\dot{\phi}^0\right)^2+I_0\left(\dot{w}^0\right)^2\Big\}dx\end{aligned} \tag{7}$$

where

$$\begin{aligned}A_n&=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}b\bar{C}_{11}^{(k)}z^n dx\quad(n=0,1,2),\\B_0&=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}bK\bar{C}_{55}^{(k)}dx,\\I_n&=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}b\rho^{(k)}z^n dx\quad(n=0,1,2)\end{aligned} \tag{8}$$

where k is the layer number.

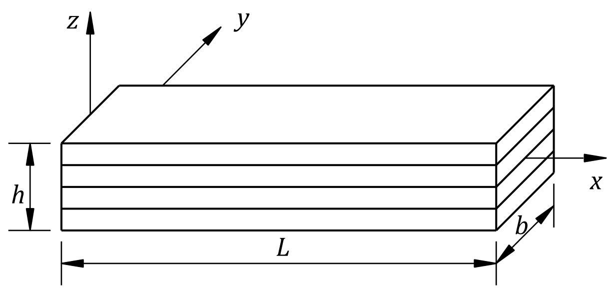

Fig. 1. Geometry and dimensions of the laminated composite beam and the co-ordinate system.

The work done by axial compressive force P_0 acting on the beam at its ends can be given by

$$V=\frac{1}{2}\int_0^L P_0\left(w^0_{,x}\right)^2dx \tag{9}$$

Assume the solutions to $u^0(x,t)$, $w^0(x,t)$,and $\phi^0(x,t)$ as:

$$\begin{aligned}u^0(x,t)&=\sum_{i=1}^{m}\varphi_i(x)u_i(t),\\w^0(x,t)&=\sum_{i=1}^{m}\psi_i(x)w_i(t),\\\phi^0(x,t)&=\sum_{i=1}^{m}\theta_i(x)\phi_i(t)\end{aligned} \tag{10}$$

where $u_i(t)$, $w_i(t)$ and $\phi_i(t)$ are the generalized nodal displacements, and $\varphi_i(x)$, $\psi_i(x)$ and $\theta_i(x)$ are trigonometric functions varying depending on the end conditions of the beams, and m is the number of trigonometric series. In Table 1, trigonometric functions selected to provide support conditions for the beams considered in the study are given. Simply-supported (S-S), clamped-clamped (C-C) and clamped-free (C-F) end conditions are considered for different laminated composite beams.

If the work and energy expressions are written in the Lagrange equation, taking into account the trigonometric functions given in Table 1, the equation of motion for the L length beam is given by

$$\mathbf{M\ddot{X}} + (\mathbf{K_e} - P_0\mathbf{K_g})\mathbf{X} = \mathbf{F} \tag{11}$$

where **M**, $\mathbf{K_e}$, $\mathbf{K_g}$ and **F** are the global mass, stiffness, geometric stiffness matrices and load vector, respectively.

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{M}_{13} \\ \mathbf{M}_{12}^T & \mathbf{M}_{22} & \mathbf{M}_{23} \\ \mathbf{M}_{13}^T & \mathbf{M}_{23}^T & \mathbf{M}_{33} \end{bmatrix},\quad \mathbf{K_e} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \mathbf{K}_{13} \\ \mathbf{K}_{12}^T & \mathbf{K}_{22} & \mathbf{K}_{23} \\ \mathbf{K}_{13}^T & \mathbf{K}_{23}^T & \mathbf{K}_{33} \end{bmatrix},\quad \mathbf{K_g} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},\quad \mathbf{F} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{F}_2 \\ \mathbf{0} \end{Bmatrix} \tag{12}$$

where Eq. (12) for each term is clearly defined in the Appendix. For free vibration of the beam without axial loading, ignoring $\mathbf{K_g}$ matrix and assuming $\mathbf{X} = \mathbf{u}e^{i\omega t}$ and **F**=0 in Eq. (11), we have the following eigenvalue problem:

$$(\mathbf{K_e} - \omega^2\mathbf{M})\mathbf{u} = \mathbf{0} \tag{13}$$

where ω denotes the natural frequencies of the beam. The natural frequencies of the beam can be obtained by non-trivial solutions of Eq. (13).

3. Results

In this section, numerical results of free vibration analysis of laminated composite beams with various boundary conditions are presented. Analytical results were obtained with the help of a program written in MATLAB. Simply-supported (S-S), clamped-clamped (C-C) and clamped-free (C-F) end conditions are considered for different laminated composite beams. Laminates are supposed to have equal thicknesses and made of the same orthotropic materials whose properties are followed:

Material I (Nguyen et al., 2017):
E_{11}/E_{22}= open, $G_{12} = G_{13} = 0.6E_{22}$, $G_{23} = 0.5E_{22}$, $\nu_{12} = 0.25$
Material II (Nguyen et al., 2017):
E_{11}/E_{22}= open, $G_{12} = G_{13} = 0.5E_{22}$, $G_{23} = 0.2E_{22}$, $\nu_{12} = 0.25$
Material III (Nguyen et al., 2017):
E_{11}= 144.8GPa, E_{22} = 9.65GPa, G_{12}= G_{13} = 4.14GPa, G_{23} = 3.45GPa, $\nu_{12} = 0.3$, ρ = 1389kg/m^3

Table 1. Trigonometric functions.

Boundary Conditions	$\varphi_i(x)$	$\psi_i(x)$	$\theta_i(x)$
S-S	$\cos\frac{i\pi}{L}x$	$\sin\frac{i\pi}{L}x$	$\cos\frac{i\pi}{L}x$
C-F	$\sin\frac{(2i-1)\pi}{2L}x$	$1-\cos\frac{(2i-1)\pi}{2L}x$	$\sin\frac{(2i-1)\pi}{2L}x$
C-C	$\sin\frac{2i\pi}{L}x$	$\sin^2\frac{i\pi}{L}x$	$\sin\frac{2i\pi}{L}x$

For convenience, the following normalized terms are used:

$$\text{For Materials I and II} \rightarrow \bar{\omega} = \frac{\omega L^2}{h}\sqrt{\frac{\rho}{E_{22}}},\qquad \text{For Materials III} \rightarrow \bar{\omega} = \frac{\omega L^2}{h}\sqrt{\frac{\rho}{E_{11}}} \tag{14}$$

In Table 2, the number of terms to be used in the analytical solution was investigated. As can be seen, sufficient accuracy is obtained with 10 terms.

The normalized fundamental frequencies are presented for cross-ply laminated beams in Table 3. The results of higher-order shear deformation theory given by Mantari and Canales (2016) and Nguyen et al. (2017) are assumed. As seen, the results are in good agreement. As the slenderness (L/h) increases, the frequencies increase, too. Here, we again observed that the agreement between the results is good for symmetrical lay-ups. However, the difference between the results becomes greater for thicker beams with unsymmetrical lay-ups.

Table 4 shows the normalized fundamental frequency for symmetric (0/θ/0) and unsymmetric (0/±θ/0) composite beams with various boundary conditions. As the fiber angle (θ) increases, the normalized natural frequencies decrease. The analytical results are consistent with the literature.

Table 2. Normalized fundamental frequencies of laminated composite beams with various boundary conditions according to the number of terms to be used in analytical solution.

Beams	m	Boundary Conditions		
		S-S	C-C	C-F
Material III (0/90/90/0) L/h = 15	2	2.5023	4.7584	0.9304
	4	2.5023	4.6798	0.9269
	6	2.5023	4.6524	0.9259
	8	2.5023	4.6384	0.9255
	10	2.5023	4.6298	0.9252
	12	2.5023	4.6240	0.9252
	16	2.5023	4.6167	0.9252
Material I (0/90) L/h = 10 E_{11}/E_{22} = 40	2	6.883	13.483	2.545
	4	6.883	13.286	2.536
	6	6.883	13.219	2.534
	8	6.883	13.185	2.533
	10	6.883	13.164	2.532
	12	6.883	13.150	2.532
	16	6.883	13.132	2.532

Table 3. Normalized fundamental frequencies of (0/90/0) and (0/90) composite beams (Material I, E_{11}/E_{22}= 40).

B.C.	Beams	Theory	L/h				
			5	10	20	30	50
S-S	(0/90/0)	Present	9.205	13.661	16.355	17.056	17.452
		Nguyen et al. (2017)	9.208	13.614	16.338	17.055	17.462
		Mantari and Canales (2016)	9.208	13.610	-	-	-
	(0/90)	Present	5.953	6.883	7.201	7.265	7.300
		Nguyen et al. (2017)	6.128	6.945	7.219	7.274	7.302
		Mantari and Canales (2016)	6.109	6.913	-	-	-
C-F	(0/90/0)	Present	4.182	5.499	6.070	6.196	6.263
		Nguyen et al. (2017)	4.234	5.498	6.070	6.198	6.267
		Mantari and Canales (2016)	4.221	5.490	-	-	-
	(0/90)	Present	2.342	2.532	2.588	2.599	2.604
		Nguyen et al. (2017)	2.383	2.543	2.591	2.600	2.605
		Mantari and Canales (2016)	2.375	2.532	-	-	-
C-C	(0/90/0)	Present	10.621	19.328	29.709	34.332	37.708
		Nguyen et al. (2017)	11.607	19.728	29.695	34.268	37.679
		Mantari and Canales (2016)	11.486	19.652	-	-	-
	(0/90)	Present	9.069	13.164	15.489	16.073	16.399
		Nguyen et al. (2017)	10.027	13.670	15.661	16.154	16.429
		Mantari and Canales (2016)	9.974	13.628	-	-	-

Table 4. Normalized fundamental frequency for symmetric and unsymmetric composite beams with various boundary conditions (Material I, E_{11} / E_{22} = 40).

L / h	Beams	S-S		C-F	
		Present	Mantari and Canales (2016)	Present	Mantari and Canales (2016)
5	(0/30/0)	9.3770	9.4651	4.2540	4.3218
	(0/45/0)	9.3142	9.3801	4.2264	4.2855
	(0/60/0)	9.2570	9.2946	4.2029	4.2519
	(0/30/-30/0)	9.3029	9.4194	4.2096	4.2821
	(0/45/-45/0)	9.1779	9.2928	4.1464	4.2129
	(0/60/-60/0)	9.0727	9.1699	4.0977	4.1548
	(0/90/0/90)	7.8579	7.7822	3.3337	3.3187
10	(0/30/0)	13.8742	13.8823	5.5764	5.5791
	(0/45/0)	13.7875	13.7795	5.5427	5.5412
	(0/60/0)	13.7190	13.6889	5.5184	5.5116
	(0/30/-30/0)	13.6945	13.7306	5.4905	5.4982
	(0/45/-45/0)	13.4676	13.5092	5.3913	5.3987
	(0/60/-60/0)	13.3053	13.3371	5.3248	5.3289
	(0/90/0/90)	10.2652	10.2007	3.9182	3.9002

Fig. 2 shows variation of the normalized fundamental frequencies with the slenderness for $(0/90)_s$ beam with properties of Material II and different end conditions. When the slenderness increases, the normalized fundamental frequencies increase, too. Effect of the slenderness is more pronounced on the results for C-C end conditions. For thicker beams (L / h < 10), we can say that the slenderness is more effective on the results.

Fig. 3 shows the effect of material anisotropy (E_{11}/E_{22}) on the normalized fundamental frequencies for $(0/90)_s$ and (0/90) composite beams with simple ends. As seen, normalized fundamental frequencies increase with increasing E_{11}/E_{22}. Any change in E_{11}/E_{22} is more effective on the results for $(0/90)_s$ beam compared to (0/90) beam. We can also see here, $(0/90)_s$ beam has greater normalized fundamental frequencies than those of (0/90) beam.

Figs. 4-6 show variation of the normalized fundamental frequencies according to fiber angles of beams with different boundary conditions having different layer arrangement are given. As the fiber angle (θ) increases, the normalized natural frequencies decrease. The values obtained for (0 / θ) are greater than those obtained for (θ) and $(\theta / -\theta)_s$. The results are the same for (θ) and $(\theta / -\theta)_s$

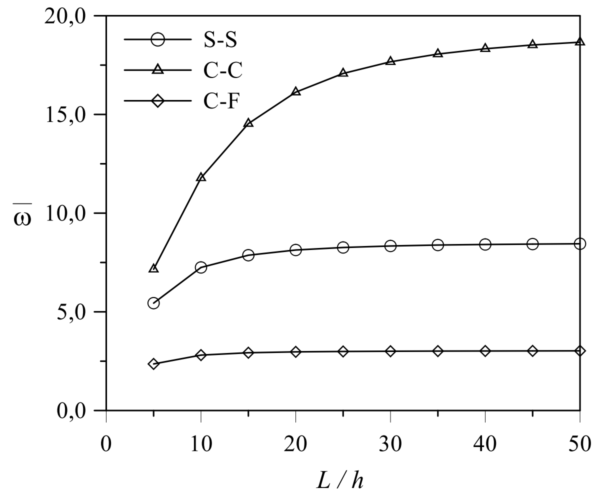

Fig. 2. Variation of the normalized fundamental frequency with span-to-depth ratio for $(0/90)_s$ laminated beam (Material II, E_{11}/E_{22} = 10).

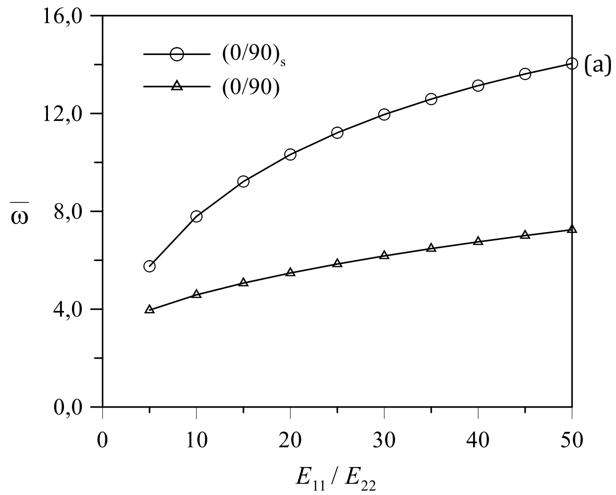

Fig. 3. Variation of the normalized fundamental frequency with E_{11}/E_{22} for (0/90) and $(0/90)_s$ simple beams (Material I, L/h=10).

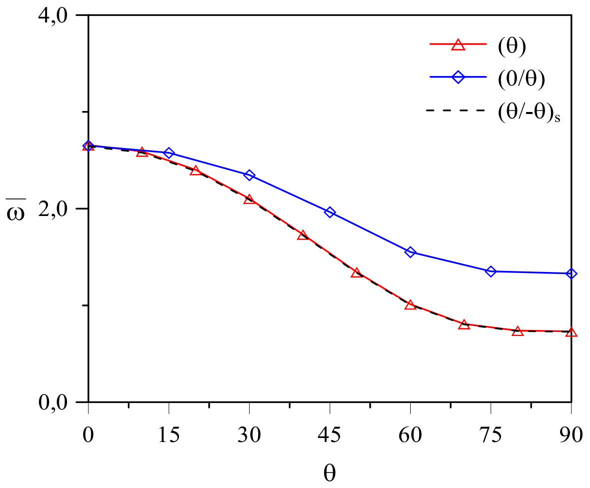

Fig. 4. Variation of the normalized fundamental frequency of simple beams according to fiber angle (θ) (Material III, L / h = 15).

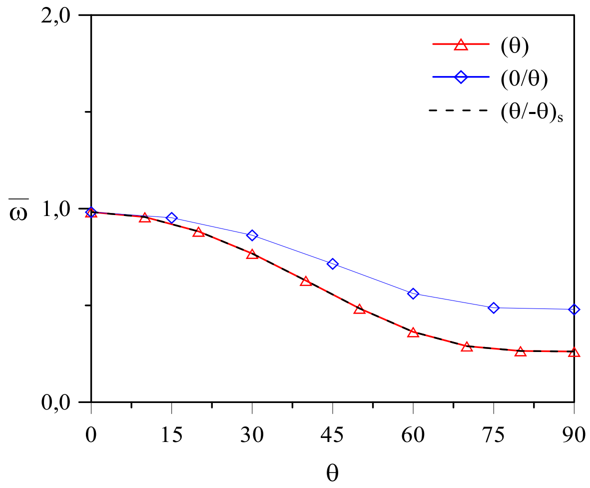

Fig. 5. Variation of the normalized fundamental frequency of cantilever beams according to fiber angle (θ) (Material III, L / h = 15).

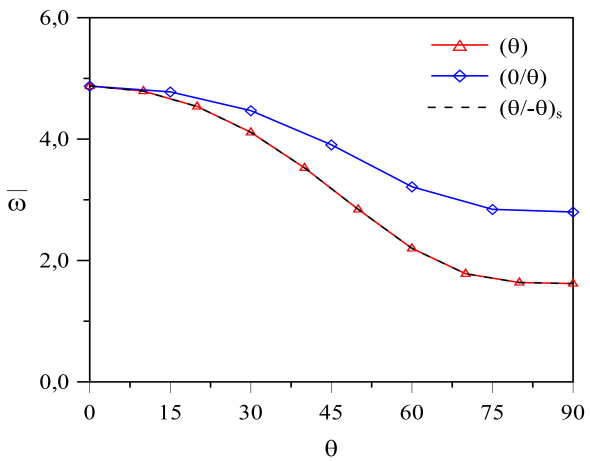

Fig. 6. Variation of the normalized fundamental frequency of fixed-ended beams according to fiber angle (θ) (Material III, L / h = 15).

4. Conclusions

Free vibration analysis of layered composite beams is performed by using an analytical method based on trigonometric series. Based on the first-order shear deformation beam theory, the governing equations are derived from the Lagrange's equations. Appropriate trigonometric series functions are selected to satisfy the end conditions of the beam. Navier-type solution is used to obtain natural frequencies. Natural frequencies are calculated for different end conditions and lamina stacking. The results of the study are quite compatible with the literature. According to results of the study:

- The slenderness (L/h) increases, the normalized fundamental frequencies increase, too.
- As the fiber angle (θ) increases, the normalized natural frequencies decrease.
- For thicker beams (L / h < 10), we can say that the slenderness is more effective on the results.
- As seen, normalized fundamental frequencies increase with increasing E_{11}/E_{22}.

Appendix A. *Elements of the global mass matrices*

$$\mathbf{M}_{11} = \mathbf{M}_{22} = \begin{bmatrix} \frac{L}{2} I_0 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{L}{2} I_0 \end{bmatrix}_{mxm}$$

$$\mathbf{M}_{12} = \mathbf{M}_{12}^{\mathrm{T}} = \mathbf{M}_{23} = \mathbf{M}_{23}^{\mathrm{T}} = 0$$

$$\mathbf{M}_{13} = \mathbf{M}_{13}^{\mathrm{T}} = \begin{bmatrix} \frac{L}{2} I_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{L}{2} I_1 \end{bmatrix}_{mxm}$$

$$\mathbf{M}_{33} = \begin{bmatrix} \frac{L}{2} I_2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{L}{2} I_2 \end{bmatrix}_{mxm} \tag{A1}$$

Appendix B. *Elements of the global stiffness matrices*

$$\mathbf{K}_{11}=\begin{bmatrix}\frac{1^2\pi^2}{2L}A_0 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{m^2\pi^2}{2L}A_0\end{bmatrix}_{mxm}$$

$$\mathbf{K}_{12}=\mathbf{K}_{12}^{\mathbf{T}}=0$$

$$\mathbf{K}_{13}=\begin{bmatrix}\frac{1^2\pi^2}{2L}A_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{m^2\pi^2}{2L}A_1\end{bmatrix}_{mxm}$$

$$\mathbf{K}_{23}=\mathbf{K}_{23}^{\mathbf{T}}=\begin{bmatrix}\frac{1\pi}{2}B_0 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{m\pi}{2}B_0\end{bmatrix}_{mxm}$$

$$\mathbf{K}_{22}=\begin{bmatrix}\frac{1^2\pi^2}{2L}B_0 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{m^2\pi^2}{2L}B_0\end{bmatrix}_{mxm}$$

$$\mathbf{K}_{33}=\begin{bmatrix}\frac{L}{2}B_0+\frac{1^2\pi^2}{2L}A_2 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{L}{2}B_0+\frac{m^2\pi^2}{2L}A_2\end{bmatrix}_{mxm} \quad \text{(A2)}$$

Appendix C. *Elements of the geometric stiffness matrices*

$$\mathbf{G}_{22}=\begin{bmatrix}\frac{1^2\pi^2}{2L} & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{m^2\pi^2}{2L}\end{bmatrix}_{mxm} \quad \text{(A3)}$$

Appendix D. *Load vector*

$$\mathbf{F}_2=\begin{bmatrix}\frac{Lq}{1\pi}-\frac{Lq\cos(1\pi)}{1\pi}\\ \vdots\\ \frac{Lq}{m\pi}-\frac{Lq\cos(m\pi)}{m\pi}\end{bmatrix}_{mx1} \quad \text{(A3)}$$

Publication Note

This research has previously been presented at the 3rd International Conference on Advanced Engineering Technologies (ICADET'19) held in Bayburt, Turkey, on September 19-21, 2019. Extended version of the research has been submitted to Challenge Journal of Structural Mechanics and has been peer-reviewed prior to the publication.

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