

Original Paper

The Risk-Return Behavior of Real Estate Mezzanine Investment (REMI) – The Singapore Experience

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Abstract

REMI is a new financial instrument for Asia's real estate market offering superior returns than those for the typical commercial bank loans. The resultant risk exposure is relatively high. With recent and robust growth of the Singapore real estate market, there is the fast-growing real estate investment trust market. This paper examines the REMI structure, the measurement and characteristics of its risks and returns via a forward-looking binomial asset tree (BAT) model. Risk neutral pricing probability is adopted to construct the BAT tree. TRs are measured by the probability weighted average returns and discussed under different scenarios. REMI bears more risk than typical commercial bank loans, resulting in higher interest rates than pure equity. Different risk issues focus on two major sources - the financial LTV ratio risk and the real estate and capital markets risk. Empirical analysis involves a rigorous discrete-time forecasting of the market rent and capital value expectations of Singapore's prime office sector, given the conditions and assumptions unique to this market. This paper fulfils the need to close the gap concerning the REMI structure and performance in the steady state, utilizing reliable, authoritative information and data sources.

Keywords

REMI (real estate mezzanine investment), Binomial asset tree (BAT) model, Real estate and capital markets risk, Financial LTV ratio risk, bank's senior debt, Junior debt, Equity owner, Direct real estate asset

1. Introduction

For decades, investors have utilized varying combinations and structures of debt and equity to finance real estate investments. Real estate mezzanine investment (REMI) was introduced in the advanced economies in the early 1990s, when real estate capital became scarce, prompting significant investment opportunities for alternative capital structures. REMI became an important source of capital in Singapore, one of East Asia's rapidly growing economy, for direct commercial real estate acquisitions, development and refinancing. Traditional first mortgage providers had become reluctant to finance projects at loan-to-value (LTV) ratios in excess of 65%. REMI is debt capital that gives the lender (investor) the rights to convert to an ownership in the direct real estate asset if the loan is not paid back in time and in full. It is generally subordinated to a bank's senior and junior debts and is senior only to the equity owner's position in the direct real estate asset. As REMI is provided to a borrower quickly with little due diligence on the part of the investor and with little or no collateral on the physical real estate asset, such REMI is aggressively priced with a substantial spread over a bank's loan rate. The challenge for the REM investor is to price the REMI appropriately on a risk-adjusted return principle, to provide adequate compensation for the risk taken. This paper is prompted by three key motivations:

- REMI is a relatively new financial innovation in Asia. Many issues relating to how it is structured are not rigorously examined. An in-depth examination should throw new light on the REMI over traditional sources of financing.
- Owing to its short history, traditional empirical methods cannot be adopted while modern derivative theory may offer more REMI insights.
- Singapore's relatively stable real estate market albeit its speculative boom periods of 1994-1996 and 2006-2008, provides a good context to examine the REMI risk-return behavior.

As an intermediate debt piece in the capital structure, REMI provides a return exceeding that of senior debt. The increased return comes at the expense of increased risk because REM investors are risk averse. If today's price is below expectation then REM investors should be remunerated for bearing the increased risk. To price REMI general, the expected values need to be adjusted for the REM investor's risk preferences and with discounted rates that vary between investors. However, an individual's risk preference is difficult to quantify. In the complete market and with no arbitrage opportunities, the probabilities of future values can be adjusted once and for all, such that the future values incorporate all the investor's risk premia. The resulting probability distribution denotes the risk-neutral probabilities, whereby every asset can be priced simply by taking its expected payoff (Ho et al., 2003 and 2007). Three pertinent research question can be posed:

- What are the REMI risky factors?
- How will these factors affect the REMI return within the context of the Singapore real estate market?
- What is the risk-adjusted return once the risky factors are taken into consideration?

Hence, this paper thus adopts the discrete-time binomial asset tree model in association with risk

neutral pricing to discuss the *ex-ante* REMI. The empirical treatment involves the rigorous discrete-time forecasting of Singapore's prime office sector rent and capital value, given its market conditions and assumptions. Subsequently, the REMI total returns are discussed under the probability-weighted average cash flow approach. In particular, the series of natural default probabilities are envisaged, corresponding to the respective REMI original interest rates.

2. The Related Literature

Holding period returns (HPRs) for mezzanine loan investment is shown by sensitivity and simulation analyses to generate high double digit expected (Ho & Sing, 2003). Its initial 6- year anticipated return is consistent with the achievable annual rate of HPRs of 25%-40% for mezzanine loan investment in the US commercial mortgage market. Mezzanine loan investment offers high expected risk-adjusted returns a potential investment opportunity in the Singapore office sector. Mezzanine's HPR excess return of 100 basis point relative to the primary debt's HPR, may well be at least 100 basis point to adequately compensate the mezzanine loan investor. Some limitations of the study:

- In the discounted cash flow (DCF) framework for simulating the projected holding period returns, excessive high rates of return are obtained towards the end of the loan period.
- The lack of mezzanine debt "comparables" in Singapore is a significant problem but not too severe.
- Due to the confidential nature of direct real estate asset, firms seem unwilling to disclose their asset underwriting and operational parameters but choose to give a close estimate.
- The study assumes minimal transaction cost that does not take into consideration other costs that may be relevant in the computation of returns. Therefore, actual returns may be even lower as the impact of the other cost is omitted from the analysis.

Nevertheless, it is noteworthy that the Government of Singapore Investment Corporation (GIC) suffers the large write-down of its US\$575 million mezzanine loan investment in 2009, and the additional investment equity loss of US\$100 million (Chan, 2010). Both investments are backed by Manhattan's "Stuyvesant Town and Peter Cooper Village", the 80-acre 11,000-unit mixed use apartment complex on 1st Avenue, between 14th Street and 23rd street in Manhattan. Such a large direct real estate complex, built in the 1940s, is purchased for US\$5.4 billion in 2006 by Tishman Speyer Properties and the private equity powerhouse BlackRock unit. These two owners owe hundreds of millions of US\$ to their tenants by New York's Court of Appeals ruling. Unable to raise rents, these two owners default on their loan and to declare bankruptcy when loan restructuring fails. This large apartment complex plunged by >half to less than US\$2 billion. Therefore, GIC's global real estate portfolio slumps by more than 20 per cent in S\$ terms for the financial year ended on 31 March 2009. However, GIC said in September 2009 that it recovers more than half of those foregoing losses.

The REMI denotes that layer of financing between a company's senior debt and its equity. REMI is a unique debt capital that grants the lender-investor the right(s) to convert that debt capital to an equity

ownership if this debt capital is not paid back on time and in full. Structurally, the REMI is subordinate to the senior debt but that the REMI is senior to equity or common stock. As the REMI is provided to the borrower with limited due diligence on the part of the lender-investor and with little or no collateral, such REMI is aggressively priced with a higher required investment return. The return may be in the form of higher interest rate or equity participation. Compared to equity, REMI may offer the advantages of a lower transaction cost, no management control and a predefined exit arrangement. When the mezzanine investor earns much of its returns that are tied directly to the performance of the borrowing company (instead of through equity ownership), the investor then participates in the success or failure of that company. The returns are limited to the REMI life arrangement. This way, the REMI can eliminate outside ownership and management control issues that often concern entrepreneurs, and that the REMI does not dilute shareholders' equity (Ho et al., 2003 and 2017).

Although there are disparities among REMIs in the capital market, there are four key common characteristics:

- REMI is a junior debt that is subordinate to the senior debt.
- Repayment is a bullet type, i.e., the loan principal is repaid at maturity.
- Owing to subordination, REMI risk is higher than that of senior loan. Therefore, the REM investor demands a higher yield, compared to the senior debt yield.
- REMI has an inherent yield that includes a cash interest, which is higher than that of the senior debt cash interest. REMI's cash interest can be a fixed or floating rate. Besides the cash interest, the REM investment yield consists of an equity component. Such an equity component grants the REM investor the right(s) to take over the direct real estate asset from the original owner, if and only if the REMI interest is not or fully paid up.

Institutional and private investors find REMIs to be relatively secure investment vehicles because of the privilege of having a first call or priority position over the borrower and the equity investor (Ho & Sing, 2003). From the investor's perspective, the REM investor is preferred to the equity investor because if the borrower defaults, then the REM investor has the ability to foreclose and pay off the first mortgagee, and so owns the direct real estate asset for a lower transaction cost. The REM investor can achieve higher returns that are adjusted for its high risk. From the borrower's perspective, the REM debt capital is more flexible than bank debt, and that such debt capital is less expensive and dilutive than equity. However, private REMI securities are the lowest ranking debt obligation in a borrower's capital structure and that such REM securities contain a very loose covenant package. REMI securities can be used by a borrower to achieve higher gearing (i.e., LTV ratio) levels and returns on the equity structure.

There are different forms of REM investment with different functions. On subordinated debt, the most straightforward case, the REM investor provides a subordinated debt to the direct real estate asset owner. The investor usually receives a fixed-income yield for operational, fully leased direct real estate assets that generate adequate cash flow to service a mortgage, and that provide a return to the equity

owner. Sponsors seek REMI to leverage their returns or limit their at-risk equity.

On subordinated debt with delayed payment, interest payments on private REMI securities involve a cash-pay portion and a pay-in-kind (PIK) portion. The total stated interest rate return usually ranges between 14% and 16%, with the cash-pay portion generally ranging between 12% and 14% while the remainder of the interest portion is in the PIK. Such an investment structure is arranged by REM borrowers, who do not want to disburse cash flow during the original real estate development life-cycle stage.

On the subordinated debt with equity warrants, the equity kicker is a contingent common equity by way of warrants or a conversion option, to which registration rights are typically attached. Warrants are the most common form of the equity component of a REMI issue. The exercise price of the warrant is nominal or at least substantially below the market value of the borrower-company's common stock. The warrant holds some value that is at least equal to the difference between the market value of the common stock and the exercise price. Such warrants have at least a ten-year term each and represent a minority stake to the issuer. The REM investor may require a "put" option on the warrant and on any common stock purchased with the warrant. The equity kicker is adopted in real estate development projects in the (pre) construction stage, with well-developed plans and budgets for development and subsequent stabilization through to lease up. Sponsors seek REMI to fund a portion of the construction costs and to leverage their return or to free up equity.

The performance participating junior mortgage of a REMI is used for non-stabilized or value-added direct real estate assets, wherein the cash flows have not stabilized or wherein the direct real estate asset is undervalued for some identifiable reason. Sponsors seek this REMI type to execute the value-add investment strategy in order to enhance cash flows.

On the REMI market, Watkins et al. (2003) provide a comprehensive review. As a financing innovation, REMI emerged in the early 1990's. During the 1980's, a typical real estate deal is financed with a combination of senior debt and equity, as the senior lenders provide a high leveraged mortgage to tax-induced investors, thereby limiting the need for REMI. Primary lenders do not desire junior mortgages because a junior mortgagee is likely to raise legal obstacles to the senior lender's remedies in the event of default. This REMI use has no claim on the underlying direct real estate asset but secured through a pledge by the borrowers for their equity.

In the early 1990's, many senior debt holders experienced difficulties in foreclosing mortgaged direct real estate assets that are also subject to a junior mortgage. Banks then adopt a more conservative approach to lending while the senior debtors are only willing to provide loans up to a certain loan-to-value (LTV) ratio, with interest rates softening in the last ten years. An increasing gap emerges in the capital market structure between borrowers and traditional lenders. Such gap creates risks for new investments in the form of constrained liquidity while opportunities emerge for investors to earn higher risk-adjusted returns through investment vehicles, designed to exploit the gap. REMI so provides an alternative financing means to raise capital. The REMI market can take the pressure off the

CMBS (commercial mortgage backed securities) issuers, the rating agencies, the B-piece buyers and direct real estate asset. The REMI market can place the mezzanine equity risks with the emerging and appropriate institutions, entering the market.

REMI can be construed to be “a range of risks rather than a vehicle or structure” (Petch, 1997). Table 1 outlines three major types of REMI and the securitized REMI. Each type has different LTV ratios that expose them to different risk factors with different expected returns. Stabilized direct real estate assets are main candidates for the REMI as their cash flows can support a LTV ratio greater than that of the typical senior debt. Two primary situations for mezzanine investment pertain to a buyer, who seeks financing related to acquiring a direct real estate asset while the owner wants to take equity out of his direct real estate asset. In other words, the owners of stabilized direct real estate assets seek the REMI to leverage their returns and to limit their “at-risk” capital (Watkins et al., 2003).

Debt financing ought to be combined with equity to arrive at an optimal financing point, whereby any increase of the debt to equity ratio is considered risky, resulting in a fall in the profitability of the investment. Various models are developed to estimate the optimal point of financing for, e.g., the capital asset pricing model. McDonald (2007) examines the optimal leverage when REMI is available, and he finds that investors may use REMI even if the REMI interest rate exceeds the target after-tax rate of return on equity. Nevertheless, real estate developers and investors have continually used REMIs in order to possibly reach the optimal point of the debt-to-equity ratio. A limit on the loan principal issued is typically imposed by banks and financial institutions to curb any lending amounting to 100% of the loan principal. Then, the investors and developers are required to make up for the shortfall in the required loan principal through secondary financing.

Table 1. Main Types of Real Estate Mezzanine Investment (REMI)

Mezzanine Investment Type	Property Characteristics	Typical Deal Structures	Total Return Expectations*	Key Risk Issues
Stabilized	<ul style="list-style-type: none"> Existing cash flow Limited lease-up risk Minor rehabilitation or repositioning 	<ul style="list-style-type: none"> 70% to 85% LTV piece No participation in cash flow or residual value 3 to 7-year term Exit through refi or mortgage amortization. Cash flow sweep/lockbox 	14% to 18% IRR	<ul style="list-style-type: none"> Severe value decline Magnitude and timing of cash flow Interest rate risk Quality of underwriting Management control
Value – Added	<ul style="list-style-type: none"> Some existing cash flow Moderate rehabilitation, repositioning. Moderate to substantial lease-up, releasing required Completed property will represent 75% to 80% loan to value based upon the total capital structure 	<ul style="list-style-type: none"> 70% to 95% LTC piece 10% to 15% interest rate, with participation in cash flow and/or residual value 18 month to 3-year term Exit through refi or sale Value creation should allow return of 100% of capital through refi. 	18% to 25% IRR	<ul style="list-style-type: none"> Severe value decline Magnitude and timing of cash flow Interest rate risk Quality of underwriting Exit timing Management control
Development	<ul style="list-style-type: none"> No existing cash flow To-be-built property Completed property will represent 75% - 80% loan to value based upon the total capital structure 	<ul style="list-style-type: none"> 70% to 95% LTC piece Participation in cash flow and residual value 3 year term Exit through refi, sale, or “presale” Value creation should allow return of 100% of capital through refi. 	20% to 30% IRR	<ul style="list-style-type: none"> Significant value decline Magnitude and timing of cash flows Financial risk Quality of underwriting Exit timing Development risk Construction risk Management control

Securitized	<ul style="list-style-type: none"> • Mortgages securitized by a pool of properties. • Existing cash flow • Stabilized underlying assets 	<ul style="list-style-type: none"> • 70% to 75% LTV tranche of CMBBS • 10-year term 	20% to 25% IRR	<ul style="list-style-type: none"> • Severe value decline • Financial risk • Quality of underwriting • Management quality • Cross-defaulted first loss • Management Control
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*Returns presume leverage at the property level but not at the “fund” or investor level. Returns are net of 1% asset management fee and 20% of profits typically paid to sponsor.

Source: The Muldavin Company.

Authors, 2019.

2.1 REMI Default and Remedy

REMI has the priority of cash flows in between the first mortgage lenders and the equity owners. In the event of borrower default, REM investors have the option to assume the first mortgage obligation or alternatively, the REM investors can choose to walk away from the bad investment without obligation. There are usually three REMI scenarios outlined herewith:

- Scenario 1. If the cash flow after the REMI interest is positive, implying that the NOI (net operating income) is enough to cover both the interest of the senior loan and the REMI. The REM investor collects the deemed interest plus the principal at the end of the REMI’s term.
- Scenario 2. If the cash flow after the REMI interest is negative but that the cash flow after the senior loan interest is positive, the implication is that the REMI is in default while the associated senior loan is safe. In “Scenario 2”, the REM investor takes over the direct real estate asset, and the cash flow to the REM investor is then that cash flow after netting off the senior loan interest quantum, but adding on the residual capital value after deducting the senior loan at the end of the REMI’s loan term.
- Scenario 3. If the cash flow after the senior loan interest is negative, implying that the senior loan and the REMI are in default. In “Scenario 3”, the direct real estate asset is liquidated and the REM investor gets back the residual value of the direct real estate asset, after deducting the associated senior loan quantum. If the capital value of the direct real estate asset under “Scenario 3” are even lower than the senior loan principal, then the REM investor gets nothing.

In practice, there is an inter-creditor agreement between the senior mortgage lender and the REM investor, with the threshold issue relating to the REM investor’s ability to realize its collateral. It is therefore that ability to take over the borrower’s position and to become the owner of the direct real estate asset. The REMI’s success or failure may well depend upon the terms of the inter-creditor agreement with the mortgage lender, because the REMI ultimately has the mere right to step into the shoes of the borrower in the event of problems. Typical provisions can those outlined below. In a typical REMI structure, the mortgage (senior) borrower is a bankruptcy-remote single-purpose entity (SPE), in the form of a partnership or a limited liability company, and with the following key features:

- Notification of non-payment or default on the first mortgage - the senior lender must give notice to the REM investor of any default under the senior loan.
- The right to cure any default on the first mortgage - the REM investor wants to protect itself by taking over the direct real estate asset, and by not allowing the senior lender to foreclose.

- The senior lender takes no action if the borrower defaults under the REMI i.e. there have been no cross-default provision in the senior loan terms.

2.2 The REMI Key Market-Wide Risks

REMI are similar to those found in other real estate investments but that they incorporate debt and equity risk characteristics, depending on the particular REMI type and structure (Ballard & Muldavin, 2000; Watkins et al., 2003). The key market risk factors consist of the unavoidable market-wide real estate and capital markets risk, affecting the REMI return volatility. The capital market risk denotes the risk that capitalization rates increase and that capital values decline, leading to the investors' inability or unwillingness to pay off their financed positions. Real estate market risk denotes the market-wide risk that real estate market conditions change for the worse and that market rents decline, leading to the inability to pay off the in-place interest obligations. It is argued that REM investors are oversimplifying the real estate market dynamics.

In contrast to the early 1990's, real estate markets are in a state of relative supply and demand balance, enabling REM investors to comfortably predict stable or strong real estate market conditions for the next several years (Rosen & Anderson, 1999). Many real estate markets seem to be moving back and forth around their peak and equilibrium positions, as supply seeks to meet growing, changing demand. Enhancing information availability to all real estate market participants should help to avoid any sustained overbuilding in real estate markets. The implication is that the real estate markets are more efficient and less volatile than the situation historically (Mueller, 2000). The impact of a normal economic downturn on real estate markets is likely to be mild (Louargand, 2000).

Other risks are non-market wide like financial risk, and such risks are highly constrained in terms of being hedged or mitigated like the risk on the quality of underwriting and tenant risk. Tenant risk: denotes that risk when tenants fail to make timely rental payment. It is usually mitigated via a tenancy deposit. Risk on the quality of underwriting denotes that risk, controlled through conducting careful direct real estate asset valuation from several independent appraisers. Interest rate risk denotes the risk from rising interest rates, which in turn increases default probability. The interest rate risk is hedged via interest rate derivatives. Financial risk is a highly specific non-market wide risk due to the REMI being inherently levered but the REMI merely forms a small slice of the capital structure (typically between 5% and 20%). Financial risk is subordinate to other financing means such that the full mezzanine principal loss occurs before the first dollar loss occurs to the senior position. The smaller the piece of the capital structure that is represented by the REMI, then the more severe the REM principle loss becomes.

2.3 REMI Pricing

REMI is like any other investment opportunity and before investing, it is essential to understand the expected (*ex ante*) risks and return from asset pricing models. The capital asset pricing models of Sharpe (1964), Linter (1965) and Mosin (1966) envisage the systematic risk, i.e. market-wide risk, is to be reflected in the return premium and is therefore the primary determinant of asset price. Ross (1976) and Roll (1977) criticize the early single factor models while Roll and Ross (1980) provide an

alternative view, with more variables entering the return generating process. While the REMI return expectation is subject to the common factors in the macro economy, the return varies significantly based upon the structure of a particular REMI. Required return rises as the level of lease-up risk increases and that the returns rise as the loan-to-value ratio rises. The required return is influenced by the REMI type and size, the financial strength of the direct real estate asset and the borrower as well as the certainty of the exit strategy. When evaluating a REMI strategy, the investor has to determine whether or not the increased yield(s) justify the commensurate risk(s) (Ballard & Muldavin, 2000).

The REMI success also depends on the manager's ability to identify correctly those situations where the risk of losing the REMI's principal is limited, and where the potential for equity or for the accrued interest appreciation is high. The deal team targets REMIs in smaller companies that may have volatile performance, less experienced management, fewer liquidity options and the need for additional capital. Success for such companies may be subject to factors over which the company's management team has little or no control, including changes in technologies, markets, competition, government regulations and the health of the economy. While the REMI portfolio may have numerous REMIs, the portfolio performance may be adversely affected by the results of a few investments. Additionally, the REMI deal team structures some control on its REMIs through board participation, representation rights and stringent loan documentation. Typically, the REMI deal team is to be a minority shareholder in each company within the REMI portfolio. The deal team is therefore unable to exercise full REMI management control.

2.4 REMI Forward Looking Pricing

The challenge is how to price the REMI to compensate for the risk undertaken by investors? So far, there is virtually no formal valuation model for pricing REMI. We need a forward-looking measure of risks and so examine the REMI. Common *ex ante* approaches include the Monte Carlo risk simulation model, the vector auto regression (VAR) model and the discrete-time binomial asset tree Model. The Monte Carlo risk simulation model first proposed by Metropolis and Ulam (1949), takes into account the distributions and the associated probabilities for the input variables and the model generates a probability distribution of future values. Such simulation provides a range of possibilities for the future outcomes. However, the limitation is that the results are only as good as the input variables, and we need to pre-specify the unique distributions of the deployed input variables. The VAR model is commonly used for forecasting systems with respect to the interrelated time series. The VAR model sidesteps the need for structural modeling by treating every endogenous variable in the system as a function of the lagged values of all the endogenous variables in the system. Advocated by Sims (1980) to be a theory-free method to estimate economic relationships, and similar to Monte Carlo simulation model, the VAR model is limited by its inputs.

Another model that is less impacted by input variables is the discrete time-based binomial asset tree model by Cox et al. (1979). An important assumption is that the probability of each price change follows the risk-neutral probability. By simulating asset price on a "discrete time" basis, the next period

asset value is estimated through multiplying the upward and downward factors with their respective risk-neutral probabilities for the two nodal branches. Being risk neutral implies that investors value risk at a constant value, and that they accept exactly the same interest rate for all assets. However, actual market prices are affected by the willingness to pay for the risk undertaken. Implementing the discrete-time binomial asset tree model with real world probabilities is the resolution (Cox & Rubinstein, 1985; Baz & Strong, 1997). Although such a model avoids the inputs and focuses on the characteristics of the output itself, the drawback is its “discrete-time” basis. Therefore, the ability to forecast an accurate probability of default is limited and is only possible to forecast the “jump point” when the default is likely to happen (Ho, 2007).

3. The Discrete-Time Binomial Asset Tree (DTBAT) Model

The DTBAT model first constructs the office rental-expectation binomial tree from a unique real estate market analysis (REMA) of the Singapore prime office space under “The Data” section of this paper, with the starting nodal rent set at S\$12 per sq ft per month (psfpm). Subsequent quarter’s upward and downward rents are forecasted by multiplying this S\$12 psfpm by the upward and downward factors as shown in Table 2, with the associated risk neutral probabilities. The process is repeated for 16 quarters, assuming a 4-year term for the senior loan and the REMI. Secondly, the office capital value (CV)-expectation binomial tree is constructed in a similar manner with the starting nodal CV of S\$2,200 psf. (See Appendices I & II for details of the two discrete-time binomial asset trees of the prime office market rent and CV in Singapore.)

Then for each node, the net operating income (NOI) is estimated, taking account of the prime office market rent, the assumed LTV ratio and the interest rates. The NOI is compared with the senior loan interest and the REMI interest to see whether or not any default occurs (see Appendices 1, 1A). As afore mentioned in the “REMI *Default and Remedy*” sub-section, the REMI leads to the earlier three “Scenarios”. The probability-weighted average cash flow for each path of the DTBAT model can be modeled. At maturity date, the prime office market CV DTBAT is matched with the respective nodes from the prime office market rental tree, to estimate the last REMI cash flow. Total return is measured as the yield to maturity (YTM) of the weighted-average cash flow. For e.g. if the NOI of the direct real estate portfolio follows Scenario 1 and no default occurs, then the REMI YTM is equal to its interest rate. If any of the default scenarios occur, then the YTM is lower than the interest rate. The default risk is measured by the spread between the YTM and the interest rate YTM.

4. The Data

The steady state Singapore real estate market provides the appropriate context for examining the REMI risk-return behavior. In addition, there are 20 real estate investment trusts (REITs) listed on the Singapore Stock Exchange, which has a total market capitalization of over S\$46 billion. Private real estate property funds that include Morgan Stanley, Goldman Sachs, Macquarie, Lehman Brothers, ING,

AIG and Pacific Star, are actively invested in the Singapore real estate market. With total investment sales of over S\$20 billion and S\$40 billion for 2006 and 2007 respectively, and assuming that 10% of the investments are funded by the REMI market, the market size is large at around S\$2-4 billion.

4.1 Office Real Estate Market Analysis (REMA)

On the office REMA, Figs 1 & 2 depict the quarterly average market rent and CV (capital value) of Singapore's prime office sector from 1993 to 2007. The data is obtained from the international real estate consultancy, DTZ Leong Research Asia, Singapore. The data indicates that during the study period the Singapore prime office sector has experienced several cycles, enabling the estimation of the upward and downward factors and their respective risk neutral probabilities. The data indicates that market rents and CVs tend to move in the same direction, and that such a relationship is problematic under the REMI's default risk. So, when the direct real estate sector goes into a severe downturn, the resulting low cash flow from a direct real estate asset leads to rising defaults for that REMI's asset. The REM investor is prompted to take over the direct real estate asset owner's equity. The severe weakening of the direct real estate asset's CV results in a very low or even zero equity for the REMI (Ho, 2007 and 2017).

Table 2 presents the associated data descriptive statistics. Quarterly average rent has a mean of S\$7.39 psfpm, with a standard deviation of 2.32. To estimate the up and down factors (u and d respectively), the quarterly rental growth factor ($1 + \text{growth rate}$) is divided into 2 groups, i.e. greater or smaller than 1, and then the average of each group is estimated. To normalize the growth factors, the average of each group is divided by the square root of their product. The risk neutral probability is estimated via the number of upward growths versus the downward growth. The estimation method is repeated for the prime office CV utilizing the DTZ Leong quarterly data (see Appendix III details for information).

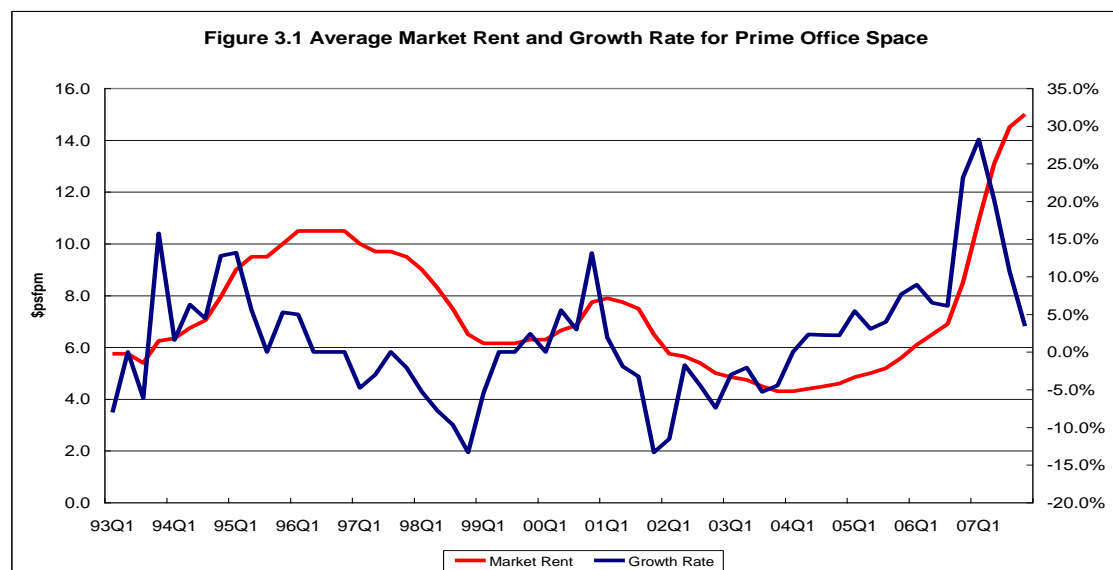


Figure 1. Average Sector Rent & Growth Rate for Singapore's Prime Office

Source: DTZ Leong Research Asia and Authors, 2019.

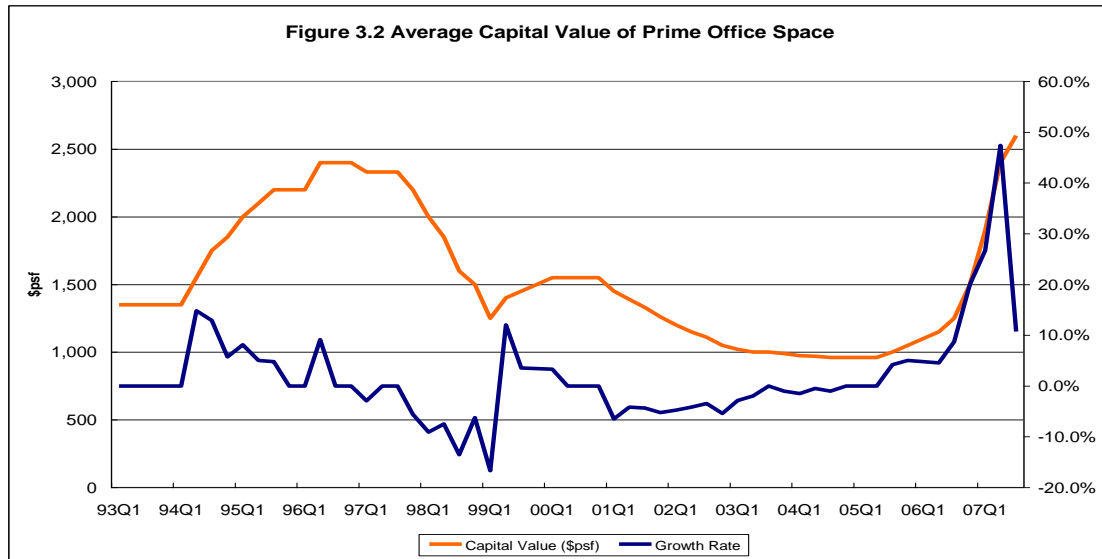


Figure 2. Average Sector Capital Value (CV) for Singapore’s Prime Office

Source: DTZ Leong Research Asia and Authors, 2019.

From Table 2 and for the prime office rent, the upward growth factor is estimated to be 1.07 with a risk neutral probability of 58.3%. The downward growth factor is estimated to be 0.93 with a probability of 46.6%. The implication is that for the 72 quarters of rents, about 53.4% of the quarters experience a rental increase from the previous quarter, and with the average increase of 7.7%. The remaining 46.6% of the quarters experience a rental decrease of rent from the previous quarter, and with the average decrease of 7.2%. The growth factor numbers imply that Singapore’s prime office rents have been highly volatile along a slight upward trend. The prime office CVs average S\$1,565 psm and with the standard deviation of 528. The associated upward growth factor is 1.07 while its upward risk neutral probability is 50.0%. Prime office CV growth conforms to the mean-reversion process but with highly volatile changes quarter-on-quarter.

Table 2. Descriptive Statistics for Prime Office Sector Quarterly Average Rent & CV

Ave Rent (S\$psfpm)	7.39	Ave CV (S\$psf)	1,565
Std dev of Rent	2.32	Std dev of CV	503
Growth Factor (<i>u</i>)	1.07	Growth Factor (<i>u</i>)	1.07
Growth Factor (<i>d</i>)	0.93	Growth Factor (<i>d</i>)	0.93
Risk Neutral Probability (<i>p</i>)	58.3%	Risk Neutral Probability (<i>p</i>)	50.0%

Source: DTZ Leong Research Asia and Authors; 1990-2007, 2019.

On prime office natural vacancy, Grenadier (1995) and Khor (2000) define Singapore’s prime office sector to be an equilibrium level of space inventory, attributable to a matching process between landlord and tenant. Office landlords hold an optimal buffer stock of prime office space to meet future

leasing contingencies. Such a process is akin to the concept of natural unemployment rate in that the natural vacancy rate arises because of imperfect market information, which gives rise to friction in the prime office sector. Central to Khor's findings is that Singapore's prime office sector natural vacancy rate fluctuates around the 10% level. It is found that the prime office sector natural vacancy rate is between 10% and 12%. Other domestic survey findings on market sentiment indicate that the majority of building landlords and real estate consultants hold the common perception that the prime office sector natural vacancy rate is about 10%. Therefore, the natural vacancy of 10% is adopted in this paper and for its three planned "Scenarios".

4.2 Market Assumptions

Once the market rent of the Singapore prime office sector and its natural vacancy rate are available, then the revenue from the direct real estate portfolio is estimated. Operating expenses for a direct real estate asset are estimated to be S\$1 psfpm for service charge, typical of the office sector, while a prevailing 10% property tax is imposed on income in Singapore. After netting off operating expenses from revenue is the net operating income (NOI) (see Appendix 1A). Domestic commercial banks usually require the borrower to hedge interest rate risk for the senior loan, and that the fixed interest rate of 4.0% p.a. typical of the domestic commercial banking sector is paid every quarter. Similarly, the REMI fixed interest rate is paid every quarter.

5. Results and Findings

This section discusses the empirical analysis to enable the discrete-time, binomial asset tree model estimation. In particular and based on the market assumptions, a series of natural default probabilities is envisaged corresponding to the respective REMI interest rates. The impact of LTV ratio pertaining to the senior loan and the REMI on the total return (TR) is examined. In general, a stable spread (of about 1.36%) exists between the REMI's original interest rate and its real TR. At different LTV ratios, such a spread tends to be stable for each different LTV ratio. The spread increases as the senior-loan LTV ratio increases and the spread conforms to a "staircase shape". The results are generally consistent in that the market risk (i.e., real estate market risk and capital market risk), and the financial risk (i.e., the LTV ratio concerning the senior loan and the REMI), are the main risk factors affecting the REMI total return.

5.1 The REMI Discrete Default Probability

Initially a prime office portfolio is assumed that is 65% financed by that senior bank loan, based on a consensus among direct real estate investors and pertaining to an LTV ratio of 60% to 70% for a typical bank loan on a Singapore prime office building. 20% of that bank loan is typically financed by REMI. The interest rate of the senior loan is fixed at 4% p.a. It is observed that the REMI TR in terms of its interest rate ranges from 5.0% to 8.0% p.a. Given the estimated NOI assumptions in the foregoing real estate market analysis (REMA) and market assumptions, the market rent is estimated at which the borrower is to default on the REMI. Then focusing on the lowest boundary of the binomial asset tree

(BAT) of the prime office market rent (see Appendices I & 1A), the default probability for a given REMI interest rate is estimated. However, owing to its “discrete time” nature, a main limitation of the BAT model is that it merely forecast the “discrete” default probability for a range of inputs. Accordingly, for that range of the REMI interest rate of between 5.0% and 8.0% p.a., there are three default probability categories as presented in Table 3 below.

Based on the common senior loan’s LTV ratio specific to the Singapore prime office sector, the default probabilities are deemed to be the natural default probabilities for the domestic REMI. The natural default probability is attributed to the real estate and capital markets risk. In particular, a high REMI interest rate of 7.1% to 8.0% p.a. is required for the high default probability of 17.4%, while a 150 bps lower REMI interest rate is required for the lower default risk at 7.2%. An even lower REMI interest rate, close the senior loan interest rate requires the relatively lowest default probability of 3.0%.

Table 3. Default Probability under Different REMI Interest-Rate Ranges

<u>REMI Interest Rate, p.a.</u>	<u>Default Probability</u>
7.1%-8.0%	17.4%
5.5%-7.0%	7.2%
Below 5.0%	3.0%

Source: Authors, 2019.

5.2 The REMI TR (Total Return)

The REMI TR is measured by the yield to maturity (YTM) of the weighted average cash flow from the BAT paths. Figure 3 below depicts the REMI YTM and with the REMI interest rate as the X-axis. The spread between the REMI interest rate and the REMI TR is plotted, to see how much the REMI YTM drops from the original REMI interest rate, owing to the default risk. From Figure 3, the YTM of REMI increases as the original interest rate increases but it is lower than the original interest rate owing to default risk. One meaningful finding from Figure 3 is that the spread between the REMI interest rate and the REMI YTM, is stable (at around 1.34%-1.38% p.a.) for the different interest rates, owing to default risk. In particular, the other sets of the LTV ratio, concerning the senior loan and REMI, show that the spread is still generally stable.

It is explicit that the REMI default probability is stable once the market risk (i.e., the real estate and capital markets risk) and the financial risk (represented by the LTV ratio for the senior loan and the REMI), are controlled. Similar to the natural default probability, the stable spread between the REMI interest rate and the REMI YTM, are regarded to be the natural default spread that is specific to the Singapore prime office sector. Therefore, under a common structure, the REMI for such a prime office sector is to generate the REMI TR, which is about 1.36% lower than the original REMI interest rate.

5.3 REMI TR with Different Senior Loan LTV Ratios

The REMI TR is next examined in relation to different senior loan LTV ratios. As expected, the REMI TR falls as the senior loan LTV ratio rises, Figure 4 below depicts the REMI YTM as against the different senior loan LTV ratio. The REMI’s 20% LTV ratio and the REMI interest rate of 6.0% p.a. are assumed. Then the REMI TR of 5.8% is very close to the original REMI interest rate of 6.0%, when the senior loan LTV ratio is relatively low at 45%. As the senior-loan LTV ratio rises, the REMI YTM falls. For a senior-loan LTV ratio of 75%, the REMI’s spread between the REMI original interest rate and the REMI YTM rises to over 3.0%. It is clear that the financial risk, owing to the senior loan’s inherent leverage, is a significant factor affecting the REMI TR. Thus, the higher the senior loan inherent leverage, then the higher the financial risk.

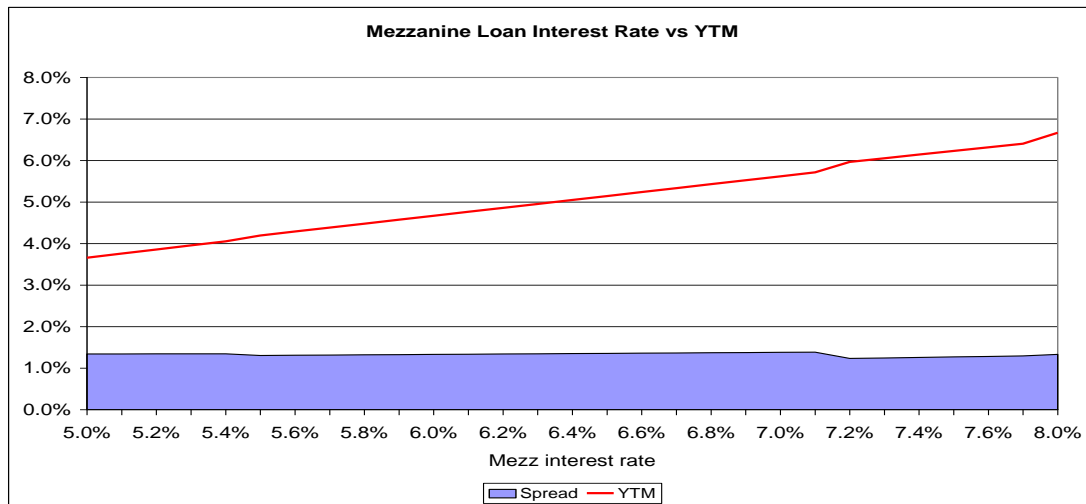


Figure 3. Total Return for Mezzanine Investment with Fixed LTV Ratios

Source: Authors, 2019.

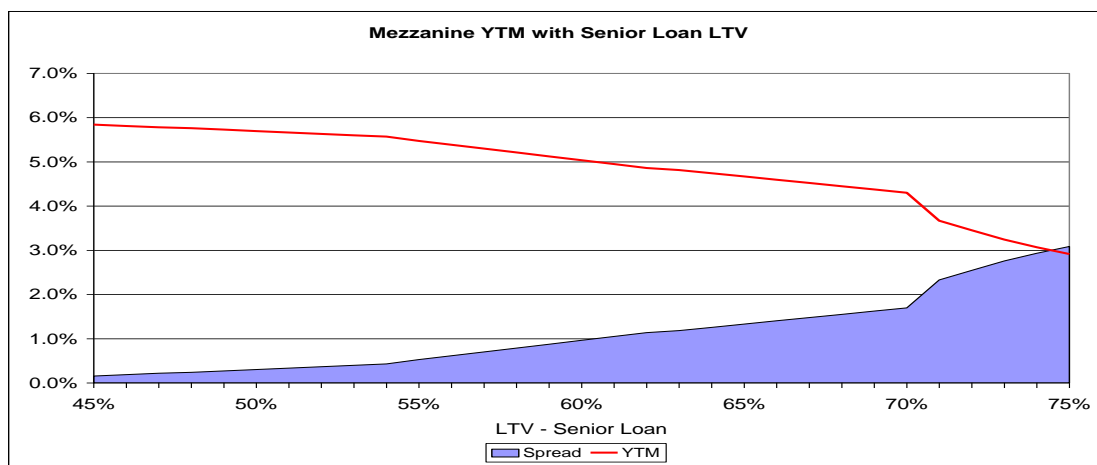


Figure 4. Total Return of Mezzanine with Different Senior Loan LTV

Source: Authors, 2019.

5.4 REMI TRs with Different Senior Loan LTV Ratios and Different REMI Interest Rates

To examine the joint effect of financial risk, owing to the senior loan leverage and the different REMI interest rate on the REMI TR, the results in Figure 5 below are so depicted. Consistent with the foregoing results, the REMI TR as measured by the REMI YTM, falls as the senior loan TV ratio rises. The spread between the REMI interest rate and the REMI YTM is found to change with the different senior-loan LTV ratio (see Figure 6).

Such a spread conforms to a “staircase shape” 3-D (dimensional) plot with roughly the same spread for the different REMI interest rates, given the the senior loan LTV ratio’s narrow range. The spread also increases as the senior loan LTV ratio rises. It is therefore inferred that the financial risk, measured by the existing senior loan LTV ratio, is the main cause of the REMI default probability, given the constant market risk.

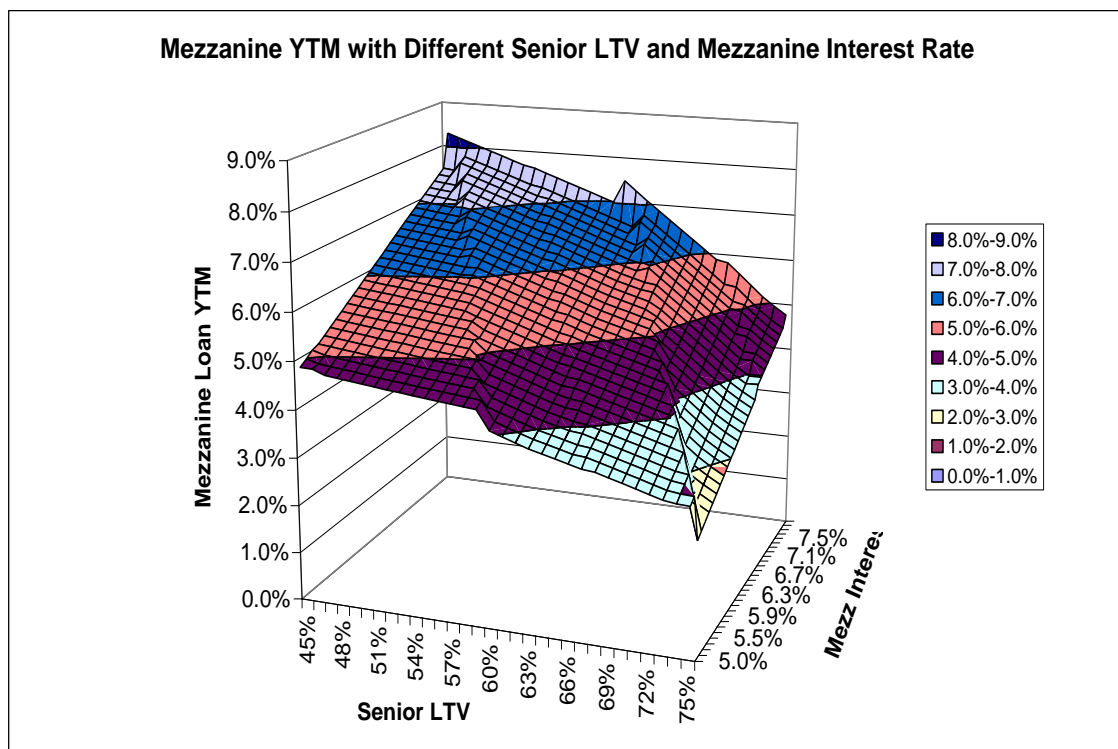


Figure 5. REMI TR with Different Senior Loan LTV Ratios and REMI Interest Rates

Source: Authors, 2019.

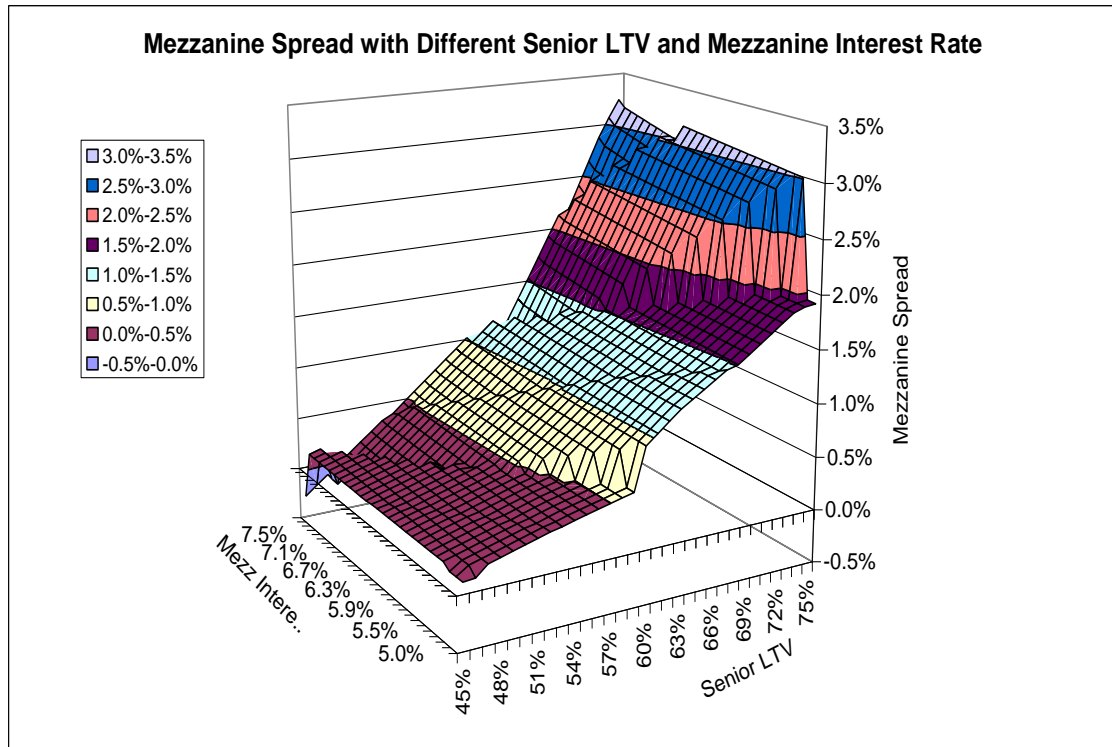


Figure 6. Spread between the REMI Interest Rate and the REMI TR under Different Senior Loan LTV Ratios and REMI Interest Rates

Source: Authors, 2019.

6. Conclusion

REMI is a new financial instrument for the real estate market in Asia although it would provide superior returns than those for usual typical commercial bank loans. The resultant risk exposure becomes relatively high. The Singapore real estate market has experienced robust growth over the past several years, with a fast-growing real estate investment trust (REIT) market and the teeming emergence of private equity fund investments. As a result, this paper is appropriately motivated to examine the risk-return behavior of mezzanine investment.

A discrete-time binomial asset tree model in association with risk neutral probabilities is estimated for the *ex ante examination* of the REMI. The empirical analysis involves a rigorous discrete-time forecasting of the market rent and capital value expectations of the prime office market in Singapore, given the conditions and assumptions unique to this market. Subsequently, the total return (TR) for mezzanine investment is investigated under a probability-weighted average cash flow approach.

In particular, a series of natural default probabilities is envisaged, corresponding to the respective REMI-investment interest rates. The impact of loan-to-value (LTV) ratio pertaining to the senior loan and the REMI on the total return for the mezzanine investment is examined. It is found that a generally stable spread (of about 1.36%) exists between REMI's original interest rate and its real total return. When this paper looks at different LTV ratios, the results show that such a spread tends to be stable for

each different LTV ratio. This spread increases as the senior-loan LTV ratio increases and it follows a “staircase shape” 3-D (3-dimensional) plot. The results are generally consistent in that the market risk, i.e., real estate market risk and capital market risk, and the financial risk, represented by the LTV ratio concerning the senior loan and the mezzanine investment, are the main risk factors that affect the return for REMI.

This paper introduces a unique valuation model that examines the *ex ante* risk-return behavior of REMI specific to Singapore’s prime office market. It introduces a rigorous straightforward model approach easily implemented in industry by adopting risk neutral pricing in a discrete-time binomial asset tree model. The results affirm that when we value a REMI, we need to be merely concerned with the inherent leverage that arises from the existing senior loan. A higher return would therefore be required to compensate for the higher financial risk and real estate market risk on the risk-adjusted return principle. While limited to Singapore, this paper can be extended to include other key cities and their real estate markets in Asia. A cross-city comparative study can be conducted to evaluate and contrast the risk-return behavior of REMI across the Asian region, in particular the key capital cities of interest.

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Appendix I: Binomial Tree for Market Rent

Binomial Tree:	
	Market Rent Capital Value
u	1.0773 1.08055
d	0.928236507 0.92545
p	58% 50%

Quarter

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
																	39.5
																	34.0
																	31.6
																	29.3
																	27.2
																	25.3
																	23.5
																	21.8
																	20.2
																	18.8
																	17.4
																	16.2
																	15.0
																	13.9
																	12.9
																	12.0
																	11.1
																	10.3
																	9.6
																	8.9
																	8.3
																	7.7
																	7.1
																	6.6
																	6.1
																	5.7
																	5.3
																	4.9
																	4.6
																	4.2
																	3.9
																	3.6
																	0.0018
																	0.00031
																	0.00053
																	0.00091
																	0.00155
																	0.00266
																	0.00456
																	0.00782
																	0.01341
																	0.02298
																	0.0394
																	0.06754
																	0.11579
																	0.1985
																	0.34028
																	0.58333
																	1.000
																	0.41667
																	0.17361
																	0.07234
																	0.03014
																	0.01256
																	0.00523
																	0.00218
																	0.00091
																	0.00038
																	0.00016
																	6.6E-05
																	2.7E-05
																	1.1E-05
																	4.8E-06
																	2E-06
																	8.3E-07

LTV	65%	45%	46%	47%	48%	49%	50%	51%	52%	53%	54%	55%	56%	57%	58%	59%	60%
YTM	4.67%	5.84%	5.81%	5.78%	5.76%	5.73%	5.70%	5.66%	5.63%	5.60%	5.57%	5.47%	5.39%	5.30%	5.2%	5.13%	5.04%
Interest Rate	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	1%	1%	1%	1%	1%
Mezz LTV	20%	10.0%	11%	11.0%	12%	12.0%	13%	13.0%	14%	14.0%	15%	15.0%	16%	16.0%	17%	17.0%	18%
YTM	5%	5%	0.05379	0.05375	0.05371	0.05368	0.05364627	0.05361786	0.04962103	0.04916606	0.04887387	0.04858221	0.04830911	0.04805286	0.0478196	0.04758505	0.04737095
		1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%
		40%	43%	46%	49%	52%	56%	59%	63%	67%	71%	75%	79%	84%	89%	94%	100%
Mezz Interest	6%	5.00%	5.10%	5.20%	5.30%	5.40%	5.50%	5.60%	5.70%	5.80%	5.90%	6.00%	6.10%	6.20%	6.30%	6.40%	6.50%
YTM	5%	3.7%	3.8%	3.9%	4.0%	4.1%	4.2%	4.3%	4.4%	4.5%	4.6%	4.7%	4.8%	4.9%	5.0%	5.0%	5.1%
		1.34%	1.34%	1.34%	1.34%	1.35%	1.31%	1.31%	1.32%	1.32%	1.33%	1.33%	1.34%	1.34%	1.35%	1.35%	1.36%
		1.25%	1.28%	1.30%	1.33%	1.35%	1.38%	1.40%	1.43%	1.45%	1.48%	1.50%	1.53%	1.55%	1.58%	1.60%	1.63%
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Interest Mezz		6%	6%	6%	6%	6%	6%	6%	6%	6%	6%	6%	6%	6%	6%	6%	6%
		1	2	3	0	0	0	0	0	0	0	0	0	0	0	0	0
		1	2	3	4	5	6	7	0	0	0	0	0	0	0	0	0
	9.84	7.12															
Default Prob Mezz	3.0%																
Default Prob Senior Loan	0.1%																

Appendix II: Binomial Tree for Capital Value

Quarter

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
																7,599	
															6,508	7,032	6,508
															6,023	6,023	5,574
															5,574	5,158	5,158
															5,158	4,774	4,774
															4,774	4,418	4,418
															4,418	4,089	4,089
															4,089	3,784	3,784
															3,784	3,502	3,502
															3,502	3,241	3,241
															3,241	2,999	2,999
															2,999	2,776	2,776
															2,776	2,569	2,569
															2,569	2,377	2,377
															2,377	2,200	2,200
															2,200	2,036	2,036
															2,036	1,884	1,884
															1,884	1,744	1,744
															1,744	1,614	1,614
															1,614	1,493	1,493
															1,493	1,382	1,382
															1,382	1,279	1,279
															1,279	1,184	1,184
															1,184	1,096	1,096
															1,096	1,014	1,014
															1,014	938	938
															938	868	868
															868	804	804
															804	744	744
															744	688	688
															688	637	637

