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Topological vacuum fluctuation and Dvoretzky's theorem – Mathematical proofs in the context of the dark energy density of the universe*

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Starting from the initial triality of physics, namely mathematical philosophy, transfinite set theory and number theory we drive the inevitability of a topological quantum vacuum fluctuation of spacetime resulting in the fundamental reality of pair creation and annihilation. Subsequently we give a simple but strong mathematical proof of Dvoretzky's marvellous theorem on measure concentration, thus making dark energy and accelerated cosmic expansion not only an astrophysical measurement and observational reality, but also a plausible topological-geometrical fact of a pointless Cantorian actual universe akin to the Penrose fractal tiling space. This space is described accurately via the von Neumann-Conne noncommutative geometry using their golden mean dimensional function and the corresponding bijection of E-infinity theory. The said theory was developed by the authors of the present paper and their group and is based on and starts from the pioneering efforts of the Canadian physicist G. Ord and the French astrophysicist L. Nottale.

Keywords: *Fractal spacetime, E-infinity theory, Quantum vacuum fluctuation, Platonic quantum set theory, Dvoretzky's theorem, Golden mean number system, G. Ord, L. Nottale, Pointless Geometry, Penrose fractal tiling, von Neumann continuous geometry, G. 'tHooft-Weltman-Wilson fractal spacetime, A. Connes noncommutative geometry, E-infinty bijection.*

1. Prelude

Despite the fact that the present authors would like to think of themselves as open minded free thinkers, they are willing to admit that they have a certain bias towards a powerful old classical idea, namely that the Pythagorean theory of cosmic music of numbers [1-7] is the beginning of modern science as documented by E-infinity Cantorian spacetime, which is at the centre of the present work [8-15]. For this reason we labelled this section as 'prelude' in lieu of 'prologue' but this is not the main or only reason for this section in which we would like to stress our scientific conviction that the

best point to start a fundamental theory for the physical universe is neither physics nor cosmology. Controversial or not, it is our considered and deeply seated honest opinion that transfinite set theory [8-12],[14-25] and scientific-mathematical philosophy of "being and nothingness" is the right starting point [25-28] for an a priori reappraisal of centuries old and equally deep seated misconceptions in fundamental modern physics and cosmology [29-50]. To do justice to this point we

*) This paper is dedicated to the memory of three exceptional early pioneers of the subject matter transacted here, namely J. von Neumann, A. Dvoretzky and N. Umov.

need to devote the entire paper to it, which is not our intention at this stage and it will be considered at a later time. Never the less, it is prudent that we give a brief summary of the rationale behind what we just indicated. This may be aptly stated in the following condensed form:

a. There is truly a world of difference logically and physically between first being a zero and being empty or being nothingness [8],[26-27]. Set theory is the most accurate language made by man almost for heaven to make waterproof descriptions of the preceding immensely fundamental notions. In set theory a zero is a zero set, which means a bag called set full of zeros for instance like a bag full of white sand [8-9],[42]. Empty on the other hand is the same bag but without the white sand or even air inside this bag-universe [28-32]. So far so good but nothingness it an entirely different situation, namely when even the bag disappears and then there is really nothing to talk about and there are no answers because there are no questions. We allege that most if not all paradoxes of modern physics and quantum cosmology are the result of confusing these three fundamental notions and while the combined theories of noncommutative geometry [33-45], Penrose tiling [25-50] and E-infinity Cantorian theory has found exact mathematical resolutions to the undue fatal confusion between these three entirely different notions, modern science went generally in a different way and invented instead some ad hoc imprecise notions and laboured to find logic and reality for the puzzles of quantum waves, Aether, state vector reduction and the like which belongs to the tool kit of quantum physics [8-9],[25]. The big exceptions here are the work of von Neumann, Connes, Penrose and more recently Beck [35-39].

b. There are some diverging views about what physics, i.e. physical reality is and how far it is from mathematics. These views range from the difference between theoretical and mathematical physics to the point where some scientists consider that any correct mathematics must be found again in physics including paradoxical decomposition [24],[30]. Here we take the mildly radical view that at the fundamental quantum physics and cosmology level, the demarcation line between physics and pure mathematics is non-existence or at a minimum blurred [24-44].

c. It is a major part of the present authors' work to look at unification and this developed into a mainstream direction, namely the unification of all the fundamental interactions [32-42]. However as time went by we found that our quest for unification evolved seemingly in a natural way to be a unification of art and science with mathematical music at the very beginning and at the very centre of our research and the authors would like to stress how seriously they look upon this superficially surprising and yet on deep reflection, not so surprising outcome of a long time research which took between all the three of them almost five decades of dedicated hard work [2-50].

Last but not least, a note on the references that is more often than not a bare minimum for the background of a paper. To hold to this tradition in the present context would be a drop in the ocean. For this reason we included a relatively large amount of background and supporting books, reviews and papers [1-87]. In particular references [50-87] are not of the utmost essentiality to the main message of the present work, but for a deep understanding they are indispensible and the authors

would strongly recommend consulting the magnificent and monumental work of Penrose [71] whenever a clarification of a general nature is required.

2. Introduction

The present paper has two opposing objectives, namely to be a brief bird's eye view on the vast subject of E-infinity and its relation to high energy physics and quantum cosmology but this is coupled with a second aim of really concentrating on a deep understanding of the maths behind the physics of vacuum fluctuation [9],[46] and present a new proof of one of the most remarkable theorems of measure concentration in high dimensional manifolds due to the outstanding Ukrainian-Israeli mathematician Ayreh Dvoretzky [49]. This theorem sheds a welcome light on cosmic dark energy, which is arguably one of the greatest present mysteries in cosmology if not in all of physical sciences [18-23]. It must be said that one of the most gratifying results of the authors' current research is to be able to assert that dark matter and pure dark energy are realities that exist side by side with the ordinary energy [29],[33],[42] and that this result affirms those of painstaking modern measurements and observations. It also proves the correctness of very early research by the Russian physicist N. Umov [50-51] who appears to have anticipated all these modern results even before the advent of relativity and quantum physics [51].

3. E-infinity Cantorian theory and the observation dependent many dimensions of spacetime

E-infinity spacetime as well known, is an infinite dimensional hierarchal space made entirely of the union and intersections of elementary triadic random Cantor sets. In this sense its dimensional size could be computed as follows [11],[15],[25],[45]

$$V = \sum_{n=o}^{\infty} n\phi^n \tag{1}$$

where $f = \left(\sqrt{5} - 1\right)\Big/2 = 0.618033989$ is the Hausdorff dimension of the famous Mauldin-Williams random triadic Cantor set. Writing equation (1) explicitly it is easy to sum the infinite convergent series and find [11],[15]

$$\begin{aligned} D &= (1)(f) + (2)(f^2) + (3)(f^3) \ldots\ldots \\ &= 4 + f^3 \\ &= 4.2360679\ldots\ldots \end{aligned} \tag{2}$$

More interestingly one finds that (2) may be written in a closed form to mean that it is a centre of gravity, i.e. mean value of a Borel mix as thought of probably for the first time by J.A. Wheeler in his famous bucket of dust analogy [11],[15]. In other words one finds that

$$\begin{aligned} D &= \langle n \rangle \\ &= (1+f)\Big/(1-f) \\ &= 4 + f^3 \end{aligned} \tag{3}$$

where n = 0, 1, 2, 3 …. may be thought of as the topological infinitely many dimensions of the E-infinity space while f^n are the corresponding statistical weights [11],[15]. In this sense $\langle n \rangle$ is an average topological dimension of E-infinity spacetime manifold. We could do something similar to all Hausdorff dimensions of the entire space and arrive at an average Hausdorff dimension and find that it it [11],[15]

$$\begin{aligned} \langle d_c \rangle &= \frac{1}{f(1-f)} \\ &= 4+f^3 \end{aligned} \tag{4}$$

In fact taking any Hausdorff dimension $d_c^{(o)}$ for the elementary Cantor sets we find that it will be singled out to be equal f if and only if $\langle n \rangle \circ \langle d_c \rangle$ [11],[15]. More over we find that the bijection formula of E-infinity theory

$$d_c^{(n)} = \left(1/d_c^{(o)}\right)^{n-1} \tag{5}$$

would give the same value, namely $4+f^3$ if and only if we set $d_c^{(o)} = f = \left(\sqrt{5}-1\right)/2$ and n = 4 , i.e. [11],[15]

$$\begin{aligned} d_c^{(4)} &= \left(1/f\right)^{4-1} \\ &= \left(1/f\right)^3 \\ &= 4+f^3 \end{aligned} \tag{6}$$

In other words for $d_c^{(o)} = f$ E-infinity Cantorian space-time does not only look like Einstein's four dimensional spacetime but it looks four dimensional on all scales, i.e. it is unlike the original D = 4 of A. Einstein in being scale invariant and every so called point in this space is again four dimensional which means it is pointless space as envisaged for maybe the first time in J. von Neumann's continuous geometry [11],[15]. This name continuous geometry may be misleading because the continuity here is referring to the dimensionality meaning it is not only 1, 2, 3, …. but includes non-integer dimensions, which is the hallmark of a fractal geometry [11],[15]. To see the scale invariance of $4+f^3$ nothing could be simpler or clearer than writing it in the continued fraction representation as [11],[15]

$$\begin{aligned} \langle n \rangle &= \langle d \rangle \\ &= 4+\cfrac{1}{4+\cfrac{1}{4.....}} \\ &= 4+f^3 \end{aligned} \tag{7}$$

We conclude this very short section by mentioning the important discovery of an equation very similar to (7) but related not to Einstein's D = 4 but rather to E. Witten's M-theory with D = 11 where it was found that D = 11 could also be made scale invariant, self similar fractal by adding to 11 the Hardy quantum probability of two entangled quantum particles, namely f^5 and finding that [42]

$$\begin{aligned} D(\text{fractal Witten M-theory}) &= \left(1 \big/ f^5 \right) \\ &= 11 + f^5 \\ &= 11 + \cfrac{1}{11 + \cfrac{1}{11 \ldots\ldots}} \end{aligned} \tag{8}$$

The importance of equations (7) and (8) for quantum physics and dark energy research cannot be sufficiently stressed.

4. The pre-quantum particle is the zero set and the pre-quantum wave is the empty set

Now we come to the most important postulate of E-infinity theory in its way to establish an alternative quantum mechanics based on platonic transfinite set theory [52]. In short it was found that the zero set is a miraculous representation of what intuition and experiments may agree upon as a pre-quantum particle while the empty set is what must be regarded in this case as the quantum pre-quantum wave. Topologically and geometrically this is a charmingly convincing picture because by invoking elementary cobordism, an empty set is the surface of a zero set which is quite reminiscent of the pilot wave theory of D. Bohm [53-55]. Let us now regress a little and recall that the zero set follows from A. Connes' dimensional function is fixed by two dimensions, a topological dimension appropriately zero and a second dimension which is a Hausdorff dimension to be expected, namely f. This bi dimension [8-9],[52]

$$D\left(o\right)^{o}\left(o;f\right) \tag{9}$$

is the remarkably simple representation of a pre-quantum particle in our E-infinity theory, ergo the platonic transfinite quantum set theory. Recalling that the empty set is given be dimension minus one $\left(-1\right)$ as per the deductive dimensional theory of K. Menger and P. Urysohn then it was easy to reason that the corresponding bi-dimensional representation of the pre-quantum wave is [8-9],[52]

$$D\left(-1\right)^{o}\left(-1;f^2\right) \tag{10}$$

Now it is our first remarkable fact about the pre-quantum particle and the pre-quantum wave spans the very same space-time average Hausdorff dimension $\left\langle d \right\rangle$. To show this a simple calculation based on the general form of equation (4), namely [8-9],[52]

$$\langle d \rangle = \frac{1}{\left(d_c\right)\left(1 - d_c\right)} \tag{11}$$

shows that setting the zero set pre-quantum particle in (11) leads to [8-9],[52]

$$\begin{aligned} \langle d \rangle_f &= \frac{1}{f\left(1 - f\right)} \\ &= \frac{1}{f\, f^2} \\ &= 4 + f^3 \end{aligned} \tag{12}$$

and setting $d_c = f^2$ in the same equation, the empty set quantum wave leads to the same dimension namely [8-9],[52]

$$\begin{aligned} \langle d \rangle_{f^2} &= \frac{1}{f^2\left(1 - f^2\right)} \\ &= \frac{1}{f^2\, f} \\ &= 4 + f^3 \end{aligned} \tag{13}$$

Now we are able to proceed to derive the mathematical consistency and inevitability of the topological vacuum fluctuations as well as give a new proof of Dvoretzky's theorem. However let us first recall, without much ado, that it was reasoned in many previous publications that not only the zero set Hausdorff dimension f and the corresponding empty set Hausdorff dimension is f^2have these remarkable physical interpretations. In fact this golden mean system goes on in the same vein and shows that the topological Unruh temperature so vital for the Rindler space is given by f^4 [8-9],[31],[46] and one step more, i.e. f^5 is the exact Hardy quantum probability which was confirmed experimentally and finally for now, f^6 turned out to be the Barbero-Immirzi parameter needed to remove some subtle conflicts between superstrings theory and loop quantum gravity theory [31]. Considering all of the above we hope it becomes understandable that our group regard the golden mean number system as the lingua franca of nature.

5. The topological quantum vacuum fluctuation

Quantum vacuum fluctuation is basically a mysterious but fundamental theme in the foundation of high energy physics, quantum field theory and quantum cosmology and constitutes a large part of the research connected to black hole physics and more [31]. In this section we will attempt, using simple topological arguments based on what we have explained so far to take the mystery of a quantum physical concept which seems surreal from a classical view point used to think that nothing like pair creation could come out of nothing such as spacetime vacuum and replace this with the crystal clear understanding of zero, empty and nothingness gained from our platonic transfinite quantum set theory upon which our corresponding new quantum theory is based. To go ahead with our proposed program

it is useful and very instructive to rewrite f and f^2 of the pre-quantum zero set particle and pre-quantum empty set wave as a deviation from their mean value. Thus using the mean value

$$\left(f+f^2\right)/2=1/2 \tag{14}$$

we can write that [56-57]

$$f=\left(1/2\right)+\text{boost} \tag{15}$$

and

$$f^2=\left(1/2\right)-\text{boost} \tag{16}$$

where this boost is given by $\left(1+k\right)/10$ as in the E-infinity interpretation of Penrose twistors theory [56-57] and k is the by now familiar 'tHooft renormalon [13]

$$k=f^3\left(1-f^3\right)=0.18033989 \tag{17}$$

In other words we write

$$\begin{aligned} f^2&=\left(1/2\right)+\left(1+k\right)/10 \\ &=\left(1/2\right)+0.118033989 \\ &=0.618033989 \end{aligned} \tag{18}$$

and

$$\begin{aligned} f^2&=\left(1/2\right)-\left(1+k\right)/10 \\ &=\left(1/2\right)-0.118033989 \\ &=0.3819660116 \end{aligned} \tag{19}$$

Next let us compute the spacetime dimension for the mean values

$$\left\langle d_c^{(o)}\right\rangle=\left\langle d_c^{(-1)}\right\rangle=1/2 \tag{20}$$

Inserting in equation (11) we find that [8-9],[11]

$$\begin{aligned} \left\langle d\right\rangle_{\frac{1}{2}}&=\frac{1}{\left(1/2\right)\left(1-\left(1/2\right)\right)} \\ &=\frac{1}{\left(1/4\right)} \\ &=4 \end{aligned} \tag{21}$$

In other words the entire fine structural details of our spacetime vanishes on average when we disregard the boost and equivalently scale invariance. By contrast our accurate golden mean geometry and topology of E-infinity Cantorian spacetime is sizzling with activity corresponding to what we perceive as vacuum fluctuation. Let us analyse this global picture step by step. It is a trivial matter to see that two zero sets f intersecting do create an empty set following

$$\phi \otimes \phi \equiv \phi^2 \tag{22}$$

This trivial point corresponds in the real spacetime to pre-quantum pair annihilation [46]. The reversed process is equally trivial namely when an empty set f^2 spontaneously disintegrates into two zero sets [8-9], i.e.

$$f^2 \to f;f \tag{23}$$

This is the simplest process going on with the highest quantum probability. Further more two zero sets in a Cantorian spacetime can spontaneously produce a Hardy quantum entanglement. This is because the latent quantum probability of Cantorian spacetime is the inverse of its $4+f^3$ Hausdorff dimension which is given by [8-9],[11]

$$1/\left(4+f^3\right)=f^3 \tag{24}$$

Consequently the quantum probability of two particles and the space become simply

$$\begin{aligned} P &= \left[P\left(\text{spacetime}\right)\right]\otimes\left[P\left(\text{two pre-quantum particles}\right)\right] \\ &= \left(f^3\right)\otimes\left(f^2\right) \\ &= f^5 \end{aligned} \tag{25}$$

This is the exact value of the famous Hardy quantum entanglement [2-20] which was experimentally confirmed sometime ago in numerous very accurate experiments.

There are many other interpretations for different scenarios leading to f^5 for instance a single pre-quantum particle f subjected to a bath of the topological Unruh temperature f^4 leading to f^5 [8-9],[31].

6. Vacuum fluctuation and the three cosmic energy densities

We saw that f and f^2 are the most important but not the only representations of the zero and the empty set because we have also the topological component corresponding to them, namely the zero and the minus one apart of the fact that our universe is not one dimensional, nor in fact only four dimensional but for reasons which we and many other scientists discovered, it is a high dimensional world manifold which is five dimensional topologically on average [2-52]. Seen that way we can start pondering the topological volume of a fractal universe. Moving along these lines of thought it becomes obvious that we must have two kinds of volumes. The one is highly correlated leading to being easily observed and measured directly via its energy content and which is spanned by the pre-

particle zero set. The energy density of this set was shown to be simply the product of half of Hardy's quantum entanglement with the mass and the square of the speed of light [8-9],[19]

$$E(o)=\left(f^5/2\right)mc^2 \tag{26}$$

where $g(o)=\left(f^2/2\right)$ is obviously the ordinary energy density of our cosmos. On the other hand there is another five dimensional topological volume that is uncorrelated, i.e. formed by union of five empty sets quantum wave, namely $5f^2$ which results in being the home of uncorrelated not easilynor directly detected energy density equal to [33-34]

$$5f^2/2\left(mc^2\right) \tag{27}$$

This is what we call dark cosmic energy density and is clearly related to the pre-quantum wave. The subdivision of this dark energy density into pure dark energy and dark matter energy will not be discussed in this work and was the subject of numerous previous publications.

As far as the present paper is concerned we urge the reader to notice that quantum wave and spacetime are both empty sets with increasing degrees of emptiness. This is best understood via three concentric circles the first of which is housing the zero set quantum particle. The second represents the surface of this zero set pre-quantum particle is the empty set pre-quantum wave. In turn the surface of the pre-quantum wave is a complex multi-fractal starting with f^3 and going on to $f^{¥}=0$ which is the real nothingness we talk about in the prelude of this paper, i.e. section 1.

7. A mathematical proof of Dvoretzky's theorem and the dark energy density of the cosmos

In essence the theorem of measure concentration of Dvoretzky asserts that for longer than four dimensions the volume of a manifold is located near to the surface and amounts to about 96% of it while the interior of the manifold constant is the rest of about 4% [49],[53]. This theorem alone seems like the ultimate mathematical proof that nature or the creator of our cosmos is a highly gifted, first class pure mathematician specialized in topology and geometry [23],[29],[71]. In what follows we give yet another mathematical proof for this amazing theorem using our E-infinity theory in the context of the physics of dark energy theory [51-53].

Let us assume that the old bosonic string theory is a sufficiently accurate theory and therefore the universe is described by a maximum of 26 dimensions. Now we should remember two things, that 26 are also the largest dimension in Gross et al heterotic superstrings [69] which is a truly very accurate description of nature and second in E-infinity 26 are transfinitely corrected (i.e. harmonically in the sense of Pythagoras cosmic music of numbers) by adding a 'tHooft renormalon to 26 and getting [13],[19-20],[23],[33]

$$\begin{aligned} D &= 26+k \\ &= 26.18033985 \end{aligned} \tag{28}$$

Now and as is well known, there are two types of strings, closed strings which live in 26 dimensions and open strings which need 25 dimensions only. The difference is of course due to the end points of the open strings and thus the lower critical dimension must be according to E-infinity theory equal to [33],[86-87]

$$\begin{aligned} D &= 25+k \\ &= 25.18033989 \end{aligned} \tag{29}$$

From equations (28) and (29) we see that the relative dimensional density of the two types of strings is simply [86-87]

$$\begin{aligned} \frac{25+k}{26+k} &= 0.961803389 \\ &= \left(96+k\right)/100 \end{aligned} \tag{30}$$

which we take to be a measure for the weak effect of end points representing the correlated pre-quantum particles and that means this ratio given by equation (3) is effectively the uncorrelated dark energy density [33],[86-87]

$$\begin{aligned} g(D) &= \frac{25+k}{26+k} \\ &= \left(96+k\right)/100 \end{aligned} \tag{31}$$

Consequently, the ordinary dark energy density must be

$$\begin{aligned} g(o) &= 1 - 0.961833989 \\ &= 0.03819660112 \\ &= \left(10\right)\left(f^2\right) \\ &= \left(3.819660112\right)\% \\ &= \left(4 - k\right)\% \end{aligned} \tag{32}$$

Noting that $4-k$ is the fractal dimension of ‘tHooft-Veltman-Wilson fractal renormalization spacetime we see that

$$g(D)+g(o)=\left(100\right)\% \tag{33}$$

Setting $k \to o$ in equations (31) and (33) we find that there is a dimensional measure concentration in our total core space of E-infinity [52],[58] which is found from the reconstruction equation of $\bar{a}_o = 137 + k_o$ of E-infinity theory [49].Consequently we have

$$\sum_{i=1}^{4}\bar{\alpha}_i = \bar{\alpha}_1+\bar{\alpha}_2+\bar{\alpha}_3+\bar{\alpha}_4$$
$$=60+30+9+1$$
$$=100 \qquad (34)$$

where the inverse couplings $\bar{a}_i$ are explained in many relevant previous publications [11],[15],[29],[48]. Thus segregating 96 from the 100 leaves only 4 dimensions for the ordinary point-like pre-quantum particles attached to the open strings. This is exactly the result predicted y Dvoretzky's theorem [49],[53].

Conclusion

At the beginning there was the music and in more than a metaphorical sense its beauty and number theoretical logic unifying emotions and pure thoughts and from that, everything else follows. This is not an over the top romantic view of science but a scientific model of thinking with a long tradition starting before Plato and did not end with Dirac but rather than that it will remain for ever a source of inspiration and faith in life and Heisenberg's central order. We hope to have shown that at the root of reality there are deeper realities, namely the reality of imagination, pure mathematics and scientific philosophy and armed with these realizations neither vacuum fluctuation nor dark energy is a mystery because in principle the entire creation is comprehensible. Perhaps the only mystery is that there is something instead of nothing but we are grateful for this mystery and we hope there will be forever and ever something instead of nothing.

Having come that far the reader should see for himself how trivial state vector reduction, i.e. wave collapse should be resolved without the maze of needless algebraic operation and hardly comprehensible terminology and hieroglyphic notions. If we agree and the writers have already all agreed that the pre-quantum wave is an empty set, then any attempt to observe the empty set will lead to definitely making it non-empty. Since the nearest non-empty set to the empty set is the zero set quantum particle, its measurement, i.e. observation instantly causes the conversion of a pre-quantum wave to a pre-quantum particle. Nothing but really nothing could be clearer nor more trivial than that.

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