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Exploring Angles, Lines and Triangles Through Project-Based Learning

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Table of Contents

Abstract

This project was created by a teacher for teachers, providing a hands-on learning module for a geometry classroom. The project-based learning module emphasizes collaboration and communication as part of the task. Topics include congruence and similarity as well as theorems involving lines, angles and triangles. Students will also apply geometric methods to solve design problems. Recommendations for how implement each phase of the project have been provided by the author. Additionally, scoring rubrics have been provided. The project-based learning module is aligned to the New York State Next Generation Standards for Mathematics, specifically geometry standards throughout secondary grade levels.

Introduction

Problem Statement

There is a relative scarcity of research-based curriculum resources that provide a solid foundation for creating student-centered hands-on opportunities within a high school Geometry classroom. Although much has been written about the effectiveness of student-centered hands-on learning with regard to student motivation, educators need a framework for developing and implementing successful project-based lessons. The purpose of this thesis is to provide secondary school teachers with a framework to support implementing a project-based curriculum in the geometry classroom. Teachers will find a flexible, hands-on learning module contained in this document.

Significance of Problem

Teaching in the 21st century has shifted significantly from direct instruction. Teachers no longer simply impart knowledge on to students with direct instruction. Teachers are now facilitators of student learning (Kuhlthau, 2015). This shift has challenged both new and veteran teachers to rethink how they present key concepts in their classrooms. Allowing students to design their own questions and construct their own understanding has been shown to be an effective tool for learning.

Unfortunately, the time, research and preparation that is required to design a research-based, project-based curriculum can be prohibitive to teachers. The interactive learning module provided in this curriculum project will provide teachers with both a generalized outline for designing a project-based lesson, as well as a fully function Project based learning (PBL) that is ready to be immediately implemented.

Purpose

The goal of this thesis is to support teachers in their implementation of Project Based Learning (PBL) so that students can better understand the concepts of surface area and volume and their practical applications. PBL, by its very nature, integrates many 21st century skills such as critical thinking, collaboration, communication and creativity, which are all essential elements of PBL. With a focus on significant content and on-going revision and reflection, students will have the opportunity to gain key mathematical concepts as well as necessary ‘soft skills’. Larmer and Mergendoller (2012) define soft skills in parallel with 21st century skills, including, problem solving and innovation.

This PBL curriculum requires a culminating product where students will collaboratively create a scale model bridge from a predetermined list of supplies. The final product will incorporate students’ own research and design. The curriculum chapter will detail the daily tasks, expectations and outcomes that will be required of students for this PBL. These elements will align closely with peer review, classroom tested PBL elements.

Project based learning is very much student focused and as such students have substantially more voice and choice in how and what they learn. Teachers go beyond the role of simply imparting knowledge. They are facilitators of learning that help students make sense of not only what they are exploring, but how it relates to real life.

Literature Review

Many studies have been conducted that discuss the effectiveness of project-based learning on student motivation, such as Pamela Beres’(2011) *Project- Based Learning and its Effect on Motivation In the Adolescent Mathematics Classroom*. Additionally, much has been researched and discussed about the Legacy Cycle which provides the foundational ideas for project-based learning. Harris and Klein (2007) also found that presenting students with a relevant, real-world problem helps to stimulate student engagement. Both of these ideas are critical to the effectiveness of the curriculum present here.

Project Based Learning Definitions

Project-based learning terminology may be unfamiliar for teachers who have never used PBL as an instructional practice. Larmer and Mergendoller (2012) layout the critical elements of PBL in *8 essentials for project-based learning.* They conclude that every good project needs to incorporate significant content. This content should be based in common core standards as well as National Council of Teachers of Mathematics (*NCTM*) *Standards*. Also, students are more likely quickly to engage with projects if there is an entry event. This Entry Event can come in many forms such as a video, a guest speaker or a field trip. Beyond gaining students interest it is imperative that a PBL has a Driving Question. The driving question can be compared to a light at the end of the tunnel. Students need to know what they should be focusing on and *why* the project is important. The Driving Question should be "provocative, open-ended and complex" (2012).

Larmer and Mergendoller (2012) go on to state that projects should be "a product of students' choice" (2012). This is better known as voice and choice, meaning that students are more investing in their own learning if they have some say is what they are doing.

The Interactive Learning Module

The learning module below contains all the necessary directions, rubrics and handouts to run a successful bridge building project. Answer keys can be found in the appendix. This project can be easily modified for both time and skill levels. The full course can take up to nine days to complete, but can be pared down to be completed in as few as four classes. Activates such as indepth research and reflective journals enhance the learning, they are optional. Before each day's activities below, the author has provided insight and guidance specific to that day's activity. When implementing this project for the first time, the author advises that the teacher create exemplars. This will help guide students as well as providing the teacher valuable hands-on experience.

Exploring Lines, Angles and Triangles – Toothpick Bridge Challenge

This curriculum project includes a complete set of resources for classroom teachers. It is aligned with content standards and curriculum being taught in Geometry classrooms. This chapter consists of all the materials necessary to complete a full 9-day unit. The first handout for students is below. It gives them an overview of this hands-on project.

	NYS Next Generation Standards for Mathematics
GEO-G.SRT.5	Use congruence and similarity criteria for triangles to solve problems algebraically and geometrically.
GEO-G.CO.9	Prove and apply theorems about lines and angles.
GEO-G.CO.10	Prove and apply theorems about triangles.
GEO-G.MG.1	Use geometric shapes, their measures, and their properties to describe objects.
GEO-G.MG.3	Apply geometric methods to solve design problems.

Bridge Design and Construction Project

For this project you and your group will research, design, build and test a bridge. Below are some of the criteria that are required for this project as well as a calendar of due dates to keep you and your group on schedule.

- Groups must be made up of 2-3 people.
- Each group will construct one bridge.
- Each group will be given a finite amount of supplies that they must keep track of.
- Due dates for each part of this project must be met. Absence from school is not an excuse. If you need to stay after school to complete your project, please plan for that.
- The last 20 minutes of each class will be dedicated to clean up and completing a journal entry for the group detailing the progress you made that day.

Date	Tasks to be completed each day
Day1	- Form a group of 2-3 people - Complete "Bridge Building" exercise - Research bridge designs - Complete Journal #1
Day 2	- Choose final bridge design - Complete bridge blueprint sketches - Complete Journal #2
Day 3	- Begin constructing your bridge - Complete Journal #3
Day 4	- Continue constructing your bridge - Complete Journal #4
Day 5	- Continue constructing your bridge - Complete Journal #5
Day 6	- Continue constructing your bridge - Complete Journal #6
Day 7	- Continue constructing your bridge - Complete Journal #7 - Sign up for a date/time to test your bridge
Day 8	- Begin bridge testing - Final Journal
Day 9	- Finish bridge testing - Final Journal

Bridge Requirements:

Geometry skills and the strengths of designs are put to the test every day. Whether building a house or a chair, geometry is required. The challenge is often the same – build the strongest object with the least materials.

For this project, teams of up to three students will construct a bridge out of only toothpicks and glue. Each toothpick bridge will be designed to hold as much weight as possible before collapsing. Teams will research the style of bridge, create a three-view diagram of a bridge, build, test and complete a presentation about the project and its connection to geometry and the design process.

The ultimate goal is to build a bridge that is strong but is light as possible. The bridge with the greatest ratio of load weight to bridge weight will win the contest.

Materials and Process Overview:

Teams are provided a predetermined number of toothpicks, approximately 250, and Elmer's Glue to build their bridge. Each bridge must span a gap length of 10 inches. As students are designing their bridge, dimension requirements must be met. The top of the bridge must be flat at some point in the design. This will be where the weight is loaded for testing the bridge strength.

Teams will sketch each part of the bridge design on graph paper. All sides must be measured, all angles must be determined. Since the top and bottom of the bridge will be parallel, angle relationships can and should be used. After laying wax paper over the graph paper, teams can begin to measure, cut and glue toothpicks together. **Be careful not to glue the wax paper to the bridge.**

In order to test the bridge, a bungee cord will be looped over the top or through the bridge. Attached to the bungee cord will be a bucket. The bucket will be filled with water, sand or measurable weights until the bridge collapses or it passes the ultimate test and holds the entire bucket. The bucket will be weighed with a scale and this weight will be recorded.

Required Bridge Dimensions	
Length	12in – 14in
Width	2in – 4in
Height	2in – 6in

Teachers Note

Materials:

- 1-2 rolls of standard sized wax paper
- Making tape
- Regular Elmer's glue
- 250 round toothpicks per group
- ¼" ruled 8.5" x 11" graph paper
- 1" ruled poster sized graph paper
- Scissors
- 1 small bucket about .5 gallon
- 1 bungee cord with hooks, approximately 12"-18"
- Mass to add to the bucket, water, sand, weights, etc.
- 1 digital scale to weigh each bridge (grams)
- 1 scale to measure the mass used to test the bridge (Kg), for example a bathroom scale

Day 1 – Introduction and Investigation

The first day of this project is broken up into several smaller parts. This project will be introduced with two brief videos. The first video shows students competing in a bridge building competition. The second video gives an overview of how to construct a toothpick bridge.

Competition Video – "Toothpick Bridges SMCC 2013-14" – https://youtu.be/4BfuxZwBn_o

Build Video – "Toothpick Bridge Start to Finish" – https://youtu.be/0wj1_mOxT1Q

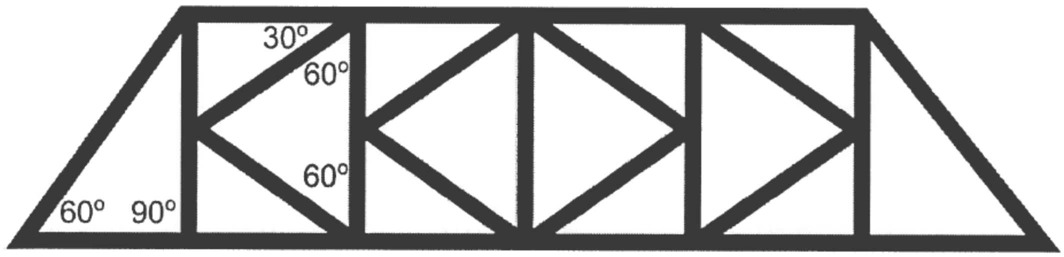
After viewing the videos student groups should be formed. Groups of 2-3 tend to work best. This will ensure that progress is maintained throughout the duration of the project despite absences or other unforeseen issues.

When groups have been formed students will discuss what properties bridges should have that ensure their strength is sufficient. Each group will make a list of as many properties as they can think of. Then the teacher will lead a class discussion. Each group must provide some criteria that they agreed upon. It’s important for the teacher to ensure that the guided discussion includes truss type bridges, parallel lines, and triangles as major components. Each group will then complete “Bridge Type” worksheet. This work sheet provides a sample of four different types of truss bridges. Students must use their knowledge of parallel lines, transversals, and triangle properties to fill in all missing angles.

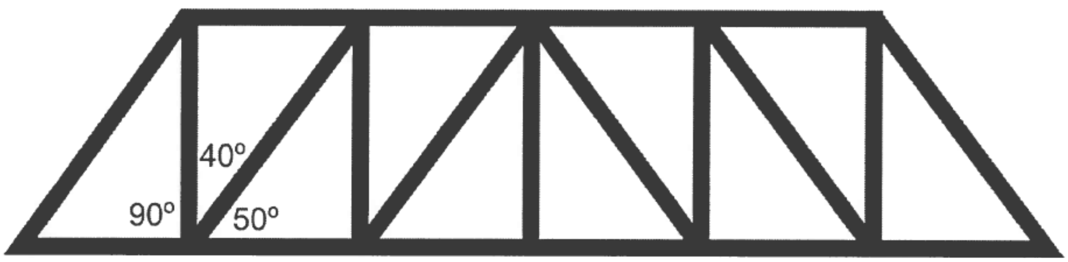
Toothpick Bridge Challenge – Angle Measurement

In each bridge design below label all missing angles. Use your knowledge of triangle properties, parallel lines and transversals. Assume triangles that appear to be congruent are.

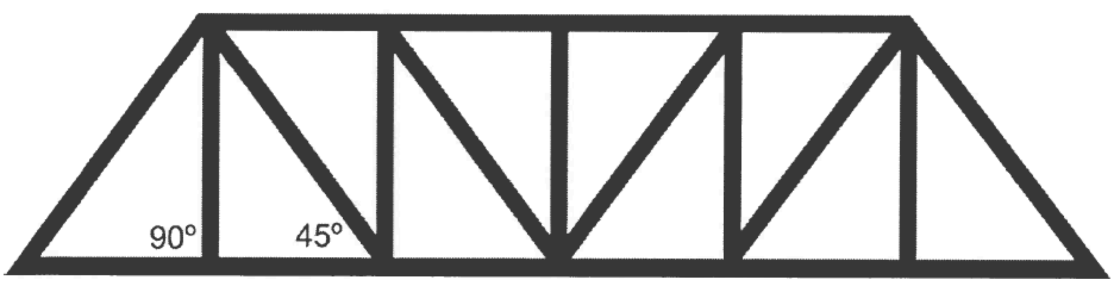
K-Truss



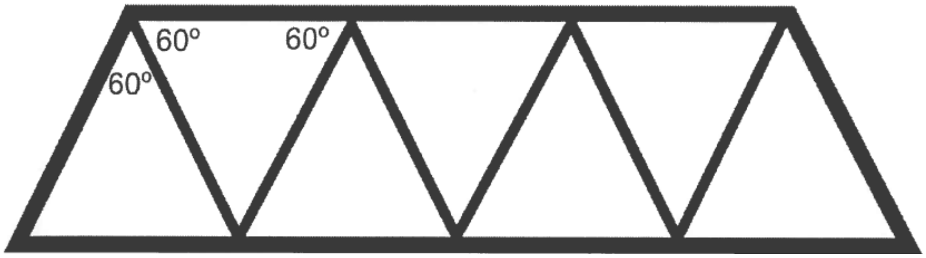
Howe Truss



Pratt Truss



Warren Truss



When student groups have completed the bridge building activity, they will begin researching specific bridge designs. It is important to ensure students are researching truss type bridges and not suspension bridges. While researching bridges, each group will select three different designs that they consider to be best for this competition. The three designs they choose will be roughly sketched on the “Bridge Research” sheet. Each groups design sheet will be collected and vetted by the teacher to ensure they can meet the criteria of the project. The teacher should review each group’s potential designs. Written comments should be made that will assist students in selecting components of each rough sketch that could be incorporated into the groups final design.

Toothpick Bridge Challenge – Bridge Research

Use the internet to research three different bridge designs. Each design should focus on the geometric shapes and designs discussed in class. List the name of each bridge if applicable. List the geometry specific to each bridge. For example, if a bridge uses right triangles be sure to discuss that design element.

Design #1

Name of bridge type:____________________________

Describe the geometry found in this design.

Sketch the bridge.

Design #2

Name of bridge type:____________________________

Describe the geometry found in this design.

Sketch the bridge.

Design #3

Name of bridge type:____________________________

Describe the geometry found in this design.

Sketch the bridge.

At the conclusion of each class students should complete a guided journal entry. This entry will log what was completed for the day and what their goals are for the next work session. This can be a paper journal or digital.

Sample Journal Prompt

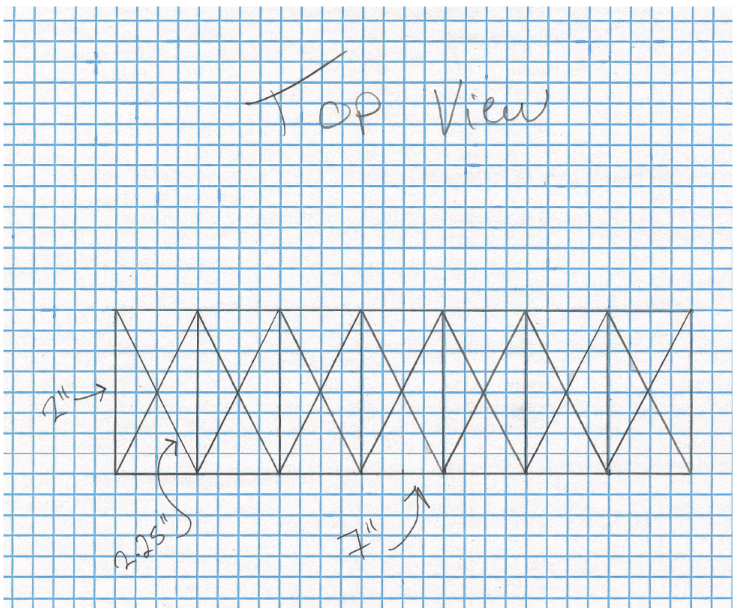
Group Name:
Group Members Names:

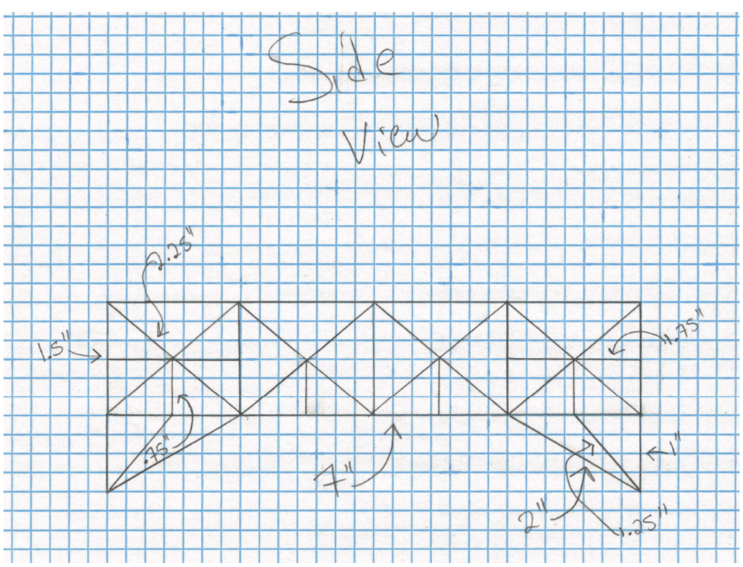
1.
2.
3.

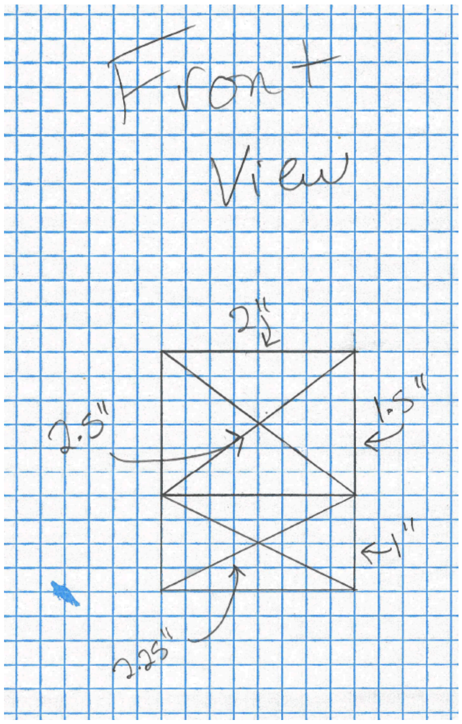
Date: Daily Goal(s):
How will your groups goal(s) be achieved? List specific tasks for each group member:
Reflect… Did you achieve your goals? Explain why or why not. • Yes • No List your goal(s) for the next class.

Day 2 – Design Selection and Formal Sketch

Each student group should review the notes and comments made by the teacher on their rough sketches. Students should be given the opportunity to ask clarifying questions. Each group will then decide which elements their final bridge design will include. When the group comes to a consensus, they will draft a half-scale blueprint. It will be helpful for the teacher to provide an exemplar for each view of a half-scale drawing. The teacher could use designs from a previous year, or the exemplars attached here.







The half-scale drawing can be completed on standard ¼ inch ruled, 8.5” x 11” graph paper. Each blueprint must be precisely drawn and include three views, a side-view, top-view, and front-view. Each view should be drawn on a separate piece of graph paper. This is a good opportunity to encourage each group member to complete one of the views. The attached rubric that is provided to the students will ensure they are completing all the necessary components of the sketches. All three views must be present. Length, width and height must be correctly labeled on all sketches. At least one of angle must be properly measured with a protractor and labeled in the sketch. A straight edge and pencil must be used on all parts of the blueprint. Before students move on to draft their full size blueprint the teacher must ensure that corresponding measurements of all three views are in agreement. The teacher should also make sure that overall length, width and height fall within the required bridge dimensions.

When the teacher has approved a groups half-scale drawing, that group will then use their blueprint to draft a full-scale drawing. This full-scale drawing is best done on 1” ruled poster sized graph paper. All three views can be sketched on the same paper. Ensure that the students are appropriately spacing each of the three views. This is the final sketch that will be used to construct the actual bridge. It has been found to be most effective for students to cut apart each of the three views when complete. Again, this will allow each group member to maximize their time by assembling a different section of the bridge. If each group member builds simultaneously the number of days required to construct the bridge can be reduced from five to three.

At the conclusion of each class students should complete a guided journal entry. This entry will log what was completed for the day and what their goals are for the next work session.

Days 3 through 6 – Bridge Construction

The number of days required to complete bridge construction varies greatly and depends on a number of factors. Have students cut apart the full-size blueprint or make additional copies. This will allow multiple group members to simultaneously construct. At the beginning of each class student groups should review their previous journal entry to review and set a daily goal.

When constructing the bridge students will need the following materials: full-size blueprint, wax paper, masking tape, scissors, toothpicks and glue. Students will proceed as follows.

1. Gently tape one layer of wax paper on top of the full-size sketch. The paper should cover the side view.
2. Place a few toothpicks along one of the sketch lines making sure to slightly overlap toothpicks.

3. Use a small amount of glue to secure the toothpick together. Sometimes it helps to use an extra toothpick to apply the glue. Only use enough glue to secure toothpicks to each other. Too much glue will make it difficult to peel off the wax paper. It will also make the bridge unnecessarily heavy.
4. Continue adding a few toothpicks at a time until all lines of the side view are covered with toothpicks.
5. Allow the glue to dry.
6. When glue is fully dried, carefully peel the wax paper away from the toothpicks. Do not try to pull the toothpick off the wax paper.

Depending on the bridge design students should have 3 or 4 parts to their bridge. Two walls, a floor and potentially a roof will be constructed. When all parts are dried groups will need to assemble their parts. They will need to prop up walls and the roof, then secure each part with glue. Tape may be used to hold the bridge together during final construction but should be removed when glue is dry. No tape should remain on the fully constructed bridge.

At the conclusion of each class students should complete a guided journal entry. This entry will log what was completed for the day and what their goals are for the next work session.

Days 7 and 8 – Bridge Testing

Each group will have measurements and results recorded on a rubric by the teacher. It could be possible to complete this in one day if multiple trials were conducted simultaneously.

Toothpick Bridge Challenge – Scoring Rubric

This scoring rubric contains three main parts. Each part is scored separately and contributes to your overall final project grade.

PART I – Design Drawings

	Advanced (15 points)	Proficient (10 points)	Developing (5 points)	Max 15 points
Full Scale Drawing	Front, side and top views all present. Length, width and height label where appropriate. Straight edge used for all parts of schematic.	Front, side and top views all present. Length, width and height missing on some parts of the diagram. Straight edge not used on all parts of schematic.	Missing some of the front, side and top views. Missing all length, width and height measurements. Straight edge not used on any parts of schematic.	
Half Scale Drawing	Front, side and top views all present. Length, width and height label where appropriate. Straight edge used for all parts of schematic.	Front, side and top views all present. Length, width and height missing on some parts of the diagram. Straight edge not used on all parts of schematic.	Missing some of the front, side and top views. Missing all length, width and height measurements. Straight edge not used on any parts of schematic.	
Part I Final Score				

Part II – Bridge Real-World Functionality

The part II score will be determined by two variables:

L = the load held by the bridge in grams
W = weight of the bridge itself in grams
The **functionality score** is computed using the following formula:

$$\left(\frac{Load}{Weight}\right)$$

Score Recording Table	
Load held by your bridge	
Weight of your bridge	

Using the values from the table on the left, calculate your **Part II Score** below.

Be sure to use the equation $\left(\frac{L}{W}\right)$

Your part II score will be determined by your finishing position relative to all other bridges tested.

1st place – 55 points 2nd place – 54 points 3rd Place – 53 points
4th Place – 50 points 5th place – 45 points 6th place – 40 points
7th place – 35 points 8th and 9th place – 30 points 10th place and below – 25 points

After plugging all values into the **Part II** equation, use the score table on the previous page to find your **Part II Score**. Record your score below.

Part II Final Score	

Part III – Reflection

Go to our Google Classroom page and complete the **"Bridge Project Reflection"**. Please answer all questions thoughtfully in complete sentences.

Bridge Project Reflection	Max 15 Points
Part III Final Score	

Total Final Score: **Part I + Part II + Part III**	__________ / 100

The testing area should be prepared as follows. First two desks should be set 10 inches apart. Students will be provided with a bucket, bungee cord and mass to add to the bucket. Student will conduct their own testing. The testing procedure is as follows.

1. Students will use a digital scale to mass their bridge in grams. This measurement will be recorded in a spreadsheet as well as the rubric.
2. Students must decide how to attach the bungee cord and bucket to their bridge.
3. Students will slowly add mas to the bucket until the bridge fails. As soon as the bridge fails, they must stop adding mass.
4. The mass in the bucket is tallied and recorded in the spreadsheet as well as the rubric.

PLACE	Name	Bridge Weight	Load Held	Load/Weight
1	Dylan G, Seth	23.6	6700	283.8983051
2	Tichon, Sabah	22.7	5660	249.339207
3	Morgan, English, Antonio	34.9	7820	224.0687679
4	Bella, Antony, Angel	20.7	4010	193.7198068
5	Jonah, Ty, Eli	38.4	4850	126.3020833
6	Mason, Michael	21	2030	96.66666667
7	Adri, Abbie, Sam	31.5	1200	38.0952381

Validity

This Project Based Learning curriculum was used in a high school geometry classroom and was reviewed by a veteran teacher for feedback. The veteran teacher had significant experience infusing state standards in mathematics into PBL learning. The author's recommendations for how to implement each phase of this PBL are as follows:

- Day 1 – Introduction and Investigation: The author recommends that teachers hook students by showing the two short videos of other classes testing toothpick bridges. When grouping students, it is best for each group to have a mix of high, middle and low-level students. All students will have an opportunity to contribute to this project due to its hands-on curriculum. It is important to stress the journal aspect of this project. It will helps students to set and maintain goals throughout the process.
- Day 2 – Design Selection and Formal Sketch: The author recommends providing students with exemplars for this phase of the bridge design. It is important to stress good blueprint habits. Students should use a pencil and straight edge when sketching all views of the diagram. The teacher should be checking that students are staying within the measurement parameters of the project rules. Students should begin with the half scale drawing. The teacher should then approve their sketch before they begin their full size sketch.
- Days 3 through 6 – Bridge Construction: The veteran teacher suggests that students divide the construction work among the group. Every member should be building a section of the bridge if it is physically possible. This will ensure that construction is completed within the proper timeframe. The journal goals will assist with keeping students focused. It is important to review the previous days goals when setting new goals for the future. One important factor to consider is the space necessary for the projects to dry. Plan ahead to ensure there is a space to spread out student projects. Also, there should be a designated area for each group’s supplies.
- Days 7 and 8 – Bridge Testing: The author recommends having a signup sheet for groups to test their bridge. This will eliminate confusion and help to streamline the process. An

important aspect of testing is to allow students to control the process. They should be weighing, setting up, and testing without intervention from the teacher. As each group weighs their bridge the teacher should enter measurements in the spreadsheet. This spreadsheet could be displayed in the room so the class can have live updates of each bridge. While groups are testing their bridge, the teacher should be filling out the scoring rubric. Additionally, to ensure all group members are participating in the testing phase, each group could potentially have someone explain their design to the rest of the class. The scoring rubric attached assigns some points based on how well a bridge performs, but this could be changed to suit any classroom environment.

Conclusion

Geometry students are in a unique position to have the opportunity to use mathematics in real world circumstances. The discovery and application of geometric principles can have a profound effect on a student's learning. This hands-on learning project is meant to guide student's insight into why math is important and how it plays a role in the world around them. Communication and collaboration are critical skills that students must practice and refine. Again, these soft skills are incorporated into this learning module. Classroom planning can be a daunting, time consuming task. This Project Based Learning module will help to alleviate the time crunch that teachers currently face. It is the hope of the author that teachers will use this fully functional learning module to enhance student understanding of Next Generation Learning Standards for mathematics.

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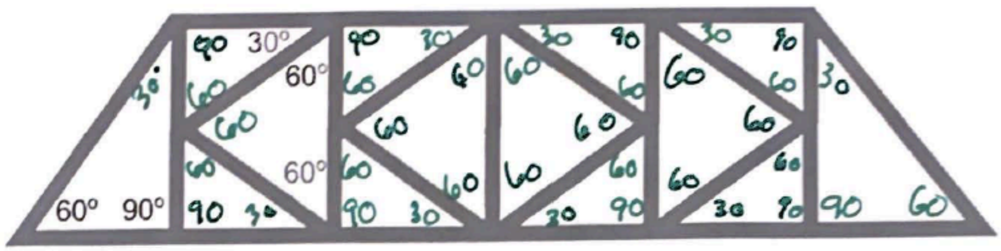
Appendix

Toothpick Bridge Challenge – Angle Measurement Key

IN each bridge design below label all missing angles. Use your knowledge of triangle properties, parallel lines and transversals. It can be assumed triangles that appear to be congruent are.

K-Truss



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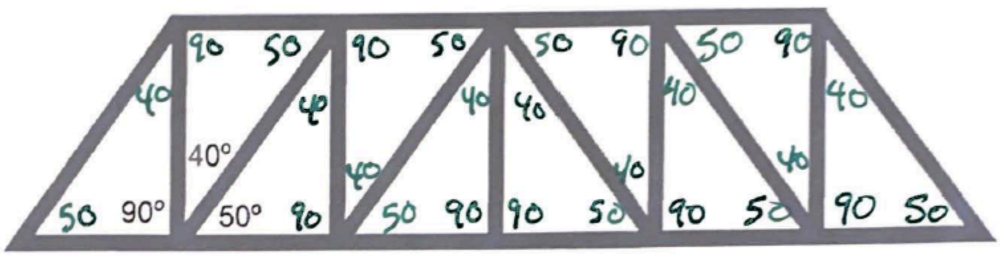
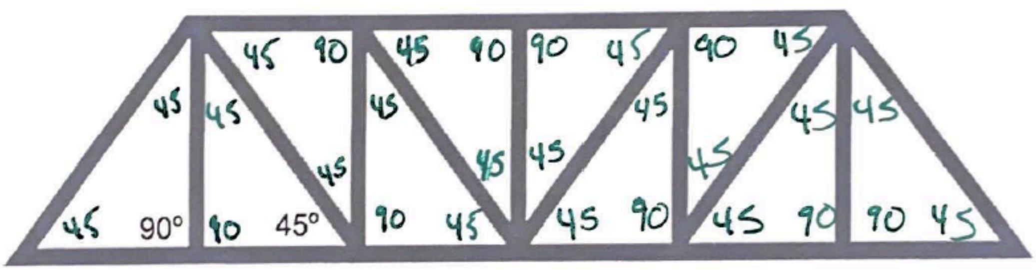


Pratt Truss



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