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Solving Systems of Equations and Inequalities using Technology and Virtual Manipulatives

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Abstract

This curriculum project was developed to assist teachers to utilize virtual manipulatives and technology in the New York State Next Generation Algebra I curriculum, specifically systems of equations and systems of inequalities. Technology has become an integral part of students' lives and research has shown that incorporating technology in the classroom can impact student learning. This curriculum project includes lessons from the Algebra I curriculum through a variety of technological applications.

Keywords: Virtual Manipulatives, Concrete Manipulatives

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Induction

Twenty years ago, only 20% of public-school teachers felt comfortable using technology in their teaching (Rosenththal, 1999; Neiss, 2005). However, teachers have had to shift their instructional practices to integrate technology within the context of the academic subject areas. There had been a tendency to only look at the technology being used and not how it is impacting the curriculum being taught. It has become clear that our focus should be on analyzing how the technology is incorporated into the classroom learning environment (Mishra & Koehler, 2003; Mishra and Koehler, 2005). As students in the classroom are more versed with technology, new tools and resources are available to educators, but teachers are not always prepared to integrate technology in lessons efficiently and effectively (Onal, 2016).

The idea of using virtual manipulatives aligns with students having access to their own personal device such as a Chromebook. This is referred to as a one-to-one classroom, meaning students can watch videos, take notes, or practice concepts on their devices. Some teachers have struggled to make the adjustments needed for a one-to-one classroom instruction (Hora and Holden, 2013). Research has shown this is a result of teachers not understanding how to adapt different manipulatives and technologies for their own teaching style (Hora & Holden, 2013). If educators struggle with how to integrate technology into the classroom effectively, they may also struggle with how to positively affect student learning (Dalal, Achambault & Shleton, 2017). Thus, this curriculum project was designed to provide teachers with an additional instructional resource to support conceptual understanding, student learning and thus student learning, during the algebra 1 unit of when solving a system of equations and a system of inequalities.

Literature Review

Technology can be used as an instrument to foster student motivation (Nicol, Owens, LeCoze, MacIntyre, & Eastwood, 2018) and could be very effective when utilized properly in a mathematics classroom. For example, a computer can easily create a graph that students can manipulate and interact with, providing an opportunity to create a deeper level of mathematical reasoning. (Schoenfeld, 2014). Utilizing technology in the mathematics classroom supports abstract understanding (Kirikçilar & Yildiz, 2018).

Technological Pedagogical Content Knowledge

Pedagogical Content knowledge (PCK) is the intersection of content and pedagogy, which experienced mathematics teachers naturally combine during instruction. Figure 1 shows how content and pedagogy combine to create PCK. When technology is combined with both content and pedagogy, then the intersection theoretically creates what is referred to as Technological Pedagogical Content Knowledge (TPCK). This construct "emphasizes the connections, interactions, affordances, and constraints between and among content, pedagogy,

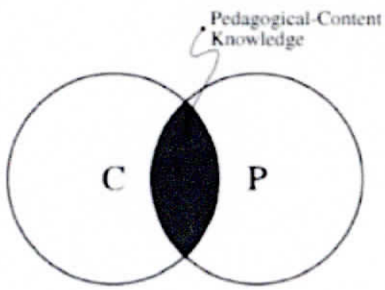


Figure 1: *The Two Circles of Pedagogical Knowledge and Content Knowledge Joined creating Pedagogical Content Knowledge (Mishra & Koehler, 2006)*

and technology" (Mirsha & Koehler, 2006, p.1024) and is accepted as the framework that represents how teachers teach specific content with technology effectively (Patahuddin, Lowrie, & Dalgarno, 2016). Figure 2 shows the representation of this concept. TPCK is the basis of engaging teaching with technology and requires: (a) knowledge of the representation of concepts using technologies; (b) pedagogical techniques that use technologies in constructive ways to teach content; (c) an understanding of what makes concepts difficult or easy to learn and how technology can help address some of the challenges that students encounter; and (d) knowledge of how technologies can be incorporated to build upon existing knowledge and to develop new methods and strengthen old ones (Mirsha & Koehler, 2006). Technology has the potential to have a significant impact on student learning, yet teachers need to know how to apply it effectively to have the greatest impact on students (Saralar, Işiksal-Bostan & Akyüz, 2018).

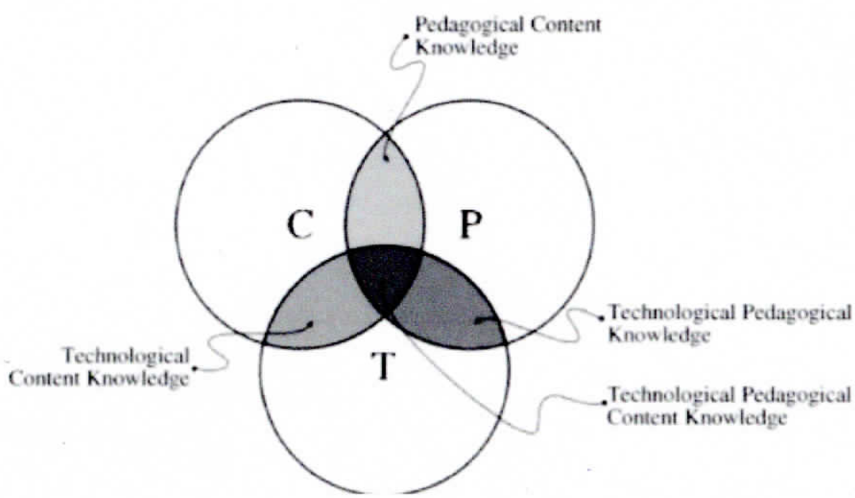


Figure 2. *Pedagogical Technological Content Knowledge. The Three Circles, Content, Pedagogy, and Technology, Overlap to Lead to Four More Kinds of Interrelated Knowledge (Mishra & Koehler, 2006)*

Virtual manipulatives

The concept of using manipulatives in mathematics education is not a new concept. Manipulatives are objects that are designed for a learner to perceive a mathematical concept by altering or moving it. The use of manipulatives provides ways for students to learn concepts through hands-on experiences and can provide opportunities for deeper understanding. The evolution of technology and its use in the classroom has led to hands on manipulatives being replaced with virtual manipulatives. The definition of virtual manipulatives includes both, static and dynamic visual representations of concrete manipulative (Spicer, 2000; Moyer. P., Bolyard, J., & Spikell, M., 2002). Static virtual manipulatives are basically pictures such as visual images, pictures, drawings on a white board, etc. These types of manipulatives cannot be used in the same ways that traditional concrete manipulatives can. Virtual manipulatives are dynamic and can be manipulated in the same ways that concrete manipulatives can but with a computer mouse or by using a touch screen, turning a static virtual manipulative into a dynamic one. A student can slide, flip, and turn the dynamic virtual manipulative as if it was a three-dimensional object. (Moyer, Bolyard, and Spikell, 2002). As technology has become prominent in the classroom the definition of virtual manipulatives has evolved into "an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that present opportunities for constructing mathematical knowledge" (Moyer and Bolyard, 2016). Virtual manipulatives are presented on computer screens, on touch screens of all sizes and through an ever-changing variety of viewing and manipulation devices. Therefore, the definition of virtual manipulatives has been used interchangeably and the universal use of technology by students in the classroom provide more incentive to investigate the practicality of virtual manipulatives.

Materials are generally most effective in the 7-11 years age group. They were still effective, but less so, in an age group consisting of 12 years and older (Larkin, 2016; Carbonneau, Marley, & Selig, 2013). As students use manipulatives in the classroom, they are able to better establish connections between objects and mathematical representation. Older students may view the use of virtual manipulatives with technology as more advance than static (concrete) manipulatives because they allow for exploration and a deeper understanding of the content. However, the issue of age appropriateness of technology is one that must be factored into the decision making process. "For older populations, offering learning aids that distinguish a student from their general education peers can have a stigmatizing effect, and negatively affect the student's academic performance" (Finnegan & Austin, 2012; Satsangi, Bouck, Taber-Doughty, Boefferding & Roberts, 2016). Students are best served by learning concepts by actual manipulation of physical materials. Motivation is best accomplished when there is an active involvement with physical objects (Hunt, A., Nipper, K., & Nash, L., 2011). This leads to the abundant use of technology by students and allow for the investigation of the usefulness of virtual manipulatives without the stigmatizing affects that would hinder the learning process for students.

Curriculum

This curriculum plan includes multiple standards across the Common Core State Standards (CCSS) and New York State Next Generation Math Standards for Algebra 1 standards using a variety of technology and manipulatives to engage students in solving systems of equations and inequalities. The virtual manipulatives that are used in this curriculum project are evidenced in the Desmos Teacher Activities. Students are first introduced to the concept of a system in eighth grade. The expectation is that students are to master the following standards from the CCSS (NYSED, 2011):

8.EE.8a Understand the solutions to a system of two linear equation in two variables correspond to points of intersection of their graphs.

8.EE.8b. Solve systems of two linear equation in two variables algebraically.

8.EE.8C. Solve real-world and mathematical problems leading to two linear equations in two variables.

Solving systems of equations and inequalities are required skills in Algebra 1, or ninth grade. Expanding from the standards listed above, the expectation is that students in grade 9 can master the following standards (NYS Next Gen, 2017).:

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context;

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions;

A-REI.6 Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables;

A-REI.10 Understand that the graph of an equation in two variables is the set of all its' solutions plotted in the coordinate plane;

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane.

Each lesson was designed for students to use computers to access the online applications and programs. The curriculum was designed to provide mathematics teachers with resources that can be incorporated into their instruction. The programs that are used provide students with the opportunity to delve into content on a deeper level. The lessons included were taught sequentially. Students need a basic understanding of graphing an equation prior to the lessons. Links are provided in the lessons to access the technological tools. Also, screenshots are provided of what is seen when students go to the specific websites provided. When accessing the website, the text will be much clearer than the snapshot of the screen.

Lesson 1

Lesson Title:	Solving Systems by Graphing
Standards:	**A. REI. C. 6** – Solve systems of linear equations exactly and approximately (e.g. with graph), focusing on pairs of linear equations in two variables
Learning Goals:	I can solve a system of equations whose solution requires graphing two lines on the same coordinate grid. I can classify which systems of equations have one solution, no solution, or an infinite number of solutions.
Technology / Manipulatives Used:	• Laptop / Chromebook / Tablet • Edpuzzle Online Notes (https://edpuzzle.com/media/5e9b75cf6e2c1f3ef3ce5e63) • Desmos Teacher Activity (https://student.desmos.com/?prepopulateCode=rb3xpc) • Quizizz Practice Assessment (https://quizizz.com/join/quiz/5e66e73210ea86001c66924d/start?from=soloLinkShare&referrer=5e667f551d5b8f001c1a8354)
Timeline:	55 minutes 5 minutes – Check and go over homework, Hand out Note Sheet 15 minutes – Student watch lesson on solving a system of equations note video through Edpuzzle. Introduces the number of solutions when graphing a system of equations 20 minutes – Have students complete the Desmos Activity to test systems of equations that have zero, one, or an infinite number of solutions and explain why this is true. 15 minutes – Explain Quizizz activity and have student complete the activity in small groups * Teacher should be moving through the classroom while students working on the activities to offer support to those who need individual assistance.

Day 1: Solving systems of equations graphically   Date ____________

A system of linear equations is ____

____

A solution of a system of linear equations is ___________________________________

____

Points of Intersection (POI) are the same thing as the solutions of a system.

No solution means ____

____

A system of equations has infinitely many solutions when ___________________

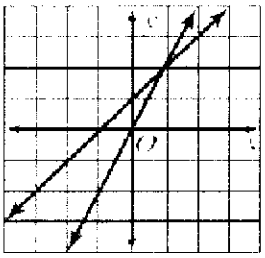
____

Practice:

Vocabulary and Key Concepts

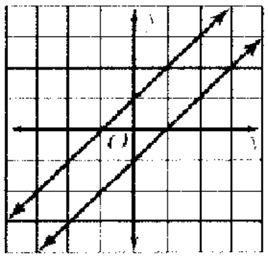
Numbers of Solutions of Systems of Linear Equations

different slopes



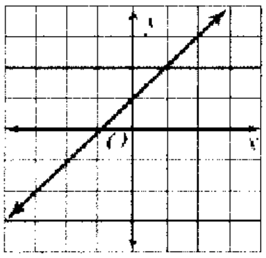
The lines [ ] so there is [ ] solution.

same slope
different y-intercepts



The lines [ ] so there are [ ] solutions.

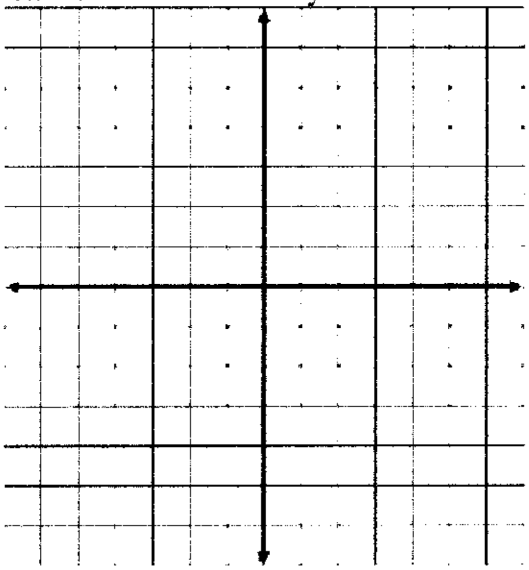
same slope
same y-intercept



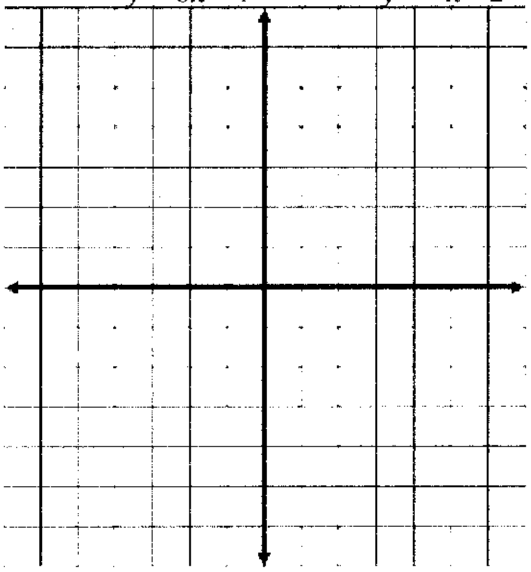
The lines are [ ] so there are [ ] solutions.

Practice

$y = 3x - 6$ $y = 6x - 4$

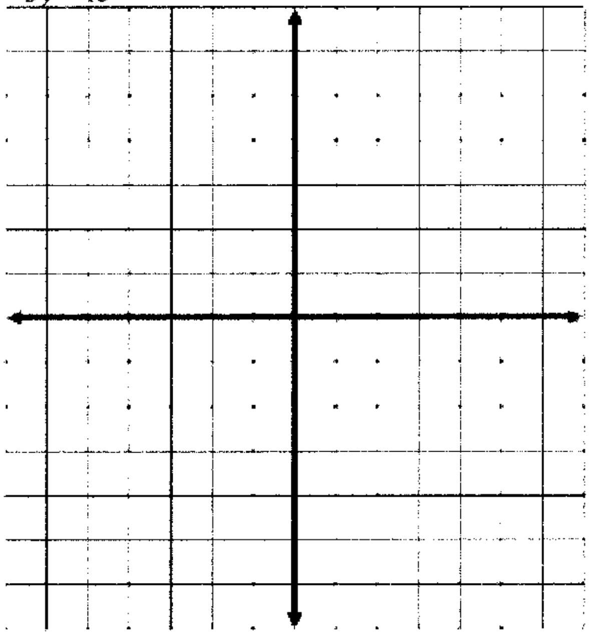


$y = 6x - 1$ $y = -x - 2$



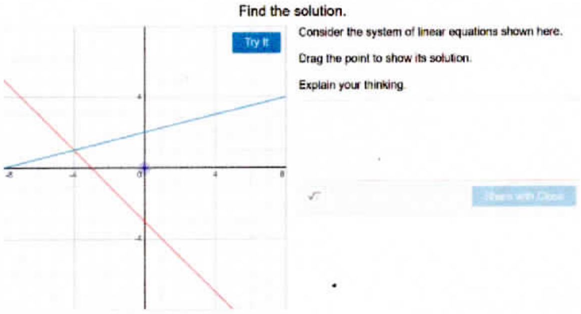
$y = 2x - 5$

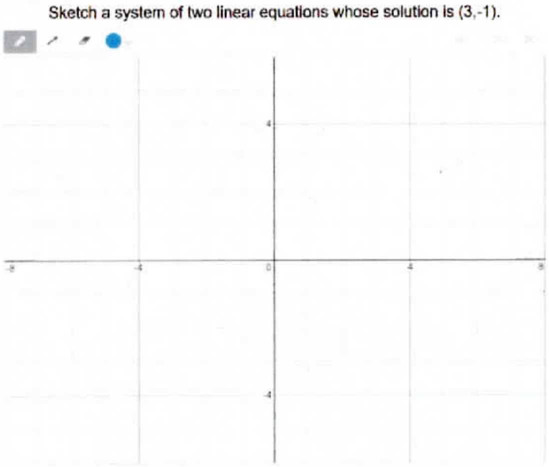
$6x - 3y = 15$

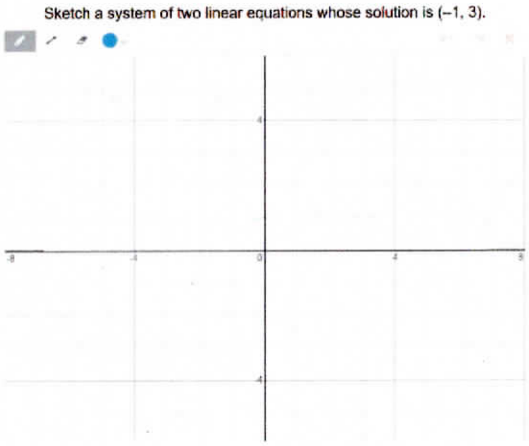


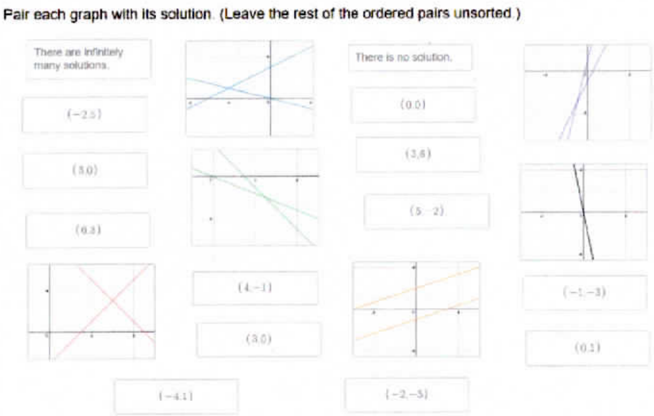
Online Activity #1: Desmos Lesson – Graphing a system of equations Screen Shots

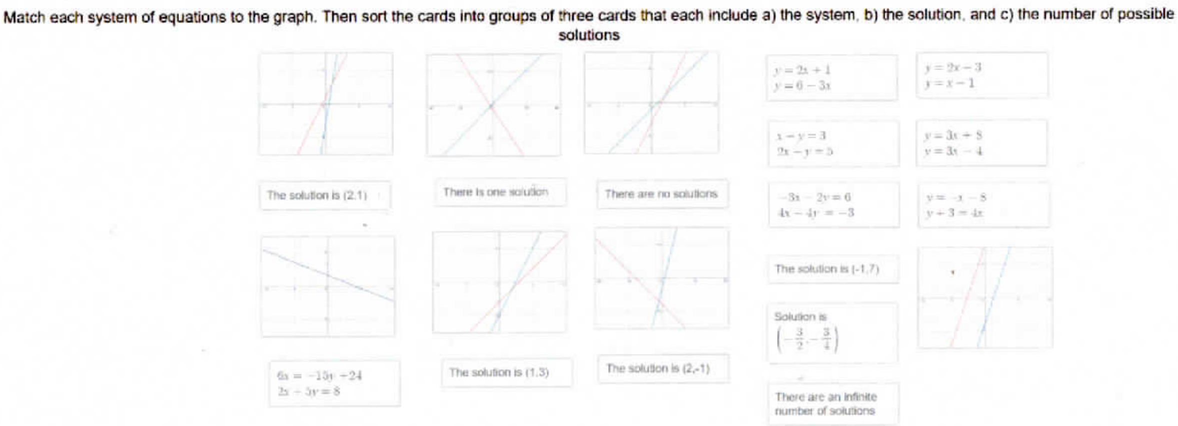
Students need to graph a pair of lines that goes through a specific point that is provided. The images are from https://student.desmos.com/?prepopulateCode=rb3xpc





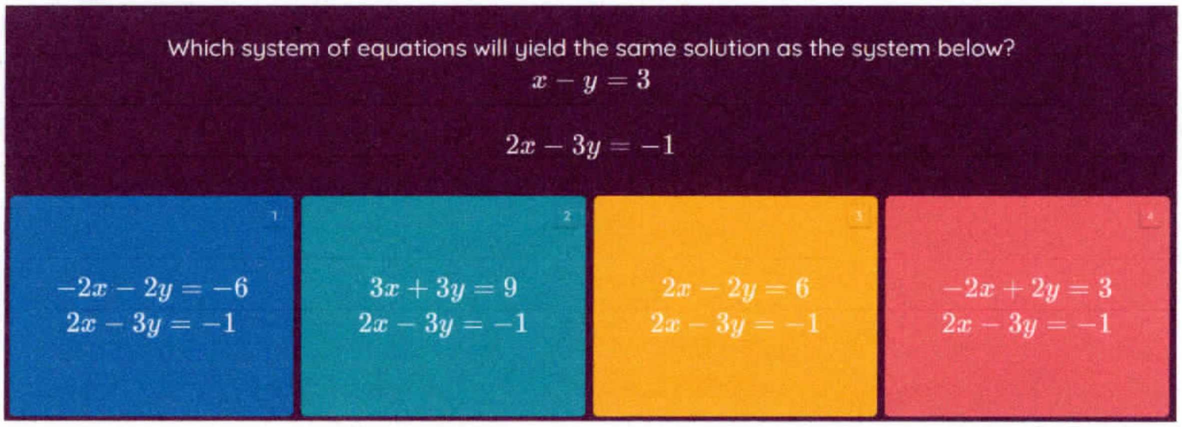






Quizizz Activity and Screen Shots

Quizizz (can be done individually or as a class) as it provides every student an opportunity to answer a question that is projected onto a screen (either a laptop or on the board in front of the room). Students do not need to a specific code for the game. They are required to read each question and answers available to select the correct answer. Students get instant feedback when time expires (if the timer option is chosen) if they were correct or not. Teachers can receive statistics on how each student performed on each question and provide additional support to those who may need it. The images and questions are from (link).



This is what the student sees on their screen or projected in the front of the room.

Here are the questions included in the Quizizz activity.

QUIZIZZ ✔ All changes saved

Quiz Editor ⊕ New question Teleport

Question 1 Edit

Which system of equations does NOT have the same solution as the system below?

$3x + 5y = 10$

$-x + 4y = -3$

answer choices

- $30x + 50y = 100$, $-10x + 40y = -30$
- $18x + 30y = 60$, $-6x + 24y = -18$
- $6x + 10y = 20$, $-3x + 12y = -3$
- $-15x - 25y = -50$, $-\frac{1}{2}x + 2y = -\frac{3}{2}$

45 Seconds

Question 2 Edit

Which system of equations will yield the same solution as the system below?

$x - y = 3$

$2x - 3y = -1$

answer choices

- $-2x - 2y = -6$, $2x - 3y = -1$
- $-2x + 2y = 3$, $2x - 3y = -1$
- $2x - 2y = 6$, $2x - 3y = -1$
- $3x + 3y = 9$, $2x - 3y = -1$

45 Seconds

Question 3

Edit

Lizzy has 30 coins that total $4.80. All of her coins are dimes, *D*, and quarters, *Q*. Which system of equations models this situation?

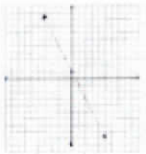
answer choices

- $D + Q = 4.80$
  $.10D + .25Q = 30$
- $D + Q = 30$
  $.10D + .25Q = 4.80$
- $D + Q = 30$
  $.25D + .10Q = 4.80$
- $D + Q = 4.80$
  $.25D + .10Q = 30$

45 Seconds

Question 4

Edit



How many solutions are there to the following system of equations?

$$y + 2x = 1$$
$$y - 1 = -2x$$

answer choices

- One Solution
- No Solution
- Infinite Solutions

45 Seconds

Question 5

Edit

How do we know if a system of equations has no solution?

answer choices

- The lines are not in slope-intercept form (y=mx+b)
- The lines have the same slope and y-intercept
- The lines are perpendicular
- The lines are parallel

45 Seconds

Quiz Editor

⊕ New question   Teleport

Question 6

Edit

Which system of equations has the same solution as the system below?

$2x + 2y = 16$

$3x - y = 4$

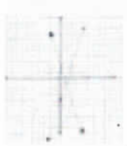
answer choices

- $2x + 2y = 16$, $6x - 2y = 8$
- $6x + 6y = 48$, $6x + 2y = 8$
- $x + y = 16$, $3x - y = 4$
- $2x + 2y = 16$, $6x - 2y = 4$

45 Seconds

Question 7

Edit



What is the solution to the following system of equations?

answer choices

- (-1,-1)
- (1,1)
- (1,-1)
- (-1,1)

30 Seconds

Quiz Editor   New question   Teleport

Question 8   Edit

Which of these systems has an infinite number of solutions?

answer choices

- $y = -2x + 1$
  $y = -2x - 3$
- $y = 3x + 4$
  $-12x + 4y = 16$
- $x + 2y = 10$
  $2x + 4y = 10$
- $y = 2x + 6$
  $4x - 2y = 8$

60 Seconds

Question 9   Edit

Is (-1,5) a solution to this system of equations?

answer choices

- $x + y = 4$
  $x = -1$
- $y = -x + 4$
  $y = -\frac{1}{5}x$
- $y = 5$
  $y = x - 6$
- $y = x + 6$
  $x = 2y + 7$

45 Seconds

Question 10   Edit

Which ordered pair is the solution of the system?

$6x - 6y = 2$

$3x + 9y = -7$

answer choices

- $\left(\frac{2}{3}, -\frac{1}{3}\right)$
- $\left(\frac{1}{3}, \frac{2}{3}\right)$
- $\left(-\frac{2}{3}, \frac{1}{3}\right)$
- $\left(-\frac{1}{3}, -\frac{2}{3}\right)$

Lesson 2

Lesson Title:	Graphing Linear Inequalities
Standards:	**A.REI.D.12** – Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solutions set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Learning Goals:	I can identify solutions to two-variable inequalities. I can determine whether a given value is a solution to a given linear inequality. I can determine the inequality symbol correctly to determine which portion of the coordinate grid is shaded to represent the solution.
Technology / Manipulatives Used:	• Laptop / Chromebook / Tablet • Edpuzzle Online Notes (https://edpuzzle.com/media/5e9b77d91dd7413f018e380a) • Desmos Teacher Activity (https://student.desmos.com/?prepopulateCode=6wfc4m) • Quizzizz Practice Assessment (https://quizizz.com/join/quiz/5e7d5b81508b28001c8bf911/start?referrer=5e667f551d5b8f001c1a8354)
Timeline:	55 minutes 5 minutes – Check and go over homework, Hand out Note Sheet 15 minutes – Student watch lesson on solving a system of inequalities note video through Edpuzzle. Introduces how to graph a single inequality and how graph changes based upon inequality symbol. The final example put two inequality graphs together and students will see what happens when inequality graphs overlap and how to find a point in the solution set. * 20 minutes – Students will participate with the Desmos activity in small groups/partners. They are given a system of inequalities on a graph and have to answer questions for their partner to guess which system of inequalities they have chosen. Similar to the children's game "Guess Who". Once the round is over students have to provide a point that works in each inequality only and also a point that works for both inequalities. * 5 minutes – Reflection on describing parts of a graphing a system of inequalities 10 minutes – Quizizz activity on systems of Inequalities, students will complete this as a whole group. * Teacher should be moving through the classroom while students working on the activities to offer support to those who need individual assistance.

Graphing 1st degree inequalities in 2 variables Date____________________

To graph Inequalities:

1)____

2)____

3)____

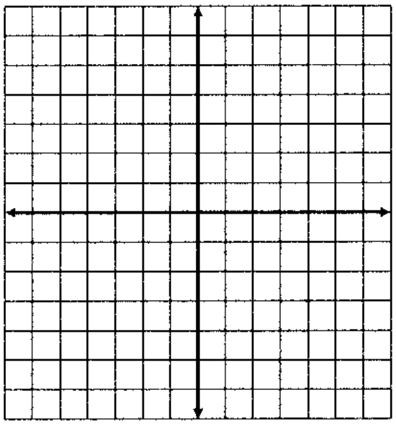
4)____

5) ** The Golden Rule of Inequalities – When multiplying or dividing by a negative, flip the inequality sign.

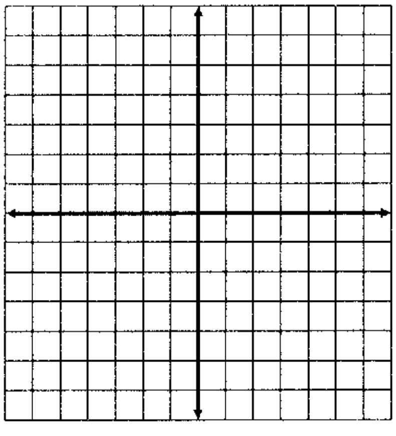
Practice: Graph the following inequalities

NOTE: $>$ or $<$ is dashed line
$\geq$ or $\leq$ is solid line

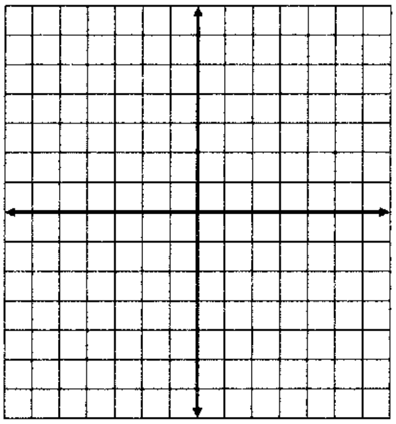
1. $y = 2x-1$



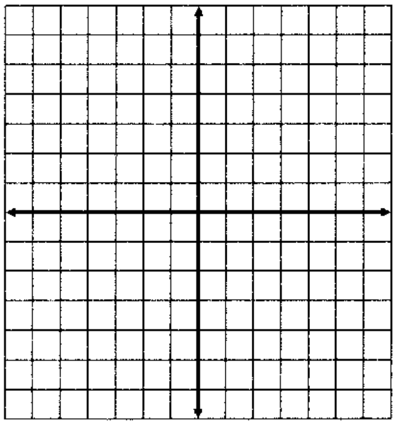
2. $y < 2x$



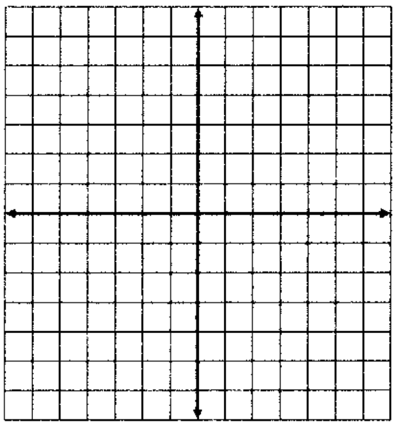
3. $y \leq 2x - 1$



4. $y > 2x$ 1



5. $y \geq x - 1$



6. $x \geq 3$
$y < 2$

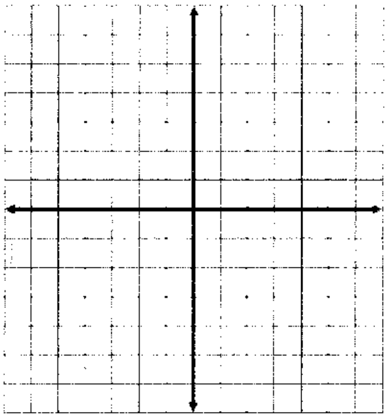
Name a point that works for only $x \geq 3$:

Name a point that works for only $y < 2$:

____

Name a point that works for both inequalities:

____



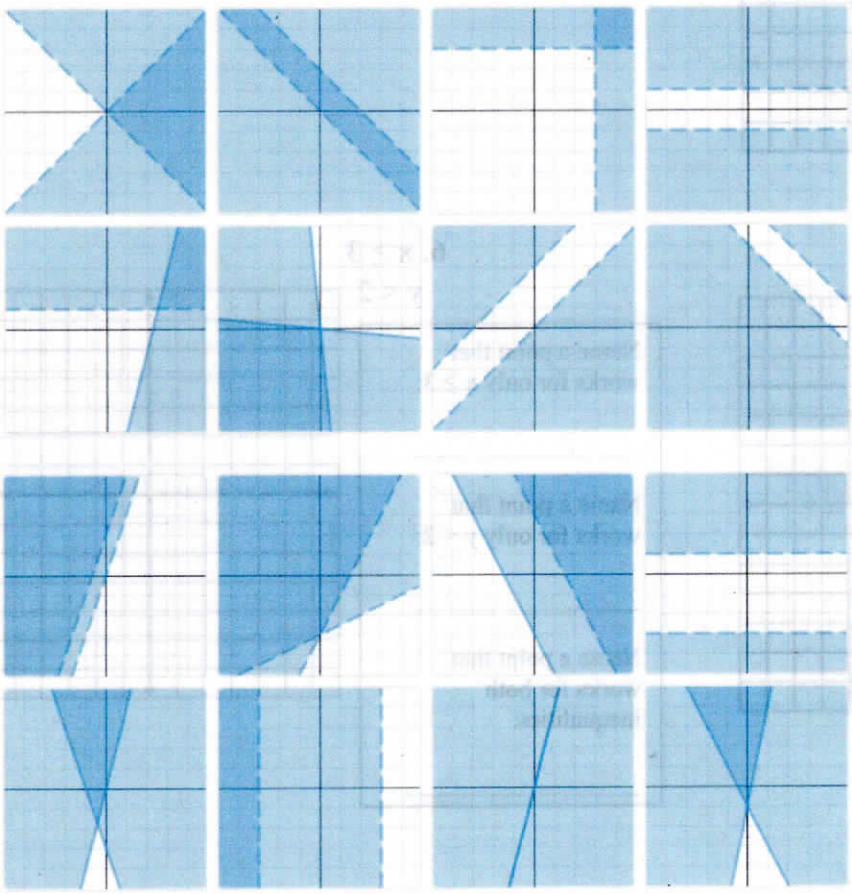
Desmos Application Investigation Screen Shots:

This application is designed to spark vocabulary-rich conversations about systems of linear inequalities. Key vocabulary that will be discussed in this lesson includes, but are not limited to shade, region, above, below, overlap, solution set, parallel.

In the exercise students will notice graph features from the vocabulary listed above. A class discussion can evolve from this to discuss their use of academic language during this exercise and how the graphs interact with each other on the graph. Students will begin by selecting a graph of inequalities from a group and the student they are paired with begin to ask questions to figure out which inequality graph their opponent had chosen. Sample questions that might ask their partner:

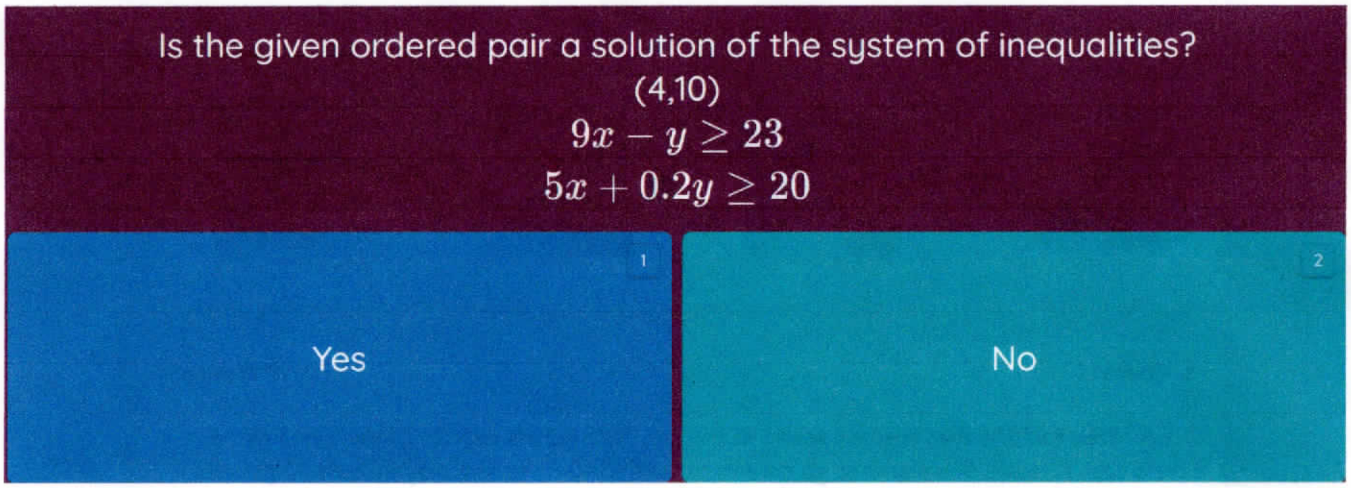
- Does your system of inequalities include the origin in the solution set?
- Do your inequalities have two dotted (dashed) lines?

The Cards



Quizizz Activity and Screen Shots:

Quizizz (can be done individually or as a class) as it provides every student an opportunity to answer a question that is projected onto a screen (either a laptop or on the board in front of the room). Students do not need to a specific code for the game. They are required to read each question and answers available to select the correct answer. Students get instant feedback when time expires (if the timer option is chosen) if they were correct or not. Teachers can receive statistics on how each student performed on each question and provide additional support to those who may need it. The images and questions are from (https://quizizz.com/join/game/U2FsdGVkX19gp1b0XM%252BhG3s8WLzhsEL4kEIefvL2YuoZ4njuVyHr4CDh3hNJAjI3?gameType=solo).



This is what the students see on their screen or projected in the front of the room.

These are the questions that were included in the activity.

Find a quiz   Create a new quiz

Start a multiplayer game

Play Live   Assign HW

Solo Practice

Practice

Preview (8 questions)   HIDE ANSWERS

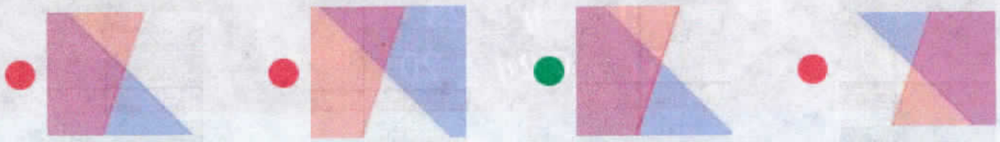
Question 1   45 seconds

Q. Which graph shows the solution of the given set of inequalities?

$y < -x + 2$

$y \geq 3x - 2$

answer choices



Question 2   60 seconds

Q. Gretchen has $50 that she can spend at the fair. Ride tickets cost $1.25 each and game tickets cost $2 each. She wants to go on a minimum of 10 rides and play at least 12 games.

Which system of inequalities represents this situation when r is the number of ride tickets purchased and g is the number of game tickets purchased?

answer choices

- $1.25r + 2g < 50$ $r \leq 10$ $g > 12$
- $1.25r + 2g \leq 50$ $r \geq 10$ $g \geq 12$
- $1.25r + 2g \leq 50$ $r \geq 10$ $g > 12$
- $1.25r + 2g < 50$ $r \leq 10$ $g \geq 12$

Find a quiz   Create a new quiz

Play Live   Assign HW   Practice

Question 3   60 seconds

Q. State the system of inequalities represented by the graph.

answer choices

- $y > 2x - 2$
 $y \leq -3x + 3$
- $y \geq 2x - 2$
 $y > -3x + 3$
- $y < 2x - 2$
 $y \geq -3x + 3$
- $y \leq 2x - 2$
 $y < -3x + 3$

Question 4   60 seconds

Q. Which point is a solution to both inequalities?

answer choices

- (2,2)
- (2,0)
- (0,2)
- (1,0)

Question 5   30 seconds

Q. Which of the following is true of the graph of $y \geq -x + 1$

answer choices

- The line is solid, and the shading is above the line
- The line is dashed, and the shading is above the line
- The line is solid, and the shading is below the line
- The line is dashed, and the shading is below the line

Find a quiz   Create a new quiz

Play Live   Assign HW   Practice

Question 6   45 seconds

Q. The graph of which of the following is shaded above the line?

answer choices

- $x + y < 9$
- $y - x < 9$
- $x + y < -9$
- $x - y < 9$

Question 7   60 seconds

Q. Is the given ordered pair a solution of the system of inequalities?

(4,10)

$$9x - y \geq 23$$
$$5x + 0.2y \geq 20$$

answer choices

- Yes
- No

Question 8   60 seconds

Q. Which point is a solution of the following system?

$y > x$ $y < 3x - 4$

#1 (1,2)   #2 (3,4)   #3 (3,9)

answer choices

- #1 only
- #1 and #2
- #1 and #3
- #2 only

Lesson 3

Lesson Title:	Solving Systems of Equations Algebraically (Elimination (Addition) Method and Substitution Method)
Standards:	**A. REI. C. 6** – Solve systems of linear equations exactly and approximately (e.g. with graph), focusing on pairs of linear equations in two variables
Learning Goals:	I can analyze and solve problems whose solutions requires solving systems of linear equations in two variables. I can solve systems of two linear equations in two variables algebraically.
Technology / Manipulatives Used:	• Laptop / Chromebook / Tablet • Edpuzzle Online Notes (https://edpuzzle.com/media/5e8501141c2ebf3fa45499b1) • Desmos Teacher Activity (https://student.desmos.com/?prepopulateCode=mk7axf)
Timeline:	55 minutes 5 minutes – Check and go over homework. Hand out Note Sheet 15 minutes – Student watch lesson on solving a system of equations algebraically note video through Edpuzzle. Solving systems through the elimination method and the substitution method. 20 minutes – Have students complete the Desmos activity to test systems of equations that includes some review from graphing systems of equations, then a card sort that helps students determine which method of solving the system would be easiest. * 15 minutes – Students that complete Desmos lesson, will continue to work on examples in note packet, reinforcing algebraic methods to solve a system of equations. * Teacher should be moving through the classroom while students working on the activities to offer support to those who need individual assistance.

Solving systems by elimination Date________________

We said that solving a system of linear equations means to find the

____ between the lines.

There are two methods to solve a system of equations algebraically:

1)____

2)____

Method 1: Addition (Elimination)

The purpose of the addition method is to ________________________ one of the variables by adding (combining) the two equation. IT DOESN'T MATTER WHICH ONE YOU ELIMINATE.

To Eliminate one of the variables, we must have the same___________________

and opposite _________________________

Example: Solve the following system of equations using addition/elimination.

1) $5x - 6y = 32$
   $3x + 6y = 48$

2) $2x + 5y = -22$
$10x + 3y = 22$

Solving systems of equations by substitution Date___________________

Method : SUBSTITUTION

We also use the Substitution Method to solve Systems of Equations. The purpose is to solve for ____________________ variable (either x or y) in one equation.

Then, we ____________________________________ that variable in other equation.

*****NOTE: SOLVE FOR THE EASIER VARIABLE*****

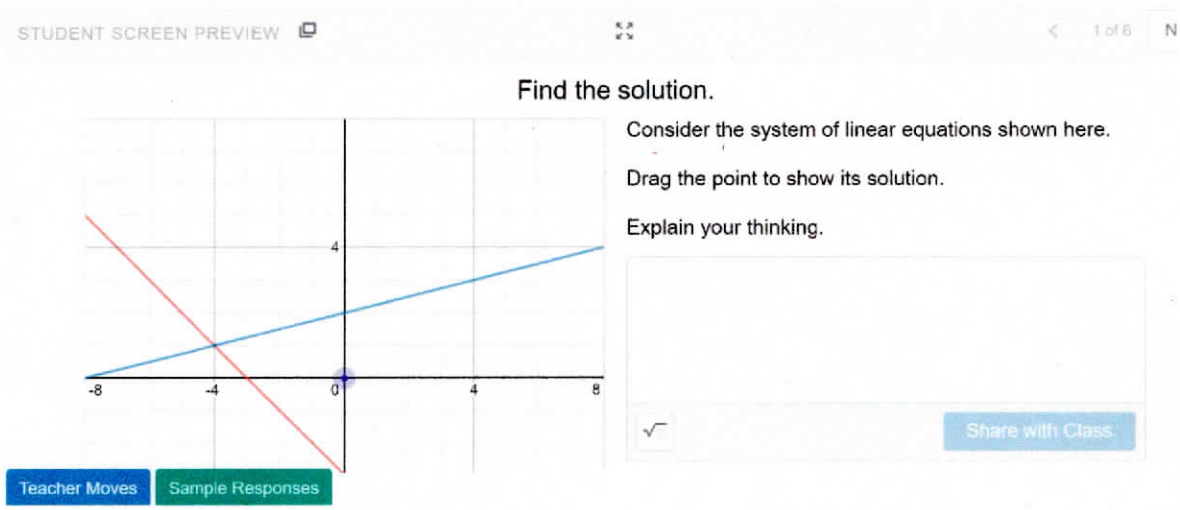
Practice: Solve the System of Equations Using Substitution.

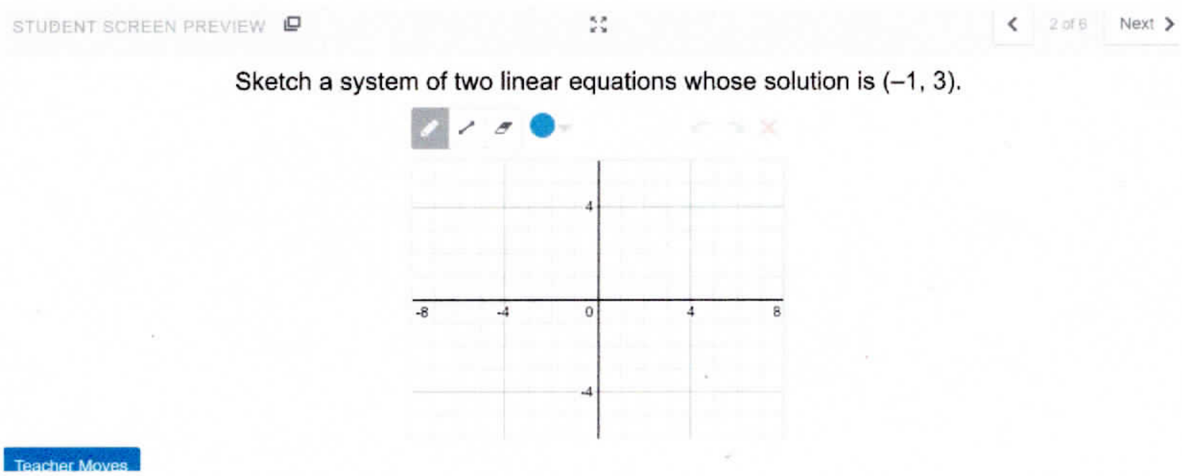
1) $3x - y = -4$
   $Y = x + 2$

Desmos Application Screen Shots:

In this activity, students practice what they have learned about solving systems of linear equations. The activity begins with a review of the graphical meaning of a solution to a system. Later in the lesson, students consider which algebraic method is most efficient for solving a given system between substitution and elimination. Finally, students practice solving equations using substitution and elimination.

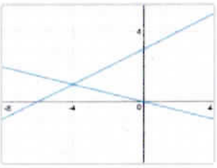
Before students begin this activity, students should have experience solving systems of linear equations graphically and algebraically.





Pair each graph with its solution. (Leave the rest of the ordered pairs unsorted.)

$(-2,-5)$ | $(4,-1)$ | $(3,6)$ | $(3,0)$ | $(3,0)$

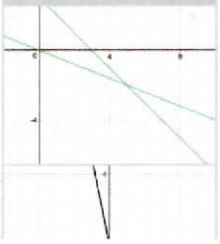


$(-4,1)$

There are infinitely many solutions.

$(6,3)$

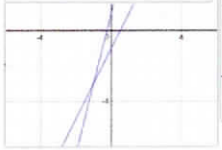
3 CARDS



$(5,-2)$

$(-2,5)$

$(-1,-3)$



There is no solution.

$(0,1)$

$(0,0)$

Teacher Moves

Sort these systems according to the algebraic method you would prefer to use.

$3x - 2y = -7$
$2x + 5y = 8$

$y = 3x - 1$
$2x + 4y = 24$

Solve by Elimination

$x + 3y = 7$
$2x + 3y = 11$

Solve by Substitution

$y = 2x + 1$
$y = 6 - 3x$

Teacher Moves

Explain

Explain the thinking behind the way you sorted the cards on the previous screen.

In particular, when do you prefer solving by substitution? When do you prefer solving by elimination?

√ Share with Class

How This Worked in the Classroom

The project of incorporating technology into the curriculum of systems of linear equations and inequalities worked to varying degrees. Students were able to spend more time understanding how inequality graphs changed depending upon the sign and which side of the inequality to shade. Students had mixed reactions by the practice questions in Quizizz. Students that process information at a slower rate, felt timed questions in Quizizz forced them to rush through a problem without being able to check their work. However, students enjoyed the Desmos lessons because it allowed for them to work at their own pace, without losing their focus. They were able to move through the content more efficiently.

Students have desks in groups (3 or 4) and for the activities that required student interactions with each other, students worked with their desk mates. Also, having students build their confidence through independent activities such as the online notes or the teacher activity through Desmos. The teacher was able to connect with students individually that required additional instruction.

Teachers may want to consider notes as a class and then break off into small group settings for activities. This would ensure that students are hearing the same material at the same moment. It would eliminate the possibility of students skipping through the video or watching the video at a quicker speed, allowing for more time on the class activity with the virtual manipulatives. In the Desmos Lesson, these manipulatives for graphing inequalities or graphing two lines could be done without technology if needed.

Conclusion

However, in the light of remote instruction during the COVID-19 pandemic. Having additional opportunities for students to develop a deeper understanding of material outside of the traditional classroom environment is crucial. The hope is that teachers can use virtual manipulatives and this as an exemplar to continue to integrate virtual manipulatives into our mathematics instruction on a regular basis. Providing technological resources to students, will promote engagement in the content and develop a deeper understanding with the material.

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Day 1: Solving systems of equations graphically Date____________

A system of linear equations is two or more linear equations using the same variables

A solution of a system of linear equations is any ordered pair that makes all of the inequalities in the system true.

Points of Intersection (POI) are the same thing as the solutions of a system.

No solution means lines are parallel. They have the same slope, different y-intercepts

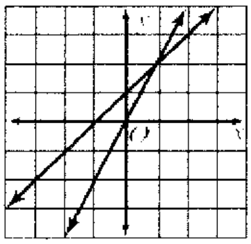
A system of equations has infinitely many solutions when the two lines have the same slope and same y-intercept.

Practice:

Vocabulary and Key Concepts

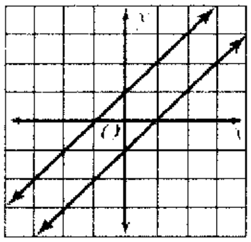
Numbers of Solutions of Systems of Linear Equations

different slopes



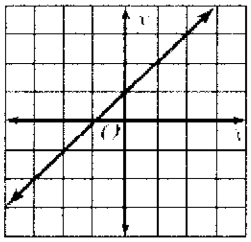
The lines intersect so there is 1 solution.

same slope
different y-intercepts



The lines parallel so there are ∅ solutions.

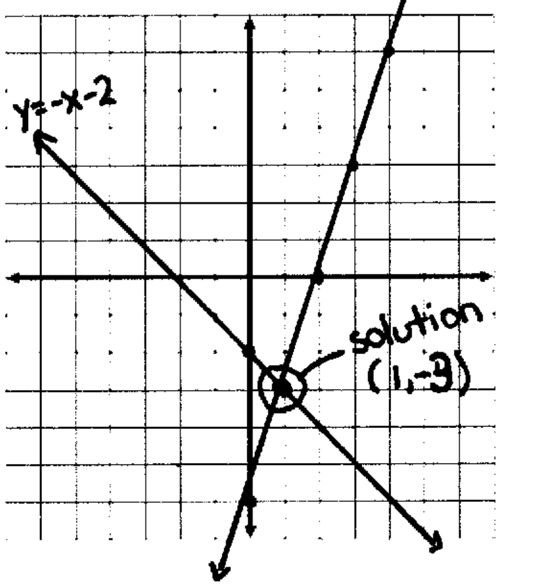
same slope
same y-intercept



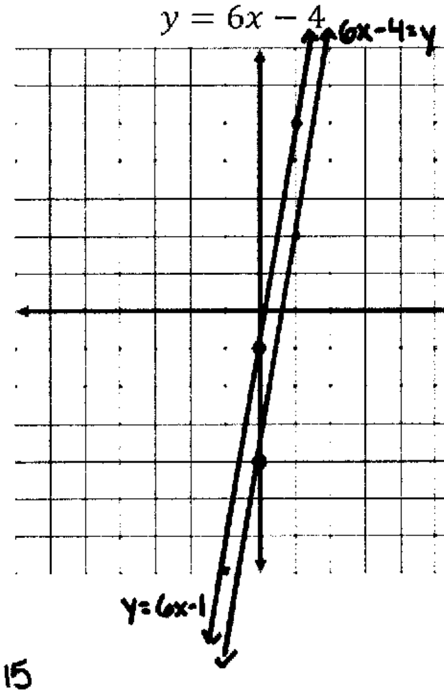
The lines are the same so there are infinitely many solutions.

Practice

$y = 3x - 6$
$y = -x - 2$

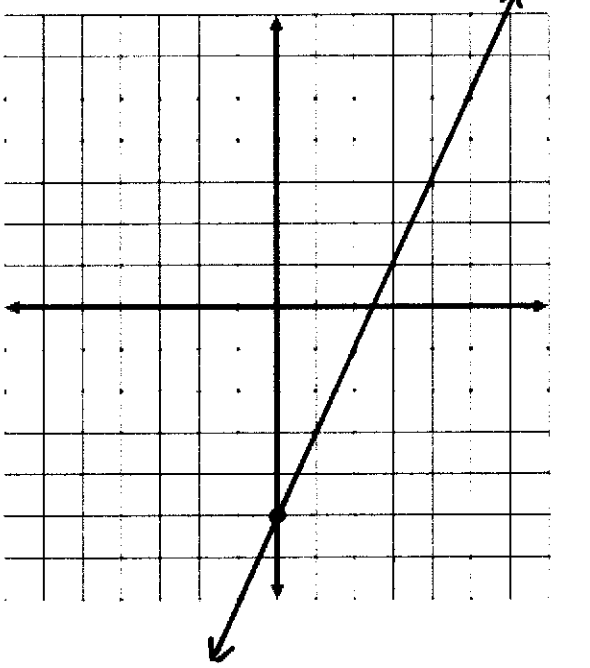


$y = 6x - 1$
$y = 6x - 4$



No solution

$y = 2x - 5$
$6x - 3y = 15$



$6x - 3y = 15$
$-6x \qquad -6x$

$\frac{-3y}{-3} = \frac{-6x}{-3} + \frac{15}{-3}$

$y = 2x - 5$

infinitely many solutions

Graphing 1st degree inequalities in 2 variables Date_______________

To graph Inequalities:

1) make sure the inequality has 'y' alone on the left side of eq.

2) if sign is > or ≥, shade up / if sign is ≤ or <, shade down

3) graph second inequality <repeating steps 1 and 2.

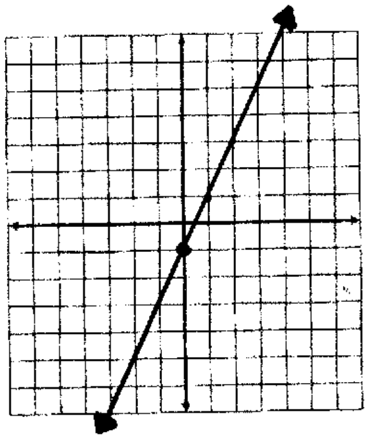
4) label the overlapping shaded are with an S.

5) ** The Golden Rule of Inequalities - When multiplying or dividing by a negative, flip the inequality sign.

Practice: Graph the following inequalities

1. $y = 2x-1$

NOTE: > or < is dashed line
≥ or ≤ is solid line



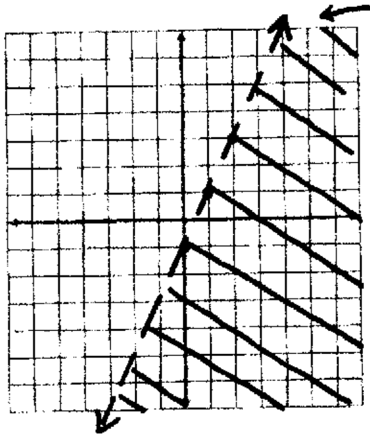
X	Y
0	-1
1	1
2	3

$m = \frac{2}{1}$

b = (0,-1)
y-int

Ck

2. $y < 2x-1$



Dotted or Dashed

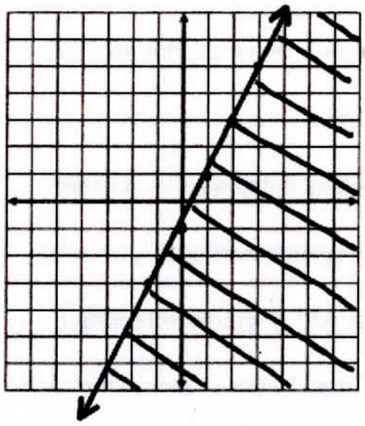
$m = \frac{2}{1}$

b = (0,-1)
y-int

Ck. Test point (0,0)

y < 2x-1
0 < 2(0)-1
0 < 0-1
0 < -1
X

3. $y \le 2x - 1$



$m = \frac{2}{1} = 2$

$b = (0, -1)$

Ck. Test Point (0,0)

$y \le 2x - 1$

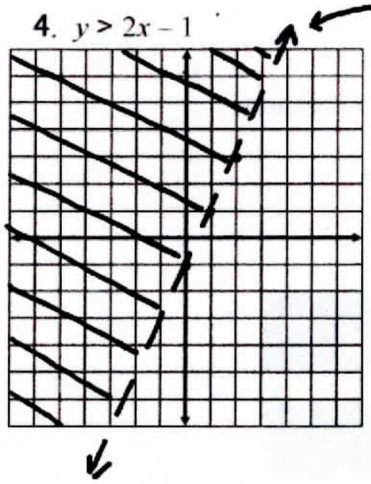
$0 \le 2(0) - 1$

$0 \le 0 - 1$

$0 \le -1$

×

4. $y > 2x - 1$ ← Dotted/Dashed



$m = \frac{2}{1} = 2$

$b = (0, -1)$

Ck. Test Point (0,0)

$y > 2x - 1$

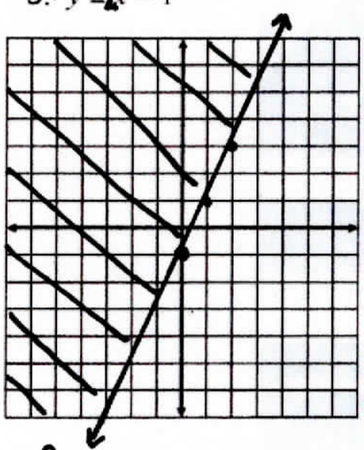
$0 > 2(0) - 1$

$0 > 0 - 1$

$0 > -1$

✓

5. $y \ge 2x - 1$



$m = \frac{2}{1} = 2$

$b = (0, -1)$

6. $x \ge 3$
$y < 2$

Name a point that works for only $x \ge 3$:

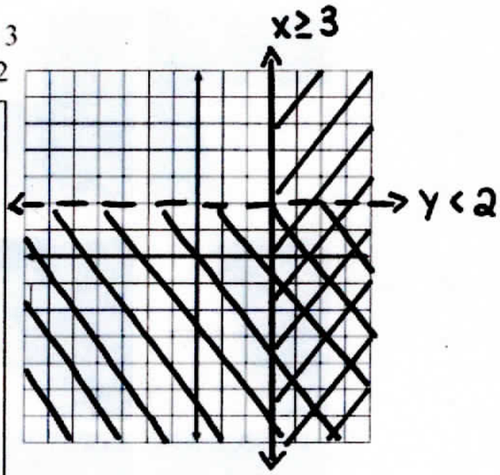
(5,5)

Name a point that works for only $y < 2$:

(0,0)

Name a point that works for both inequalities:

(6,0)



Solving systems by elimination Date_______________

We said that solving a system of linear equations means to find the

solution between the lines.

There are two methods to solve a system of equations algebraically:

1) Elimination

2) Substitution

Method 1: Addition (Elimination)

The purpose of the addition method is to cancel one of the variables by adding (combining) the two equation. IT DOESN'T MATTER WHICH ONE YOU ELIMINATE.

To Eliminate one of the variables, we must have the same coefficients

and opposite signs.

Example: Solve the following system of equations using addition/elimination.

1) $5x - 6y = 32$
$+ \; 3x + 6y = 48$

$8x = 80$

$\frac{8x}{8} = \frac{80}{8}$

$x = 10$

$5x - 6y = 32$

$5(10) - 6y = 32$

$50 - 6y = 32$

$-50 \qquad -50$

$-6y = -18$

$\frac{-6y}{-6} = \frac{-18}{-6}$

$y = 3$

Ck

$3(10) + 6(3) = 48$

$30 + 18 = 48$

$48 = 48$

✓

solution → (10, 3)

2) $2x + 5y = -22$
$10x + 3y = 22$

$-5(2x + 5y = -22) \rightarrow -10x - 25y = 110$

$$\begin{array}{r} -10x - 25y = 110 \\ \oplus\ 10x + 3y = 22 \\ \hline -22y = 132 \end{array}$$

$\frac{-22y}{-22} = \frac{132}{-22}$

$y = -6$

$10x + 3y = 22$

$10x + 3(-6) = 22$

$10x - 18 = 22$

$+18 \quad +18$

$10x = 40$

$\frac{10x}{10} = \frac{40}{10}$

$x = 4$

Solution $(4, -6)$

Ck. $2x + 5y = -22$

$2(4) + 5(-6) = -22$

$8 - 30 = -22$

$-22 = -22$

Solving systems of equations by substitution Date________________

Method : SUBSTITUTION

We also use the Substitution Method to solve Systems of Equations. The purpose is to solve for __one__ variable (either x or y) in one equation.

Then, we __substitute__ that variable in other equation.

*****NOTE: SOLVE FOR THE EASIER VARIABLE*****

Practice: Solve the System of Equations Using Substitution.

1) $3x - y = -4$

$y = x + 2$

$y = \boxed{x+2}$

$3x - y = -4$

$3x - (x+2) = -4$

$3x - x - 2 = -4$

$2x - 2 = -4$

$+2 \quad +2$

$\frac{2x}{2} = \frac{-2}{2}$

$x = -1$

$y = x + 2$

$y = -1 + 2$

$y = 1$

Solution

$(-1, 1)$

Check $3x - y = -4$

$3(-1) - 1 = -4$

$-3 - 1 = -4$

$-4 = -4$

✓