Recent Trends in Social Network Analysis

Report on the 31st Dokkyo International Forum 2019

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1. Introduction

The 31st Dokkyo International Forum 2019 was held at Dokkyo University on November 16-17, 2019. During the first day, Dr. Steglich made a keynote speech with seven presenters followed. During the second day, we had the panel discussion. Approximately 130 people participated in the forum. The present report aims to provide a self-contained review of this forum. It is organized as follows: In Section 2, a brief overview of the two major statistical models is presented. After that, we present the reports describing the first and second days of the forum. In the last section, abstracts of all presentations are listed.

2. Brief Review of Statistical Models used in Social Networks Analysis

2.1. Exponential Random Graph Models (ERGMs) and Stochastic Actor-oriented Models (SAOMs)

In social network analysis, there are two major statistical models, namely, Exponential Random Graph Models (ERGMs) and Stochastic Actor-oriented Models (SAOMs). In this section, we clarify the similarities and differences between these two models.

Their common features are as follows: both models examine factors, which determine an existing network. The factors can include small network configurations (for example, reciprocity, transitive triplets, and others), as well as individual attributes. These are included in a model as independent variables. Despite these similarities, the theoretical settings behind the two models are different. ERGMs are used to estimate the probability function of a network. In turn, in SAOMs, the myopic optimizing behavior for agents is estimated, and this behavior creates the whole network.

To enable a deeper understanding of these concepts, we provide a more detailed explanation for both ERGMs and SAOMs using mathematical notations.

Concerning a basic ERGM, we first define undirected networks.[1] Let $N \equiv \{1, 2, \cdots, n\}$ be a set of nodes, and E be a set of all possible (undirected) edges, i.e., $E \equiv \{\{i, j\}: i, j \in N,\ i \neq j\}$. Then, let $|E|$ be a total number of elements in E.[2] To simplify the expressions, we relabel an element in E and express E as $\{e_1, e_2, \cdots, e_{|E|}\}$. Let us assume that $e_i \in E$ is a random variable that takes 0 or 1. The value 1 means that the corresponding edge exists, otherwise, it does not. Network g is defined by a subset of edges (E), in which each of them takes 1,

1 In the present report, we use the term "network." However, the term "graph" is also commonly used in mathematical expressions related to networks. Usually, these two terms are interchangeable.
2 Nodes can be referred to as "vertices" or "points." Edges can be denoted as links" or "lines." Directed edges are also called "arcs." In a more specific context, "edges" can be denoted as "ties." In Section 2, the term "edge" is used consistently.
3 This definition is provided by Frank and Strauss (1986). In addition, the matrix expression of a network is also widely used, i.e., $\boldsymbol{g} \equiv \{g_{ij}\}$, where g_{ij} is a (i, j) element of a matrix and a directed edge from node i to node j. This expression could be better for understanding of SAOMs in the present report.

i.e., $g \equiv \{e_i : e_i \in E$ and $e_i = 1\}$.[3] Finally, let G be a set of networks g.

According to Besag et al. (1974) and Frank and Strauss (1986), a probability function of the network $P(G = g)$ is expressed as follows:

$$P(G = g) = c^{-1}\, Exp\, Q(g)$$

with

$$c = \sum_{g \in G} Exp\, Q(g)$$

where

$$Q(g) = \log P(G = g) - \log P(G = g^0)$$

and g^0 is a fixed network, for example, an empty network. It should be noted that $Q(g)$ is a weight of network g in the probability function; and c^{-1} is an appropriate weight defined so that $P(G = g)$ satisfies the properties of the probability function. Furthermore, by applying the inclusion-exclusion principle, it can be shown that:

$$Q(g) = \sum_{e_i \in E} e_i \cdot \theta_i + \sum_{\substack{e_i, e_j \in E \\ e_i \neq e_j}} e_i \cdot e_j \cdot \theta_{i,j} + \sum_{\substack{e_i, e_j, e_k \in E \\ e_i \neq e_j \neq e_k}} e_i \cdot e_j \cdot e_k \cdot \theta_{i,j,k} + \cdots + e_1 e_2 \cdots e_{|E|} \theta_{1,2,\ldots,|E|}$$

where $\theta_{\{i,\ldots,k\}}$ is a coefficient of the corresponding (possible) network configuration $\{e_i,\cdots,e_k\}$. It should be noted that the existence of a network $\{e_i,\cdots,e_k\}$ means that all $e_i,\cdots,e_k$ take 1, and then, $\theta_{\{i,\ldots,k\}}$ could affect $Q(g)$. Evidently, if $\theta_{\{i,\ldots,k\}} = 0$, and then, the existence of network (or configuration) $\{e_i,\cdots,e_k\}$ does not affect $Q(g)$ regardless of whether the network exists or not. As a result, all possible network configurations can be included in the weight function $Q(g)$ (Robins, Pattison, et al. 2007:178-88).

Obviously, an ad-hoc introduction of the network configuration into a model is undesirable. In this sense, Frank and Strauss (1986) show that when simple and reasonable dependences among edges are assumed, only k-star and triangle configurations are possible in $Q(g)$.[4] In other words, sufficient information for a probability of a network is the number of dyads, stars, and triangles, as follows:

$$P(G = g) = c^{-1} \cdot Exp\left\{ \sum_{k=1}^{|E|-1} \theta_k s_k + \theta_t\, t \right\}$$

where s_k is a number of k-stars, and t is a number of triangles.

In addition, we can include other factors related to node attributes. Let $\boldsymbol{A}$ be a set of node attributes, and $\boldsymbol{a}$ be an observed value of $\boldsymbol{A}$; then, we have the following formula:

$$P(G = g | \boldsymbol{A} = \boldsymbol{a}) = c^{-1} \cdot Exp\left\{ Q(g) + \sum_k \eta_k \cdot s_k(g, \boldsymbol{a}) \right\} \tag{1}$$

where η_k is the coefficient of kth variable related to node attributes, $s_k(g, \boldsymbol{a})$,[5] and c^{-1} is an appropriate weight defined so that $P(G = g|X = x)$ satisfies the properties of the probability function (Robins, Pattison, et al. 2007:185). It should be noted that in general, any term in $Q(g)$ is also expressed according to the notation of $s_k(g, \boldsymbol{a})$. Then, a simple expression is obtained as follows:

4 This dependence is referred to as Markov dependence, which implies that the probabilities of two edges are dependent when they share the same node. The definition of k-star implies that there is k periphery in the star. By definition, if k is equal to one, then k-star means an edge.

5 Information related to a network can be included as this variable, e.g., the sum of attribute values of nodes connected by a concerned node directly.

$$P(G = g|\boldsymbol{A} = \boldsymbol{a}) = c^{-1} \cdot Exp\left\{\sum_k \eta_k \cdot s_k(g, \boldsymbol{a})\right\} \tag{2}$$

where η_k is the coefficient of kth variable in which both variables related to network configurations and node attributes can be included.

SAOMs are described as follows. Nodes are interpreted as agents which make a choice for creating or cutting an edge myopic optimally. As agent i changes an edge to agent j, a network is constructed by "directed" edges.

A probability that actor i changes an edge to actor j is based on a preference function corresponding to agent i. Evidently, both a position of an agent in a network g and attributes $\boldsymbol{a}$ can affect their preference function. Similarly, as variables in ERGMs, we can express these variables as $s_{i,k}(g, \boldsymbol{a})$. Then, we have the following preference function for agent i :

$$f_i(g, \boldsymbol{a}) = \sum_k \beta_k \, s_{i,k}(g, \boldsymbol{a}) \tag{3}$$

where β_k is the coefficient of a variable $s_{i,k}(g, \boldsymbol{a})$. It should be noted that, in general, as the value of these variables differ among various agents, $s_{i,k}(g, \boldsymbol{a})$ has subscript i. However, it is assumed that the coefficient β_k is common to all agents, and thereby it does not have subscript i. In SAOMs, it is also assumed that the network is changed only by one edge. Such a change is called a "ministep" (Snijders 2005:224). To define a ministep, let $g^{\pm ij}$ be a network, which is different from network g with respect to only one edge from i to j.[6] Then, the probability of transition from network g to $g^{\pm ij}$, i.e., $P(g^{\pm ij}|g, \boldsymbol{a})$ is formalized as follows:[7]

$$P(g^{\pm ij}|g, \boldsymbol{a}) = c^{-1} \cdot Exp\{f_i(g^{\pm ij}, \boldsymbol{a})\} = c^{-1} \cdot Exp\left\{\sum_k \beta_k \, s_{ik}(g^{\pm ij}, \boldsymbol{a})\right\} \tag{4}$$

where c^{-1} is an appropriate weight defined so that $P(g^{\pm ij}|g, \boldsymbol{a})$ satisfies the properties of the probability function.[8] In SAOMs, choices of agents to change an edge are formalized according to the above probability function.

2.2. Estimation and Markov Chain Monte Carlo (MCMC) Methods

MCMC methods are used for estimating both ERGMs and SAOMs. Roughly speaking, MCMC is a method to estimate parameters in a probability function using computer simulations. If the dynamics are described by desirable Markov chains that satisfy irreducibility and aperiodicity, it has a unique stationary distribution. By comparing statistics from the stationary distribution and observed statistics, we can estimate the parameters of a probability function.

In SAOMs, as the longitudinal data are examined, an initial network and an end network are obtained based

6 If i is equal to k, then we define $g^{\pm ii} = g$. This formulation is described in Snijders (2011:146), but not in Snijders (1996), Snijders (2001), and Snijders (2005). However, this difference may be subtle and insignificant. It depends on whether an explicit explanation of the rate function exits of not. The rate function describes the probability that agent i changes its edge. Therefore, we have to exclude the situation $g^{\pm ii} = g$ after the rate function selects the situation "agent i changes its edge."

7 See Snijders (2005) for the detailed explanation in which a probabilistic choice model proposed by Maddala (1983) is used. Choices of agents for edges in a network are corresponding to choices for better goods in the probabilistic choice model.

8 Here, c^{-1} is described as $\sum_j Exp\{f_i(g^{\pm ij}, \boldsymbol{a})\}$.

on the observed data. The dynamics between two networks are described by Equation (4) and are reproduced by computer simulation. Based on the observed initial network, we can create a path from the simulated networks to the end network, which corresponds to the observed end network. Using this simulation, we can estimate the vector of coefficients, $\boldsymbol{\beta}$. From a given initial coefficient vector of $\boldsymbol{\beta_0}$, the estimated coefficient vector, $\boldsymbol{\beta_t}$, is adjusted to make the statistics in a simulation closer to those in the observed end network. This method is referred to as the method of moments.[9] It should be noted that, in this case, the transition probability, i.e., $P(g^{\pm ij}|g)$ for any ij, is assumed to be stationary; however, stationarity of networks is not assumed (Snijders 2011:147).

In ERGMs, the probability of networks, i.e., $P(G = g|\boldsymbol{A} = \boldsymbol{a})$, is focused on. There are no dynamics described in SAOMs. However, it is known that particular artificial dynamics enable an estimation of the parameters in the probability function. Let g^{-e_i} be a network, which is different from network g with respect to only an edge, e_i. We consider the following dynamics: a transition from g to g^{-e_i} occurs with probability:

$$\min\{1, \frac{P(G=g^{-e_i}|\boldsymbol{A}=\boldsymbol{a})}{P(G=g|\boldsymbol{A}=\boldsymbol{a})}\}.$$

According to Equation (2), we have the following:

$$\min\left\{1, \frac{Exp\{\sum_k \eta_k \cdot s_k(g^{-e_i}, \boldsymbol{a})\}}{Exp\{\sum_k \eta_k \cdot s_k(g, \boldsymbol{a})\}}\right\}.$$

It should be noted that in the above formulation, c^{-1} is canceled out, and the calculation becomes easier than the original Equation (2).[10] The point is that from an initial network, the above transition produces various networks. When two networks are apart enough from each other, then, these two networks are considered as being generated from Equation (2), i.e., the original probability function, independently. In addition, this transition is a desirable Markov chain that satisfies irreducibility and aperiodicity and has unique stationary distribution.

Only one observed network is required for estimation in ERGMs. This is because (i) it is assumed that the realized network is stationary (or in equilibrium), and (ii) based on the desirable Markov chains, the unique stationary distribution can be obtained from any initial network. As we can compare the simulated and observed networks, we can use the method of moments described in SAOMs (Snijders 2002). It should be noted that it is assumed that the probability function, i.e., $P(G = g|\boldsymbol{A} = \boldsymbol{a})$ is stationary, and the observed network is also stationary (or in equilibrium).[11]

The basic features for both ERGMs and SAOMs are summarized in Table 1. The features of ERGMs are based on the fact that the whole network g is considered. Consequently, only one network is sufficient for analysis; however, its stationarity is required. It is not necessary to assume that a node is a decision maker, and a simple non-directed edge is assumed in the simplest model. With regard to SAOMs, the features are based on a fact that agents make choices for edges. This induces the transition of networks. Therefore, longitudinal data are needed. An agent i's decision to create an edge to agent j induces a directed edge in the simplest model.

9 See Snijders (1996), Snijders (2001), and Snijders (2005) for a detailed explanation about estimation methods.

10 It should be noted that $\frac{Exp\{\sum_k \eta_k \cdot s_k(g^{-e_i}, \boldsymbol{a})\}}{Exp\{\sum_k \eta_k \cdot s_k(g, \boldsymbol{a})\}} = Exp\{\sum_k \eta_k \cdot [s_k(g^{-e_i}, \boldsymbol{a}) - s_k(g, \boldsymbol{a})]\}$, and $\sum_k \eta_k \cdot [s_k(g^{-e_i}, \boldsymbol{a}) - s_k(g, \boldsymbol{a})]$ is the change in the value of the network statistics. This is called as "change statistics." In this sense, only the change statistics determine the dynamics of networks.

11 See Snijders (2002), Hunter et al. (2008), Koskinen and Snijders (2013), and Robins et al. (2007) for detailed explanation about the estimation methods.

Table 1: Basic Features of ERGMs and SAOMs

	ERGMs	SAOMs
Main focus on the model	Whole network	Choice of agents for an edge
Data	One network data	Longitudinal network data
Stational Network	Yes	No (but stational law of decision makings)
Node as a decision maker	No	Yes (myopic rational behavior)
Direction of edge	Non-directed	Directed

To implement these estimations, there are packages available in R. The package "ergm" is used for ERGMs. There are an online tutorial (Morris et al. 2019) and textbooks (Harris 2014; Luke 2015; Suzuki 2017) explaining how to work with this package. In addition, there is PNet as a stand-alone application.[12] The package "RSiena" is used for SAOMs.[13] There is the official website where a manual (Ripley et al. 2020), tutorial files, and datasets are listed.[14] There are also tutorial textbooks (Luke 2015; Suzuki 2017).

2.3. Extensions

Evidently, there are various extensions for the most basic model described in the previous subsections. Directed edges are introduced into ERGMs in earlier studies (Frank and Strauss 1986; Wasserman and Pattison 1996). Non-directed edges are also included in SAOMs (Ripley et al. 2020:5.8; Snijders and Pickup 2017). SAOMs can be used for analyzing the cross-sectional network data (Snijders and Steglich 2015).[15] ERGMs are extended to be applied to an analysis of the longitudinal network data (Krivitsky and Handcock 2010; Leifeld and Cranmer 2019; Snijders and Koskinen 2013).[16]

In ERGMs, the probability function of a whole network is considered; however, it is characterized by the edge-based transition, $\frac{P(G=g^{-e_i}|\boldsymbol{A}=\boldsymbol{a})}{P(G=g|\boldsymbol{A}=\boldsymbol{a})}$, that has already been described in the previous subsection. This means that such an edge-oriented transition creates an observed network if the observed network is stationary (or in equilibrium). If the edge is directed, the estimated function in ERGMs corresponds to the preference function in SAOMs. Even in the case of non-directed edges, we can interpret that a pair of actors makes myopic optimizing choices for their edge in ERGMs (Block, Stadtfeld, and Snijders 2016:5; Jackson, Rogers, and Zenou 2017:85). In this sense, it can be said that these two models are close to each other.

There is also a qualitative extension in which parameters based on the behavior of actors, such as smoking, alcohol drinking, and academic performance, are also included. For SAOMs, a preference function for

12 See documents in "http://www.melnet.org.au/pnet" for more information.
13 It should be noted that RSiena and SAOMs are sometimes used as interchangeable. See also Section 1 "General information" in Ripley et al. (2020).
14 The site address is "https://www.stats.ox.ac.uk/~snijders/siena/".
15 A detailed comparison between two models for the cross-sectional data is conducted by Block, Stadtfeld, and Snijders (2016).
16 Snijders and Koskinen (2013) introduce edge-oriented dynamics for creating longitudinal ERGMs. Leifeld, Cranmer, and Desmarais (2018) introduce the network statistics corresponding to the previous networks into the probability function for the current network. This is implemented as a package of "btergm" in R. Krivitsky and Handcock (2010) focus on the differences between two networks and construct a probability function for them. This is implemented as a package of "STERGM" in R (Krivitsky and Goodreau 2019). The comparison between SAOMs and "btergm" was already discussed (Block et al. 2018, 2019; Leifeld and Cranmer 2019).

behavior is added. As a ministep assumption is imposed on the behavior dynamics, the similar dynamics for the edge change are applied to the change of behavior (Steglich, Snijders, and Pearson 2010).[17] Coevolution between networks, such as a friend network, and the behavior has been examined (Lomi et al. 2011; Mercken et al. 2009; Steglich, Snijders, and West 2006). In these models, there are two dependent variables related to a network and behavior.

Similarly, two (and more) different networks can be examined simultaneously if we introduce two (and more) dependent variables related to each different network (Ellwardt, Steglich, and Wittek 2012). This can also be applied to the two-mode network dynamics (Snijders, Lomi, and Torló 2013).[18] For ERGMs, similar extensions are also followed. As an edge-based transition, $\frac{P(G=g^{-e_i}|\boldsymbol{A}=\boldsymbol{a})}{P(G=g|\boldsymbol{A}=\boldsymbol{a})}$ characterizes the probability function for a whole network, we can consider the attribute based transition $\frac{P(\boldsymbol{A}=\hat{\boldsymbol{a}}|G=g)}{P(\boldsymbol{A}=\boldsymbol{a}|G=g)}$ where $\hat{\boldsymbol{a}}$ indicates the only kth attribute of agent i; a_{ik} is different from $\boldsymbol{a}$ by one unit, $\hat{a}_{ik} = a_{ik}+1$ or a $\hat{a}_{ik} = a_{ik}-1$. This new model is referred to as autologistic actor attribute model (ALAAM) (Daraganova and Robins 2013). Multiple one-mode and two-mode networks can also be examined simultaneously in ERGMs (Wang 2013).

Finally, we comment on the difficulty of estimation related to a network. Let us assume that we have only one network, and here, we need to impose the stationary assumption both for a theoretical model and an observed network, which is described in explanation corresponding to ERGMs in Section 2.2. It should be noted that if we have the longitudinal data in which the beginning network and the end network are obtained, then we can focus on the transition dynamics, and an observed network's stationarity is not needed. This is a case of SAOMs. The additional assumption for a stationary network in ERGMs tends to cause a problem that the MCMC algorithms sometimes converge to degenerate graphs, such as an empty network or complete one, or do not converge (Handcock 2003). In this case, we cannot obtain meaningful estimations. This is called as "degeneracy" or "near-degeneracy."

It is known that particular network configurations, i.e., geometrically weighted edgewise shared partner (GWESP) and geometrically weighted dyadic shared partner (GWDSP) (Hunter 2007; Hunter et al. 2008) are effective for preventing the degeneracy problem.[19]

However, with regard to this degeneracy, there is more serious criticism on ERGMs; i.e., estimation of parameters is not computationally feasible, or near-Bernoulli random graph in which all edges are created independently, is obtained as a result of estimation (Bhamidi, Bresler, and Sly 2011; Chatterjee and Diaconis 2013).[20] Counter arguments for this are proposed by Schweinberger et al. (2019). They state that the key

17 According to this formulation, there is a restriction for "dependent behavioral variables such that it must take nonnegative integer values; e.g., 0 and 1, or a range of integers like 0,1,2 or 1,2,3,4,5" (Ripley et al. 2020:4.1.3). However, recently, a continuous behavioral variable can be included into SAOMs (Niezink, Snijders, and van Duijn 2019).

18 The two-mode network is a network that expresses the relationship between agents and affiliations.

19 GWESP (or GWDSP) corresponds to the weighted sum of k-triangle (or k-two-path), which is introduced by Robins, Snijders, et al. (2007). Both k-two-path and k-triangle form complicated shapes. At first, it seems that it is an ad-hoc introduction of network configurations. However, both k-two-path and k-triangle satisfy the partial conditional dependence (Pattison and Robins 2002) that is a natural extension from Markov dependence. Roughly speaking, the Markov dependence defined as follows: if two edges share a common "node," then these two edges are dependent according to. Furthermore, if two edges share a common "edge," then these two edges are dependent according to the partial conditional dependence. Therefore, both k-two-path and k-triangle forms are natural and reasonable network configurations under the partial conditional dependence.

20 See also other studies that attempt to overcome this difficulty (Chandrasekhar and Jackson 2012, 2016; Jackson et al. 2017).

assumption for the negative claim (Bhamidi et al. 2011; Chatterjee and Diaconis 2013) is that given a fixed number of sufficient statistics (or independent variables), the number of nodes can be increased. However, GWESP (or GWDSP) does not satisfy the condition of a fixed number of sufficient statistics (or independent variables), as GWESP (or GWDSP) is defined as the weighted sum of k-triangle (or k-two-path) in which it is possible to include arbitrary number of sufficient statistics. Therefore, we cannot apply the negative results to ERGMs straightforwardly. In addition, Schweinberger et al. (2019) emphasize that the theoretical models examined in Bhamidi, Bresler, and Sly (2011) and Chatterjee and Diaconis (2013) are very simplified, i.e., there is no additional structure, such as temporal structure, nodes attributes, and so on. Moreover, Schweinberger et al. (2019) state that ERGMs with an additional structure are well-behaved.

To conclude this section, we provide a list of sources of useful information for further investigation of ERGMs and SAOMs. Carrington, Scott, and Wasserman (2005) is a good book for both ERGMs and SAOMs. Lusher, Koskinen, and Robins (2012) for ERGMs. Snijders (2011), and Snijders (2017) also presented good surveys. In addition, there is a large amount of information available online for both SAOMs and ERGMs.[21]

3. Report on the International Forum.

3.1. Keynote Speech and Four Applied Researches

In the beginning of the forum, the keynote speech was made by Dr. Steglich where he emphasized that the network analysis based on statistical models was suitable to examine micro-macro links. After the theoretical formalization and arguments, the empirical analysis was also discussed. In this analysis, SAOMs were used for examining a coevolution of a network and an agent's behavior, as mentioned in Section 2.3 of the present report. The data contained 75 students enrolled in an MBA program in Italy, an advice seeking network among them, and their average examination achievements. A focused macro phenomenon was an achievement segregation. This segregation was explained by the following factors: trend (rewiring, preferential drift, etc.), control (gender, experience, etc.), selection (homophily of achievements, etc.), and influence (assimilation of achievements, etc.). These micro mechanisms were described as explanatory variables in SAOMs. It was also demonstrated the effect of trend was 26% ; that of control was 11% , 11% of selection, 47% of influence, and 5% of others. This means that by using social network analysis, it is possible to clarify micro mechanisms underlying macro phenomena.[22]

After the keynote speech, there were four presentations related to applied research works. First, Dr. Fujimoto presented the results of the social network analysis on young men who have sex with men (YMSM) conducted with the purpose of preventing the human immunodeficiency virus (HIV) infections. Collaboration and competition networks built across social and health venues in Chicago and Houston were examined simultaneously by using ERGMs. In this regard, it can be outlined that this was a research based on multiplex networks, as described in Section 2.3 of the present report. Competition among venues tended to occur when the two venues had similar competitive relationships with other venues. That is, roughly speaking, "the competitor of my competitor was my competitor." Considering collaborative relationships, there was also an interesting tendency that could be described as follows: "the collaborator of my collaborator was my

21 For SAOMs, we recommend to visit "https://www.stats.ox.ac.uk/~snijders/siena/" and click "RSiena script" to find a large number of scripts for learning SAOMs. For ERGMs, we recommend to visit https://github.com/statnet/Workshops/wiki and click "ergm tutorial" to find a good practical tutorial.

22 The same data set is used in Lomi et al. (2011). The relative importance of effect is discussed in Indlekofer and Brandes (2013).

competitor." In addition, Dr. Fujimoto made a brief introduction of the Young Men's Affiliation Project of HIV Risk and Prevention Venue (YMAP) in which the network data corresponding to YMSM was also obtained and examined comprehensively.

Then, Dr. Kim presented her research on the firm's intraorganizational network structures and their influence in IT sector. This research was different from the preceding two presentations, as the network structure in question was a predictor variable rather than an outcome variable. Therefore, neither ERGMs nor SAOMs were used. Rather, a variety of research methods in social network analysis was demonstrated.[23] The level of clustering in a network was calculated through two steps. First, an individual's clustering coefficient was computed as the proportion of his or her collaborators who were themselves directly linked to each other. The clustering coefficient of the overall network was the average of this measure across all individuals in the network and can range from 0 (no clustering) to 1 (completely clustered). The level of connectedness in a network was measured by the proportion of dyads that are connected by a path of any length. The dependent variable was defined by the absorption of external technologies, namely, if a firm absorbed a technology from external companies, then it was coded as 1, otherwise, 0. The logistic regression was used to perform the estimation. The results suggest that a highly clustered network had a positive effect on external technology absorption, whereas highly connected one had a negative effect.

In turn, Dr. Wang's presentation was dedicated to the multiplex network analysis which included the agent, organization, and affiliation networks. It was conducted by using an extended ERGMs, as described in Section 2.3. The network data contained 97 researchers working on cancer-related research and 82 laboratories in France. Here, high performance researchers were referred to as "Big fishes," and large laboratories as "Big ponds." The obtained results were as follows. There was no association found between high performance researchers and large laboratories. However, there was a tendency that large laboratories (Big ponds) were collaborating with other large laboratories. Moreover, it was observed that advice ties among high performance researchers (Big fishes) were created not directly, but through big laboratories (Big ponds).

The presentation prepared by the author of this report (Fujiyama) was also related to multiplex network analysis using SAOMs. Study-conversation, non-study-conversation, and advice networks across students in a Japanese university were examined based on the three concepts of the self-determination theory. Autonomy was expressed by initiating a tie; the relatedness was defined by an existing tie, and competence was determined by being requested advice from other students. The obtained results were as follows: during the spring semester, a study-conversation tie caused another study-conversation tie reciprocally. However, this reciprocal effect became weak in the fall semester. Therefore, non-study-conversation and advice ties were effective measures for enhancing the study-conversation tie in the fall semester.

3.2. Research Works and Comments from the Related Areas

There were three presentations corresponding to the related area. First, Dr. Sato clarified the relationship between a social network and social capital from the viewpoint of sociology. He emphasized that conceptual confusion on social capital was entangled by introducing the utility function into the relationship between social capital and social network. That is, a social network can be formalized as an input of the utility

23 While Dr. Kim examined how the properties of entire networks influence the performance of the network, the effects of the social structure on the behavior of each agent were examined in the field of economics. Game theoretic models were developed, and empirical analysis was also conducted (Ballester, Calvó-Armengol, and Zenou 2006; Calvó-Armengol, Patacchini, and Zenou 2009; Fujiyama 2014, 2020; Liu, Patacchini, and Zenou 2014; Patacchini, Rainone, and Zenou 2017).

function, and social capital can be defined as its output. The point is that if utility functions of agents differ between each other, then valuation corresponding to the same social capital (combined with a social network) also differs.[24]

Dr. Igarashi reviewed the history of social psychology for examining social relationships based on methodological individualism. There is a fact that social network analysis, including sociometry, was initially developed in this field, but social psychologists had not used it widely since the cognitive revolution in the 1970s. Now social psychology was restarting examination of the social networks based on the relational data by using advanced modeling and statistical methods. Expected contributions from social psychology can be summarized into the three categories: “motivation” for creating a tie, “perception” of an existing network, and “cooperation” related to social networks. Social psychological approach is a way of scrutinizing human sociality as a matter of information processing at an individual level. This can be applied to social network analysis effectively.

Then, M.A. Maejima made a brief introduction about a web service using social network analysis for working persons. He is a researcher in Sansan that is a leading company in this field in Japan. Social networks were created by co-ownership of the same business card. Outstanding features of the business card were “universality” (very common among business persons), “accuracy” (smaller probability of containing false/inaccurate information), “real-time” (a scanned date is recorded), and “accompanies face-to-face interaction” (exchanges occur in a face-to-face communication). Many services and technologies were mentioned in this presentation. With regard to social network analysis, relatively simple indices, such as the degree centrality and local clustering coefficient were employed. More sophisticated methods, such as detailed network visualization and the analysis based on statistical models, e.g., ERGMs and SAOMs, were not widely used at this moment, as it was difficult to derive straightforward information for prediction (and/or decision making). It was deemed that sophisticated analysis methods generate an excessive amount of information, which makes handling them difficult.

3.4. Panel Discussion

During the second day of the forum, we had the panel discussion. First, we clarified the difference between ERGMs and SAOMs, as described in Section 2. After that, we initiated discussions corresponding to each presentation of the previous day to obtain a deeper understanding of them.

The first discussion was dedicated to the micro-macro linkages proposed in the presentation by Dr. Steglich. In ERGMs, the probability of a network is formalized directly. In SAOMs, nodes are assumed as decision makers, which create a whole network. Consequently, micro-macro linkages can be examined by SAOMs directly. However, in the case of semantic networks in which a node is not a decision maker, ERGMs are more appropriate. In addition, if we define additional assumptions, the results of ERGMs can be interpreted by the viewpoint of the micro-macro linkages.[25]

In Dr. Wang’s presentation, advanced (or complicated) ERGMs were used. There is a following practical way for preventing the degeneracy problem at a possible extent: (i) constructing a model based on the simplest one to develop a more complicated one gradually and (ii) relying on social theory. If the social theory is valid, then it captures the main factors of a social phenomenon in question. The key point is to identify variables corresponding to the main factors carefully and to include them into ERGMs step by step.[26]

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24 See also Sato (2013) for the review of social capital.
25 See also the argument in Section 2.3.

In Dr. Sato's presentation, the role of the utility function was emphasized. In this regard, his arguments correspond to SAOMs in which the preference (i.e., utility) function is modelized directly. However, he also emphasized that the unintentional creation of a tie is also very common in social network analysis. For example, we can meet others by chance in an alumni association or other social events. This is different from the concept of intentional tie creating. If such a way of unintentional tie creating prevails over intentional tie creating, then ERGMs are more applicable. The balance is very important. In addition, as intentional tie creating induces dynamics of a network, examination of network dynamics is also important. For example, it is known that a structural hole disappears in the long run through a profit maximizing behavior related to filling holes (Buskens and van de Rijt 2008).

In an empirical analysis based on social network analysis, the social theory behind estimations is crucial. We made a separate discussion on this topic. Dr. Fujimoto outlined that research works on public health are interdisciplinary. Considering biological mechanisms was also included in her research project. "Niche overlap" (from organizational ecological theory), "growth commensalism" (from organizational evolutional theory), and "structural equivalence" (from social network theory) were mentioned in her presentation made on the first day.

In Dr. Kim's presentation performed on the previous day, the behavior of firms was examined. In the panel discussion, Japanese well-organized supply chain networks were considered as a topic. To examine this topic, we should take into account of the differences from exchanges in an ordinal market. The differences between short-term and long-term perspectives are also important. Dr. Sato also added the comment that transaction costs were crucial to evaluate these differences.

With regard to the presentation by the author of this report (Fujiyama), Dr. Igarashi made the comment that due to the fact that the three concepts in the self-determination theory were defined as "basic psychological needs," it was necessary to make additional explanation about the relationship between these concepts and formulation of tie creating. The reply was as follows: in the self-determination theory, if someone satisfied the own basic psychological needs, then he/she became more active in an organization. Therefore, when the tie creating behavior allowed fulfilling the needs, then, it facilitated students becoming more active, e.g., making (or creating a tie of) study-related conversation, etc.

In the panel discussion, Dr. Igarashi made the comment on a node as an individual. The advantage of social psychology is that various aspects of individuals can be introduced. A real individual perception system is different from perfect perception, such as a computer. A biased perception is common in real social situations. For example, there is a possibility that an individual who has lower power, tends to care about a network structure and to utilize benefits from it. In other words, a higher-powered individual can control others without social networks. Interaction between an existing network and perception can bring changes to their behavior (including tie creating). In another example, loneliness perception is also interesting. Beyond the isolated node, loneliness can be defined as the gap between the ideal number of ties and the real number of them. Consequently, social psychology has the potential to contribute to social network analysis by introducing various aspects of individuals.

According to the presentation by M.A. Maejima held on the previous day, relatively simple network indices were widely applied to business services. He made additional comments with this regard. First, as the number

26 See also the argument in Section 2.3. In this regard, GWESP and GWDSP based on the partial conditional dependence are considered as important factors for capturing significant effects. In addition, if co-evolution among different networks is essential, then a more complicated network model is desirable to avoid the degeneracy problem, as it introduces an additional structure into ERGMs.

of working persons included in the business-card network is more than 10,000, a complex network index is not applicable from the viewpoint of calculation cost. In addition, degree centrality is highly correlated to other complex centrality notions in many cases. However, he also emphasized that it was important to translate complex notions of social network analysis into commonly used words. These efforts are important to spread these notions among working persons.

In modern Japanese society, it can be noted that many important social issues are related to the economy, local community, and education. The analysis results presented by Dr. Kim and M.A. Maejima corresponded to the field of economy. Dr. Fujimoto's project on public health is an effort to improve people's health in local communities (Houston and Chicago) by collaborating among organizations and people. Dr. Sato's arguments on social capital are also directly related to the local community. Dr. Wang's research is focused on enhancing outcomes within the academic community through building networks among affiliations and researchers. From the viewpoint of improving activities within communities, the results of these research works can provide valuable feedback to the problems in local communities in Japan. The analysis results presented by the author of this report (Fujiyama) are directly related to education in Japan. The arguments raised by Dr. Igarashi from social psychology can serve as a fundamental consideration with regard to working persons in business organizations and students in schools. For conducting all these investigations appropriately, we have to be conscious to account for the micro-macro linkage, as discussed in the keynote speech made by Dr. Steglich. Based on these efforts, a more desirable society will be realized.

4. Abstracts

In this section, all abstracts in this forum are listed.

"Social Network Mechanisms: Part1: Assessing Evidence on The Micro Level. Part2: Accounting for Macro Level Outcomes"

Christian Steglich (University of Groningen; Linköping University)

When studying the micro-macro link, there typically are multiple micro-level mechanisms that - jointly or separately - could account for the same macro phenomenon. For example, the observed homogeneity bias in a network (a macro level phenomenon) could be caused by (micro level) mechanisms of social influence, or social selection, or could be an artifact of shared social contexts. In this presentation, best practices are suggested if the aim is to hold a micro level mechanism accountable for a macro level phenomenon.

In Part 1 of this presentation, the focus is on empirically assessing the validity of a social network mechanistic explanations in empirical data sets. These mechanisms (e.g., reciprocity, homophily, preferential attachment, or triadic closure) leave their traces in network data sets, and these traces can be analyzed with the tools of statistical network modeling to infer type and strength of the mechanisms.

In Part 2 of the presentation, counterfactual procedures are proposed for studying the explanatory power of specific network mechanisms for generating specific network-level outcomes. A principled way of comparing the explanatory power of alternative explanatory micro mechanisms is sketched.

The method is illustrated with educational social network data. The macro phenomenon under study is achievement segregation, operationalized as network autocorrelation of a performance measure in an advice seeking network. The main competing micro-level mechanisms are, on the one hand, three mechanisms of performance-related advice seeking and, on the other hand, three "conjugate" mechanisms of performance due to advice-seeking. Control mechanisms include endogenous dependencies as well as individual and contextual factors.

The method highlights that techniques of simulation-based statistical inference naturally combine with the concept of generative models in the counterfactual simulation tradition. By offering a wide range of calibration algorithms, they facilitate the empirical study of the micro-macro link.

"Applications of Social Network Analysis to HIV/STI Research"

Kayo Fujimoto (The University of Texas)

In the United States, young men who have sex with men (YMSM) have an elevated rate of HIV infection. YMSM bear the highest disease burden, as HIV epidemic overlaps with other STIs such as syphilis epidemic. Little is known about combined network and behavioral factors that drive disease infection among YMSM. As a biomedical intervention, Tenofovir disoproxil fumarate with emtricitabine pre-exposure prophylaxis (PrEP) was approved by the U.S. Food and Drug Administration (FDA) in 2012 to reduce the number of new HIV infections. However, PrEP uptake among the highest risk groups in the U.S. faces implementation challenges. We conducted the "Young Men's Affiliation Project (YMAP)," to advance the utility of social network analysis by introducing a new way of addressing fundamental questions about social networks in relation to disease transmission and health-related behavior. We applied exponential random graph models to examine multilevel HIV/STI transmission networks. We demonstrate the utility of social network analysis for understanding how infectious diseases are spread through "risk/protective networks," and to illustrate the application of innovative social network methodologies to facilitate, improve, and expand the capability of social network analysis in the field of HIV/STI research.

"The Configuration of Investor Networks within Firms and Their Capacity to Absorb External Technologies"

Ji Youn (Rose) Kim (University of Kentucky)

Evolving technological landscapes often make it essential for incumbent firms to revitalize their technological core by absorbing the technologies of new ventures. We explore how an incumbent firm's internal inventor network configuration influences its ability to assimilate and absorb new venture technologies. We find that incumbents that have internal inventor network configurations that are highly clustered, based on prior inventor collaborations, are more likely to build on new venture technologies. These clusters create a trusting environment with a common vernacular that facilitates individuals' translating and sharing with their coworkers the technological insights that they have derived through their interactions with outside sources. Moreover, any insularity that may result from having cohesive clusters is mitigated by internal competition between clusters. In contrast, having a highly connected inventor network decreased the extent to which our sample incumbent firms absorbed new venture technologies. Broadly distributed access to internal technology combined with a strong social identity that results from high levels of connectedness can lead to insularity and a "not invented here" ethos. We also find that highly connected internal inventor networks diminish any advantage that incumbent firms gain from having corporate venture capital relationships with new ventures in terms of absorbing their technologies.

“Multilevel Network Analysis Using Exponential Random Graph Models”

Peng Wang (Swinburne University of Technology)

Understanding social structure from a multilevel perspective provides additional insights on how micro and macro level structures are affected through meso level interactions. Representing complex multilayered systems as networks enables the utilization of a suite of methods and tools for network analysis in a great range of research fields, such as management, public health, social-ecological systems, consumer behavior and governance. Exponential random graph models (ERGM) view the overall network structure as collective results from local network processes represented by configurations and their counts as graph statistics. Within each of the ERGM configuration, variables about network ties and nodal attributes are considered interdependent reflecting the interdependent nature of networks and social processes. Expending ERGM to multilevel networks explains complicated within level structure through associations with network structures and nodal attributes at a different level, while reveal the functional and strategic positions nodes at different levels serve. Key insights enabled by multilevel ERGMs are demonstrated by a set of models on data collected on medical research elites and their affiliated laboratories in France. The features and flexibilities of the general multilevel network data structure and the ERGM framework are illustrated with research and publications in a range of fields.

“Multiplex Network Dynamics in a Japanese University Class: Study, Advice, and Non-study Ties”

Hideki Fujiyama (Dokkyo University)

Some positive attitudes relating to intrinsic motivation are essential for college education, as undergraduate students must develop their independent thinking and learning abilities. The self-determination theory accords important insights into this matter. According to this theory, the three concepts, autonomy, relatedness, and competence, are vital for the development of academic abilities.

Social network analysis is a useful tool for investigating the microprocesses. The three concepts can be expressed through social network analysis. First, relatedness can be measured directly through a tie in a network. Second, autonomy is expressed in the form of a directed tie. If subjects initiate ties then they have autonomy over the creation of that tie. Third, competence is expressed through a specific kind of a tie. For example, an advice relationship comprises an advice seeker and an adviser, where the adviser’s knowledge or ability is relied upon; therefore, his/her competence increases.

Data were collected from an undergraduate class at a Japanese university during the spring and fall semesters of 2013, 2014, 2015, 2016, and 2018. Stochastic actor-oriented models were employed to examine the co-evolution of study-conversation, non-study-conversation, and advice networks.

The results of the study revealed the robust effect of autonomy. A student who initiated one relationship was also likely to initiate another. The effect of competence was also found and it induced the creation of different kinds of ties. Especially, in the fall semester, the association among study conversations was not significant. On the other hand, non-study conversations and/or advice relationships had a significant association with study conversations. This implies that social gathering in a fall seminar is one of the desirable measures for enhancing students’ study conversations.

“Social Capital and Social Networks”

Yoshimichi Sato (Tohoku University)

I argue that utility functions of actors embedded in social networks convert the social networks to social capital. There are two confusions in the study of the relationship between them. First, some researchers say that social capital is equivalent to social networks, while the others say that it is not. Second, studies of the effects of social capital report mixed results. For example, Ronald Burt points out that social networks rich in structural holes provide better opportunities for entrepreneurs, while James Coleman shows that closed networks among high school students and their parents are efficient surveillance system for the parents. Alejandro Portes reports that new immigrants coming to their ethnic town in the host country enjoy benefits provided by people in the town, but they find it difficult to leave the town when they seek better opportunities outside of it. The same social capital provides benefits to newcomers but becomes an obstacle to them.

I argue that including utility functions of actors in social networks in theoretical frameworks about the relationship between social capital and social networks gives us a better explanation of the abovementioned social phenomena that seem to be contradictory to each other. If we assume that entrepreneurs have different utility functions than those held by parents of high school students, we can explain why they prefer social networks rich in structural holes, while parents of high school students try to make closed social networks. New immigrants had high utility being embedded in their ethnic town. However, as their command of English becomes better, they get to know how to live in their host country, and they find better opportunities outside of the town, they begin to think that their embeddedness in the town becomes their fetters.

In my presentation at the forum, I will give more examples to show that theoretical frameworks with utility functions contribute to a better understanding of the relationship between social capital and social networks.

“Psychological Underpinnings of Social Network Analysis”

Tasuku Igarash (Nagoya University)

Since the 1930s, social psychology has had a long tradition to theorize social networks through the perspective of relational dynamics and their consequences, such as sociometry, group dynamics, small group performance, to name a few. However, social psychologists have struggled to handle relational data mainly due to the lack of knowledge about advanced statistical approaches under the assumption of the methodological individualism. The cognitive revolution in the 1970s caused a significant change in the research field to allow researchers to scrutinize human sociality as a matter of information processing at an individual level and has created several stimulating research topics in the network field, such as how individuals select others for cooperation, why individuals are motivated to form social ties, and how individuals perceive the social world. However, the paradigm shift also caused the diminishing interests in network dynamics, and consequently, social psychologists are being a lag behind in the recent advancements of social network modeling. The restoration of network-related theories and methodologies is widely expected in the field of social psychology nowadays.

“Trends and Issues of Web Service Development Using Social Network Analysis”

Naoki Maejima (Sansan, Inc.)

In recent years, web services applying social network analysis (SNA) have been developed all over the world. In this report, I give a brief introduction to such trends and our products. “Sansan” is a cloud-based contact management tool for corporations which centers scanning business-cards. This tool also offers an experimental feature named “Business-person type analysis,” which provides relational analysis about both intra- and inter-organizational network and summarize users’ ego-network characteristics as users' type. This app infers intra-organizational networks from co-ownership of the same business-cards and calculates network index such as degree centrality or local clustering coefficient of the users. These analyses can be used for staffing, organizing new teams or self-reflection. However, what kind of relationship does this network capture? As compared to the calendar event co-occurrence network of which were constructed for each event type, it was found that this network relatively captures the official collaborative relationship rather than an informal one. I also mention some issues in creating web services based on SNA. In parallel with web service development, our team is challenging to conduct empirical research, including various themes such as the relationship between weak ties and job mobility or heterogeneity of network formation among industries.

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