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Application of iterated filtering to stochastic volatility models based on non-Gaussian Ornstein-Uhlenbeck process

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ABSTRACT

Barndorff-Nielsen and Shephard (2001) proposed a class of stochastic volatility models in which the volatility follows the Ornstein–Uhlenbeck process driven by a positive Levy process without the Gaussian component. The parameter estimation of these models is challenging because the likelihood function is not available in a closed-form expression. A large number of estimation techniques have been proposed, mainly based on Bayesian inference. The main aim of the paper is to present an application of iterated filtering for parameter estimation of such models. Iterated filtering is a method for maximum likelihood inference based on a series of filtering operations, which provide a sequence of parameter estimates that converges to the maximum likelihood estimate. An application to S&P500 index data shows the model perform well and diagnostic plots for iterated filtering ensure convergence iterated filtering to maximum likelihood estimates. Empirical application is accompanied by a simulation study that confirms the validity of the approach in the case of Barndorff-Nielsen and Shephard's stochastic volatility models.

Key words: Ornstein–Uhlenbeck process, stochastic volatility, iterated filtering.

1. Introduction

Barndorff-Nielsen and Shephard (2001) proposed a continuous-time stochastic volatility model (BN-S model), in which the logarithm of the asset price $y(t)$ is assumed to be the solution of the following stochastic differential equation:

$$dy(t) = \left(\mu + \beta\sigma^2(t)\right)dt + \sigma(t)dB(t), \tag{1}$$

where $(B(t))_{t\geq 0}$ is the Brownian motion, $\mu \in R_+$ is the drift parameter, $\beta \in R_+$ is the risk premium. Latent instantaneous volatility process $(\sigma^2(t))_{t\geq 0}$ is determined by the stochastic differential equation

$$d\sigma^2(t) = -\lambda\sigma^2(t)dt + dz(\lambda t), \tag{2}$$

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where $\lambda \in R_+$ and $\left(z(\lambda t)\right)_{t\geq 0}$ is pure jump Lévy process with stationary, independent and positive increments, and $z(0)=0$. The process $\left(z(\lambda t)\right)_{t\geq 0}$ is called *Background Driving Lévy Process* (BDPL) of the process $\left(\sigma^2(t)\right)_{t\geq 0}$. Figure 1 presents examples of the pair of the processes $\left(z(\lambda t)\right)_{t\geq 0}$ and $\left(\sigma^2(t)\right)_{t\geq 0}$. There are several important features of such a stochastic volatility process defined by (2), some of which will be outlined in Section 2 on the basis of a series of Barndorff-Nielsen and Shephard papers (Barndorff-Nielsen and Shephard, 2001, 2002, 2003).

A great number of estimation techniques have been proposed to estimate BN-S model. In their introductory paper (Barndorff-Nielsen and Shephard, 2001), Barndorff-Nielsen and Shephard employed a nonlinear least squares estimation and suggested other possible methods: Bayesian inference, quasi-likelihood inference by means of Kalman filter (for more details of Kalman filter implemented for BN-S model, see Szczepocki (2018)), estimation equations (Sørensen, 2000) and indirect estimation (Gourieroux, Monfort and Renault, 1993). In the following years much work on estimation was devoted to the Bayesian Markov Chain Monte Carlo (MCMC) approach: Roberts *et al.* (2004), Griffin and Steel (2006, 2010), Gander and Stephens (2007a,b), Frühwirth-Schnatter and Sögner (2009). Hubalek and Posedel (2006, 2011) proposed an estimator based on martingale estimating functions. Taufer, Leonenko and Bee (2011) introduced a characteristic function-based estimation method. Raknerud and Skare (2011) implemented an indirect inference method based on approximate Gaussian state space representation. Andrieu *et al.* (2010) proposed to use Particle Markov Chain Monte Carlo (PMCMC) estimation method, which combines particle filter with Bayesian inference. Chopin *et al.* (2013) proposed SMC^2 algorithm, which substantially extends PMCMC. James *et al.* (2018) also used PMCMC for *OU-Gamma Time Change* version of BN-S model.

In this paper we propose estimation based on iterated filtering. It is relatively a new class of methods for maximum likelihood inference introduced by Ionides *et al.* (2006) and substantially modified by Ionides *et al.* (2015). It is based on a series of filtering operations which provide a sequence of parameter estimates that converges to the maximum likelihood estimate. In the discussion on (Andrieu *et al.*, 2010) Anindya Bhadra (one of co-authors of Ionides *et al.*, 2011) showed some results from applying the iterated filtering to a single example of BN-S model. However, he applied the initial version of iterated filtering (IF1) from Ionides *et al.* (2006). In this paper we employed the second generation version of iterated filtering (IF2) from Ionides *et al.* (2015).

The paper is organized as follows. Section 2 presents background material on Barndorff-Nielsen and Shephard stochastic volatility model. Section 3 presents iterated filtering. Section 4 contains simulation results on estimation and Section 5 applications to real data. Section 6 gives concluding remarks.

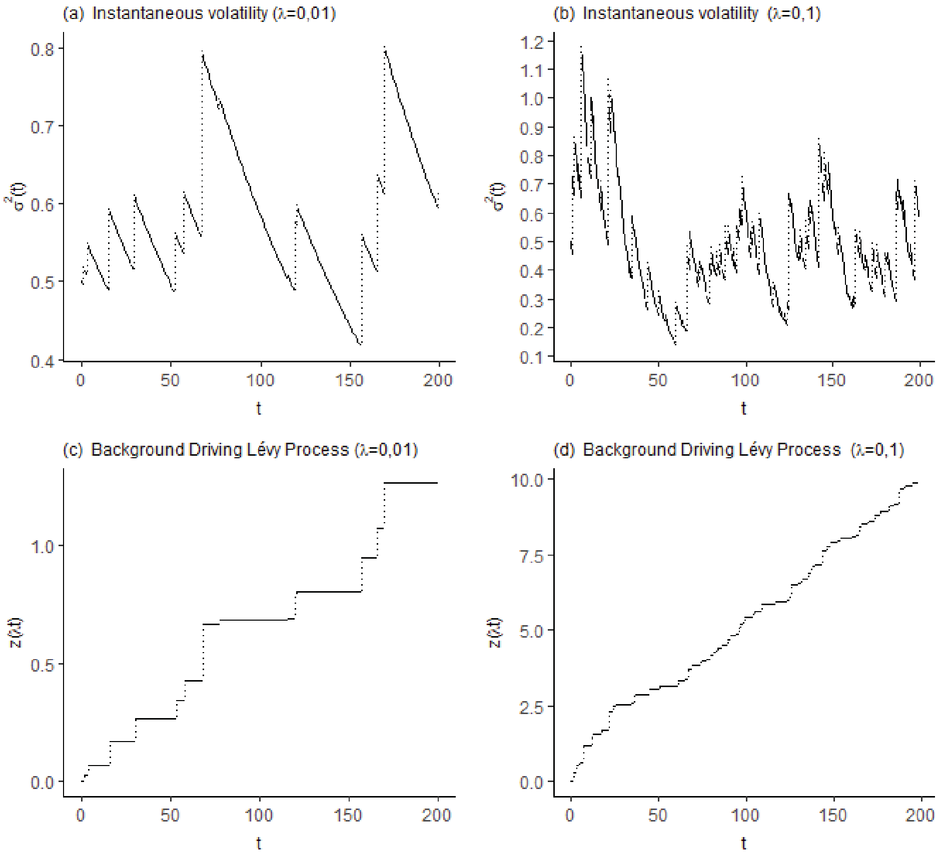

Figure 1. Two simulations of instantaneous volatility process with Gamma marginal (a) and (b), and corresponding Background Driving Lévy Process (c) and (d)

Source: Own work using R software.

2. Barndorff-Nielsen and Shephard stochastic volatility model

BN-S model has several important features which makes it very important for financial modelling. Firstly, instantaneous volatility $\left(\sigma^2(t)\right)_{t\geq 0}$ moves up by jumps according to $\left(z(\lambda t)\right)_{t\geq 0}$ and tails off exponentially at the rate λ. Thus, memory of the volatility process depends strictly on the rate λ. High values of λ result in high jumps, which are quickly discounted. On the contrary, a small value leads to a small jump but the process tails off slowly. Figure 1 shows the impact of λ on the volatility process.

Secondly, the time index of the process $\left(z(\lambda t)\right)_{t\geq 0}$ in (2) is chosen deliberately so that marginal distribution of $\sigma^2(t)$ does not depend on λ. Barndorff-Nielsen and Shephard (2001) proved that for any one-dimensional self-decomposable distribution

D there is a stationary Ornstein-Uhlenbeck process $\left(\sigma^2(t)\right)_{t\geq 0}$ and Lévy process $\left(z(\lambda t)\right)_{t\geq 0}$ satisfying equation (2), for which marginal distribution of $\sigma^2(t)$ is D. The class of self-decomposable distribution includes many distributions important in financial econometrics: gamma, normal-inverse Gaussian, inverse Gaussian, tempered stable, variance gamma, symmetric gamma, the Euler's gamma, Mexiner. (Schoutens, 2003) is a comprehensive reference text on financial application of self-decomposable distributions.

Thirdly, although instantaneous volatility $\left(\sigma^2(t)\right)_{t\geq 0}$ has discontinuous paths (due to jumps), integrated volatility

$$\sigma^{2*}(t) = \int_0^t \sigma^2(u)du \tag{3}$$

has continuous paths. Consequently, the resulting process of the logarithm of the asset price $y(t)$ also has continuous paths. One advantage of stochastic volatility of Ornstein-Uhlenbeck type is that many important process characteristics are analytically tractable. For example, integrated volatility has a simple structure

$$\sigma^{2*}(t) = \frac{1}{\lambda}\left(z(\lambda t) - \sigma^2(t) + \sigma^2(0)\right). \tag{4}$$

Finally, the implication of the formula (1) is that log-returns observed at time *n=1,..., T* (we assume that time differences $\Delta_n = t_n - t_{n-1}$ are fixed and equal Δ) take the form:

$$y_n = \int_{(n-1)\Delta}^{n\Delta} dy(t) = y(n\Delta) - y((n-1)\Delta) \tag{5}$$

and have conditional Normal distribution

$$y_n = N(\mu\Delta + \beta\sigma_n, \sigma_n^2) \tag{6}$$

where $\sigma_n^2 = \sigma^{2*}(n\Delta) - \sigma^{2*}((n-1)\Delta)$. This discretely observed volatility σ_n^2 (*n=1,..., T*) was called actual volatility by Barndorff-Nielsen and Shephard (2001). Marginal distribution of y_n is a location scale mixture of normals. Thus, returns may capture empirical facts such as skewness and thick tails. Moreover, when $\Delta \to +\infty$ marginal distribution of y_n tends to normal distribution. Hence, non-normality of returns vanishes under temporal aggregation, which is another empirical fact observed in financial data.

BN-S model has attracted much interest and research in mathematical finance and financial econometrics. Nicolato and Venardos (2003) studied equivalent martingale measures and provided closed-form prices for European call options for BN-S model. The minimal entropy martingale measure and numerical option pricing for BN-S model are investigated in (Benth and Karlsen, 2005) and (Benth and Meyer-Brandis, 2005). Hubalek and Sgarra (2009) provided option pricing by Esscher transform. Benth *et al*. (2003) considered Merton's portfolio optimization problem in a Black and Scholes market with stochastic volatility of BN-S type. Benth *et al.* (2007) provided explicit evaluation of the variance swaps. Hubalek and Sgarra (2011) developed a semiexplicit valuation formula for geometric Asian options.

3. Iterated filtering

3.1. General remarks

Iterated filtering (Ionides *et al.* 2006, 2015) are methods for maximum likelihood inference for state space models (SSMs). These models are also known as partially observed Markov Processes (POMP) or hidden Markov models (HMMs). SSMs consist of a pair of processes: (X_n, Y_n). The former is a Markov process (*state process*) which is not observed directly but may be estimated through the latter (*observation processes*). The observations of Y_n are assumed to be conditionally independent given the X_n (for details, see Durbin And Koopman, 2012). SSMs are very flexible and have been widely applied in economics, medicine, biology, mechanical system monitoring, patter recognition (see Chapter 1 in Cappé *et al.,* 2008 for examples). However, estimation for SSMs is very challenging because likelihood functions are analytically intractable in general.

Iterated filtering is one of the few if not the only available *likelihood-based* (based on the likelihood function for the full data)*, simulation*-based (dynamics of the model is captured only via a simulator), frequentist (based on frequency interpretation of probability) methods for SSMs. Iterated filtering has been successfully applied to perform parameter estimation in SSMs, mostly in the context of biological applications (King et al., 2008, He et al., 2009, Bhadra et al., 2011) but also in economic modelling (Bretó, 2014).

The key idea behind iterated filtering is to replace the model we are interested in, which have constant parameters, with a similar model but with parameters that take a random walk in time. This extra variability smooths the likelihood surface and counteracts particle depletion. Over multiple repetitions of the filtering procedure (made by means of a particle filter), the variance of this random walk goes to zero and the augmented model approaches the original one. As an output of iterated filtering, the algorithm provides a sequence of updated parameter estimates that converge to the maximum likelihood estimate. Iterated filtering algorithms use basic sequential Monte Carlo techniques (also known as bootstrap particle filter, Gordon et al., 1993). Thus, they have the property that they do not need to evaluate the transition density of the latent Markov process. Algorithms with this property have been called plug-and-play (Ionides et al., 2006) or simulation-based. It is vitally important in the case of BN-S model, for which the transition density takes no explicit form. The plug-and-play methodology is relatively recent and have been developing rapidly because of its less restrictive requirements. Examples of plug-and-play methodologies that follow the Bayesian paradigm are Approximate Bayesian Computation (Toni et al., 2009) and Particle Markov Chain Monte Carlo (Andrieu et al., 2010).

There are two generations of iterated filtering which are typically abbreviated by IF1 and IF2. The first was introduced by Ionides et al. (2006) and theoretically justified by Ionides et al. (2011). Later, Lindström et al. (2012) improved numerical performance of IF1 and Doucet et al. (2013) expanded it to include smoothing algorithm. The second generation was initiated by Ionides et al. (2015) and later supported by theoretical study of Nguyen (2016). Although both generations of iterated filtering recursively perform filtering through the augmented model, the theoretical justifications of these algorithms are essentially different. IF1 approximates the Fisher score function, whereas IF2 combines the idea of data cloning (Lele et al., 2007), with convergence of an iterated Bayes map (Nguyen, 2016). Ionides et al. (2015) showed that IF2 outperforms IF1 in empirical examples.

Convergence of iterated filtering IF2 to the maximum likelihood estimate has been shown under some regularity conditions (see Ionides et al., 2015 and Nguyen, 2016, for details). The conditions are rather technical so, in practical applications, convergence of algorithm should be assessed via diagnostic plots (Bretó, 2014).

In this paper, we use implementation of iterated filtering provided by the software package POMP (King *et al.*, 2010) written for the R statistical computing environment (R Development Core Team, 2010).

3.2. Implementation of the BN-S model

Barndorff-Nielsen and Shephard (2001) presented their model in state space model representation with y_n as an observation process and actual volatility as a state process. Conditional distribution of observation process given the state process $y_n \mid \sigma_n^2$ is given by the formula (6). The transition density is not available in explicit form. Griffin and Steel (2007) showed that the actual volatility can be written as

$$\sigma_n^2 = \frac{1}{\lambda}\left[\eta_{n,2} - \eta_{n,1} + \left(1 - e^{-\lambda\Delta}\right)\sigma^2\left((n-1)\Delta\right)\right], \tag{7}$$

where

$$\eta_n = \begin{bmatrix} e^{-\lambda\Delta}\int_0^{\Delta} e^{\lambda t}dz(\lambda t) \\ \int_0^{\Delta} dz(\lambda t) \end{bmatrix} \tag{8}$$

is a vector of random jumps, which is a pair of stochastic integrals with respect to the BDLP $(z(\lambda t))_{t\geq 0}$. The instantaneous volatility process from equation (7) may be discretized by recursion

$$\sigma^2(n\Delta) = \sigma^2((n-1)\Delta)e^{-\lambda\Delta} + \eta_{n,1}. \tag{9}$$

In this paper, we use the series representation from Barndorff-Nielsen and Shephard (2001) given by

$$\eta_n = \begin{bmatrix} e^{-\lambda\Delta}\sum_{j=1}^{\infty} W^{-1}\left(\frac{a_{i,j}}{\lambda\Delta}\right)e^{-\lambda\Delta} \\ \sum_{j=1}^{\infty} W^{-1}\left(\frac{a_{i,j}}{\lambda\Delta}\right) \end{bmatrix} \tag{10}$$

where for each *j* (*j=1,2,…*) $a_{i,j}$ are the arrival times of a Poisson process of intensity 1, and $r_{i,j} \sim$ U(0, 1), independent of the $a_{i,j}$. W^{-1} denotes the inverse of the tail mass function

$$W^+(x) = \int_x^{+\infty} w(y)dy, \tag{11}$$

where $w(y)$ is a density the Lévy measure of the Lévy-Khintchine representation for z(1) (see chapter 8 in Schoutens (2001) for detailed information of simulation techniques for Lévy processes). The only special case where the sums in (10) have only a finite number of non-zero terms is the gamma marginal distribution of instantaneous volatility. In other cases sums need to be truncated. In the case of the gamma distribution for instantaneous volatility process: $\sigma^2(t) \sim gamma(\nu,\alpha)$ ($\nu > 0$ is the scale parameter and α is the precision parameter) the inverse of the tail mass function W^{-1} takes the form (Barndorff-Nielsen and Shephard, 2001):

$$W^{-1}\left(\frac{a_{i,j}}{\lambda\Delta}\right) = \max\left\{0, \frac{1}{\alpha}\ln\left(\frac{\nu\lambda\Delta}{a_{i,j}}\right)\right\} \tag{12}$$

which is zero for $a_{i,j} \geq \nu\lambda\Delta$.

There is no agreement in the literature on how to choose a marginal distribution. In the rest of the paper we follow Roberts *et al.* (2004), Griffin and Steel (2006), Frühwirth-Schnatter and Sögner (2009), Raknerud and Skare (2011), Chopin *et al.* (2013) and use the gamma marginal distribution.

4. Simulation study

Since convergence of iterated filtering IF2 to the maximum likelihood estimates in the case of BN-S model is difficult to prove analytically, we checked the performance of the method in a simulation study. We considered 4 scenarios with different combinations of parameters. Values of the parameter were taken from Barndorff-Nielsen and Shephard (2002) and Creal (2008). We simulated 500 realizations of each scenario of length T=1000 observations. We run iterated filtering algorithm using J=100 and J=200 iteration with M=5000 particles. Table 1 presents mean errors (MEs) and mean standard errors (MSEs) obtained in the study. For the purpose of comparison, Table 1 reports also MEs and MSEs for the quasi-likelihood inference based on the Kalman filter. Thus, we set $\mu = \beta = 0$ and assessed precision only for volatility parameters: λ – the persistence parameter, ξ – the expected value of marginal distribution ($E(\sigma^2(t)) = \xi = \nu / \alpha$) and the standard deviation of marginal distribution ($\sqrt{Var(\sigma^2(t))} = \omega = \sqrt{\nu} / \alpha$).

Table 1. MEs and MSEs of the estimators

Parameters	KF		IF2 (J=100)		IF2 (J=200)	
	ME	MSE	ME	MSE	ME	MSE
$\lambda = 0.01$	0.066	0.261	0.021	0.163	0.013	0.121
$\xi = 0.5$	0.061	0.166	0.042	0.159	-0.032	0.143
$\omega = 0.25$	0.093	0.13	-0.046	0.186	-0.011	0.012
$\lambda = 0.05$	0.056	0.219	0.015	0.126	0.011	0.109
$\xi = 0.5$	-0.011	0.142	0.045	0.166	0.039	0.132
$\omega = \sqrt{0.4}$	0.072	0.146	0.051	0.232	-0.086	0.123
$\lambda = 0.1$	0.011	0.119	-0.005	0.086	0.019	0.021
$\xi = 0.5$	0.063	0.242	-0.021	0.166	-0.012	0.159
$\omega = 0.25$	0.091	0.246	0.051	0.131	0.013	0.011
$\lambda = 0.1$	0.013	0.145	0.009	0.026	0.019	0.021
$\xi = 0.5$	-0.051	0.171	-0.032	0.146	-0.022	0.169
$\omega = \sqrt{0.4}$	0.093	0.381	0.046	0.322	0.023	0.186

Source: Own work.

The results indicate that the proposed iterated filtering IF2 algorithm is quite reliable. For a smaller number of iterations J=100, the estimators seem to be biased but they become more precise as J increases. Both versions of IF2 outperform quasi-likelihood inference.

5. Empirical example

We estimate models by using Standard & Poor's 500 index (S&P500) daily data for the period 9.10.2012-30.09.2016. S&P500 index is one of the most important American stock market index. It is based on the market capitalizations of 500 large companies listed on the NYSE or NASDAQ. Data consist of 1001 closing values and 1000 log-returns. Table 2 and Figure 2 present data.

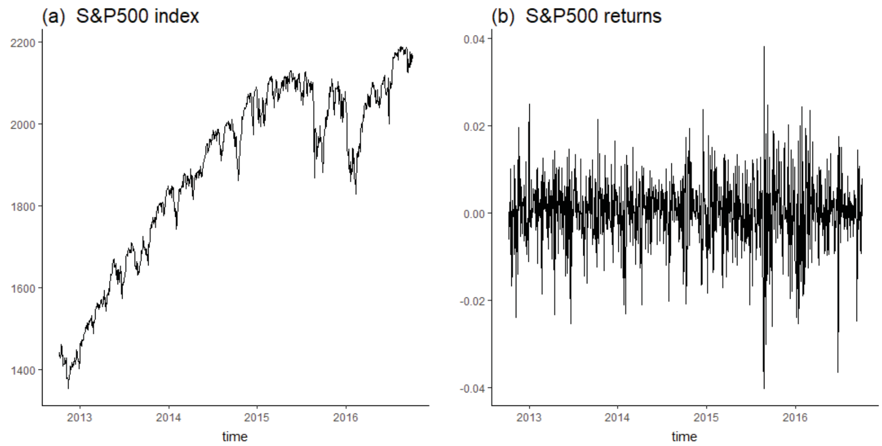

Figure 2. S&P500 daily index (a) and log-returns (b)

Source: Own work using R software.

Table 2. Descriptive statistics of S&P500 daily log-returns

Mean	Standard deviation	Skewness	Kurtosis	Quantiles		
				25%	50%	75%
0.0004	0.0083	-0.3830	5.0486	-0.0036	0.0005	0.0050

Source: Own work.

We run the iterated filtering algorithm with J=200 iteration. Each of iteration uses the bootstrap particle filter with M=5000 particles. Results of estimation are presented in Table 3. The drift parameter μ is close to zero. As may be expected from financial theory, the risk-premium coefficient β is positive. The estimated average actual volatility ξ and standard deviation ω correspond to gamma distribution with the scale parameter ν=1.571 and the precision parameter α=14.124. Figure 3 presents diagnostic plots for iterated filtering. These plots suggest that the likelihood has in fact been maximized by iterated filtering in our analysis of log-returns of S&P500 index.

Table 3. Estimation results for the log-returns of the S&P500 index

Parameter	μ	β	λ	ξ	ω
Estimates	-0.001	0.051	0.026	0.111	0.089

Source: Own work.

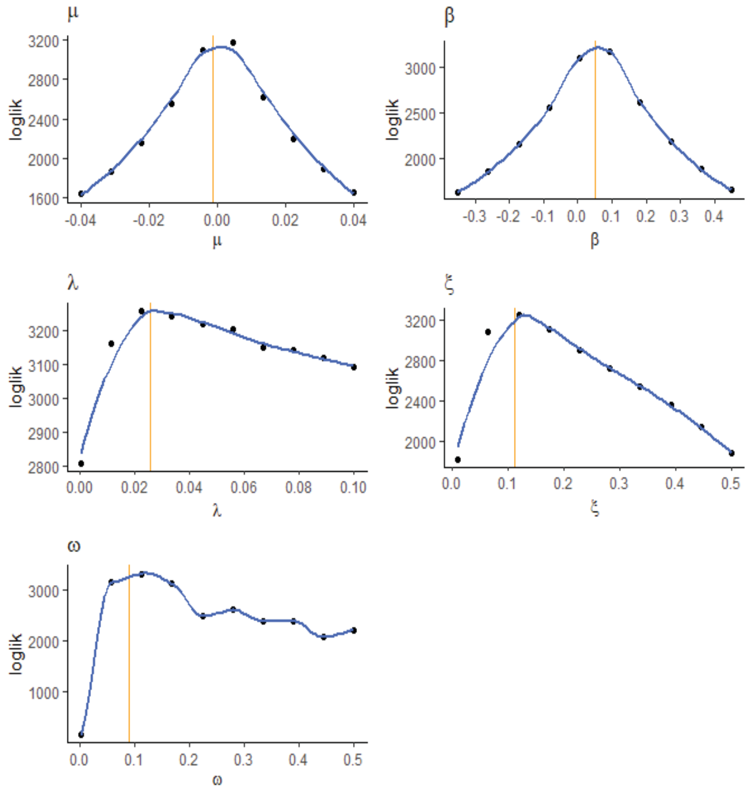

Figure 3. Diagnostic plots for iterated filtering: sliced likelihoods for the inquired parameters. For each plot the likelihood surface is explored along one of the parameters, keeping the other parameters fixed at the point which iterated filtering algorithm converges to. Points show the likelihood estimate obtained with 2,000 particles and the curves result from smoothing the likelihood evaluations with local quadratic regression. The vertical lines show iterated filtering estimates.

Source: Own work using R software.

6. Conclusions

In this article, we presented estimation of a class of stochastic volatility models where the volatility follows an Ornstein–Uhlenbeck process driven by a positive Lévy process via iterated filtration. This class of models, introduced by Barndorff-Nielsen and Shephard (2001), and therefore typically abbreviated to BN-S, has several important features, which aroused great interest in financial modelling for this class of stochastic volatility models.

From a theoretical point of view, the estimation method proposed in this article is convenient because it only requires to simulate the state process and to evaluate conditional density of the observation process given the simulated values of the state process. This feature, also known as plug-and-play property, is crucial for BN-S models, for which transition density is not available as a closed-form expression. Iterated filtration provides likelihood-based inference based on frequentist probability, which may be seen as competitive to plug-and-play methods that are based on Bayesian paradigm such as Particle Markov Chain Monte Carlo or Approximate Bayesian Computation. In this article, we exploited the second generation of iterated filtration IF2 introduced by Ionides *et al.* (2015), which outperforms the first generation IF1 in the rates of convergence to maximum likelihood estimates.

The results of the simulation study confirmed the validity of the approach in the case of BN-S model. In an application of the proposed method to S&P500 daily data, we presented, apart from estimates of parameters, also diagnostic plots for iterated filtering to ensure convergence to maximum likelihood estimates.

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