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The positional MEF-TOPSIS method for the assessment of complex economic phenomena in territorial units

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ABSTRACT

In this paper, the authors propose a new methodological approach to the construction of a synthetic measure, where the objects are described by variables with strong asymmetry and extreme values (outliers). Even a single extreme value (very large or very small) of a variable for the object may significantly affect the attribution of an excessively high or low rank in the final ranking of objects. This dependence is particularly apparent when using the classical TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method. The aim of the study is to present the application potential of the positional MEF-TOPSIS method for the assessment of the level of development of complex economic phenomena for territorial units. In the positional TOPSIS method, the application of the spatial median of Oja, which limits the impact of strong asymmetry, is proposed. In order to weaken the influence of extreme values, the Mean Excess Function (MEF) is used, by means of which it is possible to identify the limits of extreme values and establish model objects. The proposed approach is used to assess the financial self-sufficiency of Polish municipalities in 2016. The study finally compares the results of applications of positional MEF-TOPSIS and the classic and positional TOPSIS methods.

Key words: synthetic measure, TOPSIS, spatial median of Oja, Mean Excess Function.

1. Introduction

The complex nature of the economics phenomena taking place in the real world causes many problems in the research for territorial units. Complex economics phenomena (e.g. financial self-sufficiency, socio-economic development, standard of living) include many various problems, which are often difficult to identify and quantify. Therefore, these phenomena cannot be measured directly, but they can only be evaluated based on different criteria and variables. As there are many aspects to the

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process, various analyses are carried out. One type of analysis is the assessment of the phenomenon level by a synthetic measure. In this case, the use of the classical statistical methods imposes some restrictions, which often lead to the excessive simplification of the analysis. In such studies the non-classical multi-criteria quantitative methods are very useful. Therefore, the authors proposed a novel hybrid approach to the construction of a synthetic measure in the study.

The procedure for constructing the synthetic measure is a multi-stage process. One of the most important stages is selecting variables for studies. It is a very complicated issue, especially when unusual data values (e.g. extreme values) appear or strong asymmetry occurs within variables. It may result from the specifics of the complex phenomenon studied. These anomalous observations have a crucial impact on the results of the research. The occurrence of even only one problematic value of the variable for the object may significantly affect the attribution of the incorrect (excessively high or excessively low) rank in the final ranking of objects. This also leads to incorrect identification of types of the complex economics phenomena on its basis. Therefore, it is necessary to seek optimal methods to identify extreme values and develop new methodological approaches which are resistant to these phenomena.

In this study the authors propose a novel hybrid methodological approach to the construction of the synthetic measure, where the objects are described by the variables with extreme values and strong asymmetry. The aim of the study is to present the application potential of the positional MEF-TOPSIS method to assess the development level of the complex economics phenomena for territorial units. The proposed method is based on the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method (Hwang and Yoon, 1981). The TOPSIS method is very useful in constructing the ranking of objects described by many variables. The synthetic measure is constructed on the base of the distances from the model values (positive ideal solution and negative ideal solution). In the case of data set with unusual variables, the assumption that the maximum and minimum values of the variables are model values leads to excessive remoteness from typical values of the considered variables and consequently narrows the range of variability of the constructed synthetic measure. The problem may be solved by application of the Mean Excess Function (MEF) for identifying the limits of extreme values and establishing the model objects. As a result, the influence of extreme values (outliers) was reduced, whereas the spatial median of Oja was used in order to limit the impact of strong asymmetry. This novel hybrid approach was used in the assessment of financial self-sufficiency of local administrative units in Poland in 2016. The research hypothesis was that the construction of a synthetic measure for complex economic phenomena, described by variables with extreme values, using positional MEF-TOPSIS allows to perform more accurate classifications of objects and to distinguish more homogeneous types than approaches using the classical and positional TOPSIS methods.

2. Methods

The classical TOPSIS method was first presented by Hwang and Yoon (1981) and is the most established technique for solving Multi-Criteria Decision Making (MCDM) problems. TOPSIS is based on the idea that the best object should have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution. The main assumption of the method is that the variables monotonically increase or decrease. TOPSIS was further developed by Yoon (1987) and also Hwang, Lai and Liu (1993). Nowadays, many different extensions of TOPSIS exist. They are based on triangular fuzzy numbers (Chen, 2000), interval data (Jahanshahloo, Lotfi and Izadikhah, 2006), interval-valued fuzzy sets (Chen and Tsao, 2008), interval type-2 fuzzy sets (Chen and Lee, 2010), interval-valued intuitionistic fuzzy sets (Li, 2010) and multi-granularity linguistic assessment information (Liu, Chan and Ran, 2013), positional notation (Wysocki 2010, Kozera, Łuczak and Wysocki, 2016).

Extended versions of TOPSIS have solved many methodological problems in the assessment of the development level of the complex economics phenomena. The interval TOPSIS method is employed when determining the variable values of the object precisely is difficult and the values can be presented by means of intervals, i.e. two extreme variable values, which are minimum and maximum (Łuczak, 2015). The fuzzy TOPSIS method allows for the construction of the synthetic measure and the linear ordering of the objects described by means of both metrical and non-metrical (ordering) variables, owing to the transformation of the ordering characteristics into fuzzy numbers, which is one of the ways to strengthen the measurement scale (Wysocki and Łuczak, 2009). Furthermore, many hybrid approaches and their application have been presented. The approaches combine TOPSIS with the following methods: AHP (Kusumawardani and Agintiara, 2015), Pareto and genetic algorithm method (Taleizadeh, Niaki and Aryanezhad, 2009), SAW and GRA (Wang, Zhu and Wang, 2016), POT (Łuczak, Just and Kozera, 2018). A broad review of different versions of the TOPSIS method and their application was carried out by Behzadian et al. (2012); Velasquez and Hester (2013); Mardani, Jusoh and Zavadskas (2015); Afsordegan et al. (2016); Nădăban, Dzitac and Idzitac (2016), Zavadskas et al. (2016).

The novel hybrid approach to the construction of the synthetic measure, proposed by the authors, combines Technique for Order of Preference by Similarity to Ideal

Solution (TOPSIS) and the Mean Excess Function (MEF). The procedure based on the modified positional MEF-TOPSIS method includes seven main stages:

Stage 1. Selection of variables on the complex phenomenon and identification of extreme values by application of the Mean Excess Function,

Stage 2. Determination of the impact direction of variables in relation to the complex phenomenon,

Stage 3. Normalization of the variable values with utilization of the spatial median of Oja,

Stage 4. Calculation of the positive ideal solution and negative ideal solution,

Stage 5. Calculation of the distance of each object from positive and negative ideal solutions,

Stage 6. Calculation of values of the synthetic measure,

Stage 7. Ranking classification of objects and identification of the types.

The first stage is the selection of variables of the complex phenomenon and the identification of extreme values. The selection of variables for objects (e.g. territorial units: countries, regions, states, districts, municipalities) is to be carried out based on substantive and statistical analysis. The set of variables describing the complex phenomenon (e.g. financial self-sufficiency, socio-economic development, standard of living) for territorial units is usually characterized by strong asymmetry or includes extreme values. In the real data studied, the choice of a suitable threshold of extreme values is frequently a very difficult task. In order to identify extreme values, an approach based on the Extreme Value Theory (EVT) was used. The EVT is a powerful and robust theory for studying the tail behaviour of a distribution of variable. Two approaches are used in the EVT to model extreme values. The first approach is based on the Block Maxima Model (BMM), estimating the distribution of extremes. The second is based on the Peaks over Threshold Model (POT), estimating the tail of the distribution of the variable. The Mean Excess Function plot is useful for determining the appropriate thresholds for extreme values of the variable in the POT. The MEF is also a convenient visual tool for examining whether a variable has a specific distribution (Chen et al., 2015). In the research, the MEF allows a threshold (limit) of extreme values to be established.

In the POT (see e.g. McNeil, 1999, Echaust, 2014), the tail of the distribution of the variable is modelled using the Generalized Pareto Distribution (GPD), while the beginning of this tail is determined by specifying a threshold value (ul_k). In this approach, the starting point for the considerations is the conditional distribution of excess over ul_k of random variable X_k (k^{th} variable), which is defined by the formula:

$$F_{ul_k}(y_k) = P(X_k - ul_k \leq y_k | X_k > ul_k) = \frac{F(y_k + ul_k) - F(ul_k)}{1 - F(ul_k)}, \quad (1)$$

where: $y_k = x_k - ul_k > 0$, F – an unknown distribution function of random variable X_k. According to the Pickands–Balkema–de Haan theorem, for a sufficiently large ul_k, the distribution function F_{ul_k} is definite and well approximated by the GPD with the distribution function:

$$G_{\xi,\beta}(x_k - ul_k) = \begin{cases} 1-(1+\xi(x_k - ul_k)/\beta)^{-1/\xi}, & \xi \neq 0 \\ 1-\exp(-(x_k - ul_k)/\beta), & \xi = 0 \end{cases}, \quad (2)$$

where: $\beta > 0,\ x_k - ul_k \geq 0$ for $\xi \geq 0$ and $0 \leq x_k - ul_k \leq -\beta/\xi$ for $\xi < 0$. This distribution has two parameters: ξ – the shape parameter determining the thickness of the tail and β – the scale parameter. The positive values of the shape parameter mean that the distribution has fat tails. It is connected with an increased probability of extreme variable values. In turn, negative values of the shape parameter denote that the distribution has the finite right endpoint. The choice of the threshold value ul_k is very important, because it affects the obtained values of the GPD parameter estimators. If N is the number of observations, N_{ul_k} is the number exceeding ul_k, the estimator of the distribution function F is calculated from the following formula:

$$\hat{F}(x_k) = 1 - \tfrac{N_{ul_k}}{N}\left(1+\hat{\xi}\tfrac{(x_k - ul_k)}{\hat{\beta}}\right)^{-1/\hat{\xi}}. \quad (3)$$

Selecting the ul_k threshold should take into account the specifics of the variable and their number. The threshold selection methods have been described, for example, by Coles (2001). One of the methods is the analysis of the stability of the GPD parameters estimates. This method was used in POT-TOPSIS by Łuczak, Just and Kozera (2018). The next method is based on the analysis of the graph of the Mean Excess Function. In this method, the starting point is the conditional expected value:

$$E(X_k - ul_k | X_k > ul_k) = \frac{\beta(ul_k)}{1-\xi}, \quad \xi < 1. \quad (4)$$

Since $\beta(ul_k)$ depends linearly on ul_k, the empirical estimator of the conditional expected value also must depend linearly on ul_k. Therefore, the graph of the Mean Excess Function:

$$\left\{\left(ul_k, \frac{1}{N_{ul_k}}\sum_{i=1}^{N_{ul_k}}(x_{ik} - ul_k)\right) : ul_k < x_{k\max}\right\} \quad (5)$$

after exceeding ul_k should be linear. The lower limit (ll_k) of the variable is determined by performing calculations for the values of the variable with a negative coefficient.

Identification of even one variable with extreme values (or even one value) does not allow the use of a classical approach to the construction of a synthetic measure. In the second stage, the impact direction of variables in relation to the complex phenomenon is determined. The selected variables have a positive (stimulating) or negative (de-stimulating) influence on the phenomenon. Variables that have a stimulating influence, contribute to increasing the phenomenon level. These variables are called stimulants. Variables that have a de-stimulating influence, decrease the phenomenon level. The destimulating variables are called destimulants. Destimulants should be converted into stimulants with the use of a negative coefficient transformation:

$$x_{ik} = a - b \cdot x_{ik}^{D}, \tag{6}$$

where: x_{ik}^{D} – value of the k^{th} variable, identified as a destimulant $(k \in I_D)$, in the i^{th} object (i = 1, …, N), x_{ik} – value of the k^{th} variable (k = 1, …, K) converted into a stimulant, a, b – constants establish arbitrarily (e.g. a = 0 and b = 1).

The third stage is the normalization of variable values. There are many different ways to normalize the value of variables and these methods have different properties. The choice of the best approach for variables of the complex economic phenomena is not simple and requires innovative methods and approaches. In the case of the assessment of a complex phenomenon of units, variables with extreme values or characterized by an asymmetrical distribution of their values are often observed. Therefore, to solve this problem, the modified median standardization was proposed using the spatial median of Oja (cf. Lira, Wysocki and Wagner, 2002). The spatial median of Oja is resistant to variables with strong asymmetry (Oja, 1983, Ronkainen, Oja and Orponen, 2002). Additionally, for limiting the influence of extreme values of variables, threshold values of variables ul_k and ll_k (k = 1, …, K) were applied in the formula of the modified median standardization:

$$z_{ik} = \begin{cases} \dfrac{ll_k - m\widetilde{e}d_k}{1.4826 \cdot m\widetilde{a}d_k} & \text{for } x_{ik} \le ll_k \\ \dfrac{x_{ik} - m\widetilde{e}d_k}{1.4826 \cdot m\widetilde{a}d_k} & \text{for } ll_k < x_{ik} < ul_k \\ \dfrac{ul_k - m\widetilde{e}d_k}{1.4826 \cdot m\widetilde{a}d_k} & \text{for } x_{ik} \ge ul_k \end{cases}, \tag{7}$$

where: x_{ik} – value of the k^{th} variable in the i^{th} object, $m\widetilde{e}d_k$ – Oja's median vector (θ) component corresponding to the k^{th} variable, $m\widetilde{a}d_k = med_i\left|x_{ik} - m\widetilde{e}d_k\right|$ – median absolute deviation of k^{th} variable values from the median component of the k^{th} variable, 1.4826 – a constant scaling factor corresponding to normally distributed data, $\sigma \approx E\left(1.4826 \cdot m\widetilde{a}d_k\left(X_1, X_2, ..., X_K\right)\right)$, $\sigma -$ standard deviation (see, e.g. Młodak 2006). An alternative version of the spatial median was given by Weber (1909).

The median standardization is calculated for Winsorized data. Winsorization is the transformation of a variable by limiting its extreme values. In the process of Winsorization a specified number of extreme values of a variable is replaced with a constant (smaller or bigger) value. The constants are established based on the MEF plot. The authors propose to adopt threshold values of variables ul_k and ll_k $(k = 1, 2, \ldots, K)$ as the constants in Winsorization.

In the fourth stage, the positive ideal solution (PIS) and the negative ideal solution (NIS) were calculated (Hwang and Yoon, 1981):

$$\text{PIS} \qquad A^+ = \left(\max_i(z_{i1}), \max_i(z_{i2}), \ldots, \max_i(z_{iK})\right) = \left(z_1^+, z_2^+, \ldots, z_K^+\right), \tag{8}$$

$$\text{NIS} \qquad A^- = \left(\min_i(z_{i1}), \min_i(z_{i2}), \ldots, \min_i(z_{iK})\right) = \left(z_1^-, z_2^-, \ldots, z_K^-\right). \tag{9}$$

The PIS are the best values of variables, which are stimulant or are transformed into stimulant, whereas the NIS are the worst values of normalized variables.

Next, Manhattan distances (L_1 distances) for each object from the PIS (A^+) and the NIS (A^-) were calculated based on (stage 5):

$$d_i^+ = \sum_{k=1}^{K} \left| z_{ik} - z_k^+ \right|, \quad d_i^- = \sum_{k=1}^{K} \left| z_{ik} - z_k^- \right|. \tag{10}$$

The sixth stage involves calculation of values of the synthetic measure with the use the Hwang and Yoon's formula (1981):

$$S_i = \frac{d_i^-}{d_i^- + d_i^+} \quad (i = 1, \ldots, N). \tag{11}$$

The higher the values of the synthetic measure, the better the development level of the complex phenomenon and vice versa.

The calculated values of the synthetic measure are the basis of ranking the objects and identification of their typological classes (stage 7). Class identification can be carried out by different statistical methods or in an arbitrary manner. In the study the arbitrary approach based on a division of synthetic measure into eight classes is proposed, assuming:

Class I (extremely high level)	$S_i \in \langle 0.875, 1.000 \rangle$
Class II (very high level)	$S_i \in \langle 0.750, 0.875)$
Class III (high level)	$S_i \in \langle 0.625, 0.750)$
Class IV (medium-high level)	$S_i \in \langle 0.500, 0.625)$
Class V (medium-low level)	$S_i \in \langle 0.375, 0.500)$
Class VI (low level)	$S_i \in \langle 0.250, 0.375)$
Class VII (very low level)	$S_i \in \langle 0.125, 0.250)$
Class VIII (extremely low level)	$S_i \in \langle 0.000, 0.125)$

The classes of the level of financial self-sufficiency for municipalities were evaluated by statistical criteria. For this purpose, measures of homogeneity were applied. It is a concept related to the degree of similarity of objects in the same class. The idea of the measures is based on distances of objects from the centre of gravity of a class (cf. Młodak, 2006):

$$d_{ic} = \sum_{k=1}^{K} \left| x_{ik} - v_{kc} \right| \quad (c = 1,\ldots, C), \tag{12}$$

$$\bar{d}_c = \frac{1}{N_c} \sum_{i \in P_c} d_{ic}, \tag{13}$$

$$H_{MO} = \frac{1}{r} \sum_{c=1}^{C} \bar{d}_c, \tag{14}$$

where: d_{ic} – intra-class distances for each object in c^{th} class from the centre of gravity of the c^{th} class ($c = 1,\ldots, C$), C – the number of typological classes, $v_c = (v_{1c}, v_{2c}, \ldots, v_{Kc}) = \left(\underset{i \in P_1}{\text{med}}(x_{ik}),\ \underset{i \in P_2}{\text{med}}(x_{ik}),\ \ldots,\ \underset{i \in P_C}{\text{med}}(x_{ik}) \right)$ – the centre of gravity of the c^{th} class (median of its elements), P_c – a set of subscripts of objects belonging to the c^{th} class, $\bar{d}_c$ – the partial mean measure of homogeneity of c^{th} class, N_c – the number of objects in c^{th} class, H_{MO} – the total mean measure of homogeneity, r – the number of non-empty classes.

Also, the total inter-clusters homogeneity measure is based on the idea of Hubert and Lewin (cf. 1976) is proposed by the authors:

$$H_O^c = \frac{\bar{d}_c - \min_i(d_{ic})}{\max_i(d_{ic}) - \min_i(d_{ic})}, \quad H_O^c \in \langle 0,1 \rangle, \tag{15}$$

$$H_O = \frac{1}{r} \sum_{c=1}^{C} H_O^c, \tag{16}$$

where H_O – the total measure of homogeneity. Also, the total mean intra-class distance is a useful measure for assessing homogeneity of clusters: $\bar{d} = \frac{1}{N} \sum_{i=1}^{N} d_{ic}$. The lower the value of the measures $(H_{MO}, H_O, \bar{d})$, the more homogeneous the classes.

3. Results of research

The proposed approach was used to assess the level of financial self-sufficiency of municipalities (*N*=2412) in Poland in 2016. The study was based on statistical data from 2016 coming from the Central Statistical Office of Poland (*Local Data Bank*). In the first stage of the study, five indicators (variables) were selected based on a substantive and statistical analysis. These were the following indicators: x_1 – own income per capita (in PLN), x_2 – share of own income in total income (in %), x_3 – transfer income (including specific grants and the general subsidy) per capita (in PLN), x_4 – share of tax income (tax bill of agriculture, forestry, real estate, from transport fund of civil law, income from taxation, income from mining fee) in current income (budgetary revenue other than income property) (in %), x_5 – self-financing rate (share of operating surplus/deficit and capital income in capital expenditure).

Table 1. Descriptive statistics and threshold values of the indicators of municipalities in Poland in 2016

Specification	Variables				
	x_1	x_2	x_3	x_4	x_5
Mean	1575.14	38.17	2442.17	15.04	191.10
Median	1382.34	35.98	2441.40	14.01	141.59
Max	45340.71	95.04	4521.20	59.74	18507.88
Min	508.17	13.08	1163.22	2.24	-134.22
St. dev.	123.45	13.20	550.34	6.86	487.88
Mad	352.63	9,30	412.50	3.99	40.65
Range	44832.54	81.96	3357.98	57.50	18642.10
Skewness	20.35	0.58	0.18	1.34	29.95
Ex. kurtosis	667.11	-0.15	-0.37	3.30	1026.09
ll_k	657.275	17.268	1310.765	3.840	73.015
ul_k	2239.219	65.852	3688.543	27.673	270.690

Descriptive statistics of the indicators are presented in Table 1. The greatest volatility, measured by the range, standard deviation and median absolute deviation, was found for x_1, x_3 and x_5. Positive skewness was observed for all indicators, with extremely high skewness noticed for x_1 and x_5. The distributions of three indicators x_1, x_4 and x_5, demonstrated positive kurtosis. This means that extreme values in the indicators appear more frequently that in the normal distribution. In order to limit the influence of extreme values, the limits of extreme values of indicators were established based on analysis of the Mean Excess Function graph (Table 1). The calculations were performed with package *fExtremes* in *R* (Wuertz, Setz and Chalabi, 2017). The results of the analysis indicated the occurrence of very fat right tails of the distribution of indicators x_1 (estimation of shape parameter 0.42) and x_5 (estimation of shape parameter 0.75). The distribution of Winsorized data demonstrated small skewness. Moreover, kurtosis for indicators x_1, x_4, x_5 was close to normal distribution.

In the second stage, it was assumed that four indicators have a stimulating effect (x_1, x_2, x_4, x_5) and one indicator (x_3) has a de-stimulating effect on the level of financial self-sufficiency of municipalities. The indicator, which was a destimulant, was converted into an opposite indicator type.

In next stage, the values of variables were standardized by the modified Oja's median standardization. The spatial median of Oja was calculated with *OjaNP* package in *R* (Fischer et al., 2015). The standardized values of indicators allowed the authors to calculate the distances of each municipality considered from the PIS and the NIS using the Manhattan distance. In the sixth stage, the values of the synthetic measure were calculated using the positional MEF-TOPSIS method. This allowed the authors to identify eight types of municipal financial self-sufficiency levels in Poland in 2016 (Table 2). The proposed approach (approach I) was compared with the classical TOPSIS (approach II), MEF-TOPSIS (approach III) and positional TOPSIS by Wysocki (approach IV).

Table 2. Typological classification of municipalities in Poland in terms of the level of financial self-sufficiency in 2016

Class	Level of financial self-sufficiency	S_i	Approaches							
			I		II		III*		IV**	
			N_c	%	N_c	%	N_c	%	N_c	%
I	extremely high	⟨0.875, 1.000⟩	32	1.3	0	0.0	11	0.5	0	0.0
II	very high	⟨0.755, 0.875)	194	8.0	0	0.0	112	4.6	2	0.1
III	high	⟨0.625, 0.750)	359	14.9	0	0.0	425	17.6	12	0.5
IV	medium-high	⟨0.500, 0.625)	431	17.9	2	0.1	496	20.6	43	1.8
V	medium-low	⟨0.375, 0.500)	517	21.4	1	0.0	562	23.3	240	10.0
VI	low	⟨0.250, 0.375)	521	21.6	1	0.0	525	21.8	827	34.3
VII	very low	⟨0.125, 0.250)	294	12.2	135	5.6	248	10.3	976	40.5
VIII	extremely low	⟨0.000, 0.125)	64	2.7	2273	94.2	33	1.4	312	12.9

* MEF-TOPSIS with Winsorized data. ** – positional TOPSIS with standardization using Oja's spatial median and pseudo distances for each object from the PIS and the NIS calculated using median absolute deviation.

The rankings obtained by means of the applied methods indicate differences (Wilcoxon rank sum test) in the values of synthetic measures and the arrangement of municipalities into classes (Tables 2, 3). A similar classification was created only for the positional MEF-TOPSIS and the MEF-TOPSIS methods. The values of synthetic measures for these methods did not differ significantly (at the significance level of 0.1, Wilcoxon rank sum test). These methods are resistant to the occurrence of outliers. The use of the MEF graphs analysis and the determination of the values of PIS and NIS on this basis resulted in greater ranges of variability in the values of synthetic measures. The synthetic measures values fall within the following intervals: ⟨0.003, 0.961⟩ and ⟨0.006, 0.940⟩, respectively. This allowed eight types of municipalities to be determined (including eight levels of financial self-sufficiency of municipalities, from extremely low to extremely high). The application of the classical and positional TOPSIS methods was associated with obtaining the values of the synthetic measures from the intervals ⟨0.011, 0.515⟩ and ⟨0.023, 0.827⟩, respectively. The synthetic measure values in the classical and positional TOPSIS methods are more concentrated and have stronger skewness than in the positional MEF-TOPSIS and the MEF-TOPSIS methods. In the case of the classical TOPSIS method, almost all municipalities (99.8%) were qualified to classes representing an extremely low level or a very low level of financial self-sufficiency. The use of the positional TOPSIS method allowed to distinguish seven levels of financial self-sufficiency of municipalities, from extremely low to very high. In this case, almost all municipalities (almost 98%) were qualified to classes representing levels of financial self-sufficiency from extremely low to medium-low. Despite the indicated differences in the distribution of the values of synthetic measures obtained for the applied methods, the high values of Spearman's and Kendall's rank correlation coefficients of the synthetic measures pointed to a high agreement of the linear ordering results. However, the values of the synthetic measure obtained in classical and positional TOPSIS approaches, especially the values close to zero, do not allow for a meaningful identification of types of financial self-sufficiency of municipalities.

Table 3. Descriptive statistics of the synthetic measures of financial self-sufficiency of municipalities in Poland according to approaches

Specification	Approaches			
	I	II	III	IV
Max	0.961	0.515	0.940	0.827
Min	0.003	0.011	0.006	0.023
Range	0.959	0.504	0.935	0.804
Median	0.452	0.082	0.459	0.237
Mean	0.467	0.085	0.468	0.249
Skewness	0.172	3.309	0.076	0.673
Ex. kurtosis	-0.774	40.106	-0.774	0.818

The classes of the level of financial self-sufficiency for municipalities were evaluated by statistical criteria. Measures of homogeneity were calculated for this purpose (Table 4). Values of the calculated measures indicate that the use of the positional MEF-TOPSIS and the MEF-TOPSIS methods allowed to identify municipality classes characterized by better homogeneity than using the classical and positional TOPSIS methods. It should be added that the highest homogeneity was recorded for classes obtained with the positional MEF-TOPSIS method.

Table 4. Values of homogeneity measures according to approaches

Specification	Approaches			
	I	II	III	IV
H_{MO}	760.9	6775.5	840.5	3734.7
$\bar{d}$	621.2	963.5	622.2	643.4
H_O	0.017	0.215	0.019	0.186

On the basis of the analyses carried out, there is no reason to reject the research hypothesis (the construction of a synthetic measure for complex economic phenomena, described by variables with extreme values, using positional MEF-TOPSIS allows to perform more accurate classifications of objects and to distinguish more homogeneous types than approaches using the classical and positional TOPSIS methods).

4. Conclusion

The proposed positional MEF-TOPSIS method (using Oja's spatial median) of linear ordering of objects reduces the impact of strong asymmetry and extreme values of variables describing objects. The Mean Excess Function to identify extreme values and establish model objects (PIS and NIS) was used in this approach for this purpose. In the case of linear ordering, the occurrence of even one outlier (very large or very small) for an object can significantly affect the assignment of an excessively high or low rank in the final classification of objects. This is particularly evident when the classical TOPSIS method is used. Using the positional TOPSIS with standardization based on Oja's spatial median and pseudo distances for each object from the PIS and the NIS, calculations using median absolute deviation improve the classification of objects. The reason is that in the classical TOPSIS method the squared deviations of each multi-variable object from the PIS and the NIS are calculated and aggregated, whereas in the positional TOPSIS the median from absolute deviations is used, which enables locating the centre of the set of absolute differences between each multi-variable object and the PIS and the NIS. In turn, it makes it possible to limit the impact of outliers on the construction of the synthetic measure. Similar rankings and classifications of objects gave the positional MEF-TOPSIS and the MEF-TOPSIS while in the case of the application of the first method, classes are characterized by greater homogeneity.

The typology of municipalities in Poland in 2016 created on the positional MEF-TOPSIS basis well reflects the inter-class differences in financial self-sufficiency of municipalities. It includes eight classes of municipalities, spanning from an extremely low to extremely high level of financial self-sufficiency.

The research showed that the construction of a synthetic measure for complex economic phenomena, described by variables with extreme values, using the positional MEF-TOPSIS allows to perform more correct classifications of objects and to distinguish more homogeneous types than approaches using the classical and positional TOPSIS methods.

The authors recommend using the positional MEF-TOPSIS in the assessment of the development level of complex economics phenomena for territorial units described by variables with extreme values. In order to establish limits in the procedure of Winsorization, the authors recommend using the Mean Excess Function graphs analysis to determine the threshold of extreme values along with other statistical methods and substantive criteria to avoid mechanical and excessive Winsorization. The Winsorization based on only one criterion can lead to improper placement of objects in classes.

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