

$B_c \rightarrow B_{s(d)}$ form factors from lattice QCD

Laurence J. Cooper,^{1, a} Christine T. H. Davies,² Judd Harrison,^{2, b} Javad Komijani,^{2, 3} and Matthew Wingate¹
(HPQCD Collaboration)^c

¹*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, CB3 0WA, UK*

²*SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, UK*

³*Department of Physics, University of Tehran, Tehran 1439955961, Iran*

We present results of the first lattice QCD calculations of $B_c \rightarrow B_s$ and $B_c \rightarrow B_d$ weak matrix elements. Form factors across the entire physical q^2 range are then extracted and extrapolated to the physical-continuum limit before combining with CKM matrix elements to predict the semileptonic decay rates $\Gamma(B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell) = 52.4(2.5) \times 10^9 \text{ s}^{-1}$ and $\Gamma(B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell) = 3.10(21) \times 10^9 \text{ s}^{-1}$. The lattice QCD uncertainty is comparable to the CKM uncertainty here. Results are derived from correlation functions computed on MILC Collaboration gauge configurations with a range of lattice spacings including 2+1+1 flavours of dynamical sea quarks in the Highly Improved Staggered Quark (HISQ) formalism. HISQ is also used for the propagators of the valence light, strange, and charm quarks. Two different formalisms are employed for the bottom quark: non-relativistic QCD (NRQCD) and heavy-HISQ. Checking agreement between these two approaches is an important test of our strategies for heavy quarks on the lattice. From chained fits of NRQCD and heavy-HISQ data, we obtain the differential decay rates $d\Gamma/dq^2$ as well as integrated values for comparison to future experimental results.

I. INTRODUCTION

The semileptonic weak decays $B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell$ and $B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell$ proceed via tree-level flavour changing processes $c \rightarrow sW^+$ and $c \rightarrow dW^+$ parametrised by the Cabbibo-Kobayashi-Maskawa (CKM) matrix of the Standard Model. Associated weak matrix elements can be expressed in terms of form factors which capture the non-perturbative QCD physics. Precise determination of the normalisation and the q^2 dependence of these form factors from lattice QCD will allow a novel comparison with future experiment to deduce the CKM parameters V_{cs} and V_{cd} . Lattice studies of other semileptonic meson decays that involve tree-level weak decays of a constituent charm quark include [1–6]. Precise determination of these CKM matrix elements is critical for examining the second row unitarity constraint

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1. \quad (1)$$

This will complement other unitarity tests of the CKM matrix. It is possible LHCb could measure $B_c^+ \rightarrow B_s^0 \bar{\mu} \nu_\mu$ using Run 1 and 2 data. For example, normalising by $B_c^+ \rightarrow J/\psi \bar{\mu} \nu_\mu$ would yield a constraint on the ratio V_{cs}/V_{cb} . Due to CKM suppression, a measurement of $B_c^+ \rightarrow B^0 \bar{\mu} \nu_\mu$ is likely to require many more B_c^+ decays.

A lattice study of the $B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell$ and $B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell$ decays involves the practical complication of a heavy spectator quark. Care must be taken in placing such a particle on the lattice to avoid large discretisation effects. We consider two formalisms for the b quark. A valence

NRQCD [7, 8] b quark, a formalism constructed from a non-relativistic effective theory, is used to simulate with physically massive b quarks. A complementary calculation uses HPQCD’s heavy-HISQ method [9–11]. Here, all flavours of quark are implemented with the HISQ [12] formalism. This is a fully relativistic approach which involves calculations for a set of quark masses on ensembles of lattices with a range of fine lattice spacings, enabling a fit from which the physical result at the b quark mass in the continuum can be determined. The method with an NRQCD bottom quark also uses HISQ for the charm, strange and down flavours. This study will demonstrate the consistency of the NRQCD and heavy-HISQ approaches by comparing the form factors extrapolated to the physical-continuum limit.

In the limit of massless leptons, the differential decay rates for $B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell$ and $B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell$ are given by

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V|^2}{24\pi^3} |\mathbf{p}_2|^3 |f_+(q^2)|^2, \quad (2)$$

where V is the relevant associated CKM matrix element V_{cs} or V_{cd} and f_+ is one of two form factors that parametrise the continuum weak matrix element

$$\begin{aligned} \langle B_{s(d)}(\mathbf{p}_2) | V^\mu | B_c(\mathbf{p}_1) \rangle &= f_0(q^2) \left[\frac{M_{B_c}^2 - M_{B_{s(d)}}^2}{q^2} q^\mu \right] \\ &+ f_+(q^2) \left[p_2^\mu + p_1^\mu - \frac{M_{B_c}^2 - M_{B_{s(d)}}^2}{q^2} q^\mu \right]. \quad (3) \end{aligned}$$

The 4-momentum transfer is $q = p_1 - p_2$, and only the vector part of the $V - A$ weak current contributes since QCD conserves parity. The contribution of f_0 to the decay rate is suppressed by the lepton mass and hence irrelevant for the decays to $e\bar{\nu}_e$ and $\mu\bar{\nu}_\mu$. The phase space is sufficiently small to disallow decays to $\tau\bar{\nu}_\tau$. Form factors are constructed from the matrix elements that are

^aL.J.Cooper@damtp.cam.ac.uk

^bJudd.Harrison@glasgow.ac.uk

^cURL: <http://www.physics.gla.ac.uk/HPQCD>

TABLE I: Parameters for the MILC ensembles of gluon field configurations. The lattice spacing a is determined from the Wilson flow parameter w_0 [13] given in lattice units for each set in column 2 where values were obtained from [14] on sets 1 to 5 and [15] on set 6. The physical value $w_0 = 0.1715(9)$ was fixed from f_π in [16]. Sets 1 and 2 have $a \approx 0.15$ [fm], and sets 3 and 4 have $a \approx 0.12$ [fm]. Sets 5 and 6 have $a \approx 0.09$ [fm] and $a \approx 0.06$ [fm] respectively. Sets 1, 3, 5 and 6 have unphysically massive light quarks such that $m_l/m_s = 0.2$. Sets 1 to 5 were used in the NRQCD calculation of the form factors. The heavy-HISQ calculation used sets 3, 5 and 6.

set	w_0/a	$N_x^3 \times N_t$	n_{cfg}	am_l^{sea}	am_s^{sea}	am_c^{sea}
1	1.1119(10)	$16^3 \times 48$	1000	0.013	0.065	0.838
2	1.1367(5)	$32^3 \times 48$	500	0.00235	0.0647	0.831
3	1.3826(11)	$24^3 \times 64$	1053	0.0102	0.0509	0.635
4	1.4149(6)	$48^3 \times 64$	1000	0.00184	0.0507	0.628
5	1.9006(20)	$32^3 \times 96$	504	0.0074	0.037	0.440
6	2.896(6)	$48^3 \times 144$	250	0.0048	0.024	0.286

TABLE II: The HISQ valence quark masses for the light, strange and charm flavours for each of the sets described in Table I. For the light quarks, the values for the valence quarks are identical to the masses of the light sea quarks. The masses for the valence strange quarks and the valence charm quarks were tuned in [15] using w_0 (Table I) to fix the lattice spacing. The fourth and fifth columns give the valence charm quark masses for the calculations with NRQCD and HISQ spectator quarks respectively. In the calculation with NRQCD spectator quarks slightly different am_c^{val} values were used for historical reasons. Our fits allow for mistuning of the charm quark mass.

set	am_c^{val}		NRQCD spectator	HISQ spectator
	am_l^{val}	am_s^{val}		
1	0.013	0.0705	0.826	-
2	0.00235	0.0677	0.827	-
3	0.0102	0.0541	0.645	0.663
4	0.00184	0.0507	0.631	-
5	0.0074	0.0376	0.434	0.450
6	0.0048	0.0234	-	0.274

obtained by fitting the appropriate lattice QCD 3-point correlator data. By calculating correlators at a range of transfer momenta on lattices with different spacings and quark masses, continuum form factors at physical quark masses are obtained and then appropriately integrated to offer a direct comparison with decay rates that could be measured in experiment.

In this study, we begin with Sec. II in which details of the lattice calculations are described. Sec. II A reports on the parameters and gauge configurations used to generate the propagators. Next, Sec. II B explains how the correlators are subsequently constructed for the two different treatments of the heavy spectator quark, as well as how the correlator data is fit to extract the matrix elements. Our non-perturbative renormalisation method

TABLE III: The bottom quark masses, NRQCD action parameters c_j , and values for the tadpole improvement u_0 were obtained from [8]. The final columns gives the different momenta for the strange and light quarks considered in the NRQCD calculation implemented with twisted boundary conditions.

set	am_b^{val}	c_1, c_6	c_5	c_4	u_0	$ a\mathbf{q} $
1	3.297	1.36	1.21	1.22	0.8195	0 0.1243 0.3730 0.6217
2	3.25	1.36	1.21	1.22	0.8195	0 0.3649
3	2.66	1.31	1.16	1.20	0.8341	0 0.1 0.3 0.5
4	2.62	1.31	1.16	1.20	0.8341	0
5	1.91	1.21	1.12	1.16	0.8525	0 0.0728 0.364 0.437

required to obtain the form factors is set out in Sec. II C. Sec. III presents results of the lattice calculations. Correlator fits are examined in Sec. III A, whilst Sec. III B discusses results for the renormalisation of the local lattice vector current. In Sec. III C, the form factor data for the cases of an NRQCD spectator and a HISQ spectator are plotted alongside. Sec. IV is concerned with the methodology and results from fitting the form factor data. An extrapolation of the form factors to physical-continuum point is presented in Sec. IV D and Sec. IV E shows how the form factors depend on the mass of the spectator quark. Finally, in Sec. V we give our conclusions.

II. LATTICE CALCULATION

A. Parameters and Set-up

We use ensembles with $2 + 1 + 1$ flavours of HISQ sea quark generated by the MILC Collaboration [17–19] and described in Table I. The Symanzik-improved gluon action used is that from [20], where the gluon action is improved perturbatively through $\mathcal{O}(\alpha_s)$ including the effect of dynamical HISQ sea quarks. The lattice spacing is identified by comparing the physical value for the Wilson flow parameter $\omega_0 = 0.1715(9)$ fm [16] with lattice values for ω_0/a from [14] and [15]. Our calculations feature physically massive strange quarks and equal mass up and down quarks, with a mass denoted by m_l , with $m_l/m_s = 0.2$ and also the physical value $m_l/m_s = 1/27.4$ [5]. For sets 1 to 5 in Table I, strange propagators were re-used from [21], a study of the pseudoscalar meson electromagnetic form factor. Light propagators were re-used from [22], an extension of [21] to the pion. The valence quark masses used for the HISQ propagators on these gluon configurations are given in Table II. The valence strange and charm quark masses used here were tuned in [15, 21], slightly away from the sea quark masses to yield results that more closely correspond to physical values. The propagators were calculated using the MILC code [23].

We work in the frame where the B_c^+ is at rest, and

TABLE IV: Heavy quark masses and momenta used for the heavy-HISQ calculation. The momenta are in the (1 1 1) direction.

set	am_h^{val}			$ a\mathbf{q} $			
3	0.663	0.8		0	0.1	0.3	0.5
5	0.450	0.6	0.8	0	0.07281	0.218	0.364 0.437
6	0.274	0.450	0.6 0.8	0	0.143	0.239	0.334

momentum is inserted into the strange or down valence quark through twisted boundary conditions [24, 25] in the (1 1 1) direction. The values of the momenta used are given in Tables III and IV. The periodic boundary conditions of the fermion fields are modified by phases θ_i

$$\psi(n + N_x \hat{i}) = e^{i\pi\theta_i} \psi(n) \quad (4)$$

so that the usual lattice momenta $q_i = 2\pi k_i/aN_x$, for integers k_i , are shifted by $\pi\theta_i/aN_x$. The corresponding q^2 is then constructed by taking q_0 to be the difference in energies of the lowest lying initial and final states.

The coefficients of operators corresponding to relativistic correction terms in the NRQCD action are given in Table III. The valence b quark masses used for the NRQCD propagators are also given there. The values were taken from [8], where the b quark mass was found by matching the experimental value for the spin-averaged kinetic mass of the Υ and the η_b to lattice data. For the calculation with an NRQCD spectator bottom quark, we use sets 1 to 5 in Table I.

Bare heavy quark masses am_h used for the heavy-HISQ method are shown in Table IV. The selection of heavy quark masses follows [11]. As well as sets 3 and 5, the heavy-HISQ calculation makes use of a lattice finer than the five sets featuring in the calculation with an NRQCD spectator, set 6 in Table I. This is motivated by the necessity to avoid large discretisation effects that grow with (am_h) (as $(am_h)^4$ at tree-level) whilst gathering data at large masses that will reliably inform the limit $m_h \rightarrow m_b$.

B. Correlators

1. NRQCD spectator case

For the case of an NRQCD spectator quark, random wall source [26] HISQ propagators with the mass of the charm quark are calculated and combined with random wall source NRQCD b propagators to generate B_c^+ 2-point correlator data. 2-point correlators for $B_{s(d)}^0$ are generated similarly. The strategy of combining NRQCD random wall propagators and HISQ random wall propagators to yield 2-point correlators was first developed in [27]. NRQCD propagators are generated by solving an initial value problem. This is computationally very fast compared to calculating rows of the inverse of the quark matrix.

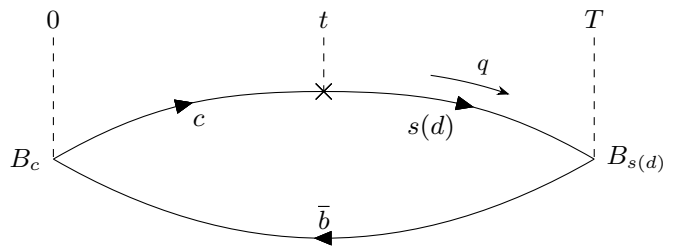


FIG. 1: 3-point correlator $C_{3\text{pt}}(t, T)$. The flavour-changing operator insertion is denoted by a cross at timeslice t and the total time length of the 3-point correlator is T . The random wall source for the b and s/d propagators is at the timeslice of the $B_{s(d)}$ interpolator.

The 3-point correlator needed here is represented diagrammatically in Fig. 1. A HISQ charm quark propagator is generated by using the random wall bottom quark propagator as a sequential source. Following Appendix B in [28], and excluding a spacetime-dependent sign, the sequential source is given by the spin-trace

$$\text{Tr}_{\text{spin}} \left\{ \Gamma \Omega^\dagger(\mathbf{x}, 0) S_b^{\text{RW}}(\mathbf{x}, 0) \right\}, \quad (5)$$

where Γ is the gamma matrix structure at the operator insertion, S_b^{RW} is the random wall NRQCD propagator, and

$$\Omega(x) := \prod_{\mu=1}^4 (\gamma_\mu)^{\frac{x_\mu}{a}} \quad (6)$$

is the space-spin matrix which transforms the naive quark field to diagonalise the HISQ action in spin-space.

2. HISQ spectator case

The case of a HISQ spectator quark proceeds similarly with the only difference being the use of a HISQ propagator instead of an NRQCD propagator for the bottom quark. Again, the charm propagator uses the spectator bottom quark propagator as a sequential source. Multiple masses are used for the spectator quark, each requiring a different charm propagator for the 3-point correlator. Fig. 2 shows the heavy-charm pseudoscalar meson masses that arise from calculations with the am_h values in Table IV. On set 6, the finest lattice considered, we reach a value for M_{H_c} that is 80% of the physical B_c mass.

The same strange and light random wall HISQ propagators on sets 3 and 5 are used in both the NRQCD and the heavy-HISQ calculations, thus the data on these lattices in the two approaches will be correlated. However, the effect of these correlations is small in the physical-continuum limit since the heavy-HISQ data on sets 3 and 5 are the furthest away from the physical b quark mass point, and hence these correlations are safely ignored.

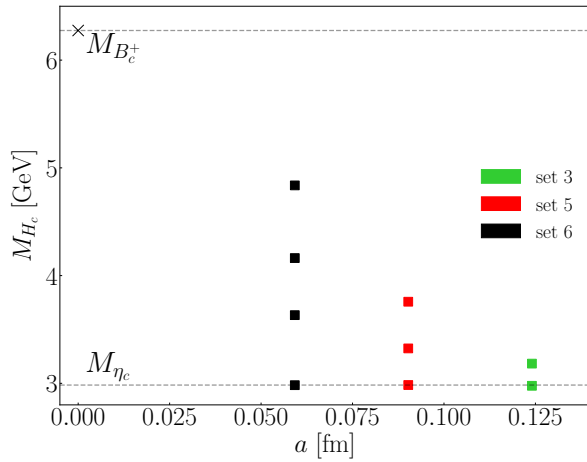


FIG. 2: The mass M_{H_c} of the heavy-charm meson is plotted against lattice spacing for each of the values of am_h used in the heavy-HISQ calculation. Obtained from fitting the correlators as described in Eq. (7), M_{H_c} is a proxy for the bare lattice heavy quark mass am_h . The continuum-physical point is denoted by a cross at $a = 0$ [fm] and $M_{H_c} = M_{B_c^+}$. Note that the y -axis scale begins near M_{η_c} .

3. Fitting the correlators

The correlator fits minimise an augmented χ^2 function as described in [29–31]. The functional forms for the 2-point and 3-point correlators

$$\begin{aligned}
C_{2\text{pt}}^{B_{s(d)}}(t) &= \sum_i a[i]^2 e^{-E_a[i]t} - \sum_i a_o[i]^2 (-1)^t e^{-E_{a_o}[i]t} \\
C_{2\text{pt}}^{B_c}(t) &= \sum_j b[j]^2 e^{-E_b[j]t} - \sum_j b_o[j]^2 (-1)^t e^{-E_{b_o}[j]t} \\
C_{3\text{pt}}(t, T) &= \sum_{i,j} a[i] e^{-E_a[i]t} V_{nn}[i, j] b[j] e^{-E_b[j](T-t)} \\
&\quad - \sum_{i,j} (-1)^{T-t} a[i] e^{-E_a[i]t} V_{no}[i, j] b_o[j] e^{-E_{b_o}[j](T-t)} \\
&\quad - \sum_{i,j} (-1)^t a_o[i] e^{-E_{a_o}[i]t} V_{on}[i, j] b[j] e^{-E_b[j](T-t)} \\
&\quad + \sum_{i,j} (-1)^T a_o[i] e^{-E_{a_o}[i]t} V_{oo}[i, j] b_o[j] e^{-E_{b_o}[j](T-t)}
\end{aligned} \tag{7}$$

follow from their spectral decomposition and include oscillatory contributions from the staggered quark time-doubler. The matrix elements are related to the fit parameters $V_{nn}[i, j]$ through

$$V_{nn}[0, 0] = \frac{\langle B_{s(d)} | J | B_c \rangle}{\sqrt{2E_{B_{s(d)}}} 2E_{B_c}}, \tag{8}$$

where J is the relevant operator that facilitates the $c \rightarrow s(d)$ flavour transition. The pseudoscalar mesons of interest are the lowest lying states consistent with their

quark content, so we are only concerned with the matrix elements for $i = j = 0$ since we restrict $E[k] \leq E[k+1]$ by using log-normal prior distributions for the energy differences. The presence of $i, j > 0$ terms are necessary to give a good fit and to allow for the full systematic uncertainty from the presence of excited states to be included in the extracted $V_{nn}[0, 0]$. On each set, the 2-point and 3-point correlator data for both $c \rightarrow s$ and $c \rightarrow d$ at all momenta are fit simultaneously to account for all possible correlations. The matrix elements and energies are extracted and form factor values determined, along with the correlations between results at different momenta.

C. Extracting The Form Factors

The Partially Conserved Vector Current (PCVC) Ward identity allows for a fully non-perturbative renormalisation of the lattice vector current. Since the same HISQ action is used for the c and $s(d)$ quarks that couple to the W^+ in both the NRQCD and heavy-HISQ approaches, we have the PCVC identity

$$\partial_\mu V_{\text{cons}}^\mu = (m_c - m_{s(d)})S, \tag{9}$$

relating the conserved (point-split) $c \rightarrow s(d)$ lattice vector current and the local lattice scalar density S . We choose a local lattice operator V_{loc}^μ , thus Eq. (9) must be adjusted by a single renormalisation factor Z_V associated with that operator, giving

$$q_\mu \langle B_{s(d)} | V_{\text{loc}}^\mu | B_c \rangle Z_V = (m_c - m_{s(d)}) \langle B_{s(d)} | S | B_c \rangle. \tag{10}$$

Since Z_V is q^2 independent, in principle Z_V need only be found at zero-recoil where q^μ has only a temporal component [3]. This avoids the need to calculate 3-point correlators associated with the spatial components of the vector current matrix element that appear in Eq. (10) for $\mathbf{q} \neq \mathbf{0}$. However, in practice, it is preferable to determine f_+ near zero-recoil through the spatial components of the vector current matrix element, albeit with the additional cost in computing 3-point correlators with the corresponding insertion.

As in [2], we combine Eqs. (3) and (10) to give a determination

$$f_0(q^2) = \langle B_{s(d)} | S | B_c \rangle \frac{m_c - m_{s(d)}}{M_{B_c}^2 - M_{B_{s(d)}}^2} \tag{11}$$

of f_0 solely in terms of the scalar density matrix element. We use Eq. (11) and calculation of the vector current matrix element to determine f_+ and f_0 for the full q^2 range following [3, 32]. Thus, we will calculate matrix elements of both the local scalar density $J = S$ and the local vector current $J = V$.

Once f_0 is determined, f_+ is obtained using Eq. (3) for $\mu = 0$ to yield

$$f_+(q^2) = \frac{Z_V \mathcal{V}^0 - q^0 f_0(q^2) \frac{M_{B_c^+}^2 - M_{B_s^0}^2}{q^2}}{p_2^0 + p_1^0 - q^0 \frac{M_{B_c^+}^2 - M_{B_s^0}^2}{q^2}}, \tag{12}$$

where \mathcal{V}^μ is the vector current matrix element, except at zero-recoil where the denominator vanishes and f_+ cannot be extracted. We find that using Eq. (12) near zero-recoil is problematic since both the numerator and denominator grow from 0 as q^2 is decreased from the maximum value at zero-recoil. For the case where the spectator is an NRQCD b quark, we instead use Eq. (3) with $\mu = i \neq 0$

$$f_+(q^2) = \frac{-\frac{Z_V \mathcal{V}^i}{q^i} + f_0(q^2) \frac{M_{B_c^+}^2 - M_{B_s^0}^2}{q^2}}{1 + \frac{M_{B_c^+}^2 - M_{B_s^0}^2}{q^2}}. \quad (13)$$

This method gives much smaller errors near to zero-recoil. Although mathematically equivalent to Eq. (12), extracting f_+ through Eq. (13) does not suffer an inflation of error near zero-recoil since both the numerator and denominator are non-zero for all physical q^2 . However, since \mathcal{V}^i appears explicitly in Eq. (13), 3-point correlators with an insertion of V^i need to be calculated. For the case of the spectator NRQCD b quark, the use of Eq. (13) is straightforward except that it requires inversions of the charm quark propagator from a different sequential source (see Eq. (5)) to allow for insertion of the current $V^i = \gamma^i \otimes \gamma^i$ in the mixed NRQCD-HISQ 3-point function. Collecting \mathcal{V}^i at non-zero 3-momentum transfer in the NRQCD calculation will also test for any q^2 dependence of Z_V that would appear as a discretisation effect.

Using Eqs. (11) and (12) or (13), form factor data at a variety of lattice spacings, light quark masses and momenta are obtained from the energies and matrix elements.

1. NRQCD spectator case

For the case of an NRQCD spectator quark, the form factor extraction is complicated by the energy offset as a consequence of the subtraction of the b quark rest mass inherent in the NRQCD formalism. Whilst physical energy differences are preserved with NRQCD quarks, energy sums are not. Consequently, Particle Data Group (PDG) [33] values are used where necessary. For example, we take

$$M_{B_c}^2 - M_{B_s(d)}^2 = \left(E_{B_c}^{\text{sim}}(|a\mathbf{q}|=0) - E_{B_s(d)}^{\text{sim}}(|a\mathbf{q}|=0) \right) \times \left(M_{B_c}^{\text{PDG}} + M_{B_s(d)}^{\text{PDG}} \right) \quad (14)$$

when extracting the form factors. We use interpolating operators $\bar{c}\gamma_5 b$ and $\bar{s}\gamma_5 b$ ($\bar{d}\gamma_5 b$) for $J^P = 0^-$ pseudoscalars B_c^+ and $B_{s(d)}^0$ respectively.

2. HISQ spectator case

For the case of a HISQ spectator quark, we work only with local scalar and vector currents. Expressed in the spin-taste basis, we use $\gamma_5 \otimes \gamma_5$ for the $H_{s(d)}$ interpolating operator and two different operators, $\gamma_5 \otimes \gamma_5$ and $\gamma_5 \gamma_0 \otimes \gamma_5 \gamma_0$, for the H_c interpolator. The first of these, $\gamma_5 \otimes \gamma_5$, makes a tasteless 3-point correlation function when the scalar density operator $\mathbb{1} \otimes \mathbb{1}$ is used. The second, $\gamma_5 \gamma_0 \otimes \gamma_5 \gamma_0$, allows for a tasteless 3-point correlation function when we use the local temporal vector current operator $\gamma_0 \otimes \gamma_0$ [3]. This requires the calculation of two H_c 2-point functions with the two different choices of operator at both the source and the sink. The difference in masses between these two different tastes of H_c meson is tiny and, although consistently taken care of, it has no impact on the calculation.

III. RESULTS

A. Correlators

Figs. 3 and 4 provide samples of the correlator data from the NRQCD and heavy-HISQ calculations respectively. The quantity plotted is the effective simulation energies, which we define by the two-step log-ratio

$$aE_{\text{sim,eff}} = \frac{1}{2} \log \left(\frac{C(t)}{C(t+2)} \right). \quad (15)$$

This ratio is preferable to an effective energy defined using $C(t)/C(t+1)$ since the ratio in Eq. (15) better suppresses the oscillatory contributions in Eq. (7). Error bars are present in the figure but mostly too small to observe. We exclude t_{min}/a data points from the beginning and end points of the correlators in our fits to reduce the contributions from excited states.

For each of the cases of an NRQCD and HISQ spectator quark, we fit all of the correlator data to Eq. (7) on each set simultaneously to obtain the correlations between the fitted parameters. Consequently, the correlator fits involve a large covariance matrix. Without extremely large statistical samples of results small eigenvalues of the covariance matrix are underestimated [34, 35] and this causes problems when carrying out the inversion to find χ^2 . We overcome this by using an SVD (singular-value decomposition) cut; any eigenvalue of the covariance matrix smaller than some proportion c of the biggest eigenvalue λ_{max} is replaced by $c\lambda_{\text{max}}$. By carrying out this procedure, the covariance matrix becomes less singular. These eigenvalue replacements will only inflate our final errors, hence this strategy is conservative. The SVD cut reduces the $\chi^2/\text{d.o.f.}$ reported by the fit because it lowers the contribution to χ^2 of the modes with eigenvalues below the SVD cut. In order to check the suitability of the SVD cut, we must test the goodness-of-fit from a fit where noise (SVD-noise) is added to the data to restate the size of fluctuations expected from the modes

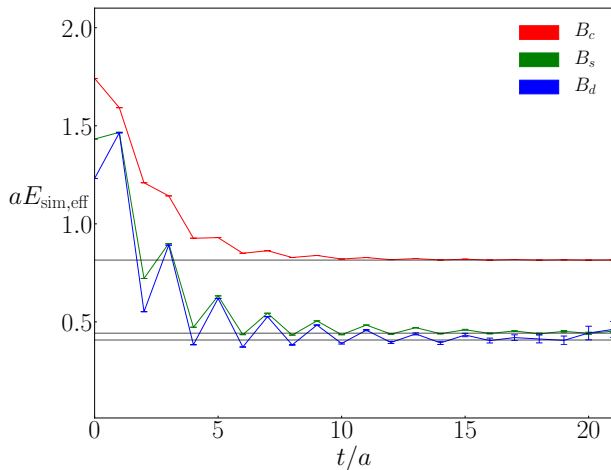


FIG. 3: Effective simulation energies (Eq. (15)) of 2-point correlators with an NRQCD spectator quark on set 5 for $|\alpha\mathbf{q}| = 0.427$. The black lines with grey error bands show the energies extracted from fitting the correlators in the simultaneous fit of fine lattice data with all the 3-point correlators and all the momenta. The B meson energies shown here are offset from their physical values as a consequence of applying the NRQCD formalism to the constituent b quark.

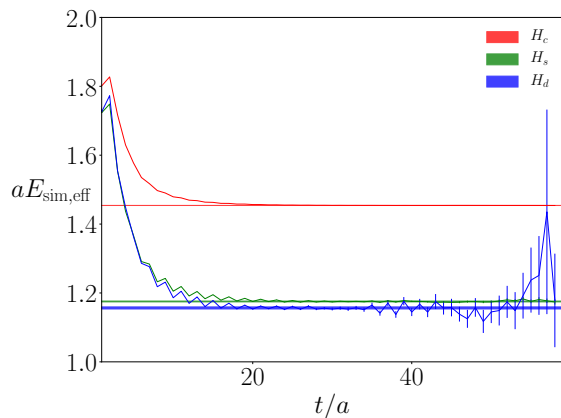


FIG. 4: Effective simulation energies (Eq. (15)) of 2-point correlators with a HISQ spectator quark on set 6 at zero twist with $am_h = 0.8$. The horizontal bands show the energies extracted from the full simultaneous correlator fit.

below SVD cut, as described in Appendix D of [35]. The $\chi_{\text{SVD-noise}}^2/\text{d.o.f.}$ is used to check the goodness of fit for both cases of spectator quark.

Many fits were carried out with different SVD cuts, number of exponentials N , and positions $t_{\text{min}}^{2\text{pt}}$ and $t_{\text{min}}^{3\text{pt}}$ of the first timeslice where the correlators are fit. We selected the fit of the correlators on each lattice for form factor extraction based on the $\chi^2/\text{d.o.f.}$ and Q -value.

The parameters used in the fits of correlators with an NRQCD spectator quark are presented in Table V. The parameters given in bold are those used for our final fits.

TABLE V: Input parameters (see text for definition) to the correlator fits for the calculation with NRQCD spectator quarks together with fits including variations of t_{min}/a , N and SVD cut. Bold entries indicate those fits used to obtain our final result. Other values are used in tests of the stability of our form factor fits to be discussed in Sec. IV B.

set	SVD cut	$t_{\text{min}}^{2\text{pt}}/a$	$t_{\text{min}}^{3\text{pt}}/a$	N	$\chi_{\text{SVD-noise}}^2/\text{dof}$
1	0.1	2	2	6	1.00
	0.1	2	3	4	1.00
2	0.075	6	2	6	1.00
	0.075	6	2	5	1.10
3	0.1	6	3	6	1.00
	0.075	4	2	6	1.00
4	0.025	4	3	6	1.00
	0.075	4	2	6	0.95
5	0.05	6	2	6	1.00
	0.3	4	3	6	1.00

TABLE VI: Input parameters (see text for definition) to the heavy-HISQ correlator fits together with fits including variations of t_{min}/a , N and SVD cut. Bold entries indicate those fits used to obtain our final result. Other values will be used in tests of the stability of our form factor fits in Sec. IV C.

set	SVD cut	$t_{\text{min}}^{2\text{pt}}/a$	$t_{\text{min}}^{3\text{pt}}/a$	N	$\chi_{\text{SVD-noise}}^2/\text{dof}$
3	0.025	6	2	4	0.94
	0.025	6	2	3	1.05
	0.075	6	2	4	0.90
5	0.025	4	2	4	0.95
	0.025	4	2	3	0.94
	0.075	4	2	4	0.96
6	0.025	6	3	4	0.95
	0.025	4	2	3	0.99
	0.05	6	3	4	0.95

Other values are used in tests of the stability of our form factor fits to be discussed in Sec. IV B.

We fit the heavy-HISQ correlator data to Eq. (7) on each set simultaneously, including correlations between data with different values of twist, heavy quark mass, and $H_{s/d}$ final state. Values for t_{min}/a , the chosen SVD cut, the number of exponentials used in Eq. (7) and the resultant value of χ^2/dof including SVD noise are given in Table VI. We also include in Table VI fits using variations of these parameters. Form factor fit coefficients obtained using combinations of these variations are shown in Figs. 19 and 20 in Sec. IV C and demonstrate that our results are insensitive to such choices.

B. Vector Current Renormalisation Z_V

In this section we give our results for the renormalisation factor Z_V for the vector current (Eq. (10)) and test

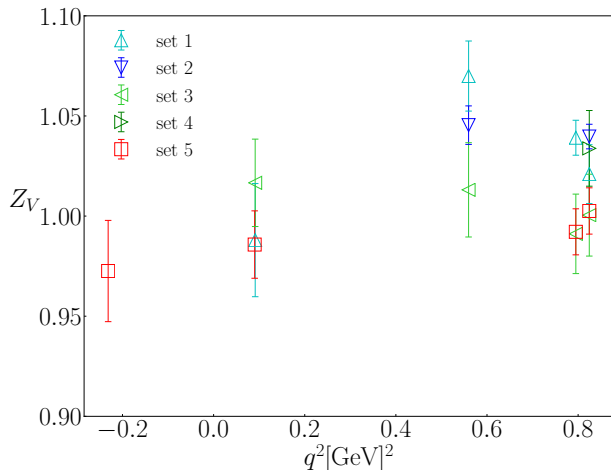


FIG. 5: Z_V for the $c \rightarrow s$ vector current evaluated at different q^2 from the calculation with an NRQCD spectator quark using Eq. (10).

TABLE VII: Z_V obtained at zero-recoil using an NRQCD spectator b quark .

set	$c \rightarrow s$	$c \rightarrow d$
1	1.021(15)	1.041(18)
2	1.0397(61)	1.021(17)
3	1.000(20)	1.004(22)
4	1.034(19)	0.983(20)
5	1.003(12)	0.958(20)

for dependence of Z_V on q^2 (for the case of an NRQCD spectator) and on the spectator quark mass (for the case of a HISQ spectator).

The vector current renormalisation factor Z_V computed at different momentum transfer with NRQCD b quarks shows no significant dependence on q^2 on each set, demonstrated by Fig. 5. Mild lattice spacing dependence is observed, however. For each momenta, we use the Z_V found at the corresponding q^2 from Eq. (10).

The Z_V factor in Eq. (10) is associated only with the local vector current operator and should be independent of the spectator quark. Z_V values obtained in the different calculations are tabulated in Tables VII, VIII and IX. Good agreement is seen on set 5 at zero-recoil between the results with NRQCD and heavy-HISQ spectator quarks. Dependence on the mass of the spectator

TABLE VIII: Z_V for $c \rightarrow s$ obtained at zero-recoil using a HISQ spectator quark with different values of the heavy quark mass m_h .

set/ am_h	0.274	0.450	0.6	0.663	0.8
3	-	-	-	1.026(32)	1.029(36)
5	-	1.006(17)	1.003(19)	-	1.000(20)
6	0.997(14)	0.994(17)	0.995(19)	-	0.995(22)

TABLE IX: Z_V for $c \rightarrow d$ obtained at zero-recoil using a HISQ spectator quark with different values of the heavy quark mass m_h .

set/ am_h	0.274	0.450	0.6	0.663	0.8
3	-	-	-	1.016(47)	1.019(50)
5	-	1.009(23)	1.004(25)	-	1.000(27)
6	0.996(22)	0.993(25)	0.994(28)	-	0.995(32)

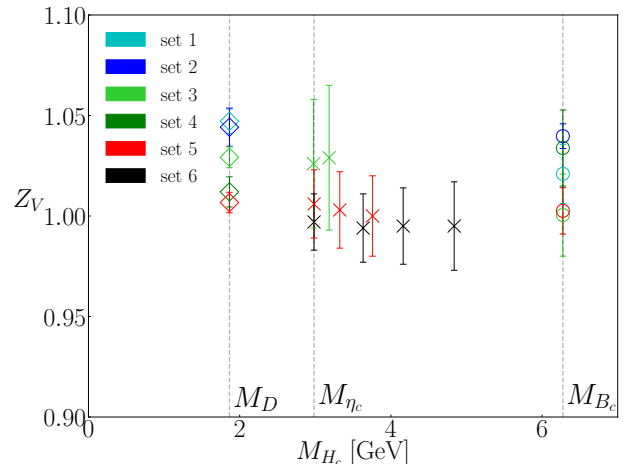


FIG. 6: Z_V of the $c \rightarrow s$ vector current from both the NRQCD and heavy-HISQ calculations are plotted alongside values from $D \rightarrow K$ [3]. The NRQCD data is marked with circles, the heavy-HISQ data is marked with crosses, and finally the $D \rightarrow K$ values are given by diamonds. As expected, no significant dependence on the spectator mass is observed.

quark is displayed in Fig. 6. The plot includes values from the analogous calculation for the $D \rightarrow K$ case [3]. For $D \rightarrow K$, a charm quark decays into a strange quark, as in $B_c \rightarrow B_s$, but here the spectator quark is a light quark, much less massive than the heavy spectator quark in $B_c \rightarrow B_s$. The Z_V from $B_c \rightarrow B_s$ and $D \rightarrow K$ in Fig. 6 are nevertheless in good agreement, demonstrating negligible dependence on the mass of the quark spectating the $c \rightarrow s$ transition.

It is also of interest to compare vector current renormalisation factors for different masses of quark featuring in the current. For example, [36] calculates the local $\bar{s}\gamma_\mu s$ vector current renormalisation factor from an $\eta_s \rightarrow \eta_s$ 3-point correlation function at $q^2 = 0$ on the 2+1+1 MILC ensembles. This gave very precise values and it was possible to fit Z_V to a perturbative expansion in α_s (including the known first-order term) along with discretisation effects. The fit is plotted in Fig. 7 alongside Z_V for $c \rightarrow s$ values determined in this study. This plot shows differing behaviour as a function of a^2 . The value for Z_V , determined non-perturbatively, is a combination of the underlying perturbative series in α_s evaluated at a scale related to the lattice spacing and discretisation effects that depend on how it was determined. Since the underlying perturbative series is common to differ-

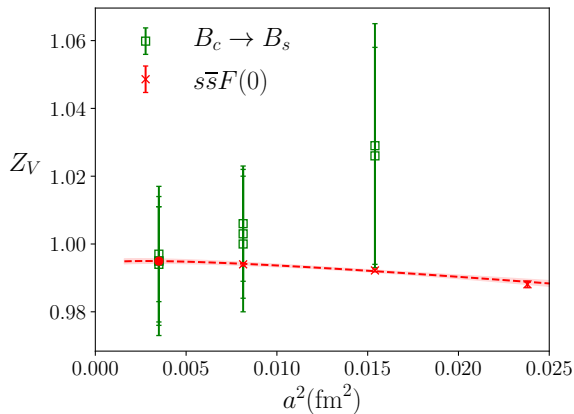


FIG. 7: Z_V from the $s \rightarrow s$ vector current from [36] (red crosses) and the $c \rightarrow s$ vector current from the heavy-HISQ calculation given here (green squares). The curve is the fitted perturbative expansion, including discretisation effects, detailed in [36]. The red circle is an extrapolated value at the lattice spacing associated with the superfine lattice.

ent determinations, comparison will reveal the differing discretisation effects. Fig. 7 shows this in the comparison of our Z_V values for the local $\bar{s}\gamma_\mu c$ current with those determined for the local $\bar{s}\gamma_\mu s$ current. In the limit of vanishing lattice spacing, where discretisation effects vanish, the renormalisation factors are in agreement.

One might worry that the large errors appearing in Fig. 7 for the $\bar{s}\gamma_\mu c$ renormalisation factors determined here would carry forward into our determination of the form factor f_+ . However, the vector current matrix element at zero recoil, which contributes the dominant error in Z_V , is highly correlated with the vector matrix elements at non-zero recoil. These correlations cancel in the ratio $\mathcal{V}^0/\mathcal{V}^0(q_{\text{max}}^2)$ appearing when using Eq. (10) to construct the renormalised current $Z_V\mathcal{V}^0$ appearing in Eqs. (12) and (13). Hence, the uncertainty in the renormalisation factor is not a large contribution to our final uncertainty in the form factors.

C. Form Factors

Fig. 8 provides an example of the extracted values for the form factor f_+ , comparing results from the NRQCD and heavy-HISQ spectator calculations. The lines on the figures connect data on the same set at a given am_h value and are present as a guide only. The spread of the heavy-HISQ data for different heavy quark masses is small, and the NRQCD and heavy-HISQ results are in good agreement on the fine lattice. Discretisation effects are more noticeable for the case of an NRQCD spectator quark, especially on the coarsest lattices, sets 1 and 2. We believe that they result from the B_c meson in the calculation since the effects are comparable to those seen in the B_c meson decay constant study with NRQCD b quarks

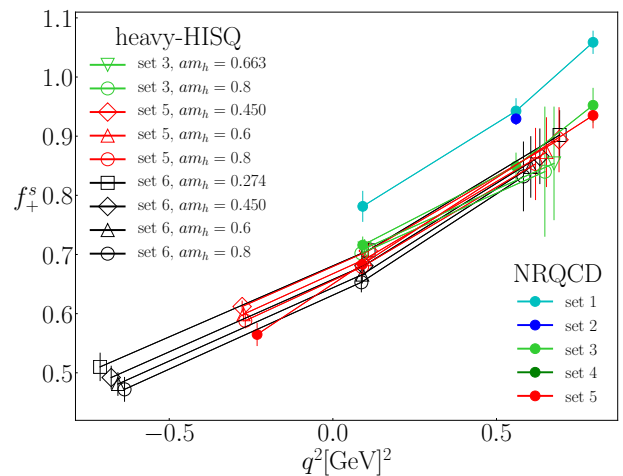


FIG. 8: f_+ form factor data for $B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell$ from both the NRQCD and heavy-HISQ approaches. The NRQCD form factor data is given by filled circles; the heavy-HISQ data, by open circles. Data points on a given set and for a given heavy quark mass are joined by lines to guide eye.

in [37]. Data points outside the physical region of momentum transfer are unphysical but nevertheless aid the fit.

IV. DISCUSSION

A. z Expansion

The four form factors, f_0 and f_+ for each of the $B_c \rightarrow B_s$ and $B_c \rightarrow B_d$ processes, at all momenta on all the lattices, are fit simultaneously to a functional form which allows for dependence on the lattice spacing a and bare quark masses. The fit is carried out using the *lsqfit* package [38] that implements a least-squares fitting procedure. As is now standard, we map the semileptonic region $0 < q^2 < (M_{B_c} - M_{B_{s(d)}})^2$ to a region on the real axis within the unit circle through

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (16)$$

so that the form factors can be approximated by a truncated power series in z . Here we choose the parameter t_0 to be 0 so that the points $q^2 = 0$ and $z = 0$ coincide. The parameter t_+ is in principle the threshold for production of mesons, the lightest being $D + K$, from the $c\bar{s}$ current in the t -channel. It is convenient here, however, to work with $t_+ = (M_{B_c} + M_{B_{s(d)}})^2$, but this gives a very small range for z because then $t_+ \gg t_-$. To correct for this we rescale z .

The rescaling factor that we use is $|z(M_p^2)|^{-1}$, where M_p is the mass of the nearest $c\bar{s}$ or $c\bar{d}$ meson pole (we use the same mass for both vector and scalar form factors for convenience). For $B_c \rightarrow B_s$ we take M_p as the mass of

the vector meson D_s^* and for $B_c \rightarrow B_d$, the mass of D^{*0} . Thus, we define

$$z_p(q^2) = \frac{z(q^2)}{|z(M_p^2)|}. \quad (17)$$

z_p then has a range more commensurate to that for the corresponding D decay and the polynomial coefficients in z_p are $\mathcal{O}(1)$. Coefficients of the conventional expansion in terms of z can easily be obtained from the expansion in z_p . Using z_p also avoids introducing large heavy mass dependence through the z transform in the heavy-HISQ case, which otherwise would require large $\Lambda_{\text{QCD}}/M_{H_c}$ coefficients in the heavy-HISQ fit. Note that in the case of the heavy-HISQ spectator, the B -meson masses above in t_+ are replaced by the appropriate heavy meson masses at each value of am_h (see Sec. IV C).

B. NRQCD Form Factor Fits

The form factor results from the calculation with NRQCD spectator quark are fit to

$$f(q^2) = P(q^2) \sum_{n=0}^N b^{(n)} z_p^n. \quad (18)$$

Here, the dominant pole structure is represented by a factor $P(q^2)$ given by $(1 - q^2/M_{\text{res}}^2)^{-1}$ with M_{res} the mass of the relevant $c\bar{s}$ or $c\bar{d}$ meson (the vector meson for f_+ and the scalar for f_0). We take the values of M_{res} from current experiment [33]: $M_{D_s^*} = 2.112$ GeV, $M_{D_{s^*}^0} = 2.317$ GeV, $M_{D^*} = 2.01027$ GeV, and $M_{D_s^*} = 2.300$ GeV. We do not include uncertainties in these values since $P(q^2)$ is a purely fixed factor designed to remove much of the q^2 -dependence from the form factors. For our lattice results uncertainties enter $P(q^2)$ from the uncertainty in our determination of q^2 in physical units, including that from the determination of the lattice spacing.

$P(q^2)$ multiplies a polynomial in z_p , and the polynomial coefficients are

$$\begin{aligned} b^{(n)} = & A^{(n)} \left\{ 1 + B^{(n)} (am_c/\pi)^2 + C^{(n)} (am_c/\pi)^4 \right. \\ & + \kappa_1^{(n)} \frac{\delta m_l^{\text{sea}}}{10m_s^{\text{tuned}}} + \kappa_2^{(n)} \frac{\delta m_s^{\text{sea}}}{10m_s^{\text{tuned}}} + \kappa_3^{(n)} \frac{\delta m_c^{\text{sea}}}{m_c^{\text{tuned}}} \\ & \left. + \kappa_4^{(n)} \frac{\delta m_s^{\text{val}}}{10m_s^{\text{tuned}}} + \kappa_5^{(n)} \frac{\delta m_c^{\text{val}}}{m_c^{\text{tuned}}} + \kappa_6^{(n)} \frac{\delta m_b^{\text{val}}}{m_b^{\text{tuned}}} \right\}. \end{aligned} \quad (19)$$

The parameters $\kappa_j^{(n)}$ allow for errors associated with mistunings of both sea and valence quark masses. The term accounting for mistuning of valence strange quarks is included only for the $B_c \rightarrow B_s$ transition. The tuned masses m_s^{tuned} and m_c^{tuned} are the valence quark masses that yield physical η_s and η_c meson masses respectively in the sea of 2+1+1 flavours of sea quark. Values for

m_s^{tuned} and m_c^{tuned} were obtained from [15]. Also, m_l^{tuned} is fixed by multiplying m_s^{tuned} by the physical ratio

$$\frac{m_l}{m_s} = \frac{1}{27.18(10)} \quad (20)$$

obtained from [39]. For the b quark, we take tuned values¹ of the quark mass from Table XII in [8].

For each of the sea and valence quark flavours, δm^{sea} and δm^{val} are given by

$$\begin{aligned} \delta m^{\text{sea}} &= m^{\text{sea}} - m^{\text{tuned}} \\ \delta m^{\text{val}} &= m^{\text{val}} - m^{\text{tuned}}, \end{aligned} \quad (21)$$

giving estimates of the extent that the quark masses deviate from the ideal choices in which appropriate meson masses are exactly reproduced.

For prior values on the parameters in Eq. (19), we use 0(1) for $A^{(n)}$, $B^{(n)}$ and $C^{(n)}$, and 0.0(5) for $\kappa_{(j)}$. The power series in Eq. (18) is truncated to include up to the z_p^3 term. Fits without a pole, i.e. $P(q^2) = 1$, yield no statistically significant discrepancies. This is not surprising since the poles are far away from the physical region of q^2 , and so the pole effect on the form factor can be reasonably absorbed into the polynomial. Finally, the kinematic relation

$$f_0(0) = f_+(0) \quad (22)$$

is imposed on the fit as a constraint (we have tested that removing this constraint makes very little difference to the fit in fact and $f_+(0) - f_0(0)$ is zero to well within 1σ).

Constraints on $b^{(n)}$ from unitarity, as in the BCL [40] and BGL [41] expansions, are unnecessary here since the full range of physical momentum transfer can be reached and so extrapolation in q^2 , which may benefit in accuracy from imposing these constraints, is not required. Hence, more complicated fit forms that impose additional physical constraints are not expected to be appreciably advantageous.

In Figs. 9 and 10, we demonstrate that the form factors in the physical-continuum limit are insensitive to the choice of the parameters in the fits of the correlators. As can be seen in the figures, the coefficients in the fits of the form factors are stable, within their uncertainties, as the correlator fits on different sets are varied.

The fitted form factors from the NRQCD spectator case exhibit errors no greater than 4% across the entire physical range of q^2 when tuned to the physical-continuum limit. Figs. 11, 12, 13 and 14 show the results on all the lattices along with the fitted function for the form factors in the physical-continuum limit.

¹To ensure consistency, we convert values from [8] in lattice units to physical units by using the lattice spacing determined in [8] from the $\Upsilon(2S - 1S)$ splitting.

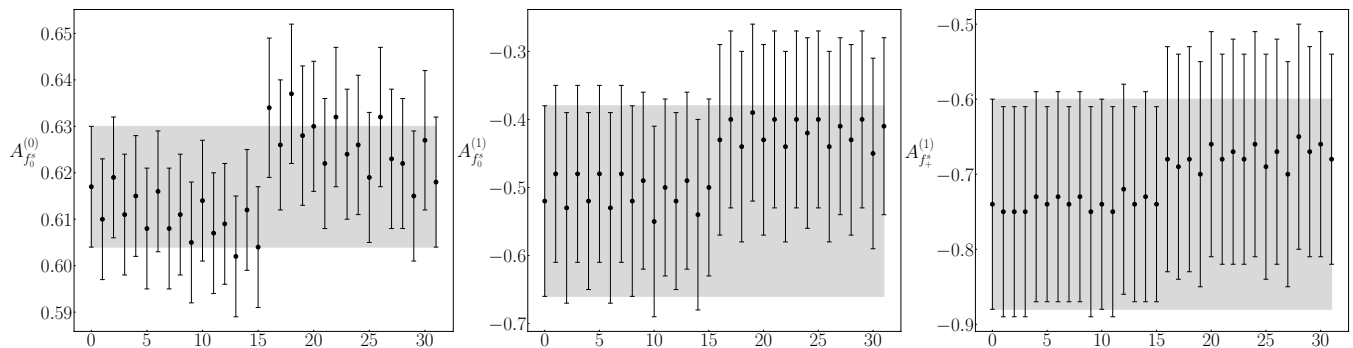


FIG. 9: z_p expansion coefficients, for the calculation with an NRQCD spectator quark, computed using the variations of correlator fit parameters listed in Table V for the $B_c \rightarrow B_s$ form factors. The integer x coordinate of each result is given by $n_{\text{var}}^{(1)} + 2n_{\text{var}}^{(2)} + 4n_{\text{var}}^{(3)} + 8n_{\text{var}}^{(4)} + 16n_{\text{var}}^{(5)}$ where $n_{\text{var}}^{(i)} = 0, 1$ corresponding to the first and second fits respectively listed in Table V of set i .

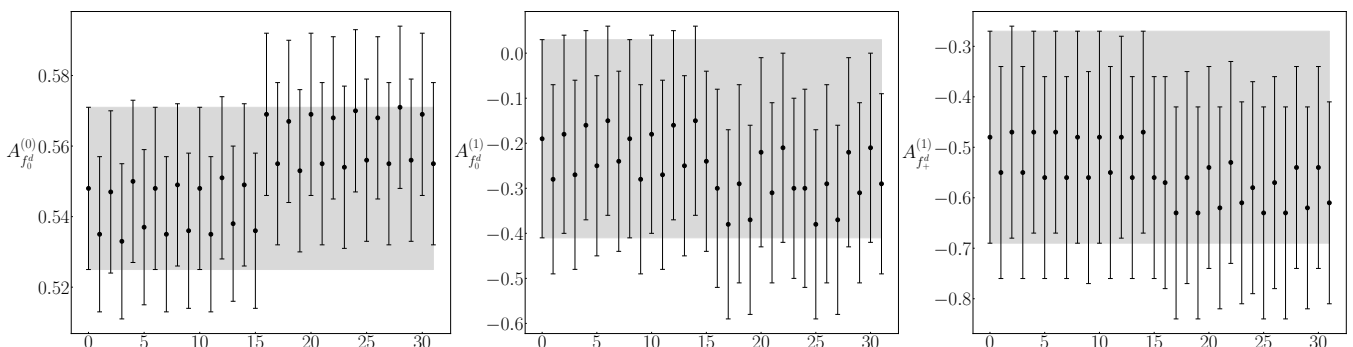


FIG. 10: z_p expansion coefficients, for the calculation with an NRQCD spectator quark, computed using the variations of correlator fit parameters listed in Table V for the $B_c \rightarrow B_d$ form factors. The x coordinate is the same as that in Fig. 9.

TABLE X: A selection of f_0 and f_+ fit parameters from our fit to Eq. (19) with an NRQCD- b quark, demonstrating the leading order momentum and lattice spacing dependence. Note that the discretisation effects in the f_0 and f_+ fits are allowed to vary independently of each other with separate $B^{(0)}$ parameters. In practice, as the Table shows, the fit returns very similar values.

	f_0^s	f_0^d	f_+^s	f_+^d
$A^{(0)}$	0.617(13)	0.548(23)	0.617(13)	0.548(23)
$A^{(1)}$	-0.52(14)	-0.19(22)	-0.74(14)	-0.48(21)
$A^{(2)}$	-0.63(63)	0.05(74)	-0.29(72)	0.12(77)
$B^{(0)}$	1.44(38)	1.45(46)	1.44(38)	1.45(46)

The z_p^0 and z_p^1 behaviour of the form factors is well resolved by our fit to Eq. (19), as well as the $(am_c/\pi)^2 z_p^0$ discretisation effect. Table X summarises the corresponding parameters from the fit. After fitting, other parameters show errors comparable to the width of their prior and are consistent with 0. In particular, quark mass mistuning coefficients simply return their prior value.

C. Heavy-HISQ Form Factor Fits

We take a similar approach to fitting the form factor results for the case of a heavy-HISQ spectator. Now we have results at multiple heavy-quark masses and the conversion from q^2 to z -space (Eq. (16)) uses the values of M_{H_c} and M_{H_s} or M_{H_d} , as appropriate, from our calculation. We then rescale z at each m_h as described in Sec. IV A (Eq. (17)). This rescaling gives a similar z -range for each m_h and avoids introducing spurious dependence on m_h that comes simply from the z -transform.

The heavy-HISQ results are then fit to a form that is a product of $P(q^2)$ and a polynomial in z_p as for the NRQCD case. We now require a fit form for the polynomial coefficients that accounts for $(am_h)^{2n}$ discretisation effects as well as physical dependence on m_h , however. Motivated by HQET we express this physical heavy mass dependence as a power series in $\Lambda_{\text{QCD}}/M_{H_c}$. The form

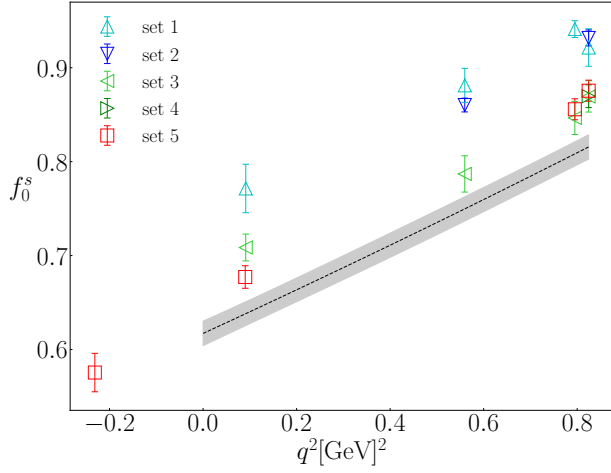


FIG. 11: Lattice results and fitted f_0 form factor data for $B_c^+ \rightarrow B_s^0 \bar{l} \nu_l$ with an NRQCD b quark. The grey band shows the fitted form factor tuned to the limit of vanishing lattice spacing and physical quark masses.

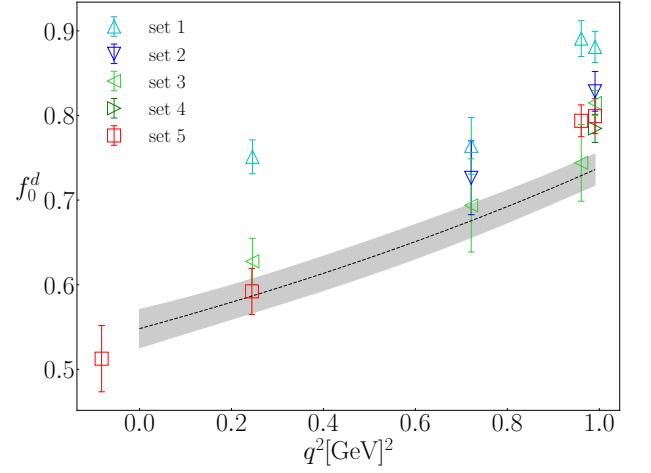


FIG. 13: Lattice results and fitted f_0 form factor data for $B_c^+ \rightarrow B^0 \bar{l} \nu_l$ with an NRQCD b quark. The grey band shows the fitted form factor tuned to the limit of vanishing lattice spacing and physical quark masses.

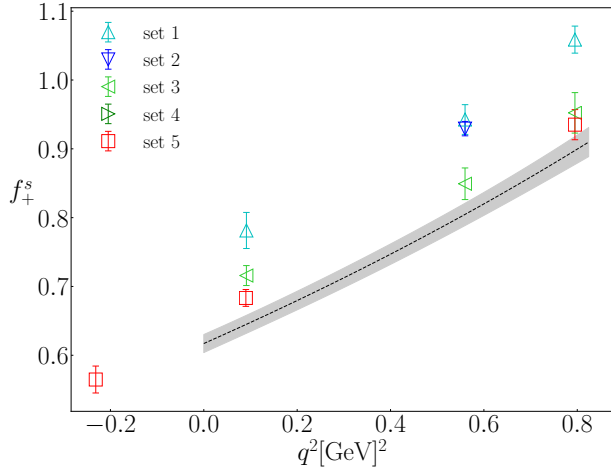


FIG. 12: Lattice results and fitted f_+ form factor data for $B_c^+ \rightarrow B_s^0 \bar{l} \nu_l$ with an NRQCD b quark. The grey band shows the fitted form factor tuned to the limit of vanishing lattice spacing and physical quark masses.

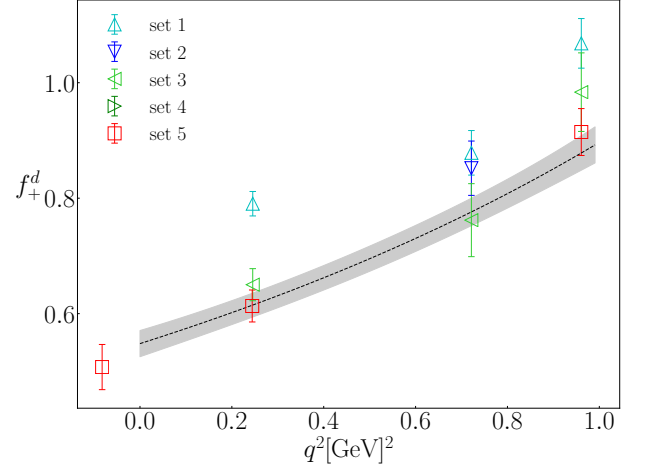


FIG. 14: Fitted f_+ form factor data for $B_c^+ \rightarrow B^0 \bar{l} \nu_l$ with an NRQCD b quark. The grey band shows the fitted form factor tuned to the limit of vanishing lattice spacing and physical quark masses.

factor data from the heavy-HISQ approach is fit to

$$f(q^2) = P(q^2) \sum_{n,i,j,k=0}^3 A_{ijk}^{(n)} z_p^n \times \left(\frac{am_c}{\pi}\right)^{2i} \left(\frac{am_h}{\pi}\right)^{2j} \Delta_{H_c}^{(k)} \mathcal{N}_{\text{mis}}^{(n)}, \quad (23)$$

where, for $k = 0$, $\Delta_M^{(k)} = 1$ and, for $k \neq 0$,

$$\Delta_{H_c}^{(k)} = \left(\frac{\Lambda_{\text{QCD}}}{M_{H_c}}\right)^k - \left(\frac{\Lambda_{\text{QCD}}}{M_{B_c}}\right)^k \quad (24)$$

where we take $\Lambda_{\text{QCD}} = 500\text{MeV}$. The mistuning terms

are given by

$$\begin{aligned} \mathcal{N}_{\text{mis}}^{(n)} = & 1 + \frac{\delta m_c^{\text{val}}}{m_c^{\text{tuned}}} a_n + \frac{\delta m_c^{\text{sea}}}{m_c^{\text{tuned}}} b_n \\ & + \frac{\delta m_s^{\text{val}}}{10m_s^{\text{tuned}}} c_n + \frac{\delta m_s^{\text{sea}}}{10m_s^{\text{tuned}}} d_n + \frac{\delta m_l}{10m_s^{\text{tuned}}} e_n, \end{aligned} \quad (25)$$

where we only include the term proportional to δm_s^{val} for the $B_c \rightarrow B_s$ case. $P(q^2)$, δm and the tuned masses have the same definitions as in the NRQCD case (Sec. IV B). In the physical continuum limit, this form collapses to $P(q^2) \sum_n z_p^n A_{000}^{(n)}$. Again we apply the constraint $f_0(0) = f_+(0)$ in the continuum limit (by fixing $A_{000}^{(0)}$ to be the same in the two cases).

Results for the extrapolated form factors are given in

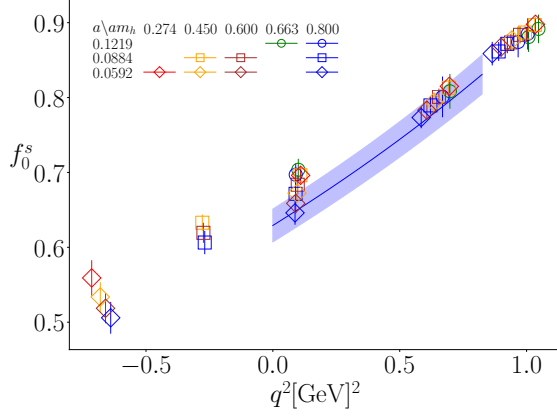


FIG. 15: Heavy-HISQ form factor results for f_0^s together with the fitted curve at the physical point with its error band.

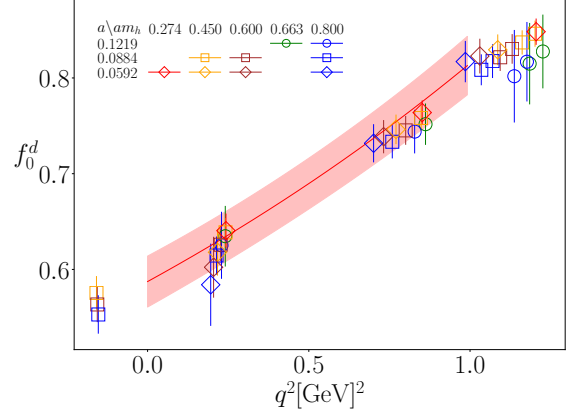


FIG. 17: Heavy-HISQ form factor results for f_0^d together with the fitted curve at the physical point with its error band.

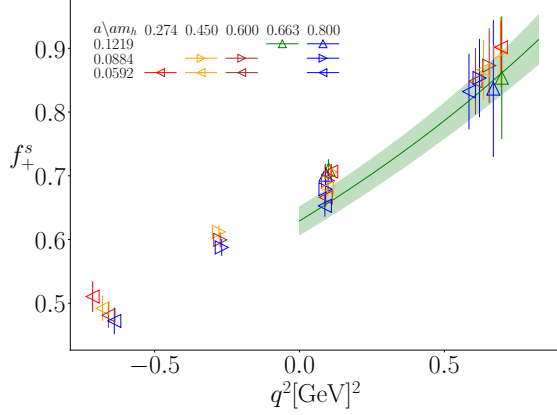


FIG. 16: Heavy-HISQ form factor results for f_+^s together with the fitted curve at the physical point with its error band.

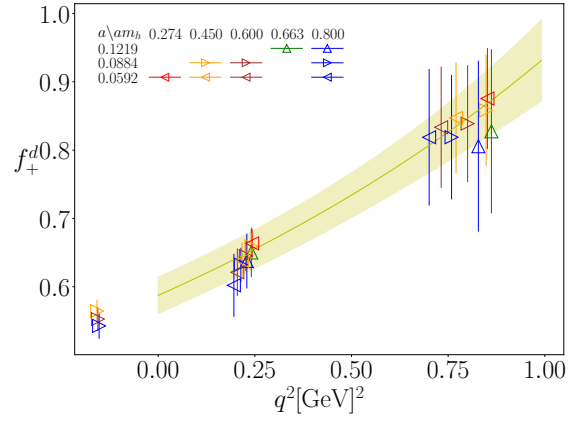


FIG. 18: Heavy-HISQ form factor results for f_+^d together with the fitted curve at the physical point with its error band.

Figs. 15, 16, 17 and 18 together with the corresponding lattice data. For the $B_c \rightarrow B_s$ case a_n and c_n take prior values 0(1) and b_n , d_n and e_n take prior values 0(0.3) to reflect the fact that they enter through loop effects. In the $B_c \rightarrow B_d$ case we take prior values of 0(1) for a_n and e_n and 0(0.3) for b_n and d_n . In both cases we take prior values of 0(1) for A_{ijk}^n except for when $i = 1$ or $j = 1$ where we use a prior values of 0(0.3) to account for the HISQ one loop improvement.

As in the case of an NRQCD spectator quark, we present coefficients of the form factors fits from many different fits of the correlator data. Figs. 19 and 20, show that the coefficients are insensitive to the choice of the parameters in the fits of the correlators.

D. Chained Fit

The form factor functions tuned to the physical-continuum limit from NRQCD and heavy-HISQ are compared in Figs. 21, 22, 23 and 24 in z -space. There is good agreement across the entire physical range of z , with particularly good agreement for the more accurate $B_c \rightarrow B_s$ case.

Whilst the fit forms for the form factors from NRQCD and heavy-HISQ at Eqs. (18) and (23) differ in appearance, they both allow for effects of discretisation and mistuning of the quark masses. In the continuum limit with physical masses, the two forms collapse such that the parameters $A^{(n)}$ from Eq. (19) and $A_{000}^{(n)}$ from Eq. (23) coincide. Plotted among the functions from the heavy-HISQ and NRQCD calculations is a function arising from a ‘chained’ fit where the $A_{000}^{(n)}$ from the heavy-HISQ fit were used as prior distributions for the $A^{(n)}$ in the form

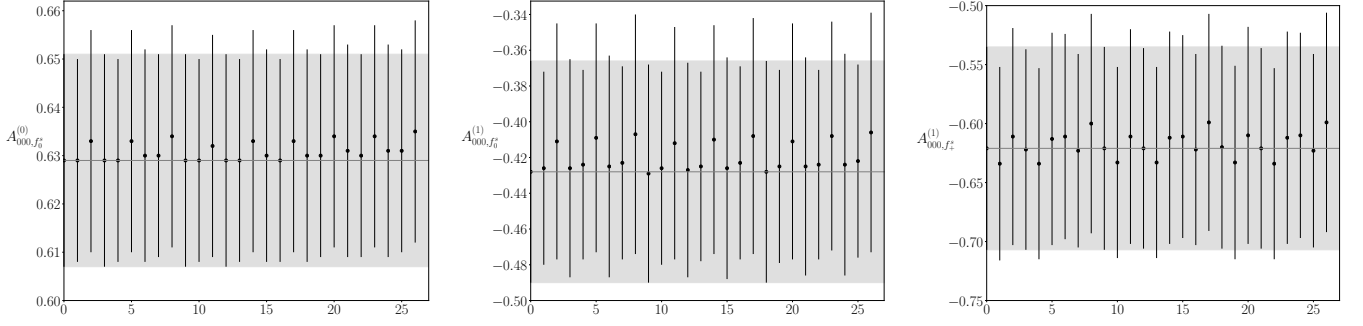


FIG. 19: z_p expansion coefficients, for the calculation with a HISQ spectator quark, computed using the variations of correlator fit parameters listed in Table VI for the $B_c \rightarrow B_s$ form factors. The integer x coordinate of each result is given by $n_{\text{var}}^{(3)} + 3n_{\text{var}}^{(5)} + 9n_{\text{var}}^{(6)}$ where $n_{\text{var}}^{(i)} = 0, 1, 2$ corresponding to original, first and second variations respectively listed in Table VI of set i .

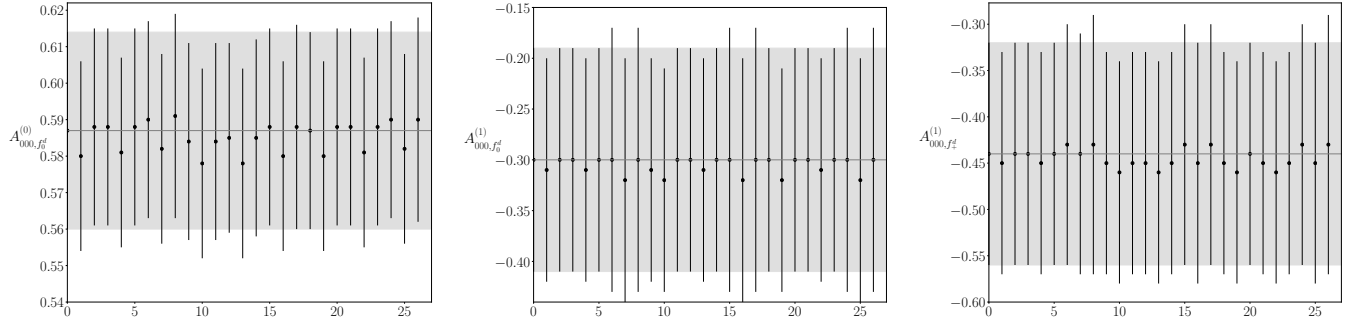


FIG. 20: z_p expansion coefficients, for the calculation with a HISQ spectator quark, computed using the variations of correlator fit parameters listed in Table VI for the $B_c \rightarrow B_d$ form factors. The x coordinate is the same as that in Fig. 19.

factor fit forms in the NRQCD study. We label this fit *NRQCD from heavy-HISQ* in Figs. 21, 22, 23 and 24. As with the separate fits for each case of spectator quark, the form factors for $B_c \rightarrow B_s$ and $B_c \rightarrow B_d$ are fit simultaneously. This chained fit has $\chi^2/\text{d.o.f.} = 1.3$ and is consistent with both the separate fits. We make our final predictions for the decay rates and values for $\Gamma|V|^2$ using the chained fit.

We include the coefficients $A_{0,+}^{(n)}$ from the chained fit in the ancillary *json* file `BcBsd_ff.json`.

E. Dependence of the form factors on the spectator quark mass

In order to build up a picture of the behaviour of form factors it is interesting to ask: how do the form factors for c to s/d decay depend on the mass of the spectator quark? We can answer that question with our heavy-HISQ calculation because we have results at a range of spectator quark masses from m_c upwards (see Fig. 2). Our form factor fits (Sec. IV C) enable us to extrapolate up to m_b . Our most accurate results are for the c to s decay case and we concentrate on that here.

Fig. 25 shows the fit curve from the heavy-HISQ results for f_+^s and f_0^s as a function of the heavy-charm meson

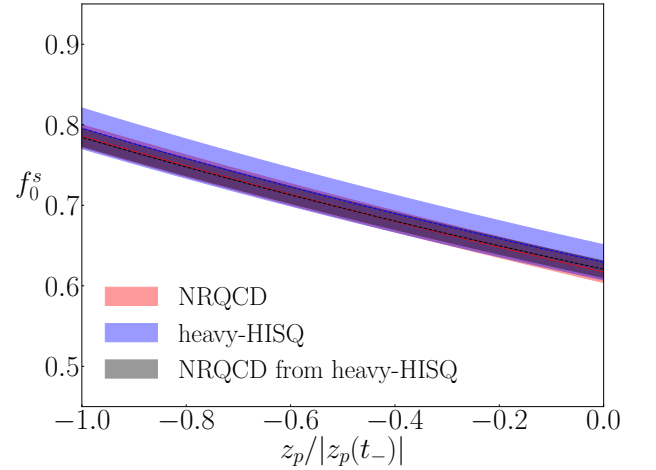


FIG. 21: Fits of f_0 for $B_c^+ \rightarrow B_s^0 \bar{l} \nu_l$ tuned to the physical-continuum limit. The form factor is plotted against $z_p/|z_p(t_-)|$.

mass (as a proxy for the spectator quark mass). The form factor curves that are plotted are those for $q^2 = 0$ (where $f_+ = f_0$) and for the zero-recoil point (q_{max}^2). At q_{max}^2 the daughter meson is at rest in the rest-frame of the H_c meson. The q^2 value at q_{max}^2 falls slowly as the

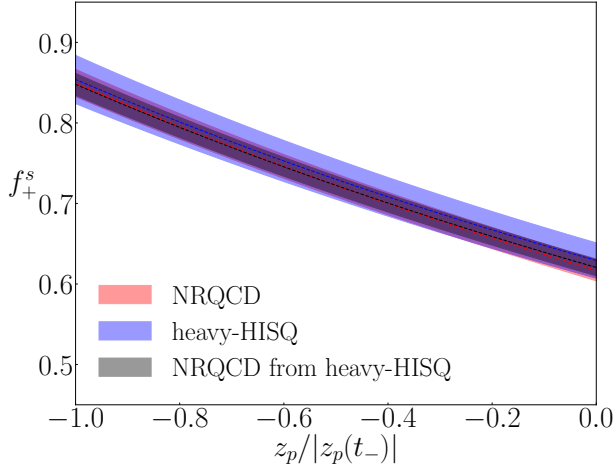


FIG. 22: Fits of f_+ for $B_c^+ \rightarrow B_s^0 \bar{l} \nu_l$ tuned to the physical-continuum limit.

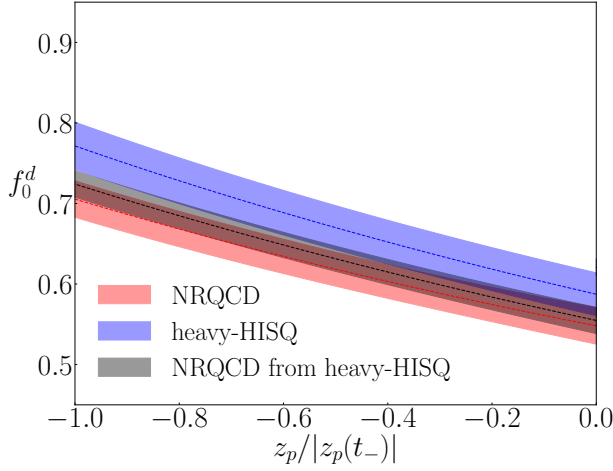


FIG. 23: Fits of f_0 for $B_c^+ \rightarrow B^0 \bar{l} \nu_l$ tuned to the physical-continuum limit.

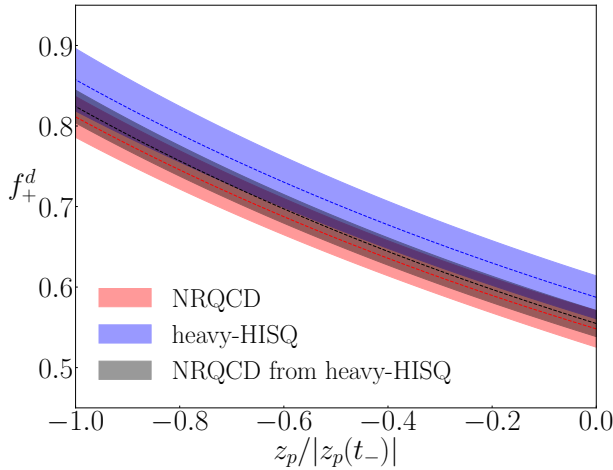


FIG. 24: Fits of f_+ for $B_c^+ \rightarrow B^0 \bar{l} \nu_l$ tuned to the physical-continuum limit.

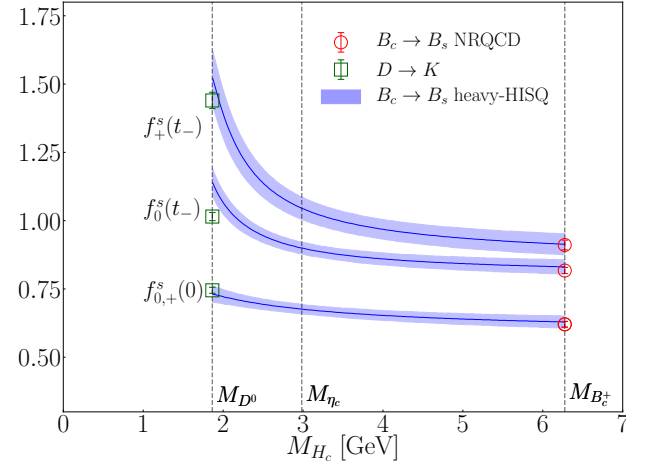


FIG. 25: Values for the physical-continuum form factors $f_0^s = f_+^s$ at $q^2 = 0$ and f_0^s and f_+^s at q_{\max}^2 are plotted against the mass of the heavy-charm pseudoscalar meson. The curve is the continuum limit of the heavy-HISQ fit function (Eq. (23)) extrapolated to the physical B_c and D masses. Note that the region in which the heavy-HISQ calculation has results is the region above M_{η_c} . See the text for a description of how the extrapolation down to the D was done. Also plotted are the form factor results for $D \rightarrow K$ [3] (green squares) as well as the NRQCD $B_c \rightarrow B_s$ result presented in this work (red circles).

heavy-quark mass increases above m_c because the mass difference between H_c and H_s mesons falls. Examining the region between M_{η_c} and M_{B_c} in Fig. 25 we see almost no dependence on the spectator mass. The form factor value that shows the most dependence is $f_+(q_{\max}^2)$. This is not surprising because f_+ shows the biggest slope in q^2 close to q_{\max}^2 and hence sensitivity to the value of q_{\max}^2 . Note that the curve from the heavy-HISQ analysis agrees to that of the b . As discussed in the previous subsection, the form factors obtained from the two calculations agree across the full q^2 range.

We can also investigate the behaviour of the heavy-HISQ fit function as m_h is taken below m_c to m_l where contact is made with results for $D \rightarrow K$ from [3]. For the form factors at $q^2 = 0$, we have $P(q^2) = 1$ and our fit form at Eq. (23) depends only on M_{H_c} . This permits a straightforward extrapolation to the point $M_{H_c} = M_D$ in the continuum limit. For the form factors at zero-recoil (q_{\max}^2), constructing the extrapolation curve is complicated by requiring the dependence of q_{\max}^2 on the mass of the spectator quark. This requires knowledge of M_{H_s} as a function of M_{H_c} . To achieve this, we fit our values of M_{H_s} taken from set 6, together with physical values from experiment [33] at $m_h = m_l, m_b$ (i.e. M_K and M_{B_s}), using a simple fit form $M_{H_s} = M_{H_c}(1 + \sum_{n=1}^4 \omega_n (\Lambda_{QCD}/M_{H_c})^n + A(a\Lambda_{QCD})^2 + B(a\Lambda_{QCD})^4)$. Here A , B and ω_n take prior values 0(2) and we do not include $a\Lambda$ terms for data from [33]. We find this fit function reproduces our data, as well as the physical

TABLE XI: Final results of the weighted integral of $|f_+(q^2)|^2$ over the physical range of squared 4-momentum transfer. Units are MeV.

	$B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell$	$B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell$
$\Gamma V ^{-2}$	$3.47(12) \times 10^{-11}$	$4.29(25) \times 10^{-11}$

values, well. Fig. 25 also shows the result of this downward extrapolation. Whilst this extrapolation below m_c is outside the region where HQET is expected to be valid, the curves nevertheless show approximately the correct amount of upward movement necessary to reproduce the $D \rightarrow K$ results in [3] for f_+ and f_0 at zero-recoil and $q^2 = 0$. The form factors at $q^2 = 0$ continue to show almost no spectator mass dependence, and this is in agreement with the $D \rightarrow K$ results.

F. Decay rate

The hadronic quantity required for determining the decay rate and branching fraction is the integral

$$\Gamma|V|^{-2} = \frac{G_F^2}{24\pi^3} \int_0^{t^-} dq^2 |\mathbf{p}_2|^3 |f_+(q^2)|^2, \quad (26)$$

where V is the CKM element V_{cs} or V_{cd} . Table XI gives values for this quantity for each of the $B_c \rightarrow B_s$ and $B_c \rightarrow B_d$ processes based on the NRQCD and heavy-HISQ chained form factor fit described in Sec. IV D. Values for different q^2 bins can also be obtained. Proceeding with the total decay rate, combining these results with existing CKM matrix values [33] $V_{cs} = 0.997(17)$ and $V_{cd} = 0.218(4)$ yields the predictions

$$\begin{aligned} \Gamma(B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell) &= 52.4(1.8)(1.8) \times 10^9 \text{ s}^{-1} \\ \Gamma(B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell) &= 3.10(11)(18) \times 10^9 \text{ s}^{-1} \end{aligned} \quad (27)$$

where the CKM matrix elements are responsible for the first errors and the second errors arise from our lattice calculations. The dominant source of lattice QCD uncertainty is the fitting of 2-point and 3-point correlators described in Sec. II B 3.

We can convert these results for the decay width into a branching fraction using the lifetime of the B_c meson, 513.49(12.4) fs [42]. This gives

$$\begin{aligned} \mathcal{B}(B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell) &= 0.0269(9)(9)(6) \\ \mathcal{B}(B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell) &= 0.00159(6)(9)(7) \end{aligned} \quad (28)$$

where now the third uncertainty is from the lifetime.

We also present the ratio of the $\Gamma|V|^{-2}$ for $B_c \rightarrow B_s$ to $B_c \rightarrow B_d$ taking correlations into account between the numerator and denominator. From the chained fit of $B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell$ and $B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell$ form factors, we obtain

$$\frac{\Gamma(B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell) |V_{cd}|^2}{\Gamma(B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell) |V_{cs}|^2} = 0.809(53). \quad (29)$$

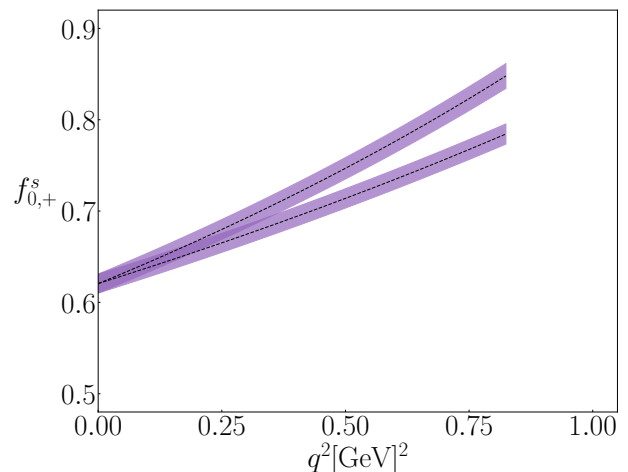


FIG. 26: Final form factors from the chained fits of f_0 (below) and f_+ (above) for $B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell$ in the physical-continuum limit, plotted against the entire range of physical q^2 . This fit is described in Sec. IV D.

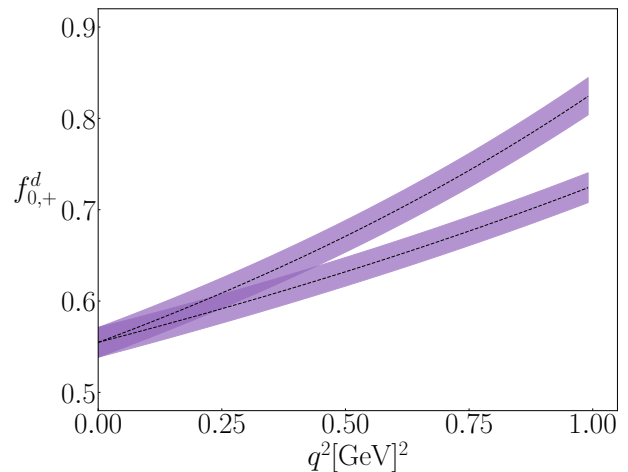


FIG. 27: Final form factors from the chained fits of f_0 (below) and f_+ (above) for $B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell$ in the physical-continuum limit, plotted against the entire range of physical q^2 . This fit is described in Sec. IV D.

In fact the uncertainty is roughly the same as if we were to treat the numerator and denominator as uncorrelated.

V. CONCLUSIONS

We have reported here the first calculations of the decay rates $\Gamma(B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell)$ and $\Gamma(B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell)$, demonstrating the success of lattice QCD in studying decays of heavy-light mesons. The use of HISQ-HISQ $c \rightarrow s(d)$ currents allows for a non-perturbative renormalisation using the PCVC. We used two different formulations for the spectator b quark, heavy-HISQ and NRQCD. Results from the heavy-HISQ calculations are in good agreement with the physical-continuum form factors derived from

the calculations using NRQCD b quarks, giving us confidence in assessing and controlling the systematic errors in each formulation. Simulating at a variety of spectator masses in the heavy-HISQ calculation has provided a check of the spectator-independence of the renormalisation procedure for the vector current. The NRQCD study also accessed Z_V away from zero-recoil to scrutinise momentum independence.

Our final form factors from the chained fit that combines both NRQCD and heavy-HISQ results are plotted against q^2 in Figs. 26 and 27.

The decay rates are predicted from our calculation with 4% and 6% uncertainty for $\Gamma(B_c^+ \rightarrow B_s^0 \bar{\ell} \nu_\ell) = 52.4(2.5) \times 10^9 \text{ s}^{-1}$ and $\Gamma(B_c^+ \rightarrow B^0 \bar{\ell} \nu_\ell) = 3.10(21) \times 10^9 \text{ s}^{-1}$ respectively. There is scope for significant improvement should future experiment demand more precision from the lattice. Such improvement would be readily achieved by the inclusion of lattices with a finer lattice in the heavy-HISQ calculation. ‘Ultrafine’ lattices with $a \approx 0.045 \text{ [fm]}$ were used in [11] to provide results nearer to the physical-continuum limit with $am_h \approx am_b$. Larger statistical samples could also be obtained on the lattices used here, at the cost of more computational resources.

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- [1] C. Aubin *et al.* (Fermilab Lattice, MILC, HPQCD), *Phys. Rev. Lett.* **94**, 011601 (2005), [arXiv:hep-ph/0408306 \[hep-ph\]](#) .
- [2] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage, and J. Shigemitsu (HPQCD), *Phys. Rev.* **D82**, 114506 (2010), [arXiv:1008.4562 \[hep-lat\]](#) .
- [3] J. Koponen, C. T. H. Davies, G. C. Donald, E. Follana, G. P. Lepage, H. Na, and J. Shigemitsu, (2013), [arXiv:1305.1462 \[hep-lat\]](#) .
- [4] G. C. Donald, C. T. H. Davies, J. Koponen, and G. P. Lepage (HPQCD), *Phys. Rev.* **D90**, 074506 (2014), [arXiv:1311.6669 \[hep-lat\]](#) .
- [5] A. Bazavov *et al.* (Fermilab Lattice, MILC), *Phys. Rev.* **D90**, 074509 (2014), [arXiv:1407.3772 \[hep-lat\]](#) .
- [6] V. Lubicz, L. Riggio, G. Salerno, S. Simula, and C. Tarantino (ETM), *Phys. Rev.* **D96**, 054514 (2017), [erratum: *Phys. Rev.* D99, no.9, 099902 (2019)], [arXiv:1706.03017 \[hep-lat\]](#) .
- [7] G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, *Phys. Rev.* **D46**, 4052 (1992), [arXiv:hep-lat/9205007 \[hep-lat\]](#) .
- [8] R. J. Dowdall *et al.* (HPQCD), *Phys. Rev.* **D85**, 054509 (2012), [arXiv:1110.6887 \[hep-lat\]](#) .
- [9] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel, and G. P. Lepage, *Phys. Rev.* **D82**, 034512 (2010), [arXiv:1004.4285 \[hep-lat\]](#) .
- [10] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel, and G. P. Lepage (HPQCD), *Phys. Rev.* **D85**, 031503 (2012), [arXiv:1110.4510 \[hep-lat\]](#) .
- [11] E. McLean, C. T. H. Davies, J. Koponen, and A. T. Lytle, (2019), [arXiv:1906.00701 \[hep-lat\]](#) .
- [12] E. Follana, Q. Mason, C. Davies, K. Hornbostel, G. P. Lepage, J. Shigemitsu, H. Trotter, and K. Wong (HPQCD, UKQCD), *Phys. Rev.* **D75**, 054502 (2007), [arXiv:hep-lat/0610092 \[hep-lat\]](#) .
- [13] S. Borsanyi *et al.*, *JHEP* **09**, 010 (2012), [arXiv:1203.4469 \[hep-lat\]](#) .
- [14] B. Chakraborty, C. T. H. Davies, P. G. de Oliveira, J. Koponen, G. P. Lepage, and R. S. Van de Water, *Phys. Rev.* **D96**, 034516 (2017), [arXiv:1601.03071 \[hep-lat\]](#) .
- [15] B. Chakraborty, C. T. H. Davies, B. Galloway, P. Knecht, J. Koponen, G. C. Donald, R. J. Dowdall, G. P. Lepage, and C. McNeile, *Phys. Rev.* **D91**, 054508 (2015), [arXiv:1408.4169 \[hep-lat\]](#) .
- [16] R. J. Dowdall, C. T. H. Davies, G. P. Lepage, and C. McNeile, *Phys. Rev.* **D88**, 074504 (2013), [arXiv:1303.1670 \[hep-lat\]](#) .
- [17] A. Bazavov *et al.* (MILC), *Phys. Rev.* **D82**, 074501 (2010), [arXiv:1004.0342 \[hep-lat\]](#) .
- [18] A. Bazavov *et al.* (MILC), *Phys. Rev.* **D87**, 054505 (2013), [arXiv:1212.4768 \[hep-lat\]](#) .
- [19] A. Bazavov *et al.* (MILC), *Phys. Rev.* **D93**, 094510 (2016), [arXiv:1503.02769 \[hep-lat\]](#) .
- [20] A. Hart, G. M. von Hippel, and R. R. Horgan (HPQCD), *Phys. Rev.* **D79**, 074008 (2009), [arXiv:0812.0503 \[hep-lat\]](#) .

- [21] J. Koponen, A. C. Zimermmane-Santos, C. T. H. Davies, G. P. Lepage, and A. T. Lytle, *Phys. Rev.* **D96**, 054501 (2017), [arXiv:1701.04250 \[hep-lat\]](#) .
- [22] J. Koponen (HPQCD), *Proceedings, 33rd International Symposium on Lattice Field Theory (Lattice 2015): Kobe, Japan, July 14-18, 2015*, *PoS LATTICE2015*, 119 (2016).
- [23] MILC Code Repository, <https://github.com/milc-qcd>.
- [24] C. T. Sachrajda and G. Villadoro, *Phys. Lett.* **B609**, 73 (2005), [arXiv:hep-lat/0411033 \[hep-lat\]](#) .
- [25] D. Guadagnoli, F. Mescia, and S. Simula, *Phys. Rev.* **D73**, 114504 (2006), [arXiv:hep-lat/0512020 \[hep-lat\]](#) .
- [26] C. Aubin, C. Bernard, C. E. DeTar, J. Osborn, S. Gottlieb, E. B. Gregory, D. Toussaint, U. M. Heller, J. E. Hetrick, and R. Sugar (MILC), *Phys. Rev.* **D70**, 114501 (2004), [arXiv:hep-lat/0407028 \[hep-lat\]](#) .
- [27] E. B. Gregory *et al.* (HPQCD), *Phys. Rev.* **D83**, 014506 (2011), [arXiv:1010.3848 \[hep-lat\]](#) .
- [28] G. C. Donald, C. T. H. Davies, R. J. Dowdall, E. Follana, K. Hornbostel, J. Koponen, G. P. Lepage, and C. McNeile, *Phys. Rev.* **D86**, 094501 (2012), [arXiv:1208.2855 \[hep-lat\]](#) .
- [29] G. P. Lepage, B. Clark, C. T. H. Davies, K. Hornbostel, P. B. Mackenzie, C. Morningstar, and H. Trotter, *Lattice field theory. Proceedings, 19th International Symposium, Lattice 2001, Berlin, Germany, August 19-24, 2001*, *Nucl. Phys. Proc. Suppl.* **106**, 12 (2002), [arXiv:hep-lat/0110175 \[hep-lat\]](#) .
- [30] K. Hornbostel, G. P. Lepage, C. T. H. Davies, R. J. Dowdall, H. Na, and J. Shigemitsu (HPQCD), *Phys. Rev.* **D85**, 031504 (2012), [arXiv:1111.1363 \[hep-lat\]](#) .
- [31] C. M. Bouchard, G. P. Lepage, C. Monahan, H. Na, and J. Shigemitsu (HPQCD), *Phys. Rev.* **D90**, 054506 (2014), [arXiv:1406.2279 \[hep-lat\]](#) .
- [32] B. Chakraborty, C. Davies, J. Koponen, and G. P. Lepage (HPQCD), *Proceedings, 35th International Symposium on Lattice Field Theory (Lattice 2017): Granada, Spain, June 18-24, 2017*, *EPJ Web Conf.* **175**, 13027 (2018), [arXiv:1710.07334 \[hep-lat\]](#) .
- [33] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev.* **D98**, 030001 (2018).
- [34] C. Michael, *Phys. Rev.* **D49**, 2616 (1994), [arXiv:hep-lat/9310026 \[hep-lat\]](#) .
- [35] R. J. Dowdall, C. T. H. Davies, R. R. Horgan, G. P. Lepage, C. J. Monahan, J. Shigemitsu, and M. Wingate (HPQCD), *Phys. Rev.* **D100**, 094508 (2019), [arXiv:1907.01025 \[hep-lat\]](#) .
- [36] B. Chakraborty, C. T. H. Davies, G. C. Donald, J. Koponen, and G. P. Lepage (HPQCD), *Phys. Rev.* **D96**, 074502 (2017), [arXiv:1703.05552 \[hep-lat\]](#) .
- [37] B. Colquhoun, C. T. H. Davies, R. J. Dowdall, J. Kettle, J. Koponen, G. P. Lepage, and A. T. Lytle (HPQCD), *Phys. Rev.* **D91**, 114509 (2015), [arXiv:1503.05762 \[hep-lat\]](#) .
- [38] G. P. Lepage, `lsqfit` Version 11.1 (github.com/gplepage/lsqfit).
- [39] A. Bazavov *et al.*, *Phys. Rev.* **D98**, 074512 (2018), [arXiv:1712.09262 \[hep-lat\]](#) .
- [40] C. Bourrely, I. Caprini, and L. Lellouch, *Phys. Rev.* **D79**, 013008 (2009), [Erratum: *Phys. Rev.*D82,099902(2010)], [arXiv:0807.2722 \[hep-ph\]](#) .
- [41] C. G. Boyd, B. Grinstein, and R. F. Lebed, *Nucl. Phys.* **B461**, 493 (1996), [arXiv:hep-ph/9508211 \[hep-ph\]](#) .
- [42] R. Aaij *et al.* (LHCb), *Phys. Lett.* **B742**, 29 (2015), [arXiv:1411.6899 \[hep-ex\]](#) .