Zang, B., Mayer, Y. D., & Azarpeyvand, M. (2020). A note on the pressure – velocity correlation and coherence normalisation. *Experiments in Fluids*, [186 (2020)]. https://doi.org/10.1007/s00348-020-03019-0

Experiments in Fluids manuscript No.
(will be inserted by the editor)

A Note on the Pressure – Velocity Correlation and Coherence Normalisation

B. Zang · **Yannick D. Mayer** · **Mahdi Azarpeyvand**

Received: date / Accepted: date

Abstract In aerodynamic and aeroacoustic research, pressure–velocity correlation and coherence have been increasingly utilised to identify and understand the motions of large scale structures and their collective effects on the surface pressure fluctuations due to the boundary layer. Nevertheless, when a relatively sharp transition from turbulent boundary layer to low turbulence free-stream takes place, it is observed that 'spurious' regions of elevated correlation and coherence level arise as a result of this transition and the use of conventional normalisation. The present study identifies this issue using measurements from a NACA 0012 airfoil and proposes a modified normalisation by taking the flow turbulence into account. The results have clearly demonstrated that the modified procedure helps successfully preclude these spuriously heightened $p-u$ correlation and coherence regions.

Keywords pressure-velocity correlation · unsteady surface pressure · hot-wire anemometry · boundary layer turbulence

1 Introduction

Modern engineering applications in flow control are becoming increasingly demanding under stringent regulatory frameworks, such as regulations to reduce CO_2 and noise emission and expanding engineering application areas of interest, for instance, modern aircraft configurations, unmanned aerial vehicles, etc. This in turn requires a more thorough investigation of the underlying physics of the flow, and in particular boundary layers. Indeed, significant progress has been made on the understanding of boundary layers in the past several decades [16], attributed partly to the development of advanced experimental and numerical techniques and partly to the advances in robust and high order analytical tools, i.e. modal decompositions, temporal and spatial correlations, wavelet and bicoherence analyses.

Spatial-temporal correlation and coherence are important parameters that can help identify and locate the large-scale structures and their collective motions in the flow, and are particularly useful in applications involving noise source identification in aeroacoustics. Recently, there has seen an increasing number of studies employing simultaneous pressure and velocity measurements to examine the underlying physics of various flow scenarios [11–13, 16, 18]. The conventional normalised pressure–velocity (referred as $p-u$ thereafter) correlation, $R_{p'u'}$, and magnitude-squared coherence, γ^2, can be calculated, respectively, as [15]:

$$R_{p'u'}(y_n,\tau) = \frac{\overline{p'(t)\,u'(y_n,t-\tau)}}{p'_{rms}\,u'_{rms}(y_n)}, \tag{1}$$

$$\gamma^2(y_n,f) = \frac{|\phi_{p'u'}(y_n,f)|^2}{\phi_{p'p'}(f)\,\phi_{u'u'}(y_n,f)}. \tag{2}$$

where p' and u' denote the pressure and velocity fluctuations, respectively, and y_n represents the wall normal distance from the surface. The subscript '*rms*' refers to the root-mean-square, and '$^-$' indicates time-averaging. ϕ represents the power spectral density (PSD) of either surface pressure or velocity fluctuations.

The direct application of the above equations on a turbulent boundary layer can lead to spurious regions of high $p-u$ correlation and coherence levels, near the edge of the boundary layer and well into the free-stream area, which could be misleading and undesirable especially when the flow structures near the edge of the boundary layer are of

---

Corresponding author: Mahdi Azarpeyvand, m.azarpeyvand@bristol.ac.uk, +44 117 3315949

B. Zang · Yannick D. Mayer · Mahdi Azarpeyvand
Faculty of Engineering, University of Bristol, Bristol,
United Kingdom.

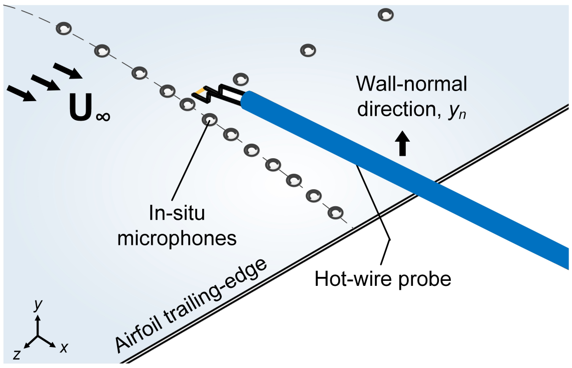


Fig. 1: Schematic of the zoomed-in view for the present experimental setup with NACA 0012 airfoil.

primary interest and importance (Chauhan *et al.* [2] and Lee *et al.* [6]). Moreover, addressing such an issue has become more pertinent and crucial with the recent surge in aeroacoustic studies for identification of the noise contributing turbulent structures, such as Afshari *et al.* [1], Garcia-Sagrado and Hynes [5], Showkat Ali *et al.* [13] and Szöke *et al.* [18], among others. The present experiment aims to examine the plausible cause of the spuriously high $p-u$ correlation and coherence regions, which will be discussed in section 3. A modified normalisation procedure, consistent for both correlation and coherence definitions, will also be proposed such that these regions can be identified and separated from the actual physical events with sufficient confidence.

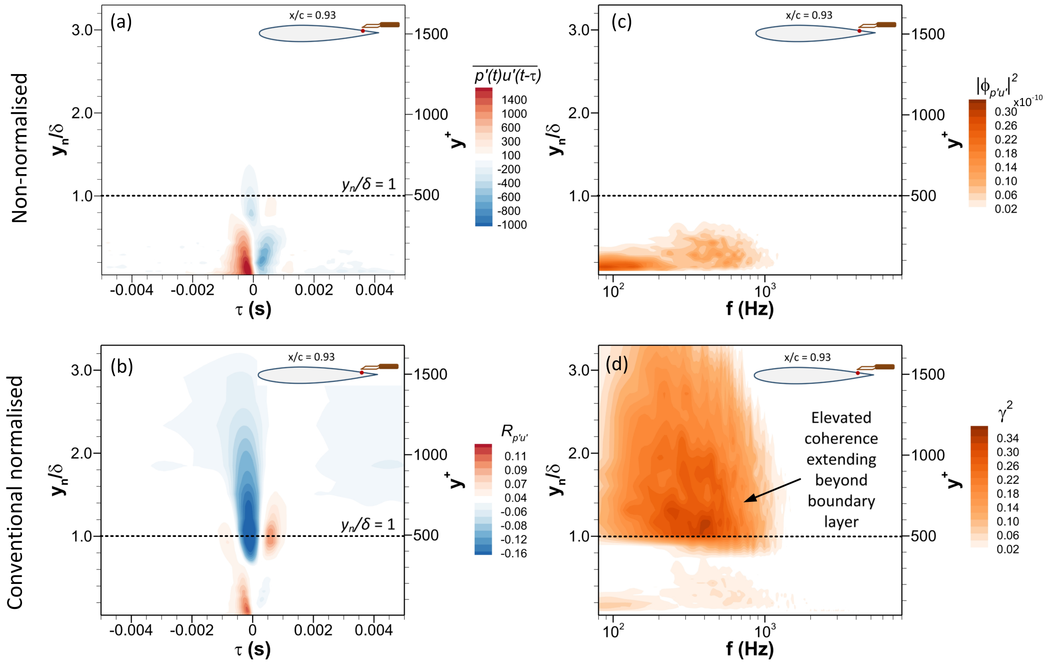
2 Experimental setup

The present experiments were performed in the closed-loop open-jet aeroacoustic wind tunnel facility at University of Bristol [7]. A NACA 0012 airfoil with a chord of $c = 300$ mm was placed at 0° angle of attack, 300 mm from the outlet of a 500 mm wide × 775 mm height nozzle. The contraction nozzle has a relatively low turbulence intensity of 0.12 %. The free-stream velocity was maintained at $U_\infty = 20\,\mathrm{m\,s^{-1}}$, which corresponds to a Reynolds number of $Re_c \approx 420\,000$.

The NACA 0012 airfoil used in the experiments has 91 condenser microphones (Knowles FG-23629-P16) to measure the unsteady wall pressure fluctuations. The microphones are placed in-situ under 0.4 mm diameter pin holes and were calibrated in both magnitude and phase with a reference G.R.A.S. 40PL free-field microphone with known sensitivity prior to the experiments. The calibration procedure follows that outlined by Mish [9] and Szöke [17]. The calibration result (not shown here for brevity) shows a relatively constant sensitivity and less than 8° of phase shift for the valid frequency range from 100 Hz to 8000 Hz in the present measurements. To achieve simultaneous pressure and velocity measurements, a Dantec 55P16 single-probe hotwire was positioned directly on top of the a surface microphone located at x/c = 0.93, traversing in the wall normal direction, y_n, as shown in the zoomed-in schematic in Fig. 1. All microphone and hot-wire measurements were sampled at a frequency of 2^{16} Hz for 16 s using a National Instruments PXIe-4499 module. Subsequently, the power spectral density (PSD) of both the surface pressure and velocity fluctuations is calculated via Welch's method using a window size of 2^{12} samples and a Hamming window with 50 % overlap resulting in a bin size of 16 Hz. The relative uncertainty of the velocity measurements, as calculated during the calibration process, was approximately ±2.5 %. Furthermore, the uncertainty of the microphone measurements was estimated to be ±1.5 %. Consequently, the uncertainty of the correlation and coherence calculations were estimated to be ±3 % and ±4 %, respectively [10], as coherence is magnitude-squa-red of the $p-u$ cross-spectral. Finally, the airfoil is tripped at 10 % of the chord on both the suction and pressure sides to achieve a fully turbulent boundary layer over the trailing edge area. A detailed description of the present experimental set-up can be found in Mayer *et al.* [8].

3 Results and discussion

To begin with, Fig. 2 shows the comparison of the $p-u$ correlation and coherence between non-normalised results obtained from the present measurements and those determined using conventional formulae defined by Eqs. 1 and 2, respectively. Recall that the pressure and velocity data are collected at $x/c = 0.933$. It is useful to first relate the $p-u$ correlation and coherence results to the development and dynamics of the flow. For the $p-u$ correlation shown in Figs. 2(a) and (c), a negligible time shift, τ, calculated from Eq. 1 indicates the the velocity fluctuations impact directly upon pressure fluctuations as the turbulent structures convecting past the measurement point, and furthermore the correlation pattern agrees well with those obtained by Garcia-Sagrado [4] in similar $p-u$ measurements. Moreover, for the $p-u$ coherence shown in Figs. 2(b) and (d), the broadband nature of the coherence within the boundary layer reflects the nature of a turbulent boundary layer, in which a cascade of different-sized eddies impact upon the surface pressure fluctuations. More importantly, when comparing the non-normalised and normalised results, it becomes evident that an elevated region can be observed for both the correlation, $R_{p'u'}$, and coherence, γ^2, starting from the edge of the boundary layer and stretching well into the free-stream region, i.e. approximately $0.8 < y_n/\delta < 3.4$, after being subjected to the conventional normalisation. Note that the boundary layer thickness is determined to be $\delta \approx 10.2$ mm, based on 99 % of the free-stream velocity under the present experimental condition and the wall unit length, y^+, has been included to better characterise the boundary layer (see Fig. 3 below). With



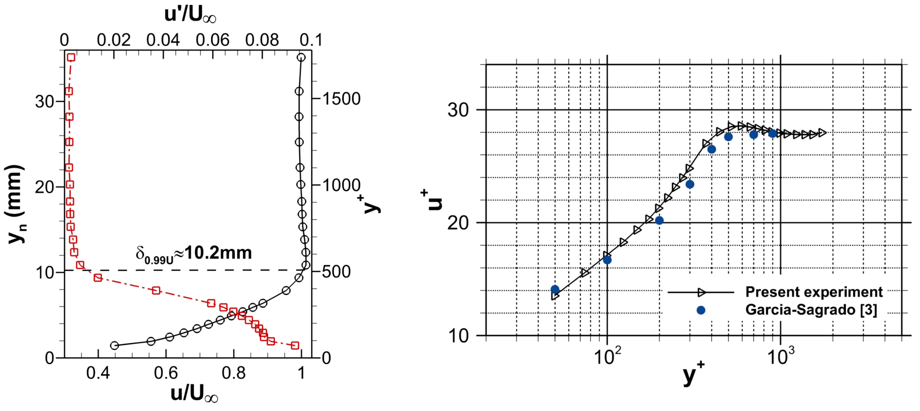
p − u correlation

p − u coherence

Fig. 2: Contour maps of the pressure–velocity correlation ($R_{p'u'}$) determined for (a) non-normalised and (b) normalised with Eq. 1, and the pressure–velocity coherence (γ^2) determined for (c) non-normalised and (d) normalised with Eq. 2, from the NACA 0012 experiments.

the correlation and coherence magnitudes even greater than those close to the wall, the extended regions of high $p-u$ correlation and coherence suggest that there exists a field of strong hydrodynamic disturbances, which contributes significantly to the surface pressure fluctuations, even at the laminar free-stream region far from the surface. However, this is well beyond the y_n/δ range identified by Chauhan *et al.* [2], where the turbulent/non-turbulent interface fluctuations take place. Furthermore, any prominent disturbances and the coherent structures possibly associated with it are likely to be in the much lower frequencies, for a uniform free-stream with a low turbulence intensity level.

It should also be remarked that during the experimental study, the authors have performed a similar set of experiments using another open-jet wind tunnel, whose free-stream turbulence level is an order of magnitude higher than the tests reported in the paper. The results (not shown here for brevity) demonstrated a reduced level of the $p-u$ correlation and coherence beyond the boundary layer, which, nevertheless, remained more accentuated as an area of high correlation and coherence levels compared to the levels observed close to the wall. Consequently the elevated $p-u$ correlation and coherence levels are likely to be spurious to a large extent. It is noteworthy that although velocity fluctuations decay rapidly after entering the free-stream region, the pressure fluctuations in the flow field persist relatively far beyond the boundary layer. Hence, the coherence and correlation between two pressure measurements are expected to extend far beyond the boundary layer, unlike the $p-u$ analyses here [14]. More importantly, it is worthwhile to mention that as can be seen from the non-normalised results in Fig. 2(a), there exists a pair of small but noticeable $p-u$ correlation structures near the edge of the boundary layer, indicating the presence of physically meaningful turbulent structures influencing the surface pressure fluctuations. Garcia-Sagrado and Hynes [5] observed a similar correlation behaviour and attributed it to the existence of large-scale turbulent structures, reaching close to the edge of the boundary layer. Another possible cause of the correlation level can be attributed to the modulation of the turbulent/non-turbulent interface induced by the large-scale motions close to the edge of the boundary layer [2, 6]. Nevertheless, a detailed analysis on the associated physical development is beyond the scope of the proposed normalisation procedure. Therefore, from the discussion above, it is evident that the elevated level of $p-u$ correlation and coherence can largely be attributed to the low turbulence level of the free-stream. Indeed, the 'co-existence' of non-physical $p-u$ correlation and coherence levels with those arising from physically important turbulent structures essentially demands a better understanding on the origin of such spurious $p-u$ correlation and coherence, and subsequently a solution to the problem.



(a) (b)

Fig. 3: Boundary layer characteristics for (a) boundary thickness and its associated turbulence intensity and (b) the log-law profile of wall unit length, y^+, against dimensionless velocity, u^+, of NACA 0012 airfoil. Note that several data points were extracted from Garcia-Sagrado [4] for validation.

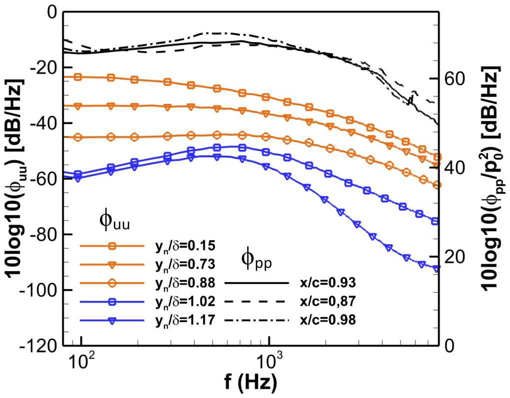
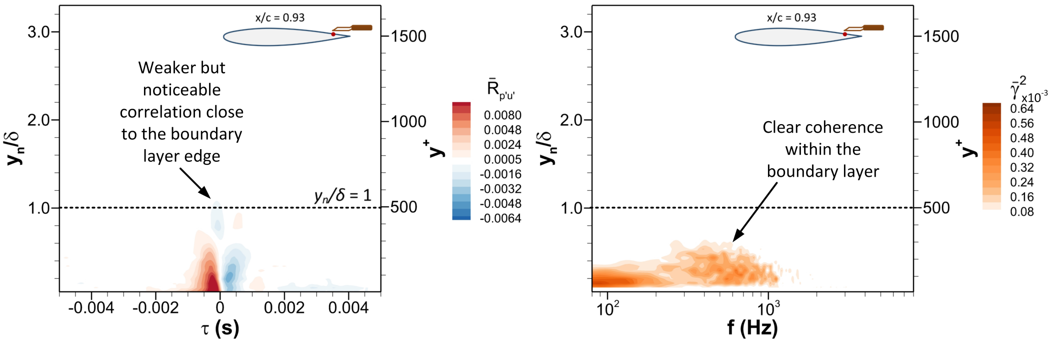


Fig. 4: Power spectra density of the surface pressure (ϕ_{pp}) and velocity (ϕ_{uu}) fluctuations at various chord-wise and wall-normal locations.

Therefore, it is necessary to first examine the boundary layer characteristics of the flow and put those into the context of the correlation and coherence definitions, in order to identify the plausible cause of the spuriously high $p-u$ correlation and coherence levels.

Firstly, Fig. 3(a) shows the velocity and turbulence intensity profiles of the turbulent boundary layer over the NACA 0012 at $x/c = 0.933$. To provide a better knowledge on the boundary layer characteristics and measurement resolution of the present experiments, Fig. 3(b) shows the log-law profile of the wall unit length, y^+ against the dimensionless velocity, u^+. Note that the wall shear stress, τ_w, was estimated from the investigations on NACA 0012 airfoil at the same 0° angle of attack and similar Reynolds number of Re_c =400 000 by Wolf *et al.* [19]. The log-law profile of the boundary layer agrees very well with the study by Garcia-Sagrado [4], reassuring the validity of the present boundary layer measurements. For a naturally developed boundary layer, a relatively abrupt change in the gradient of turbulence intensity, u'_{rms}, can be observed when the boundary layer transitions into free-stream, as shown in Fig. 3. A closer examination of the correlation from Eq.1 indicates that the magnitude of correlation increases as the flow turbulence intensity, u'_{rms}, reduces. Clearly, since the u'_{rms} is an order of magnitude smaller from $y_n/\delta \approx 0.8$ to the free-stream region as compared to that in the inner region of the boundary layer, the normalised $p-u$ correlation, $R_{p'u'}$, becomes inadvertently large in magnitude, corresponding very well with high $R_{p'u'}$ region observed in Fig. 2(a).

Secondly, Fig. 4, shows the PSD distribution of the surface pressure and velocity fluctuations at different chordwise (x/c) and wall-normal (y_n/δ) locations, respectively. Note that the reference pressure is $p_0 = 20 \cdot 10^{-6}$Pa. Zooming into the velocity spectra close to the edge of the boundary layer in Fig. 4, a shift of the energy contents towards lower frequency can be discerned, from $y_n/\delta = 0.88$ (in brown) to $y_n/\delta = 1.0$ (in blue). Expectedly, such a transition would produce an increased level of normalised coherence with that of the surface pressure PSD (in black) as the velocity spectra curve produces a 'hump' similar to that of the surface pressure spectra over a broadband frequency range of approximately 160 Hz to 1000 Hz. Indeed, Fig. 2(b) clearly illustrates a region of elevated $p-u$ coherence starting from $y_n/\delta = 0.8$, and with a frequency range of approximately 100 Hz to 1000 Hz. It is worthwhile to mention that this energy content shift of the velocity spectra close to the boundary layer has also been observed previously by Devenport *et al.* [3]. Moreover, further analyses of the $p-u$ correlation and coherence between the upstream and downstream surface pressure and the velocity results (not shown here for



(a) $p-u$ correlation (b) $p-u$ coherence

Fig. 5: Contour maps of the modified pressure–velocity (a) correlation ($\tilde{R}_{p'u'}$ in Eq. 3) and (b) coherence ($\tilde{\gamma}^2$ in Eq. 4) from the NACA 0012 experiments.

brevity) have seen similar elevated levels, that reaffirm their largely spurious nature.

From the analyses above, it appears that it is essential to take the transition from boundary layer to free-stream into account, i.e. the u'_{rms} profile, to produce physically more meaningful $p-u$ correlation and coherence plots. As a result, a rather straightforward modification is proposed, involving a multiplication by a u'_{rms}/U_∞ factor, such that the correlation, $\tilde{R}_{p'u'}$, becomes:

$$\tilde{R}_{p'u'}(y_n,\tau) = \frac{\overline{p'(t)\,u'(y_n,t-\tau)}}{p'_{rms}\,u'_{rms}(y_n)}\left(\frac{u'_{rms}(y_n)}{U_\infty}\right), \tag{3}$$

and similarly, the coherence, $\tilde{\gamma}^2$, is expressed as:

$$\tilde{\gamma}^2(y_n,f) = \frac{|\phi_{p'u'}(y_n,f)|^2}{\phi_{p'p'}(f)\phi_{u'u'}(y_n,f)}\left(\frac{u'_{rms}(y_n)}{U_\infty}\right)^2. \tag{4}$$

Note that '~' is used to differentiate the modified definitions from the convention. It should be cautiously mentioned that although $\tilde{R}_{p'u'}$ and $\tilde{\gamma}^2$ remain dimensionless, their magnitudes are not comparable to the actual level of coherence and correlation. Nonetheless, the primary motivation here is to firstly illustrate the effects of low turbulence intensity and normalisation on these quantities, and secondly to help preclude spurious regions of high $p-u$ correlation and coherence using the modified normalisation procedure. Once physically meaningful correlation and coherence are established, it can be useful to refer back to the conventional calculations for an informed comparison of normalised absolute magnitudes with other test cases and/or with other studies. Figure 5(a) and (b) show the $p-u$ correlation and coherence obtained from the modified normalisation procedures, i.e. Eqs. 3 and 4, respectively. The modification successfully suppresses the elevated regions of correlation and coherence, and accentuates the relation between pressure and velocity fluctuations within the near wall of the boundary layer. More importantly, it preserves the noticeable region of $p-u$ correlation near the edge of the boundary layer, as observed from the non-normalised results in Fig. 2(a). It is useful to highlight here that the term 'spurious' refers to the significantly higher $p-u$ cross-correlation and coherence levels observed relatively far beyond the boundary layer compared to those close to the airfoil surface, but not to exclude the actual physically important correlation and coherence features within and close to the boundary layer. In fact, as seen from Fig. 5(a), the proposed normalisation procedure proved effective in isolating the spurious regions arisen from the sharp boundary layer transition, which can help better identify and characterise the turbulent structures in the boundary layer. Also, it does not introduce extra physical terms and/or measurements which may complicate the analyses further. Lastly, it should be remarked that the choice of normalisation factors remains flow physics-dependent. For instance, both Garcia-Sagrado and Hynes [5] and Naka *et al.* [11] scaled their $p-u$ correlation with constant velocities instead. Nevertheless, a constant velocity in the $p-u$ coherence may omit the frequency information carried with the velocity fluctuations. The proposed normalisation, therefore, provides a simple and effective modification, consistent for both the $p-u$ cross-correlation and coherence calculations.

4 Conclusions

In aerodynamic and aeroacoustic studies, simultaneous measurements of pressure and velocity and determination of their coherence and correlation have been actively employed in order to identify and better understand the crucial connections between pressure fluctuations and large scale motions. Nevertheless, the present experiments on NACA 0012 suggested that one should be mindful when interpreting results from these correlation and coherence and additional care must always be exercised in isolating physical events from the non-physical disturbances arising from the analyses. For a naturally developed boundary layer, u'_{rms} could be accounted for and a simple modification to the conventional normalisation of coherence and correlation proved useful in reducing the ‘spuriously’ elevated levels of coherence and correlation as the boundary layer merges with the free-stream.

Acknowledgements The authors would like to gratefully acknowledge the financial support for the present study by EPSRC (EP/R010-846/1).

References

1. Afshari, A., Azarpeyvand, M., Dehghan, A.A., Szöke, M., Maryami, R.: Trailing-edge flow manipulation using streamwise finlets. Journal of Fluid Mechanics **870**, 617–650 (2019)
2. Chauhan, K., Philip, J., de Silva Charitha, M., Hutchins, N., Marusic, I.: The turbulent/non-turbulent interface and entrainment in a boundary layer. Journal of Fluids Mechanics **742**, 119–151 (2014)
3. Devenport, W., Burdisso, R.A., Camargo, H., Crede, E., Remillieux, M., Rasnick, M., Van Seeters, P.: Aeroacoustic Testing of Wind Turbine Airfoils. Tech. rep., Virginia Polytechnic Institute and State University and National Renewable Energy Laboratory (2010)
4. Garcia-Sagrado, A.: Boundary layer and trailing-edge noise source. Ph.D. thesis, Univerity of Cambridge, United Kingdom (2007)
5. Garcia-Sagrado, A., Hynes, T.: Wall pressure sources near an airfoil trailing edge under turbulent boundary layers. Journal of Fluids and Structures **30**, 3–34 (2012)
6. Lee, J., Sung, H.J., Zaki, T.A.: Signature of large-scale motions on turbulent/non-turbulent interface in boundary layers. Journal of Fluids Mechanics **819**, 165–187 (2017)
7. Mayer, Y.D., Jawahar, H.K., Szöke, M., Showkat Ali, S.A., Azarpeyvand, M.: Design and performance of an aeroacoustic wind tunnel facility at the University of Bristol. Applied Acoustics **155**, 358 – 370 (2019)
8. Mayer, Y.D., Zang, B., Azarpeyvand, M.: Design of a Kevlar-Walled Test Section with Dynamic Turntable and Aeroacoustic Investigation of an Oscillating Airfoil. In: 25th AIAA/CEAS Aeroacoustics Conference. AIAA 2019-2464, Delft, The Netherland. (2019)
9. Mish, P.F.: Mean loading and turbulence scale effects on the surface pressure fluctuations occurring on a NACA 0015 airfoil immersed in grid generated turbulence. Master’s thesis, Virginia Polytechnic Institute and State University, United States (2001)
10. Moffat, R.J.: Describing the uncertainties in experimental results. Experimental Thermal and Fluid Science **1**(1), 3–17 (1988)
11. Naka, Y., Stanislas, M., Foucaut, J.M., Coudert, S., Laval, J.P., Obi, S.: Space-time pressure-velocity correlations in a turbulent boundary layer. Journal of Fluid Mechanics **771**, 624–675 (2015)
12. Rife, M.C., Devenport, W.J., Simpson, R.L.: An experimental study of the relationship between velocity and pressure fluctuations in a wing-body junction. Tech. Rep. AD-A249628, Office of Naval Research (1992)
13. Showkat Ali, S.A., Azarpeyvand, M., Ilario da Silva, C.R.: Trailing-edge flow and noise control using porous treatments. Journal of Fluid Mechanics **850**, 83–119 (2018)
14. Sillero, J.A., Jiménez, J., Moser, R.D.: Two-point statistics for turbulent boundary layers and channels at reynolds number up to $\delta^+ \approx 2000$. Physics of Fluids **26**, 105109 (2014)
15. Smith, J.O.I.: Mathematics of the discrete Fourier transform (DFT): with audio applications. Julius Smith (2007)
16. Stanislas, M.: Near wall turbulence : An experimental view. Physical Review Fluids **2**(10) (2017)
17. Szöke, M.: Trailing edge noise control using active flow control methods. Ph.D. thesis, University of Bristol, United Kingdom (2019)
18. Szöke, M., Fiscaletti, D., Azerpeyvand, M.: Effect of inclined transverse jets on trailing- edge noise generation. Physics of Fluids **30** (2018)
19. Wolf, W.R., Azevedo, J.L.F., Lele, S.K.: Convective effects and the role of quadrupole sources for aerofoil aeroacoustics. Journal of Fluid Mechanics **708**, 502–538 (2012)