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Waiting in inter-temporal choice tasks affects discounting and subjective time perception[1]

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Abstract

The literature on human delay discounting behavior is dominated by experimental paradigms which do not impose actual delays. Given that waiting may be aversive even on short timescales, we present a novel delay discounting paradigm to study differences in delay discounting behavior either when real waiting is involved, or not. This paradigm retains the fundamental trade-off between rewards received versus their immediacy. We used hierarchical Bayesian modelling to decompose and test models that separate discounting and subjective time perception mechanisms. We report two experiments which also explore the magnitude effect and gain-loss asymmetry. In both experiments, we found greater discounting and non-linear (convex) time perception in the online waiting task, which required waiting after each choice, compared to a control condition where waiting was deferred until the end of the experiment. Discounting was also measured by a parameter of a hyperbolic-type model and related to reversals of preference between initial desire to wait and then not doing so. These results from our novel paradigm support the emerging view that subjective time perception plays an important role in inter-temporal choice in addition to discounting tendencies.

[1] Data and analysis code is available at https://osf.io/pz5e3/

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Introduction

A large body of research demonstrates that humans, as well as non-human animals, discount future rewards, such that sooner rewards are preferred unless a delayed reward is large enough to overcome this discounting. The extent to which individuals discount is connected to important life consequences (Chabris, Laibson, Morris, Schuldt, & Taubinsky, 2008; Mischel, Shoda, & Rodriguez, 1989; Mischel et al., 2011; Shoda, Mischel, & Peake, 1990).

In the typical delay discounting task, participants make a series of choices between an immediate reward and a larger later one. The purpose is to find points of subjective indifference between present and future values at various reward delays. The majority of the studies in this domain have used a paradigm that describes the choice options to participants. That is to say, individuals receive information about rewards and delays, but they do not experience the delays or receive the rewards. An exception is behavioral economic experiments that generally give participants real monetary payoffs, but there is evidence that choice behavior is similar for either hypothetical versus real monetary rewards (Johnson & Bickel, 2002). It is less common to make participants wait in inter-temporal choice tasks presumably out of practicality. The notable exceptions are studies with children, such as the seminal Stanford marshmallow study by Mischel and colleagues (e.g., Mischel et al., 1989; Mischel & Ebbesen, 1970; Mischel, Ebbesen, & Raskoff Zeiss, 1972; also see review by Staubitz, Lloyd, & Reed, 2018), studies with non-human animals (Hayden, 2016; Vanderveldt et al., 2016), and studies that have used real consumptions, working effort, and noise as choice of non-monetary outcomes (Augenblick et al., 2015; McClure et al., 2007; Solnick et al., 1980). A goal of the present work is to focus on understanding delay discounting behavior when participants experience the waiting.

The literature has identified many possible reasons behind the tendency to discount future payoffs, including but not limited to uncertainties associated with receiving the

promised reward in the future; inflation; expectations of changing wealth and tastes; valuation processes, time perception, impulsivity (Frederick, Loewenstein, & O'donoghue, 2002), and imperfect information about the future (Gabaix & Laibson, 2017). Among them, time perception processing has recently attracted substantial attention. Studies have demonstrated that time perception is a concave function of objective time, and thus can play a critical role in generating time discounting behavior (Han & Takahashi, 2012; Takahashi et al., 2008; Zauberman et al., 2009). Other studies found that perception of future time depends on properties of the outcomes such as their magnitude. Future time is perceived as shorter for losses than for gains (Bilgin & Leboeuf, 2010), for larger reward magnitudes (Wang et al., 2015), and when outcomes are associated with more intense emotions (Van Boven et al., 2010). Time perception is also easily manipulated. For example, Zauberman et al. found that subjective time estimates increased as more attention was given to duration (Zauberman et al., 2009). Ebert and Prelec (2007) also showed that sensitivity to future time was influenced by factors such as attention focus, time pressure, and visual cues.

Studies on time perception also document that temporal judgments are easily manipulated (Eagleman, 2008). Time perception studies, unlike inter-temporal choice experiment, do include the passage of time as an experience. These studies have found that duration is dilated by motion or sequence complexity (Brown, 1931; brown, 1995; Schiffman & Bobko, 1974), by the magnitude of the stimulus (Xuan et al., 2007), by the unpredictability of the events (Pariyadath & Eagleman, 2007), and its estimation depends on the specific emotion an individual is experiencing (Droit-Volet & Gil, 2009). However, these findings have rarely been taken into consideration in the decision-making literature. The two areas work in silos, with decision making research mainly focusing on "future" or "described" delays as a way to understand time-monetary trade-offs, whereas the focus of the time perception studies is on defining the connection between experienced time (with a known underlying objective value to the researcher) and the perceived time. Even though both areas

have revealed that time perception is easily malleable, their similarities and differences in subjective perception and subsequent impacts in decision making have been mostly ignored. With these considerations, we propose that experiencing waiting might greatly change time perception and choice trade-offs compared to trade-offs carried out when time is only described. We aim to reveal how differences in subjective time perception might lead to different time discounting behavior.

In everyday modern environments the significance of understanding waiting cannot be overestimated. A slow website may cost a business an online sale, and companies spend much effort in ameliorating the aversion of waiting (Larson & Pinker, 2000). The present study focuses on understanding differences in time discounting behavior between two tasks, one in which the waiting follows a choice in an online waiting paradigm, and a traditional task in which the delayed is described and experienced only at the end (i.e., deferred waiting). We hope to elucidate the psychological mechanisms differentiating discounting behavior in the tasks via modeling. Furthermore, we study the impact of online versus deferred waiting on well-known effects: the magnitude effect (Experiment 1), which refers to higher discounting when amounts are smaller, and the gain-loss asymmetry (Experiment 2), which is greater discounting for gains than losses. In addition, the two experiments show a novel phenomenon: preference reversals while waiting, and differences in inter-temporal choice behavior between online and deferred delay tasks.

We first review the inter-temporal choice literature and models. We then describe prior research that used experienced delays, noting shortcomings that the new experimental paradigm overcomes. We then describe a new discounting paradigm.

Discounted utility model

The most famous time discounting property is that valuation of a future reward can be traced by a curve that declines quickly at the start of a delay and flattens out with time (Ainslie, 1975; Chung, 1965; Green, Fry, & Myerson, 1994; Rachlin, Raineri, & Cross, 1991;

Wolfe, 1934). This property is frequently represented by the basic hyperbolic model: V = R / (1+*k*D), where V is the subjective value of a delayed reward, R is the amount of the delayed reward, D is the length of the delay, and *k* stands for the rate of time discounting.

This basic hyperbolic model is widely accepted for its accountability of preference time inconsistency, and its parameter *k* has been often regarded as a measure of impulsivity, with a higher *k* indicating greater impulsivity. Congruent with this, studies have shown that people who demonstrate impulsive behaviors, such as opiate dependent drug abusers (Cheng et al., 2012; Madden et al., 1997), chronic cigarette smokers (Bickel et al., 1999), and addictive gamblers (Petry & Casarella, 1999) have higher values of *k*. However, *k* cannot be taken as a pure measure of a person's impatience given the multitude of factors that can influence discounting behavior. Indeed, studies frequently use parameter *k* to track changes in discounting as a function of task characteristics (Cheng & González-Vallejo, 2014).

In order to distinguish the roles of temporal discounting and subjective time perception, Rachlin (2006) used a parameter *s* to represent subjective time in $V = R/(1+kD^s)$. However, Vincent and Stewart (2019) demonstrated that this model still suffered from interpretability of its parameters[2]. We therefore conducted our behavioral modelling using the modified Rachlin discount function proposed by Vincent and Stewart (2019) in order to more clearly separate discounting and subjective time perception contributions without losing the basic functional-form of the hyperbolic model. This modified Rachlin discount function is

$$V = R \cdot \frac{1}{(1+(kD)^s)} \tag{1}$$

[2] According to the authors, the units of *k* are mathematically contaminated by *s*. If D is measured in units of days, then the term D^s is measured in units of $days^s$. This mathematically demands that *k* is in units of $1/days^s$. This is highly problematic as the *k* values obtained from participants with different *s* values are measured in different units. This means that it is mathematically invalid to compare the *k* values from the Rachlin discount function across participants (or groups) unless all participants have identical *s* values, which is highly unlikely.

where V is the subjective value of a reward R (in cents in our study) at a delay D (in seconds in the current work). The s parameter is unitless and we assume $s > 0$; the k parameter is also assumed positive, and its units are 1/seconds (as D is in seconds in our study). The modified Rachlin discount function also retains the interpretability of k as the inverse half-life. That is, the present subjective value of a reward halves in 1/k seconds (Yoon & Higgins, 2008).

Because we focus on both subjective time and temporal discounting processes, it is important to exercise care in how we interpret model parameters. Mathematically, the k parameter is meant to capture the overall impact of objective time on value discounting, and affects all delays by the same constant. On the other hand, the exponent parameter s impacts delays differentially. That is, the parameter s controls the extent to which different levels of time are perceived as larger as or smaller than the objective units. At $s = 1$, then $(k\,\mathrm{D})^s = k\,\mathrm{D}$ equating to linear subjective time perception; for $0 < s < 1$ then $(k\,\mathrm{D})^s$ is concave with marginally decreasing subjective time as D increases. For $s > 1$ then $(k\,\mathrm{D})^s$ is convex with subjective time positively accelerating as D increases. We note that the determinants of k are usually assumed to be driven by self-control mechanisms. However, Cheng and González-Vallejo (2014) noted the ambiguity of interpreting k in data from several studies (using the traditional Rachlin's and other related models). We defer discussion of these different interpretations to the General Discussion, but advance that our experiments give support that k in Equation 1 is a measure of self-control as validated by additional behavioral measures.

Inter-temporal choice tasks with experienced waiting in the literature

Intertemporal choice tasks that involved waiting were common in non-human animal studies with subjects waiting in order to receive the selected reward (Ainslie, 1975; Chung, 1965; Wolfe, 1934). In these studies, the waiting lasted from several seconds to at most several minutes; and the rewards were usually real food. In the 1980s and 1990s, some researchers began to study inter-temporal choice in humans using tasks similar to those used

with non-human animals. These studies adopted similar time frames, used points instead of food, and included an inter-trial interval (ITI) (Flora & Pavlik, 1992; Hyten et al., 1994). For example, the study conducted by Flora and Pavlik (1992) asked human participants to choose between an immediate small set of points (which were eventually exchanged for money) and a larger set that required waiting for 15-60 seconds. In addition, a post-reinforcement delay was included after each trial to make sure that the interval between two trials would be 75 seconds regardless of which option the participant chose. In other words, participants had to wait the same period of time regardless of whether they chose an immediate option or not. Unsurprisingly, these studies found that participants exclusively selected the larger option.

More recently, new experience tasks without ITIs were developed (Johnson, 2012; Reynolds & Schiffbauer, 2004). For example, the experiential discounting task (EDT) (Reynolds & Schiffbauer 2004) included five blocks with each block designed to measure an indifferent point (an equivalent amount for a delayed reward) for a delay that ranged from 5-60 seconds. The number of trials within each block could vary from one another depending on participants' responses. As in Hyten et al.'s (Hyten et al., 1994) study, participants were required to actually wait when they selected the standard, delayed option. However, instead of including an ITI, the EDT task included an inter-block-interval to make sure that the total time spent in each block did not depend on a participant's responses. Hence, whether or not participants chose the immediate option, the total time they waited in each block (instead of trial) was constant. To make sure participants would not choose the larger, delayed option exclusively (thus fixing the issue in the Flora and Pavlik (1992) study), the larger, delayed option was made such that there was only a 35% chance that the larger reward would be delivered after waiting. The effect of probability was removed by a data normalization process in which the indifference point for each delay was divided by the indifference point of an additional block which involves probability only (i.e. the delay of the standard option is 0).

The quick discounting operant task (QDOT) also imposed a waiting period between blocks to prevent participants from ending quickly by choosing the immediate reward in all trials (Johnson, 2012). However, the QDOT task fixed the number of trials for each block using 4 trials to obtain an indifference point for each delay.

Both the EDT and QDOT tasks have been used to detect variations in time discounting due to physiological state changes. Using EDT, studies found that participants had a higher discounting rate when they were in a sleep-deprived state, or when they had ingested high levels of alcohol compared to when they were in a normal state (Reynolds & Schiffbauer, 2004; Reynolds, Ortengren, Richards, & de Wit, 2006). Johnson (2012) also reported that both the QDOT and EDT showed that cocaine-dependent individuals exhibited greater time discounting than matched controls. However, both tasks have been challenged methodologically. For example, the QDOT task includes only 4 trials (compared to more than 10 trials in most studies) to find an indifference point for each delay. In addition, since it includes an inter-block interval, the majority of normal participants tend to exclusively select the large option. On the other hand, the EDT is criticized for its inclusion of a probability dimension which confounds the effects of time and probability in valuations. In addition, Smits et al. (Smits et al., 2013) criticized that a lack of ITIs in the EDT task enables a subject to select and obtain several smaller-sooner rewards during the time of waiting for a single large-delayed reward. Hence, selecting the smaller-sooner options does not necessarily reflect myopic behavior if doing so maximizes the total reward.

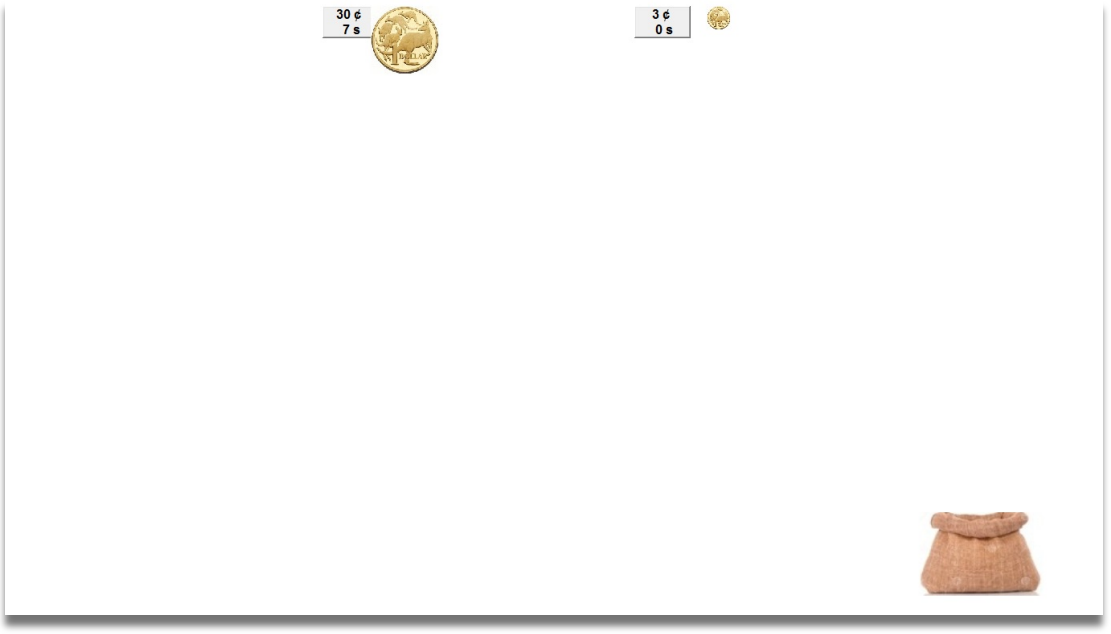
In sum, one major concern about experience time discounting tasks is the inclusion of ITIs. Including ITIs means that the overall waiting time for selecting an immediate option is the same as that of selecting a delayed-large reward, and thus there is no benefit to choosing a small reward, which motivates participants to exclusively choose the larger option. On the other hand, if ITIs are not included as in the EDT, participants could maximize the overall payoff by selecting many small-sooner options during a single waiting period of a large

reward. One way to overcome this dilemma is to exclude ITIs but fix the number of available trials. This means that choosing the smaller-sooner reward would lead to less waiting time, but at the same time lead to less reward overall. We describe a new task that excludes ITIs and makes waiting realistic without leading to ceiling or floor effects of always selecting the larger or the smaller payoffs.

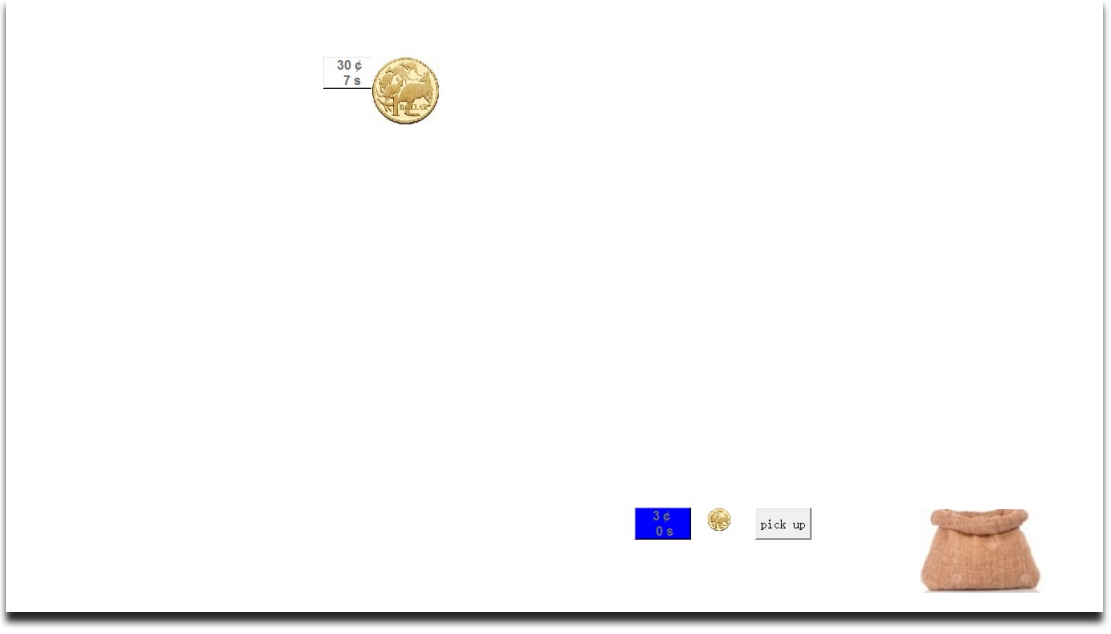
A new experience intertemporal choice paradigm: The online and deferred waiting tasks

We describe two variations of our new intertemporal choice paradigm: the *online waiting task* has waiting occurring after making each choice, and the *deferred waiting task* has all the accumulated delays experienced at the end of the experiment. Both tasks offer a fixed number of choices and require actual waiting for periods of time as described by the selected options, hence participants need to make trade-offs between reward and waiting.

Figure 1a shows the choices at the start of a trial in the online task. Each choice option is represented by a coin with its size matching the amount described in the labeled button with a labeled delay next to each coin. The smaller option is immediate, so the delay is labeled as 0 seconds (s). In addition, there is a bag positioned in the south-east corner of the screen.



(a)



(b)

Figure 1. Online waiting intertemporal choice experiment. Rewards and delays are described in labels at the start of a trial (a). After a choice is made by clicking on one of the buttons the immediate reward appears at the bottom of the screen whilst the delayed reward falls slowly to the bottom of the screen (b). Either reward is available for pickup when it is at the bottom of the screen, allowing participants the possibility of a preference reversal during the waiting process.

Participants can take as much time as they like to select one of the two options by pressing one of the two buttons. Once an option is selected, they cannot select the option

again, but instead they can collect the coin that is ready for pick up. That can be the small coin, which is immediately available, or the large coin, which can be picked up if the participant waits for it. That is, even if a participant selects an immediate option, she/he can still take the large coin if she/he decides to wait for it. In such cases, once the large coin arrives at the bottom, a second pick-up button appears near the large coin. However, a participant could only take a single coin in each trial. A new trial began after collection. The larger coin always falls with a constant speed matching its length of delay (see Figure 1b). Note, that the coin trajectory to the bag was shown to participants, the accumulated amount was not displayed.

In sum, in each trial, participants first selected an option and then picked up a coin. While waiting for the coin to fall, participants could reverse their choices. Thus, the current paradigm is analogous to the marshmallow task in which participants could first decide to wait, but then reverse their choice by eating the reward at hand, and such change could be used as an index of delay of gratification (Mischel et al., 1989; Mischel & Ebbesen, 1970; Shoda et al., 1990). But the opposite was also true, a participant could first select the immediate coin but then decide to wait for the larger coin instead.

The online task has four dependent variables: the option participants selected (the first choice), the coin participants picked up (the second choice), the number of trials in which participants reversed between first selection and picked coin (preference reversals), and the time participants took to switch their preference from the delayed to the immediate option. In the present study, we focus on the coin a participant picked up as his/her final decision. We also count preference reversals for each participant.

The deferred waiting task is identical to the traditional inter-temporal choice task except that participants are to experience waiting, equal to the total accumulated delay duration (based on their selections), at the end of the study. To keep the two tasks as similar as possible, each trial in the deferred task is displayed in a separate screen with a bag in the

south-east corner. After a participant selects a coin, the next trial is displayed in a new screen immediately without waiting. The bag is not active in the deferred waiting task. Participants are instructed that they would experience the choices they made for each trial at the end of the experiment.

Given the nature of the online waiting task, the time was restricted to delays of several seconds to 2 minutes. Compared to longer delays of days to years, tasks with short delays are unlikely to produce discounting based on factors such as risk perceptions, inflation, and expectation of changing utility. Thus, this new inter-temporal choice task has the potential to clarify the role of more basic cognitive processing such as time perception and self-control in discounting behavior.

Goals of the study

The present study aims to better understand the psychological mechanisms of time discounting, and subjective time perception when rewards and delays are small, and when delays are experienced online versus when they are deferred. As earlier stated, Experiment 1 explored differences in inter-temporal choice behavior across the online vs. deferred waiting tasks, and across low vs. high reward magnitudes. Experiment 2 aimed to replicate the findings of Experiment 1 and also assessed differences in discounting behavior between gains and losses. Self-reported impulsivity was measured in both experiments as well as preference reversals in the online waiting task. These two variables were further used to validate the interpretation of the parameter k in Equation 1.

Experiment 1

Experiment 1 investigated behavioral inter-temporal choice differences regarding task and magnitude, as well as how these differences (if any) could be traced to specific parameters in Equation 1.

To investigate which parameter(s) governs the differences associated with task and/or magnitude, collected data were fit to Equation 1 using a Bayesian hierarchical approach described below. Estimated parameters at the individual level were then compared across the four conditions. The aim was to elucidate whether delay of gratification, subjective time perception (as represented by parameters *k,* and *s* respectively) vary across tasks and magnitudes.

We also present descriptive statistics for preference reversals. A correlation analysis was further conducted to explore relations among preference reversal, self-reported impulsivity, and parameters derived from the behavioral task. The aim is thus one of validity analysis, such that convergence of measures is expected if the same construct is tapped by the different measures (Campbell & Fiske, 1959).

Method

Participants

Participants were 190 undergraduate students at a Midwest university in the USA. They were recruited through a psychology pool, for which they received academic credit for participation. Participants' ages ranged from 18-22 years. All experiments were approved by the Institution Review Board of the university and follow ethical guidelines.

Measures

The Baratt Impulsiveness Scale (Patton & Stanford, 1995)(BIS-II) was used to measure self-reported impulsivity. The scale can be found in Appendix A. The BIS-II is a 30-item scale with items such as: "I plan tasks carefully" and "I do things without thinking." The response format is a 4-point rating scale with respondents indicating at what level they agree with each item. The total score of the BIS-II ranges from 30 to 120 with higher scores indicating higher impulsiveness. The overall score can be further classified into 6 factors:

attention, motor, self-control, cognitive complexity, perseverance, and cognitive instability. Cronbach's alpha for the scale in this study was .83.

Stimuli and design

The study is a 2 (task: online vs. deferred) × 2 (magnitude: small vs. larger amounts) between-subjects design. Both the deferred and online waiting tasks were completed on a computer, and they used the same stimuli as described below.

The task consisted of a series of choices, employing a staircase procedure, in order to derive indifference points for a series of delays. The design consisted of five delays, and the choices were between a large, delayed amount that was constant, and a smaller immediate payoff that varied with fixed size increments.

The delays were 7, 15, 29, 56, and 101 (in seconds) with 10 trials for each delay. To reduce the possibility of comparison between blocks, the delays are nearly but not exactly two times each other. Table 1 displays the stimuli for 10 trials in one block for the small amount condition for which the larger, delayed amounts were fixed to 30 cents (30¢); while the smaller amounts were adjusted from 3 to 30 cents. The smaller, adjusted amounts were used for the other 4 blocks (i.e., for delays of 15, 29, 56, 101). For the large amount conditions, the fixed amount was 60 cents, while the adjusted amounts for the immediate option were two times larger than those in the small conditions. That is: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60 cents for each trial.

Table 1

Stimuli for one block in small and large magnitude conditions (Experiment 1).

Small Magnitude condition			Large magnitude condition		
Fixed Amount [a]	Delay [b]	Adjust. Amount	Fixed Amount [a]	Delay [b]	Adjust. Amount
30	7	3	60	7	6
30	7	6	60	7	12

30	7	9	60	7	18
30	7	12	60	7	24
30	7	15	60	7	30
30	7	18	60	7	36
30	7	21	60	7	42
30	7	24	60	7	48
30	7	27	60	7	54
30	7	30	60	7	60

Note. [a] amount units were cents; [b] time units were seconds

The blocks were presented using a Latin Square counterbalance technique such that any single delay follows any other delays twice. The design is shown in Table 2 in which the set of delays (7, 15, 29, 56, and 101) occurs in different positions. For example, order type D in Table 2 shows the sequence (4 5 3 1 2) describing that the smallest delay, 7 seconds, appeared in the fourth position, the second delay of 15 seconds, appeared in the fifth position, the delay of 29 seconds appeared in the third position, and delays of 56 and 101 seconds appeared in the first and second positions, respectively.

Table 2

Balanced Latin Square design for 5 delay blocks.

Order type	Sequences	Order Type	Sequences
A	1 2 4 5 3	F	5 4 2 1 3
B	3 1 2 4 5	G	3 5 4 2 1
C	5 3 1 2 4	H	1 3 5 4 2
D	4 5 3 1 2	I	2 1 3 5 4
E	2 4 5 3 1	L	4 2 1 3 5

Within each block, half of the participants had an increasing adjusting amount from trial to trial, while the other half had a decreasing adjusting amount from trial to trial. For the same participants, the same adjusting order was used across various delays. That is, if the adjusting amount was increasing for the first block, it was increasing for the other blocks. The ten delay orders as defined by the Latin Square designs were randomly assigned with the constraint that each of the ten orders was used at least four times.

Procedures

Instructions. Before the start of the experiment, participants were asked to leave their belongings and watches, and were assigned to an individual room. Participants were then randomly assigned to one of four groups: deferred small, deferred large, online small and online large. After they completed the BIS-II questionnaire participants were shown screens from the corresponding task and directions on how to complete the task.

For the online waiting task, directions clearly stated that in each trial participants could either pick up the smaller coin or wait to pick up the larger coin. For the delayed deferred task, participants were told: “After you complete all 50 trials, you will actually experience all the choices you’ve made.”, and given an example (exact instruction can be found in Appendix B). Directions for each group then emphasized that participants would receive real money for 10% of randomly selected trials at the end of the study.

The task instructions for all the four groups were kept as similar as possible. For each group, participants were told that people often face situations in which they must decide between two options according to each option’s characteristics. They were also provided a real-life decision example: waiting for a long time at a sit-down restaurant to eat a more desirable meal vs. waiting for a short time at a fast-food restaurant to eat a less desirable meal. They were then told the following task involves playing a coin-machine game in which one machine gives out larger-valued coins slowly, while the other machine gives out smaller-valued coins quickly.

Participants made two choices in each trial: choosing a coin and picking up a coin. There was no time limit for each choice. Responses for both choices and response time for the first choice were recorded. In addition, the time elapsed between participants making the first choice and picking up a coin was recorded. For the deferred waiting tasks, a participant's response for each trial and its corresponding response time were recorded.

In the online waiting task, the final screen of the choice task showed participants the amount of money they had collected (based on the 10% of trials randomly selected by the program). They were paid the collected amount and debriefed. In the deferred task, the final screen of the choice task showed the participants the amount of money they had collected (based on the 10% of trials randomly selected by the program) and the accumulated time they needed to wait (based on all of trials). Participants were paid and debriefed after waiting.

Bayesian parameter estimation and model comparison

We conducted a hierarchical Bayesian analysis for model fitting and comparisons in both experiments. The methods used were based upon those established by Vincent (2016). Briefly stated, the hierarchical Bayesian estimation approach conducts parameter estimation based upon a generative model of the choice data which explicitly models the data at the group, participant, and trial levels. This hierarchical (also known as multi-level modelling) approach has a desirable and principled shrinkage effect (Lee, 2011) where individual level parameters are shrunk towards the group mean – thus balancing what the stochastic choice data tells us about a participant's parameters and what we know about participants being drawn from a population. Simply put, this method produces more reliable parameter estimates when dealing with noisy behavioral responses. A further advantage of the hierarchical Bayesian estimation is that it produces posterior distributions of parameters and this allows for judging the level of uncertainty in the estimates from credible intervals. In the current analysis we used a Python-based implementation of Vincent (2016)'s work, PyMC3

(Salvatier et al., 2016) to conduct Markov Chain Monte Carlo (MCMC) sampling of the posterior distribution of parameters given the data.

Model comparison. To verify the necessity of including the time perception parameter s in the model, we conducted a model comparison between the two-parameter modified Rachlin model and the traditional one-parameter Hyperbolic models. We found that the modified Rachlin model outperformed the Hyperbolic models even controlling for model complexity. Therefore, we have good justification for proceeding with the modified Rachlin model. Details of this analysis can be found in Appendix C.

Parameter transformations. In order to increase the efficiency of MCMC sampling, we reparametrized the parameters k, and s (also known as 'change of variables'). Namely the MCMC sampling process operates on ln(k) and ln(s), where ln refers to the natural logarithm. These ln scaled variables are transformed back into their original variables for operation within the value function.

Hierarchical modelling approach. We split the data into 4 cells representing the 2×2 design: high vs. low reward magnitude, and deferred vs. online waiting task in Experiment 1; and of gains vs. loss, deferred vs. online waiting task in Experiment 2. We treated participants within each of the 4 cells as being a random sample from a group level distribution of parameters ψ (Lee, 2011). Therefore, hierarchical analysis was carried out on all data within a given cell – analysis of data across different cells was separate and independent from one another. Participant level parameters are denoted θ_p, for participant p, where $\theta_p = \{\ln(k_p), \ln(s_p)\}$ are treated as being samples from a group level distribution specified by the hyperparameters ψ (see below).

The Probabilistic Model. We constructed a probabilistic model to account for all choice data (not estimated indifference points) for all participants. An excellent introduction to these methods in the context of cognitive modelling is provided by Lee & Wagenmakers (2013), and this work builds upon the hierarchical estimation methods applied to delay

discounting data by Vincent (2016). The probabilistic model (based on Equation 1) is given in the set of equations below. Note that the normal distributions are parameterized by mean and standard deviation, and the subscripts represent truncation bounds on the normal distributions. The equations below are defined for each person *p* responding in trial *t*.

$$\mu_{lnk} \sim \text{Normal}\left(\ln\left(\frac{1}{30}\right), 2\right) \tag{2}$$

$$\sigma^2_{lnk} \sim \text{Exponential}(10) \tag{3}$$

$$\mu_{lns} \sim \text{Normal}(0,0.5) \tag{4}$$

$$\sigma^2_{lns} \sim \text{Exponential}(20) \tag{5}$$

$$\ln(k[p]) \sim \text{Normal}(\mu_{lnk}, \sigma^2_{lnk}) \tag{6}$$

$$\ln(s[p]) \sim \text{Normal}(\mu_{lns}, \sigma^2_{lns}) \tag{7}$$

$$\alpha[p] \sim \text{Normal}_+(1.7,3) \tag{8}$$

$$V_A[p,t] = R_A[p,t] \cdot \frac{1}{1+(k_p \cdot D_A[p,t])^{s_p}} \tag{9}$$

$$V_B[p,t] = R_B[p,t] \cdot \frac{1}{1+(k_p \cdot D_B[p,t])^{s_p}} \tag{10}$$

$$x[p,t] = V_B[p,t] - V_A[p,t] \tag{11}$$

$$P[p,t] = \epsilon + (1\text{-}2\epsilon) \cdot \frac{1}{1+\exp(-\alpha[p] \cdot x[p,t])} \tag{12}$$

$$C[p,t] \sim \text{Bernoulli}(P[p,t]) \tag{13}$$

By way of example, Equation 2 means a Normal distribution, with a mean of ln(1/30), equating to a mean half-life of 30 seconds; and a standard deviation of 2.

Explaining the model from bottom to top, we start with the likelihood function (Equation 13). Each choice response $C[p,t]$ for participant *p* on trial *t* is modelled as the outcome of a Bernoulli trial (a coin toss) with probability of choosing the later reward (choice B) equal to $P[p,t]$. Equation 13 defines a decision variable *x*, which is simply the difference between present subjective values of the delayed and immediate choices – *x*=0 corresponds to indifference between the two choices. This decision variable is mapped onto a response

probability using a simple logistic choice function (Equation 12) which incorporates a response error rate of 1% ($\epsilon = 0.01$). The use of the logistic function was arbitrary – a number of other similar psychometric functions, such as the cumulative Gaussian could have been used. The slope of the logistic function is determined by the α parameter for each participant – this essentially determines the degree of response stochasticity around the point of indifference where *x*=0. Present subjective values are given by Equations 9 and 10 based on Equation 1. These are the core equations which translate choice A (its reward value R_A and delay D_A), and same for choice B, into present subjective values. This function is influenced by the participant-level parameters (Equations 6-8). These participant-level parameters are in turn assumed to be sampled from group-level priors (or a set of hyperparameters; Equations 2-7). The hyperparameters are $\Psi = \{\mu_{lnk}, \sigma^2_{lnk}, \mu_{lns}, \sigma^2_{lns}\}$ which generate participant level parameters $\theta[p] = \{\ln(k[p]), \ln(s[p])\}$. The information for each participant for each trial contains the following elements $\{R_A, D_A, R_B, D_B, C\}$, these determine the present subjective values of choice A (immediate) and B (delayed) and thus the probability $P[p,t]$ that a participant selects the delayed larger amount option in a given trial and therefore the likelihood of the observed response $C[p,t]$.

In sum, the goal of our cognitive modelling was to arrive at a posterior distribution of parameter values, given the observed data, $P(\theta|data)$. This approach further allows for comparisons of conditions in terms of estimated parameters.

Approach with specifying prior beliefs. Specification of prior beliefs can impact the parameter estimation procedure and study conclusions, and therefore need careful thought and justification (see Appendix D). Our prior beliefs are specified at the hyperparameter level (Equations 2-5) which determine our participant level priors. Our hyperpriors were chosen so that priors over participant level ln(*k*) and ln(*s*) spanned a broad but reasonable range, and can be seen in Figure D1(a, b). Our priors were broad which allows the data to have a strong role in the final posterior distribution of parameters. This resulted in a very wide range of

discounting behaviors being plausible *a priori* (see Figure D1 c). We proceeded by making the reasonable assumption that parameters were independent, but the posterior distribution would capture any dependencies on the participant level.

Approximating the posterior distribution with MCMC. We implemented the above model (Equation 1) by adapting the procedure outlined by Vincent (2016) for our present purposes. In order to check the samples were reasonable approximations to $P(\theta|data)$, we ran 4 MCMC chains, checked these visually, as well as confirmed that the $\hat{R}$ statistic was below the 1.01 threshold (Gelman & Rubin, 1992). A burn-in period of 2,000 samples was used, with a total of 5,000 retained samples over the 4 chains.

The raw output of the parameter estimation procedure was a set of MCMC samples over all latent variables at the participant, and the group levels. These were also used as the basis for a very large number of visualizations in order to help confirm meaningful estimation of the posterior and to explore model 'fits' to data. We also conducted posterior prediction in order to evaluate model goodness of fit. We confirmed the model was a good account of the data by examining a number of metrics including Log Loss and proportion of responses correctly accounted for. These are reported below.

The parameter estimation results, for each of the cells in the design, were exported to a `.csv` file where the posterior distributions were condensed into a point estimate, namely the posterior mean. It was this data which was used as the basis for later statistical analyses comparing parameters across the experimental conditions in both experiments.

Parameter recovery. When estimating model parameters based upon experimental data, it is important to have some degree of faith that our inferred parameters are accurate and meaningful. While it is impossible to prove that this is true, the traditional method (parameter recovery) is to estimate parameters based on data for which we have the true data generating parameters. If we can successfully recover known parameters based upon simulated data generated using those parameters, then we can have some faith that estimated parameters

based upon real data are accurate. The details of this parameter recovery procedure and results are found in Appendix E, demonstrating support for our modelling approach.

Results

Model-free analysis

We included all participants in this model-free analysis. Among them, there were 5 participants in the online small group who always chose the smaller coin, and 1 participant in the online large group who always chose the larger coin. For each participant, we counted the number of trials choosing the larger coin for each delay. Descriptive statistics appear in Table 3.

Table 3

Group mean frequencies of choosing the larger coin for each delay (Experiment 1)

Delays	Deferred small (n = 50)	Deferred large (n = 50)	Online small (n = 46)	Online large (n = 44)
7s	7.920(2.029)	7.720(1.852)	5.935(3.235)	6.273(3.083)
15s	6.760(2.503)	7.140(1.980)	5.261(3.574)	5.477(3.188)
29s	6.080(2.538)	6.220(2.735)	4.413(3.550)	4.545(3.136)
56s	5.380(2.754)	5.260(2.805)	3.478(3.443)	2.818(3.208)
101s	4.840(2.691)	4.560(2.757)	2.761(3.308)	2.023(3.267)

Note. M(SD). Total 10 trials for each delay.

A repeated measure ANOVA with delay as a within-subjects factor and magnitude (small/large) and task(deferred/online) as between-subjects factors revealed that there was a significant main effect of delay, F (4, 744) = 119.673, $p < .001$, $\eta^2 = .386$, and a significant main effect of task, F (1,186) = 28.190, $p < .001$, $\eta^2 = .132$. Participants were more likely to choose the larger coin at smaller delays, and this was also the case in the deferred task than in

the online task. There was no main effect of magnitude, F (1,186) = .049, p = .824. No interaction was detected.

Parameter estimation results[3]

Before making inferences about changes in latent cognitive mechanisms through parameter fits to participant data, it is sensible to check whether the model can in fact account for behavioral data better than chance. To confirm that Equation 1 was in fact a good description of participant behaviors, we calculated the percent of correctly predicted responses for each participant (see Figure 2a for a distribution of values). The model did a very good job of accounting for participant responses, with a median of 90% of responses correctly accounted for. We also evaluated the ability of the model to fit the data using the Logarithmic Loss[4] metric (see Figure 2b). This is the appropriate metric for modelling inter-temporal choice tasks at the individual trial level, i.e. a classification task in which our model produces a probability that the participant will choose the immediate, or the delayed reward on each trial. A model with perfect prediction would have a Log Loss of zero, whereas a non-informative model predicting choices at 50% would have a Log Loss of 1. Again, we see good model performance, with a median Log Loss of 0.27 for Experiment 1.

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[3] We note that a side-effect of the Bayesian hierarchical approach is that participants who respond with near 100% consistency, either for the immediate or the delayed option, would have an apparently acceptable posterior distribution. Even though the Bayesian approach provides a reasonable posterior given our prior beliefs, we decided to exclude participants who responded with >90% consistency to either the immediate or delayed option. This resulted in 12 exclusions in the deferred waiting task (7 in the low reward condition, 5 in the high reward condition) and 20 in the online waiting task (12 in the low reward condition, 8 in the high reward condition).

[4] The Log Loss (also known as binary cross entropy) is given by $-\frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T} C[p,t]\ln(P[p,t]) + (1 - C[p,t])\ln(1 - P[p,t])$, where N is the number of participants and T is the number of trials, $C[p,t]$ is the response by participant p on trial t, and $P[p,t]$ is the model's predicted response probability for participant p on trial t.

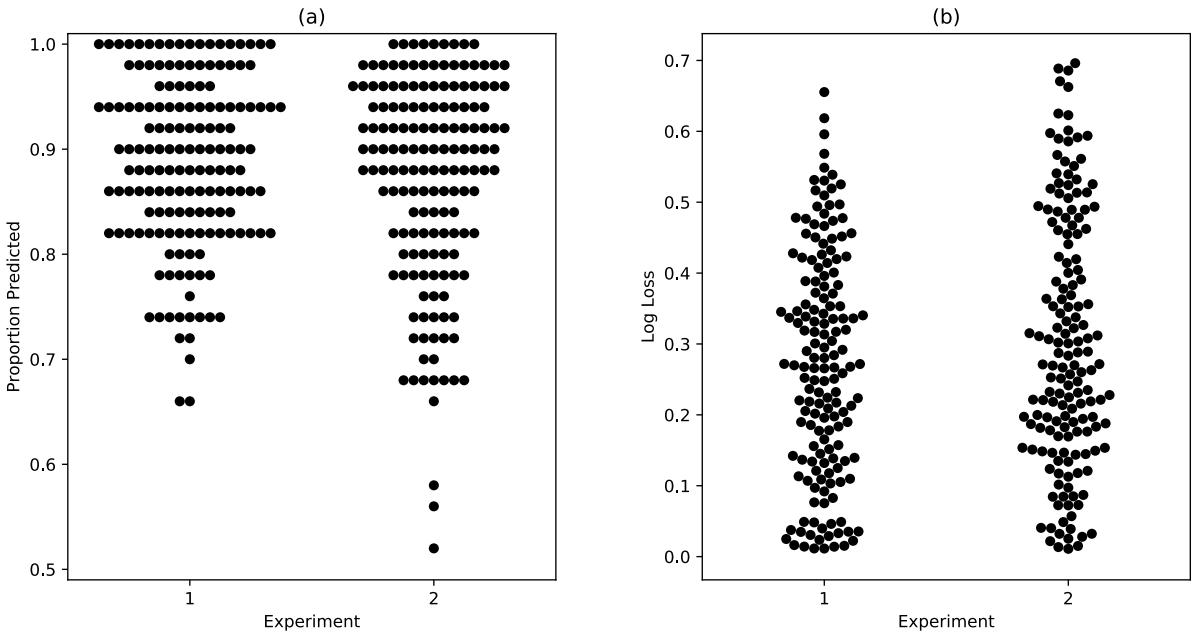

Figure 2. Goodness of fit of the model (Equation 2) to responses made in the inter-temporal choice tasks. Each point represents a participant. Left panel (a) shows that the median percent of responses predicted was 90% for Experiment 1 and 90% for Experiment 2. Right panel (b) shows that the median Log Loss was 0.27 for Experiment 1 and 0.27 for Experiment 2. It is possible to have models that have poor Log Loss scores but still predict responses well - if a small number of responses are unpredicted by the model. So, the few participants with poor Log Loss scores are likely due to a few errant responses.

We also confirmed satisfactory parameter estimation by plotting posterior predicted indifference functions alongside behavioral data. Figure 3 shows examples from 3 randomly chosen participants (columns) from each of the 4 experiment conditions (rows). We see that the posterior predicted indifference functions do a good job of separating the immediate (white points) and delayed reward choices (black points).

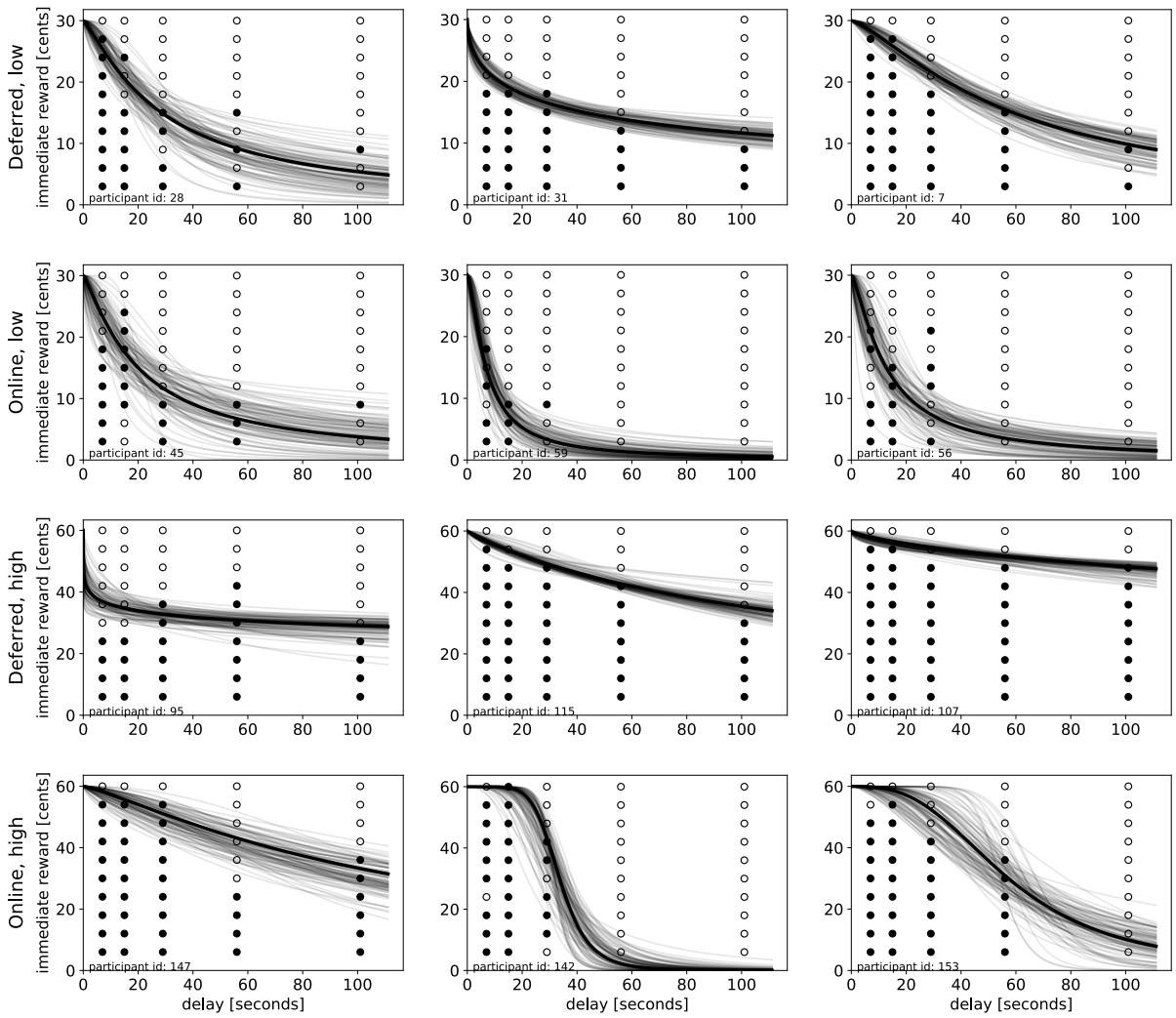

Figure 3. Example data and model fits for Experiment 1. Each panel shows data (points) and fits for an individual participant. Thick solid lines represent the discount function corresponding to the mean of the posterior parameter values, thin lines are random samples from the posterior and so the degree of precision in the parameter estimates. Each row corresponds to a condition, and each panel represents a participant randomly chosen from that condition. Empty points represent immediate choices, filled points represent delayed choices.

Descriptive statistics of estimated parameters appear in Table 4. The parameter estimates were back-transformed mean ln(k) and ln(s) to k and s.

Table 4

Group means of estimated parameters (Experiment 1). Individual participant parameter estimates were take

distribution over parameters.

	N	*k*	ln*(k)*	*s*	ln*(s)*	*α*
Deferred small	43	.041(.076)	-4.083(1.289)	.912(.335)	-.159(0.371)	1.298(1.346)
Deferred large	44	.025(.027)	-4.231(1.138)	1.191(.892)	-.055(0.679)	1.194(1.351)
Online small	34	.071(.085)	-3.445(1.463)	1.728 (.810)	.449(0.446)	.982(1.194)
Online large	36	.065(.084)	-3.490(1.385)	2.278(1.949)	.577(0.672)	1.028(1.198)

Note. M (SD).

Figure 4 shows combined parameter ln(k) and s across 4 conditions. We explored any differences in estimated parameters (posterior means for each participant) in a series of Bayesian 2×2 ANOVA's using JASP (JASP Team, 2018) with reward magnitude (low/high) and task (deferred/online) as fixed factors. We report Bayes Factors (BF_{10}), which is the ratio of the likelihood of the data fitting under the alternative model (H_1) and the likelihood of the data fitting under a null model (H_0). This latter null hypothesis model amounts to a single intercept only model with no main effects or interactions. According to (Jeffreys, 1961), a value of 1 means the probability of the data having been generated by the model and the null model are equal, values above 3 represent substantial evidence, above 10 represent strong evidence, above ~30 is very strong evidence, and above 100 is considered decisive evidence. For interested readers, we also provided a test on the Area Under Curve metric, a combined measure of discounting and subjective time, in Appendix F.

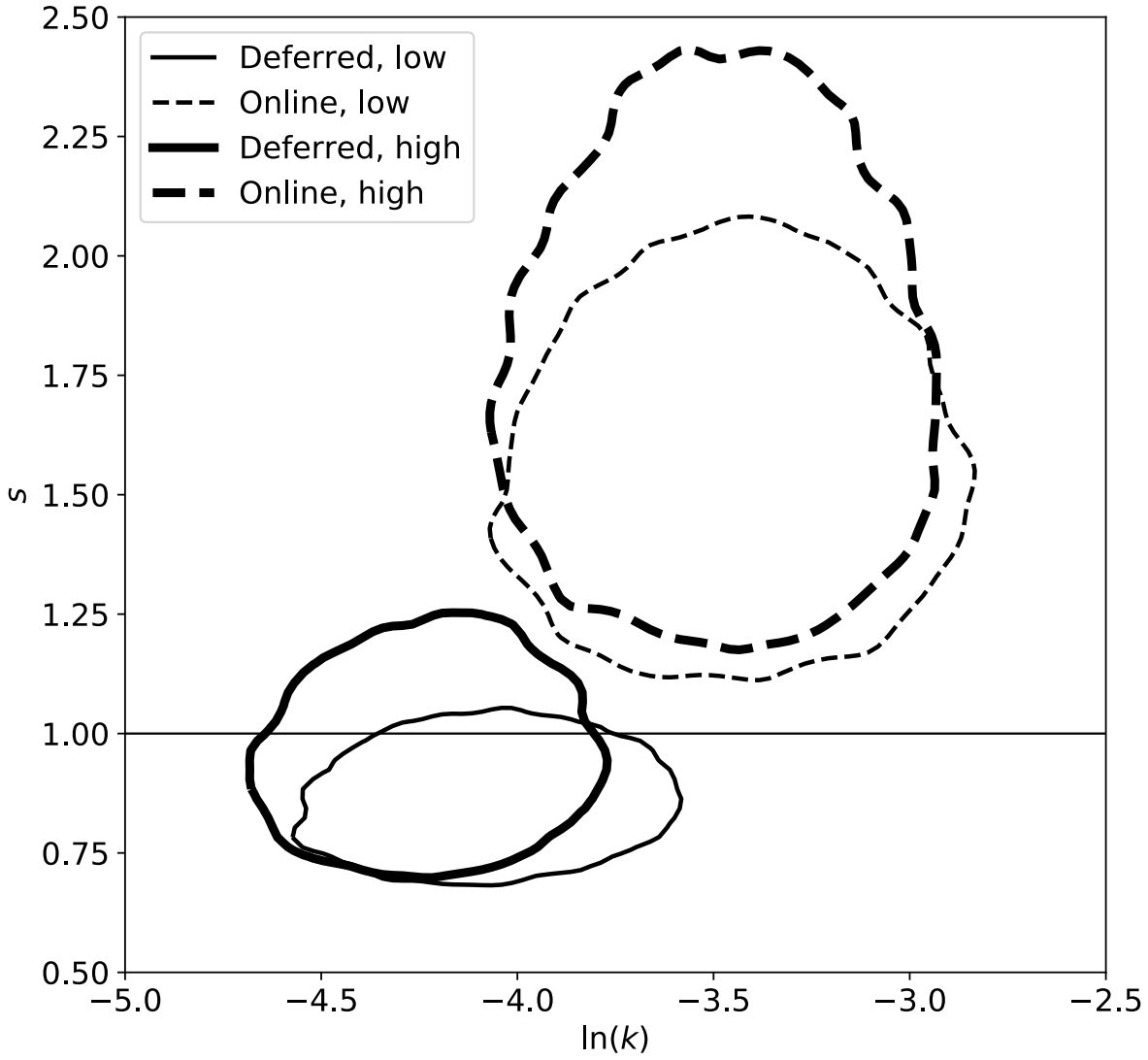

Figure 4. The 95% credible intervals of our posterior beliefs, shown as a contour plot, about group level ln(k) and s parameters for each of the 4 conditions in Experiment 1. The regions define the 95% credible interval for parameter combinations. The horizontal line at s=1 corresponds to linear subjective time perception.

Changes in *k*. We find very strong evidence for a main effect of task on ln(k), with a BF_{10} of 22.64, compared to an intercept only model. Values of ln(k) were larger in the online than in the deferred waiting task (see Table 4). In contrast, we found moderate evidence against a main effect of reward magnitude (BF_{10} = 0.189, i.e. BF_{01} = 5.293), therefore we have evidence of a lack of a magnitude effect in terms of the k parameter.

Changes in *s*. We find decisive evidence for a main effect of task on ln(*s*), with a BF_{10} of 6.99E+7, compared to an intercept-only model. Parameter *s* was higher in the online than in the deferred waiting task (see Table 4). We conducted Bayesian one sample *t*-tests to see if ln(*s*) was equal to zero (i.e. s = 1, linear subjective time perception) for each task. We found no compelling evidence for non-linear time perception in the deferred task (BF_{10} = 0.56). We found decisive evidence for non-linear time perception in the online waiting task (BF_{10} = 6.59E+7). In combination, the linearity of subjective time perception is different across the online and deferred waiting tasks.

Slope parameter *a*. We found no evidence of any effect of task and magnitude upon parameter α.

Preference reversals

In the online waiting task, there were two types of preference reversals possible: choosing the larger coin in the first choice, but picking up the small coin in the second choice (LS), and choosing the small coin in the first choice, but picking up the large coin in the second choice (SL). The former may be regarded as an indication of impulsivity, or lack of self-control, similar to the conceptualization in Mischel & colleagues' (1970) study where participants could begin by not taking the one marshmallow (exerting self-control by waiting) but then, giving into temptation, eating the one marshmallow in the hand before the larger option was available. The mean number of preference reversal trials across participants are shown in Table 5. On average, among a total of 50 trials, there were around 3.4 trials in which participants switched from a large coin to a small coin (LS) and .6 trials in which participants switched from a small coin to a large coin (SL).

Table 5

Descriptive statistics for preference reversals (Experiment 1).

	Online Small	Online Large

LS	3.640 (4.133)	3.120 (6.765)
SL	.821 (1.819)	.333 (.612)

Note. M (SD). LS: switch from the later choice to the sooner choice; SL: switch from the sooner choice to the later choice.

To study how these preference reversals were distributed across delays, frequencies, and rates of these two types of switches were calculated. As shown in Table 6, the rate of LS increased with the length of delay, whereas SL rate decreased with the length of delay. This pattern was the same for both small and large magnitude conditions. In addition, compared to the large condition, the proportion of LS was larger in the small condition, meaning that as the amount of reward increased, participants were less likely to succumb to the immediate option.

Table 6

Frequency and rate of preference reversals in all conditions of Experiments 1 and 2 by delay size

	Experiment 1				Experiment 2			
	Online small		Online large		Online gain		Online loss	
Delay	LS	SL	LS	SL	LS	SL	LS	SL
7s	41(.136)	12(.076)	21(.072)	7(.047)	32(.116)	10(.055)	21(.057)	13(.101)
15s	45(.162)	9(.049)	29(.108)	2(.012)	23(.108)	6(.024)	18(.054)	10(.059)
29s	52(.208)	5(.024)	23(.105)	3(.014)	30(.140)	7(.029)	22(.077)	7(.033)
56s	62(.283)	3(.012)	27(.180)	1(.003)	40(.253)	4(.013)	37(.159)	4(.015)
101s	54(.303)	3(.011)	31(.261)	1(.003)	42(.362)	3(.008)	51(.252)	3(.010)

Self-reported impulsivity and its relation to other measures

Correlational analysis among the following transformed variables was conducted: self-report impulsivity as measured by the BIS-II questionnaire; estimated parameters; and rate of LS as a direct measure of people's lack of self-control[5]. This latter measure was only available in the online waiting task. Pearson's correlation was computed within each task pooling participants from the small and large magnitude conditions. The correlation coefficients appear in Table 7.

[5] We also calculated the scores of six sub-scales of BIS-II and their correlations with other measures. We found no meaningful correlation with regard to the present topic. Interested readers can find the two expanded correlation tables (both for Experiment 1 and Experiment 2) in the OSF data center of the present work.

Table 7

Pearson Correlations for Deferred and Online Tasks in Experiment 1.

Deferred task				
	ln(k)	ln(s)	α	BIS
ln(s)	.061			
α	-.336*	.233		
BIS	.009	-.167	-.112	
Online task				
ln(s)	.070			
α	-.353*	.112		
BIS	.343*	.121	-.090	
LS	.626*	.007	-.269	.189

Note. * are correlations of which $BF_{10} > 3$. BIS = the overall score of self-reported impulsivity. LS = rate of switch from a large coin to a small coin.

As shown in Table 7, there was no correlation among ln(k), and ln(s) for both online and deferred waiting tasks, indicating the parameters can be interpreted as indexing discounting vs time perception mechanisms respectively. There was substantial evidence showing that self-reported impulsivity, as measured by the BIS, was positively correlated with parameter k in the online tasks, but not in the deferred tasks. However, BIS did not correlate with preference reversals.

We observed a strong correlation (r = 0.626) between ln(k) and LS, the rate of reversals from a large coin to a small coin (BF_{10} = 2.026E+6), meaning preference reversals were more common in participants who had greater discounting as measured by k. In combination, these results show convergence of measures for parameter k, and preference reversals, suggesting that parameter k represents trial by trial discounting tendencies that BIS, in contrast, is not able to capture.

Discussion of Experiment 1

One of the core results we found was that discounting behavior, as shown in the mean number of selections of delayed amounts, was lower in the online waiting task compared to the deferred waiting task. That is, participants preferred smaller but more immediate rewards more so in the online than the deferred waiting tasks. A second core result was that subjective time perception was best characterized by a convex function in the online task. That is, a unit of physical time was perceived longer as the waiting proceeded. These are compelling initial findings in our new inter-temporal choice paradigm.

We also found that preference reversals, as defined in the online waiting condition, were strongly related to the k parameter. In addition, parameters k and s showed little relationship. Thus, the current results provide evidence for k as a measure of impulsivity, separate from parameter s--a measure of time perception.

We found no magnitude effect (i.e., the effect that a larger reward is associated with a smaller discounting rate in traditional discounting tasks). That is, participants' intertemporal preferences were similar in the large and small magnitude conditions. This is presumably attributable to the choice of reward magnitudes – despite the high magnitude rewards being double the low magnitude rewards, they were both in the same order of magnitude (small and in cents).

Experiment 2

The gain-loss asymmetry in utility shows that people value losses more than gains (Kahneman & Tversky, 1979; Loewenstein & Prelec, 1992). This asymmetry is expressed as the sign effect in time discounting, referring to the observation that gains are discounted more than losses (Frederick et al., 2002; Hardisty & Weber, 2009; Loewenstein, 1988; Thaler, 1981). In the study conducted by Lowenstein (1988), participants were indifferent between receiving $10 immediately and $21 in one year; but in losses, they were indifferent between paying $10 immediately and $15 in one year (indicating that they preferred to pay $10 immediately rather than to pay $21 in one year), which is indicative of a smaller discounting rate.

Employing the same tasks and same analytical approach as that of Experiment 1, Experiment 2 investigate the gain-loss asymmetry regarding differences between the deferred and online waiting tasks. Because a loss has a greater value than a gain, we expected participants to be more willing to wait in the loss condition than in the gain condition if waiting offered them a reduced loss. Therefore, Experiment 2 was set such that waiting would lead to a higher revenue in both conditions (i.e., a larger gain or a reduced loss). That is, choices in the loss condition were between an immediate but larger loss versus a delayed but

smaller loss.[6] The experiment had task (deferred vs. online) and domain (gain vs. loss) as conditions in a between-subjects design. Similar to Experiment 1, Equation 2 was used to conduct parameter estimation to reveal mechanisms underlying response differences across conditions. Preference reversals in the online waiting task, and correlation between self-reported impulsivity and variables derived from the behavioral tasks were also analyzed.

Method

Participants

Participants were 190 undergraduate students at a Midwest university. They were recruited through a psychology pool, for which they received academic credit for participation. Participants' ages ranged from 18-22 years.

Measures

As in Experiment 1, the Barratt Impulsiveness Scale (BIS-II) was used to measure self-reported impulsivity. Cronbach's alpha for this study was .823.

Stimuli and design

The experiment was a 2×2 with task (deferred vs. online) and type of outcome (gains vs. loss) as between-subjects design. The loss conditions used stimuli identical to that of the gain conditions, except the values were negative.

For the gain conditions, the same stimuli from Experiment 1 was used. That is, the amount of the larger option was fixed to 30 cents with delays in seconds varying from 7 to 101 seconds (7, 15, 29, 56, and 101). The immediate amount was adjusted from 3 cents to 30 cents across 10 intervals. For the loss conditions, the immediate amount was fixed to -30 cents while the delayed amount was adjusted from -30 cents to -3 cents across 10 intervals.

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[6] In traditional intertemporal choice studies using losses, the design offers an immediate smaller loss versus a delayed larger one.

Note that in the loss condition, the delayed and immediate amounts were reversed compared to the gain condition. Table 8 displays the stimuli for 10 trials in one block for the gain and loss conditions.

As in Experiment 1, the blocks were counterbalanced using the Latin Square counterbalance order as shown in Table 2. Within each block, half of the participants had an increasing adjusted amount from trial to trial, while the other half had a decreasing adjusted amount from trial to trial. The order of adjusted amount (either increasing or decreasing) was fixed across blocks.

Table 8

Stimuli for one block in gain and loss conditions (Experiment 2).

Gain condition			Loss condition		
Fixed Amount [a]	Delay [b]	Adjust. Amount	Fixed Amount [a]	Delay [b]	Adjust. Amount
30	7	3	-30	7	-30
30	7	6	-30	7	-27
30	7	9	-30	7	-24
30	7	12	-30	7	-21
30	7	15	-30	7	-18
30	7	18	-30	7	-15
30	7	21	-30	7	-12
30	7	24	-30	7	-9
30	7	27	-30	7	-6
30	7	30	-30	7	-3

Note. [a] amount units were cents; [b] time units were seconds

Procedures

Incentives. In all tasks, participants were paid for 10 % of trials randomly selected at the end of the experiment. For the loss condition, participants were first given a savings account with an amount equal to $2.35 to make sure participants would receive exactly the same maximum amount of money as in the gain condition if they had decided to always choose the waiting option.

Instructions. Participants were randomly assigned to one of the four study conditions. For both the deferred and online waiting tasks, participants were told they would complete a questionnaire and a choice task. In the choice task, they would make a total of 50 choices during the study, and the incentive structure of the task was described.

In addition, participants were asked to read instructions after they completed the questionnaire. The directions for all the four groups were kept as similar as possible. The general instructions for the choice task stated that people often face situations in which they must decide between two choices according to each choice's characteristics. Both gain and loss examples were presented. They were all told that the following task was designed to study these types of decisions and the task involved playing a coin-machine game in which one machine gives out larger-valued coins slowly (or makes you pay them quickly); the other machine gives out smaller-valued coins quickly (or makes you pay them slowly).

Participants were then shown screens from the corresponding task and given directions on how to complete the task. For the online waiting task, the instructions clearly stated that in each trial participants could either pick up the smaller coin (large coin for loss condition) or wait to pick up the larger coin (small coin for loss condition). For the deferred waiting tasks, after showing the screens, participants were told: "After you complete all 50 trials, you will actually experience all the choices you've made".

Participants in each group were then reminded of the study incentives. That is, they would receive (or lose) real money for 10 % of randomly selected trials depending on gain /loss condition. As in Experiment 1, participants in the deferred waiting task were further

asked to wait the amount of time defined by their selections in all trials at the end of the study in order to receive the payments.

Results

Model-free analysis

We included all participants in this model-free analysis. Among them, there were 5 participants in the online gain group and 3 in the online loss group who always chose the smaller coin, and 1 participant in the online loss group who always chose the larger coin. For each participant, we counted the number of trials choosing the larger coin for each delay. Descriptive statistics appear in Table 9.

Table 9

Group mean frequencies of choosing the larger coin for each delay (Experiment 2)

Delays	Deferred gain (n = 48)	Deferred loss (n = 46)	Online gain (n = 46)	Online loss (n = 50)
7s	7.458(1.675)	7.870(1.809)	5.543(3.291)	7.260(2.717)
15s	6.729(1.876)	7.478(1.941)	4.261(3.666)	6.460(3.045)
29s	5.396(2.656)	6.783(2.564)	4.174(3.536)	4.400(3.188)
56s	4.750(2.855)	6.152(2.538)	2.652(3.078)	4.000(3.417)
101s	4.208(2.931)	5.609(2.840)	1.674(2.600)	3.080(3.212)

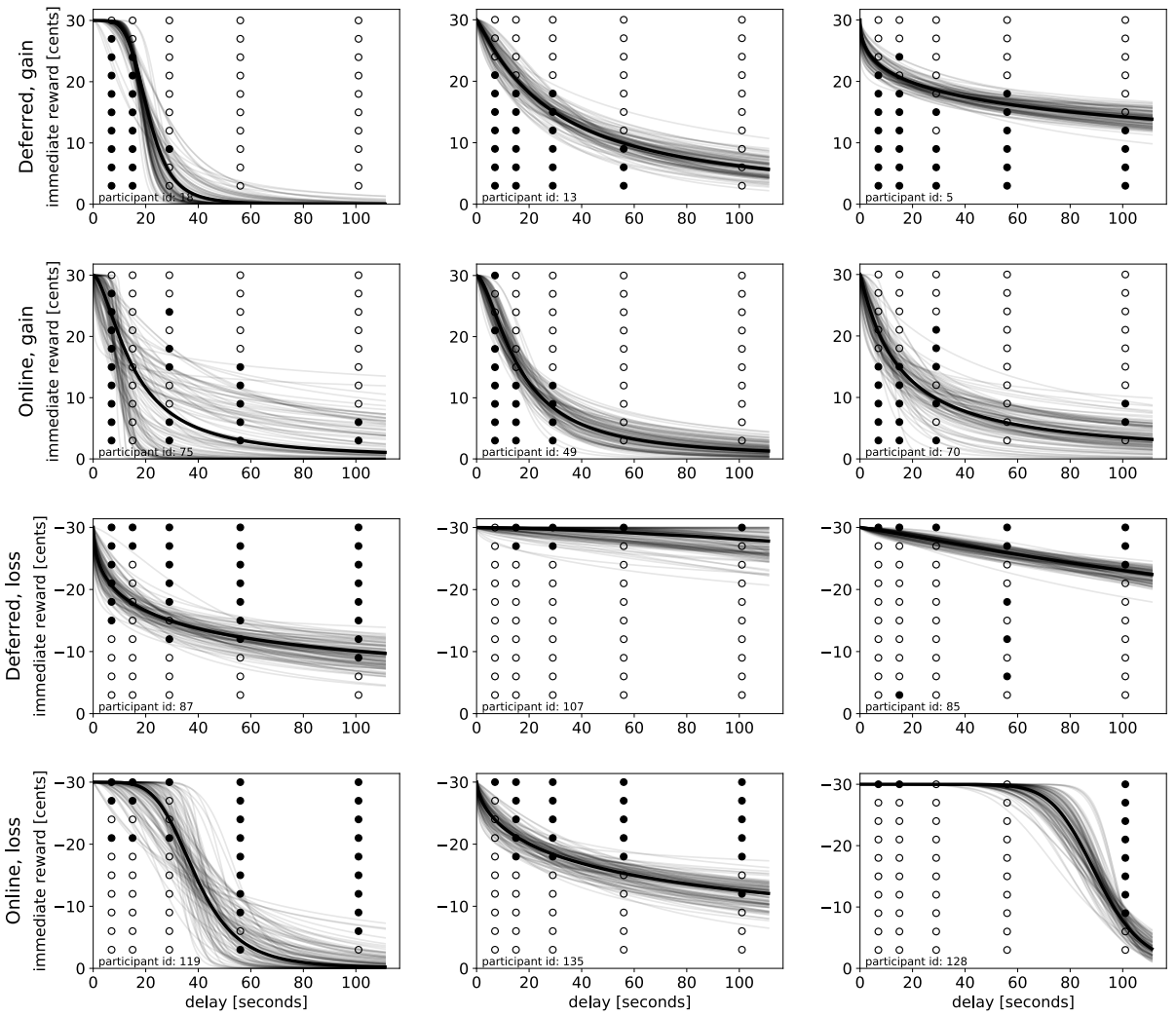
Note. M(SD).

An ANOVA with delay as a within-subjects factor and domain (gain/loss) and task(deferred/online) as two between-subjects factors revealed that there was a significant main effect of delay, F (4, 744) = 102.306, $p < .001$, $\eta^2 = .347$, a significant main effect of domain, F (1,186) = 15.844, $p < .001$, $\eta^2 = .068$, and a significant main effect of task, F (1,186) = 29.029, $p < .001$, $\eta^2 = .125$. Participants were more likely to choose the larger coin for the shorter delays (compared to the longer delays), in the deferred task (compared to the

online task) and in the loss domain (compared to the gain domain). There was no interaction between domain and task, F (1,186) = .585, p = .445

Parameter estimation results[7]

We found again that the model (Equation 1) was able to accurately predict participant responses well, with a median of 90% responses predicted and a low Log Loss score of 0.34 (see Figures 2a and 2b). This is also reflected in posterior predictive checks (see Figure 5) which compares our posterior beliefs about indifference curves to the observed behavioral data for a randomly selected group of participants.



[7] The same participant exclusion criteria from Experiment 1 was used. This resulted in 11 exclusions in the deferred waiting task (9 in the loss condition, 2 in the gain condition), and 19 in the online waiting task (6 in the loss condition, 13 in the gain condition).

Figure 5. Example data and model fits for Experiment 2. Explanation as in Figure 3.

Descriptive statistics of estimated parameters appears in Table 10. As in Experiment 1, mean ln(k) and ln(s) were back-transformed to k and s for reading simplicity. Mean parameter values for k are smaller in the deferred than in the online waiting condition, and this is also true for the s parameter.

Table 10

Group means of estimated parameters (Experiment 2). Individual participant parameter estimates were take

distribution over parameters.

	N	*k*	Ln(*k*)	*s*	Ln(*s*)	α
Deferred gain	46	.038(.057)	-4.321(1.595)	1.036 (.905)	-.145(.568)	1.481(1.194)
Deferred loss	37	.011 (.016)	-5.004(.974)	1.310(1.312)	-.009(.697)	.961(1.234)
Online gain	33	.066 (.052)	-3.081(.944)	2.232 (1.766)	.596(.616)	.930(1.207)
Online loss	44	.032 (.041)	-3.976(1.017)	3.099 (2.837)	.781(.847)	1.298(1.406)

Note. M (SD).

Estimated values of ln(k) and s are shown in Figure 6. Bayesian 2×2 ANOVAs elucidated the differences among groups and findings are discussed below. Again, the result of a test on the overall discounting as measured by AUC can be found in Appendix F.

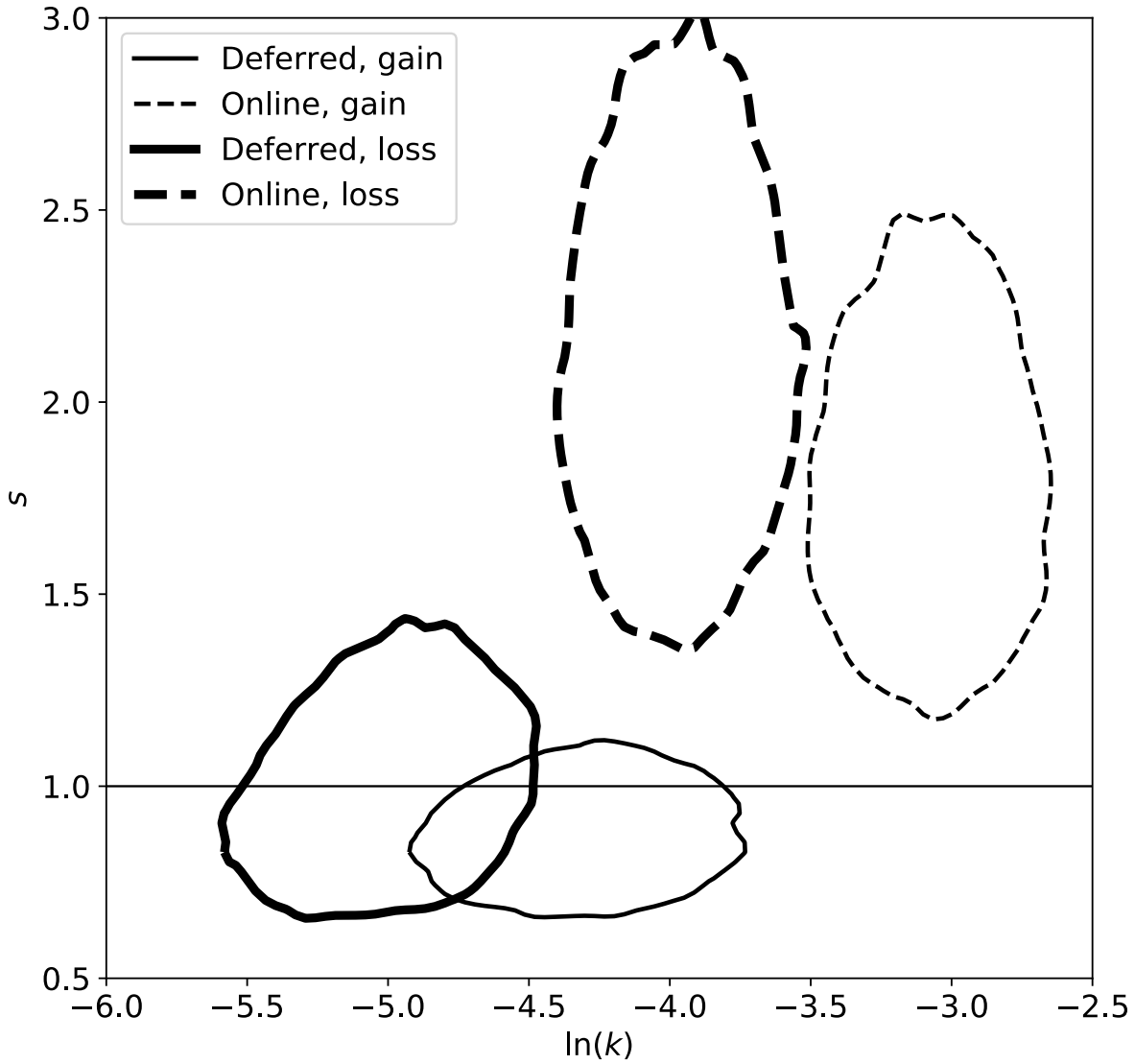


Figure 6. The 95% credible intervals of our posterior beliefs about group level ln(k) and s parameters for each of the 4 conditions in Experiment 2. The horizontal line at s=1 corresponds to linear subjective time perception.

Changes in *k*. We find decisive evidence (BF_{10} = 7.87E+6, compared to a control intercept-only model) for main effects of task and domain. ln(k) was larger in the online waiting than in the deferred task, and higher in the gain than in the loss condition. That is,

preference for smaller but immediate rewards was stronger for gains and stronger when waiting occurs online.

Changes in *s*. We find decisive evidence for a main effect of task (BF_{10} = 2.65E+8, compared to a control intercept-only model). Again, we find that time is subjectively perceived as near linear in the deferred waiting condition and non-linear in the online waiting condition. A one sample (2-tailed) *t* test revealed decisive evidence that ln(*s*) values (collapsed over gain and loss domains) was greater than 0 in the online waiting task (BF_{10} = 1.36E+9). There was strong evidence against non-linear time perception in the deferred waiting task (BF_{10} = 0.25).

Slope parameter *α*. As in Experiment 1, we found no evidence of any effect of task and domain upon parameter α.

Preference reversals

The mean number of trials in which participants switched from a large option to a small option (LS) and switched from a small option to a large option (SL) are displayed in Table 11. On average, out of 50 trials, participants switched from a large coin to a small coin in 2.6 trials, and in 1.5 trials from a small to a large coin.

Table 11

Descriptive statistics for preference reversals in gain and loss conditions (Experiment 2)

	Online Gain	Online Loss
LS	2.880 (3.777)	2.420 (2.607)
SL	1.500 (3.442)	1.467 (2.873)

Note. M (SD)

The number of preference reversal cases for each delay was also counted. The frequency and rates of LS and SL trials are shown in Table 6. Congruent with Experiment 1, the proportion of trials that participants switched from a large coin to a small coin increased with

the length of delay, while the proportion of trials participants switched from a small coin to a large coin decreased with the length of delay. This pattern was the same for both gain and loss conditions. In addition, the pattern indicated the tendency that participants were less likely to succumb to the immediate option in the loss compared to gain condition.

Self-reported impulsivity and its relation to other measures

Pearson's correlation analysis was conducted among impulsivity, estimated parameters, and rate of LS. Statistics are shown in Table 12.

Table 12

Pearson Correlations Among Measures in Experiment 2[8].

Deferred task				
	ln(k)	ln(s)	α	BIS
ln(s)	.257			
α	-.008	.231		
BIS	.173	-.081	-.119	
Online task				
ln(s)	.028			
α	-.128	.343*		
BIS	.114	.006	.106	
LS	.515*	.098	-.044	.057

Note. * are correlations of which $BF_{10} > 3$. BIS = the overall score of self-reported impulsivity. LS = rate of switch from a large coin to a small coin.

[8] As in Experiment 1, we detected no meaningful correlation between sub-scales of BIS-II with other measures. An expanded table including sub-scales of BIS-II can be found in the OSF data center of the present study.

There was weak evidence showing that estimated parameter ln(*k*) is positively correlated with ln(*s*) in the deferred task ($BF_{10} = 2.044$), but not in the online task ($BF_{10} = 0.146$). Impulsivity as measured by the BIS showed no correlation with *k* in both tasks. Rate of LS preference reversals were positively correlated with ln(*k*) just as in Experiment 1, with BF_{10} of 1.098E+4 (decisive evidence).

Discussion of Experiment 2

Experiment 2 replicated the findings of Experiment 1 where they overlap in design. Namely, we see a greater preference for smaller immediate rewards in the online waiting task and this was again associated with higher ln(*k*) values. Subjective time perception for gains in the deferred waiting task is near linear, but this becomes convex in the online waiting task. This consensus between experiments is edifying both in terms of the reliability of the Bayesian parameter estimation procedures and the consistency of the results across different groups of participants.

Experiment 2 also expands our understanding of how people behave in deferred versus online waiting tasks in inter-temporal choice. Framing rewards as reduced losses increased preference of waiting, as predicted by the gain-loss asymmetry. Losses are associated with lower ln(*k*) values, as compared to the gain domain.

Similar to Experiment 1, the analysis on preference reversals showed a significant agreement between the rates of preference reversals and parameter *k*. Table 6 shows that as the length of the delay increased, the likelihood of participants succumbing to the immediate option also increased. In addition, participants were less likely to show this type of preference reversals in the loss compared to the gain condition. In terms of BIS, self-reported impulsivity was not correlated with impulsivity measures derived from the behavioral task.

General Discussion

According to a 1980 statement by the company FedEx, “Waiting is frustrating, demoralizing, agonizing, aggravating, annoying, time consuming and incredibly expensive." (Fortune, 28 July 1980, p. 10, as cited by Maister, 1984). The experience of the passage of time can be taxing as described by Maister, and thus companies in the service sectors spend significant efforts in managing customers’ satisfaction as a function of waiting. A well-known case is Disney’s creation of the FASTPASS which allows customers to essentially wait by doing something else. Customers can purchase a FASTPASS and return to an attraction at a later designated time without further wait. The system has been successful at maintaining customers’ satisfaction and also managing waits to rides (Cope et al., 2011). A separate literature has demonstrated that individuals as well as non-human animals tend to prefer sooner, smaller rewards to delayed, larger ones. This inter-temporal choice literature thus converges on the notion that waiting is essentially undesirable.

However, few experimental studies have tested actual waiting between a selection and the receipt of a payoff. And few have also examined the extent to which a hyperbolic type of mathematical description is applicable to describing the psychological processes underlying making trade-offs with very small amounts of payoffs (in cents) and time units (in seconds). Given the relevance of these units in our current experiences with technology, for example the time it takes for a browser to display a web-page, our experiments are innovative and advance knowledge about impulsivity and time perception in discounting behavior in the “small”.

Two experiments advanced interesting results pertaining to the experience of waiting in the context of making trade-offs between time and money. Results across the experiments indicated that the online experience of time paradigm resulted in stronger preference of immediate options than those based on deferred waiting. In addition, we analyzed our inter-temporal choice data using a model (Equation 1), which allowed us to explore effects of

discount rates and subjective time perception. We discuss findings in more detail in what follows.

Waiting over short durations

Given that delay discounting studies typically examine (hypothetical) delays of days to years or decades, one might initially think that discounting processes over the timescale of seconds may be negligible. However, our everyday experiences suggest that time versus reward trade-offs are common with small time units, and thus, such trade-offs may be important to understand. Idleness and waiting even for very short durations can be highly aversive (Hsee et al., 2010) – people pay for faster internet connections or the latest mobile phone in order to save mere seconds of waiting time. Indeed, the classic marshmallow studies involved online waiting on the timescales of minutes (Mischel et al., 1989).

For the timescales of seconds to minutes, we found substantial changes in inter-temporal choice behavior between our online experienced waiting task and the deferred waiting task. This demonstrates the degree to which even small amounts of time can affect behavior when the waiting is experienced. Calculating from Tables 4 and 10 (half-life = $1/k$ based on Equation 1, meaning the amount of time it takes for a reward to half in present subjective value) shows group mean half-lives (for any reward of size A) varied between ~10 seconds to ~1 minute. The online waiting conditions demonstrated the greatest reductions of reward A (half-lives between 10 and 30 seconds). We note that these results are similar to those from the experiential discounting tasks described in the introduction (Reynolds & Schiffbauer, 2004). Using delays up to 60 seconds and rewards of cents to a few dollars, they found sleep deprived individuals had half-lives of around 140 seconds compared to non-sleep deprived controls of around 170 seconds.

Task differences

There is a growing number of studies proposing that subjective time perception contributes to the hyperbolic pattern of time discounting (Han & Takahashi, 2012; Takahashi et al., 2008; Zauberman et al., 2009), and time discounting effects such as the sign effect and magnitude effect (Bilgin & Leboeuf, 2010; Wang et al., 2015). These studies showed that decreasing time discounting is related to the effect that distant future time intervals are perceived as shorter than proximate time intervals of the same objective durations (Han & Takahashi, 2012; Takahashi et al., 2008; Zauberman et al., 2009). The sign effect and magnitude effect might be due to the fact that time is perceived as shorter when its outcomes are losses (vs. gains) and of larger magnitude (Bilgin & Leboeuf, 2010; Wang et al., 2015). While all these studies showed how perception of future time could be changed, and hence influence time discounting, the present study demonstrated that "experiencing" time differs from "imagining" future time, in subjective time perception related to discounting behavior. Consistent with the previous finding that time perception is easily influenced by factors such as attention focus, visual cues (Ebert & Prelec, 2007; Zauberman et al., 2009), the present study found that "experiencing" time made individuals perceive time as more convex compared to described time. This change from linear to convex subjective time function mapped to the heavier discounting in the online waiting task in two separate experiments.

The result that *k* was larger in the online waiting than the deferred waiting task means that overall waiting, regardless of its length of duration, resulted in greater discounting in the online waiting than in the deferred task. However, we are careful to point out that the psychological meaning of *k* can be ambiguous (as it modifies delays suggesting a time perception role); thus, the interpretation of *k* needs external validation with other behavioral markers. In our experiments, we were able to observe reversals in preference that overtly showed switches from the desire to wait and the inability to do so. Although the extent to which the reversals occurred was low, only parameter *k* related to this type of impulsivity in both experiments. Therefore, the larger value of parameter *k* might imply that the online

waiting is more aversive than deferred waiting, as indicated by a shorter half-life (smaller $1/k$) in the former than the latter. However, preference reversals can result from other factors; thus, we discuss the role of k in greater detail in what follows.

Parameter *k*, preference reversals, and self-reported impulsivity

The psychological meaning attributed to models' parameters depends on the nomological network that they are embedded in (Cronbach & Meehl, 1955). The process of construct validation of measures entails a variety of methods, including but not limited to model fit. With regard to the parameter k, past research with hyperbolic models (e.g., Myerson & Green, 1995; Rachlin et al., 1991; Rachlin, 2006) have used k as an index of impulsivity or lack of self-control because of the models' accountability of preference reversal (Ainslie, 1975). However, empirical evidence of the relation between parameter k and preference reversals is indirect, and mainly come from the finding that k is larger for people who are considered impulsive (e.g., drug abusers, chronic cigarette smokers) than for controls (Bickel, Odum, & Madden, 1999; Cheng, Lu, Han, González-Vallejo, & Sui, 2012; Madden, Petry, Badger, & Bickel, 1997). The present work was able to address the meaning of parameter k estimated from the modified Rachlin model by relating it to observed preference reversal, and self-report impulsivity.

Notably, the observed preference reversal as measured by intra-trial switches and the possible preference reversal as implied by parameter k in the online task correspond to two different constructs in the literature. The former maps the delay-of-gratification (DG) paradigms (Walter Mischel et al., 1972) in which a decision maker encounters choices between a later-larger (LL) reward and a sooner-smaller (SS) reward that is always immediately available. It captures more about an individual's ability in resisting immediate temptation. The second corresponds to the time-inconsistency construct as reflected by a hyperbolic discounting curve, which describes the effect that the subjective value of a future reward is acceleratingly increasing as it draws near. The hyperbolic discounting implies the

possibility of preference reversal in which a decision maker initially prefers an LL reward over an SS reward, but reverse their choice because the subjective value of the SS increased more quickly than that of the LL. More directly, time-inconsistency preference is evidenced by preference reversals observed from studies adopting either cross-sectional designs or longitudinal designs (Sayman & Öncüler, 2009). As pointed out by McGuire & Kable (2013), an important difference between the two is that in the delayed gratification paradigm, the subjective value of the sooner reward never changes. Therefore, even though both are frequently referred to as a measure of impulsivity or delay of gratification, the two constructs do not completely overlap. Compared to previous experienced time discounting tasks (e.g., EDT task as developed by Reynolds & Schiffbauer, 2004), an advantage of the online task in the present study is being able to measure intra-trial switches, the type of preference reversal mapping with the delayed gratification paradigm, in addition to the modelling approach. Therefore, a positive correlation between parameter *k* and the observed preference reversal suggests the two constructs are measuring something in common.

While both studies revealed that parameter k in Equation 1 was positively correlated with observed preference reversals, we are cautious in interpreting the meaning of this correlation. Previous studies showed that preference reversal may be attributed to causes other than depletion of self-control resources (Baumeister & Heatherton, 1996; Cohen et al., 2016), including affective interference (George Loewenstein, 1996), imperfect information about future events (Gabiax & Laibson 2017), change of time predictions about uncertain delays (McGuire & Kable, 2013), and even choice stochasticity (Rieskamp, 2008; Bhatia & Loomes, 2017). A particularly relevant concern for the present study is whether the observed intra-trial reversal was due to choice noise resulting from computational mistakes, and/or inattention to some elements of the decision (Rieskamp, 2008; Bhatia & Loomes, 2017). While it is difficult to differentiate observed reversals caused by choice errors from those caused by other reasons, using parameter α in the psychometric function to estimate choice

stochasticity, we found no substantial evidence of correlation between parameter α and observed preference reversal in both experiments.

Likewise, other motives underlying preference reversal are difficult to distinguish from each other. Gabiax and Laibson (2017) illustrated that a decision maker can reverse his choice when vague information about future outcomes becomes more accurate as time passes. McGuire and Kable (2013) showed that people might reverse their choice because as waiting proceeds, their prediction of the remaining length of the uncertainty waiting might increase. Both theories interpret preference reversals as a result of a rational cost-benefit tradeoff. As explained by Gabiax and Laibson (2017), such accounts do not predict the choice of commitment devices, in contrast to interpretations based on affective interference, depletion of self-control strength. However, in the present study, both outcomes and waiting time were certain and fully informed to participants, hence observed preference reversal might mainly be due to factors such as affective fluctuations, depletion of self-control, and weak decision strength. Therefore, we interpreted the positive correlation between parameter *k* and preference reversals to mean that an individual with a higher *k* might also be more likely to reverse their choices because they experienced more affective fluctuations during the waiting procedure, or he was less certain about his original choices, or he might possess less self-control resources. Given that impulsivity is a multidimensional construct (Duckworth & Kern, 2011; Evenden, 1999), this correlation between *k* and observed preference reversal serves to validate that parameter *k* captures some aspects of self-control or impulsivity. However, future studies are needed to give parameter *k* a more precise interpretation.

In contrast, the self-reported impulsivity as measured by BIS-II was correlated with *k* only for the online task in Experiment 1. No correlation was detected in the deferred tasks or the tasks in Experiment 2. In addition, self-reported impulsivity and its sub-scales were not correlated with any other variables derived from the behavioral tasks. Given the varieties of impulsivity, this result is not surprising. In agreement with the present result, in a study using

an EDT task, Reynolds et al. (Reynolds, Penfold, & Patak, 2008) found no correlation between BIS and responses in the EDT tasks. More generally, a great number of previous studies found no correlation, or a very small correlation between self-reported impulsivity and behaviorally measured impulsivity (Cyders & Coskunpinar, 2011; Duckworth & Kern, 2011; Malesza & Ostaszewski, 2016). For studies which found correlations, the correlation was typically between some specific subscales and a specific behavioral task. For example, Lange and Eggert (2015) used BIS, GO/No-Go task, and delay discounting choice task (DD) to measure participants' impulsivity. They found that DD did not correlate with any components of BIS, but the Go/No-Go task correlated with attention impulsivity and motor impulsivity.

Even though the number of trials in which participants reversed their choices was too small to be studied at the individual level, the results showed interesting patterns that agree with general findings in the time discounting literature. That is, participants were less likely to reverse their waiting in the larger (compared to the smaller) magnitude condition and in the loss (compared to the gains) domain. Preference reversals occurred more for longer delays – when the length of delay was 101 seconds and the probability of switching to the immediate option ranged from 25%-36% across the four online waiting conditions.

Gain-loss asymmetry and magnitude effect

We found a strong gain-loss asymmetry (Frederick, Loewenstein, & O'Donoghue, 2002; Hardisty & Weber, 2009) – choices of waiting were more frequent in the loss domain. Regarding subjective time perception, previous studies found that subjective time was perceived as shorter in the loss domain than in the gain domain (Han & Takahashi, 2012). In the present study, we found no substantial evidence for this. In addition, the trend is that the subjective time perception function was more convex in the loss condition than in the gain domain. We speculated that this contradiction might be related to the large difference of timescales the two studies used (delays were of weeks, months, and years in Han et al.'s study). Nevertheless, time perception of small periods might differ from that of long intervals.

For short periods of time, studies in time perception literature generally show that negative emotion is associated with slower perceived time (Droit-Volet & Gil, 2009; Langer et al., 1961; Noulhiane et al., 2007). Hence, future studies are needed to test the effect of outcomes (gain versus loss) in time perception at varying time scales. We also found that the *k* parameter decreased in the loss condition, hence capturing the reduction in discounting when compared to the gain domain.

Our unexpected failure to find a magnitude effect, normally a robust phenomenon, could be the result of two main methodological factors. First, while the *relative* difference of the magnitudes was large (double), the *absolute* difference in the magnitudes was small compared to previous studies (e.g., Green, Myerson & McFadden, 1997; Cheng & González-Vallejo, 2014). Second, the lack of a magnitude effect is actually a common observation in non-human studies of inter-temporal choice (Freeman et al., 2009; Green et al., 2004). In such studies the reward magnitudes differ by a factor of about 1.5-2 and involved short (< 16 seconds) delays. Further work is required to more firmly establish the boundary conditions of the magnitude effect.

Future studies

Based on the findings from the present work, we outlined that there are at least three research aspects future work should address: 1) Further replicate the present findings with expansion to other time units for both the online and deferred waiting tasks. Even though the present work showed similar overall pattern of discounting behaviors as traditional tasks involving long delay units (both are well described by hyperbolic models), there might be some qualitative differences in aspects such as time perception. 2) Advance our understanding of psychological meaning of related constructs (e.g., self-control, impulsivity), model parameters (e.g., *k*), and their relations with behavioral expressions (e.g., observed preference reversal, discounting behavior). 3) Validate model fitting results with results from empirical studies. For example, the present work showed time perception differs between

online and deferred waiting tasks. This result however requires validations from experimental measures of subjective time perception.

Conclusion

In summary, we outline the development of a new experimental paradigm to study inter-temporal choice preferences. We overcome some methodological limitations from previous studies, and further explore how the latent factors of delay discounting, and subjective time perception, vary across deferred vs. online waiting delays; high vs. low rewards; and gains vs. losses. We provided initial but compelling evidence that a model, Equation 1, based on adjustments to the hyperbolic forms found in previous works, captured the behavioral data well and provided meaningful parameter values that advanced our understanding of the cognitive processes underlying discounting behavior. There is an emerging view that subjective time perception should be taken more seriously in the study of inter-temporal choice (Kim & Zauberman, 2009; Zauberman et al., 2009), and the present work demonstrates that inter-temporal choice tasks with online waiting are particularly useful in further elucidating the role of subjective time perception and discounting mechanisms.

Appendix A

The Barratt Impulsiveness Scale (BIS-II)

DIRECTIONS: People differ in the ways they act and think in different situations. This is a test to measure some of the ways in which you act and think. Read each statement and put an X on the appropriate circle on the right side of this page. Do not spend too much time on any statement. Answer quickly and honestly.

① Rarely/Never ② Occasionally ③ Often ④ Almost Always/Always

1 I plan tasks carefully.	①	②	③	④
2 I do things without thinking.	①	②	③	④
3 I make-up my mind quickly.	①	②	③	④
4 I am happy-go-lucky.	①	②	③	④
5 I don't "pay attention."	①	②	③	④
6 I have "racing" thoughts.	①	②	③	④
7 I plan trips well ahead of time.	①	②	③	④
8 I am self controlled.	①	②	③	④
9 I concentrate easily.	①	②	③	④
10 I save regularly.	①	②	③	④
11 I "squirm" at plays or lectures.	①	②	③	④
12 I am a careful thinker.	①	②	③	④
13 I plan for job security.	①	②	③	④
14 I say things without thinking.	①	②	③	④
15 I like to think about complex problems.	①	②	③	④
16 I change jobs.	①	②	③	④
17 I act "on impulse."	①	②	③	④
18 I get easily bored when solving thought problems.	①	②	③	④
19 I act on the spur of the moment.	①	②	③	④
20 I am a steady thinker.	①	②	③	④
21 I change residences.	①	②	③	④
22 I buy things on impulse.	①	②	③	④
23 I can only think about one thing at a time.	①	②	③	④
24 I change hobbies.	①	②	③	④
25 I spend or charge more than I earn.	①	②	③	④
26 I often have extraneous thoughts when thinking.	①	②	③	④
27 I am more interested in the present than the future.	①	②	③	④
28 I am restless at the theater or lectures.	①	②	③	④
29 I like puzzles.	①	②	③	④
30 I am future oriented.	①	②	③	④

Patton, Stanford, Barratt (1995). *J Clin Psy*, vol. 51, pp. 768-774

Appendix B

Exact instruction of waiting procedure for the deferred task

- After you complete all 50 trials, you will actually **experience** all the choices you've made.
- For example, if you've chosen Machine 1 for bellow trial:

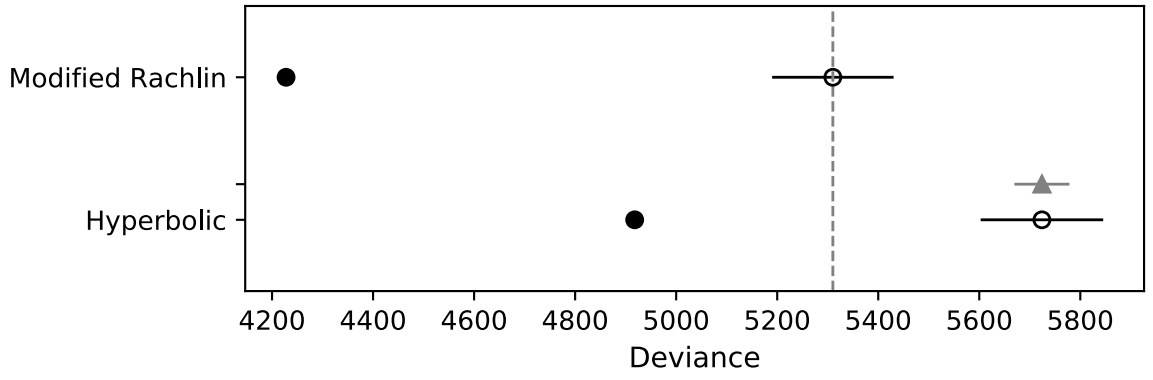
Machine 1	Machine 2:
30 ¢ 101s	18 ¢ 0 s

then you will need to wait 101 seconds to get 30 ¢. If you've chosen Machine 2, then you do not need to wait but you will only get 18¢.

Appendix C

In order to check that use of the two-parameter modified Rachlin discount functions was warranted, above and beyond the traditional Hyperbolic discount functions, we conducted model comparison, for both experiments. We found that the modified Rachlin was the best model for both experiments, even when accounting for model complexity using the Widely-applicable Information Criterion (WIAC; Watanabe, 2013) metric (see Figure C1). There are many methods of model comparison, with no agreed upon 'best' method – we chose to use the WAIC because it is 'fully Bayesian' and built into the PyMC3 software we used to conduct our Bayesian analyses with (Salvatier et al., 2016). We therefore have good justification for proceeding with the modified Rachlin discount function.

(a)



(b)

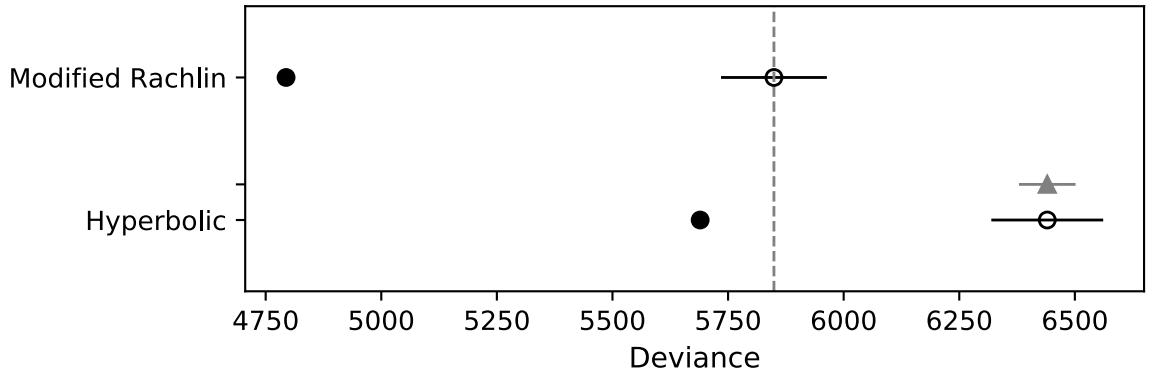


Figure C1. Results of the model comparison between the modified Rachlin and the Hyperbolic discount functions for Experiment 1 (a) and Experiment 2 (b). For both experiments, we see the modified Rachlin model has the best (lowest) WAIC values (see vertical dashed lines). The WAIC values for the hyperbolic models were worse (see triangles). Empty circles represent the WAIC, along with standard deviations. The dashed vertical line marks the model with the lowest (best) WAIC score. Filled circles represent the in-sample deviance and can be seen as a measure of goodness of fit (lower values are better) not taking model complexity into account. Grey triangles show the difference between the two models and the associated standard deviation, which is most important for determining if a model is meaningfully worse than the best model.

Appendix D

We chose priors which were informative but also relatively broad, such that the posterior over parameters is largely influenced by the data available. Figure D1 visualizes the priors at the participant level, before any data has been observed.

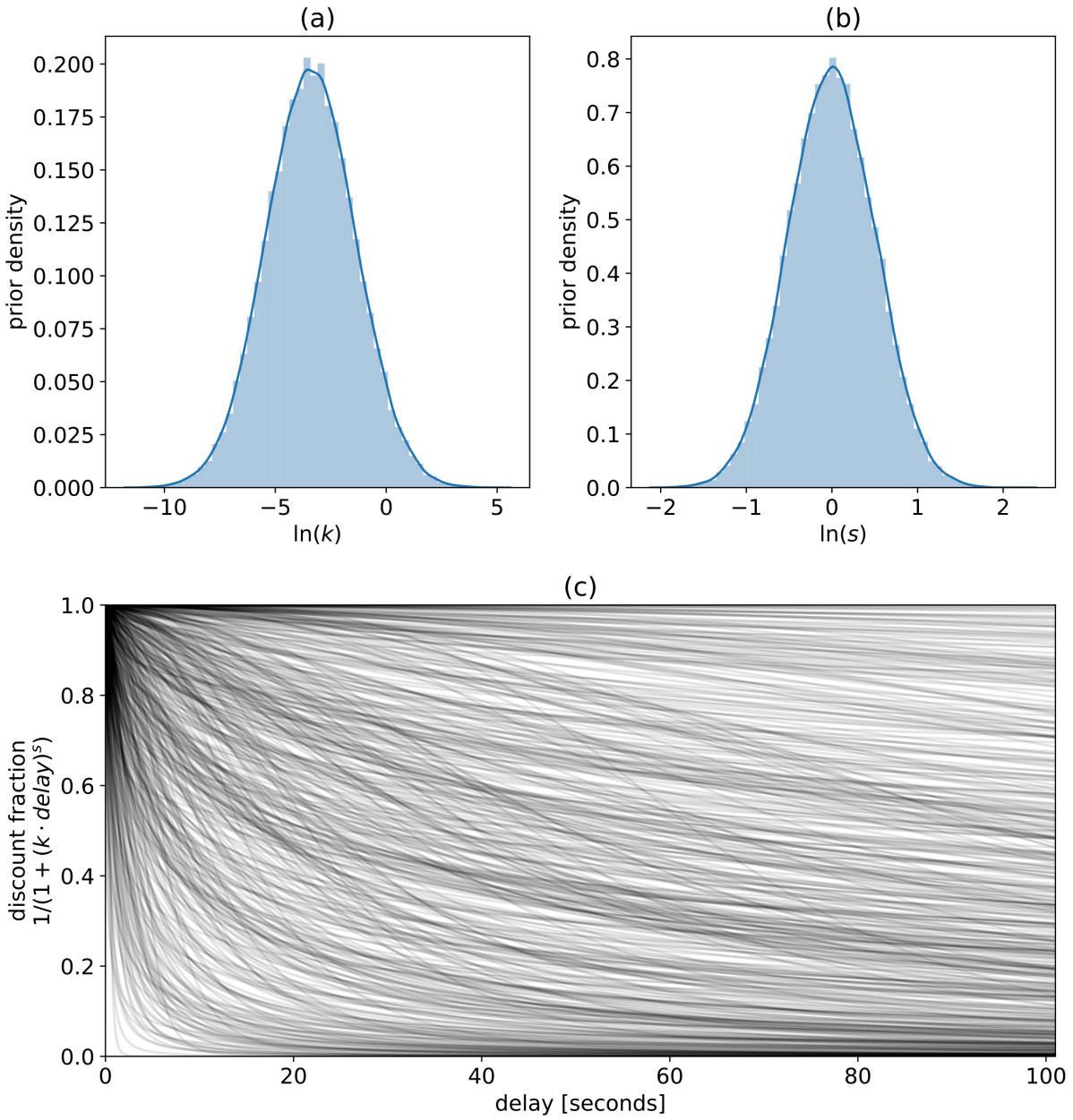


Figure D1. Demonstration of priors over parameters for both experiments. Prior distributions over ln(k) and ln(s) are shown in panels a-b. Panel c shows 500 discount functions with (k, s)

parameters sampled from these priors. Our broad priors over the relevant range of parameter values correspond to a broad spread of possible discount functions, indicating that our priors will not unduly influence our final posterior parameter estimates.

Appendix E

In order to ensure that our parameter estimates are meaningful, we conducted a parameter recovery simulation. We ran 200 simulated delay discounting experiments adhering exactly to the protocol we used with real participants, as laid out in the Methods sections. Each simulated experiment has a simulated participant with true parameter values randomly sampled from $-5 < \ln(k) < -0.5$ and $\ln(0.5) < \ln(s) < \ln(3)$. We simulated the responses using the generative model as described by Equations 2-11. More specifically, after having obtained the probability of choosing the delayed reward we generated a response as a biased coin flip (Bernoulli trial).

For each simulated experiment, we used our Bayesian inference procedure as outlined in the Methods. What we want to know from the parameter recovery is – for a given simulated participant with known true parameters, can we recover those parameters accurately using our inference procedure? Therefore, the only modification to the inference procedure described in the Methods was to use an individual participant level model.

Figure D1 shows the results of the parameter recovery. We can see that parameter recovery for $\ln(k)$ is very good in that we have low/no bias, recovered parameters are tightly correlated with their true values, and we have good precision in our posterior estimates. Parameter recovery is also good for $\ln(s)$ and does not contain any systematic bias, although the estimates have slightly higher variability around the true parameter values. We can gain further confidence by comparing our parameter estimates with posterior predictions and directly compare those to see if they are consistent with the behavioral data. Examples of such plots are seen in Figures 4 and 5. Given our exclusion criteria, removing participants with high proportions of choosing delayed or immediate choices, and checking of posterior

prediction plots against behavioral data, we have confidence that our inference procedure is capable of accurately recovering parameter values given our set of discounting questions.

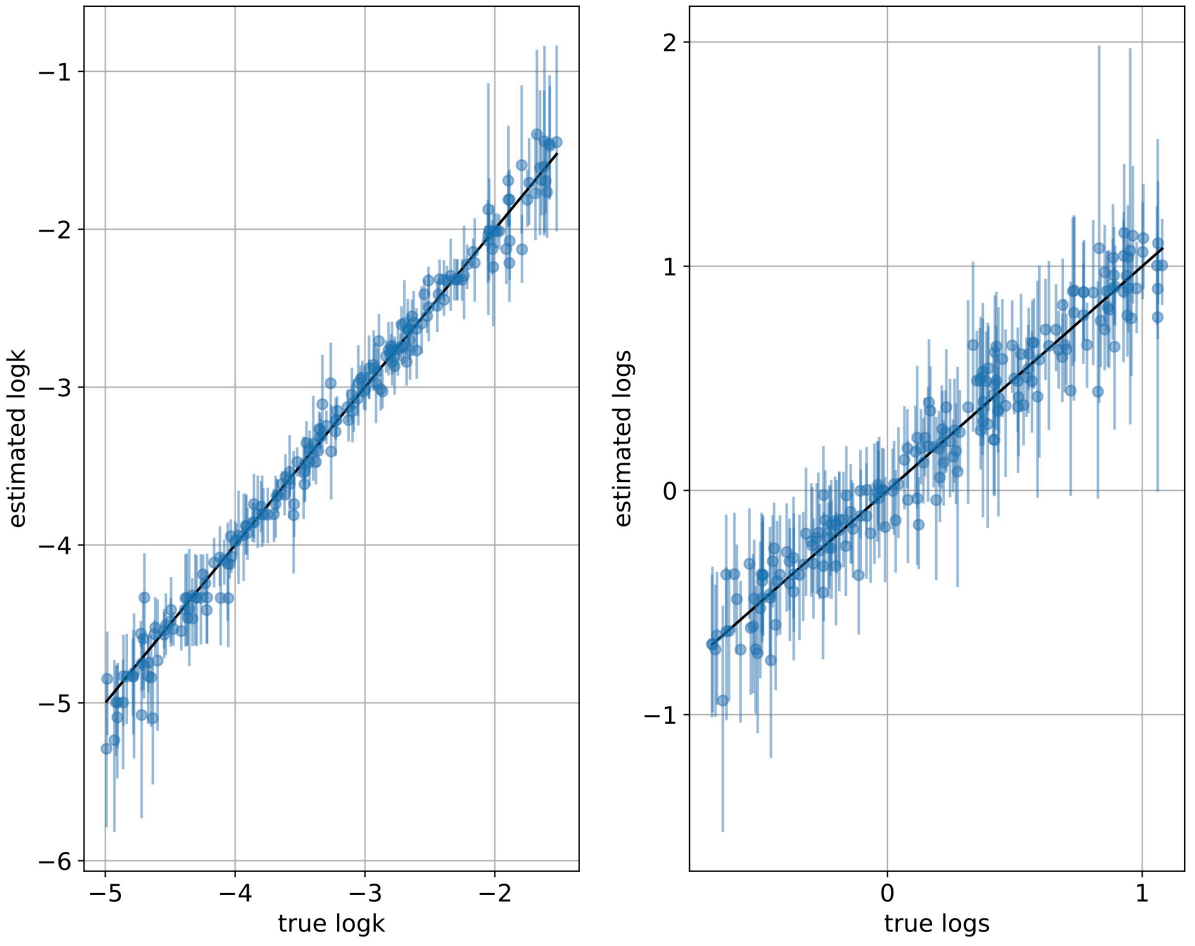


Figure E1. Results of the parameter recovery simulations. Points and error bars represent the mean and 95% credible interval of the posterior, respectively.

We also tested our ability to recover correlations in log(*k)* and log(*s*) parameters in groups of simulated participants. To do this we simulated 30 separate groups of 40 participants whose log(*k*) and log(*s*) parameters were sampled from the bivariate normal distribution

$$N\left(\left[\mu_{\log(k)}, \mu_{\log(s)}\right], \begin{bmatrix} \sigma^2_{\log(k)} & r \cdot \sigma_{\log(k)} \cdot \sigma_{\log(s)} \\ r \cdot \sigma_{\log(k)} \cdot \sigma_{\log(s)} & \sigma^2_{\log(s)} \end{bmatrix}\right)$$

where r varied in 30 uniform steps between -0.8 and +0.8. We set $\mu_{\log(k)} = \log(1/50)$, $\sigma^2_{\log(k)} = 1, \mu_{\log(s)} = 0, \sigma^2_{\log(s)} = 0.2$. For each group, our Bayesian parameter recovery procedure was applied to all simulated participants and the recovered group correlation coefficient was calculated. Figure E2 shows that over a broad range of true correlation coefficients between log(k) and log(s), we are able to recover the true underlying correlation coefficient with reasonable precision.

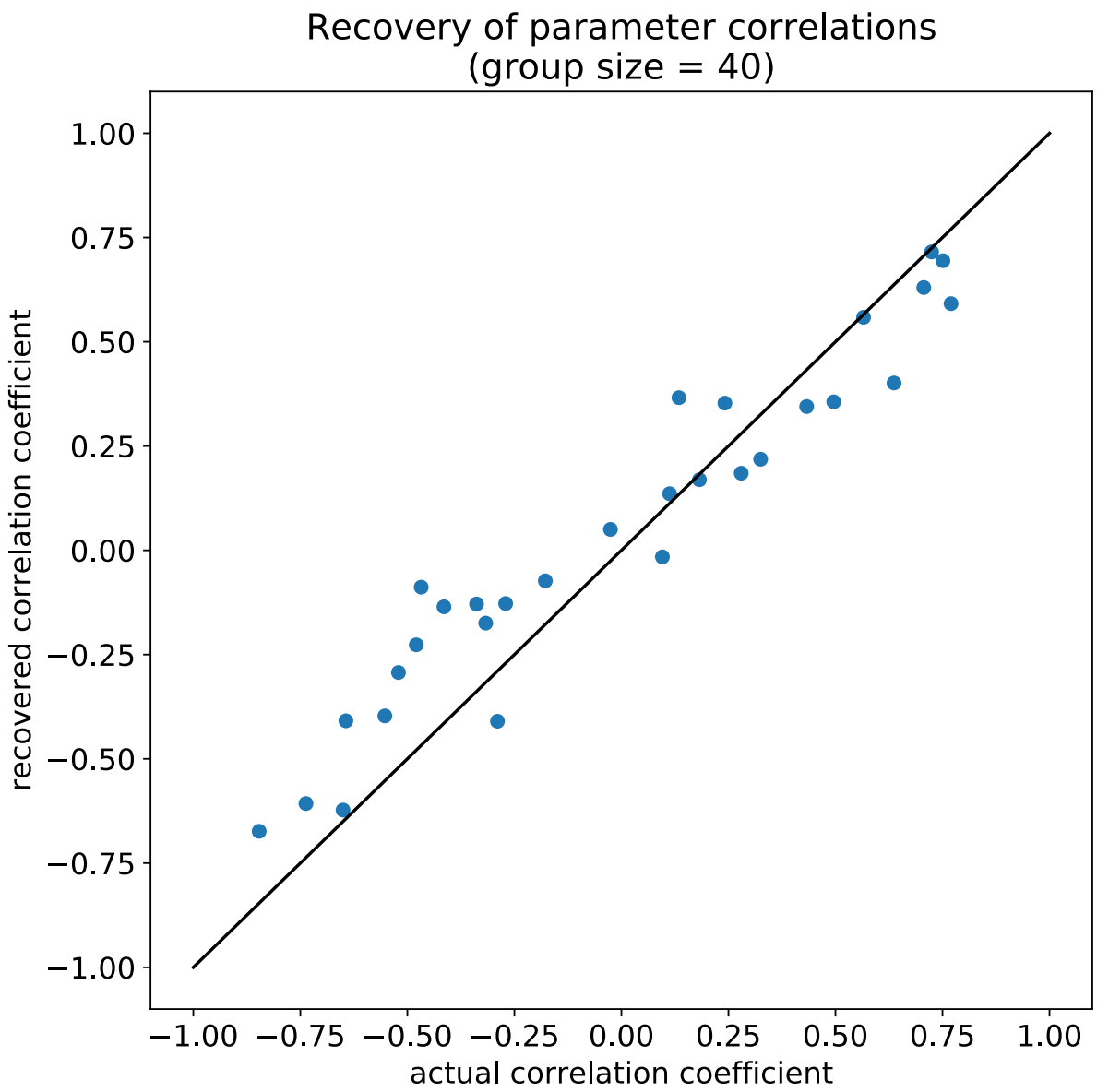


Figure E2. Results of the parameter recovery simulations. Points and error bars represent the mean and 95% credible interval of the posterior, respectively.

Appendix F

Calculation of *AUC* measure. We calculated the Area Under Curve metric (*AUC*, Myerson, Green, & Warusawitharana, 2001) for the discount fraction *f* as a function of delay $f(D,k,s) = 1/(1+(kD)^s))$ which captures the combined effects of discounting and subjective time. Note that this is a model-based approach (Gilroy & Hantula, 2018) as opposed to the originally proposed point-based method by (Myerson et al. 2001). Because we have many MCMC samples representing a distribution over *k* and *s*, this function was evaluated for all MCMC samples, giving rise to a distribution of *AUC* values for each participant. As with other variables, we took the posterior mean as our point estimate. The function was evaluated for many values, in 1-second increments, and trapezoidal integration was used to calculate the *AUC*. The *AUC* was appropriately normalized by rescaling delays between 0-1 such that someone who does not discount would have *AUC=1*.

***AUC* analysis for Experiment 1.** We found decisive evidence (BF_{10} = 78.70) for a main effect of task. That is, the data is 78.70 times more consistent with there being a main effect of task, compared to an intercept-only model. The *AUC* was lower in the online waiting task, showing strong evidence for increased discounting in the online waiting task as compared to the deferred waiting task.

***AUC* analysis for Experiment 2.** We find decisive evidence for main effects of both domain (gain/loss) and task ($BF_{10} = 6.59\times10^5$, compared to a control intercept-only model). *AUC* is lower for the online waiting task compared to the deferred waiting task (i.e. present bias is stronger in the online waiting task), in line with Experiment 1's finding. Additionally, AUC is lower in the gain condition than in the loss condition, which is consistent with the sign effect.

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